Eighth Grade Math Lesson Materials

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CITYTUTOR DC

G8 Unit 1:

Rigid Transformations and Congruence

G8 U1 Lesson 1
Describe the movement of shapes using the terms "clockwise," "counterclockwise," "translations," "rotations," and "reflections" of figures.

CITYTUTOR DC

G8 U1 Lesson 1 - Today will describe the movement of shapes using the terms “clockwise,” “counterclockwise,” “translations,” “rotations,” and “reflections” of figures.

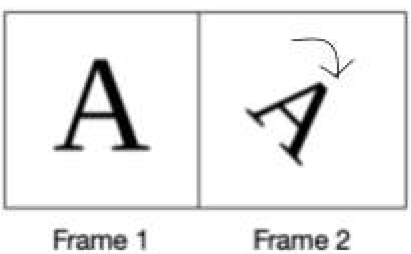
Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slide 2): Today we will and understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

As you may know, the hands of the analog clock turn to the right. That turn or circular motion is described as a “rotation.” Whenever a geometric figure rotates to the right, like the hands of a clock, the direction of that movement is described as “clockwise.”(*Draw an arrow to show a clockwise rotation. Write the words clockwise rotation.*)

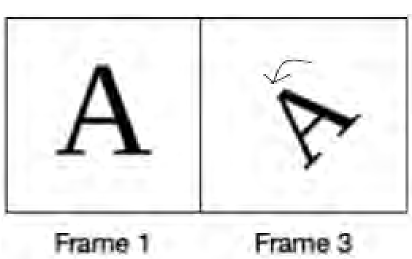
Let’s Talk (Slide 3): Now let’s talk about lines that intersect. Earlier we recalled the Vertical Angles Theorem from 7th grade. Let’s use what we know about line segments rotating 180 degrees and the properties of rigid transformations to figure why the Vertical Angles Theorem is true. Let’s start with a pair of intersecting lines *AB* and *CD* that intersect at point *E.* Let’s mark angle *AEC* on the intersecting lines and then use our tools to rotate line *AB* and line *CD* Possible Students Answers, Key Points:

- The second A is turning.
- It turns to the right.
- It’s rotating to the right.
- It’s turning clockwise.



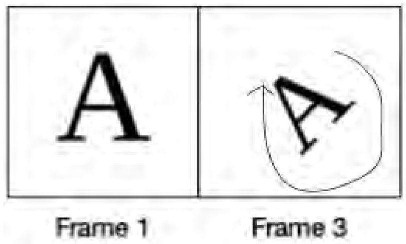
The letter in Frame 2 seems to be turning to the right, so we can describe this movement as a clockwise rotation of Frame 1. (Say and Do: *draw the arrow in the direction of the turn and write “clockwise rotation.”*)

Let’s Talk (Slide 4): How do we describe the same movement when turning to the left? When you counter something, it means you have an opposing or opposite opinion or action. So, “counterclockwise” means a figure is rotating in the opposite direction of the hands on a clock. Consider Frame 1 and Frame 3. How could you describe the movement from Frame 1 to Frame 3 using the words “rotation,” “clockwise,” and/or “counterclockwise.” Possible Students Answers, Key Points:



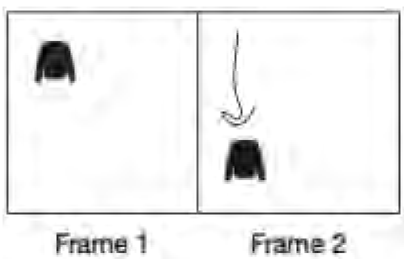
- The second A is turning.
- It turns to the left.
- It rotates to the left.
- It’s turning counterclockwise.
- Bonus: it’s turning a lot to the right.

I'll use arrows again to show the movement of the letter. In this case, Frame 3 is a counterclockwise rotation of Frame 1. (Say and Do: *draw the arrow in the direction of the turn and write "counterclockwise rotation."*)



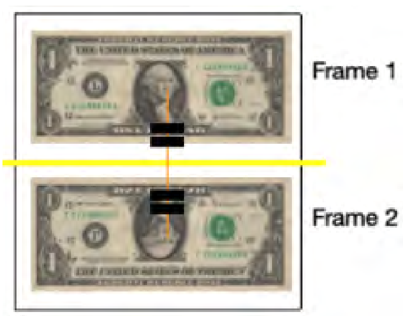
Rotations are fun because they can go in either direction. We could also say that Frame 3 is a clockwise rotation. (*Draw the arrow in the direction of a "clockwise rotation."*)

Let's Think (Slide 5): Now that we know what to call a turning figure - a "rotation" - and we can describe the direction of the turn using "clockwise" and "counterclockwise," let's explore the terms "translation" and "reflection." Take a look at Frame 1 and Frame 2. If I were going to describe the movement of the shirt from Frame 1 to Frame 2, I would say that the shirt slid down from Frame 1 to Frame 2. A geometric slide, in any direction, is also known as a "translation."



I used an arrow to show that the shirt slid down. So you can say that Frame 2 is a translation of Frame 1. (*Say: Draw an arrow to show a slide down and write the word "translation."*)

Let's Think (Slide 6): Finally, when a figure moves as if it is flipping over a straight line, it will appear as if the image created its reflection after looking in a mirror. That is why this type of move is described as a "reflection."



In this case, Frame 2 is a reflection of Frame 1. The mirror or line of reflection is a horizontal line. (Do and Say: *Use a different color or highlighter to highlight the line separating Frames 1 and 2. Write "reflection*.) When an image is reflected, the distance from the line of reflection to all corresponding parts are the same. For example, if we measured the distance from George Washington's nose in Frame 1 to the yellow line and the distance from his nose in Frame 2 to the yellow line, they should be equivalent. (*Draw a line to show the distance from the nose to the line of reflection on both sides is equal.)*

Let's Try it (Slides 7-8): Let's work on describing geometric transformations. We will work on this page together. Remember..

WARM WELCOME

Describe the movement of shapes using the terms "clockwise," "translations," "rotations," and "reflections" of figures.

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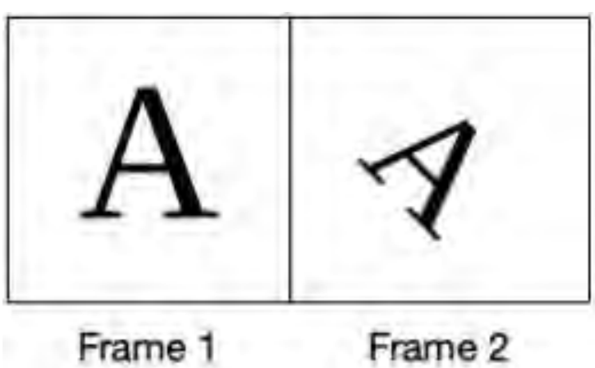
Let's Review:

The hands of an analog clock rotate clockwise.

Let's Talk:

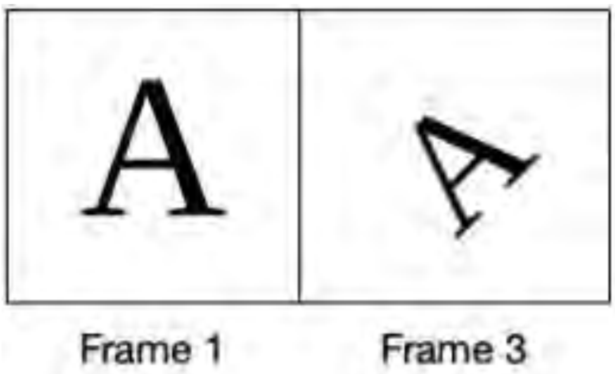
How do you describe a geometric figure that turns to the right?

How would you describe the transformation from Frame 1 to Frame 2?



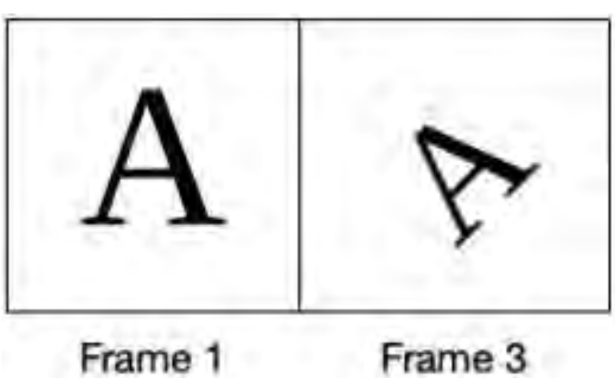
Let's Talk: How do you describe a geometric figure that turns to the left?

How would you describe the transformation from Frame 1 to Frame 3?



Let's Talk: How do you describe a geometric figure that turns to the left?

How else can you describe the transformation from Frame 1 to Frame 3?



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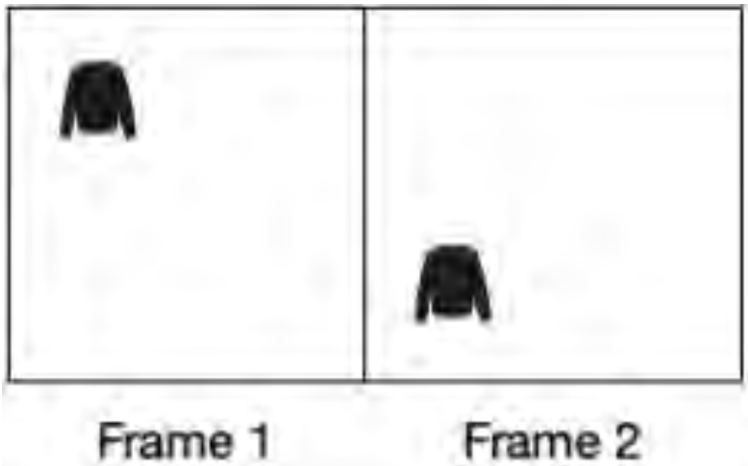
Let's Think:

How do you describe a geometric figure that slides?



Let's Think:

How do you describe a geometric figure that flips?

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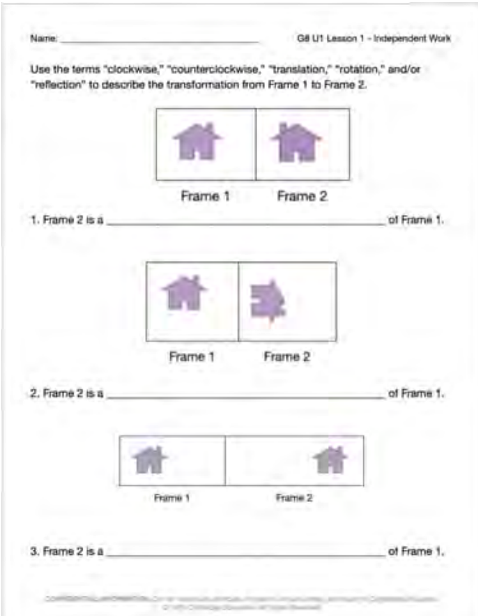
Let's Try It:

Let's practice describing the movement of shapes together.


Name: ______________________ G8 U1 Lesson 1 - Independent Work

Use the terms "clockwise," "counterclockwise," "translation," "rotation," and/or "reflection" to describe the transformation from Frame 1 to Frame 2.

Frame 1 Frame 2

1. Frame 2 is a ______________________ of Frame 1.

Frame 1 Frame 2

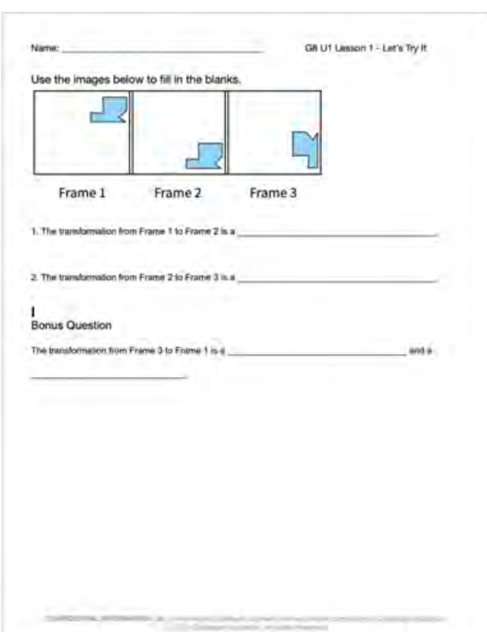
2. Frame 2 is a ______________________ of Frame 1.

Frame 1 Frame 2

3. Frame 2 is a ______________________ of Frame 1.

On your Own:

Now it's time to describe the movement of shapes on your own.


Name: ______________________ G8 U1 Lesson 1 - Let's Try It

Use the images below to fill in the blanks.

Frame 1 Frame 2 Frame 3

1. The transformation from Frame 1 to Frame 2 is a ______________________

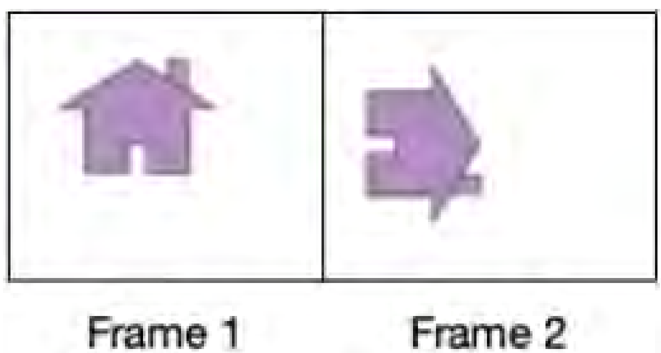
2. The transformation from Frame 2 to Frame 3 is a ______________________

Bonus Question

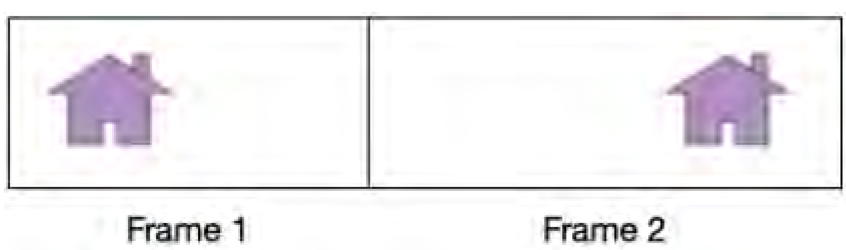
The transformation from Frame 3 to Frame 1 is a ______________________ and a ______________________

Use the terms "clockwise," "counterclockwise," "translation," "rotation," and/or "reflection" to describe the transformation from Frame 1 to Frame 2.

1. Frame 2 is a ____ of Frame 1.

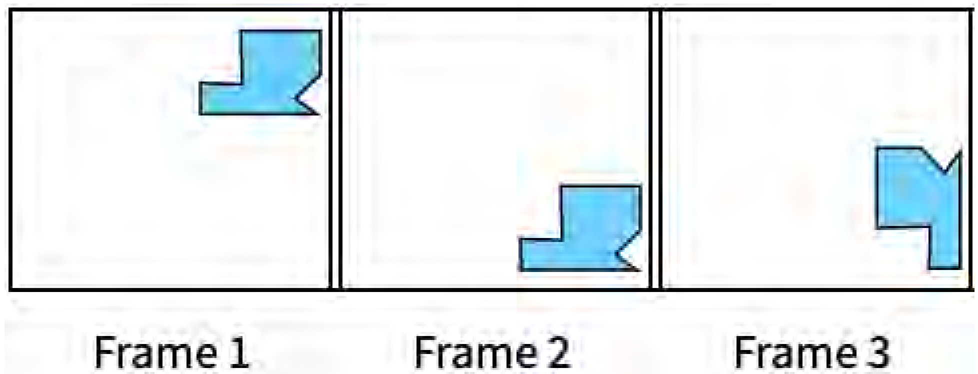


2. Frame 2 is a ____ of Frame 1.



3. Frame 2 is a ____ of Frame 1.

Use the images below to fill in the blanks.



1. The transformation from Frame 1 to Frame 2 is a _______________________________________.

2. The transformation from Frame 2 to Frame 3 is a _______________________________________.

Bonus Question

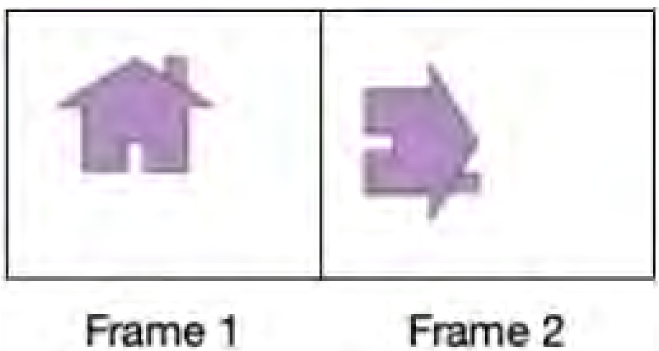
The transformations from Frame 3 to Frame 1 is a ___________________________________ and a

______________________________.

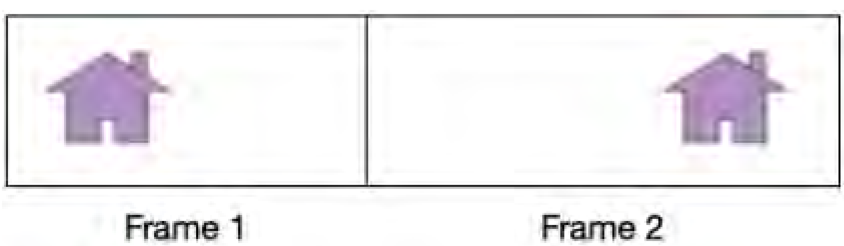
Name: Answer Key

Use the terms "clockwise," "counterclockwise," "translation," "rotation," and/or "reflection" to describe the transformation from Frame 1 to Frame 2.

1. Frame 2 is a reflection of Frame 1.



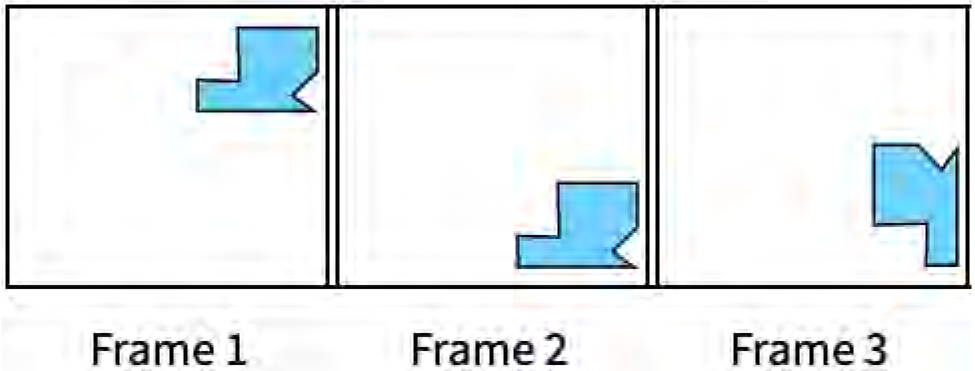
2. Frame 2 is a rotation clockwise of Frame 1.



3. Frame 2 is a translation right of Frame 1.

Name: Answer Key

Use the images below to fill in the blanks.



1. The transformation from Frame 1 to Frame 2 is a translation down.

2. The transformation from Frame 2 to Frame 3 is a rotation counterclockwise.

Bonus Question

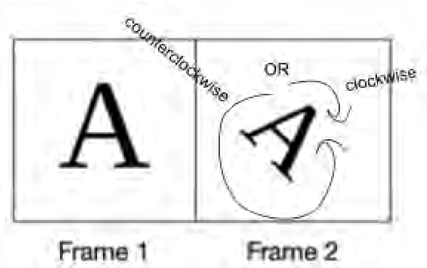
The transformations from Frame 3 to Frame 1 is a rotation clockwise and a translation up.

G8 U1 Lesson 2
Use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image.

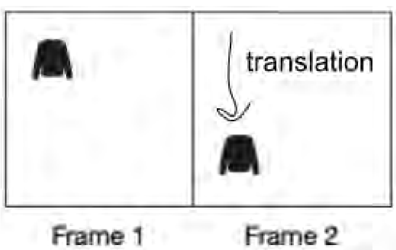
G8 U1 Lesson 2 - Use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image.

Warm Welcome (Slide 1): Tutor Choice

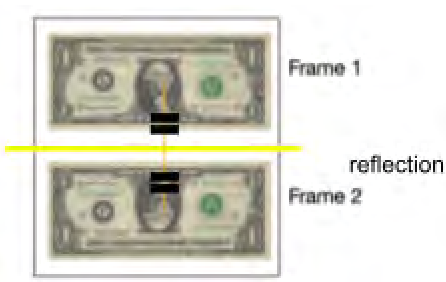
Frame the Learning/Connection to Prior Learning (Slides 2 - 5): Today we will use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image. First, let's recall the terms we use to describe rigid transformations.



Transformations that turn right, like the hands of an analog clock, or left, in the opposite direction of the hands of an analog clock, can be described as clockwise or counterclockwise rotations respectively.

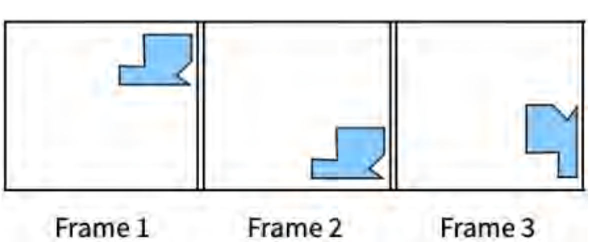


Transformations that move up and down or left and right can be described as translations.



Transformations that appear to flip over a straight line can be described as a reflection.

Let's Talk (Slide 6): Now, consider the term "image" and what it means in the real-world. When you take a picture with any camera, the result is also known as an image. Translations, rotations, and reflections are simply images created after an original figure undergoes some form of rigid motion.

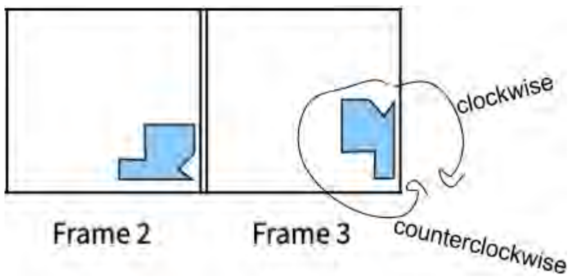


In the last lesson there was a bonus set of frames and the task asked you to describe the sequence from Frame 3 to Frame 1.

Typically the image is the final frame, but the challenge was to start with the image and map a set of transformations backwards. How would you describe how the original image transformed from Frame 3 to Frame 2? Possible Students Answers, Key Points:

- The original image in Frame 3 turned.
- The original image turned to the right.

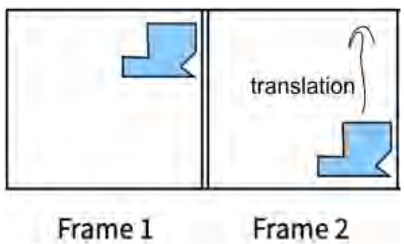
- The original image turned to the left.
- Frame 3 rotated counterclockwise or clockwise to create the image in Frame 2.



The transformation that took the original image from Frame 3 to Frame 2 was a turn to the right like the hands of an analog clock, a clockwise rotation, or a turn in the opposite direction of the hands on an analog clock, a counterclockwise rotation. (*Draw two arrows to show the possible rotations. Write the words clockwise and counterclockwise on the corresponding arrows.*)

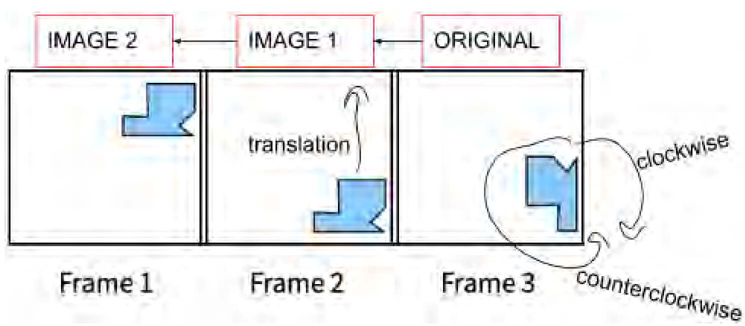
Let's Talk (Slide 7): How do we describe the movement that took the image in Frame 2 to the image in Frame 1? Possible Students Answers, Key Points:

- The image in Frame 2 moved up.
- The image in Frame 1 is a vertical translation of Frame 2.



The transformation that took Frame 2 to Frame 1 was a vertical move, a translation. *(Draw an arrow to show the movement. Write the word translation.)*

Let's Think (Slide 8): Using these terms to describe multiple transformations is known as identifying a sequence of transformations. Rather than identifying one transformation at a time, you may have a case where you are given more than one transformation and you are asked to identify the sequence of transitions. This means you need to name the transformations in the order that they occur to produce the final image.



The transformation from Frame 3 to its image in Frame 1 is a counterclockwise (or clockwise) rotation and a vertical translation, in that order. (*Draw arrows and label the appropriate terminology on each image. Write the word original above Frame 3, Image 1 above Frame 2, and Image 2 above Frame 3 with arrows to show the sequence of transformations.)*

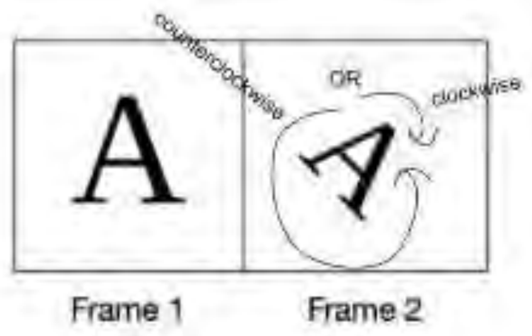
Let's Try it (Slides 7-8): Let's work on using transformation terminology to describe a sequence of transformations. We will work on this page together. Remember to write the transformations in the order that they occur.

WARM WELCOME

Use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image.

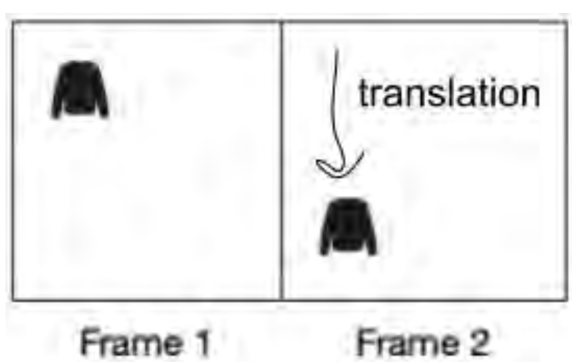
Let's Review:

Transformations that turn right and left can be described as clockwise or counterclockwise rotations like the hands of an analog clock.



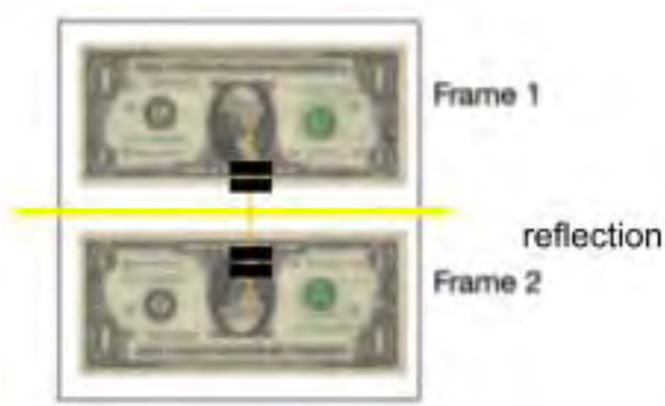
Let's Review:

Transformations that move up and down or left and right can be described as a translations.



Let's Review:

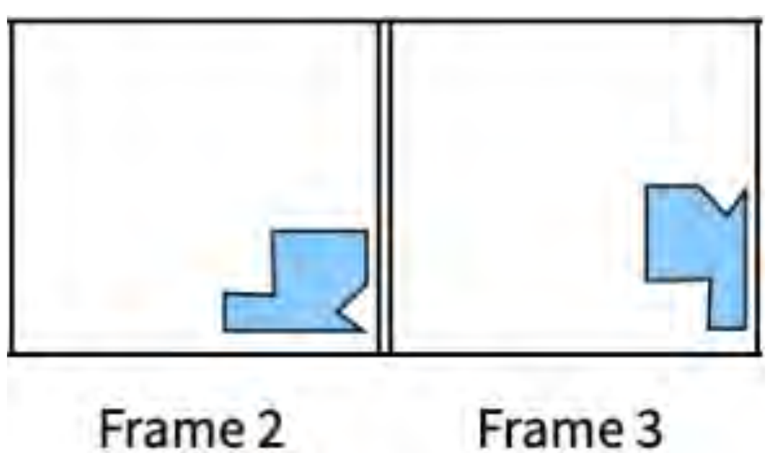
Transformations that appear to flip over a straight line can be described as a reflection.



Let's Talk:

How do you use transformation terminology to precisely describe transformations?

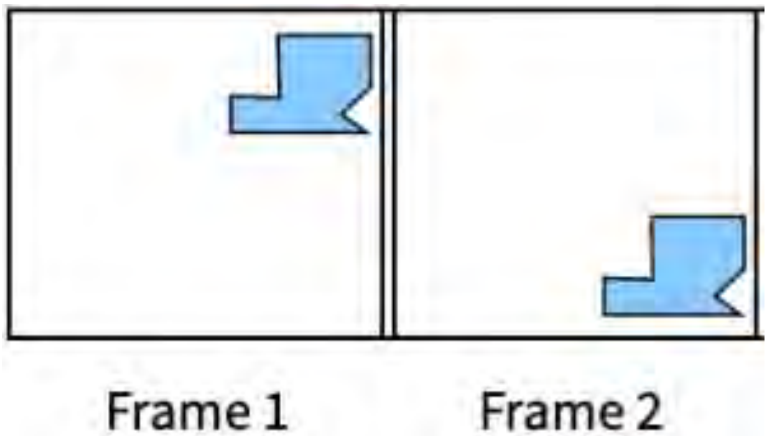
What series of transformations took Frame 3 to it's image in Frame 2?



Let's Talk:

How do you use transformation terminology to precisely describe transformations?

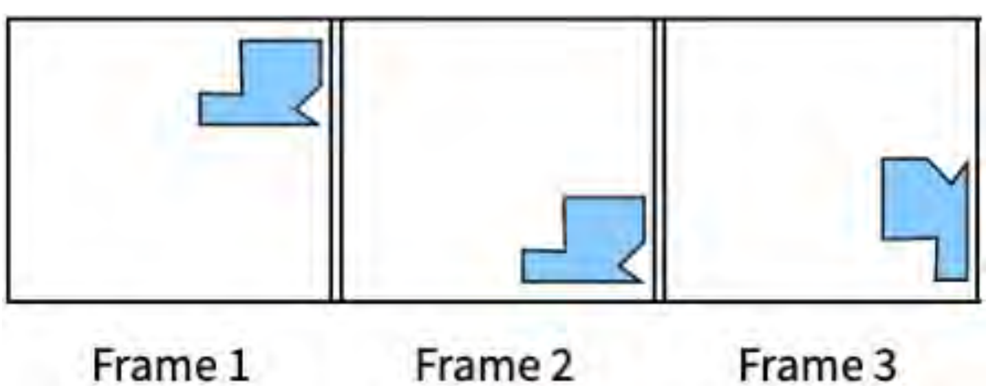
What series of transformations took Frame 2 to it's image in Frame 1?



Let's Think:

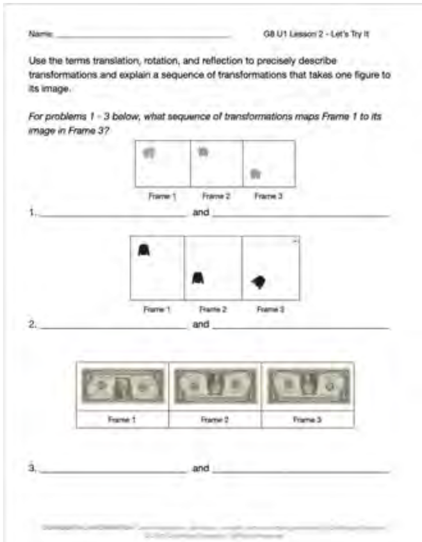
How do you use transformation terminology to precisely describe transformations?

What series of transformations from the original image in Frame 3 will create the image in Frame 1?



Let's Try It:

Let's practice using transformation terminology to identify a sequence of transformations.



Name: ____________________ G8 U1 Lesson 2 - Let's Try It

Use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image.

For problems 1 - 3 below, what sequence of transformations maps Frame 1 to its image in Frame 3?

Frame 1 Frame 2 Frame 3

1. ____________ and ____________

Frame 1 Frame 2 Frame 3

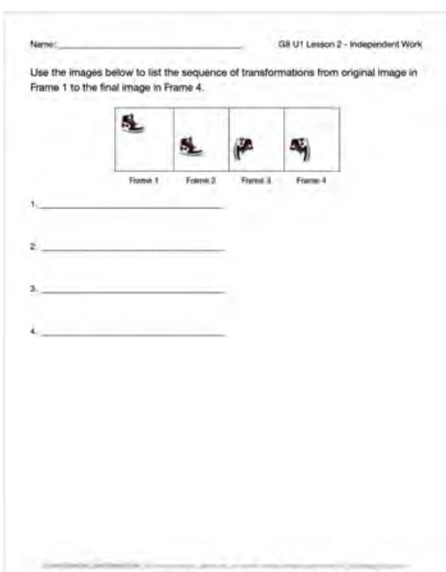
2. ____________ and ____________

Frame 1 Frame 2 Frame 3

3. ____________ and ____________

On your Own:

Now it's time to use transformation terminology to describe a sequence of transformations on your own.



Name: ____________________ G8 U1 Lesson 2 - Independent Work

Use the images below to list the sequence of transformations from original image in Frame 1 to the final image in Frame 4.

Frame 1 Frame 2 Frame 3 Frame 4

1. ____________

2. ____________

3. ____________

4. ____________

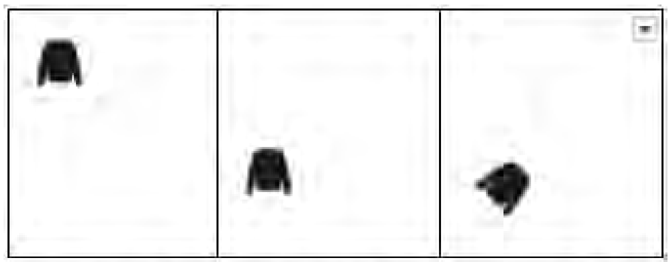
Name: ____ G8 U1 Lesson 2 - Let's Try It

Use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image.

For problems 1 - 3 below, what sequence of transformations maps Frame 1 to its image in Frame 3?

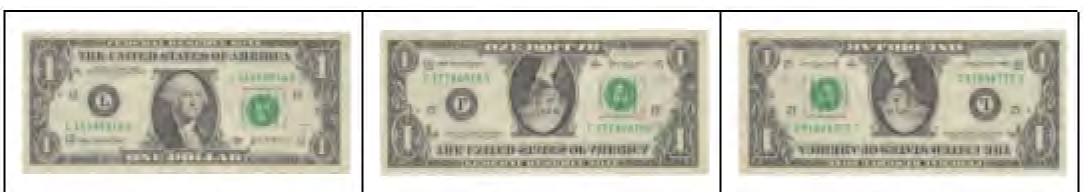
Frame 1 Frame 2 Frame 3

1. ____________________________ and ________________________________



Frame 1 Frame 2 Frame 3

2. ____________________________ and ________________________________

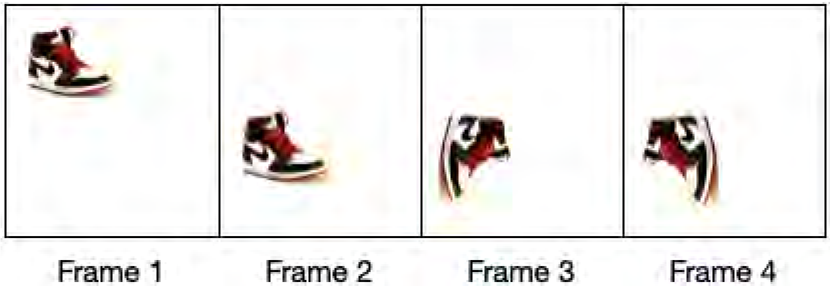


Frame 1	Frame 2	Frame 3

3. ____________________________ and ________________________________

Name: ____ G8 U1 Lesson 2 - Independent Work

Use the images below to list the sequence of transformations from the original image in Frame 1 to the final image in Frame 4.



1. ______________________________________

2. ______________________________________

3. ______________________________________

4. ______________________________________

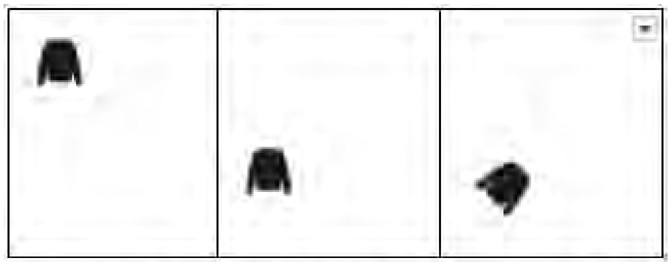
Name: ______________________________ G8 U1 Lesson 2 - Let's Try It

Use the terms translation, rotation, and reflection to precisely describe transformations and explain a sequence of transformations that takes one figure to its image.

For problems 1 - 3 below, what sequence of transformations maps Frame 1 to its image in Frame 3?

Frame 1 Frame 2 Frame 3

1. reflection and translation down



Frame 1 Frame 2 Frame 3

2. translation down and rotation clockwise

		
Frame 1	Frame 2	Frame 3

3. reflection and reflection

Name: Answer Key

Use the images below to list the sequence of transformations from original image in Frame 1 to the final image in Frame 4.

Frame 1 Frame 2 Frame 3 Frame 4

1. translation down

2. rotation clockwise

3. reflection

4. ~~________~~

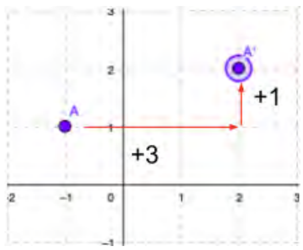
G8 U1 Lesson 3
Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

G8 U1 Lesson 3 - Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will apply transformations to points on the coordinate plane and name the coordinates of points in the image of a transformation. Recall that a translation is a movement left/right or up/down. A reflection is a movement that flips.

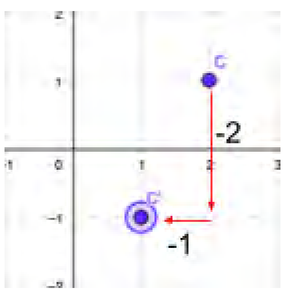
Let's Talk (Slide 4): The image of a point in a coordinate plane is named by the same letter and an apostrophe.



For example, point *A*(-1,1) was translated 3 units to the right and 1 unit up on the grid. The name of its image after the transformation is *A'*(2,2). *(Draw arrows to show the described translation and label the number of units the original image moved. Mark the image as A'.)*

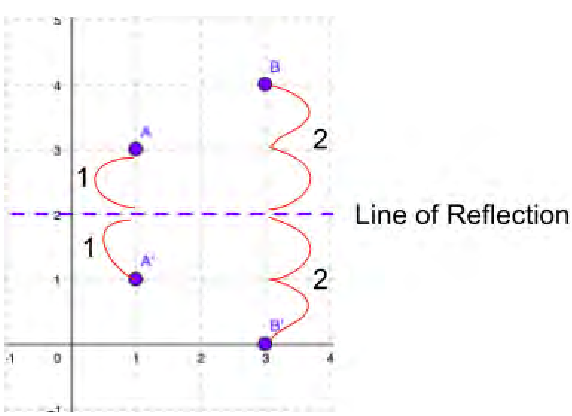
Let's Talk (Slide 5): Now, consider a similar translation. What are the new coordinates of the point *C*(2,1) after being translated 2 units down and 1 unit to the left? Possible Students Answers, Key Points:

- The new coordinates are (1,-1).
- *x* = 1 and *y* = -1
- The new name of the point is C'.



When you translate point *C*, you pass over the *x* axis into the 4th quadrant which tells us the resulting *y*-coordinate will be negative. Moving to the left impacts the *x-coordinate* only; it will remain positive because it's to the right of the *y*-axis. The image of *C* after a translation down 2 units and to the left 1 unit is *C'*(1,-1). *(Draw arrows to show the described translation. Mark the image as C'.)*

Let's Think (Slide 6): Reflections can be tricky because there are properties that must be true in order to make an image a reflection. Most important is that the distance from the line of reflection to all parts of the original figure must equal the distance from the line of reflection to all parts of the image. Let's consider how to reflect points *A* and *B* over the dashed line of reflection.



Since a reflection is a flip, we know that point *A* needs to flip over the line of the reflection. Since *A* is one unit above the line of reflection, its image, *A'*, will be one unit below the line of reflection. The same is true for point *B*. Since point *B* is two units above the line of reflection, its image, *B'*, will be two units below the line of reflection to ensure that the distances are the same from each point to the line of reflection. *(Mark the graph to show the distances from points A and B are the same to the line of reflection. Write the number of units and label the images A' and B'.)*

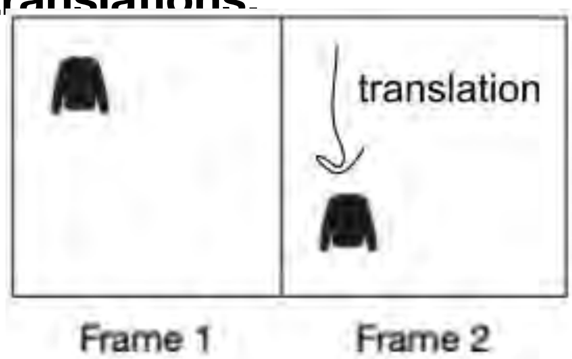
Let's Try it (Slides 7 - 8): Let's work on applying transformations and naming the coordinates of the image. We will work on this page together. Remember to ensure the distances remain the same and mark the point of the image with an apostrophe.

WARM WELCOME

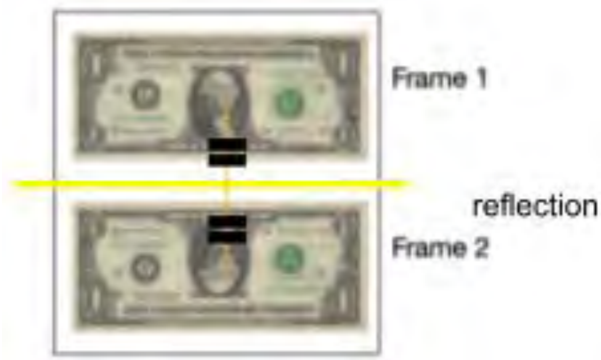
Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

Let's Review:

Transformations that move up and down or left and right can be described as a translations.



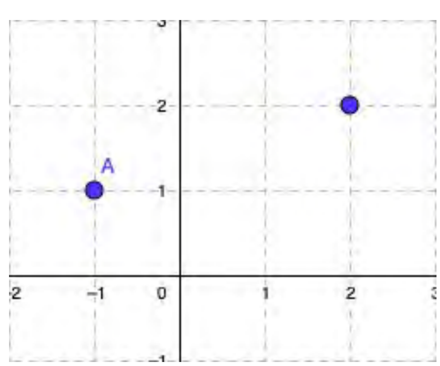
Transformations that appear to flip over a straight line can be described as a reflection.



Let's Talk:

How do you apply a transformation and label its image?

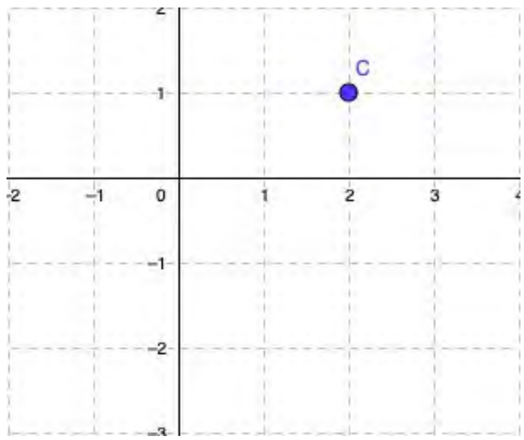
What is the name of the image of point *A* after being translated 3 units to the right and 1 unit up?



Let's Talk:

How do you apply a transformation and label its image?

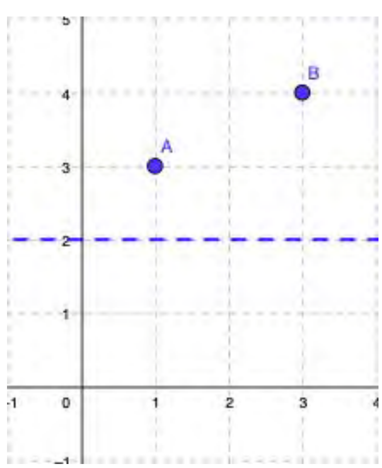
What are the new coordinates of point *C*(2,1) after being translate 2 units down and 1 unit to the left?



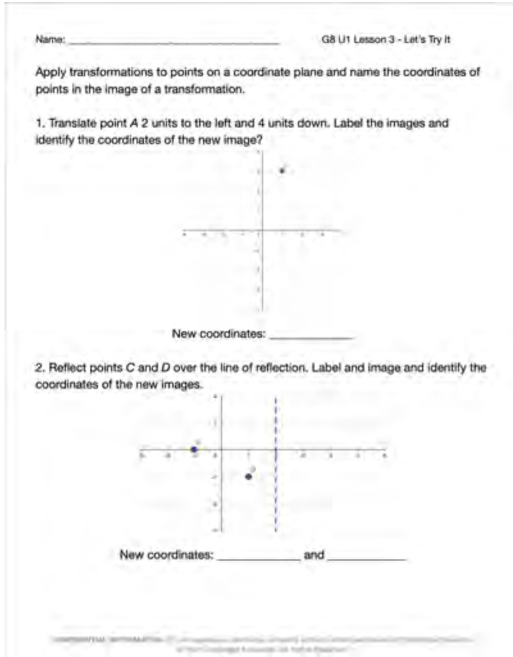
Let's Think:

How do you apply a transformation and label its image?

Reflect and label points *A* and *B* over the dashed line of reflection.



Let's Try It: Let's apply a transformation and label its image?


Name: ______________________ G8 U1 Lesson 3 - Let's Try It

Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

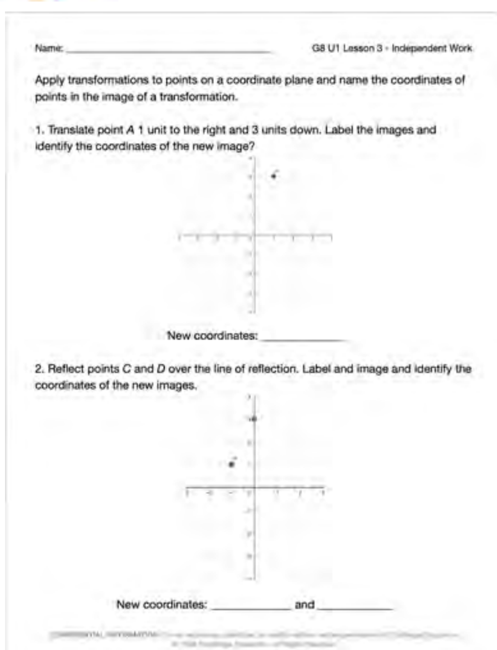
1. Translate point *A* 2 units to the left and 4 units down. Label the images and identify the coordinates of the new image?

New coordinates: __________

2. Reflect points *C* and *D* over the line of reflection. Label and image and identify the coordinates of the new images.

New coordinates: __________ and __________

On your Own: Now it's time to apply a transformation and label its image?


Name: ______________________ G8 U1 Lesson 3 - Independent Work

Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

1. Translate point *A* 1 unit to the right and 3 units down. Label the images and identify the coordinates of the new image?

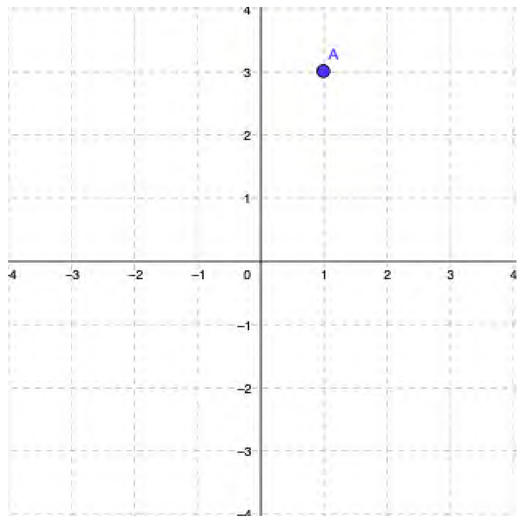
New coordinates: __________

2. Reflect points *C* and *D* over the line of reflection. Label and image and identify the coordinates of the new images.

New coordinates: __________ and __________

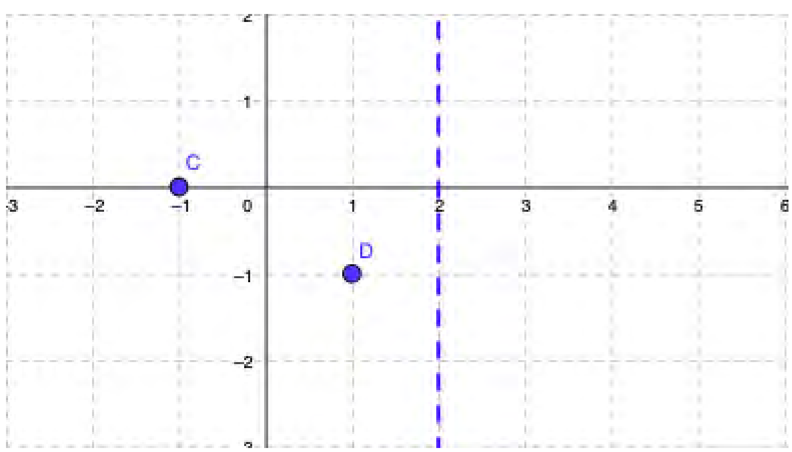
Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

1. Translate point *A* 2 units to the left and 4 units down. Label the images and identify the coordinates of the new image?



New coordinates: _____________

2. Reflect points *C* and *D* over the line of reflection. Label the images and identify the coordinates of the new points.

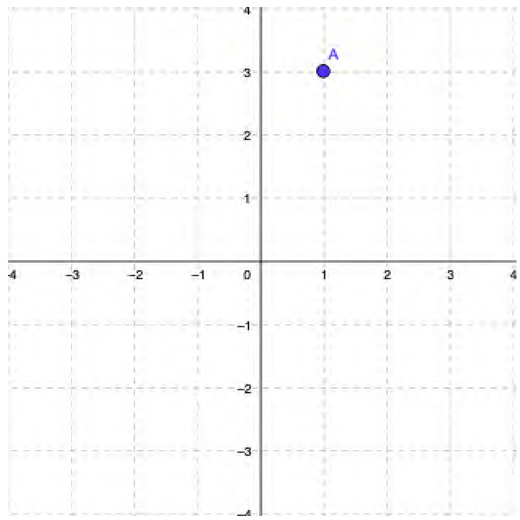


New coordinates: _____________ and ____________

Name: ____ G8 U1 Lesson 3 - Independent Work

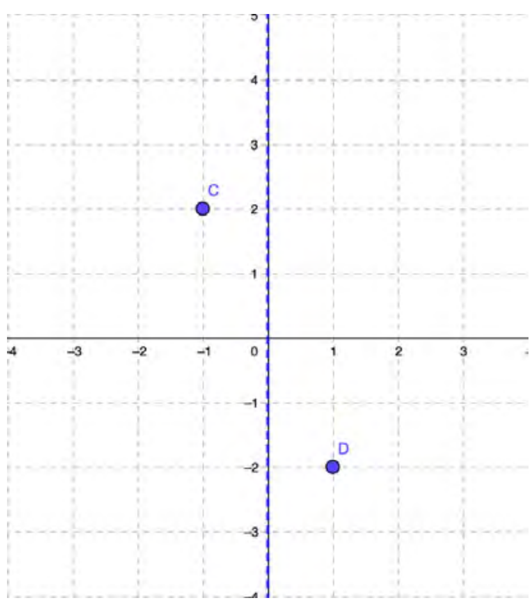
Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

1. Translate point *A* 1 unit to the right and 3 units down. Label the image and identify the coordinates of the new image?



New coordinates: _____________

2. Reflect points *C* and *D* over the dashed line of reflection. Label the images and identify the coordinates of the new points.

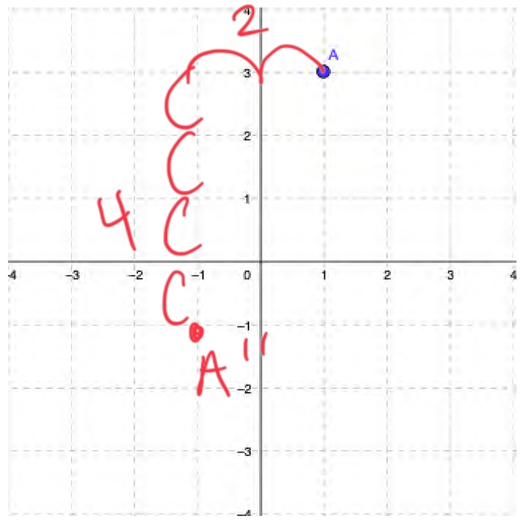


New coordinates: _____________ and ____________

Name: Answer Key

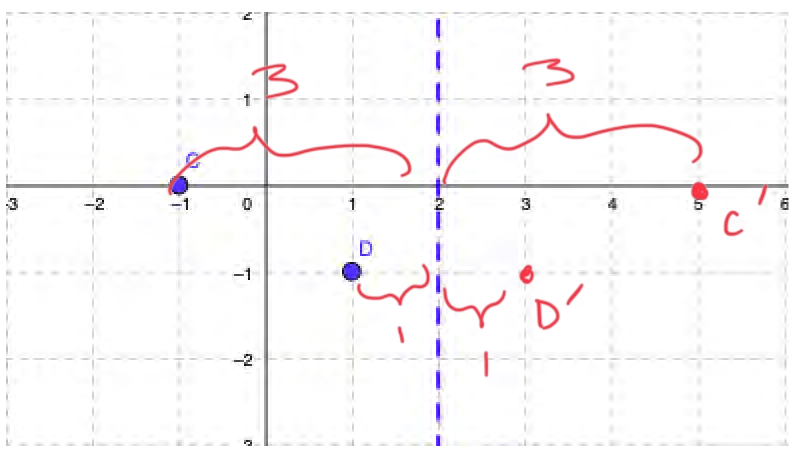
Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

1. Translate point *A* 2 units to the left and 4 units down. Label the images and identify the coordinates of the new image?



New coordinates: (-1, -1)

2. Reflect points *C* and *D* over the line of reflection. Label and image and identify the coordinates of the new images.

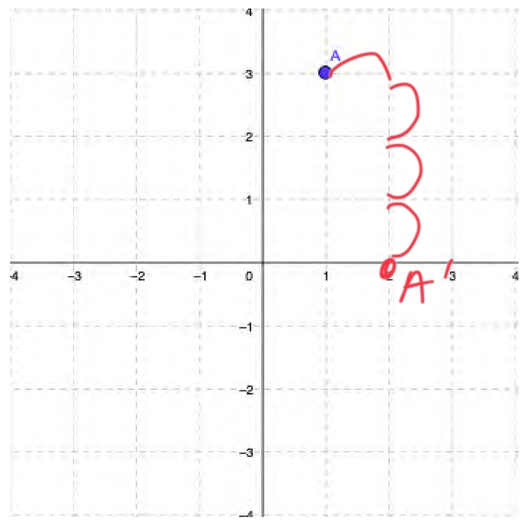


New coordinates: C'(5, 0) and D'(3, -1)

Name: Answer Key

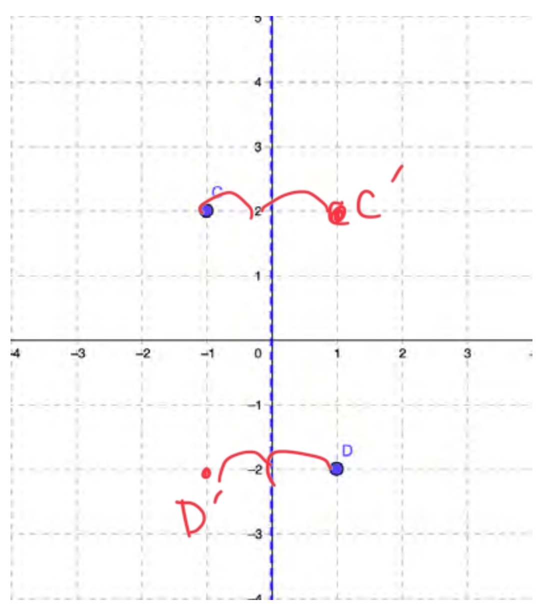
Apply transformations to points on a coordinate plane and name the coordinates of points in the image of a transformation.

1. Translate point *A* 1 unit to the right and 3 units down. Label the image and identify the coordinates of the new image?



New coordinates: A' (2, 0)

2. Reflect points *C* and *D* over the dashed line of reflection. Label the images and identify the coordinates of the new points.



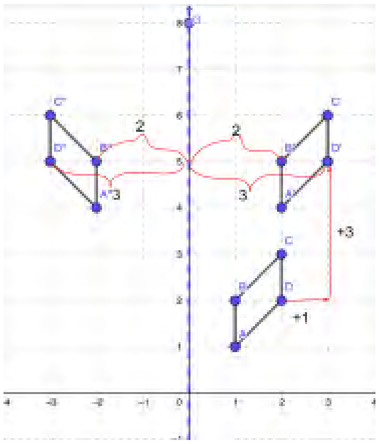
New coordinates: C' (1, 2) and D' (-1, -2)

G8 U1 Lesson 4
Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

G8 U1 Lesson 4 - Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will apply a sequence of transformations to points on a coordinate to determine whether the order of sequences has an effect on the image. First, remember that a sequence of transformations occurs when you combine multiple transformations to produce a final image. You'll name the points of the final image with multiple apostrophes.

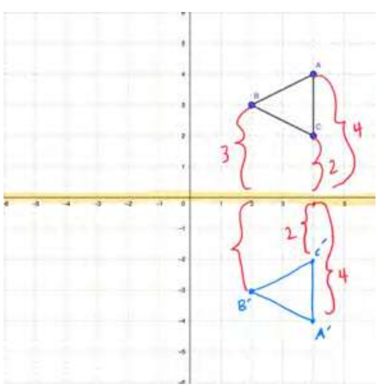


For example: In the last lesson, *ABCD* was translated one unit to the right and 3 units up creating the image *A'B'C'D'*. *(Draw arrows to demonstrate the vector by which the original image moved. Label at least one set of arrows with the number of units each point moved.)*

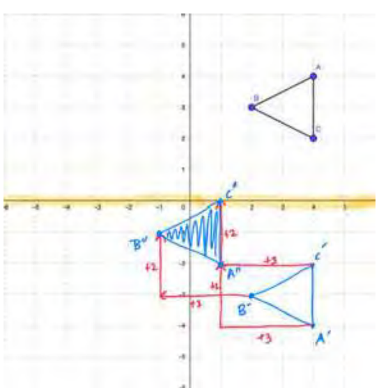
We then reflected the image over the *y*-axis. We ensured that the distance from each point to the line of reflection was the same and then we labeled each point (or vertex of the figure) *A''B''C''D''. (Mark the grid to show the distances are equal from at least two points to the line of reflection. Label the resulting vertices A'', B'', C'' and D''.)*

Let's Talk (Slide 4): Let's perform a sequence of transformations on triangle *ABC*. What do you think will happen to the new coordinates if we apply the following transformations: 1) Reflect *ABC* over the *x*-axis? 2) Translate the image 3 units left and 2 units up? Possible Students Answers, Key Points:

- The new coordinates will both be negative because the triangle will be in quadrant 4 after the reflection but some values may change after they are translated.
- The points (vertices) will be labeled A', B', and C'.

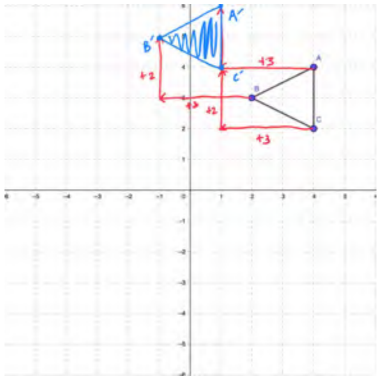


Let's apply the reflection over the *x*-axis to *ABC*. First, let's highlight or outline the line of reflection and notice how many units each vertex is away from the line of reflection. *(Highlight the line of reflection and apply the transformation to ABC. Label the vertices A'B'C'.)*

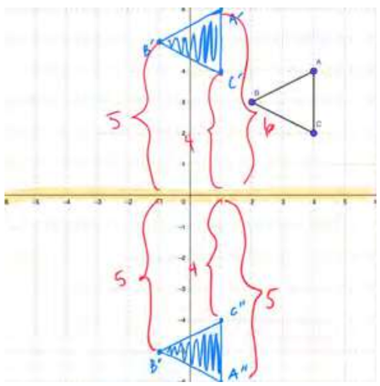


Next, let's apply the translation, 3 units to the left and 2 units up, to complete the sequence. (*Label the vertices of the image A"B"C".)*

Let's Think (Slide 5): Now, let's consider what happens if we take the same original figure, *ABC*, but switch the order of the transformations. This time we'll start by translating *ABC* 3 units left and 2 units up. After, we'll reflect the image over the *x*-axis.



Let's apply the first transformation to *ABC*, translating the triangle. *(Count with the students to translate each point and label the vertices A'B'C'. Mark the vectors using arrows or use counting arcs to demonstrate the movement.)*



Next, let's highlight or outline the line of reflection, noticing how many units each point is away from the *x*-axis to keep the distances equivalent on both sides. *(Highlight the line of reflection and draw the image. Label the vertices A''B''C''.)*

Now, let's compare *A''B''C''* in both cases after we switched the order of the transformations. What we've verified is that the order of the transformations in a sequence definitely matters since the final image, *A''B''C''*, is in a different final location.

Let's Try it (Slides 6 - 7): Let's work on applying a sequence of transformations twice, changing the order the second time, to see if it affects the final coordinates of the image. We will work on this page together. Remember to label your final vertices with the appropriate number of apostrophes. If you completed 1 transformation, there should be one apostrophe. If you performed two transformations, the final image should be labeled with two apostrophes. And so on.

WARM WELCOME

Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

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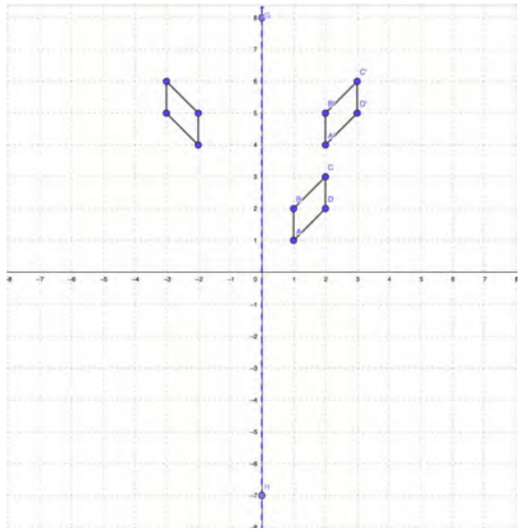
Let's Review:

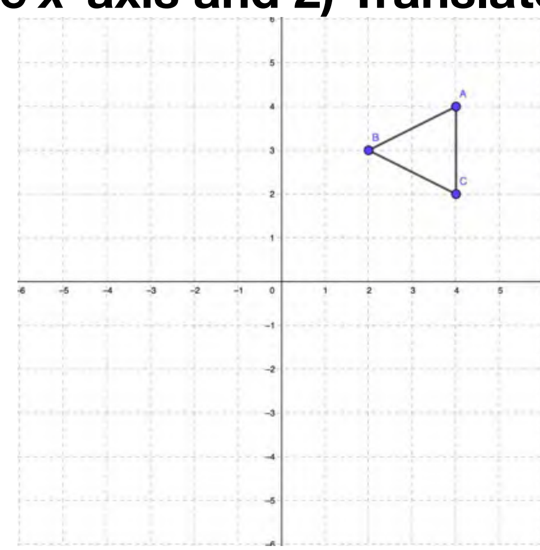
A sequence of transformations is when you combine multiple transformations to produce a final image.



Let's Talk:

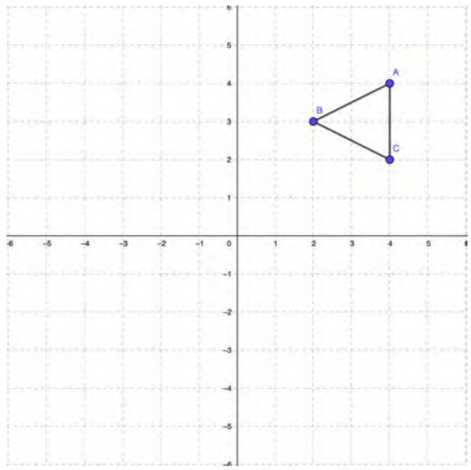
Does the order matter in a sequence of transformations?

What will happen to the coordinates of the image when you 1) Reflect *ABC* over the *x*-axis and 2) Translate the image 3 units left and 2 units up?

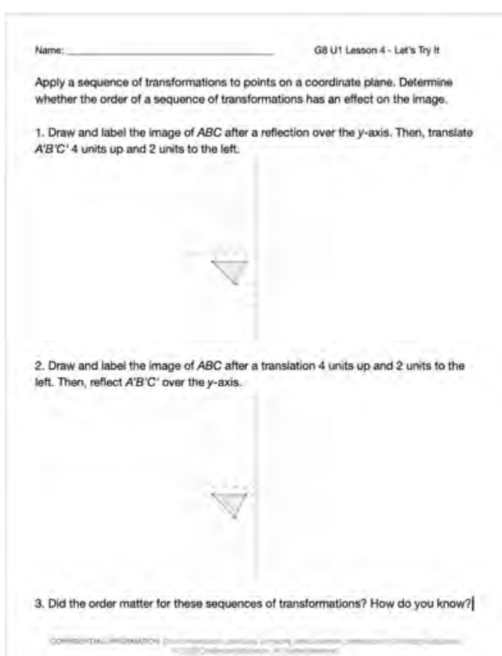


Let's Think: Does the order matter in a sequence of transformations?

Is there an impact on the coordinates of the image if we reverse the order of the transformations? 1) Translate the image 3 units left and 2 units up 2) Reflect *ABC* over the *x*-axis



Let's Try It: Let's determine if the order matter in a sequence of transformations?



Name: ______________________ G8 U1 Lesson 4 - Let's Try It

Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

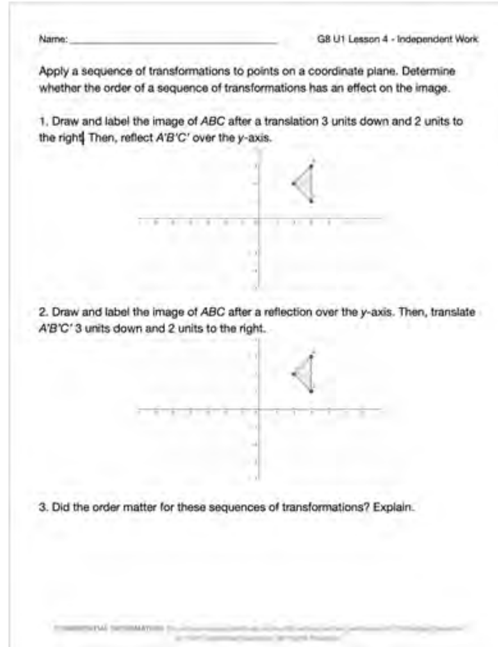
1. Draw and label the image of *ABC* after a reflection over the *y*-axis. Then, translate *A'B'C'* 4 units up and 2 units to the left.

2. Draw and label the image of *ABC* after a translation 4 units up and 2 units to the left. Then, reflect *A'B'C'* over the *y*-axis.

3. Did the order matter for these sequences of transformations? How do you know?

On your Own:

Now it’s time to determine if the order matters in a sequence of transformations?



Name: ______________________________ G8 U1 Lesson 4 - Independent Work

Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

1. Draw and label the image of *ABC* after a translation 3 units down and 2 units to the right. Then, reflect *A'B'C'* over the *y*-axis.

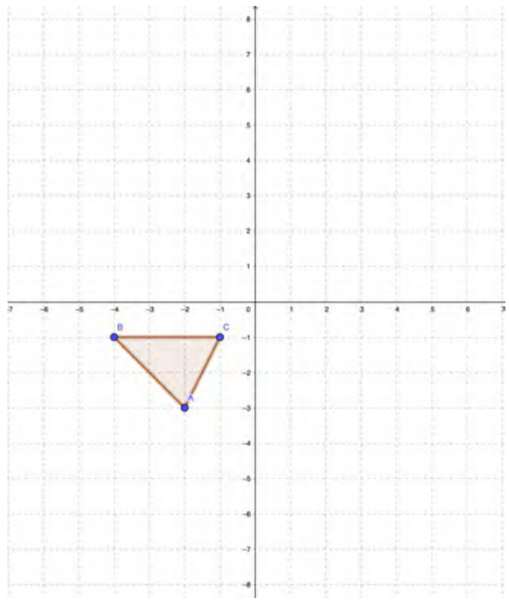
2. Draw and label the image of *ABC* after a reflection over the *y*-axis. Then, translate *A'B'C'* 3 units down and 2 units to the right.

3. Did the order matter for these sequences of transformations? Explain.

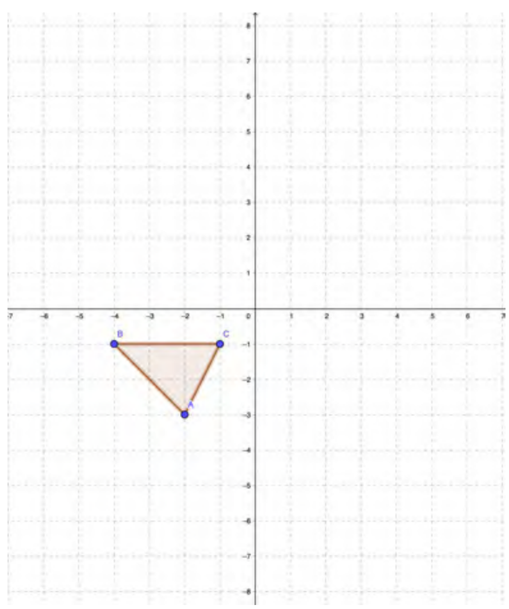
Name: ____

Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

1. Draw and label the image of *ABC* after a reflection over the *y*-axis. Then, translate *A'B'C'* 4 units up and 2 units to the left.



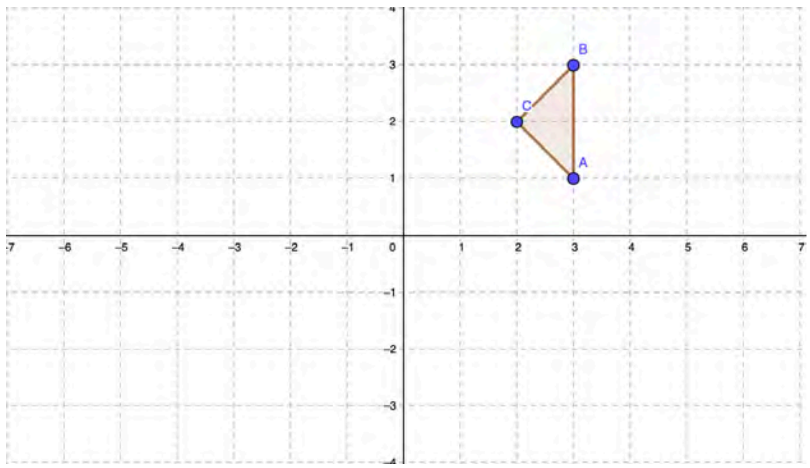
2. Draw and label the image of *ABC* after a translation 4 units up and 2 units to the left. Then, reflect *A'B'C'* over the *y*-axis.



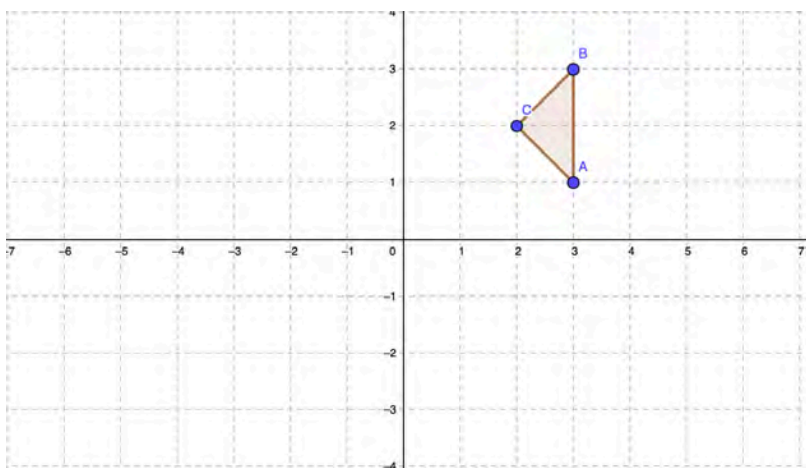
3. Did the order matter for these sequences of transformations? How do you know?

Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

1. Draw and label the image of *ABC* after a translation 3 units down and 2 units to the right. Then, reflect *A'B'C'* over the *y*-axis.



2. Draw and label the image of *ABC* after a reflection over the *y*-axis. Then, translate *A'B'C'* 3 units down and 2 units to the right.



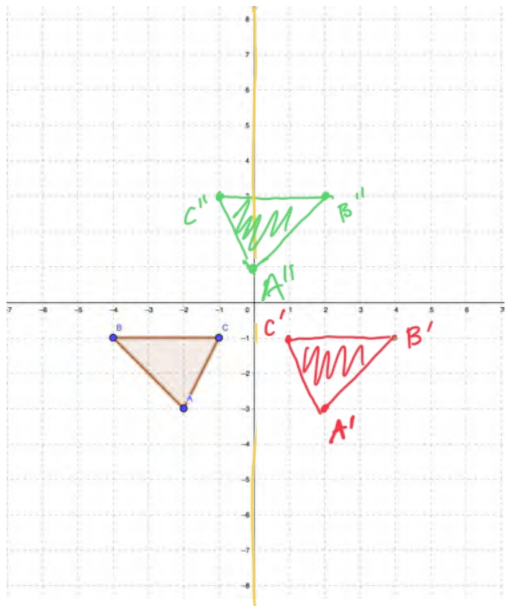
3. Did the order matter for these sequences of transformations? Explain.

Name: Answer Key

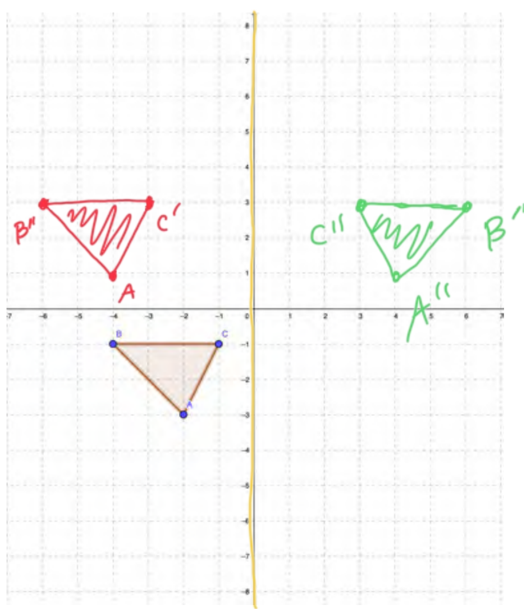
G8 U1 Lesson 4 - Let’s Try It

Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

1. Draw and label the image of *ABC* after a reflection over the *y*-axis. Then, translate *A’B’C’* 4 units up and 2 units to the left.



2. Draw and label the image of *ABC* after a translation 4 units up and 2 units to the left. Then, reflect *A’B’C’* over the *y*-axis.



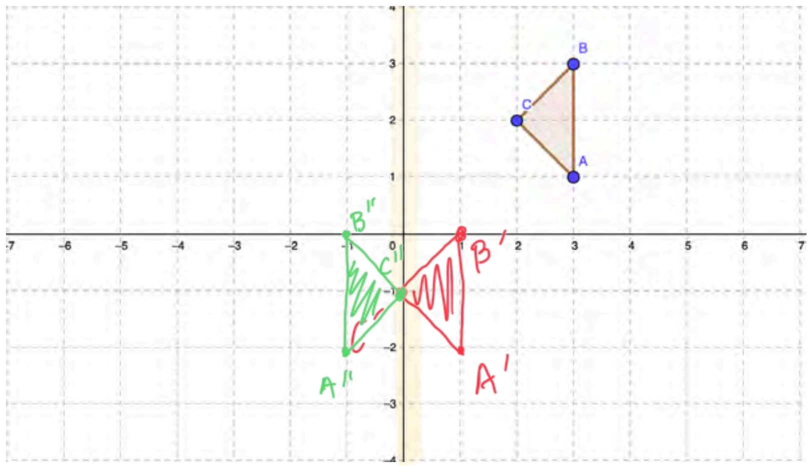
3. Did the order matter for these sequences of transformations? How do you know?

Yes, the order mattered. The final image of Δ ABC ended in different places even though it started in the same place.

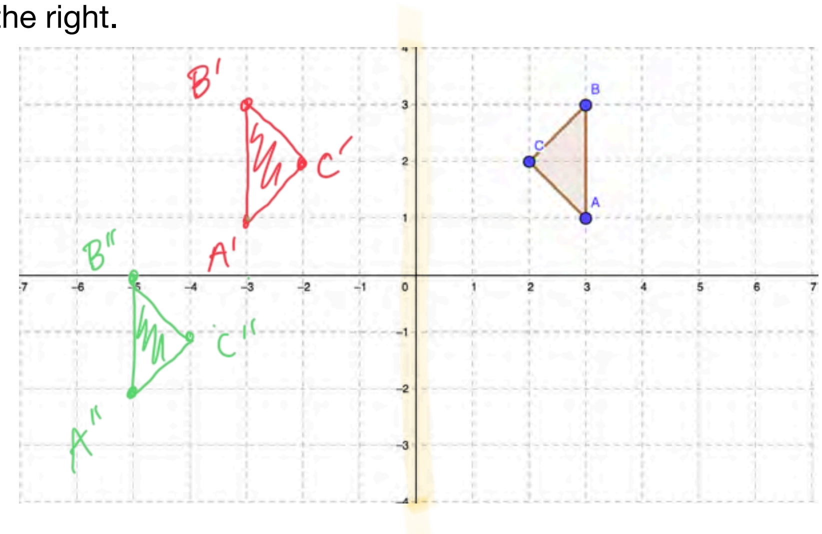
Name: Answer Key

Apply a sequence of transformations to points on a coordinate plane. Determine whether the order of a sequence of transformations has an effect on the image.

1. Draw and label the image of *ABC* after a translation 3 units down and 2 units to the right. Then, reflect *A’B’C’* over the *y*-axis.



2. Draw and label the image of *ABC* after a reflection over the *y*-axis. Then, translate *A’B’C’* 3 units down and 2 units to the right.



3. Did the order matter for these sequences of transformations? Explain.

G8 U1 Lesson 5
Compare measurements of sides and angles on a shape before and after rigid transformations.

G8 U1 Lesson 5 - Compare measurements of sides and angles on a shape before and after rigid transformations.

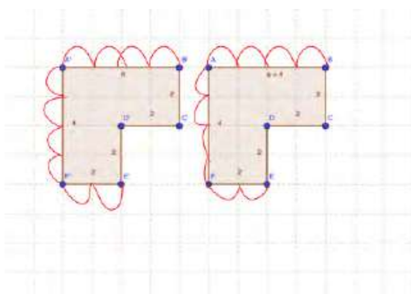
Materials: protractor

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will compare measurements of sides and angles on a shape before and after rigid transformations. First, recall that when you complete a rigid transformation, the side lengths of the original figure and its image have to be the same.

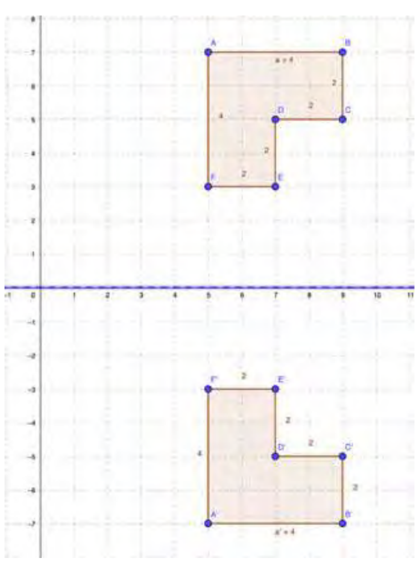
Let's Talk (Slide 4): First, let's confirm that the side lengths will be the same after a rigid transformation. Look at the polygon below. It was translated to the left 5 units. Without counting, what are the side lengths of segments *A'B'*, *E'F'*, and *F'A'*? Possible Students Answers, Key Points:

- Since the side lengths are the same in rigid transformation, the side lengths should be 4, 2, and 2 respectively.
- *A'B'* = 4 because it's the same as *AB.*
- *E'F'* = 2 because it's the same as *EF*.
- *F'A'* = 4 because it's the same as *FA*.



(Sketch counting humps to show that the three segments of the image are equal in length to the corresponding sides of or the original figure.) We've confirmed with a translation that the side lengths are the same in this rigid transformation. We can say that corresponding sides of a translated figure are the same.

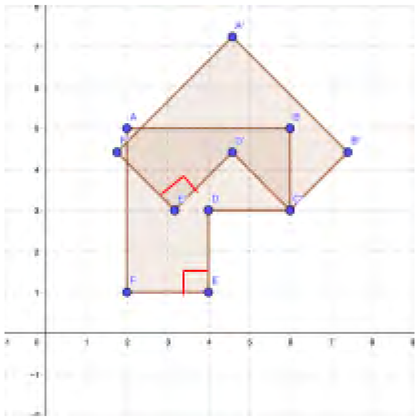
Let's Think (Slide 5): What about the side lengths of a figure that is reflected over a line? Let's use the same polygon but this time it was reflected over the *x*-axis.



Notice that like the translated figure, all of the corresponding sides are congruent.

Now that we verified, with two different rigid transformations, that the side lengths are the same once the original figure undergoes a transformation, let's consider what happens to the angles.

Let's Think (Slide 6): Let's consider what happens to the angles of a figure by rotating the same polygon 90 degrees clockwise about point *C*. Currently, all interior angles are 90 degrees which you can agree with without verifying because each angle forms a square with each unit of the grid.



We'll use a protractor to measure angle *E* in both polygons. *(Draw a right angle after you measure each angle.)* We can see that each angle appears to be the same but using the protractor helps us to verify that both angles measure 90 degrees.

Let's Try it (Slides 7 - 8): Let's work on comparing measurements of sides and angles on a shape before and after rigid transformations. We will work on this page together. Remember corresponding sides and angles of figures that undergo all three rigid transformations will be equal.

WARM WELCOME

Compare measurements of sides and angles on a shape before and after rigid transformations.

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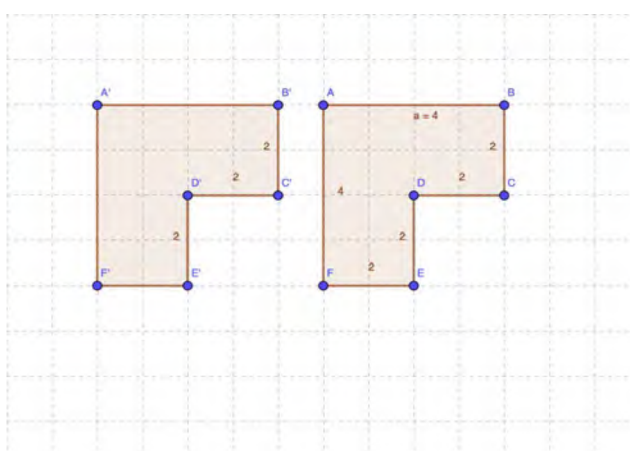
Let's Review:

When you complete a rigid transformation, the side lengths of the original figure and its image will be the same.

Let's Talk:

Can we determine if a rigid transformation occurred based on the measurements of the sides and angles of shapes?

Given that polygon *A'B'C'D'E'F'* is the image of polygon *ABCDEF* after a translation 3 units to the left, what are the lengths of segments *A'B'*, *E'F'*, and *F'A'*.



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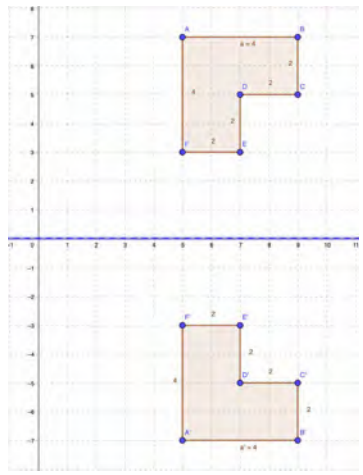
Let's Think:

Can we determine if a rigid transformation occurred based on the measurements of the sides and angles of shapes?

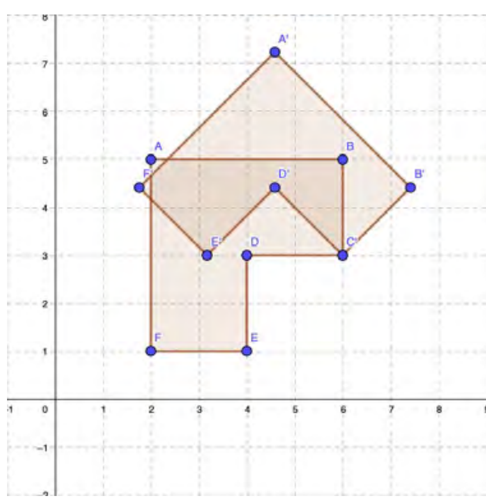
What do the images below tell us about the side lengths of a figure when it undergoes a reflection as the rigid transformation?



Let's Think:

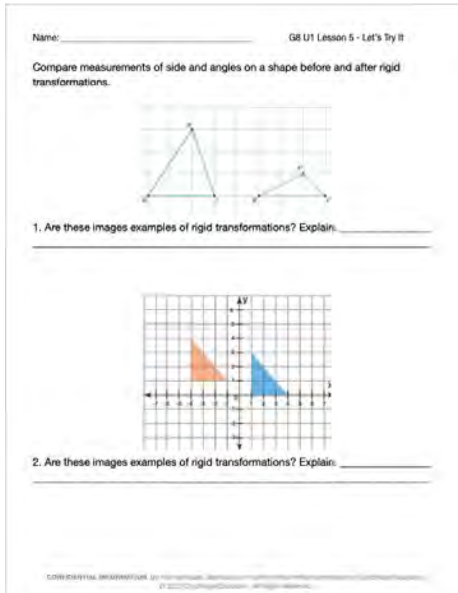
Can we determine if a rigid transformation occurred based on the measurements of the sides and angles of shapes?

What about the corresponding angles of figures before and after a rigid transformation? Are they also the same?



Let's Try It:

Let's determine if a rigid transformation occurred based on the measurements of the sides and angles of shapes.


Name: ______________________ G8 U1 Lesson 5 - Let's Try It

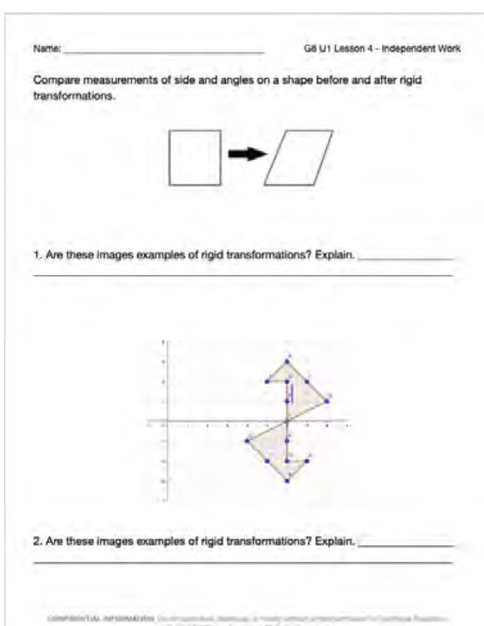
Compare measurements of side and angles on a shape before and after rigid transformations.

1. Are these images examples of rigid transformations? Explain. ____________

2. Are these images examples of rigid transformations? Explain. ____________

On your Own:

Now it's time to determine if a rigid transformation occurred based on the measurements of the sides and angles of shapes.


Name: ______________________ G8 U1 Lesson 4 - Independent Work

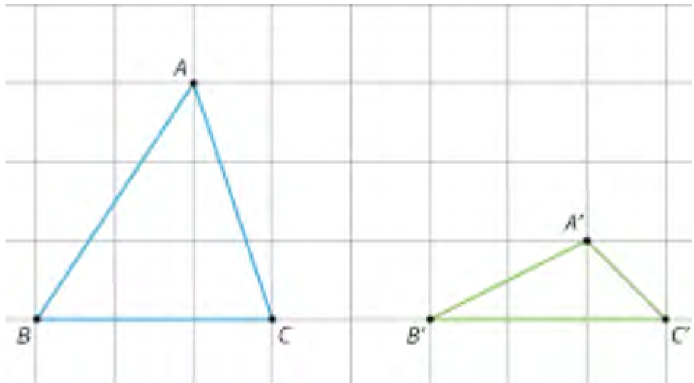
Compare measurements of side and angles on a shape before and after rigid transformations.

1. Are these images examples of rigid transformations? Explain. ____________

2. Are these images examples of rigid transformations? Explain. ____________

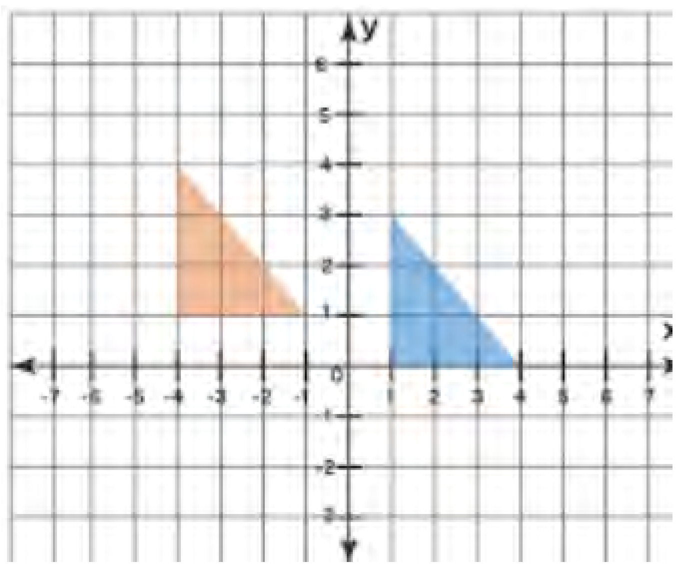
Name: ____ G8 U1 Lesson 5 - Let's Try It

Compare measurements of sides and angles on a shape before and after rigid transformations.



1. Are these images examples of rigid transformations? Explain. ___________________________

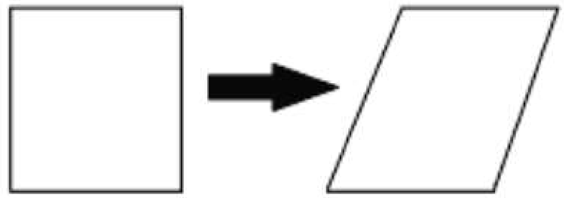
____



2. Are these images examples of rigid transformations? Explain. ___________________________

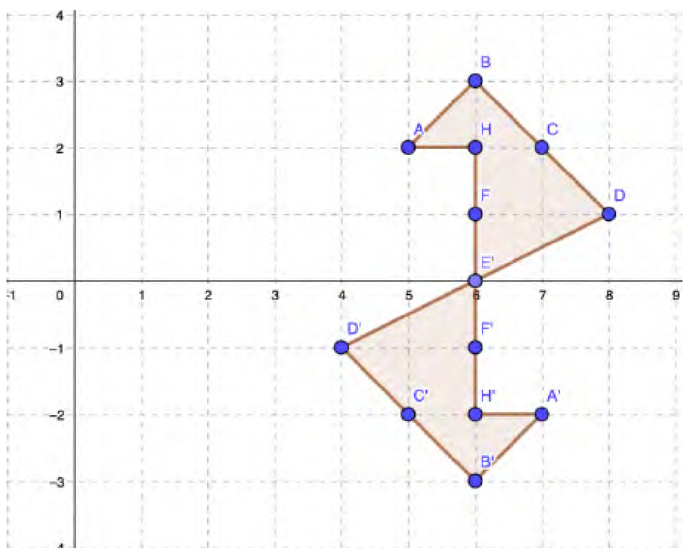
____

Compare measurements of sides and angles on a shape before and after rigid transformations.



1. Are these images examples of rigid transformations? Explain. ____________________________

____

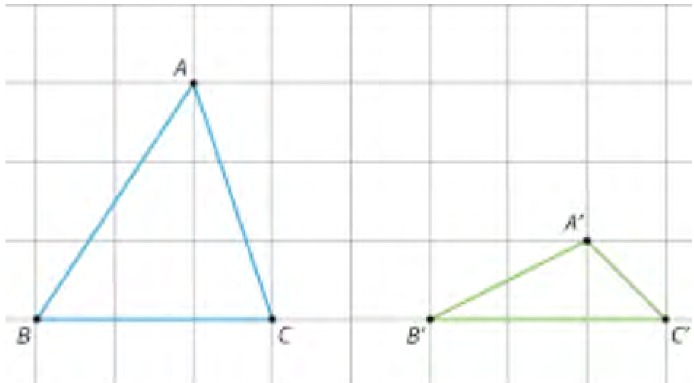


2. Are these images examples of rigid transformations? Explain. ____________________________

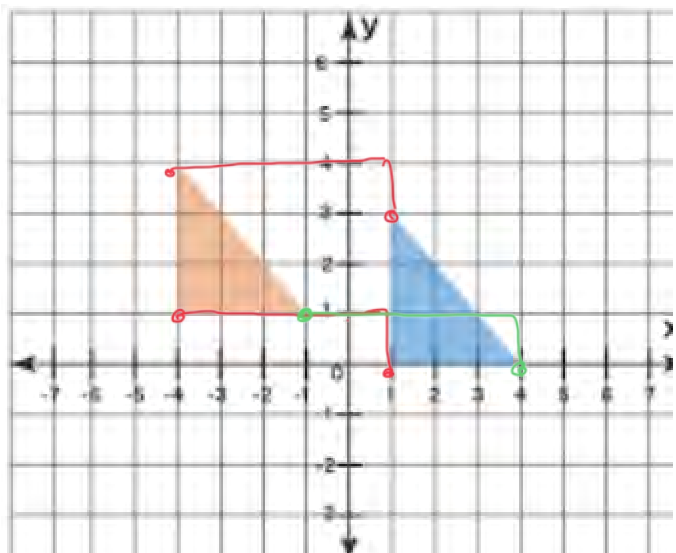
____

Name: Answer Key

Compare measurements of side and angles on a shape before and after rigid transformations.



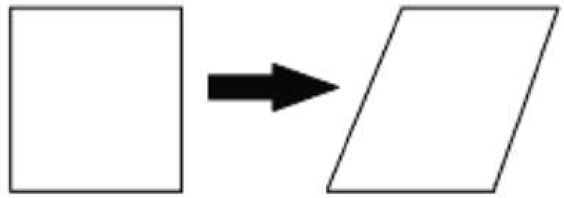
1. Are these images examples of rigid transformations? Explain. No. I cannot use a translation, reflection, or rotation to make Δ A'B'C' lay on top of Δ ABC so their measurements are not all the same.



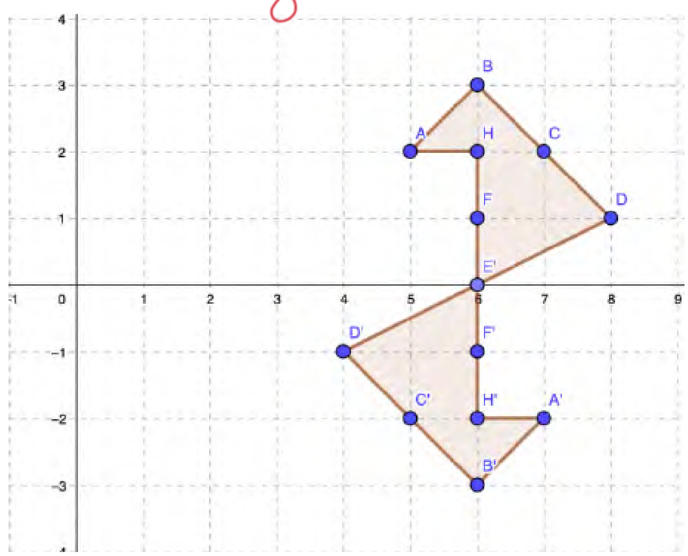
2. Are these images examples of rigid transformations? Explain. Yes. These triangles will overlap completely using a translation of 5 units right and 1 unit down [or the opposite] which means their side lengths and angles are all the same.

Name: Answer Key

Compare measurements of sides and angles on a shape before and after rigid transformations.



1. Are these images examples of rigid transformations? Explain. No. The interior ∡s of the left shape appear to be 90° while those on the right are greater or less than 90°. Since the angles are not congruent these shapes are not rigid transformations.



2. Are these images examples of rigid transformations? Explain. Yes. I can rotate polygon ABCDEFH 180° Counterclockwise and all sides and angles will match.

G8 U1 Lesson 6
Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.

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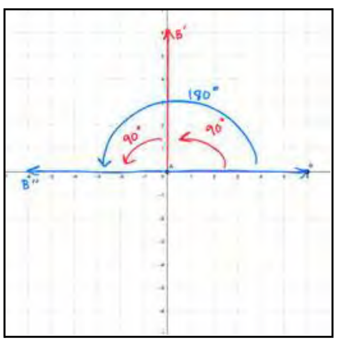
G8 U1 Lesson 6 - Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slide 2): Today we will rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment in order to generalize the outcomes of rotating a segment 180 degrees around different points. First, let's recall what it means for something to be 180 degrees. The simplest way to think of 180 degrees might be to consider a quadrant chart.

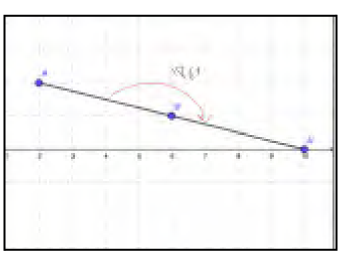
Consider the arrow pointing toward *B*. If we rotate the arrow 90 degrees counterclockwise, the image creates a 90 degree angle with the original arrow. *(Draw an arrow in the direction counterclockwise and write 90 degrees.)*



Since 90 + 90 is 180, we can rotate the arrow another 90 degrees for a full angle rotation of 180 degrees. *(Show an arc for an additional 90 degree rotation.)* The final image of the arrow makes a straight line with the original arrow. (*Draw an arrow to show 180 degrees then highlight/outline the straight line that the final arrow created with the original arrow.)*

Let's Talk (Slide 3): Now, consider the segment *AB*. Where would the image be if we rotated the segment 180 degrees around point *B*? Possible Students Answers, Key Points:

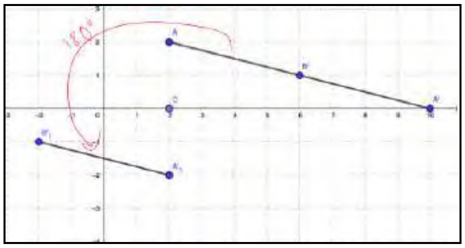
- The image would create a longer straight line.
- The image will be in line with segment *AB*.
- The image will create a longer line where *B* is the midpoint of *A* and *A*'



(*Draw an arrow to show the angle of rotation and mark it as 180 degrees.)* Notice that since *B* is on the original line segment and is the point of rotation, rotating 180 degrees in this case gives us a straight line with *B* as the midpoint.

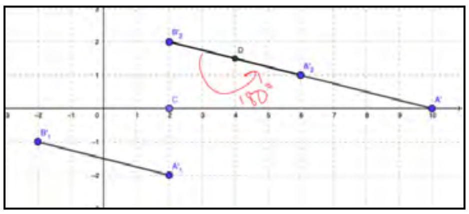
Let's Talk (Slide 4): Now let's rotate the same segment around a point *C* that is not on the original segment. What do you think will happen? Possible Students Answers, Key Points:

- The image will not create a longer line segment with the point as the midpoint.
- The image will be in a different place but the same distance from the point of rotation.



(*Draw an arrow to show the angle of rotation and mark it as 180 degrees.)* Notice that since *C* is not on the original line segment but is the point of rotation, rotating 180 degrees in this case creates a new image of *AB* that is equidistant from the point of rotation as the original segment but not on the same line.

Let's Think (Slide 5): Now that we have some idea of what happens when we rotate a segment 180 degrees around a point on the segment and a point not on the segment, let's think about what might happen if I rotate segment *AB* around its own midpoint *D*.



Remember that a 180 degree angle is a straight line angle. In this case, because the point of rotation is the midpoint of the segment being rotated, the segment turned and overlapped itself. *(Draw an arrow to show the angle of rotation and mark it as 180 degrees. Label the new point of the image as shown so they understand that the B now overlaps A and vice versa after undergoing this transformation.)*

Let's Try it (Slides 7-8): Let's work on rotating segments 180 degrees around a segment's midpoint, a point on the segment, and a point off of the segment. We will work on this page together. Remember that 1) rotating a segment 180 degrees around its midpoint creates an image that overlaps the original segment, 2) rotating a segment 180 degrees around a point on the segment creates an image that extends the length of the original segment, and 3) rotating a segment 180 degrees around a point that is not on the segment creates an image that is does not connect to the original segment but all points on both segments are equidistant to the center of rotation.

WARM WELCOME

Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.

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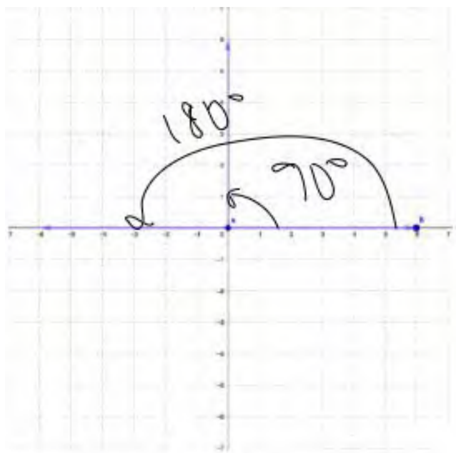
Let's Review:

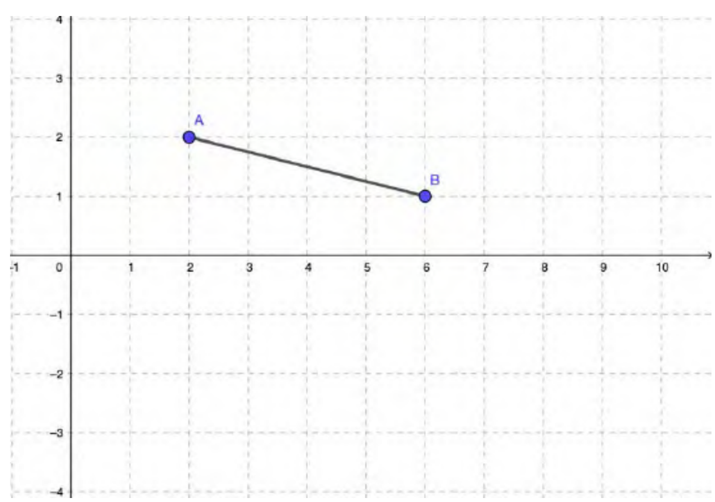
A straight line is 180 degrees.



Let's Talk:

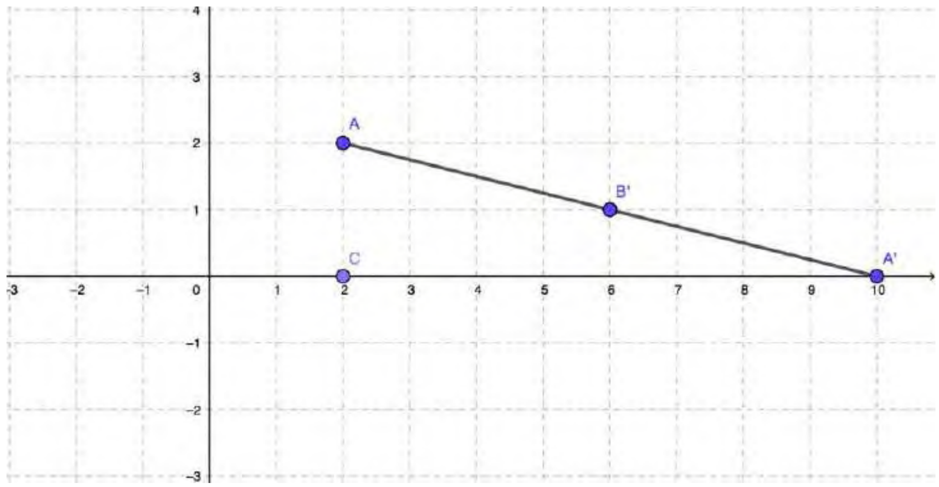
What is true about line segments rotated 180 degrees around a point?

Describe where the image of *AB* will be when it is rotated 180 degrees around *B*.



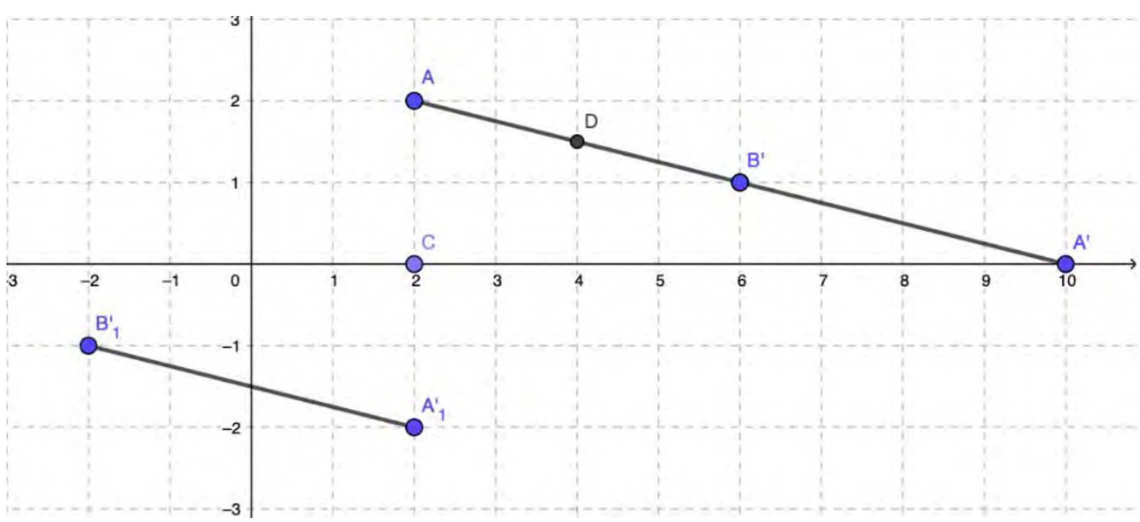
Let's Talk: **What is true about line segments rotated 180 degrees around a point?**

Now, where will the image of *AB* be when it is rotated 180 degrees around *C*?



Let's Think: **What is true about line segments rotated 180 degrees around a point?**

Where will the image of *AB* be when it is rotated 180 degrees around *D*, its midpoint?



Let’s Try It:

Let’s practice rotating segments 180 degrees about different points.

Name: ______________________ G8 U1 Lesson 6 - Let’s Try It

Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.

1. Rotate segment *AB* 180 degrees around point *B* and label the points of the image. What is true about rotating segments 180 degrees around a point on the original segment? ______________________

2. Rotate segment *AB* 180 degrees around point *D* and label the points of the image. What is true about rotating segments 180 degrees around a point NOT on the original segment? ______________________

3. Rotate segment *AB* 180 degrees around point *C*, *its midpoint*, and label the points of the image. What is true about rotating a segment 180 degrees around its midpoint? ______________________

On your Own:

Now it’s time to rotate segments 180 degrees about different points.

Name: ______________________ G8 U1 Lesson 6 - Independent Work

Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.

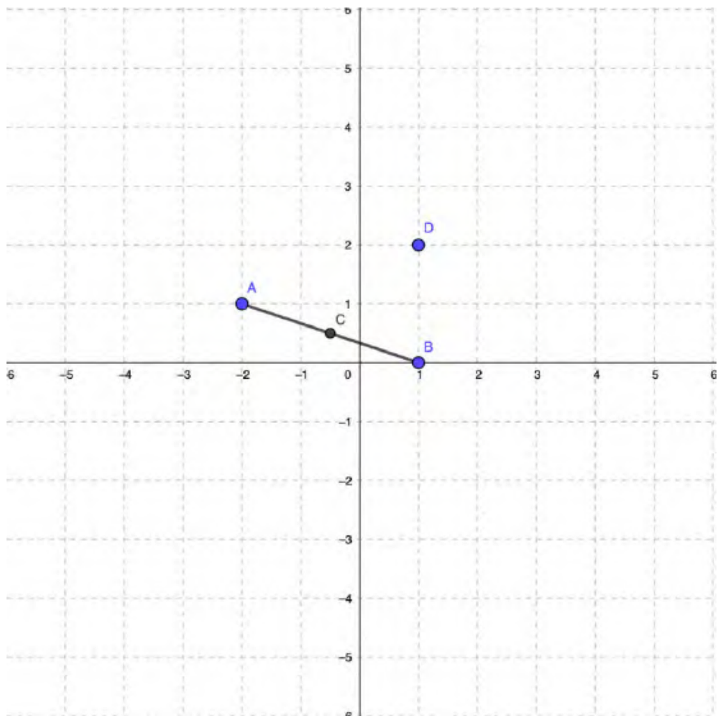
1. Rotate segment *AB* 180 degrees around point *B* and label the points of the image. What is true about rotating segments 180 degrees around a point on the original segment? ______________________

2. Rotate segment *AB* 180 degrees around point *D* and label the points of the image. What is true about rotating segments 180 degrees around a point NOT on the original segment? ______________________

3. Rotate segment *AB* 180 degrees around point *C*, *its midpoint*, and label the points of the image. What is true about rotating a segment 180 degrees around its midpoint? ______________________

Name: ____ G8 U1 Lesson 6 - Let's Try It

Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.



1. Rotate segment *AB* 180 degrees around point *B* and label the points of the image. What is true about rotating segments 180 degrees around a point on the original segment? __________________

____

2. Rotate segment *AB* 180 degrees around point *D* and label the points of the image. What is true about rotating segments 180 degrees around a point NOT on the original segment? ______________

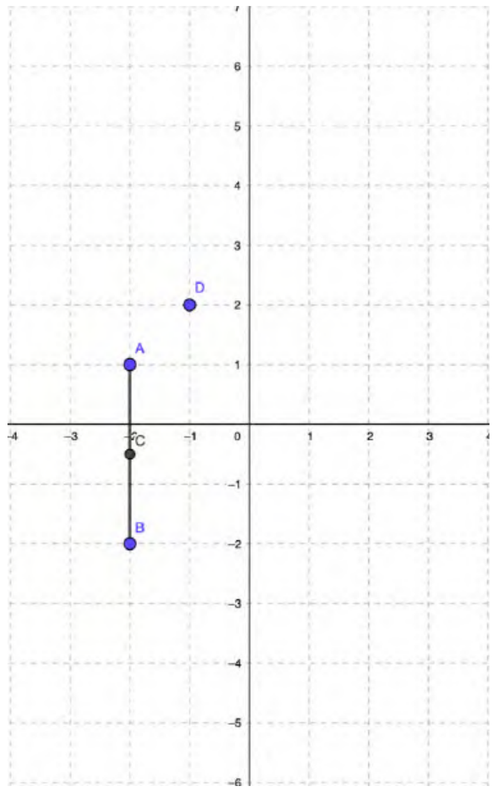
____

3. Rotate segment *AB* 180 degrees around point *C, its midpoint,* and label the points of the image. What is true about rotating a segment 180 degrees around its midpoint? ________________________

____

Name: ____

Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.



1. Rotate segment *AB* 180 degrees around point *B* and label the points of the image. What is true about rotating segments 180 degrees around a point on the original segment?_________________

____

2. Rotate segment *AB* 180 degrees around point *D* and label the points of the image. What is true about rotating segments 180 degrees around a point NOT on the original segment?______________

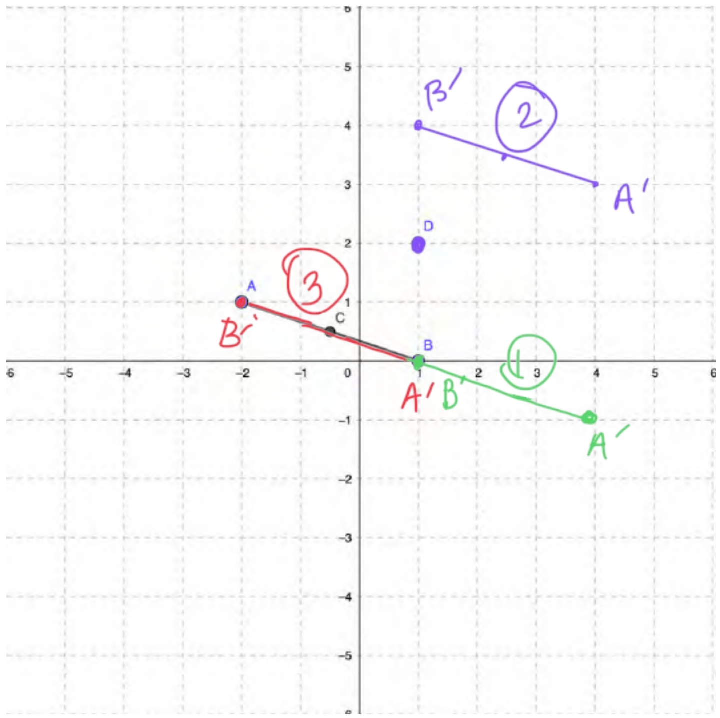
____

3. Rotate segment *AB* 180 degrees around point *C, its midpoint,* and label the points of the image. What is true about rotating a segment 180 degrees around its midpoint?_______________________

____

Name: Answer Key

Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.



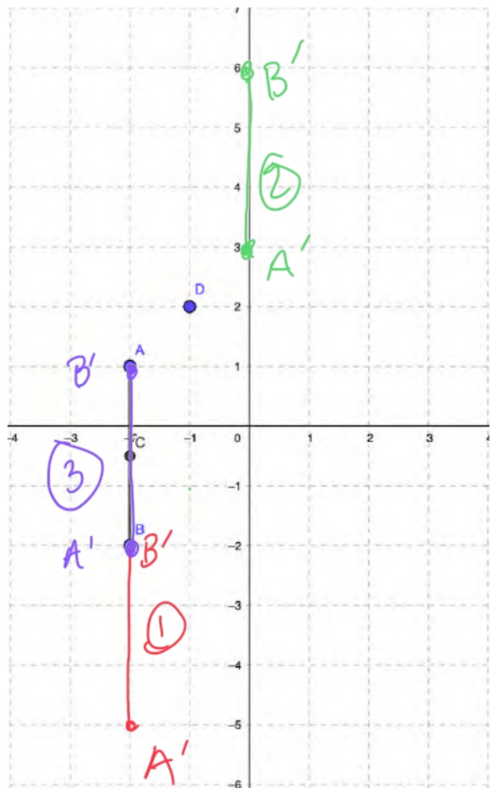
1. Rotate segment *AB* 180 degrees around point *B* and label the points of the image. What is true about rotating segments 180 degrees around a point on the original segment? The segment extends the length of the original line

2. Rotate segment *AB* 180 degrees around point *D* and label the points of the image. What is true about rotating segments 180 degrees around a point NOT on the original segment? The new segment will not be on the original segment.

3. Rotate segment *AB* 180 degrees around point *C, its midpoint,* and label the points of the image. What is true about rotating a segment 180 degrees around its midpoint? The segment will lie directly on top of itself.

Name: Answer Key

Rotate a line segment 180 degrees around its midpoint, a point on the segment, and a point not on the segment. Generalize the outcomes of rotating a segment 180 degrees around different points.



1. Rotate segment *AB* 180 degrees around point *B* and label the points of the image. What is true about rotating segments 180 degrees around a point on the original segment? The segment extends the length of the line.

2. Rotate segment *AB* 180 degrees around point *D* and label the points of the image. What is true about rotating segments 180 degrees around a point NOT on the original segment? The segment will not be on the original image

3. Rotate segment *AB* 180 degrees around point *C, its midpoint,* and label the points of the image. What is true about rotating a segment 180 degrees around its midpoint? The segment overlaps itself completely.

G8 U1 Lesson 7
Describe the effects of a rigid transformation on a pair of parallel lines.

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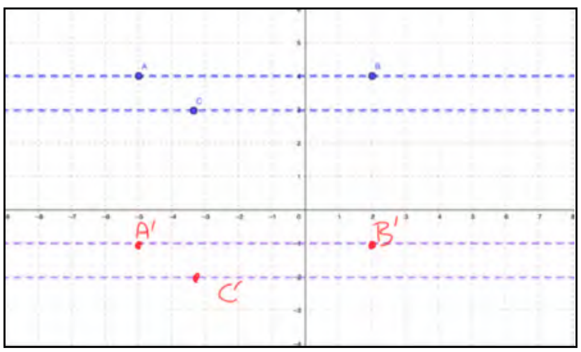
G8 U1 Lesson 7 - Describe the effects of a rigid transformation on a pair of parallel lines and understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

Warm Welcome (Slide 1): Tutor Choice

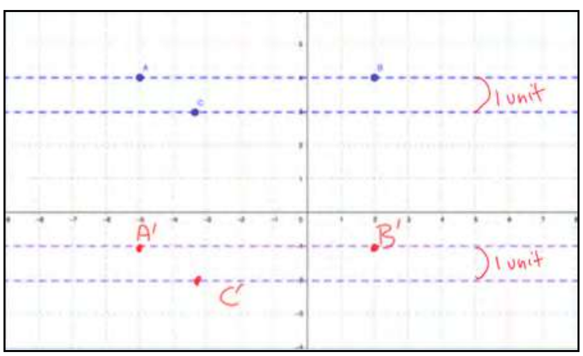
Frame the Learning/Connection to Prior Learning (Slides 2): Today we will describe the effects of a rigid transformation on a pair of parallel lines. In High School Geometry, you'll be asked to prove theorems and formulas are true. Understanding the properties of parallel lines after they undergo a rigid transformation will be helpful.

Let's Talk (Slide 3): Now, let's see what happens when we perform rigid transformations on parallel lines. Lines *AB and C* are parallel. They were translated down 4 units. What do you notice about the parallel lines and their image? Possible Students Answers, Key Points:

- The new parallel lines are 4 units below.
- The distance between the parallel lines is the same in both sets of lines.
- The new lines are still parallel.

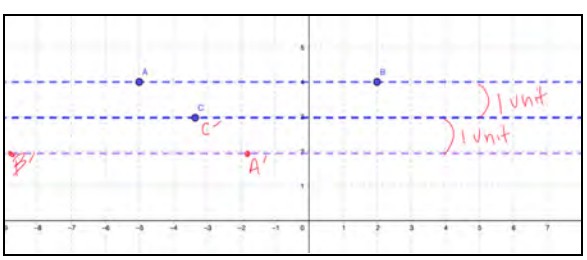


The image of the parallel lines is still parallel after the translation. *(Label the new lines A'B' and C'.)*



The distance between both sets of parallel lines is the same because the translation did not change the angles or lengths, it simply moved everything together, maintaining their original properties. *(Draw a unit arc to show each set of parallel lines has one unit of space between the lines.)*

Let's Think (Slide 4): What may happen if we rotate the original parallel lines 180 degrees about point C.



Like translations, the properties remained the same and the new set of lines are also parallel. *(Label the image pair of lines C' and A'B' and draw unit arcs to label the distances between both sets of parallel lines.)*

Let's Try it (Slides 5 - 6): Let's work on describing the effects of a rigid transformation on a pair of parallel lines. We will work on this page together. Remember, when parallel lines undergo a rigid transformation, they remain parallel and the distance between the lines is the same in all sets of lines from the original pair to the image pair.

WARM WELCOME

Describe the effects of a rigid transformation on a pair of parallel lines and understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

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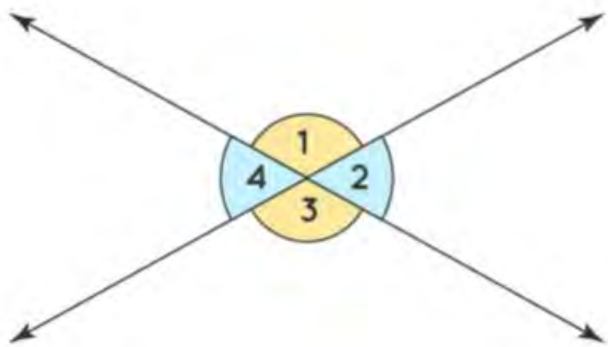
Let's Review:

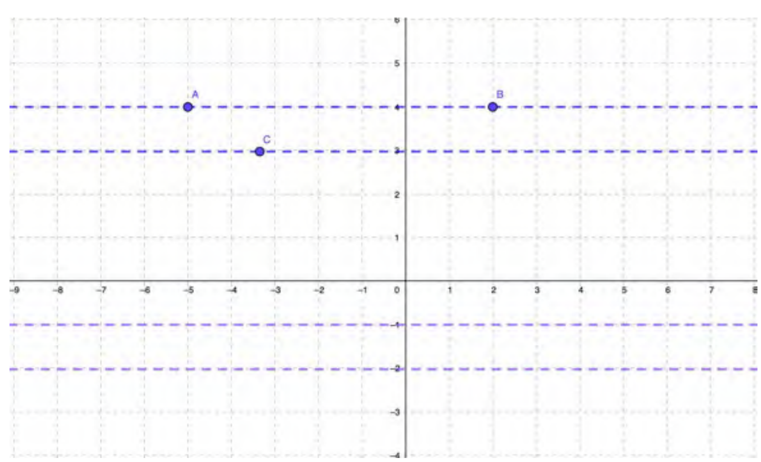
The vertical angles theorem says that angles, across from each other, that are created by a pair of intersecting lines are congruent.



Let's Talk:

What effects do rigid transformations have on parallel lines?

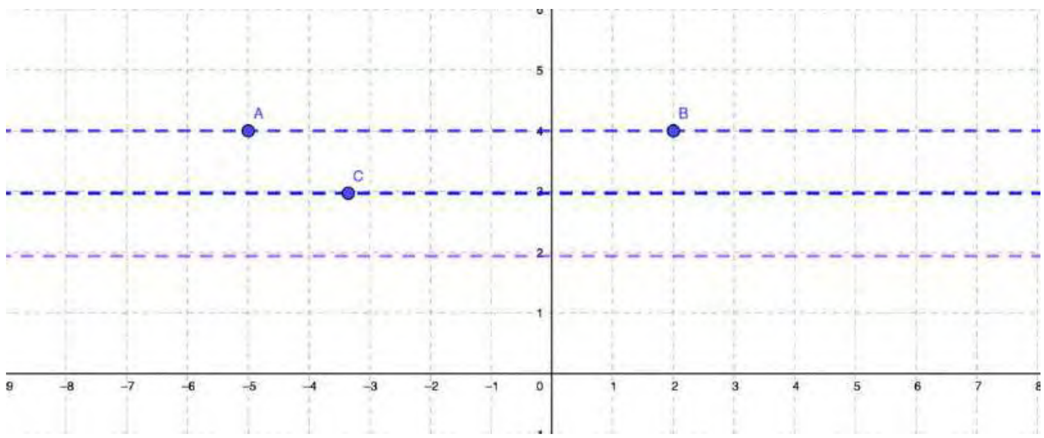
What do you notice about parallel lines *AB* and *C* and their image highlighted in purple?



Let's Think:

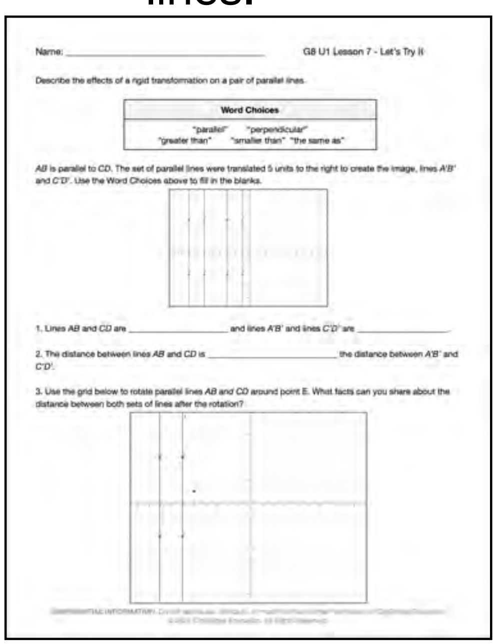
What effects do rigid transformations have on parallel lines?

What do you notice about parallel lines *AB* and *C* after they were rotated 180 degrees around point *C*?



Let's Try It:

Let's practice describing the effects of a rigid transformation on a pair of parallel lines.



Name: ______________________ G8 U1 Lesson 7 - Let's Try It

Describe the effects of a rigid transformation on a pair of parallel lines.

Word Choices
"parallel" "perpendicular" "greater than" "smaller than" "the same as"

AB is parallel to *CD*. The set of parallel lines were translated 5 units to the right to create the image, lines *A'B'* and *C'D'*. Use the Word Choices above to fill in the blanks.

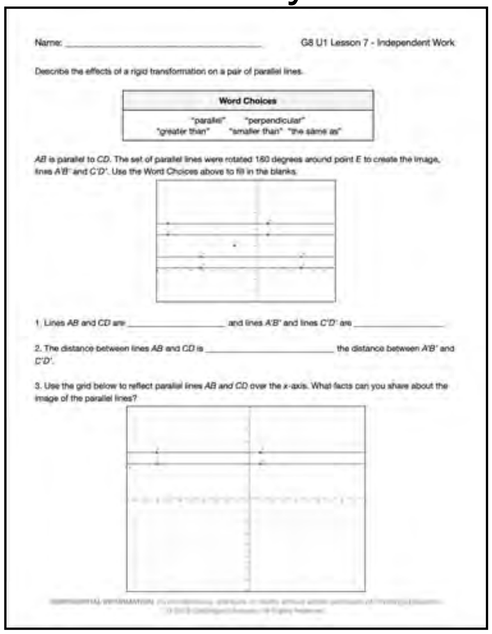
1. Lines *AB* and *CD* are ______________ and lines *A'B'* and lines *C'D'* are ______________.

2. The distance between lines *AB* and *CD* is ______________ the distance between *A'B'* and *C'D'*.

3. Use the grid below to rotate parallel lines *AB* and *CD* around point E. What facts can you share about the distance between both sets of lines after the rotation?

On your Own:

Now it's time to describe the effects of a rigid transformation on a pair of parallel lines on your own.


Name: ________________________ G8 U1 Lesson 7 - Independent Work

Describe the effects of a rigid transformation on a pair of parallel lines.

Word Choices
"parallel" "perpendicular" "greater than" "smaller than" "the same as"

AB is parallel to *CD*. The set of parallel lines were rotated 180 degrees around point *E* to create the image, lines *A'B'* and *C'D'*. Use the Word Choices above to fill in the blanks.

1. Lines *AB* and *CD* are ______________ and lines *A'B'* and lines *C'D'* are ______________

2. The distance between lines *AB* and *CD* is ______________ the distance between *A'B'* and *C'D'*.

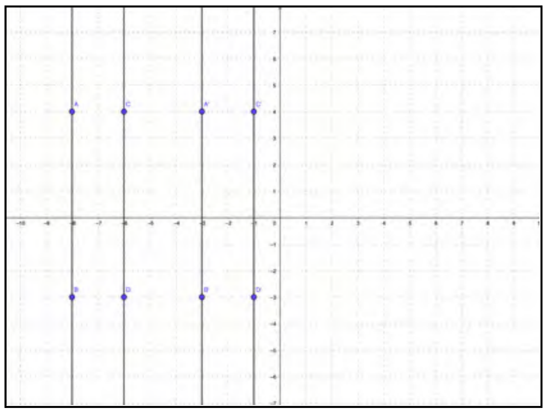
3. Use the grid below to reflect parallel lines *AB and CD* over the *x*-axis. What facts can you share about the image of the parallel lines?

Name: ____ G8 U1 Lesson 7 - Let's Try It

Describe the effects of a rigid transformation on a pair of parallel lines.

Word Choices
"parallel" "perpendicular" "greater than" "smaller than" "the same as"

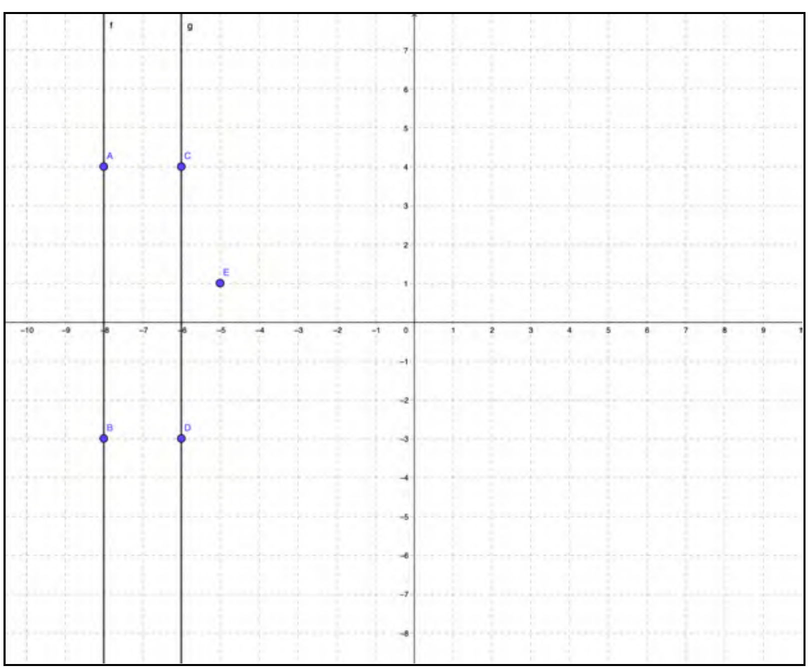
AB is parallel to *CD*. The set of parallel lines were translated 5 units to the right to create the image, lines *A'B'* and *C'D'*. Use the Word Choices above to fill in the blanks.



1. Lines *AB* and *CD* are ____________________ and lines *A'B'* and lines *C'D'* are __________________.

2. The distance between lines *AB* and *CD* is __________________________ the distance between *A'B'* and *C'D'*.

3. Use the grid below to rotate parallel lines *AB and CD* around point E. What facts can you share about the distance between both sets of lines after the rotation?

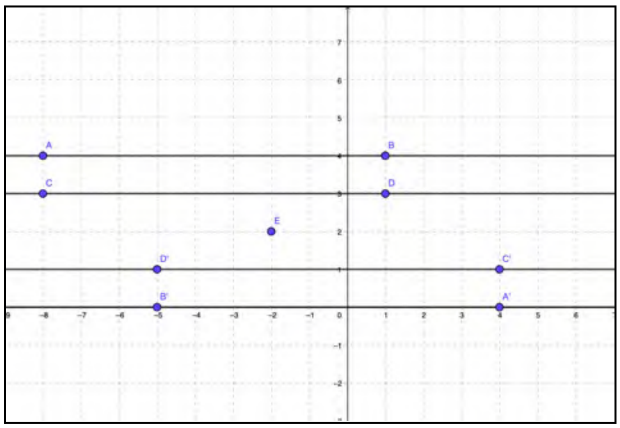


Name: ____ G8 U1 Lesson 7 - Independent Work

Describe the effects of a rigid transformation on a pair of parallel lines.

Word Choices
“parallel” “perpendicular” “greater than” “smaller than” “the same as”

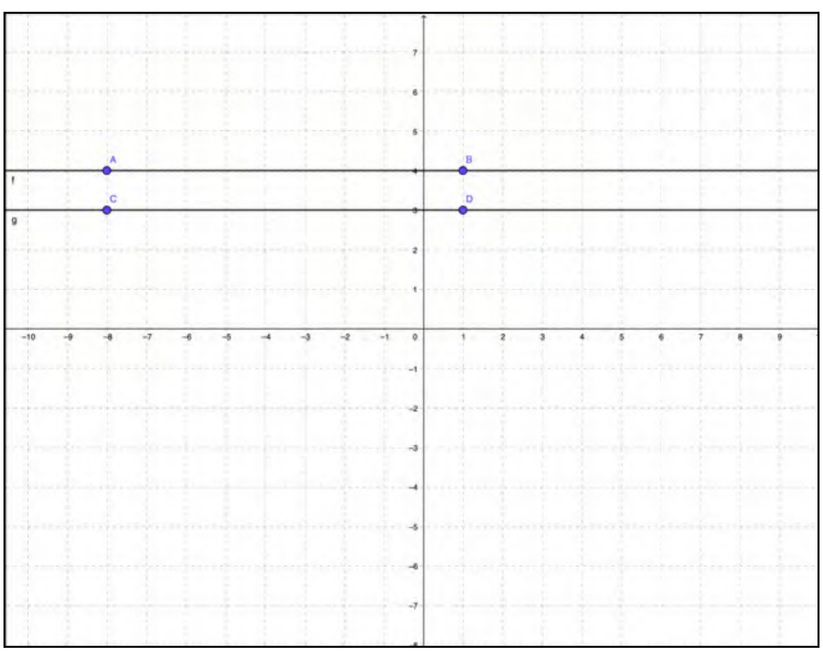
AB is parallel to *CD*. The set of parallel lines were rotated 180 degrees around point *E* to create the image, lines *A’B’* and *C’D’*. Use the Word Choices above to fill in the blanks.



1. Lines *AB* and *CD* are ______________________ and lines *A’B’* and lines *C’D’* are ____________________.

2. The distance between lines *AB* and *CD* is _____________________________ the distance between *A’B’* and *C’D’*.

3. Use the grid below to reflect parallel lines *AB and CD* over the *x*-axis. What facts can you share about the image of the parallel lines?

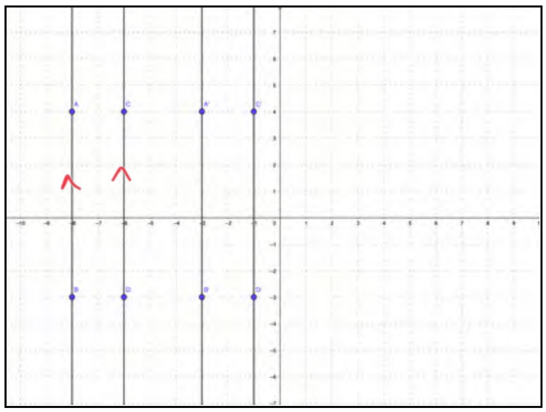


Name: Answer Key G8 U1 Lesson 7 - Let's Try It

Describe the effects of a rigid transformation on a pair of parallel lines.

Word Choices
"parallel" "perpendicular" "greater than" "smaller than" "the same as"

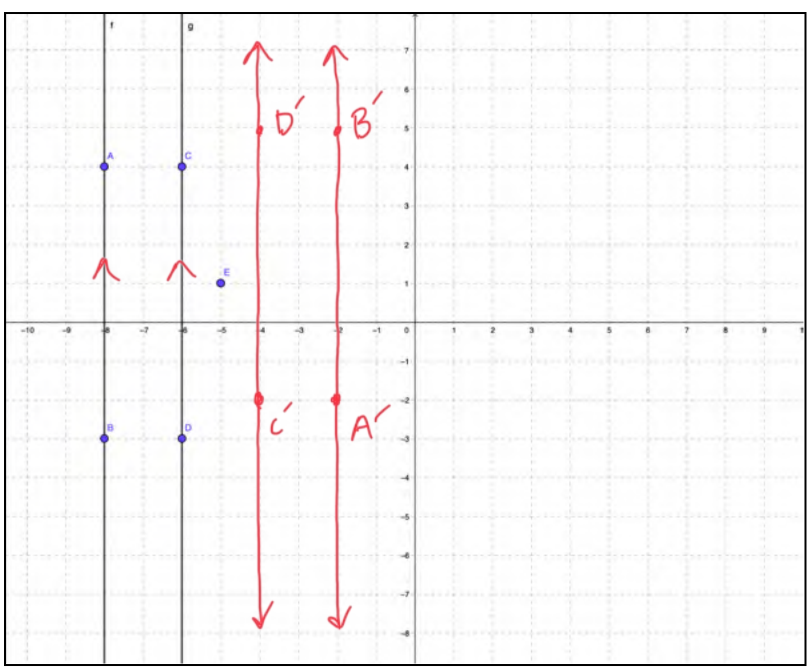
AB is parallel to *CD*. The set of parallel lines were translated 5 units to the right to create the image, lines *A'B'* and *C'D'*. Use the Word Choices above to fill in the blanks.



1. Lines *AB* and *CD* are parallel and lines *A'B'* and lines *C'D'* are parallel.

2. The distance between lines *AB* and *CD* is the same as the distance between *A'B'* and *C'D'*.

3. Use the grid below to rotate parallel lines *AB and CD* around point E. What facts can you share about the distance between both sets of lines after the rotation?



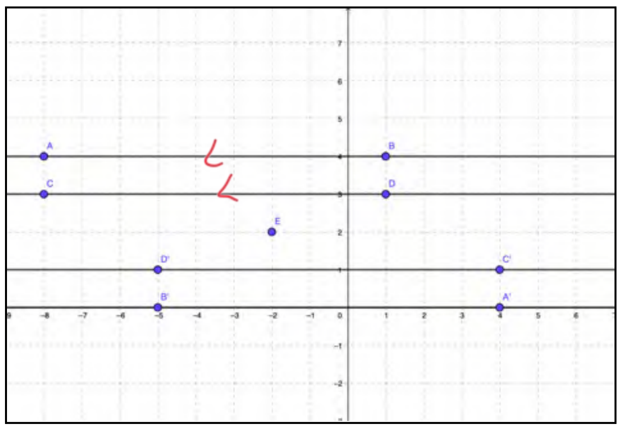
The distance between both sets of lines will be the same because rotations maintain all properties of the original figure.

Name: Answer Key

Describe the effects of a rigid transformation on a pair of parallel lines.

Word Choices
“parallel” “perpendicular” “greater than” “smaller than” “the same as”

AB is parallel to *CD*. The set of parallel lines were rotated 180 degrees around point *E* to create the image, lines *A’B’* and *C’D’*. Use the Word Choices above to fill in the blanks.

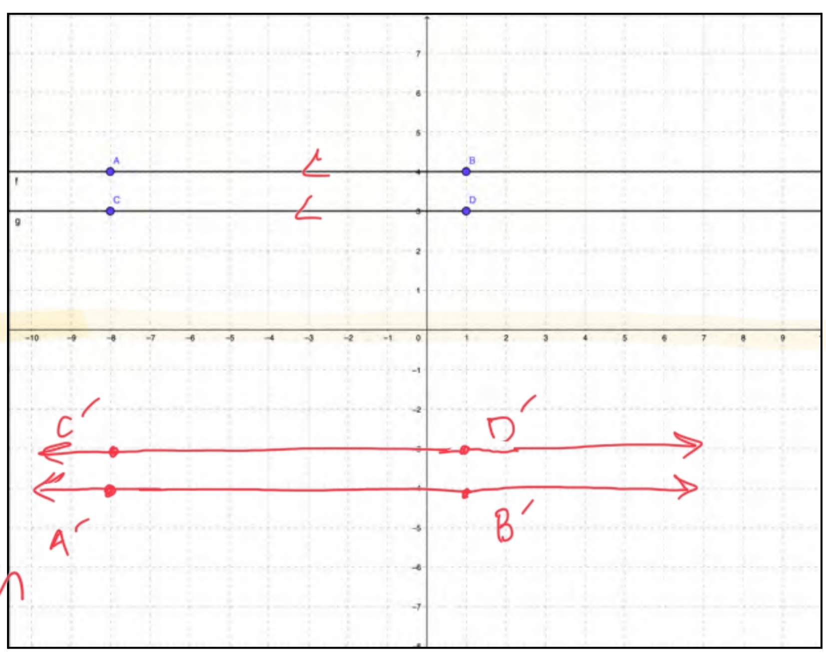


1. Lines *AB* and *CD* are parallel and lines *A’B’* and lines *C’D’* are parallel.

2. The distance between lines *AB* and *CD* is the same as the distance between *A’B’* and *C’D’*.

3. Use the grid below to reflect parallel lines *AB and CD* over the *x*-axis. What facts can you share about the image of the parallel lines?

The image of ∥ lines AB and $\overline{CD}$ is also ∥. The distance between the lines is the same in both sets.

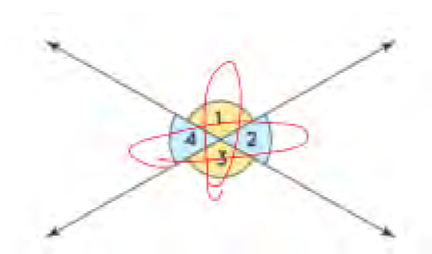


G8 U1 Lesson 8
Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

G8 U1 Lesson 8 - Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

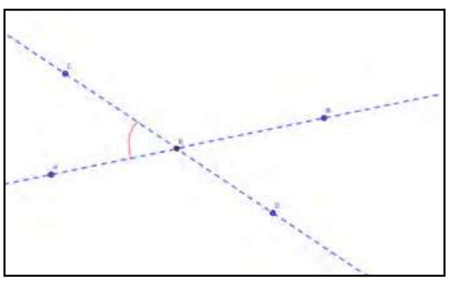
Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it. First, let's recall the vertical angles theorem that you learned in 7th grade. The vertical angles theorem is almost misleading because the angles being compared may appear horizontal to each other or diagonal to each. So what is meant in this case by vertical?



The vertical angles theorem states that angle measures that appear to be across from each other at the point of two intersecting lines, regardless of their actual orientation, are the same. In this case, angle 2 and angle 4 are vertical angles because they were created by the intersection of two lines and are across or opposite from each other on a straight line. The same can be said for angles 1 and 3. This also means that angle 4 has the same measure as angle 2 and angle 3 has the same measure as angle 1. *(Circle the vertical angles.)* But, why is that true?

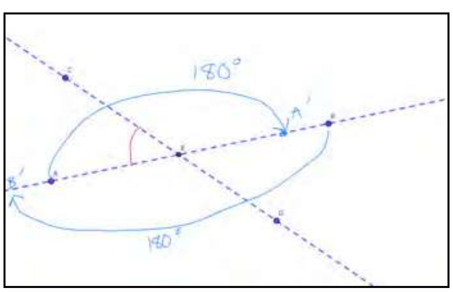
Let's Talk (Slide 4): Let's use what we know about line segments rotating 180 degrees and the properties of rigid transformations to figure out why the Vertical Angles Theorem is true. Let's start with a pair of intersecting lines *AB* and *CD* that intersect at point *E.*



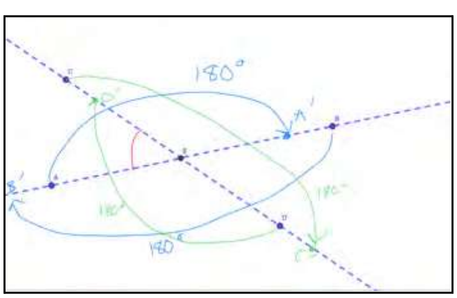
Let's mark angle *AEC* on the intersecting lines and then use our tools to rotate lines *AB* and *CD* 180 degrees about point *E*. What do you think will happen? Possible Students Answers, Key Points:

- Point *E* and point *E*' will be in the same place.
- New angles will form.

(Mark angle AEC.)

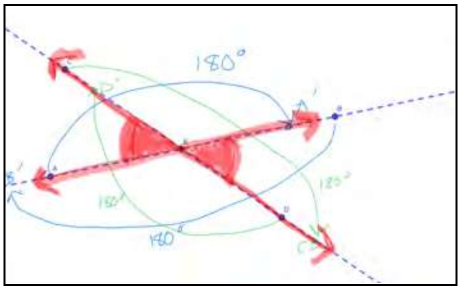


Recall that a straight line that is rotated 180 degrees about a point on the line will create an image that is on top of itself or the same line. *(Draw arcs to show the rotation of line AB and the points A and B on the line.)*



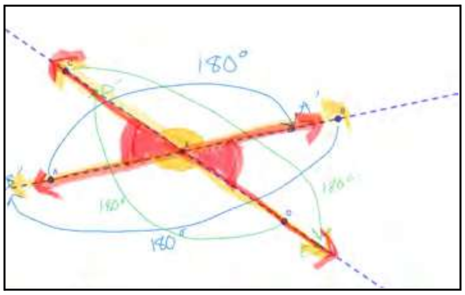
(Draw arcs to show the rotation of line CD and the points C and D on the line.) After the rotation of both lines, the images lay on top of the original lines. What does this tell us about the angle measures of angle *AEC* and angle *A'E'C'*? Possible Students Answers, Key Points:

- The angles are the same.



The measures of angle *AEC* and angle *A’E’C’* are equal because of the properties of rigid transformations which maintains the measure of angles. *(Mark the angles in question and shade in the arcs.)* This proves why the Vertical Angle Theorem is true. Angle *AEC* and angle *A’E’C’* are vertical angles, then. Given this, what is true about angle *CEB* and its image, created after the rotation of the lines? Possible Students Answers, Key Points:

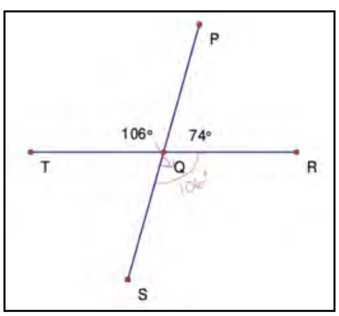
- The angles are the same because they are vertical angles.



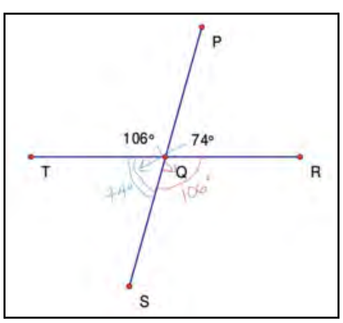
Angles *CEB* and *C’E’B’* are vertical angles and thus they are equal or congruent. *(Mark the angles in a different color and shade in the arcs.)*

Let’s Think (Slide 5): Now that we know what why the Vertical Angle Theorem is true and we know that angles formed by a pair of intersecting lines are congruent to the angle across the point of intersection, not adjacent or next to the angle, let’s solve a problem that used the Vertical Angle Theorem to find a missing angle.

When we are presented with a pair of intersecting lines and only two of the 4 angles created by the lines, we can assume the other angles using the Vertical Angle Theorem. Let’s start by finding the measure of angle *RQS*.



Angle *RQS* is congruent to angle *TQP* because they are vertical angles so angle *RQS* measures 106 degrees. *(Draw the missing angle and and arrow to show the angle it is vertical to. Then label the measure of the missing angle.*



Similarly, angle *PQR* is vertical to angle *SQT* so the angle measures must be the same. Angle *SQT* must have a measure of 74 degrees. I know - this is so simple it almost makes me wonder if it can be this easy. But it is. *(Draw the missing angle and an arrow to show the angle it is vertical to. Then label the measure of the missing angle.)*

Let’s Try it (Slides 6-7): Let’s work on applying what we understand about vertical angles. We will work on this page together. Remember that vertical angles are congruent to each other.

WARM WELCOME

Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

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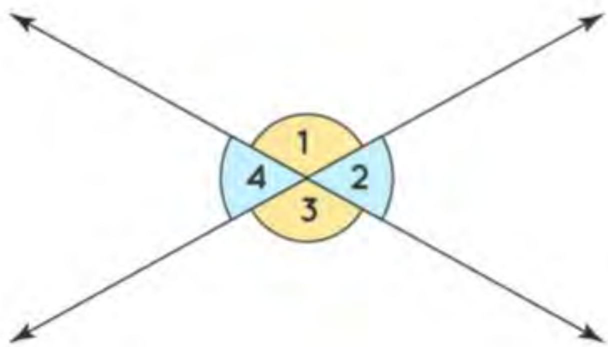
Let's Review:

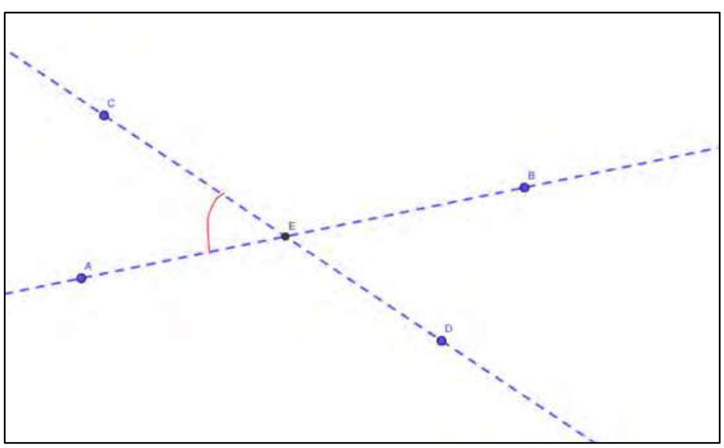
The vertical angles theorem says that angles across from each other that are created by a pair of intersecting lines are congruent.



Let's Talk:

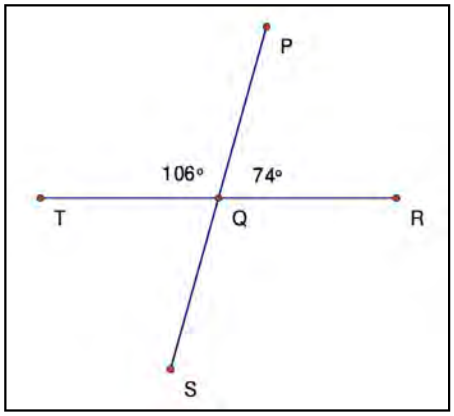
Why are vertical angles congruent?

What happens to the angles formed by intersecting lines when we rotate the lines 180 degrees about their point of intersection?

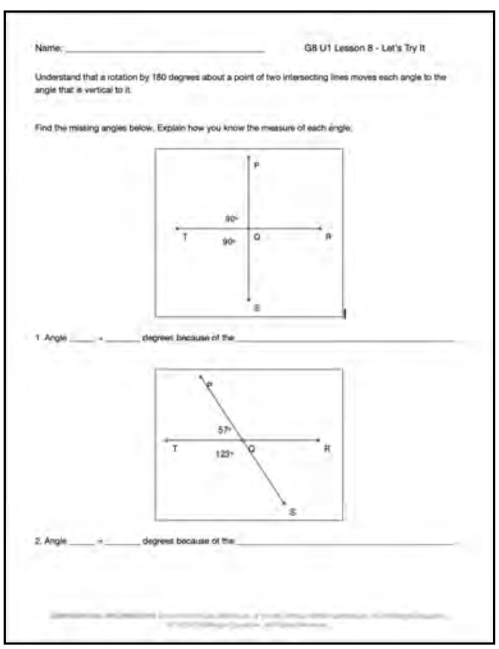


Let's Think: How do we apply the Vertical Angle Theorem?

What are the measures of angles *RQS* and *SQT*?



Let's Try It: Let's practice applying the Vertical Angle Theorem.



Name: ______________________ G8 U1 Lesson 8 - Let's Try It

Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

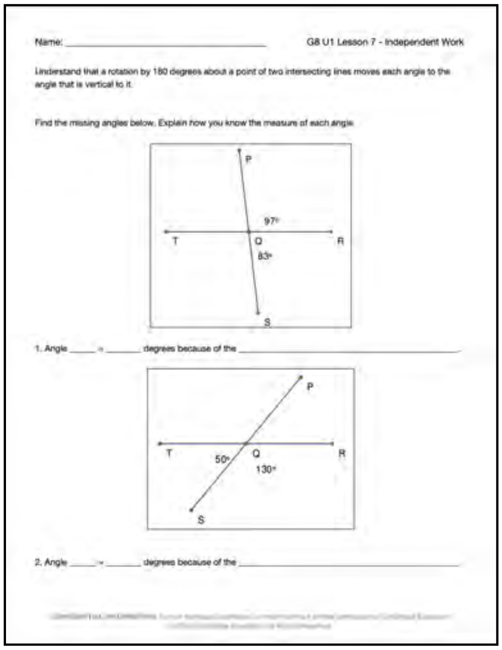
Find the missing angles below. Explain how you know the measure of each angle.

1. Angle _____ = _______ degrees because of the ______________________

2. Angle _____ = _______ degrees because of the ______________________

On your Own:

Now it's time to apply the Vertical Angle Theorem on your own.



Name: ______________________ G8 U1 Lesson 7 - Independent Work

Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

Find the missing angles below. Explain how you know the measure of each angle.

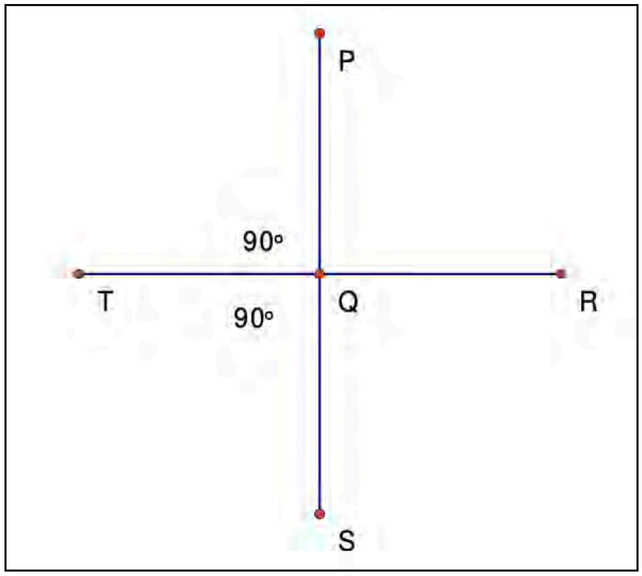
1. Angle _____ = _______ degrees because of the ______________________

2. Angle _____ = _______ degrees because of the ______________________

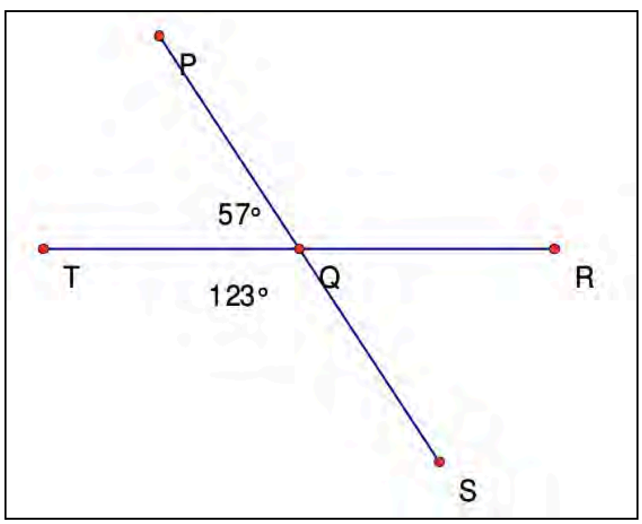
Name: ____ G8 U1 Lesson 8 - Let's Try It

Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

Find the missing angles below. Explain how you know the measure of each angle.



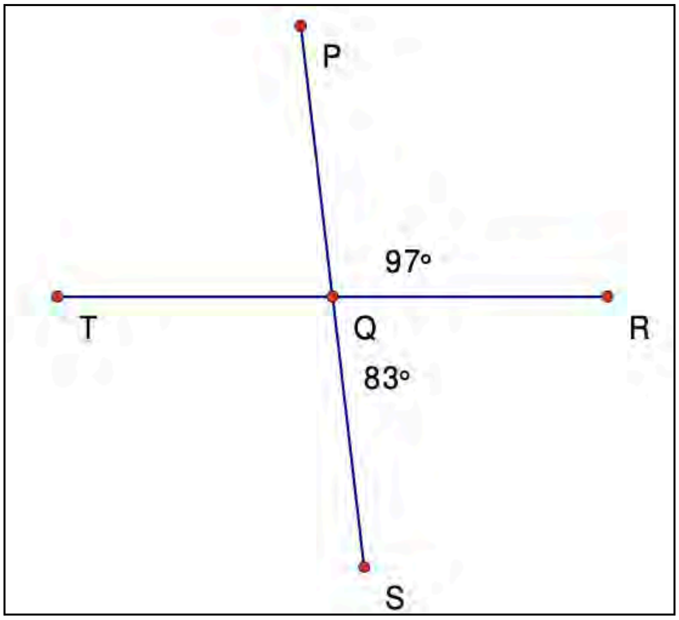
1. Angle ______ = ________ degrees because of the ____.



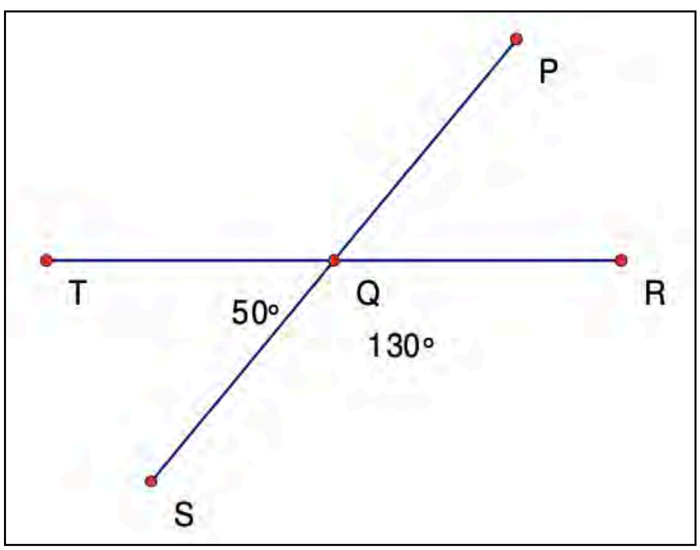
2. Angle ______ = ________ degrees because of the ____.

Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

Find the missing angles below. Explain how you know the measure of each angle.



1. Angle ______ = ________ degrees because of the ____.

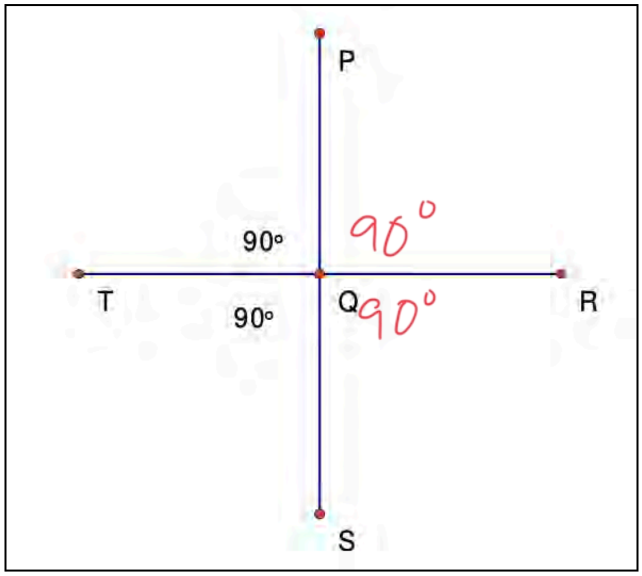


2. Angle ______ = ________ degrees because of the ____.

Name: Answer Key

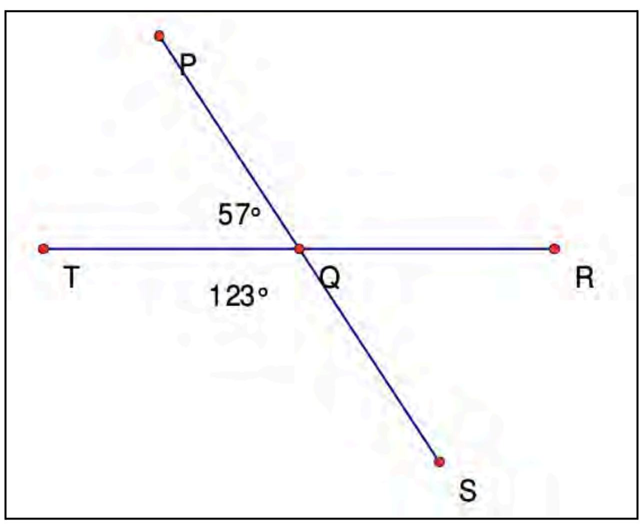
Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

Find the missing angles below. Explain how you know the measure of each angle.



1. Angle PQR = 90 degrees because of the Vertical Angle Theorem.

or RQS = 90 ״



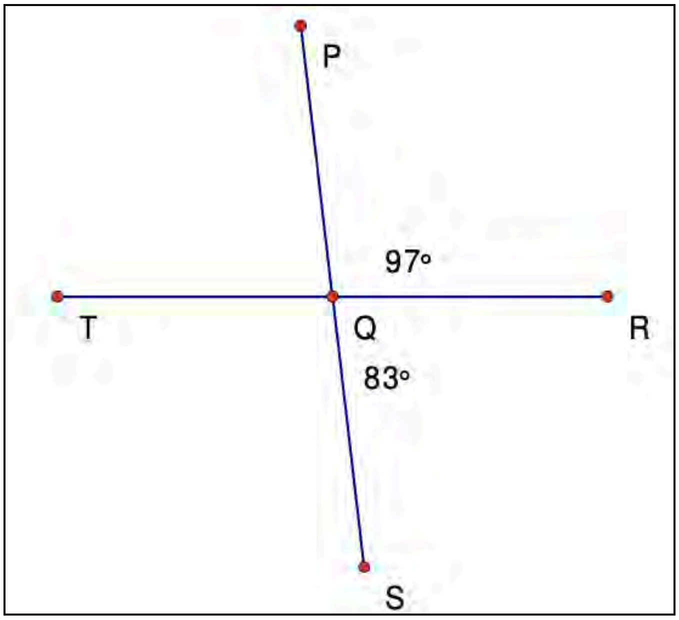
2. Angle RQS = 57° degrees because of the vertical Angle Theorem.

OR PQR = 123 ״

Name: Answer Key

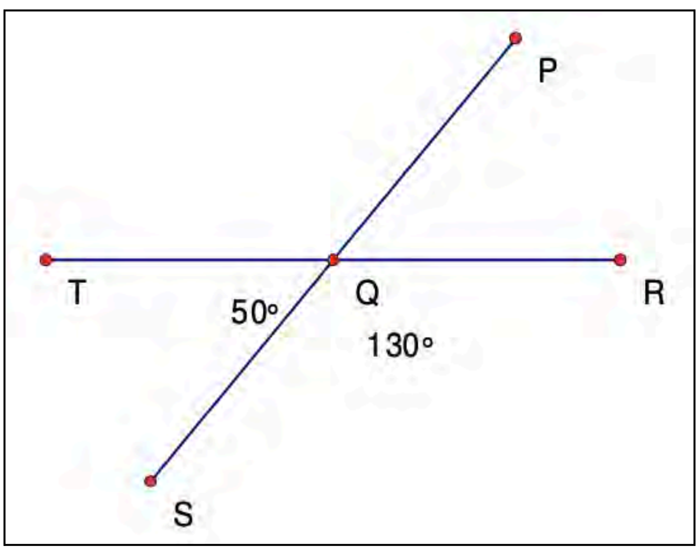
Understand that a rotation by 180 degrees about a point of two intersecting lines moves each angle to the angle that is vertical to it.

Find the missing angles below. Explain how you know the measure of each angle.



1. Angle SQT = 97 degrees because of the Vertical Angle Theorem.

OR TQP = 83 "



2. Angle PQR = 50 degrees because of the Vertical Angle Theorem.

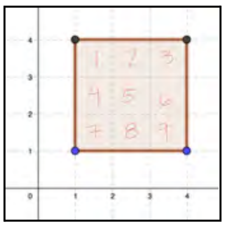
OR ∡TQP = 130 "

G8 U1 Lesson 9
Determine whether two shapes are congruent by using properties of rigid transformations and each shape's area and perimeter.

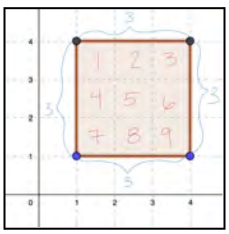
G8 U1 Lesson 9 - Determine whether two shapes are congruent by using properties of rigid transformations and each shape's area and perimeter.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will determine whether two shapes are congruent by using properties of rigid transformations and each shape's area and perimeter. First, let's remember what area and perimeter are. Area is the space inside a shape - on a square grid, the total number of units that make up the space inside the shape. Perimeter is the distance around a shape. Let's find the area and perimeter of this square.



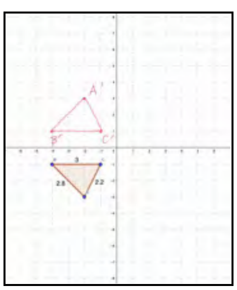
To find the area we'll count all of the boxes inside. *(Write a number in each box to demonstrate counting each box.)* The area of this square is 9 square units.



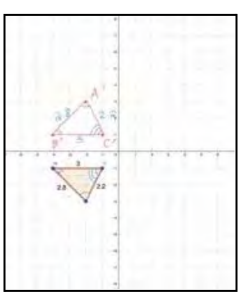
To find the perimeter we'll count the distance around the figure. *(Mark each side length with its measure.)* The perimeter of the square is 12 units.

Let's Talk (Slide 4): Now, let's recall the properties of rigid transformations by applying a reflection to a triangle we've seen before. What will be true about the angles and side lengths of triangle *ABC* after we reflect it over the *x*-axis?. Possible Students Answers, Key Points:

- The side lengths will be the same.
- The angles will be the same.



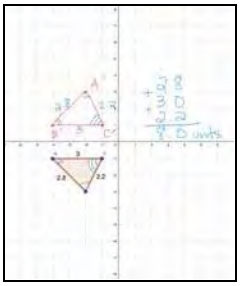
First, let's reflect triangle *ABC* over the *x*-axis and label the new vertices. *(Draw the reflection and label the vertices.)*



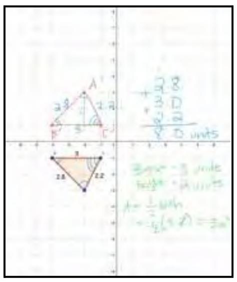
Recall that rigid transformations produce congruent corresponding sides and angles. So, we can label the side lengths by finding the corresponding side on the original shape. We can also mark and show the angles of the image without measuring them. Instead, we'll apply a different number of arcs to each angle to show they are congruent to their corresponding angle in the original shape. *(Write the side lengths and mark the angles.)*

Since triangle *ABC* underwent a rigid transformation, all of its side lengths and angles are congruent. Now let's see what's true about its perimeter and area.

Let’s Talk (Slide 5): What do you think will be true about the area and perimeter of the image of triangle *ABC*?
Possible Students Answers, Key Points:
- The perimeter will be the same since the side lengths are all the same.
- The area will be the same.



First, let’s calculate the perimeter since we know all of the side lengths of the triangle. 2.8 + 3 + 2.2 = 8 units *(Show your calculation.)* Since triangle *ABC* has the same side lengths we know that its perimeter is also 8 units without adding the distances around the figure again. So, it seems that the perimeter of a rigid transformation is the same as the perimeter of the original shape.



Now let’s calculate the area of the image. Recall that the formula to calculate the area of a triangle is *A=½(base x height)*. Triangle *A’B’C’* has an area of 3 square units. *(Show your calculation.)* Since triangle *ABC* has the same measurements we know that its area is also 3 square units without calculating again. So, it seems that the area of a rigid transformation is the same as the area of the original shape.

Let’s Think (Slide 5): We’ve just confirmed that the area and perimeter of a rigid transformation will be the same for both figures. Since this is true, that means that we can use area and perimeter along with the properties of rigid transformations to determine if two figures are congruent. In order to be congruent, all corresponding side lengths and angles must be equal. In addition, the area and perimeter of the figure will be the same. Now let’s determine which shapes are congruent using what we know about rigid transformations and area and perimeter.

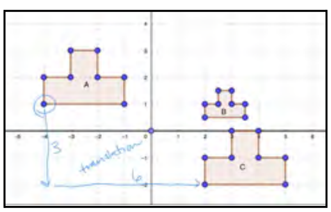
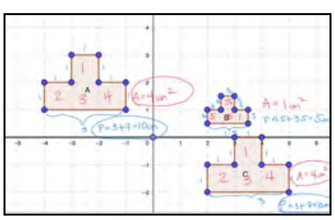


Figure C appears to be a translation 3 units down and 6 units to the right. *(Mark the grid to show the vectors used for the translation.)* It is possible that they are congruent. Figure B is not congruent to either A or C because the size is different and that doesn’t follow the properties of rigid transformations even though the angle measure appears to be the same as those in A and C. To ensure that A and C are in fact congruent, I’ll also calculate their area and perimeter.



The perimeter, or the distance around all figures is the same for figures A and C, but not for figure B. This further lets us know that figure B is not congruent to A or C but that A and C might be congruent to each other. *(Mark the figures and then show your work to calculate the perimeter. Consider highlighting the distance around to ensure you don’t miss a side.)*

The area of the figures A and C are 10 square units each. Given that they are a translation of each other and their perimeter and area are the same, figure A is congruent to figure C.

Let’s Try it (Slides 7-8): Let’s work on using the properties of rigid transformations and area and perimeter to determine if shapes are congruent to each other. We will work on this page together. Remember that shapes that undergo a rigid transformation and have the same area and perimeter are congruent to each other.

WARM WELCOME

Determine whether two shapes are congruent by using the properties of rigid transformations and each shape's area and perimeter.

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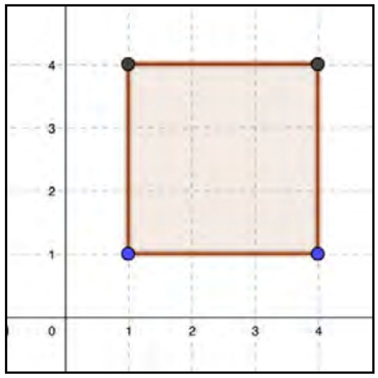
Let's Review:

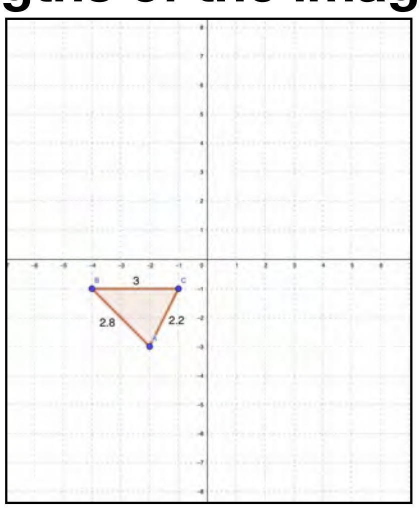
Area is the space inside of a shape and perimeter is the distance around a shape.



Let's Talk:

What do the properties of rigid transformations tell us about whether or not shapes are congruent?

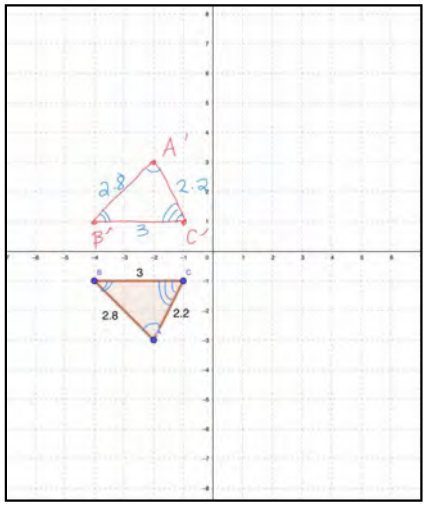
Reflect triangle *ABC* over the *x*-axis. What is true about the angles and side lengths of the image?



Let's Talk:

What do the properties of rigid transformations tell us about whether or not shapes are congruent?

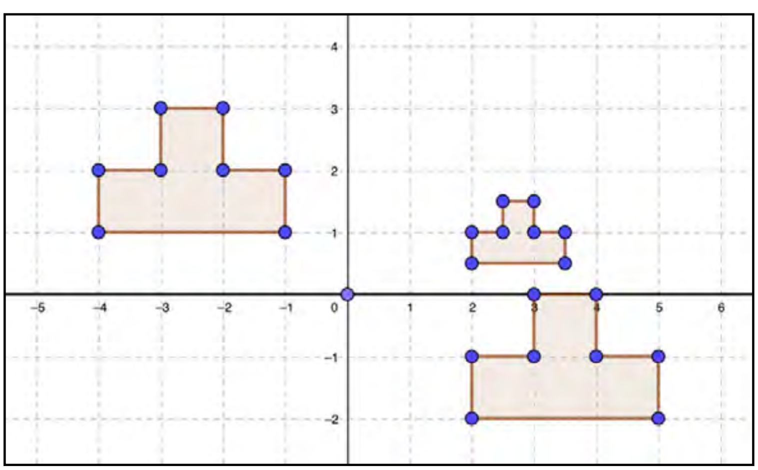
What will be true about the area and perimeter of the image of triangle *ABC*?



Let's Think:

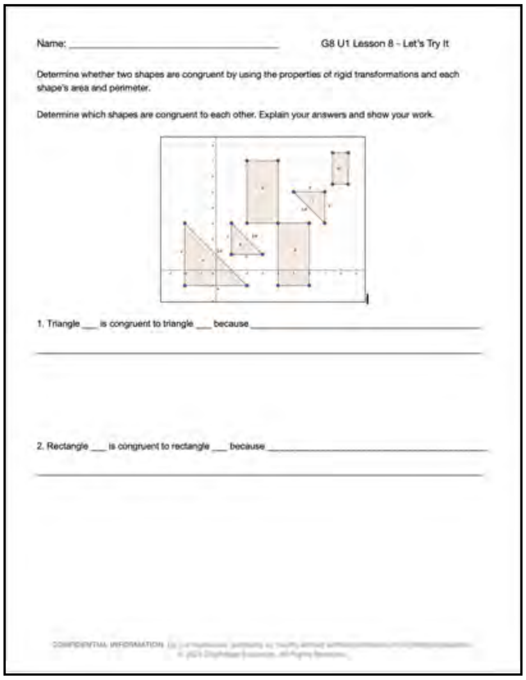
What do the properties of rigid transformations tell us about whether or not shapes are congruent?

Which shapes are congruent? Explain using what you know about the properties of rigid transformations and area and perimeter.



Let's Try It:

Let's practice determining if shapes are congruent.



Name: ______________________ G8 U1 Lesson 8 - Let's Try It

Determine whether two shapes are congruent by using the properties of rigid transformations and each shape's area and perimeter.

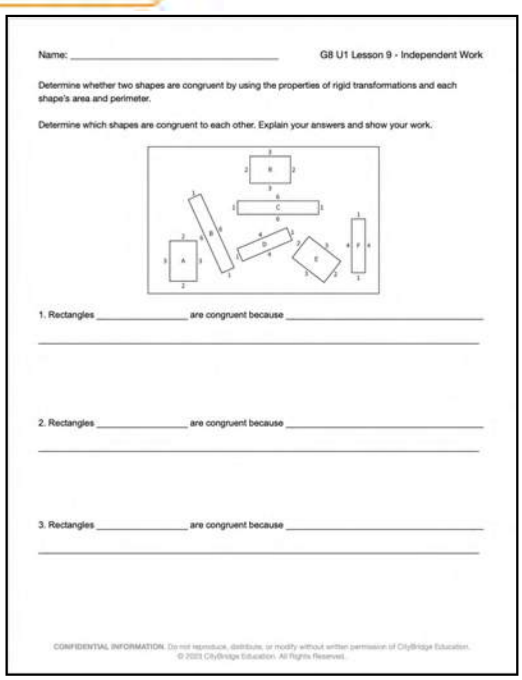
Determine which shapes are congruent to each other. Explain your answers and show your work.

1. Triangle ___ is congruent to triangle ___ because ______________________

2. Rectangle ___ is congruent to rectangle ___ because ______________________

On your Own:

Now it's time to determine if shapes are congruent on your own.



Name: ______________________ G8 U1 Lesson 9 - Independent Work

Determine whether two shapes are congruent by using the properties of rigid transformations and each shape's area and perimeter.

Determine which shapes are congruent to each other. Explain your answers and show your work.

1. Rectangles ____________ are congruent because ______________________

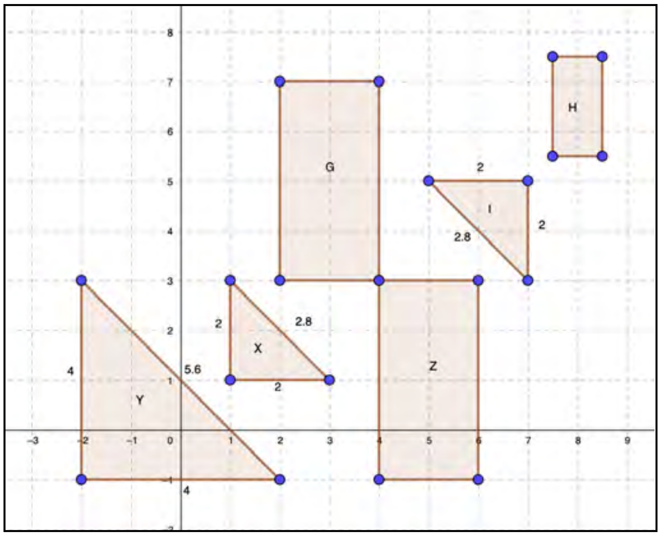
2. Rectangles ____________ are congruent because ______________________

3. Rectangles ____________ are congruent because ______________________

Name: ____ G8 U1 Lesson 9 - Let's Try It

Determine whether two shapes are congruent by using the properties of rigid transformations and each shape's area and perimeter.

Determine which shapes are congruent to each other. Explain your answers and show your work.



1. Triangle ___ is congruent to triangle ___ because ____

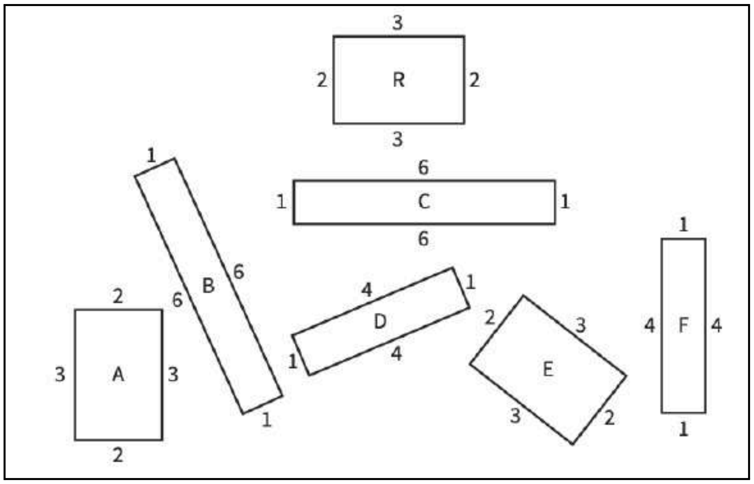
____

2. Rectangle ___ is congruent to rectangle ___ because ____

____

Determine whether two shapes are congruent by using the properties of rigid transformations and each shape's area and perimeter.

Determine which shapes are congruent to each other. Explain your answers and show your work.



1. Rectangles ___________________ are congruent because ____

____

2. Rectangles ___________________ are congruent because ____

____

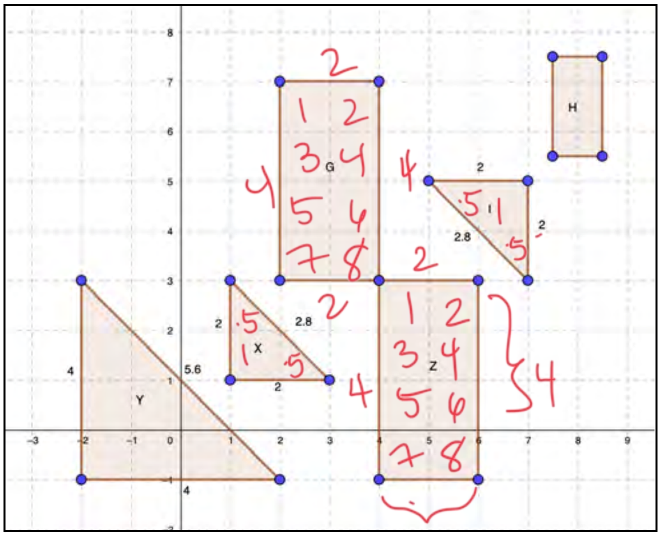
3. Rectangles ___________________ are congruent because ____

____

Name: Answer Key   G8 U1 Lesson 9 - Let's Try It

Determine whether two shapes are congruent by using the properties of rigid transformations and each shape's area and perimeter.

Determine which shapes are congruent to each other. Explain your answers and show your work.



1. Triangle X is congruent to triangle 1 because they have the same perimeter & area and can be created by a rigid trans from the other

$A_X = 2\,un^2$ $A_1 = 2\,un^2$

$P_X = 2+2+2.8 = 6.8$ units $P_1 = 2+2+2.8 = 6.8$ units

2. Rectangle Z is congruent to rectangle G because they have the same perimeter & area and can be created from a rigid transformed from the other.

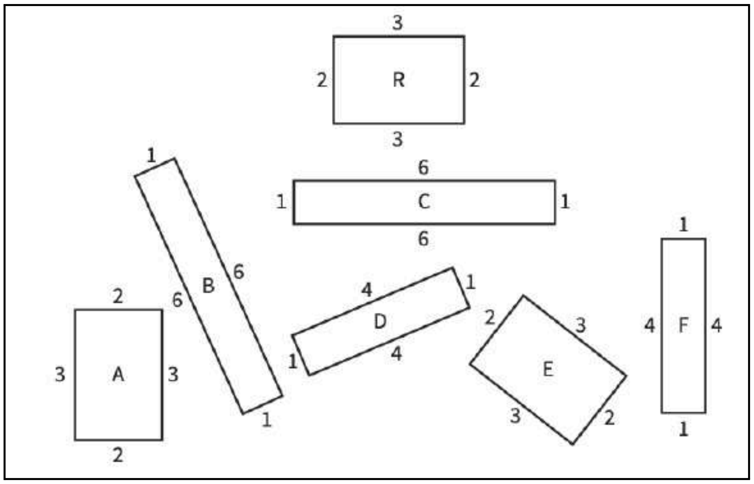
$A_Z = 8\,un^2$ $A_G = 8\,un^2$

$P_Z = 4+2+4+2 = 12$ units $P_G = 4+2+4+2 = 12$ units

Name: Answer Key G8 U1 Lesson 9 - Independent Work

Determine whether two shapes are congruent by using the properties of rigid transformations and each shape's area and perimeter.

Determine which shapes are congruent to each other. Explain your answers and show your work.



1. Rectangles B and C are congruent because they have the same area and perimeter and one can be created from the other by rigid transformations.

2. Rectangles A, R, and E are congruent because they have the same area and perimeter and they can be created from the other by rigid transformations.

3. Rectangles D and F are congruent because they have the same area and perimeter and they can be created from the other by rigid transformations.

G8 U1 Lesson 10
Use the properties of a straight angle to calculate supplementary angle measures.

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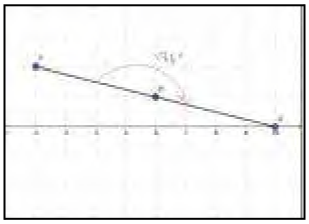
G8 U1 Lesson 10 - Use the properties of a straight angle to calculate supplementary angles.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will use the properties of a straight angle to calculate supplementary angles. First, remember that a straight angle measures 180 degrees.

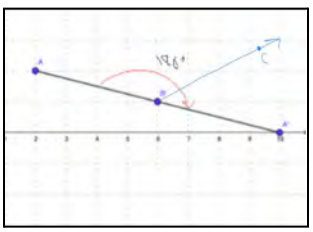


This line segment was created by rotating line segment *AB* 180 degrees about point *B*. By doing so, we found that a segment rotated 180 degrees creates an image that extends the line. Applying the rotation helped us to verify that a straight line or a straight angle measures 180 degrees.

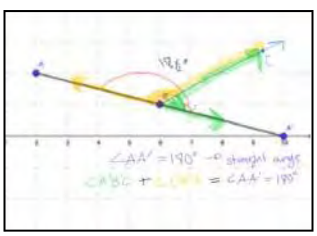
A supplementary angle uses the properties of a straight angle. The sum of angles that make a straight line are considered supplementary. Let's explore this concept.

Let's Talk (Slide 4): Let's take the same straight angle *AA'* and break it into two smaller angles. What will be true about the sum of the two smaller angles? Possible Students Answers, Key Points:

- The sum of the angles will be 180 degrees.



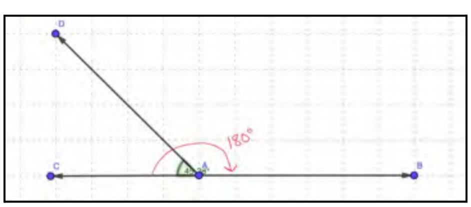
Let's use point *B* to divide our straight angle into two smaller angles. *(Draw a ray or vector B'C and label point C.)*



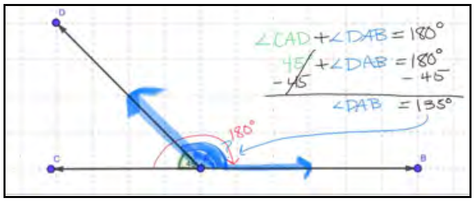
Notice that we didn't change the straight angle at all. I'll mark our two new angles so we can see that the two smaller angles combine to equal the straight angle. *(Draw the angle arcs and show a formula to demonstrate that the angles are supplementary.*

So we can say that angle *A'B'C* is supplementary to angle *CB'A* which means that the sum of the two angles equals 180 degrees.

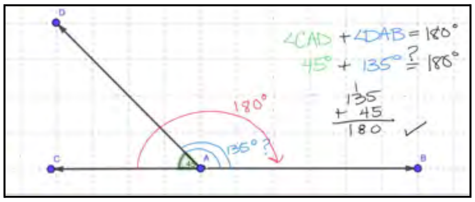
Let's Think (Slide 5): Now that we know that supplementary angles have a sum of 180 degrees, let's use this to find a missing angle on a straight angle.



First, let's notice that angle *CAB* is a straight angle. That means that angles *CAD* and *DAB* are supplementary. I am going to mark the straight angle on my paper.



Now let's use what we know about supplementary angles to calculate the missing angle *DAB*. We can say that angle *CAD* plus angle *DAB* is 180 degrees because the angles are supplementary. *(Write out the supplementary formulas.)* Let's substitute angle *CAD*'s measure in the formula since we know its value. Now we have a simple one step equation to solve. Since 180 minus 45 is 135, angle *DAB* is 135 degrees.



We can check our work by plugging both angle measures into the original formula that I wrote and verify that the sum of the angle measures is 180 degrees. In this case, our calculation is correct. Angle *CAD* + angle *DAB* is 180 degrees.

Let's Try it (Slides 6-7): Let's work on using the definition of supplementary angles to find the missing angle on a straight angle. We will work on this page together. Remember that supplementary angles are two adjacent angles on a straight line and have a sum of 180 degrees.

WARM WELCOME

Use the properties of a straight angle to calculate supplementary angles.

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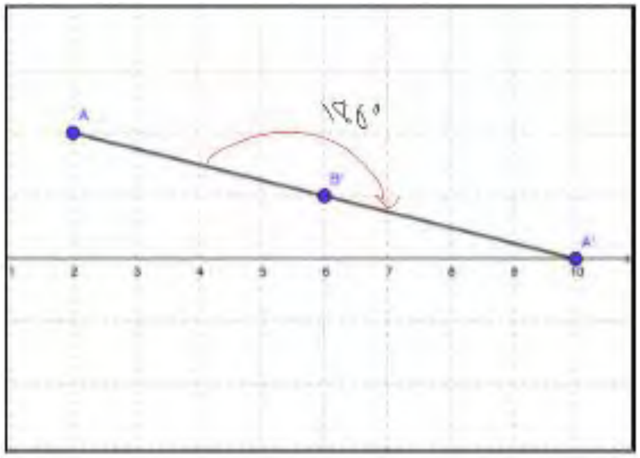
Let's Review:

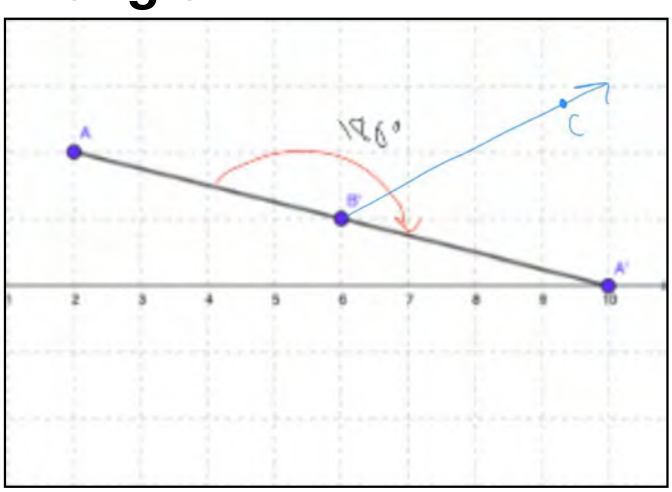
A straight angle measures 180 degrees.



Let's Talk:

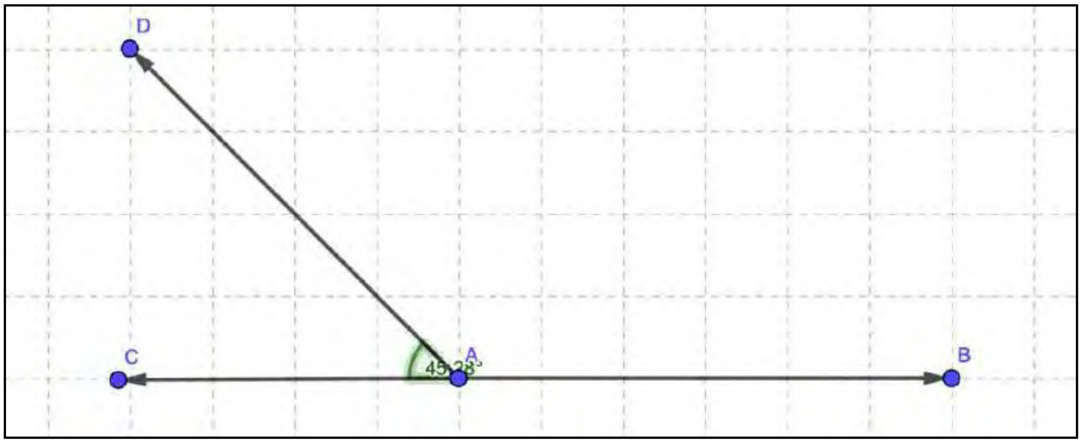
What are supplementary angles?

What is true about the sum of two smaller angles formed from a straight angle?

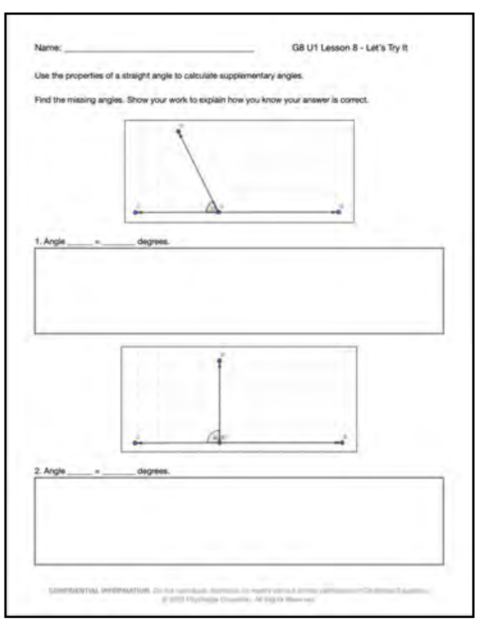


Let's Think: What are supplementary angles?

What is the value of the missing angle if angle *CAD* = 45 degrees?



Let's Try It: Let's practice calculating angles using the definition of supplementary angles.



Name: ______________________ G8 U1 Lesson 8 - Let's Try It

Use the properties of a straight angle to calculate supplementary angles.

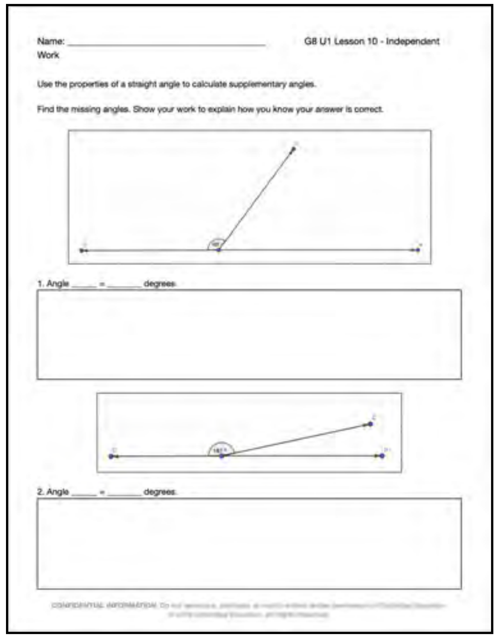
Find the missing angles. Show your work to explain how you know your answer is correct.

1. Angle _____ = _______ degrees.

2. Angle _____ = _______ degrees.

On your Own:

Now it’s time to calculate angles using the definition of supplementary angles on your own.


Name: ______________________ G8 U1 Lesson 10 - Independent Work

Use the properties of a straight angle to calculate supplementary angles.

Find the missing angles. Show your work to explain how you know your answer is correct.

1. Angle _____ = _______ degrees.

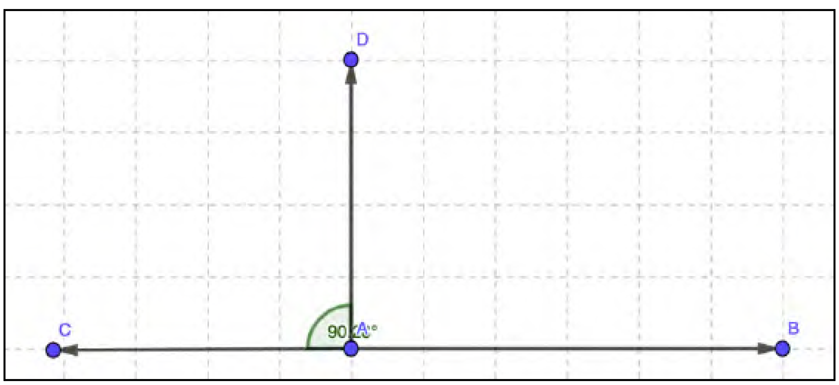
2. Angle _____ = _______ degrees.

Name: ______________________________________ G8 U1 Lesson 10 - Let's Try It

Use the properties of a straight angle to calculate supplementary angles.

Find the missing angles. Show your work to explain how you know your answer is correct.

1. Angle ______ = ________ degrees.

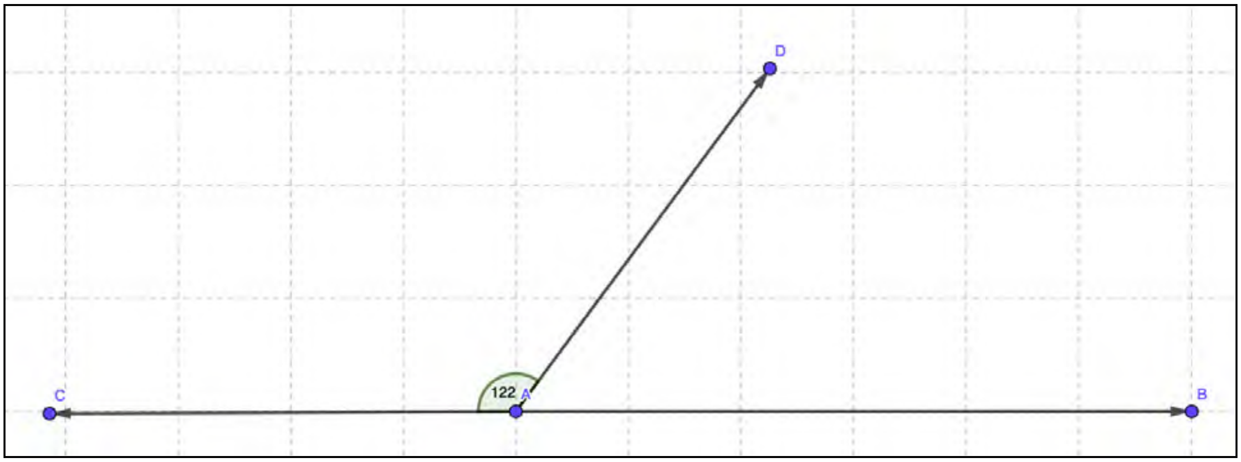


2. Angle ______ = ________ degrees.

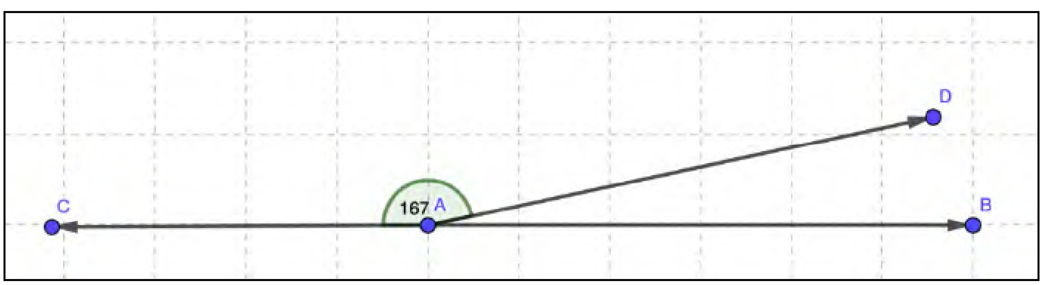
Name: ____ G8 U1 Lesson 10 - Independent Work

Use the properties of a straight angle to calculate supplementary angles.

Find the missing angles. Show your work to explain how you know your answer is correct.



1. Angle ______ = ________ degrees.



2. Angle ______ = ________ degrees.

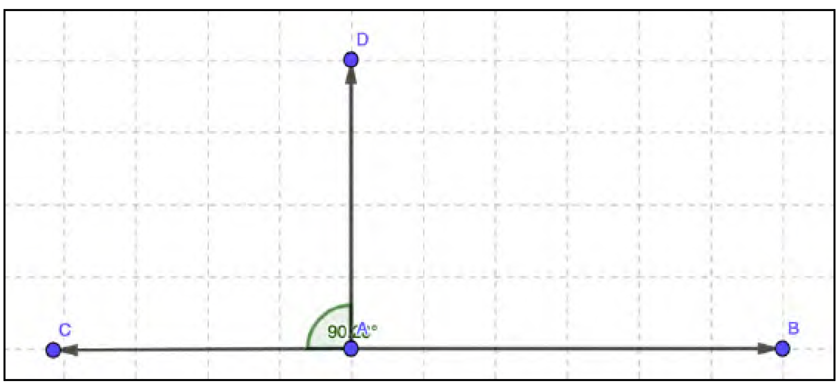
Name: Answer Key G8 U1 Lesson 10 - Let's Try It

Use the properties of a straight angle to calculate supplementary angles.

Find the missing angles. Show your work to explain how you know your answer is correct.

1. Angle DAB = 117 degrees.

$\angle CAD + \angle DAB = 180°$ → supplementary angle
$63 + \angle DAB = 180°$
$-63 \quad\quad -63$
$\angle DAB = 117°$

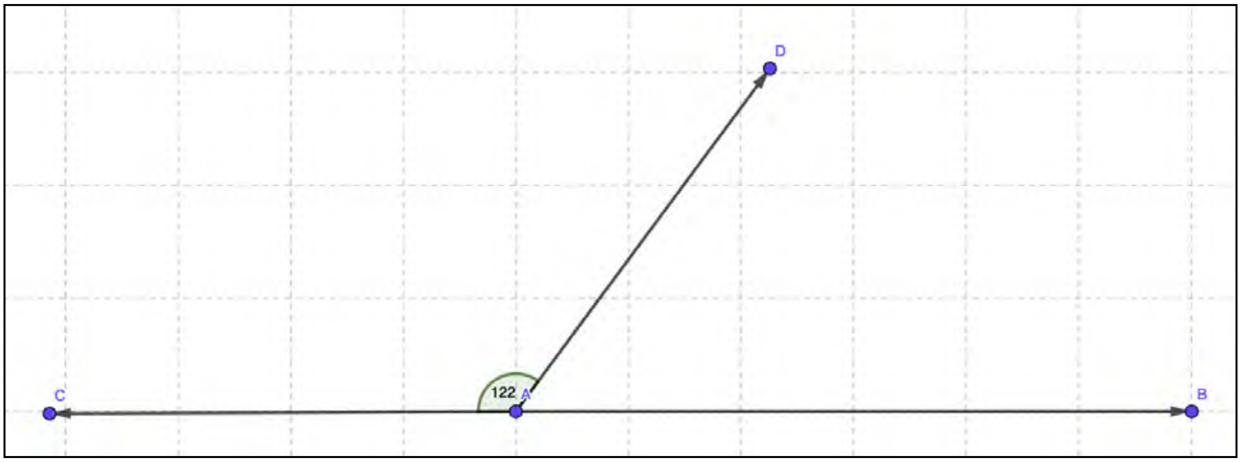


2. Angle DAB = 90 degrees.

$\angle CAD + \angle DAB = 180°$
$90 + \angle DAB = 180°$
$-90 \quad\quad -90$
$\angle DAB = 90°$

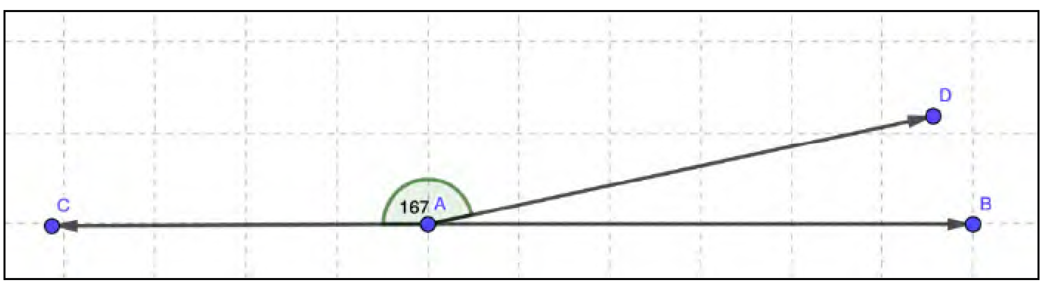
Use the properties of a straight angle to calculate supplementary angles.

Find the missing angles. Show your work to explain how you know your answer is correct.



1. Angle DAB = 58 degrees.

$\angle CAD + \angle DAB = 180°$ supplementary angles
$122 + \angle DAB = 180°$
$-122 \quad\quad -122$
$\angle DAB = 58°$



2. Angle DAB = 13 degrees.

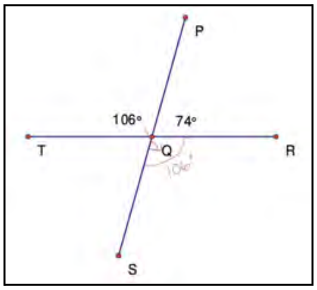
$\angle CAD + \angle DAB = 180°$
$167 + \angle DAB = 180$
$-167 \quad\quad -167$
$\angle DAB = 13°$

G8 U1 Lesson 11
Apply what you know about vertical angles and supplementary angles to calculate the measures of unknown angles.

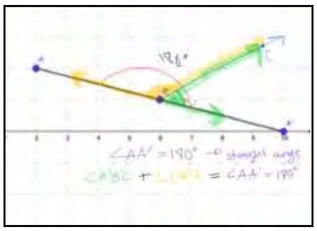
G8 U1 Lesson 11 - Apply what you know about vertical angles and supplementary angles to calculate the measures of unknown angles.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 4): Today we will apply what we know about vertical angles and supplementary angles to calculate the measures of unknown angles.



Recall that vertical angles are nonadjacent angles created by the intersection of two lines - vertical angles are congruent.

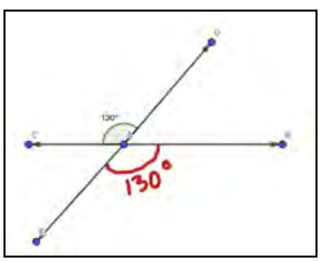


Supplementary angles are adjacent angles on a straight angle - the sum of supplementary angles is 180 degrees.

Let's Talk (Slide 5): Now let's consider how vertical angles and supplementary angles work together to help us calculate the missing angles on a pair of intersecting lines. What is the value of ∠*BAE* if ∠*CAD* is 130°?

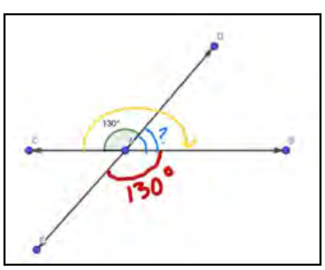
Possible Students Answers, Key Points:

- The angles are the same.

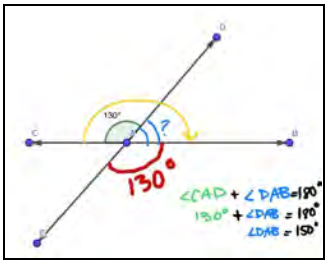


∠*BAE* is 130° because it is vertical to ∠*CAD. (Mark ∠BAE and it's value.)*

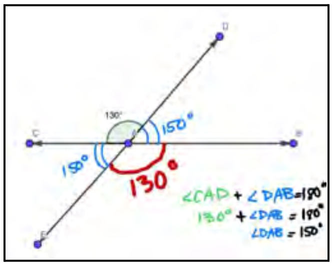
Let's Think (Slide 6): Now that we identified the vertical angles and their values, we will use what we know about supplementary angles to find the value of ∠*DAB* and ∠*EAC*.



First, notice that ∠*CAD* and ∠*DAB* are adjacent and on a straight angle. *(Mark one of the missing angles, ∠DAB, and the straight angle.)* Now we can use the definition of supplementary angles to find the missing value.



∠*CAD* + ∠*DAB* = 180° because they are supplementary angles. I will substitute 130° for ∠*CAD* to show that the missing angle, ∠*DAB*, is 150°. *(Write the equation and solve for the missing angle.)*



Since ∠*DAB* = 150°, ∠*EAC* = 150° because the angles are vertical to each other.

Let's Try it (Slides 7-8): Let's work on applying vertical angles and supplementary angles to finding the missing angles on a pair of intersecting lines. We will work on this page together. Remember that on a pair of intersecting lines, you only need one angle to find the others. Use the Vertical Angles Theorem and the definition of supplementary angles to find the missing values.

WARM WELCOME

Apply what you know about vertical angles and supplementary angles to calculate the measure of unknown angles.

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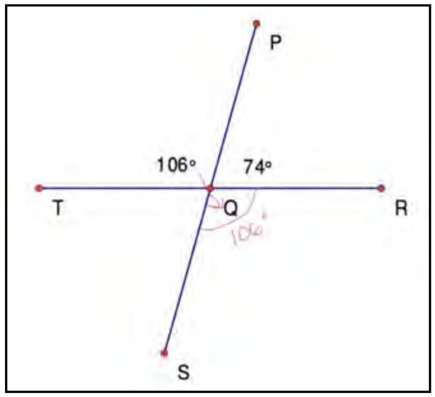
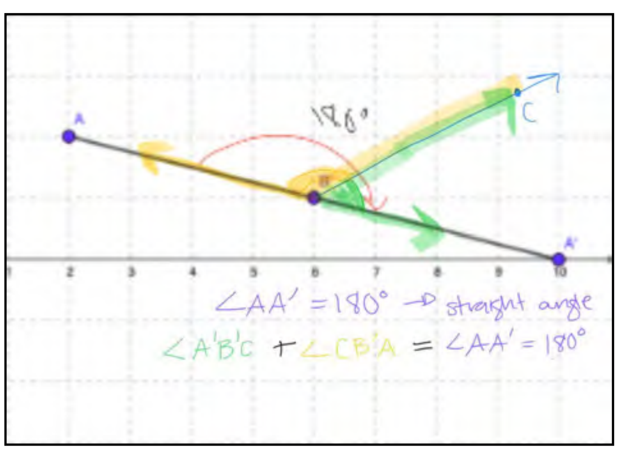
Let's Review:

Vertical angles are congruent.



Let's Review:

The sum of supplementary angles is 180 degrees.



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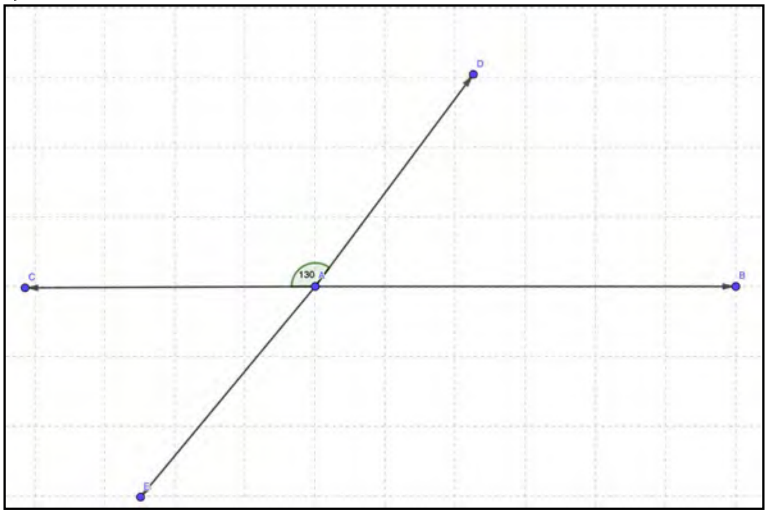
Let's Talk:

How do vertical angles and supplementary angles work together to help us calculate missing angles of a pair of intersecting lines?

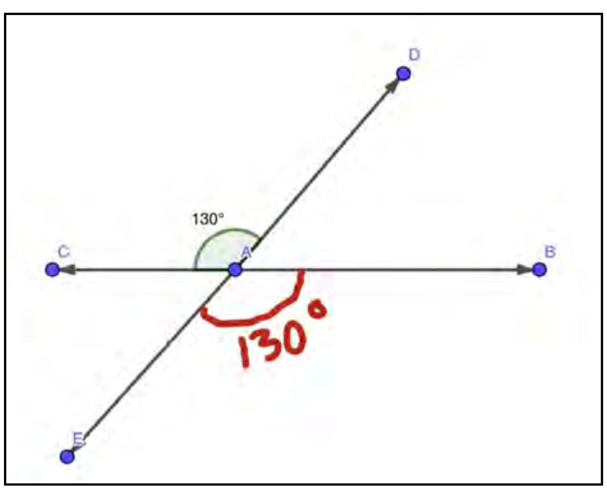
If ∠*CAD* = 130°, what is the value of ∠*BAE*?



Let's Think:

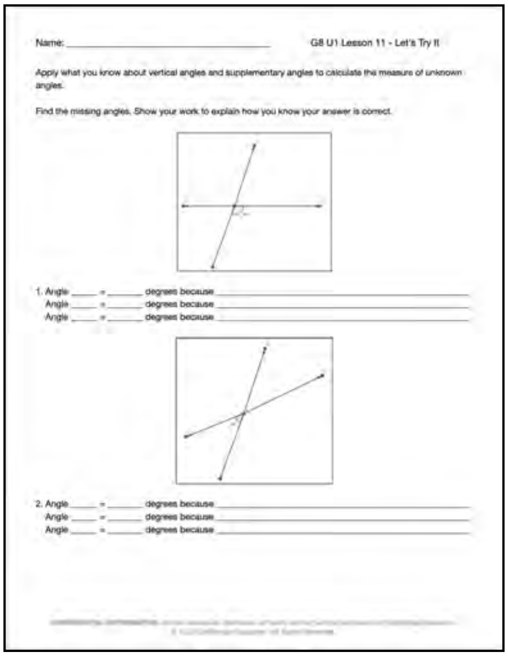
How do vertical angles and supplementary angles work together to help us calculate missing angles of a pair of intersecting lines?

What are the values of the missing angles?



Let's Try It:

Let's practice finding the missing angles of a pair of intersecting lines.


Name: ______________________________ G8 U1 Lesson 11 - Let's Try It

Apply what you know about vertical angles and supplementary angles to calculate the measure of unknown angles.

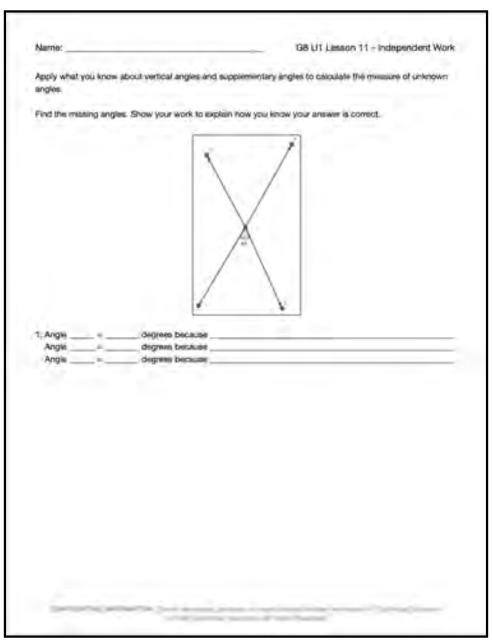
Find the missing angles. Show your work to explain how you know your answer is correct.

1. Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________

2. Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________

On your Own:

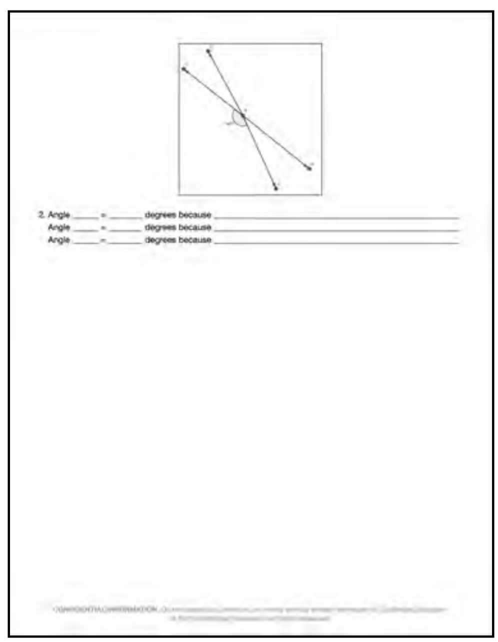
Now it's time to find the value of the missing angles of a pair of intersecting lines on your own.


Name: ______________________________ G8 U1 Lesson 11 – Independent Work

Apply what you know about vertical angles and supplementary angles to calculate the measure of unknown angles.

Find the missing angles. Show your work to explain how you know your answer is correct.

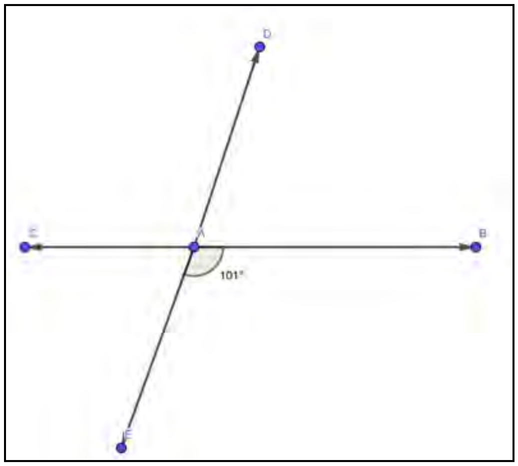
1. Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________


2. Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________
   Angle _____ = _______ degrees because ______________________

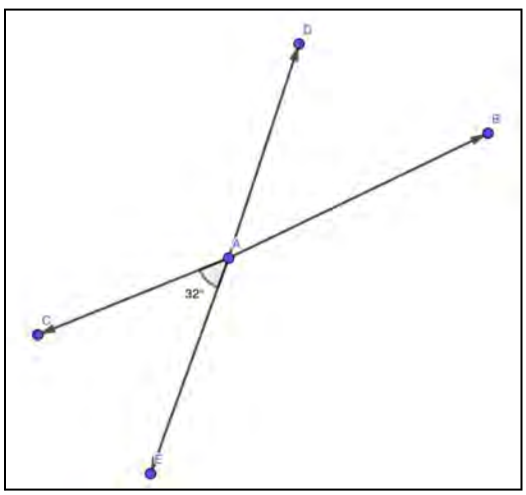
Name: ____ G8 U1 Lesson 11 - Let's Try It

Apply what you know about vertical angles and supplementary angles to calculate the measure of unknown angles.

Find the missing angles. Show your work to explain how you know your answer is correct.



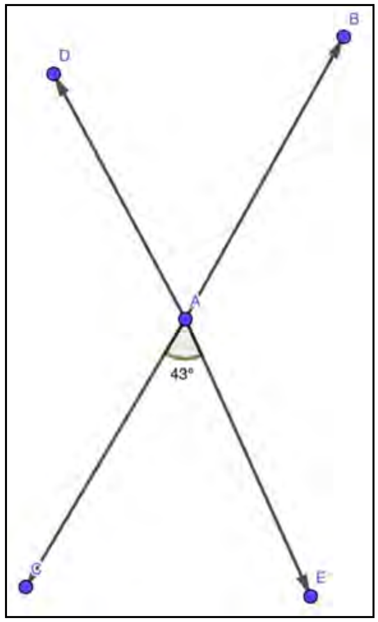
1. Angle ______ = ________ degrees because ______________________________
   Angle ______ = ________ degrees because ______________________________
   Angle ______ = ________ degrees because ______________________________



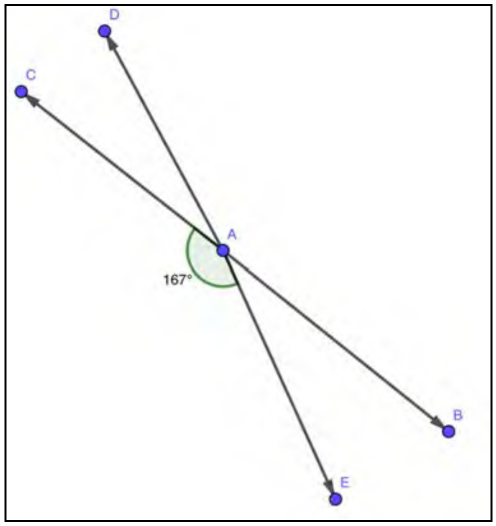
2. Angle ______ = ________ degrees because ______________________________
   Angle ______ = ________ degrees because ______________________________
   Angle ______ = ________ degrees because ______________________________

Apply what you know about vertical angles and supplementary angles to calculate the measure of unknown angles.

Find the missing angles. Show your work to explain how you know your answer is correct.



1. Angle ______ = ________ degrees because ____
   Angle ______ = ________ degrees because ____
   Angle ______ = ________ degrees because ____



2. Angle ______ = ________ degrees because ____

Angle ______ = ________ degrees because ____

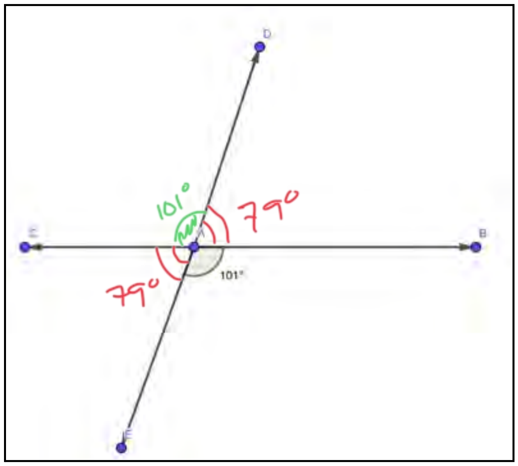
Angle ______ = ________ degrees because ____

Name: Answer Key

Apply what you know about vertical angles and supplementary angles to calculate the measure of unknown angles.

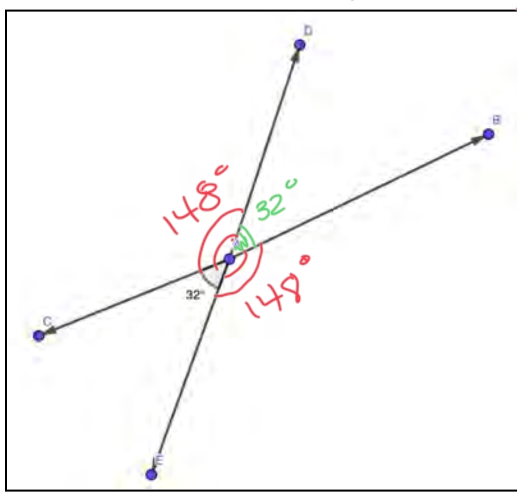
Find the missing angles. Show your work to explain how you know your answer is correct.

180
−101
79



1. Angle CAD = 101 degrees because vertical angle theorem
   Angle DAB = 79 degrees because it is supplementary to ∠CAD OR ∠BAE.
   Angle EAC = 79 degrees because it is vertical to ∠DAB OR it is supplementary to ∠CAD OR ∠BAE

180
− 32
148



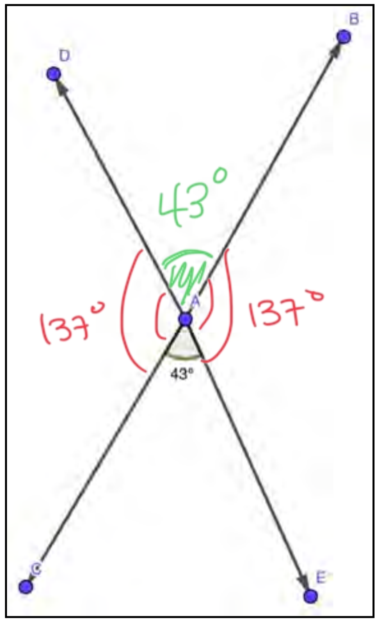
2. Angle DAB = 32 degrees because it is vertical to ∠EAC.
   Angle CAD = 148 degrees because it is supplementary to ∠EAC OR ∠DAB.
   Angle BAE = 148 degrees because it is vertical to ∠CAD OR it is supplementary to ∠EAC OR ∠DAB.

* Students only need one explanation per line. The "OR" statements are for the tutors.

Name: Answer Key

Apply what you know about vertical angles and supplementary angles to calculate the measure of unknown angles.

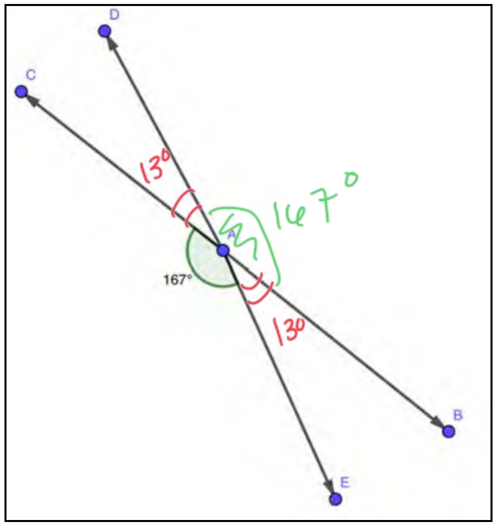
Find the missing angles. Show your work to explain how you know your answer is correct.



180
− 43
137

1. Angle DAB = 43 degrees because it is vertical to ∠EAC.
Angle BAE = 137 degrees because it is supplementary to ∠DAB OR ∠EAC.
Angle CAD = 137 degrees because it is vertical to ∠BAE OR supplementary to ∠DAB OR ∠EAC.

*Students only need one explanation per line. The "or" statements are for the tutors.



180
-167
13

2. Angle DAB = 167 degrees because it is vertical to ∠EAC.
Angle BAE = 13 degrees because it is supplementary to ∠EAC OR ∠DAB.
Angle CAD = 13 degrees because it is vertical to ∠BAE OR supplementary to ∠DAB OR ∠EAC.

G8 U1 Lesson 12
Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

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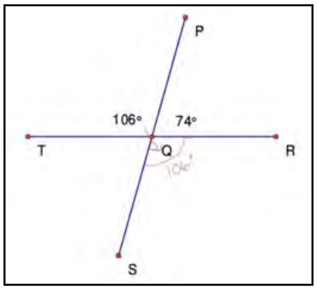
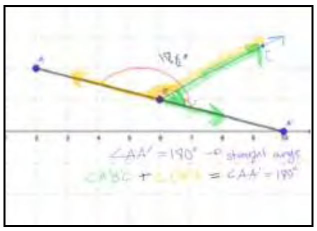
G8 U1 Lesson 12 - Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 4): Today we will calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.



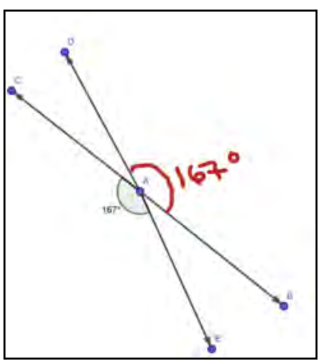
Recall that vertical angles are nonadjacent angles created by the intersection of two lines - vertical angles are congruent.



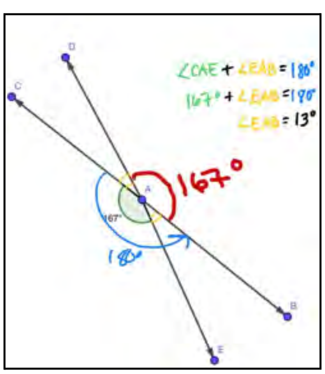
Supplementary angles are adjacent angles on a straight angle - the sum of supplementary angles is 180 degrees.

Let's Talk (Slide 5): Let's consider a pair of intersecting lines where only one of its angles is known. How will we find the other angles? Possible Students Answers, Key Points:

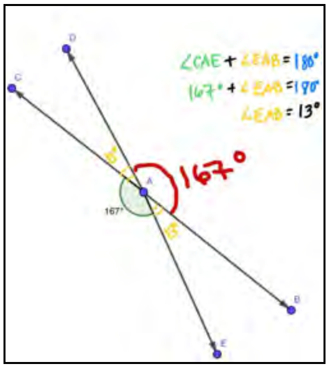
- Use the Vertical Angle Theorem to find the angle across from the given angle.
- Use the definition of supplementary angles to calculate the sum of adjacent angles on a straight line.



The given $\angle CAE$ is 167°. Since *$\angle DAB$ is vertical to $\angle CAE$ and vertical angles are congruent,* $\angle DAB$ = 167°.

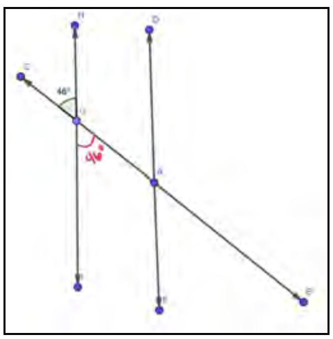


Now that we know the value of two angles, we'll use the definition of supplementary angles to find the value of an adjacent angle to one of the known angles. $\angle CAE + \angle EAB = 180°$. Substitute the known values to find the missing angle. *(Mark the angles and write the supplementary angles formula.)* $\angle EAB = 13°$.

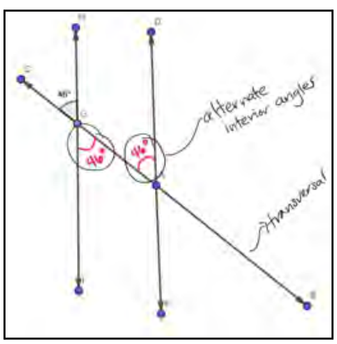


Now that we know ∠*EAB* = 13°, ∠*DAC* = 13° also because they are vertical angles.

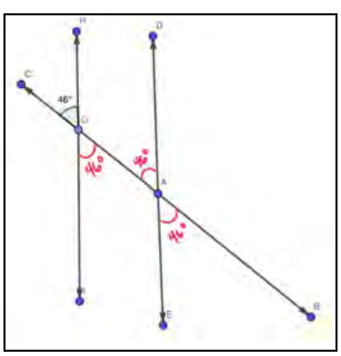
Let's Think (Slide 6): Now let's consider what happens when we intersect a pair of parallel lines with a third line. Can we use what we know about properties of parallel lines, vertical angles, and supplementary angles to find the missing values?



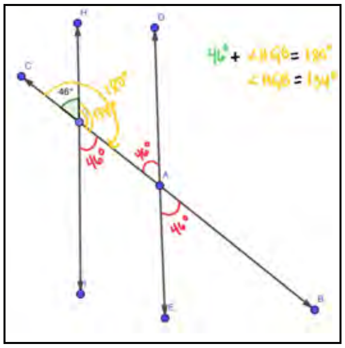
First, notice that ∠*CGH* and ∠*BGI* are vertical angles which means they are congruent. *Mark the unknown angle and write its value.)* are adjacent and on a straight angle. *(Mark one of the missing angles, ∠DAB, and the straight angle.)* Now we can use the definition of supplementary angles to find the missing value.



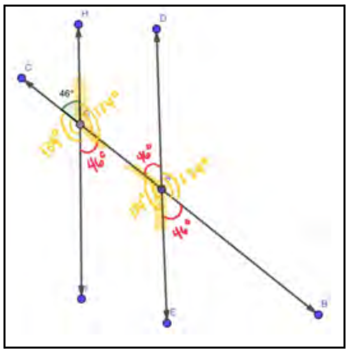
We know that the parallel lines share the same properties. So ∠*CAD* = 46° because it is congruent to ∠*CGH*, the same angle created by the line intersecting the parallel lines. *(Mark the angles.)* The intersecting line is known as a transversal. Since ∠*BGI* is vertical to ∠*CGH* and thus congruent to ∠*CGH*, ∠*BGI* is also congruent to ∠*CAD*. ∠*BGI* and ∠*BGI* are known as alternate interior angles because of their location between the parallel lines and on the transversal. Alternate interior angles are congruent. (*Label the new vocabulary.)*



Now we have enough information to find the other angles using vertical angles, supplementary angles, and alternate interior angles. ∠*CAD* is vertical to ∠*BAE* so ∠*BAE* = 46° by the Vertical Angles Theorem.



Finally, let's consider what we know about supplementary angles to find ∠*HGB* which is supplementary to ∠*CGH* because they are adjacent angles on a straight line. 180 minus 46 is 134 so ∠*HGB* = 134°.



Now we can use alternate interior angles to find ∠*EAG*. ∠*EAG* is congruent to ∠*HGB* because they are alternate interior angles so ∠*EAG* = 134°. *(Highlight or outline the alternate interior angles and write the value of ∠EAG before using vertical angles to fill in the remaining values.)* We can use vertical angles to determine that the remaining angles also have a measure of 134°.

Let's Try it (Slides 7-8): Let's work on finding unknown angle measurements when a pair of parallel lines are cut by a transversal. We will work on this page together. Remember that a transversal is a line that intersects a pair of parallel lines and alternate interior angles are created by the transversal, on alternate sides of the transversal but between the parallel lines. They cannot be adjacent or they would be supplementary and alternate interior angles are congruent.

WARM WELCOME

Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

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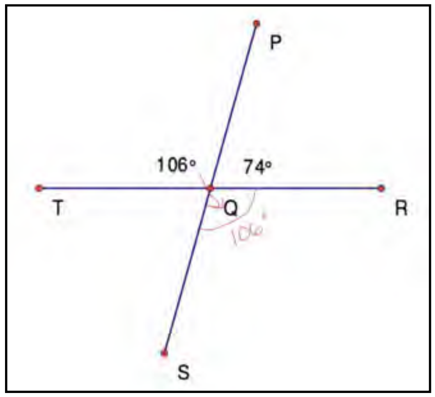
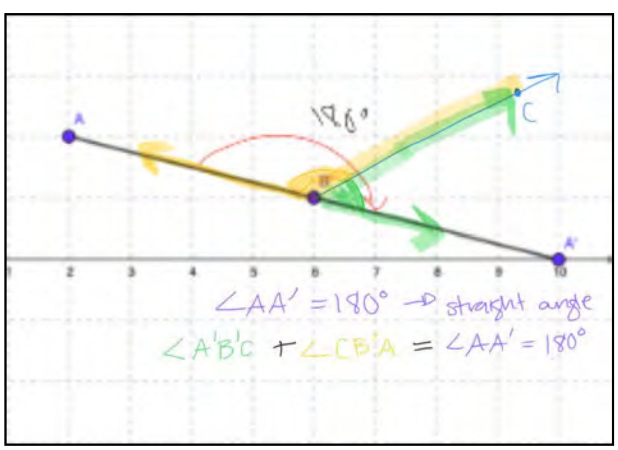
Let's Review:

Vertical angles are congruent.



Let's Review:

The sum of supplementary angles is 180 degrees.



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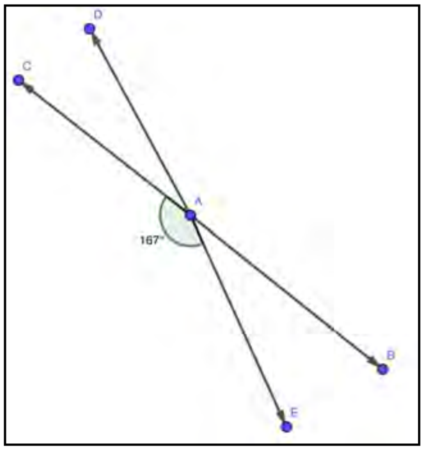
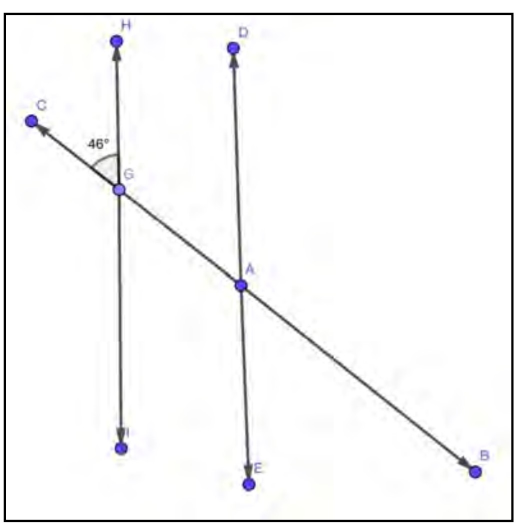
Let's Talk: How do vertical angles and supplementary angles work together to help us calculate missing angles of a pair of intersecting lines?

Calculate the value of the missing angles.



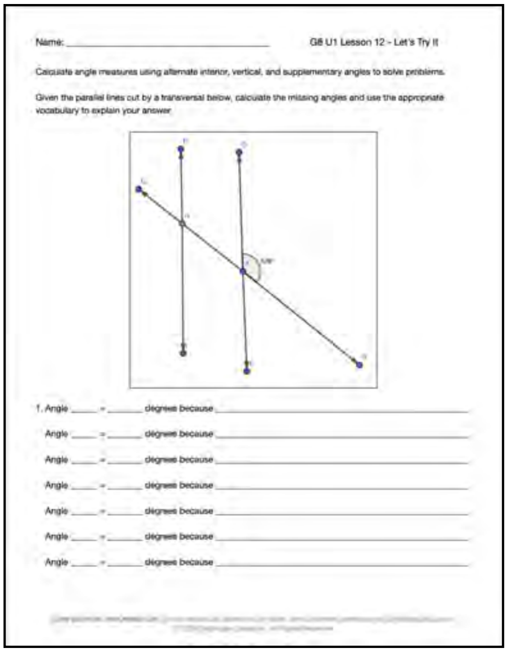
Let's Think: How do we use vertical angles and supplementary angles to calculate the unknown angles when a pair of parallel lines are cut by a transversal?

What are the values of the unknown angles?



Let's Try It:

Let's practice finding the missing angles of a pair of parallel lines cut by a transversal.



Name: ______________________ G8 U1 Lesson 12 - Let's Try It

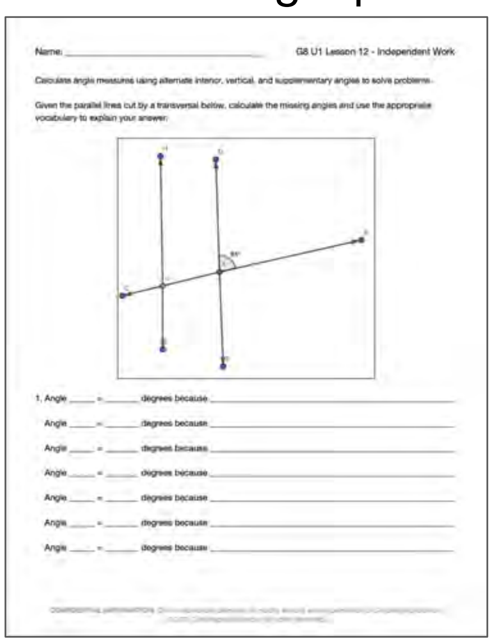
Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

Given the parallel lines cut by a transversal below, calculate the missing angles and use the appropriate vocabulary to explain your answer.

1. Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________

On your Own:

Now it's time to calculate the missing values of angles created by a transversal cutting a pair of parallel lines on your own.



Name: ______________________ G8 U1 Lesson 12 - Independent Work

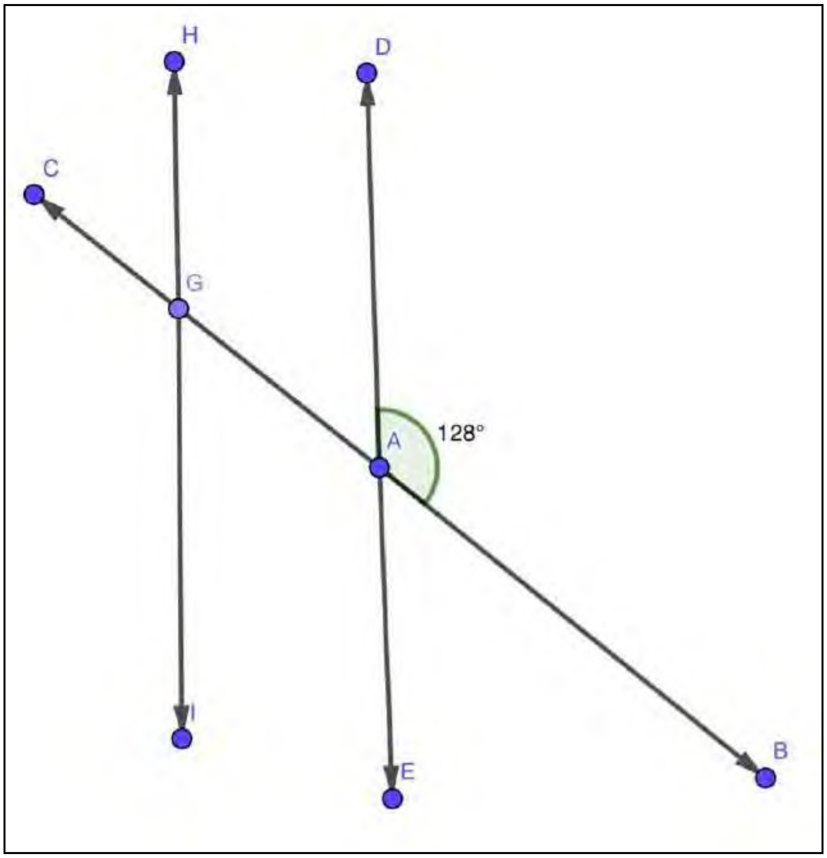
Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

Given the parallel lines cut by a transversal below, calculate the missing angles and use the appropriate vocabulary to explain your answer.

1. Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________
Angle ____ = ____ degrees because ____________________

Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

Given the parallel lines cut by a transversal below, calculate the missing angles and use the appropriate vocabulary to explain your answer.



1. Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

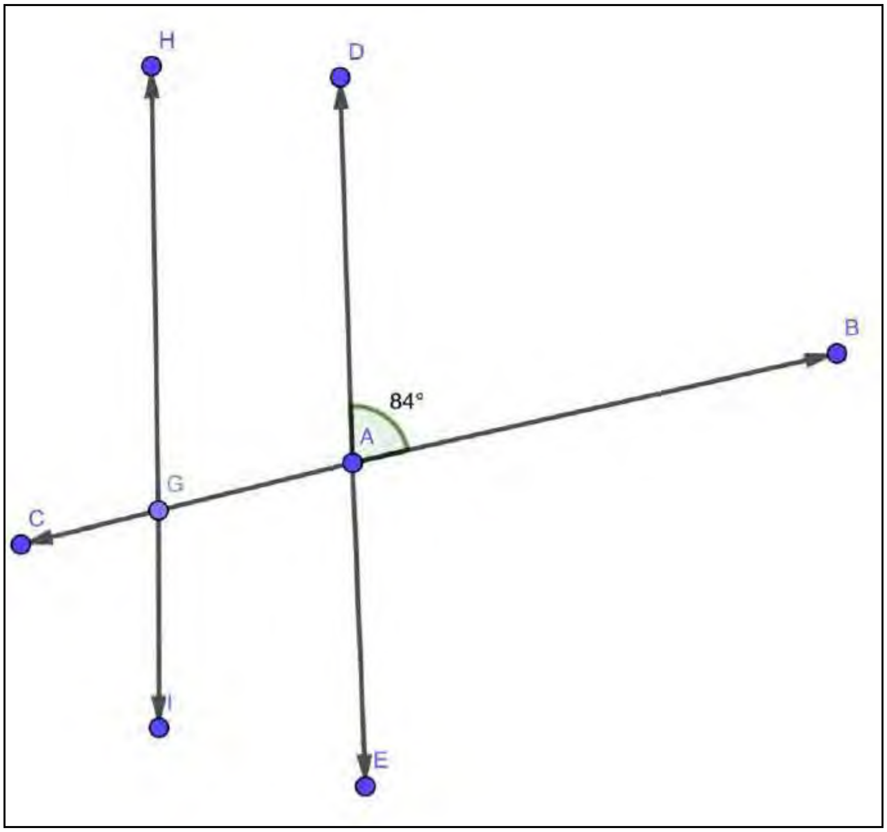
   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

Given the parallel lines cut by a transversal below, calculate the missing angles and use the appropriate vocabulary to explain your answer.



1. Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

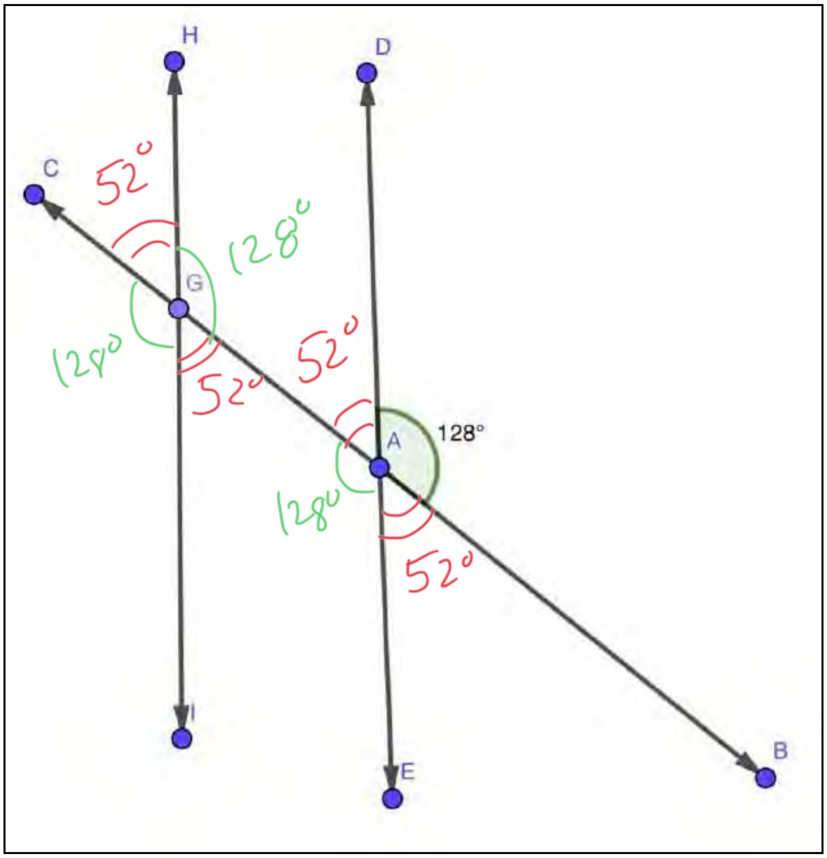
   Angle ______ = ________ degrees because ____

   Angle ______ = ________ degrees because ____

Name: ______________________________ G8 U1 Lesson 12 - Let's Try It

Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

Given the parallel lines cut by a transversal below, calculate the missing angles and use the appropriate vocabulary to explain your answer.



180
− 128
52

1. Angle GAE = 128 degrees because vertical angle theorem

Angle BAE = 52 degrees because supplementary to ∠DAB or ∠GAE

Angle CAD = 52 degrees because verticle angle theorem

Angle BGI = 52 degrees because alternate interior to ∠CAD

Angle HGB = 128 degrees because alternate interior to ∠EAC

Angle CGI = 128 degrees because vertical to ∠HGB

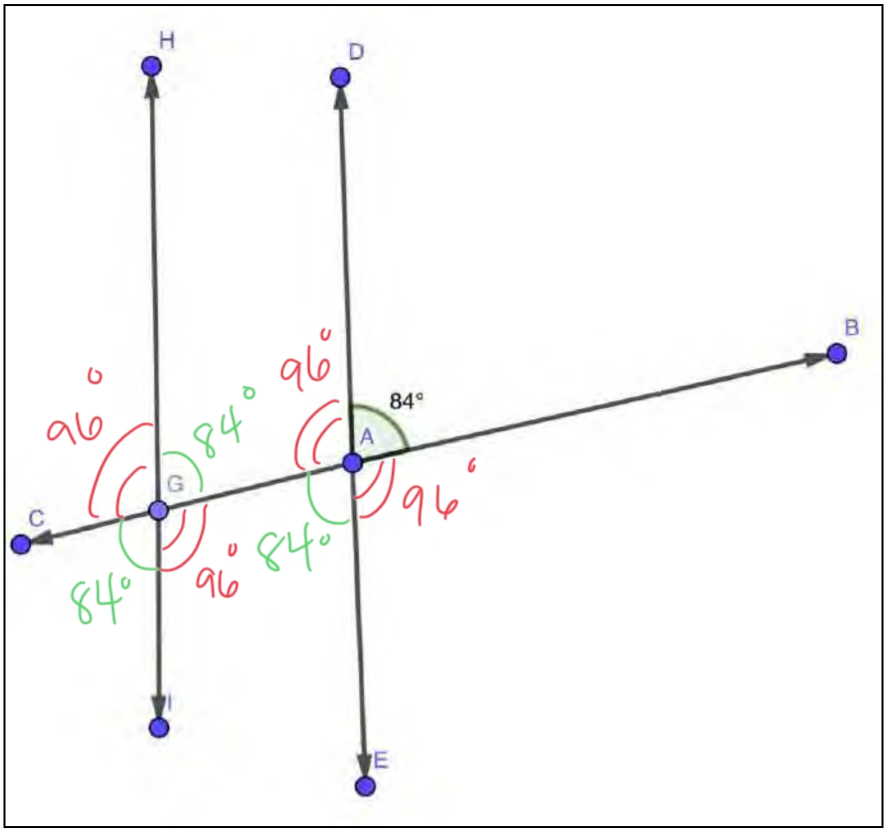
Angle CGH = 52 degrees because vertical to ∠BGI

* There are many explanations that work.

Name: Answer Key

Calculate angle measures using alternate interior, vertical, and supplementary angles to solve problems.

Given the parallel lines cut by a transversal below, calculate the missing angles and use the appropriate vocabulary to explain your answer.



180
−84
96

1. Angle BAE = 96 degrees because supplementary to given ∡

Angle DAC = 96 degrees because vertical to ∠ BAE

Angle EAC = 84 degrees because vertical to given ∡

Angle HGB = 84 degrees because alternate interior to ∠EAC

Angle BGI = 96 degrees because alternate interior to ∠DAC

Angle CGI = 84 degrees because vertical to ∠ HGB

Angle HGC = 96 degrees because vertical to ∠BGI

* There are many possible explanations.

G8 U1 Lesson 13
Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

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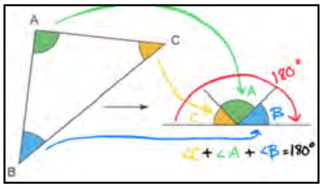
G8 U1 Lesson 13 - Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

Warm Welcome (Slide 1): Tutor Choice

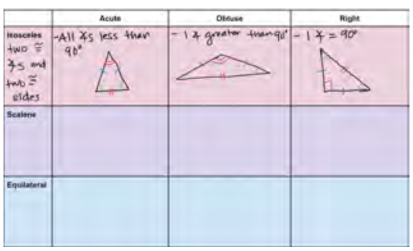
Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will calculate the measures of missing angles and exterior angles of triangles. First, let's recall that the sum of the interior angles of a triangle is 180°. In your classes you might have proved this to be true using what you know about a straight angle.



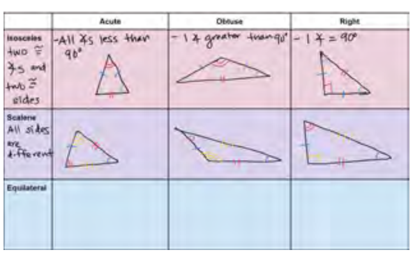
Notice that the angles of triangle *ABC* fit together like puzzle pieces on a straight angle. *(Draw arrows to show how the interior angles moved to form the straight angle.)* Even though we don't know the value of any of the triangle's interior angles, because together they form the straight angle *(Draw an arc to show the straight angle and label its measure.),* we know that the sum of the triangle's angles must be 180 degrees. *(Write a formula beneath the straight angle to show that the three angles have a sum of 180 degrees.)* Thus, the Triangle Sum Theorem.

Let's Talk (Slide 4): Let's apply this to finding the missing angles in a triangle. But first, let's talk about the properties of triangles which will help us to make decisions about the angles of a triangle. Tell me what you know about different types of triangles. Possible Students Answers, Key Points:

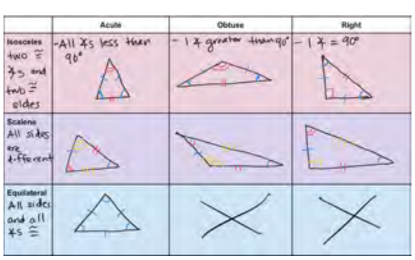
- One triangle is a right triangle because it has a 90 degree angle.
- One type of triangle has two sides and two angles that are always congruent.
- One type of triangle has everything different.



First, let's discuss isosceles triangles. Isosceles triangles have two congruent sides and two congruent angles. *(Write this in this in the space below the term "isosceles.")* They can be classified as acute (having all interior angles that measure less than 90 degrees each), obtuse (having one angle with a measure greater than 90 degrees), and right (having one angle measure exactly 90 degrees). *(Write and draw images as you describe each one.)*



Next, we have scalene triangles. Scalene triangles have all different side lengths which means all three interior angles are also different. Like an isosceles triangle, scalene triangles can be acute, obtuse, or right.

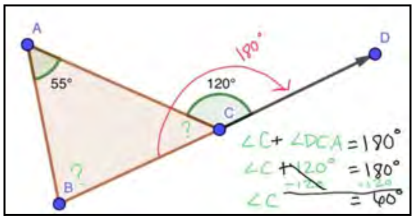


Finally, there is the equilateral triangle. Since the sum of the interior angles of a triangle is 180 degrees and an equilateral triangle has all parts equal, sides and angles, we know that all angles in an equilateral triangle will always be 60 degrees because 60 + 60 + 60 = 180. What does that tell us about classifying equilateral triangles? Possible Students Answers, Key Points:

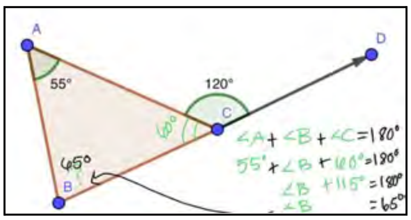
- All of the sides are all equal.
- They can't be obtuse since none of the angles are greater than 90 degrees.
- They can't be right because none of the angles equal 90 degrees.

That's right. This means that an equilateral triangle can be acute, but never right or obtuse.

Let's Think (Slide 5): Now that we recall all types of triangles, let's use their classifications and the Triangle Sum Theorem to find some missing angles.



Let's always start with what we know or the given information. I have one interior angle but I don't know anything about the side lengths so I can't be sure if any of the angles are congruent to each other. *(Write a question mark in the angle spaces where we don't know their measure.)* While that's tricky, we do know the measure of the exterior angle *DCA*. Since the exterior angle is on a straight angle with interior angle *C*, we can use what we know about supplementary angles to find the value of angle *C*. *(Draw an arc and label it to show 180 degrees.)* ∠*C* + ∠*DCA* = 180°, so ∠*C* = 60°. *(Write the equation and solve for angle C.)*



Now that we know the value of interior ∠*C*, we can use the other given angle, ∠*A*, to calculate the value of interior ∠*B*. Remember, by the Triangle Sum Theorem, ∠*A* + ∠*B* + ∠*C* = 180°. So ∠*B* = 65°.

Let's Try it (Slides 7-8): Let's work on calculating the missing angle measures interior and/or exterior to a given triangle. Remember, the Triangle Sum Theorem tells us that the sum of the interior angles of a triangle equals 180 degrees. If you only know one interior angle, consider what other information you may have that can help you to find the missing angles.

WARM WELCOME

Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

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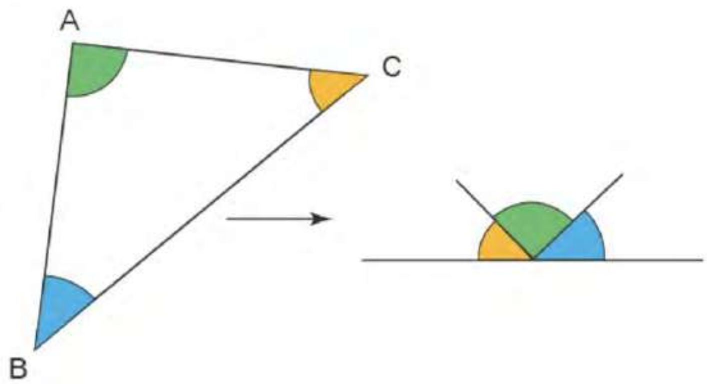
Let's Review:

The Triangle Sum Theorem tells us that the sum of all interior angles in a triangle is 180°.



Let's Talk:

How do we find the unknown angles, interior and exterior, to triangles?

Let's classify triangles and define their properties.

	Acute	Obtuse	Right
Isosceles			
Scalene			
Equilateral			

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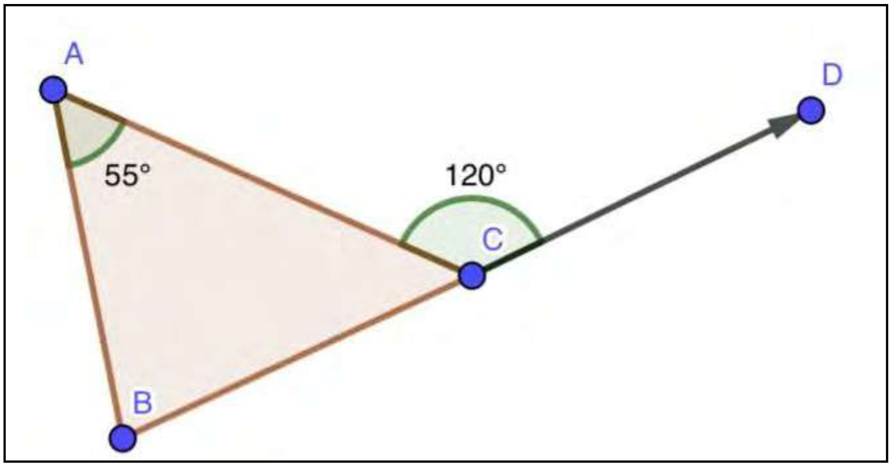
Let's Think:

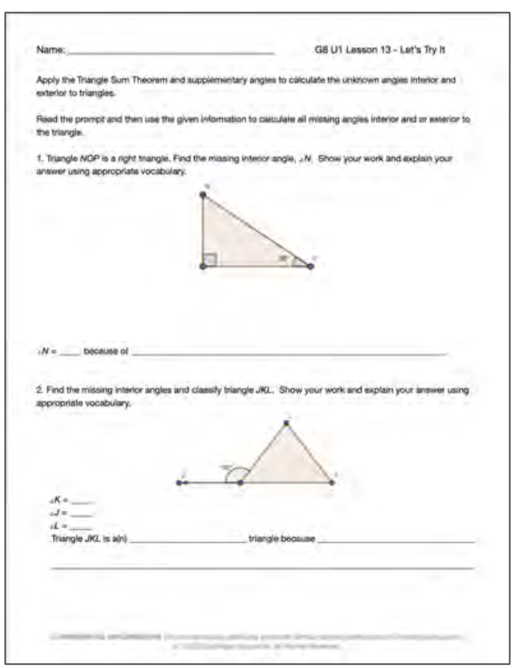
How do we find the unknown angles, interior and exterior, to triangles?

Calculate the missing angles.



Let's Try It:

Let's practice calculating the interior and exterior angles of triangles.



Name: ____________________ G8 U1 Lesson 13 - Let's Try It

Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

Read the prompt and then use the given information to calculate all missing angles interior and or exterior to the triangle.

1. Triangle *NOP* is a right triangle. Find the missing interior angle, ∠*N*. Show your work and explain your answer using appropriate vocabulary.

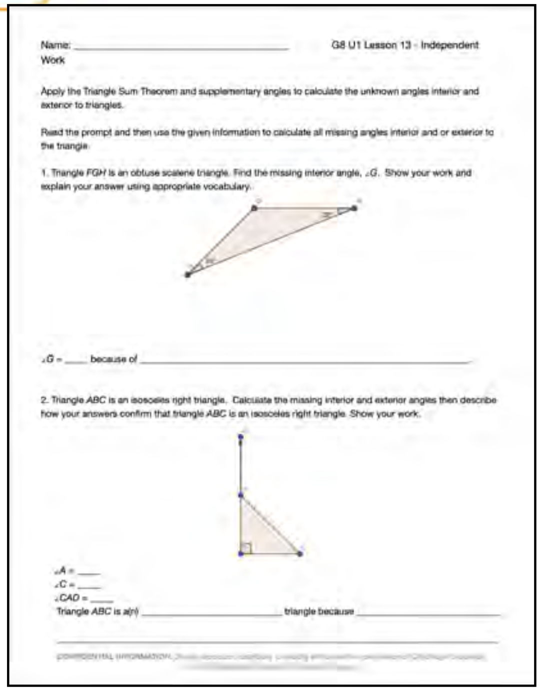
∠*N* = ____ because of ____________________

2. Find the missing interior angles and classify triangle *JKL*. Show your work and explain your answer using appropriate vocabulary.

∠*K* = ____
∠*J* = ____
∠*L* = ____
Triangle *JKL* is a(n) ____________ triangle because ____________

On your Own:

Now it's time to calculate interior and/or exterior angles of triangles on your own.



Name: ______________________________ G8 U1 Lesson 13 - Independent Work

Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

Read the prompt and then use the given information to calculate all missing angles interior and or exterior to the triangle.

1. Triangle *FGH* is an obtuse scalene triangle. Find the missing interior angle, ∠*G*. Show your work and explain your answer using appropriate vocabulary.

∠*G* = ____ because of ____

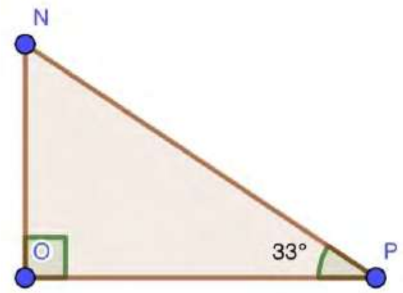
2. Triangle *ABC* is an isosceles right triangle. Calculate the missing interior and exterior angles then describe how your answers confirm that triangle *ABC* is an isosceles right triangle. Show your work.

∠*A* = ____
∠*C* = ____
∠*CAD* = ____
Triangle *ABC* is a(n) ____________________ triangle because ____________________

Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

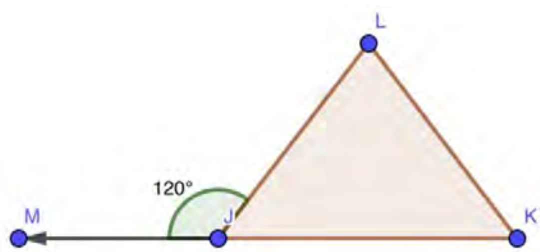
Read the prompt and then use the given information to calculate all missing angles interior and or exterior to the triangle.

1. Triangle *NOP* is a right triangle. Find the missing interior angle, ∠*N*. Show your work and explain your answer using appropriate vocabulary.



∠*N* = _____ because of ____.

2. Interior angle *K* is 60 degrees. Find the missing interior angles and classify triangle *JKL*. Show your work and explain your answer using appropriate vocabulary.



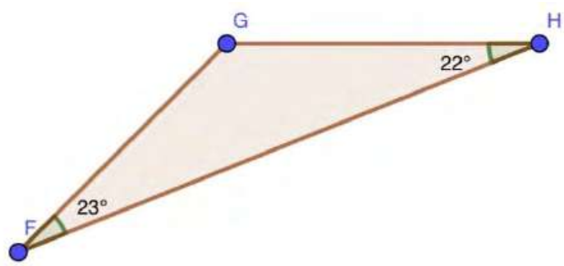
∠*J* = _____
∠*L* = _____
Triangle *JKL* is a(n) _________________________ triangle because __________________________________
____

Name: ____ G8 U1 Lesson 13 - Independent Work

Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

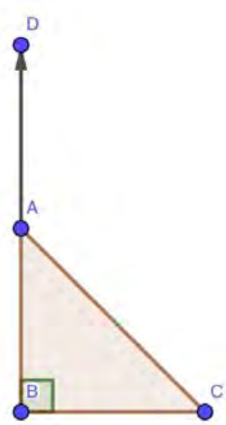
Read the prompt and then use the given information to calculate all missing angles interior and or exterior to the triangle.

1. Triangle *FGH* is an obtuse scalene triangle. Find the missing interior angle, $\angle G$. Show your work and explain your answer using appropriate vocabulary.



$\angle G$ = _____ because of ____.

2. Triangle *ABC* is an isosceles right triangle. Calculate the missing interior and exterior angles then describe how your answers confirm that triangle *ABC* is an isosceles right triangle. Show your work.



$\angle A$ = _____
$\angle C$ = _____
$\angle CAD$ = _____
Triangle *ABC* is a(n) ______________________________ triangle because _______________________________

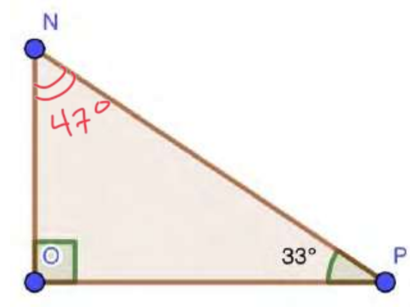
Name: Answer Key

Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

Read the prompt and then use the given information to calculate all missing angles interior and or exterior to the triangle.

1. Triangle *NOP* is a right triangle. Find the missing interior angle, $\angle N$. Show your work and explain your answer using appropriate vocabulary.

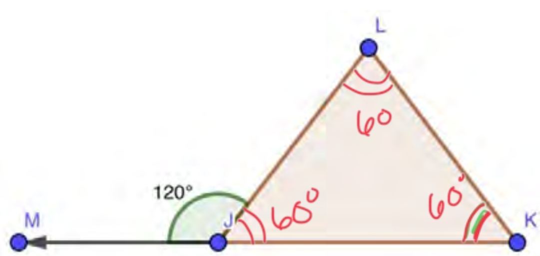
$\angle O + \angle P + \angle N = 180^\circ$
$90^\circ + 33^\circ + \angle N = 180^\circ$
$123^\circ + \angle N = 180^\circ$
$\angle N = 47^\circ$



$\angle N$ = 47° because of Δ sum theorem.

2. Find the missing interior angles and classify triangle *JKL*. Show your work and explain your answer using appropriate vocabulary.

$\angle MJL + \angle J = 180$
$120 + \angle J = 180$
$\angle J = 60^\circ$



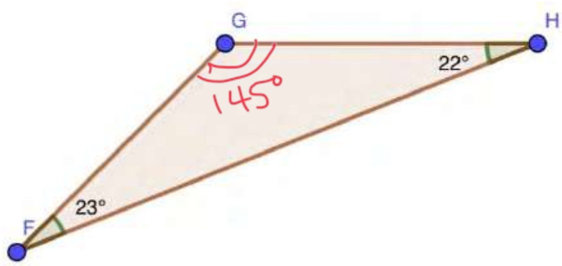
$\angle K$ = 60°
$\angle J$ = 60°
$\angle L$ = 60°
Triangle *JKL* is a(n) equilateral triangle because all ∡s are congruent.

Name: Answer Key G8 U1 Lesson 13 - Independent Work

Apply the Triangle Sum Theorem and supplementary angles to calculate the unknown angles interior and exterior to triangles.

Read the prompt and then use the given information to calculate all missing angles interior and or exterior to the triangle.

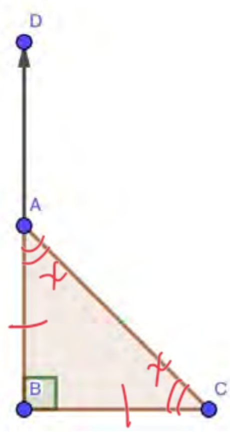
1. Triangle *FGH* is an obtuse scalene triangle. Find the missing interior angle, $\angle G$. Show your work and explain your answer using appropriate vocabulary.



$\angle F + \angle G + \angle H = 180$
$23 + \angle G + 22 = 180$
$\angle G + 45 = 180$
$\angle G = 145°$

$\angle G$ = 145° because of △ sum theorem.

2. Triangle *ABC* is an isosceles right triangle. Calculate the missing interior and exterior angles then describe how your answers confirm that triangle *ABC* is an isosceles right triangle. Show your work.



$\angle A + \angle B + \angle C = 180$
$x + 90 + x = 180$
$90 + 2x = 180$
$2x = 90$
$x = 45$

$\angle A$ = 45°
$\angle C$ = 45°
$\angle CAD$ = 145 $180 - 45 = 145$

Triangle *ABC* is a(n) isosceles right triangle because it has 1 right ∠ and two congruent ∠s.

Eighth Grade Math Lesson Materials

CITYTUTOR DC

G8 Unit 2:

Dilations, Similarity, and Introducing Slope

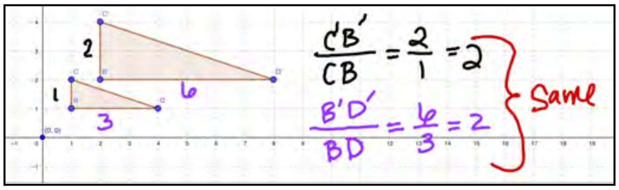
G8 U2 Lesson 1
Dilate polygons given a scale factor and the origin as the center of dilation.

CITYTUTOR DC

G8 U2 Lesson 1 - Dilate polygons given a scale factor and the origin as the center of dilation.

Warm Welcome (Slide 1): Tutor Choice

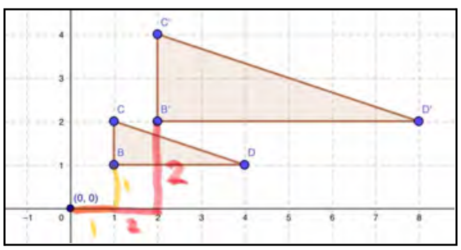
Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will dilate polygons given a scale factor and the origin as the center of dilation. Before that, let's think about 7th grade math. When you were 7th graders, you completed a unit on scale factors. Dilations combine properties of scale factors and some properties of rigid transformations.



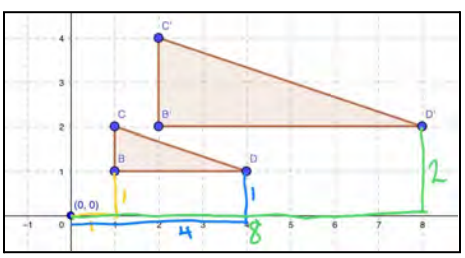
In particular, if an image is a dilation of an original polygon, then all of its side lengths will be proportional to the original polygon by the same factor.

Let's Talk (Slide 4): In addition, the distances from the center of dilation, in this case the origin of the grid (0,0), will be proportional based on the same scale factor. We already verified that the scale factor is 2. If the distance from the origin to point *B* is 1 unit to the right and 1 unit up, what should be the distances from the origin to the image of *B*, *B*'? Possible Students Answers, Key Points:

- The scale factor is 2 so the distances should be doubled in both directions.



You're all correct, since the distances have to use the same scale factor as the side lengths, we can multiply all of the distances from the origin to point *B* by 2 and then checkon the grid to see if the distances from the origin to point *B'*, the image of *B*, is double the original. *(Draw and label the distances.)* In this case, the answer is yes.

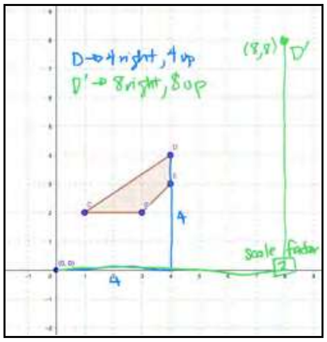


Let's go ahead and verify this for all of the points of the original polygon and image. How far away is D from the origin and then how far must its image, *D*', be from the origin? Possible Students Answers, Key Points:

- *D* is 4 units to the right and 1 unit up.
- *D*' should be 8 units to the right and 2 units up since the scale factor is 2.

You've got it! Trust me that the same is true for *C* and let's work on performing dilations given the center of dilation as the origin and some scale factor.

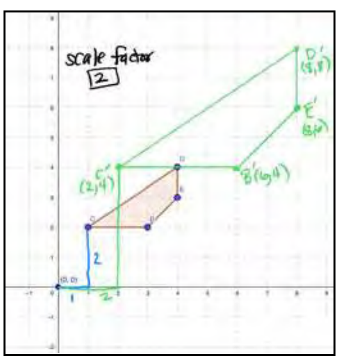
Let's Think (Slide 5): You may remember from 7th grade that anytime a scale factor is greater than 1, something is increasing in size. However, if the scale factor is less than 1, something is decreasing in size. Think about this in terms of your favorite dessert. If I multiply the size of your dessert by 1, nothing changes. You just get another of the exact same dessert. If I multiply the size of your desert by 2 or 3, your dessert will be double or triple its original size. However, if you take a factor less than 1, let's say ⅓, then you'll only get ⅓ of your original dessert, a small fraction of the dessert. Try to remember this dessert analogy so you can judge the reasonableness of your dilations in the future.



Let's start with a simple polygon *BCDE*. Let's dilate this polygon using a scale factor of 2 and the origin, (0,0), as the center of dilation. The simplest way to do this is to first identify the distance from the origin to each point on the original polygon. *(outline the number of units it takes to go right to D and up to D.)* Point *D* is 4 units to the right and 4 units up from the origin. Now that we have that, we'll use the scale factor of 2 and multiply both distances by 2 to get the image *D'* after a dilation by a scale factor of 2. *(Draw the distances that show double the original distances. Mark and label D' on the grid.)* *D*' will go to (8,8) because that is 8 units to the right and 8 units up.

Now, let's apply the same methods to plot the remaining points. Next, we'll find the image of *E*. Since *E* is 4 units to the right of the origin and 3 units up from the origin, using a scale factor of 2, *E*' should be double those distances. That is, *E*' will be 8 units to the right and 6 units up. It's new coordinates will be (8,6).

Point *B* is 3 units to the right of the origin and 2 units up. So its image, *B'*, will be double those distances: 6 units to the right and 4 units up. The new coordinates will be (6, 4).



Finally, point *C* is 1 unit to the right and 2 units up from the origin. Its image, *C*', will be double those distances because of the scale factor of 2. *C*' will be 2 units right and 4 units up from the original and have coordinates of (2,4).

The final image of the polygon is *B'C'D'E'*, a dilation with a center at the origin and a scale factor of 2.

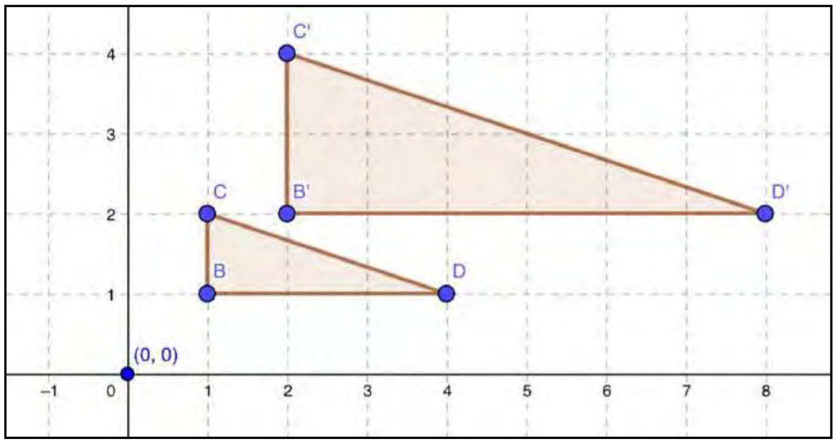
Let's Try it (Slides 7-8): Let's work on dilating polygons given a scale factor and the center at the origin. We will work on this together. Remember, a dilation with a scale factor greater than 1 gets bigger and a dilation with a scale factor less than 1 gets smaller. All of the distances and side lengths should be proportional to each other by the same scale factor.

WARM WELCOME

Dilate polygons given a scale factor and the origin as the center of dilation.

Let's Review:

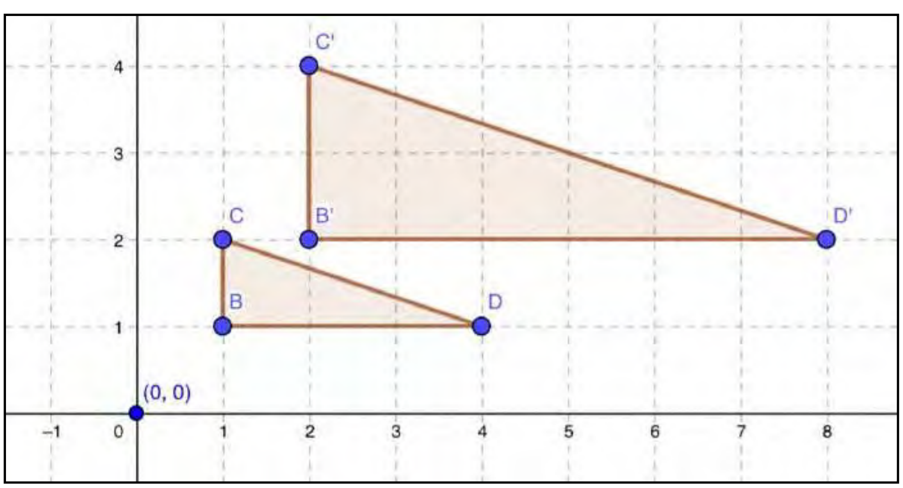
If an image is a dilation of an original polygon, all of its side lengths will be proportional to the original polygon by the same factor.



Let's Talk:

How do you dilate polygons given a scale factor and the center at the origin?

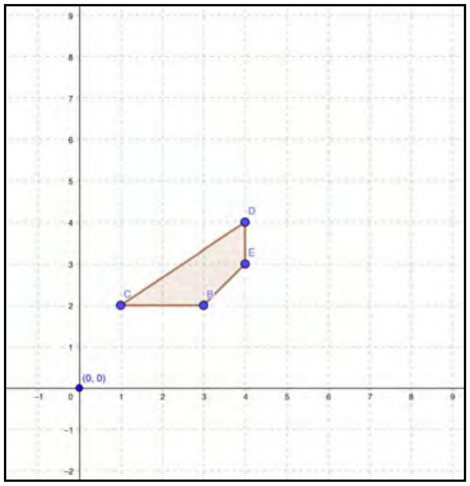
Verify that the distances from the origin to the vertices of the original polygon are proportional to the vertices of the image by 2.



Let's Think:

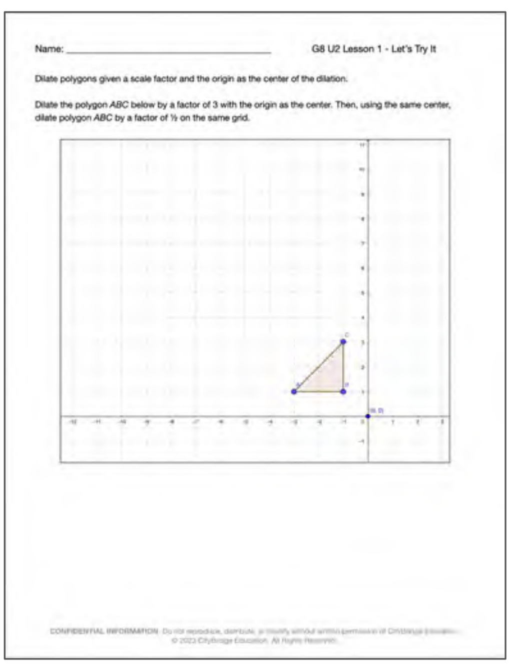
How do you dilate polygons given a scale factor and the center at the origin?

Perform a dilation with a scale factor of 2 and the center of dilation as the origin.



Let's Try It:

Let's practice dilating polygons given a scale factor and the center at the origin.



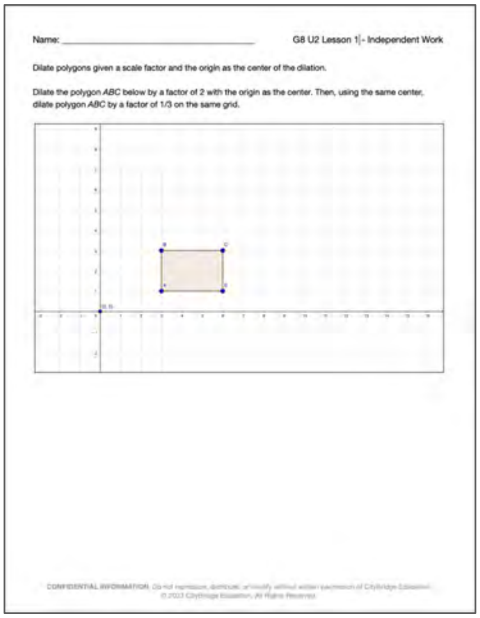
Name: ______________________ G8 U2 Lesson 1 - Let's Try It

Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 3 with the origin as the center. Then, using the same center, dilate polygon *ABC* by a factor of ½ on the same grid.

On your Own:

Now it's time to dilate polygons given a scale factor and the center at the origin on your own.

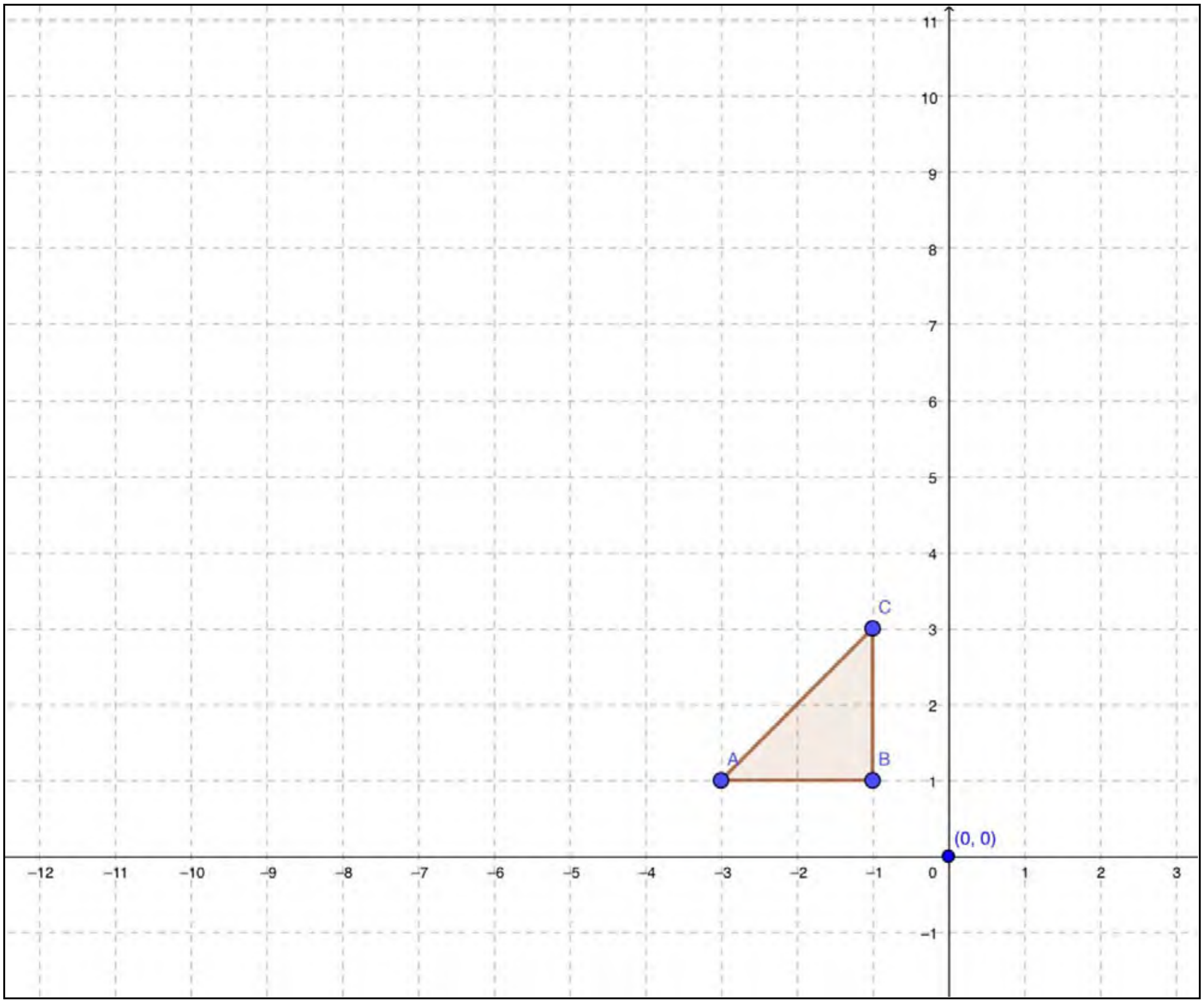

Name: ______________________________ G8 U2 Lesson 1 - Independent Work

Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 2 with the origin as the center. Then, using the same center, dilate polygon *ABC* by a factor of 1/3 on the same grid.

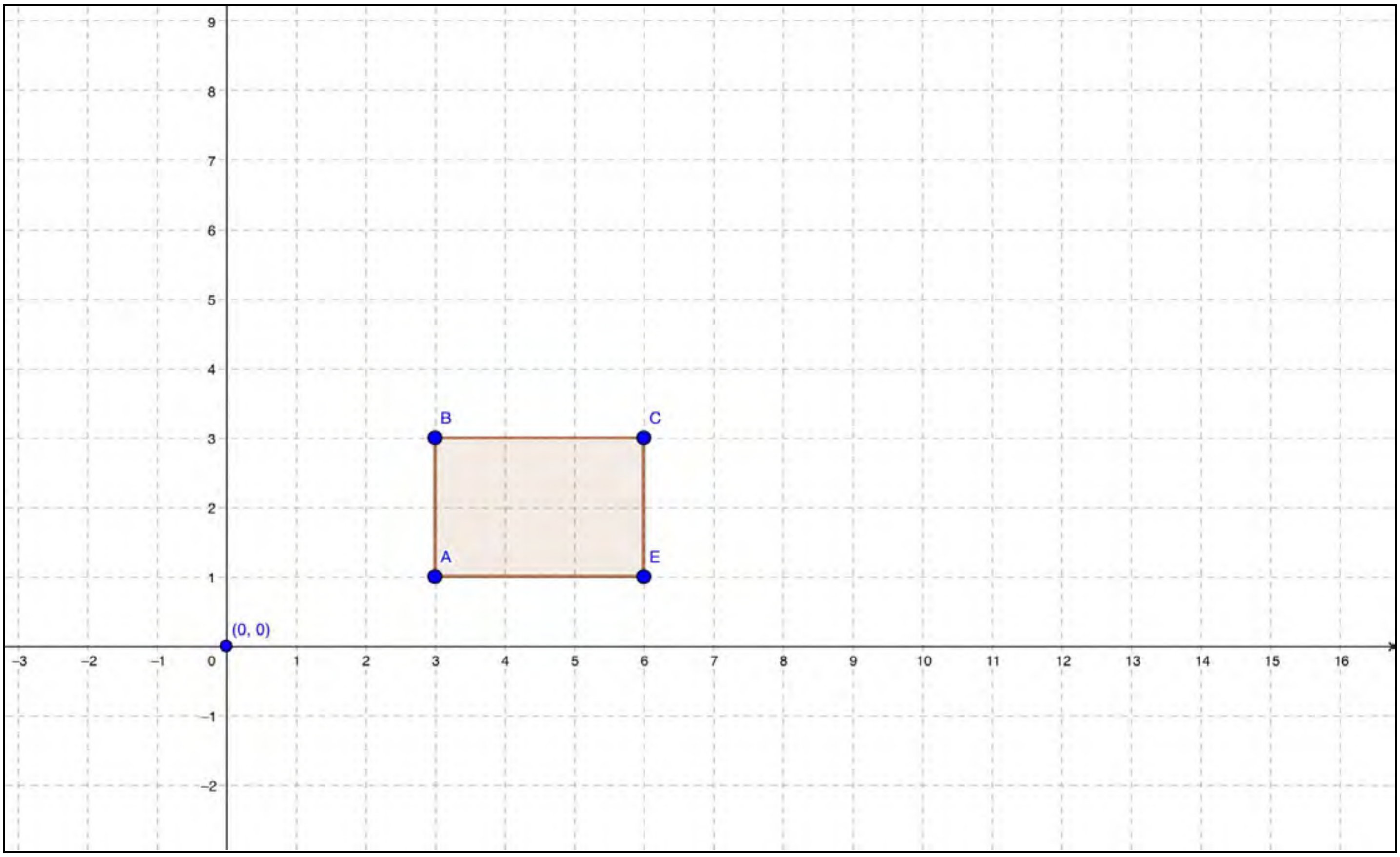
Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 3 and a center of dilation at the origin. Then, using the same center, dilate polygon *ABC* by a factor of ½ on the same grid.



Dilate polygons given a scale factor and the origin as the center of the dilation.

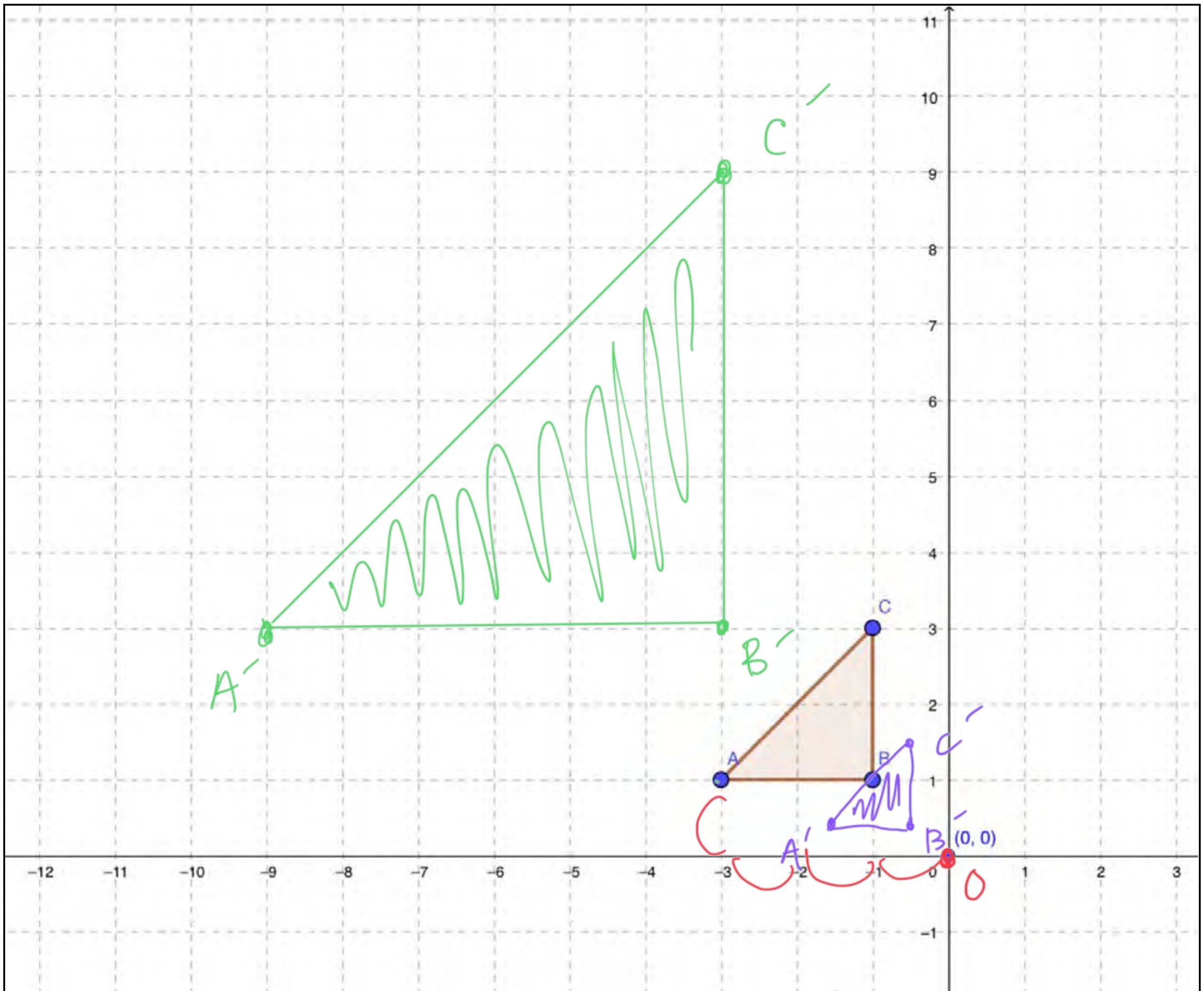
Dilate the polygon *ABCE* below by a factor of 2 and a center of dilation at the origin. Then, using the same center, dilate polygon *ABCE* by a factor of 1/3 on the same grid.



Name: Answer Key

Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 3 and a center of dilation at the origin. Then, using the same center, dilate polygon *ABC* by a factor of ½ on the same grid.

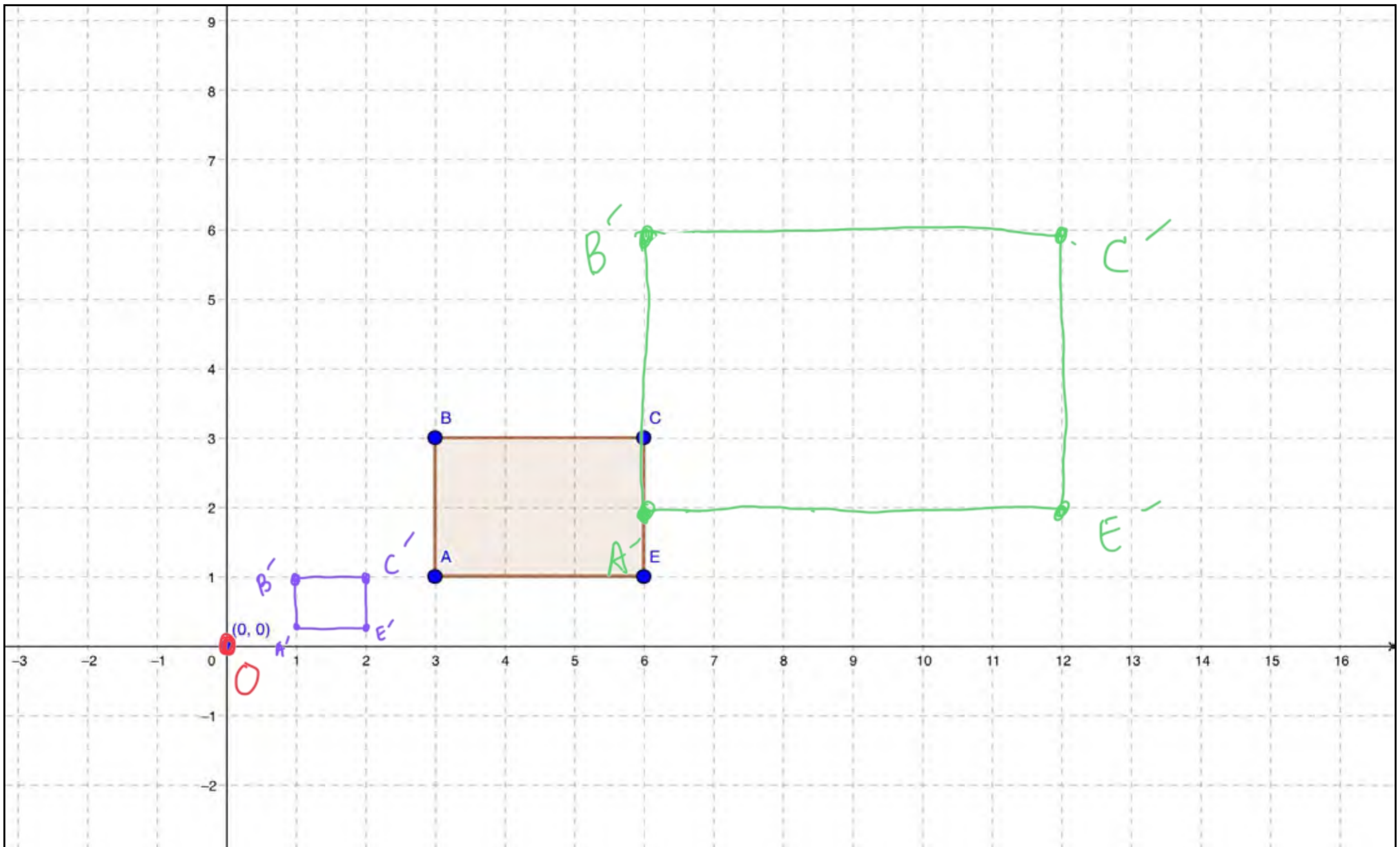


			x 3		x ½
A	3←, 1↑	A′	9←, 3↑	A″	1.5←, .5↑
B	1←, 1↑	B′	3←, 3↑	B″	.5←, .5↑
C	1←, 3↑	C′	3←, 9↑	C″	.5←, 1.5↑

Name: Answer Key

Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABCE* below by a factor of 2 and a center of dilation at the origin. Then, using the same center, dilate polygon *ABCE* by a factor of 1/3 on the same grid.



			×2		×1/3
A	3→, 1↑	A'	6→, 2↑	A'	1→, 1/3↑
B	3→, 3↑	B'	6→, 6↑	B'	1→, 1↑
C	6→, 3↑	C'	12→, 6↑	C'	2→, 1↑
E	6→, 1↑	E'	12→, 2↑	E'	2→, 1/3↑

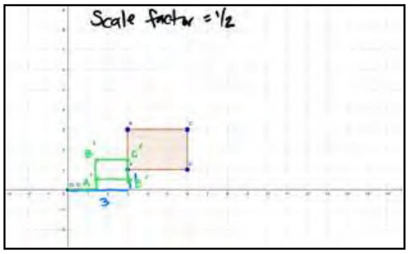
G8 U2 Lesson 2
Perform dilations given a scale factor and center of dilation that is not the origin.

CITYTUTOR DC

G8 U2 Lesson 2 - Perform dilations given a scale factor and center of dilation that is not the origin.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will perform dilations given a scale factor and center of dilation that is not the origin. First, let's recall what we know about dilations. Dilations either increase the size of something or decrease the size. If the scale factor is greater than 1, a dilation will increase the size of a figure. If the scale factor is less than 1, a dilation will shrink the size of a figure.

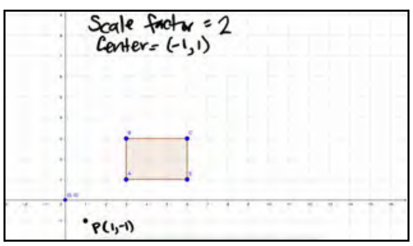


Let's take polygon *ABCE* that was dilated by a scale factor of ½ with a center at the origin. Since ½ is less than one, we knew the dilation would be smaller than the original polygon. We also knew that all of the distances from the center of dilation would be ½ of the original distances.

Let's Talk (Slide 4): Now, what will happen if we dilate a polygon by a scale factor with a center of dilation that is not the origin? Possible Students Answers, Key Points:

- The distances should still remain proportional from the center of dilation to the vertices of the original poly and from the center of dilation to the vertices of the image.

That's right, the rules are the same. So let's get right to it. We will dilate the same polygon, *ABCE*, but this time we'll use a scale factor of 2 and a center at P(1, -1).



First, we'll plot the center of dilation on the grid and call it point *P*. It's also important that we write down the given information so we don't get lost in the process. We are doubling this polygon so everything except for the angles will double.

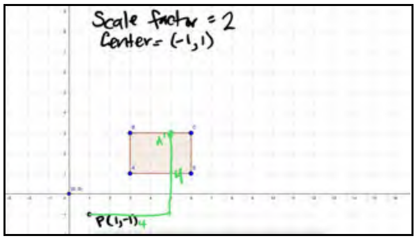
Now, let's find the distances from the center of dilation to each vertex of the original polygon. We'll use a table to help us keep track. Since our scale factor is 2, what does that tell us about the distances from the center of dilation to the image created? Possible Students Answers, Key Points:

- The distances should still remain proportional from the center of dilation to the vertices of the original polygon and from the center of dilation to the vertices of the image.

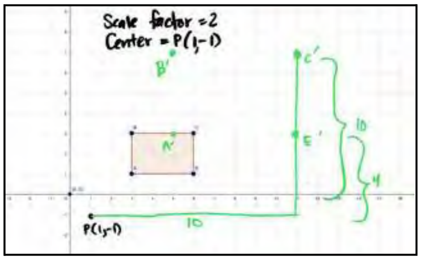
original	Distance from *P*, the center of dilation	Image	Image after a scale factor of 2: distance from *P*, the center of dilation
A	2 units right 2 units up	A'	4 units right 4 units up
B	2 units right 4 units up	B'	4 units right 8 units up
C	5 units right 4 units up	C'	10 units right 8 units up
E	5 units right 2 units up	E'	16 units right 4 units up

Let's use the table to fill in the distances. *(Walk the students through each cell of the table, first identifying the distances of the original vertices and then walking them through each one reminding them to multiply by the scale factor to get the distances from the center to the image.)*

Let's Think (Slide 5): Now, let's use the information in the table to graph the dilation of polygon *ABCE*.

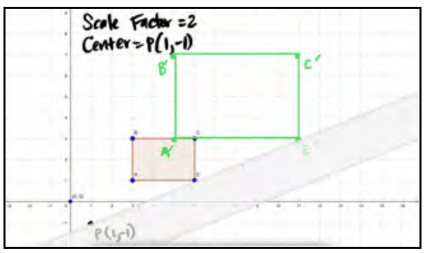


We'll start at the center of dilation and count the number of units in the table for *A'*. *(Show this on the grid and label A'.)*



Next, we'll do the same thing for the other three vertices.
Finally, we'll connect all of the image vertices to create the image, polygon *A'B'C'E'*. *(Draw the new vertices and show at least one set of legs where you count on the grid to mark the new vertex. Label the vertices.)*

You can use a ruler to check your dilations. If the rule forms a straight line through the center of dilation, the original vertex and its corresponding vertex on the image, then you have performed a successful dilation.



Let's try this with *E* and *E'*. *(Have the students use any version of a straight edge - index cards, post it notes, etc. Choose a different student to verify the straight line rule and share with their peers.)* Success! We have successfully performed a dilation with a center not at the origin.

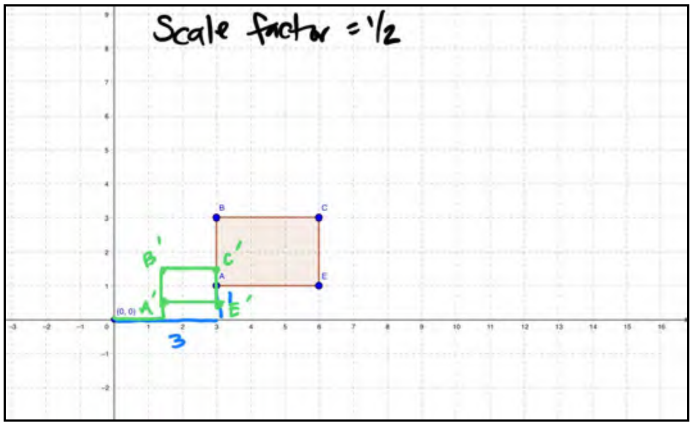
Let's Try it (Slides 7-8): Let's work on dilating polygons given a scale factor and the center not at the origin. We will work on this together. Remember, a dilation with a scale factor greater than 1 gets bigger and a dilation with a scale factor less than 1 gets smaller. All of the distances and side lengths should be proportional to each other by the same scale factor.

WARM WELCOME

Perform dilations given a scale factor and a center of dilation that is not the origin.

Let's Review:

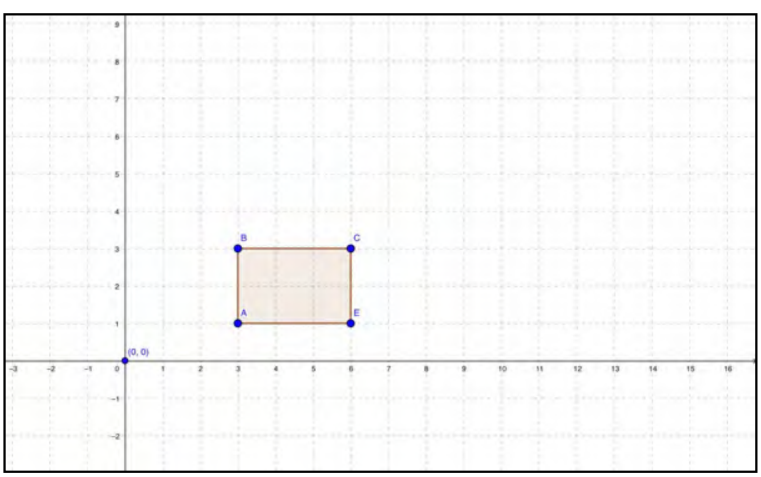
If the scale factor of a dilation is greater than 1, the polygon will increase in size. If the scale factor is less than 1, the polygon will shrink.



Let's Talk:

How do you dilate polygons given a scale factor and the center not at the origin?

Perform a dilation with a scale factor of 2 and the center of dilation at *P*(1, -1).



original	Distance from *P*, the center of dilation	Image	Image after a scale factor of 2: distance from *P*, the center of dilation
A		A'	
B		B'	
C		C'	
E		E'	

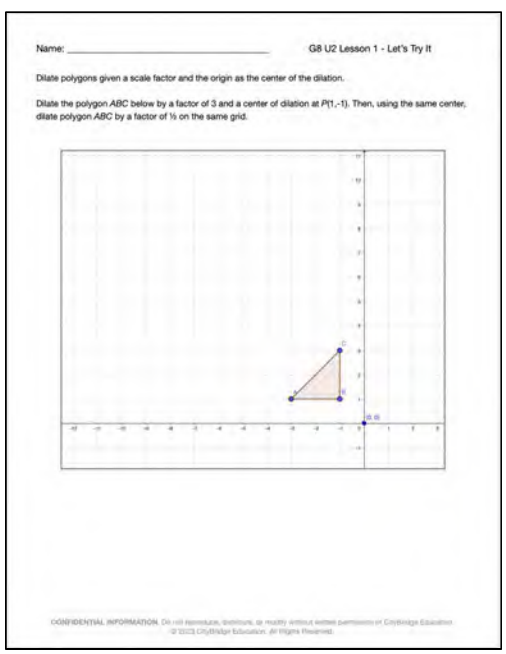
Let's Think:

How do you dilate polygons given a scale factor and the center not at the origin?

Perform a dilation with a scale factor of 2 and the center of dilation at *P*(1, -1).

Let's Try It:

Let's practice dilating polygons given a scale factor and the center not at the origin.



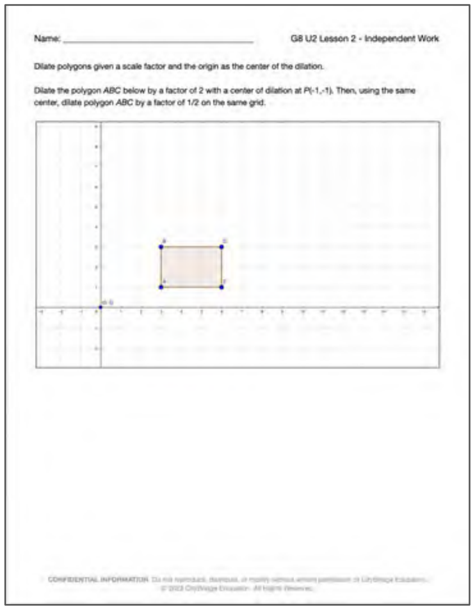
Name: ______________________________ G8 U2 Lesson 1 - Let's Try It

Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 3 and a center of dilation at *P*(1,-1). Then, using the same center, dilate polygon *ABC* by a factor of ½ on the same grid.

On your Own:

Now it's time to dilate polygons given a scale factor and the center not at the origin on your own.


Name: ______________________ G8 U2 Lesson 2 - Independent Work

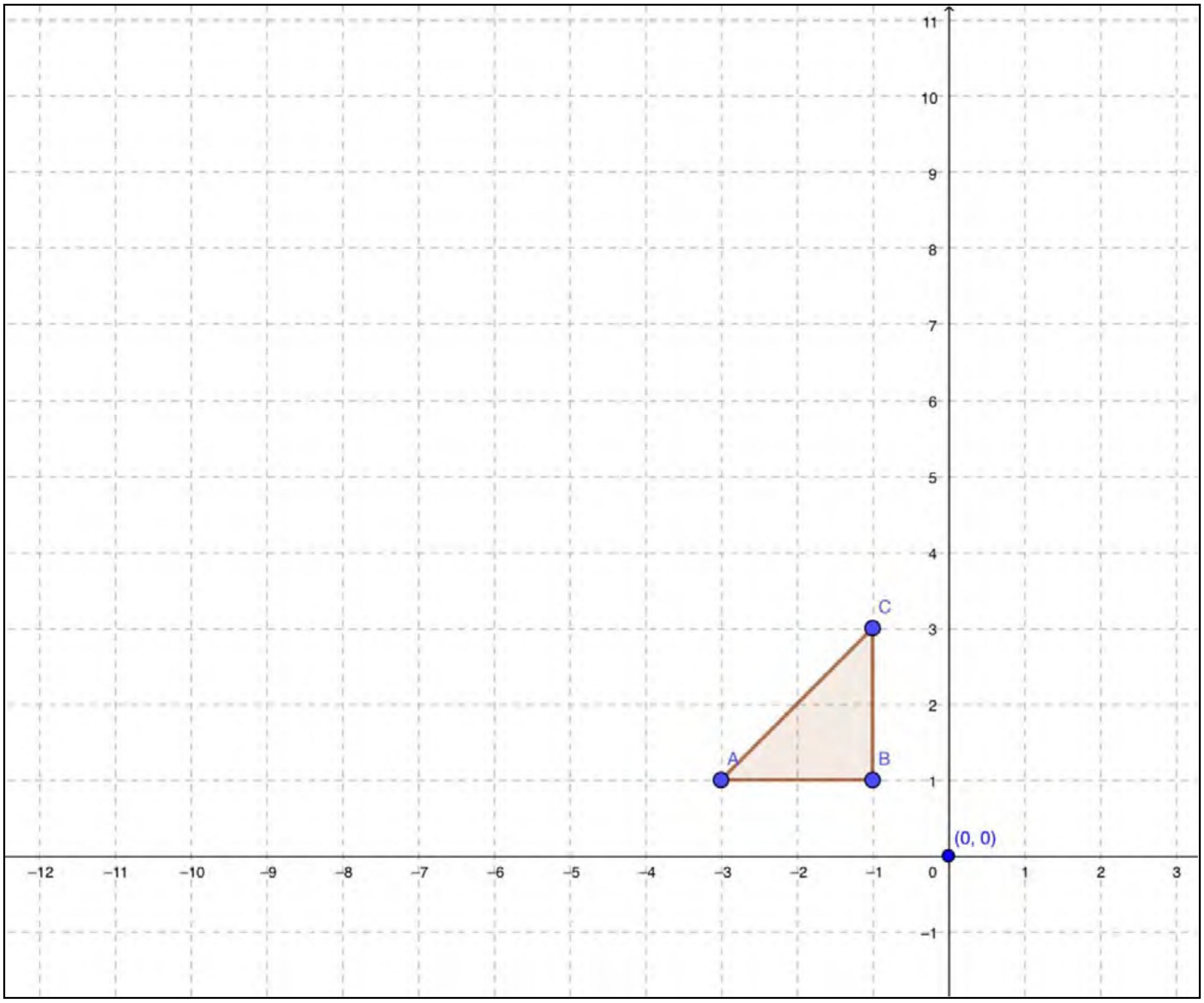
Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 2 with a center of dilation at *P*(-1,-1). Then, using the same center, dilate polygon *ABC* by a factor of 1/2 on the same grid.

Name: ____ G8 U2 Lesson 1 - Let's Try It

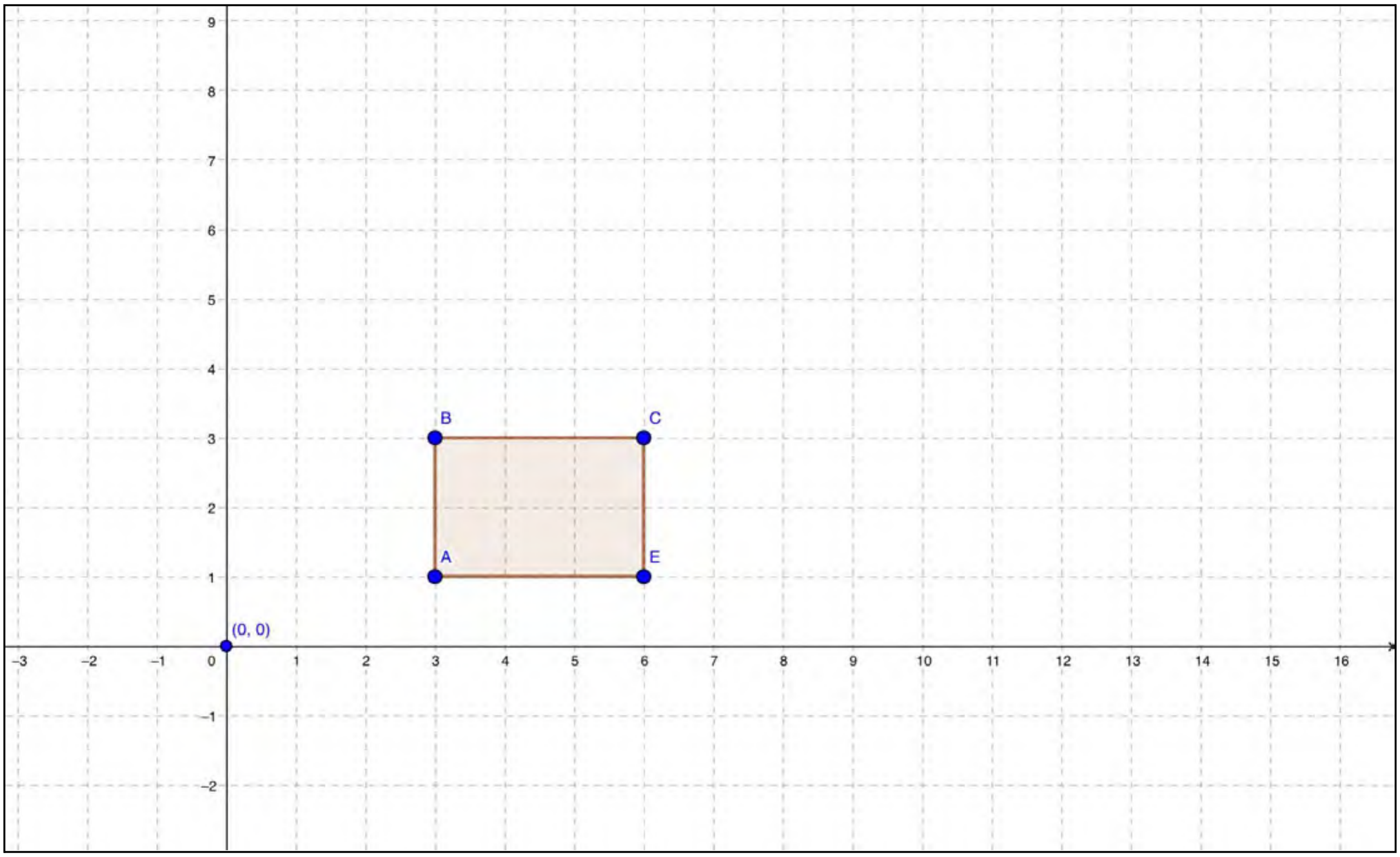
Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 3 and a center of dilation at *P*(1,-1). Then, using the same center, dilate polygon *ABC* by a factor of ½ on the same grid.



Dilate polygons given a scale factor and the origin as the center of the dilation.

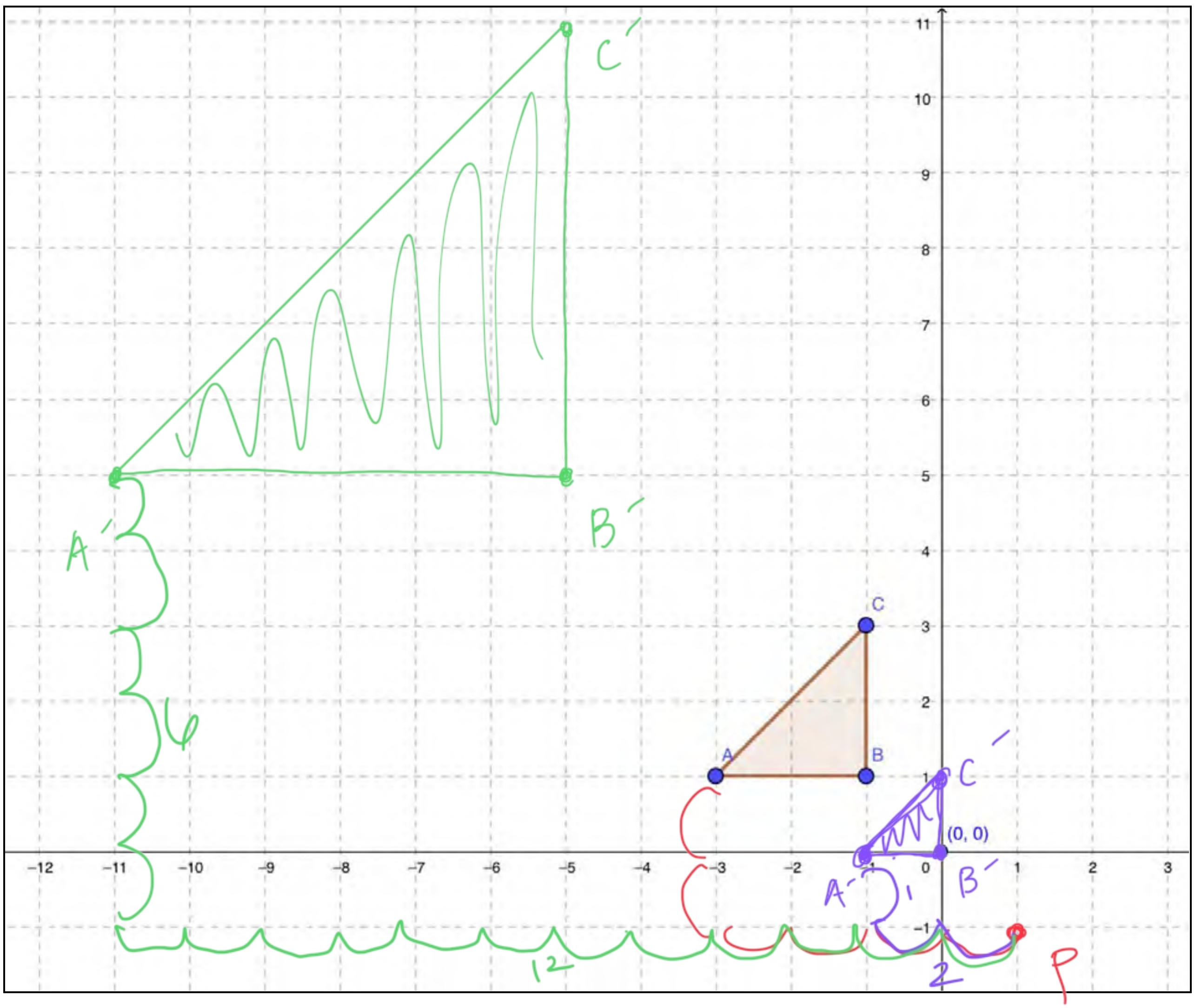
Dilate the polygon *ABC* below by a factor of 2 with a center of dilation at *P*(-1,-1). Then, using the same center, dilate polygon *ABC* by a factor of 1/2 on the same grid.



Name: Answer Key

Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 3 and a center of dilation at *P*(1,-1). Then, using the same center, dilate polygon *ABC* by a factor of ½ on the same grid.

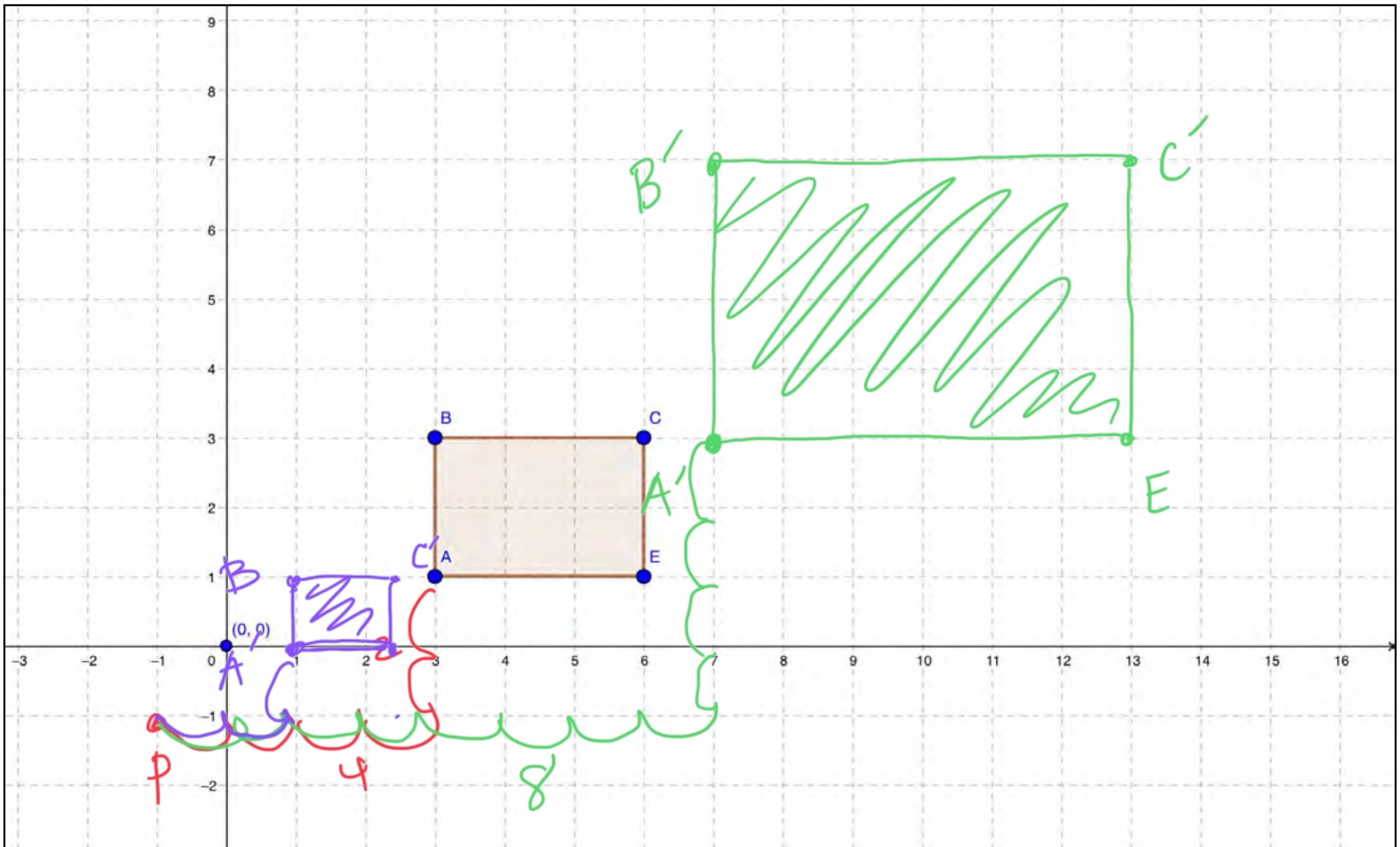


			×3		×½
A	4←, 2↑	A'	12←, 6↑	A'	2←, 1↑
B	2←, 2↑	B'	6←, 6↑	B'	1←, 1↑
C	2←, 4↑	C'	6←, 12↑	C'	1←, 2↑

Name: Answer Key

Dilate polygons given a scale factor and the origin as the center of the dilation.

Dilate the polygon *ABC* below by a factor of 2 with a center of dilation at *P*(-1,-1). Then, using the same center, dilate polygon *ABC* by a factor of 1/2 on the same grid.



			x 2		x 1/2
A	4→, 2↑	A'	8→, 4↑	A'	2→, 1↑
B	4→, 4↑	B'	8→, 8↑	B'	2→, 2↑
C	7→, 4↑	C'	14→, 8↑	C'	3.5→, 2↑
E	7→, 2↑	E'	14→, 4↑	E	3.5→, 1↑

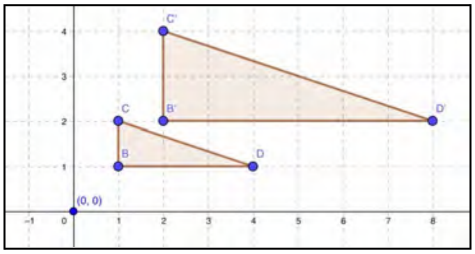
G8 U2 Lesson 3
Identify a sequence of transformations that takes one figure on top of another to explain why two figures are similar.

CITYTUTOR DC

G8 U2 Lesson 3 - Identify a sequence of transformations that takes one figure on top of another to explain why two figures are similar.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will apply a sequence of transformations to a figure to explain why two figures are similar. Similarity is different from congruence. When figures are congruent, all corresponding parts have the same measures. When figures are similar, however, only their angles have to be the same.



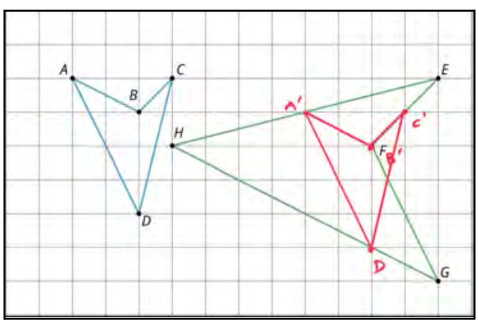
For example, triangle *B'C'D'* is the image of triangle *BCD* after a dilation with a scale factor of 2. The side lengths are proportional, though not the same. So these figures are not congruent. However, if I apply a dilation using a scale factor of ½ to triangle *B'C'D'*, it will lie directly on top of triangle *BCD*. This means that the figures are similar.

Let's explore this some more by considering a sequence of transformations.

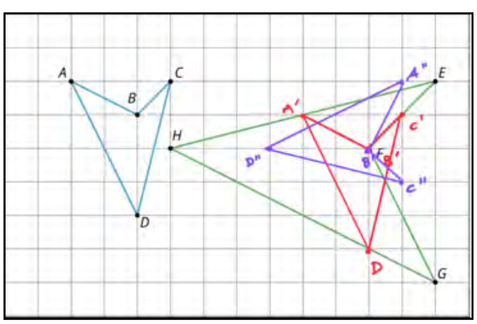
Let's Talk (Slide 4): Quadrilateral *ABCD* and quadrilateral *EFGH* are similar. What sequence of transformations could we use to show this? Possible Students Answers, Key Points:

- There are multiple possible sequences. Use the one below to demonstrate.

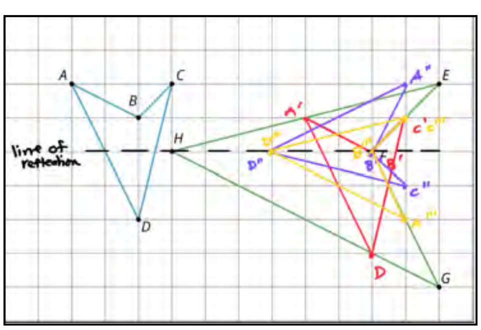
Let's Think (Slide 5): There are multiple sequences we can use. Let's try this one.



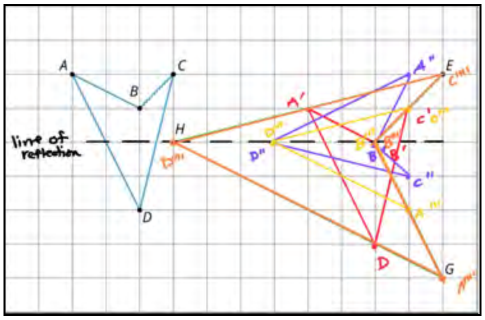
First, we'll translate all vertices 7 units right and then 1 unit down so that the image of *B* lands on top of the *F*.



Next, let's rotate *A'B'C'D'* 90 degrees around point *F*.



Then, we'll draw a line of reflection through points *H* and *F* to reflect *A''B''C''D''over that line.*



Finally, we'll dilate *A'''B'''C'''D'''* using *F* as the center with a scale factor of 2.

We've just proved that quadrilateral *ABCD* and quadrilateral *EFGH* are similar because we verified a sequence of transformations that takes one figure onto the other one.

Let's Try it (Slides 7-8): Let's work on identifying a sequence of transformations that takes one figure on top of the other to verify that the figures are similar. Remember, figures are similar if you can map one figure onto the other using a sequence of translations, reflections, dilations and/or rotations. Figures are congruent if there is no change in size.

WARM WELCOME

Identify a sequence of transformations that takes on figure on top of another to explain why two figures are similar.

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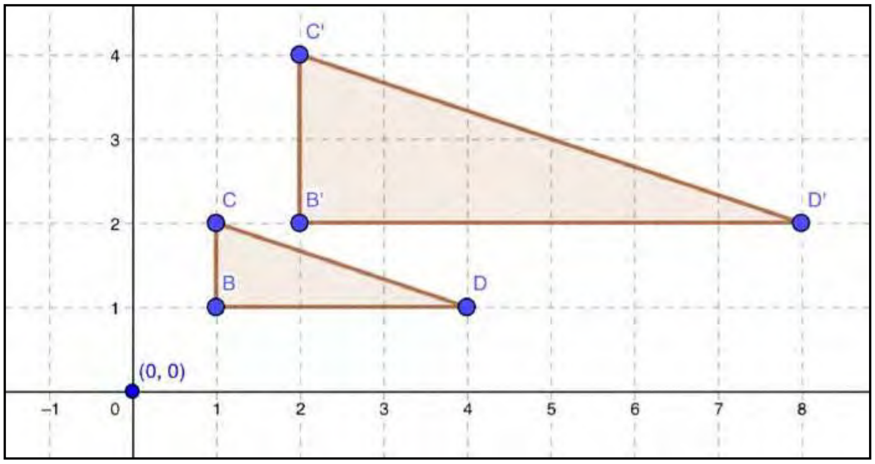
Let's Review:

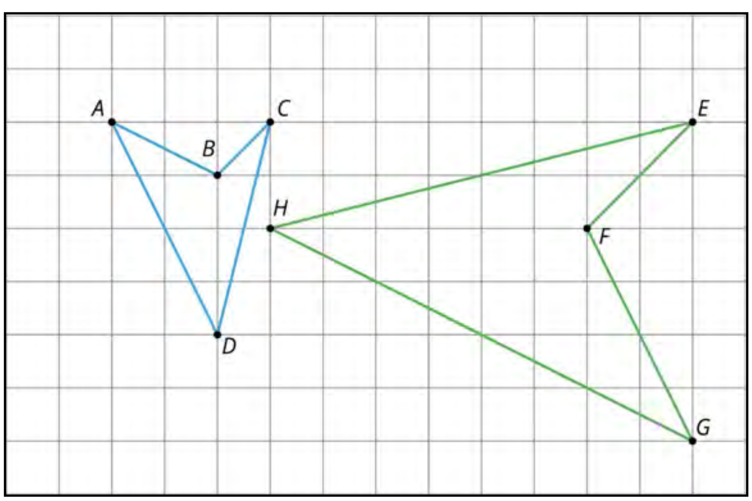
When figures are similar, only their angles have to be the same.



Let's Talk:

How can a sequence of transformations help us to identify similar figures?

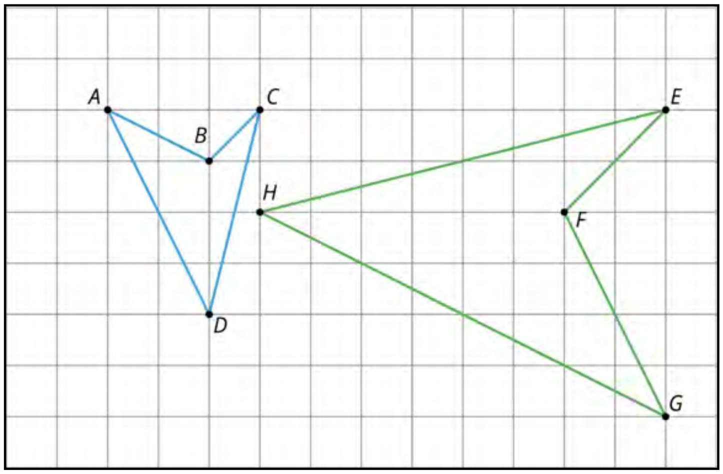
Quadrilateral *ABCD* and quadrilateral *EFGH* are similar. What sequence of transformations could we use to show this?



Let's Think:

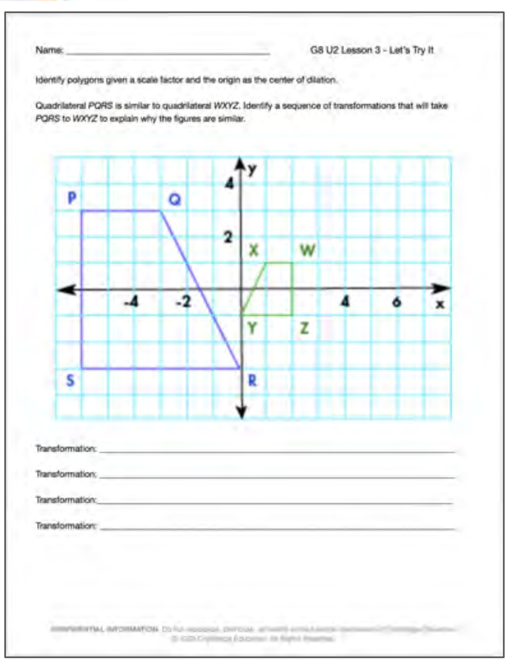
How do you dilate polygons given a scale factor and the center not at the origin?

Apply a sequence of transformations to quadrilateral *ABCD* to show that it is similar to quadrilateral *EFGH.*



Let's Try It:

Let's practice dilating polygons given a scale factor and the center not at the origin.



Name: ______________________ G8 U2 Lesson 3 - Let's Try It

Identify polygons given a scale factor and the origin as the center of dilation.

Quadrilateral *PQRS* is similar to quadrilateral *WXYZ*. Identify a sequence of transformations that will take *PQRS* to *WXYZ* to explain why the figures are similar.

Transformation: ______________________

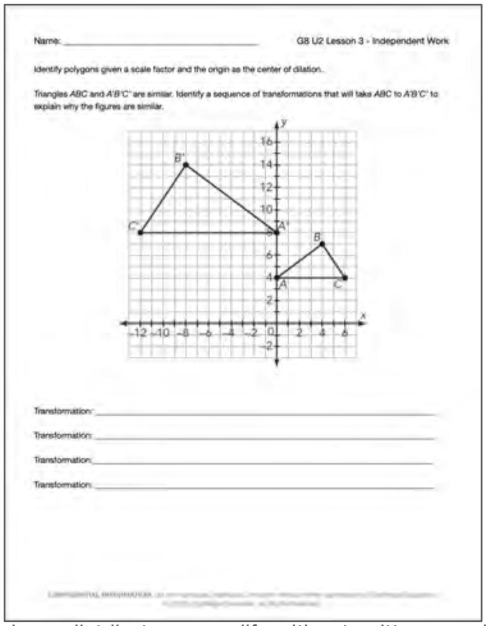
Transformation: ______________________

Transformation: ______________________

Transformation: ______________________

On your Own:

Now it's time to dilate polygons given a scale factor and the center not at the origin on your own.



Name: ______________________________ G8 U2 Lesson 3 - Independent Work

Identify polygons given a scale factor and the origin as the center of dilation.

Triangles *ABC* and *A'B'C'* are similar. Identify a sequence of transformations that will take *ABC* to *A'B'C'* to explain why the figures are similar.

Transformation: ______________________________

Transformation: ______________________________

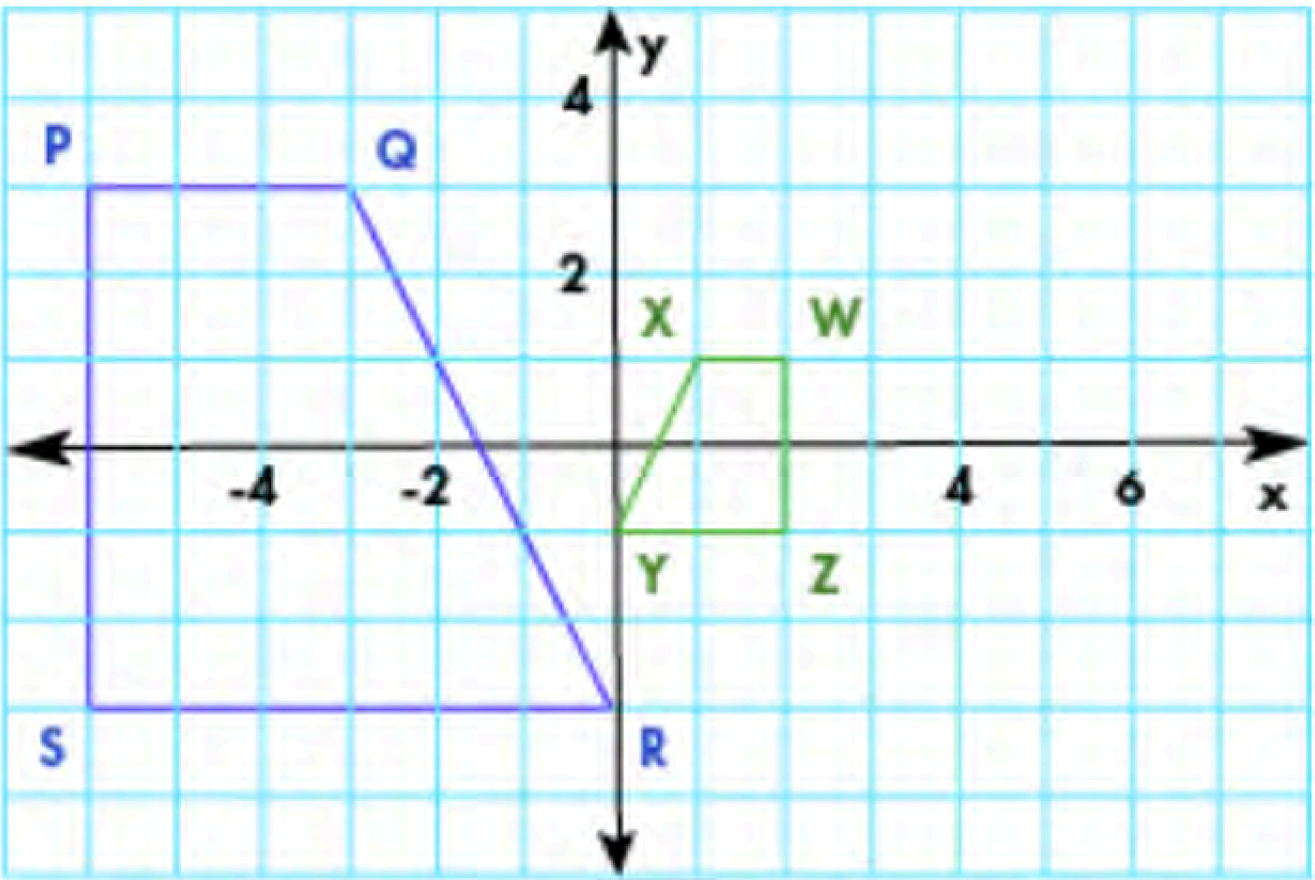
Transformation: ______________________________

Transformation: ______________________________

Name: ____ G8 U2 Lesson 3 - Let's Try It

Identify a sequence of transformations that takes one figure on top of another to explain why two figures are similar.

Quadrilateral *PQRS* is similar to quadrilateral *WXYZ*. Identify a sequence of transformations that will take *PQRS* to *WXYZ* to explain why the figures are similar.



Transformation: ____

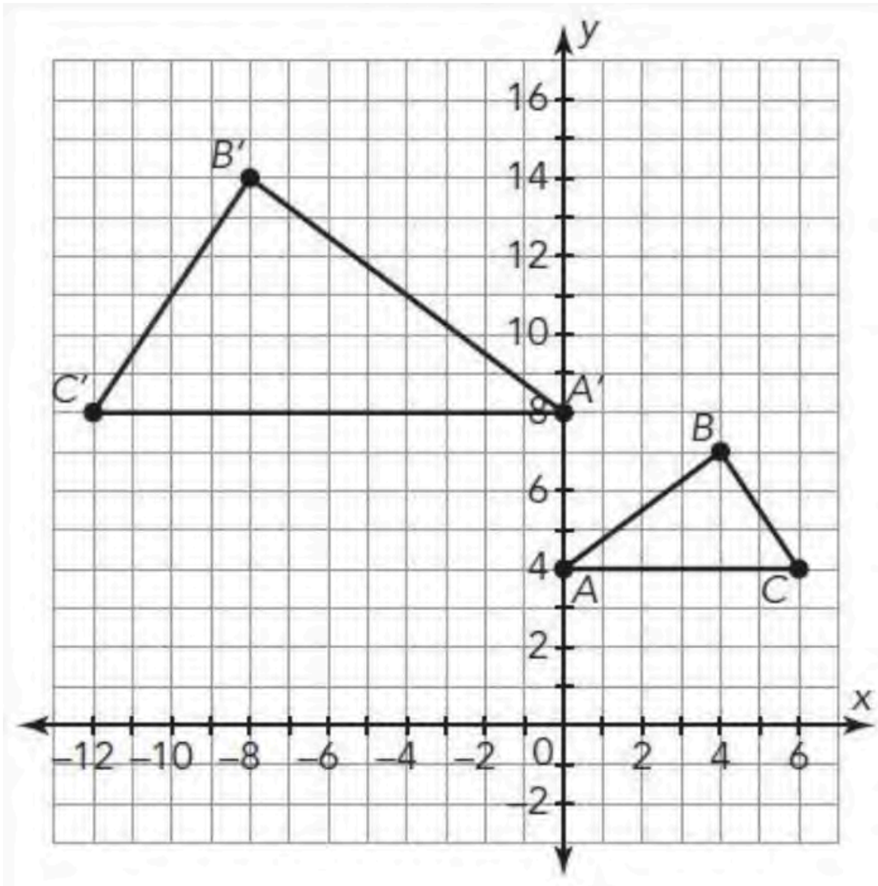
Transformation: ____

Transformation: ____

Transformation: ____

Identify a sequence of transformations that takes one figure on top of another to explain why two figures are similar.

Triangles *ABC* and *A'B'C'* are similar. Identify a sequence of transformations that will take *ABC* to *A'B'C'* to explain why the figures are similar.



Transformation: ____

Transformation: ____

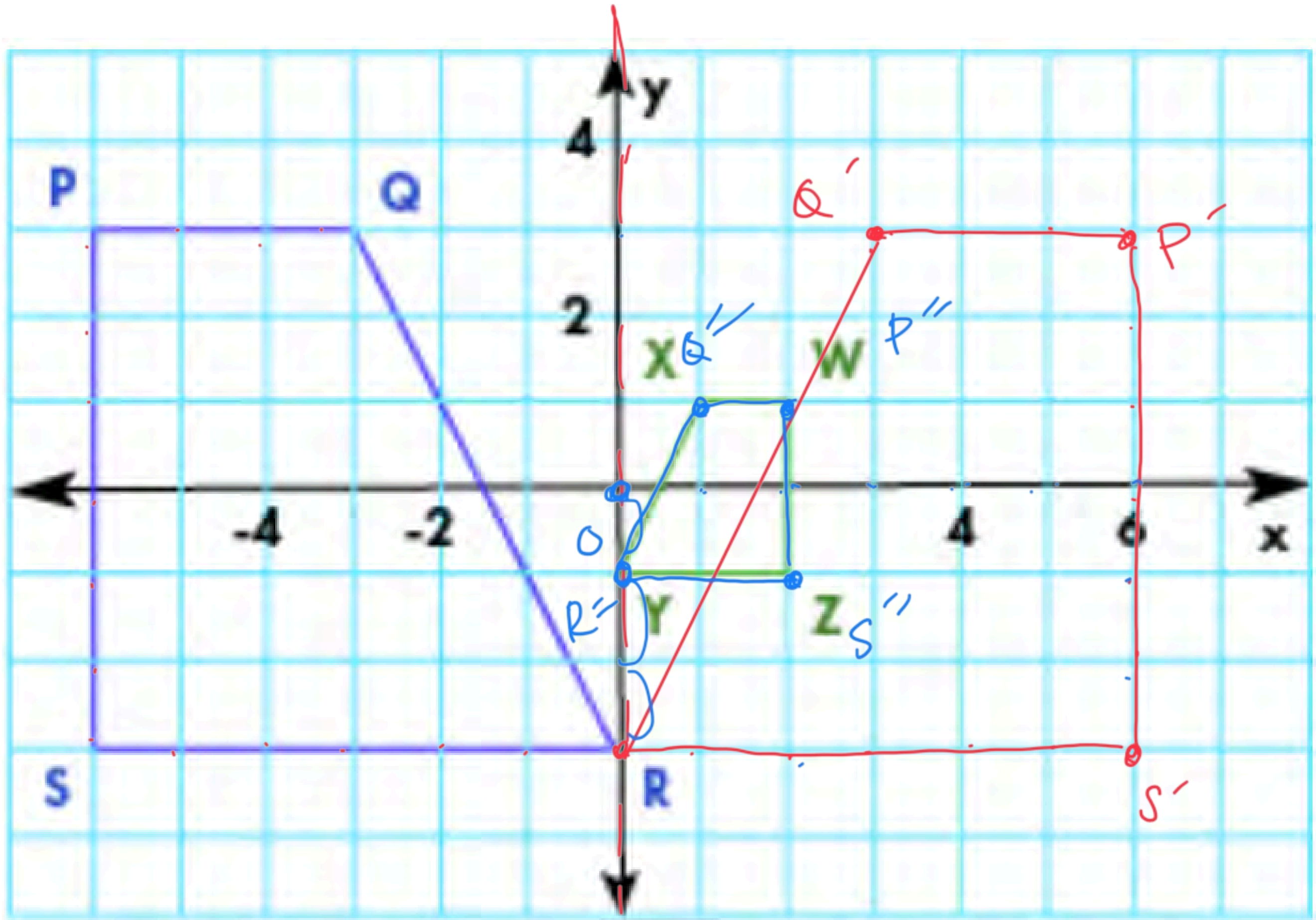
Transformation: ____

Transformation: ____

Name: Answer Key

Identify a sequence of transformations that takes one figure on top of another to explain why two figures are similar.

Quadrilateral *PQRS* is similar to quadrilateral *WXYZ*. Identify a sequence of transformations that will take *PQRS* to *WXYZ* to explain why the figures are similar.



Transformation: reflection over the y-axis

Transformation: dilation, scale factor $\frac{1}{3}$, center: origin

Transformation:

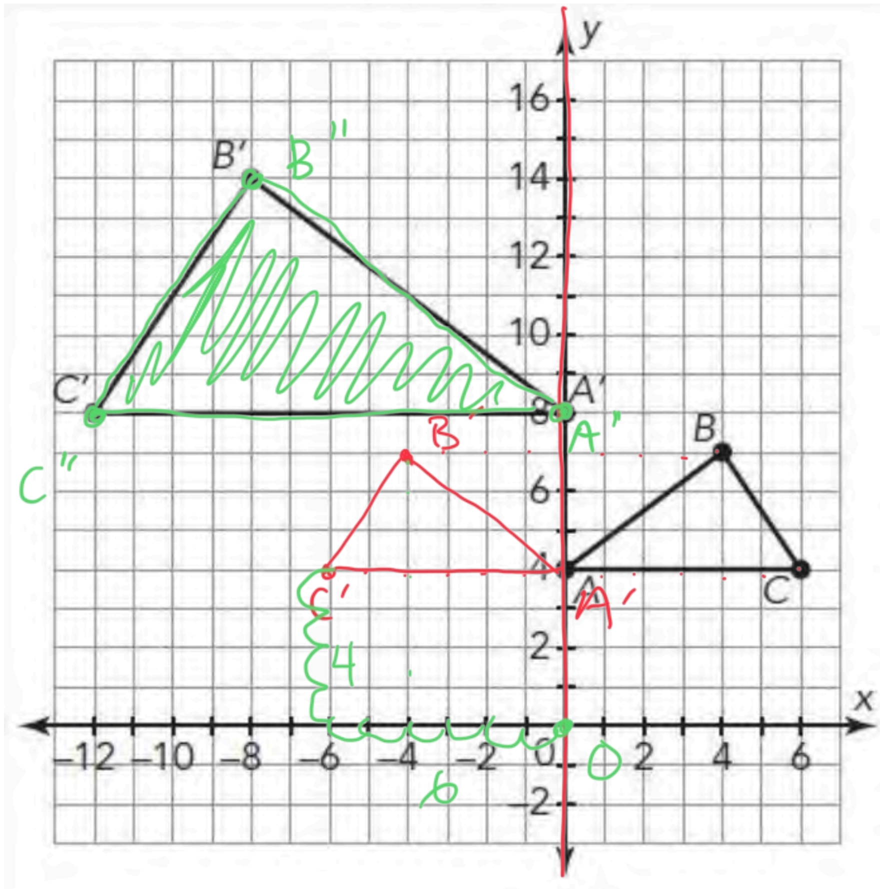
Transformation:

		$\times \frac{1}{3}$
R'	0, 3↓	0, 1↓
Q'	3↑, 3→	1↑, 1→
S'	6→, 3↓	2→, 1↓
P'	6→, 3↑	2→, 1↑

Name: Answer Key

Identify polygons given a scale factor and the origin as the center of dilation.

Triangles *ABC* and *A'B'C'* are similar. Identify a sequence of transformations that will take *ABC* to *A'B'C'* to explain why the figures are similar.



Transformation: reflection over y-axis

Transformation: dilation, scale factor 2, center = origin

Transformation:

Transformation:

			x2
A'	0, 4↑	A''	0, 8↑
B'	4←, 7↑	B''	8←, 14↑
C'	6←, 4↑	C''	12←, 8↑

The figures are similar because a sequence of transformations takes one to the other.

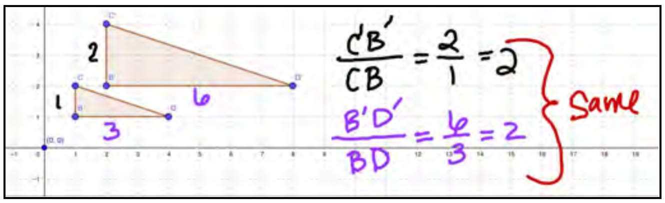
G8 U2 Lesson 4
Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

CITYTUTOR DC

G8 U2 Lesson 4 - Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

Warm Welcome (Slide 1): Tutor Choice

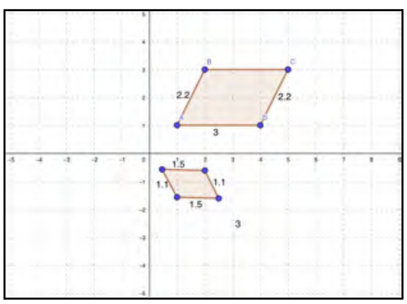
Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will demonstrate our understanding that similar polygons have congruent corresponding angles and proportional side lengths. You might have learned that some shapes can be congruent - that means all of their corresponding angles and sides are equal in measure. Rigid Transformations produce congruent figures. Dilations, however, produce similar shapes. If you remember, when you dilate a shape the corresponding angles are congruent but their side lengths are not. Instead, in similar figures, the corresponding side lengths are proportional to each other while the corresponding angles are congruent. Remember, if two shapes are similar, you can use a sequence of transformations to take one polygon to the other. Figures that are similar but have not undergone a dilation, are also congruent.



In this example of two triangles, notice that the side lengths are proportional by a scale factor of 2. The polygons are not the same size but we know from a previous lesson that these triangles are dilations of each other which means we can use what we know about rigid transformations to confirm that the shapes are similar.

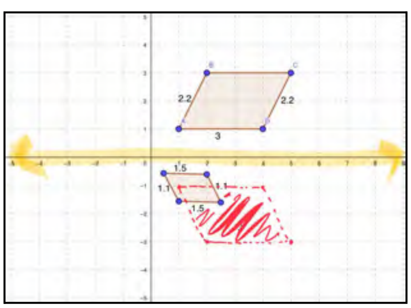
Let's Talk (Slide 4): Let's look at two more polygons that are similar. Based on what you see, how could you verify that these polygons are similar? Possible Students Answers, Key Points:

- We need to determine if one was created from the other by a sequence of transformations.



That's right! We don't need to draw this out but we can start to consider if there is a sequence of rigid transformations and/or dilations that will take one shape onto the other. What do you think may have happened? Possible Students Answers, Key Points:

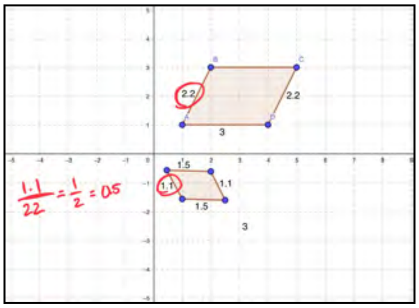
- A reflection over the *x*-axis.
- A dilation by a scale factor of 2.



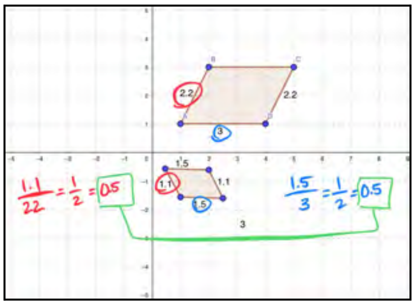
That's right. We can see that a reflection must have occurred first. *(If students are struggling, draw the line of reflection and a quick sketch of what this image will look like after a reflection.)* After the reflection, using the origin as the center and a scale factor of .5, we can take the larger polygon onto the smaller one.

Let's Think (Slide 5): Now, how did we know the side lengths were proportional by a factor of ½ or 0.5? Let's explore to better understand.

First, we'll compare the corresponding side lengths of each polygon to see if they are all proportional. If the proportion is the same for all sides of the polygons, then we have confirmed by sequence of transformations that the polygons are similar.



We will start with the smaller shape since we're considering it as the image. This also works in reverse, if needed. *(Show the work of dividing 1.1 by 2.2.)* Notice that two sets of corresponding sides share a ratio of 0.5 because 1.1 divided by 2.2 is ½ or 0.5.



In addition, since 1.5 divided by 3 is ½ or 0.5, the next two sets of corresponding sides are also proportional with a scale factor of 0.5. *(Show the math on the side of the grid.)*

We did it! We've verified if two shapes are similar, in addition to their other properties, their corresponding side lengths are proportional.

Let's Try it (Slides 7-8): Let's demonstrate our understanding that similar polygons have congruent corresponding angles and proportional corresponding sides. We will work on this page together. Remember, you can verify that figures are similar if you can identify a sequence of transformations that takes one figure onto the other. Also remember that similar figures have congruent corresponding angles and proportional corresponding side lengths.

WARM WELCOME

Understand that similar polygons have congruent angles and proportional corresponding sides.

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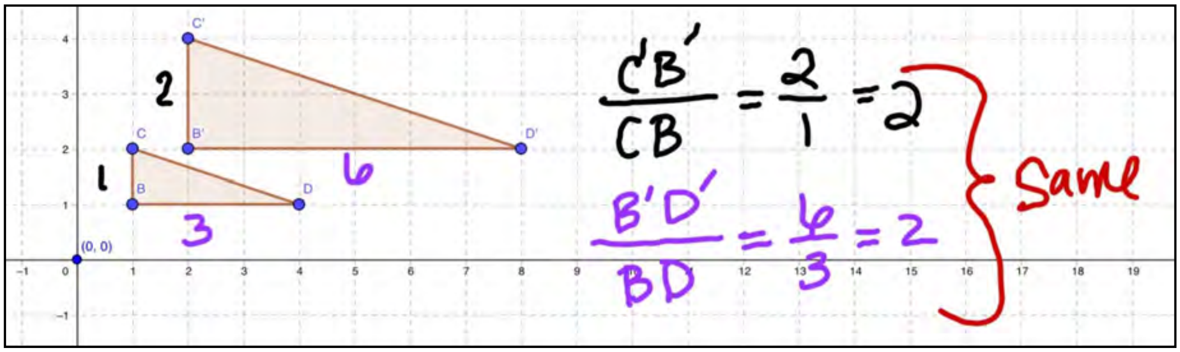
Let's Review:

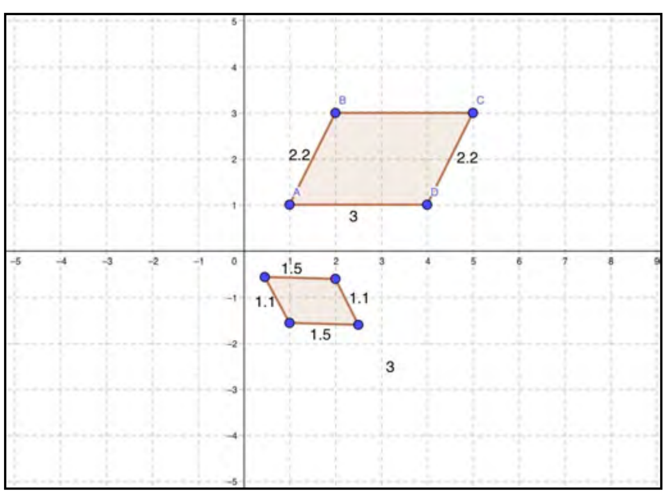
If two shapes are similar, you can use a sequences of transformations to take one polygon to the other.



Let's Talk:

How do we verify that similar polygons have congruent corresponding angles and proportional corresponding sides?

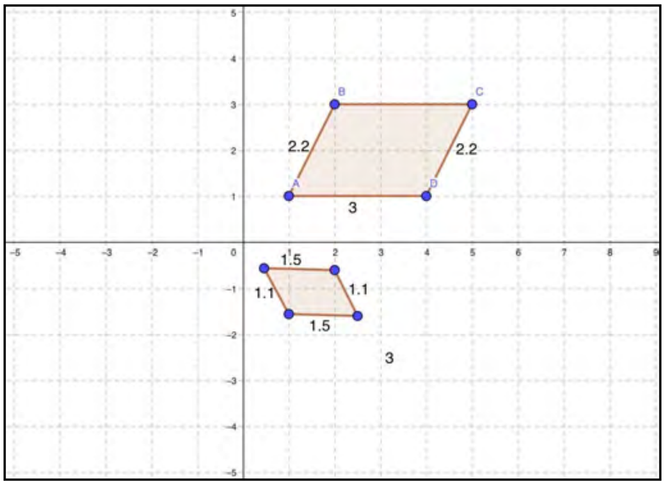
These polygons are similar. How can we verify that they are similar?



Let's Think:

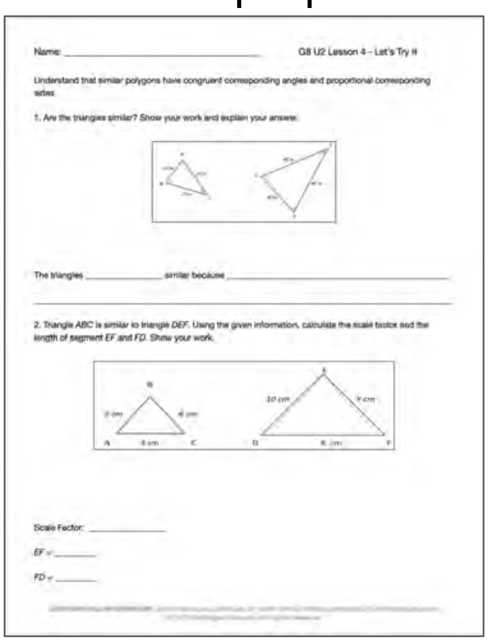
How do we verify that similar polygons have congruent corresponding angles and proportional corresponding sides?

How do we know if the side lengths of similar figures are proportional?



Let's Try It:

Let's practice verifying that similar polygons have congruent corresponding angles and proportional sides.



Name: ______________________ G8 U2 Lesson 4 – Let's Try It

Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

1. Are the triangles similar? Show your work and explain your answer.

The triangles ____________ similar because ______________________

____

2. Triangle *ABC* is similar to triangle *DEF*. Using the given information, calculate the scale factor and the length of segment *EF* and *FD*. Show your work.

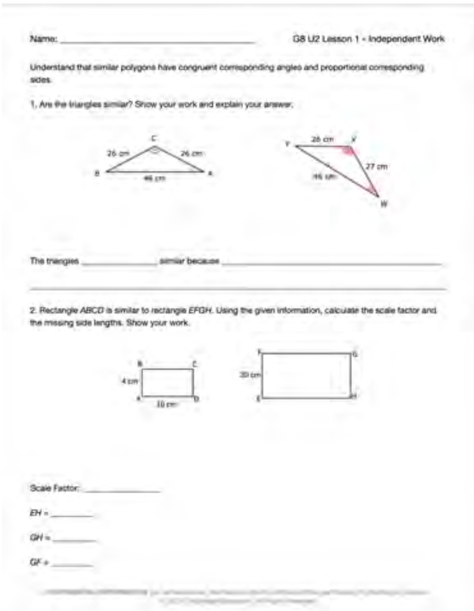
Scale Factor: ____________

EF = ________

FD = ________

On your Own:

Now it's time to verify that similar polygons have congruent corresponding angles and proportional corresponding sides on your own.



Name: ______________________ G8 U2 Lesson 1 - Independent Work

Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

1. Are the triangles similar? Show your work and explain your answer.

The triangles ____________ similar because ______________________

____

2. Rectangle *ABCD* is similar to rectangle *EFGH*. Using the given information, calculate the scale factor and the missing side lengths. Show your work.

Scale Factor: ____________

EH = ________

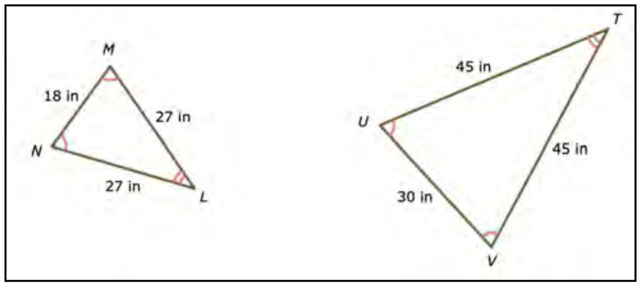
GH = ________

GF = ________

Name: ____ G8 U2 Lesson 4 - Let's Try It

Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

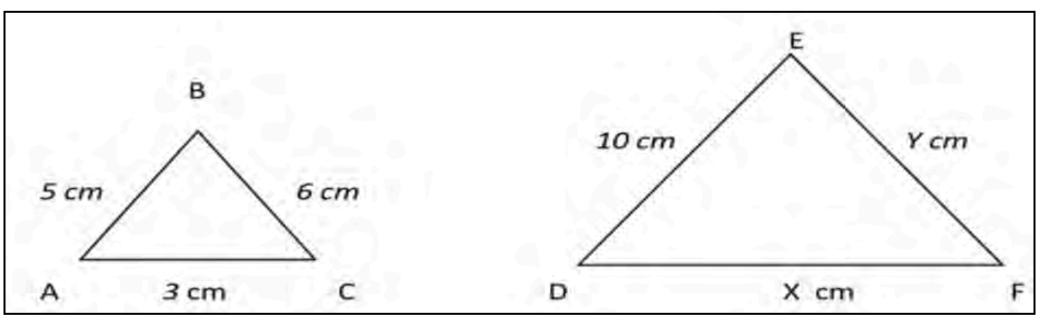
1. Are the triangles similar? Show your work and explain your answer.



The triangles _________________ similar because ____

____

2. Triangle *ABC* is similar to triangle *DEF*. Using the given information, calculate the scale factor and the length of segment *EF* and *FD*. Show your work.



Scale Factor: _________________

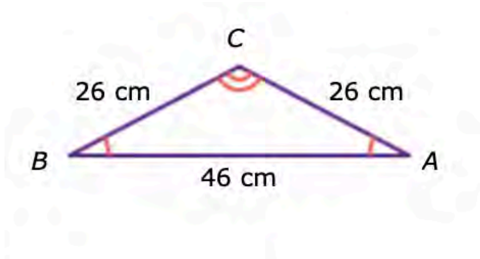
EF = __________

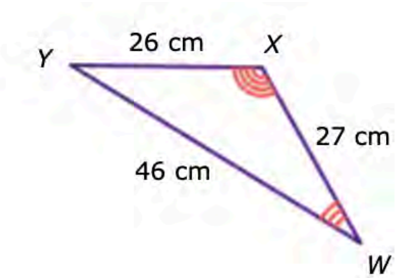
FD = __________

Name: ____

Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

1. Are the triangles similar? Show your work and explain your answer.

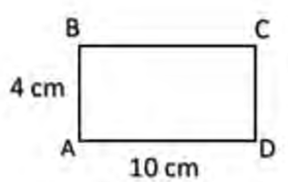


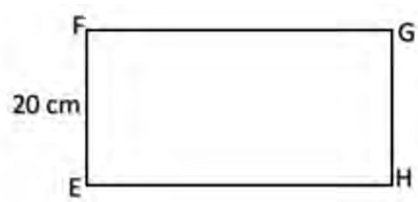


The triangles ________________ similar because ____

____

2. Rectangle *ABCD* is similar to rectangle *EFGH*. Using the given information, calculate the scale factor and the missing side lengths. Show your work.





Scale Factor: ________________

EH = __________

GH = __________

GF = __________

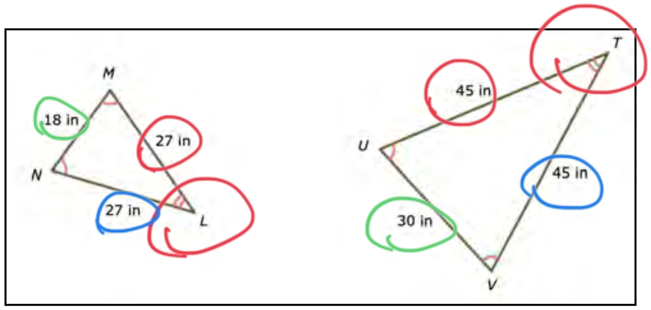
Name: Answer Key

Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

1. Are the triangles similar? Show your work and explain your answer.

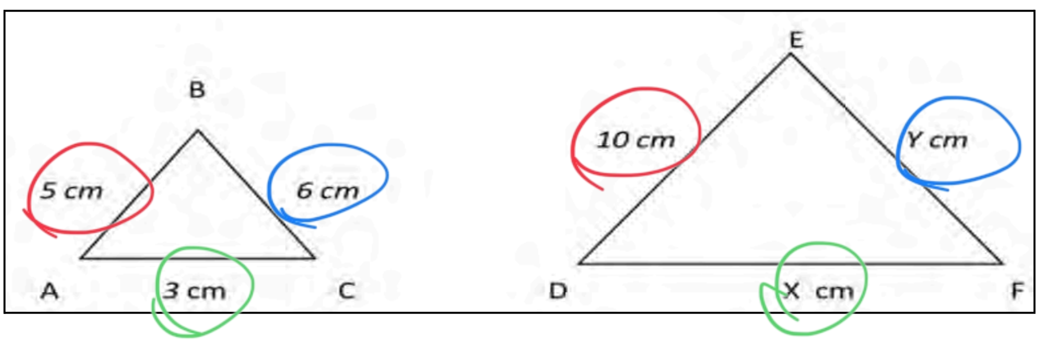
$\frac{45}{27} = \frac{5}{3}$ $\frac{30}{18} = \frac{5}{3}$

$\frac{45}{27} = \frac{5}{3}$



The triangles are similar because the corresponding ∡s are congruent and the corresponding sides are proportional.

2. Triangle *ABC* is similar to triangle *DEF*. Using the given information, calculate the scale factor and the length of segment *EF* and *FD*. Show your work.



$\frac{BIG}{Small} = \frac{10}{5} = 2$

Scale Factor: 2

$\frac{Y}{6} = 2 \Rightarrow Y = 12$

EF = 12cm

FD = 6cm

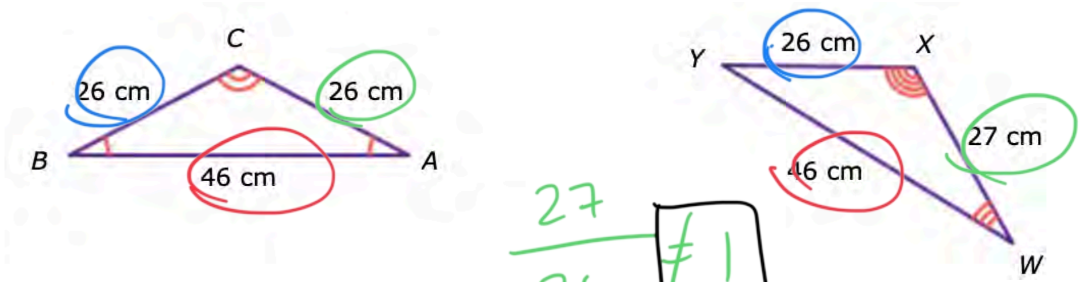
Name: Answer Key

Understand that similar polygons have congruent corresponding angles and proportional corresponding sides.

1. Are the triangles similar? Show your work and explain your answer.

$\frac{46}{46} = 1$

$\frac{26}{26} = 1$

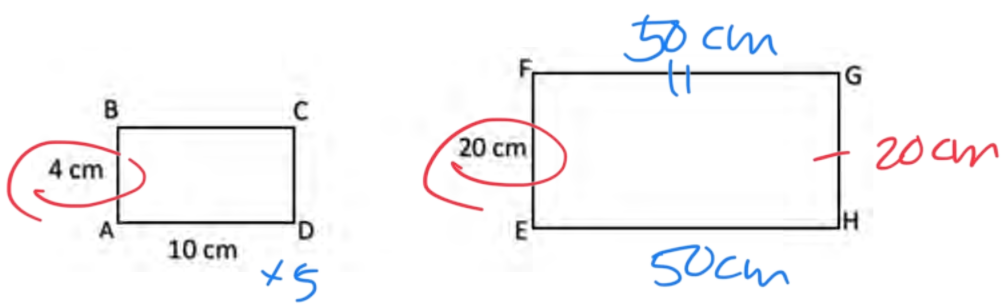


$\frac{27}{26} \neq 1$

The triangles are not similar because all corresponding sides are not proportional by the same scale factor.

2. Rectangle *ABCD* is similar to rectangle *EFGH*. Using the given information, calculate the scale factor and the missing side lengths. Show your work.

$\frac{\text{BIG}}{\text{small}} = \frac{20}{4} = 5$



* opposite sides of a parallelogram are congruent. A rectangle is a parallelogram.

Scale Factor: 5

EH = 50 cm

GH = 20 cm

GF = 50cm

G8 U2 Lesson 5
Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

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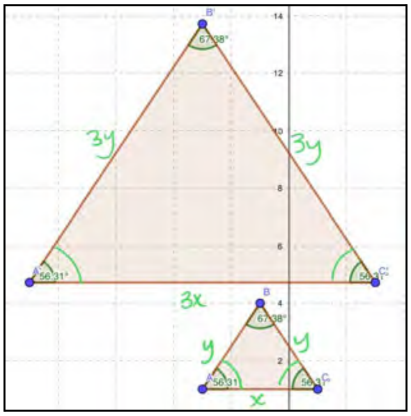
G8 U2 Lesson 5 - Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will determine that two triangles are similar by checking that two pairs of corresponding angles are congruent. In a previous lesson, you learned that figures are similar if their corresponding angles are congruent and if their corresponding side lengths are proportional.

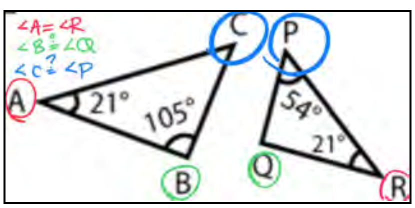


In this example, notice that the corresponding angles are congruent. *(draw an additional angle arc the base angles in each triangle and simply point out that the top angle is congruent to the corresponding top angle in the other triangle.)* Also notice that the corresponding sides are proportional - in this case the scale factor is 3.

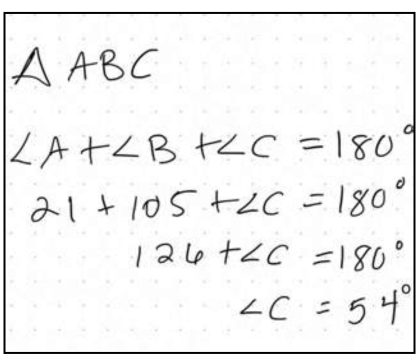
Let's Talk (Slide 4): Let's look at two similar triangles. We have some information but not all of the information we need. Based on what you can see, how do you think we could use the angles to determine if the triangles are similar? Possible Students Answers, Key Points:

- We can check to see if all of the angles are congruent/equal.
- We can use the Triangle Sum Theorem to find the missing angle. Then, we'll know if the corresponding angles are congruent.
- Misconception: They are not similar because the given angles are not the same.

Let's Think (Slide 5):



Let's first start by identifying what we believe the corresponding angles are in each triangle. I will use colors to match them up.



Next, let's use the triangle sum theorem on one triangle to find the missing angle measure. Let's choose triangle *ABC*. If the ∠*C* = 54°, then the triangles are similar because that means that ∠*Q* = 105°. *(Write the formula for the triangle sum theorem and use it to solve for angle C.)*

Finally, now that we know that ∠*C* = 54°, we also know that ∠*Q* = 105°. Since all corresponding angles are congruent, triangle *ABC* and triangle *RQP* are similar. *(Point out that the order of the letters is important when referencing congruence and similarity because they are associated with the corresponding parts.)* What we've just shown is known as angle angle similarity. The rationale is that if you know two pairs of corresponding angles are congruent then you also know that the third pair is congruent.

Let's Try it (Slides 7-8): Let's determine that two triangles are similar by checking that two pairs of corresponding angles are congruent. Remember, triangles are similar is all of their corresponding angles are congruent but you only need to check for two to verify similarity. This is known as angle angle similarity.

WARM WELCOME

Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

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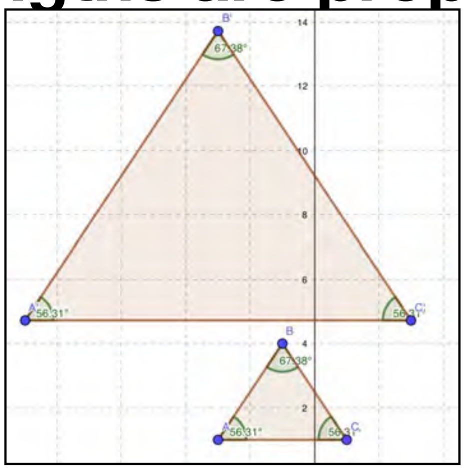
Let's Review:

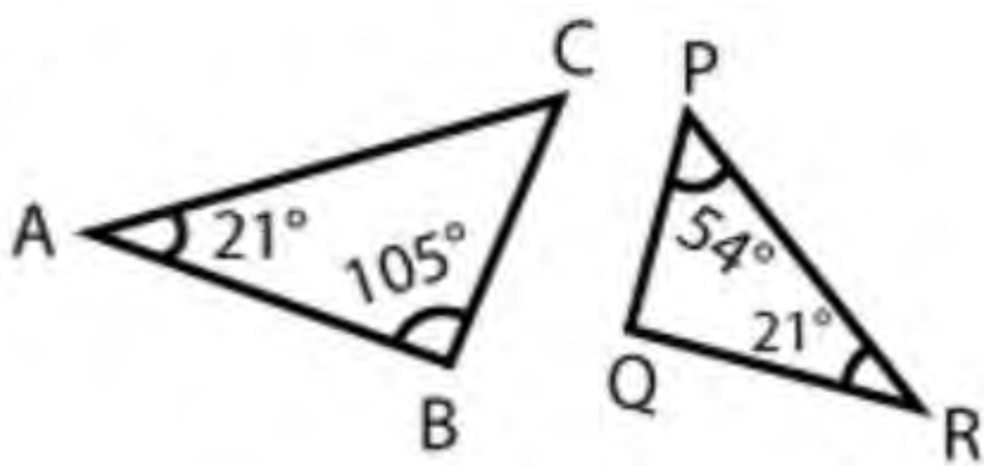
Figures are similar if their corresponding angles are congruent and their corresponding side lengths are proportional.



Let's Talk:

How do we determine if two triangles are similar using two pairs of corresponding angles?

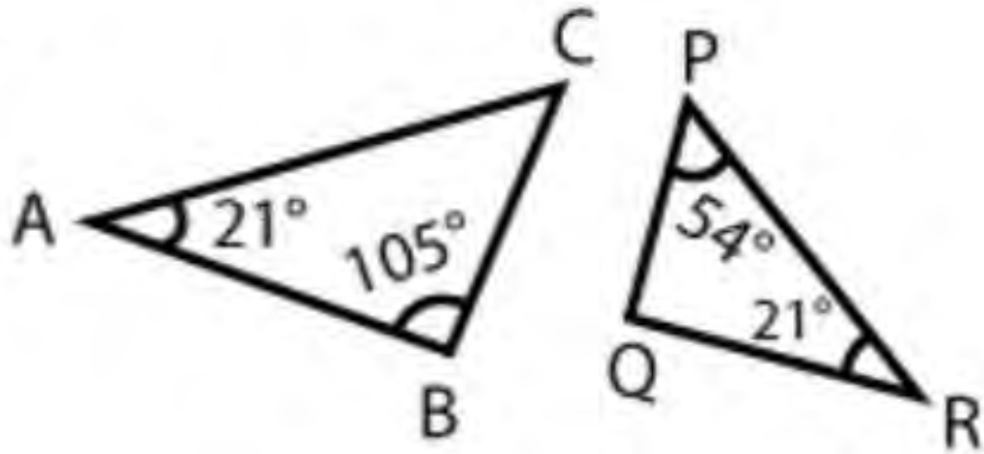
How can we verify that these triangles are similar?



Let's Think:

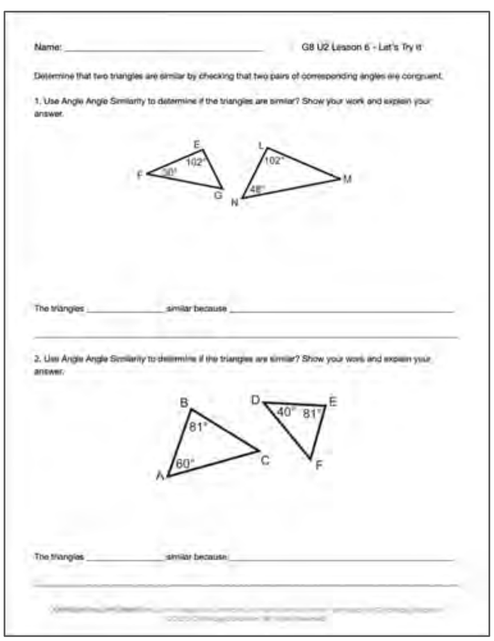
How do we determine if two triangles are similar using two pairs of corresponding angles?

How can we verify that these triangles are similar?



Let's Try It:

Let's practice determining that two triangles are similar by checking that two pairs of corresponding angles are congruent.



Name: ______________________ G8 U2 Lesson 6 - Let's Try It

Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

1. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

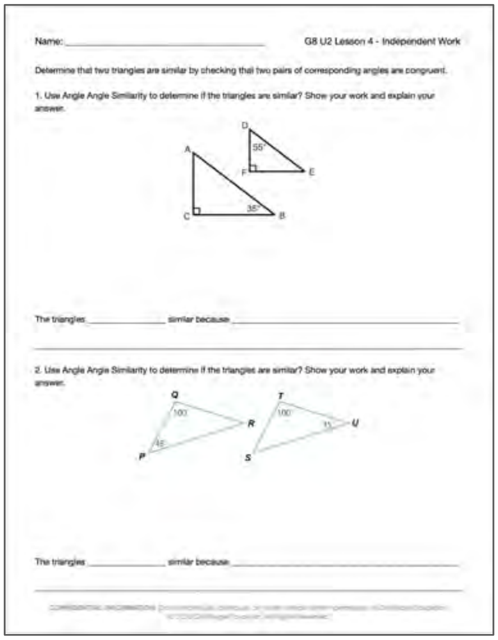
The triangles ____________ similar because ______________________

2. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

The triangles ____________ similar because ______________________

On your Own:

Now it's time to determine that two triangles are similar by checking that two pairs of corresponding angles are congruent on your own.



Name: ______________________ G8 U2 Lesson 4 - Independent Work

Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

1. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

The triangles __________ similar because ______________________________

____

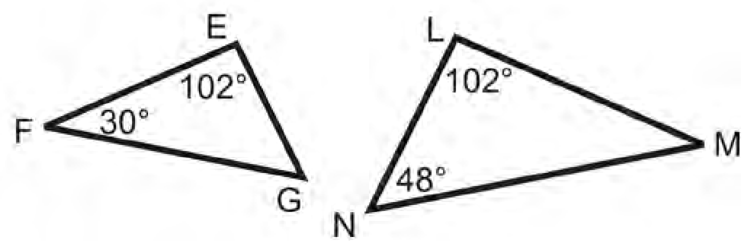
2. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

The triangles __________ similar because ______________________________

____

Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

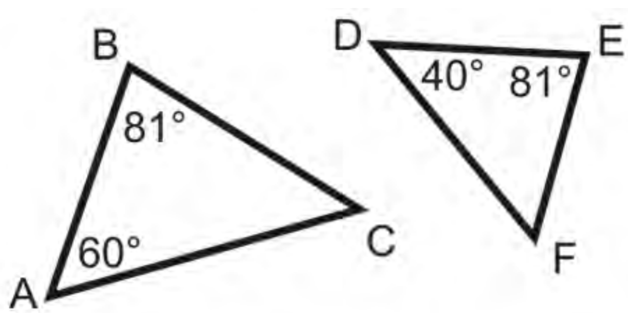
1. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.



The triangles ________________ similar because ____

____

2. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

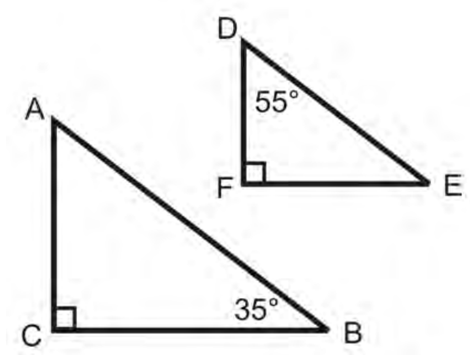


The triangles ________________ similar because ____

____

Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

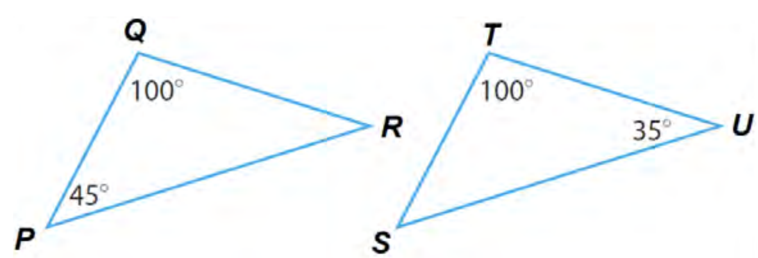
1. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.



The triangles _______________ similar because ____

____

2. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.



The triangles _______________ similar because ____

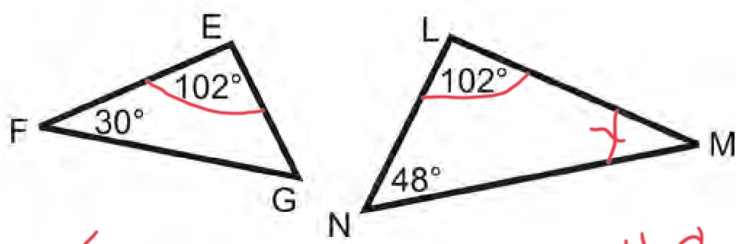
____

Name: Answer Key G8 U2 Lesson 5 - Let's Try It

Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

1. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

$\triangle FEG \approx \triangle MLN$?



$\angle E = \angle L$ ✓

$\angle F = \angle M$ ✓

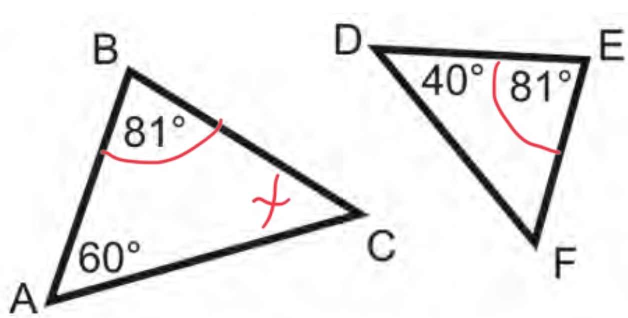
$48 + 102 + x = 180$

$150 + x = 180$

$x = 30$

The triangles are similar because at least two pairs of corresponding ∡s are NOT congruent.

2. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.



$60 + 81 + x = 180$

$141 + x = 180$

$x = 39$

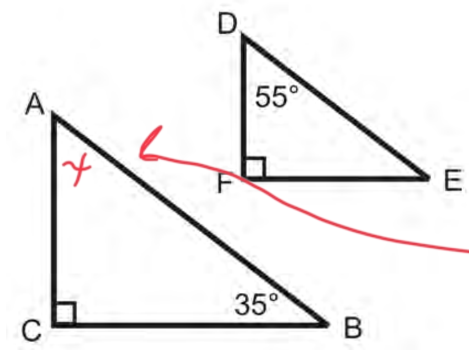
The triangles are not similar because at least two pairs of corresponding ∡s are NOT congruent.

Name: Answer Key

Determine that two triangles are similar by checking that two pairs of corresponding angles are congruent.

1. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

$\angle A = \angle D$ ✓
$\angle C = \angle F$ ✓

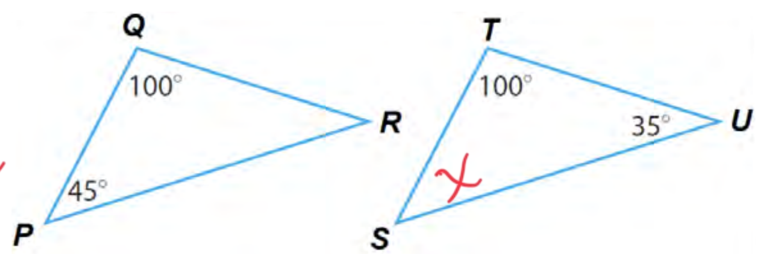


$90 + 35 + x = 180$
$125 + x = 180$
$x = 55°$

The triangles are similar because at least two pairs corresponding ∡s are congruent

2. Use Angle Angle Similarity to determine if the triangles are similar? Show your work and explain your answer.

$\angle Q = \angle T$ ✓
$\angle P = \angle S$ ✓



$35 + 100 + x = 180$
$135 + x = 180$
$x = 45$

The triangles are similar because at least two pair of corresponding ∡s are congruent.

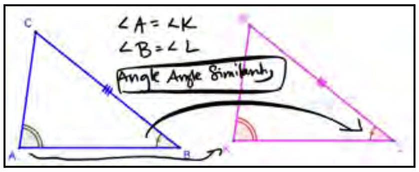
G8 U2 Lesson 6
Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

CITYTUTOR DC

G8 U2 Lesson 6 - Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

Warm Welcome (Slide 1): Tutor Choice

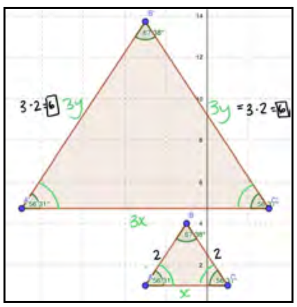
Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will calculate unknown side lengths in similar triangles using the scale factor between similar triangles.



You've learned that you can verify if two triangles are similar using angle angle similarity - that simply means you only need to know that two pairs of corresponding congruent angles in a triangle are congruent to determine if the triangles are similar. But, what about the sides of triangles?

Let's Talk (Slide 4): You also know that corresponding sides of similar figures are proportional. So what if you only know one or two sets of corresponding sides? How do you think we may find the missing sides?
Possible Students Answers, Key Points:

- We can find the scale factor of the other sides first.

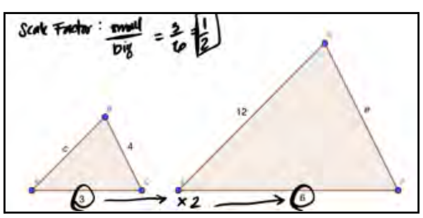


That's right. Let's consider triangle *ABC* and its image. We already know that the scale factor is 3. What if the value of *y* is 2. What is the value of the corresponding sides to *AB* and *BC*? Possible Students Answers, Key Points:

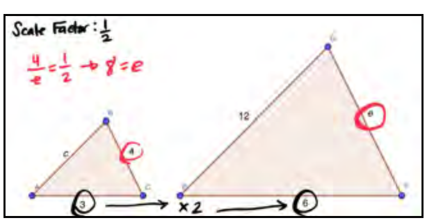
- Since the scale factor is 3, the two corresponding sides will be 6, because that's 3 x 2.

Great job! Yes - since we already knew the scale factor, we can say that the corresponding sides to side length *y* are 3 x 2 =6. *(Show you work.)*

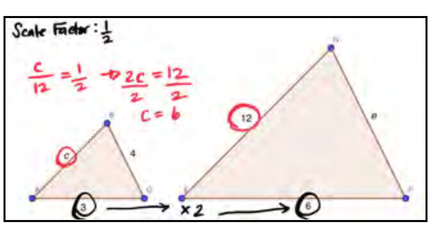
Let's Think (Slide 5): Now, let's consider a case of similar triangles where we don't know the scale factor for some missing sides. What will we do? First, we'll start by calculating the scale factor for at least one pair of corresponding sides. If we already know the triangles are similar, we don't need to verify more than one pair of sides.



In this pair of similar triangles, we know that segment AC is 3 and its corresponding segment is 6. The scale factor between those two, if we go from small to big, is ½. *(Show your work.)*



Now that we know the scale factor from the smaller triangle to the bigger triangle, we can calculate the measure of side *e*, the corresponding side to side *BC* which equals 4.



Finally, we can determine the value of *c* by using what we know about its corresponding side, *EG*, which equals 12.

Let's Try it (Slides 7-8): Let's calculate unknown side lengths in similar triangles using the scale factor between similar triangles. Remember, find the scale factor using two given corresponding sides of similar triangles first. Then you can use that scale factor to find the remaining sides.

WARM WELCOME

Calculate unknown side lengths in similar triangles using the scale factor between similar triangles

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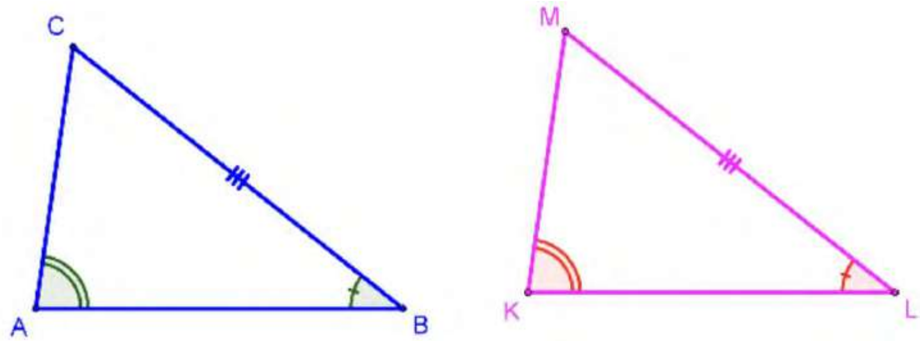
Let's Review:

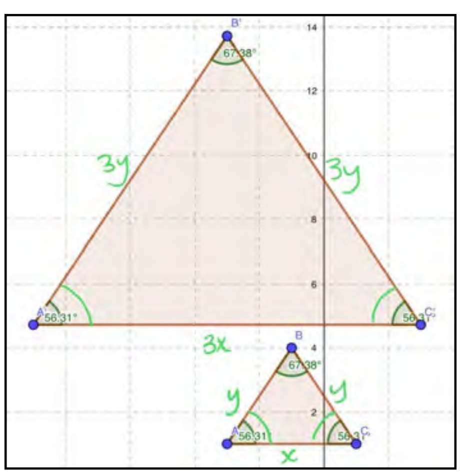
You can use Angle Angle Similarity to verify that two triangles are similar.



Let's Talk:

How do we calculate unknown side lengths in similar triangles using the scale factor between similar triangles?

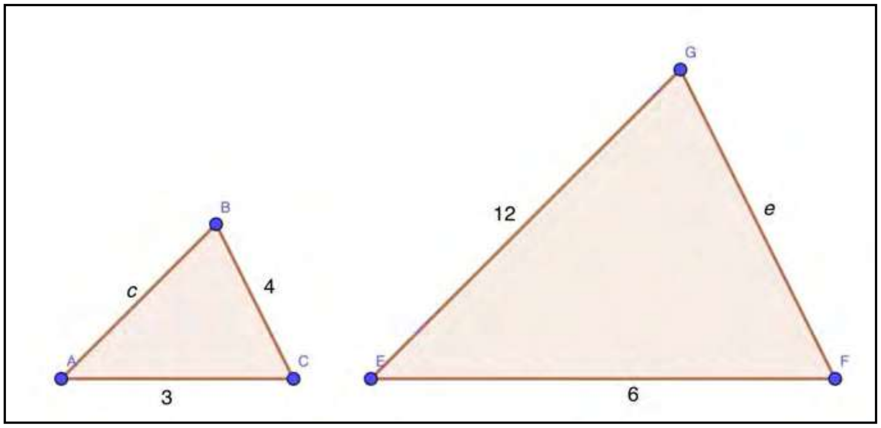
How can we verify that these triangles are similar?



Let's Think:

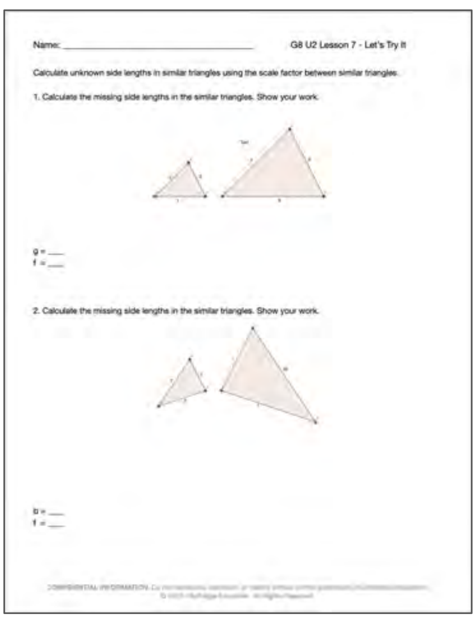
How do we calculate unknown side lengths in similar triangles using the scale factor between similar triangles?

Let's calculate the missing side length in these similar triangles.



Let's Try It:

Let's practice calculating unknown side lengths in similar triangles using the scale factor between similar triangles.



Name: ____________ G8 U2 Lesson 7 - Let's Try It

Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

1. Calculate the missing side lengths in the similar triangles. Show your work.

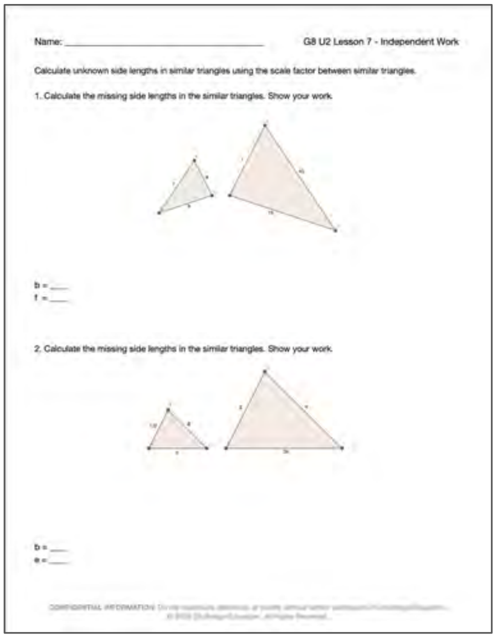
g = ___
f = ___

2. Calculate the missing side lengths in the similar triangles. Show your work.

b = ___
f = ___

On your Own:

Now it’s time to calculate unknown side lengths in similar triangle using hte scale factor between similar triangles on your own.



Name: ______________________ G8 U2 Lesson 7 - Independent Work

Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

1. Calculate the missing side lengths in the similar triangles. Show your work.

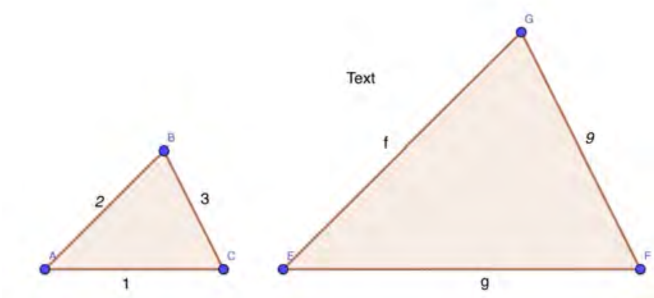
b = ____
f = ____

2. Calculate the missing side lengths in the similar triangles. Show your work.

b = ____
e = ____

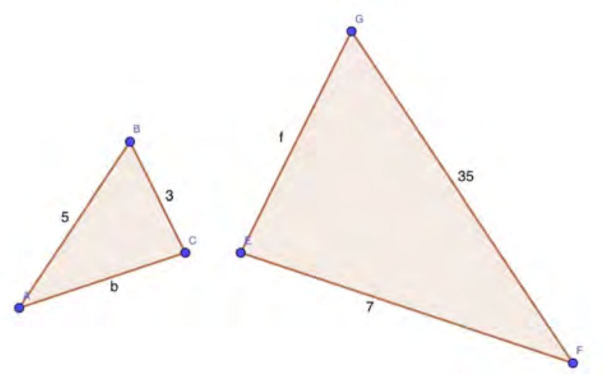
Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

1. Calculate the missing side lengths in the similar triangles. Show your work.



g = ____
f = ____

2. Calculate the missing side lengths in the similar triangles. Show your work.

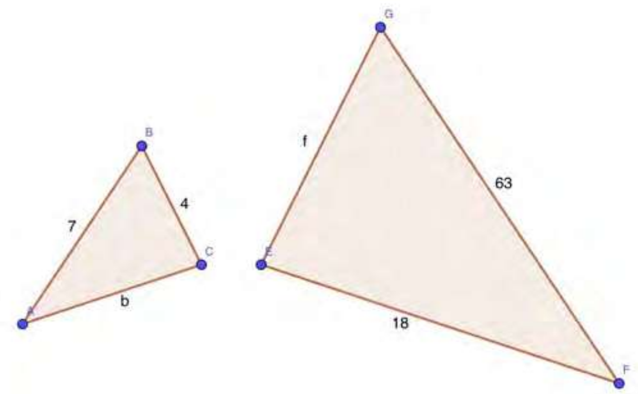


b = ____
f = ____

Name: ____ G8 U2 Lesson 6 - Independent Work

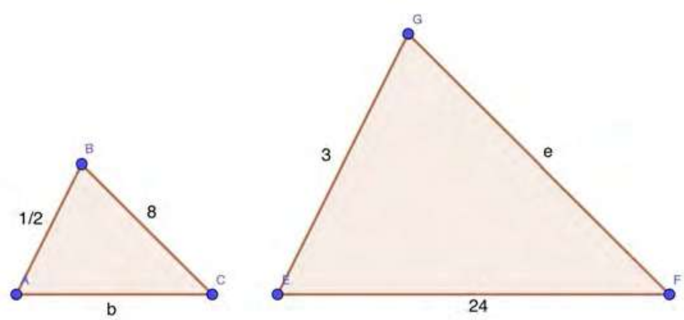
Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

1. Calculate the missing side lengths in the similar triangles. Show your work.



b = ____
f = ____

2. Calculate the missing side lengths in the similar triangles. Show your work.



b = ____
e = ____

Name: Answer Key

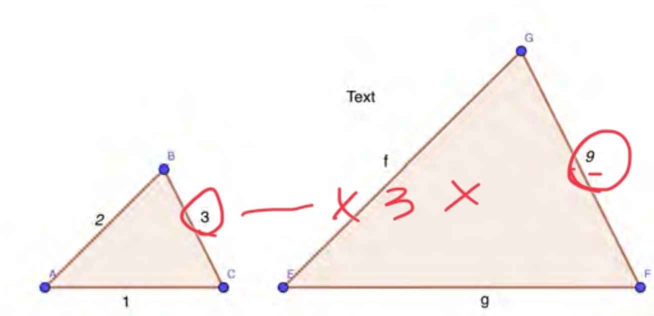
Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

1. Calculate the missing side lengths in the similar triangles. Show your work.

Scale Factor

$\frac{small}{big} = \frac{3}{9} = \frac{1}{3}$

$\frac{big}{small} = \frac{9}{3} = 3$



g = 3

f = 6

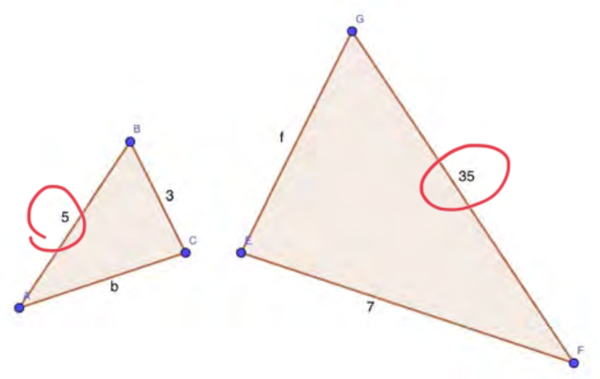
$1 \times 3 = g$

$2 \times 3 = f$

2. Calculate the missing side lengths in the similar triangles. Show your work.

$\frac{small}{big} = \frac{5}{35} = \frac{1}{7}$

$\frac{big}{small} = \frac{35}{5} = 7$



b = 1

f = 21

$b \times 7 = 7$

$b = 1$

$3 \times 7 = f$

$21 = f$

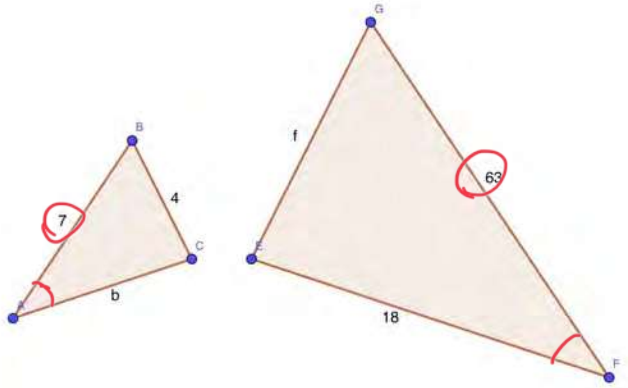
Name: Answer Key

Calculate unknown side lengths in similar triangles using the scale factor between similar triangles.

1. Calculate the missing side lengths in the similar triangles. Show your work.

$\frac{\text{small}}{\text{big}} = \frac{7}{63} = \frac{1}{9}$

$\frac{\text{big}}{\text{small}} = \frac{63}{7} = 9$



$b \times 9 = 18$

$b = 2$

$4 \times 9 = f$

$36 = f$

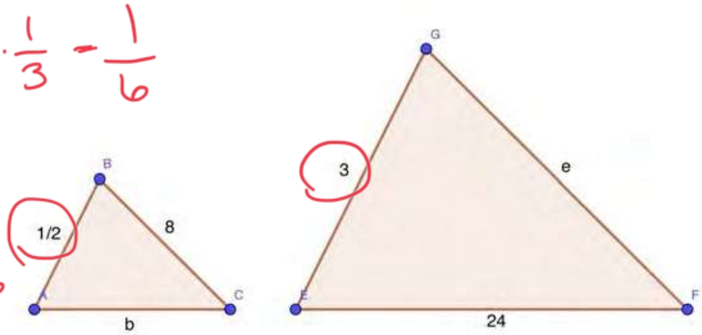
b = 2

f = 36

2. Calculate the missing side lengths in the similar triangles. Show your work.

$\frac{\text{small}}{\text{big}} = \frac{\frac{1}{2}}{3} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$\frac{\text{big}}{\text{small}} = \frac{3}{\frac{1}{2}} = 3 \cdot \frac{2}{1} = 6$



$b \times 6 = 24$

$b = 4$

$8 \times 6 = e$

$48 = e$

b = 4

e = 48

G8 U2 Lesson 7
Understand that the quotients of pairs of side lengths in similar triangles are equal.

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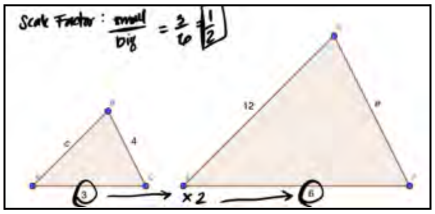
G8 U2 Lesson 7 - Understand that quotients of pairs of side lengths in similar triangles are equal.

Warm Welcome (Slide 1): Tutor Choice

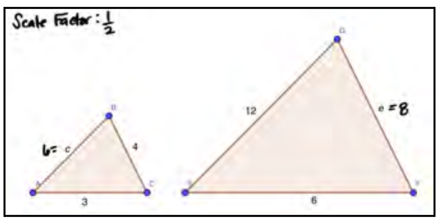
Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will demonstrate our understanding that quotients of pairs of side lengths in similar triangles are equal.



In our last lesson, we calculated a scale factor and used that scale factor to find the missing sides of similar triangles. What about quotients from the same triangle? We'll look at some triangles that we've already seen and use the measures we calculated to make a determination about their quotients.

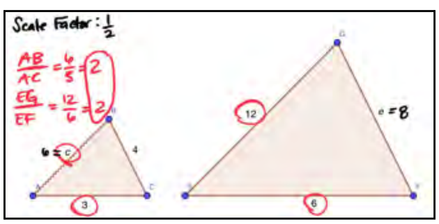
Let's Talk (Slide 4): Consider the same set of triangles. First let's find the side lengths again. If the scale factor is ½, what the value of *c* and *e*? Possible Students Answers, Key Points:

- c = 6 because 6 is half of 12.
- e = 8 because half of 8 is 4.

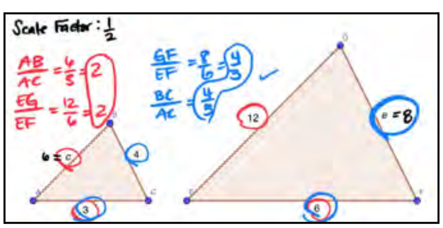


That's right! C = 6 and e = 8. Using a scale factor of 1/2, we determined the value of the missing side lengths in two pairs of triangles. *(Write the scale factor and the value of the missing sides that your students shared.)*

Let's Think (Slide 5): Now, let's use the values in the same triangle. There is something true about how the sides in each individual triangle relate to each other as compared to the same set of sides in the other triangle.



First, let's calculate the quotient of AB/AC and the corresponding sides EG/EF. *(Write the quotients and fill in their values.)* We know that 6/3 is 2 and 12/6 is 2. So the quotients of these corresponding sides is equal. Hmmm… Let's check that with a different set, just to be sure.



Let's calculate the quotients of GF/EF and BC/AC. 8/6 is 4/3 and the smaller triangle is already simplified as 4/3. Success!

Let's Try it (Slides 7-8): Let's demonstrate our understanding that the quotients of pairs of side lengths in similar triangles are equal. Remember, make sure you set up your quotes big over big or small over small since we're comparing the quotients of side lengths in one triangle to a triangle in which it is similar.

WARM WELCOME

Understand that the quotients of pairs of side lengths in similar triangles are equal.

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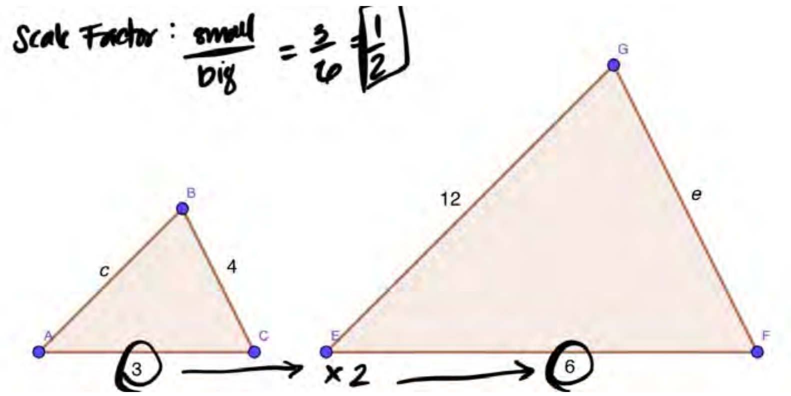
Let's Review:

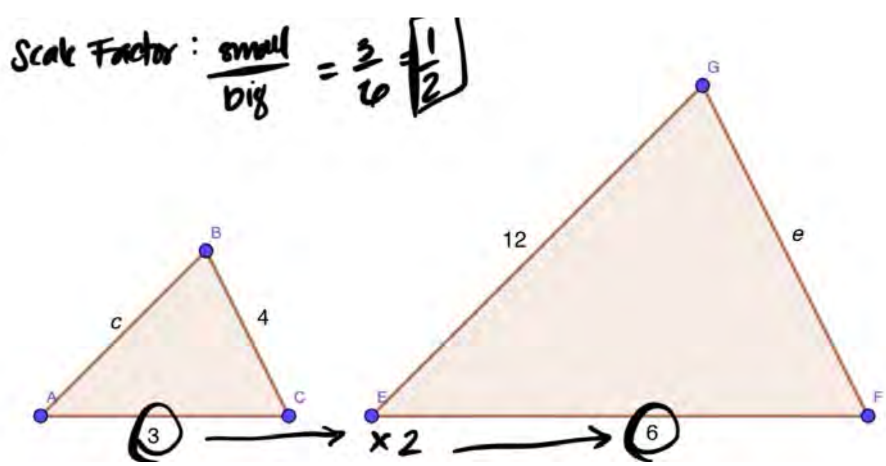
You can use a scale factor to determine the missing sides of similar triangles.



Let's Talk:

What is true about the quotients of pairs of side lengths in similar triangles?

What are the side lengths of the missing sides?



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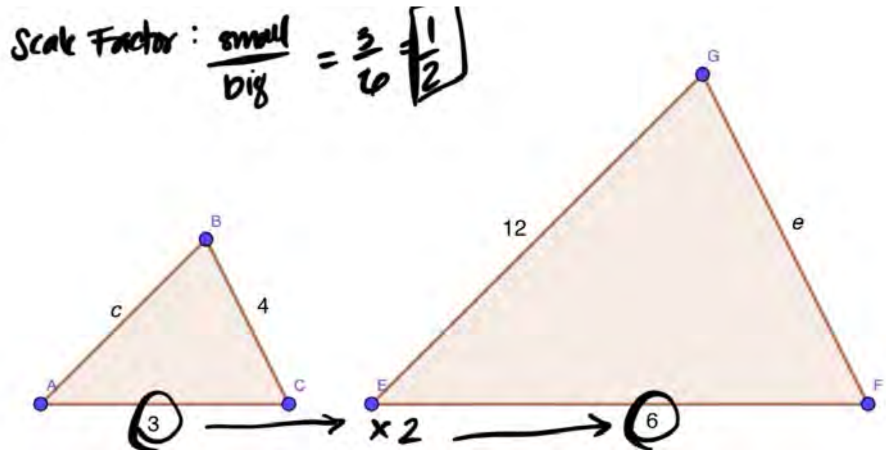
Let's Think:

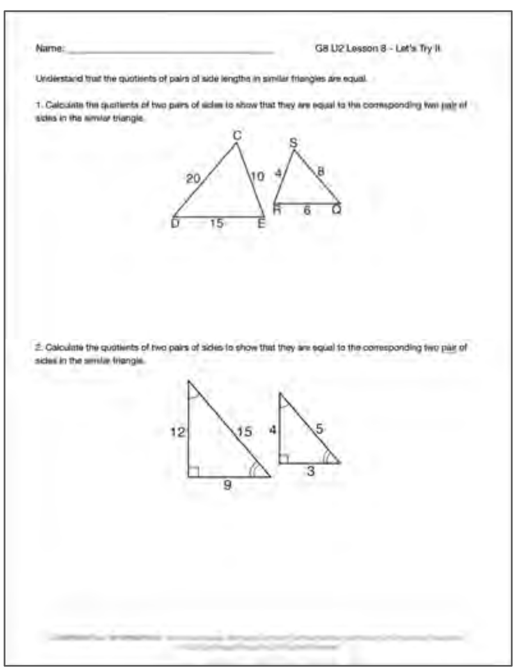
What is true about the quotients of pairs of side lengths in similar triangles?

Calculate the quotients of pairs of side lengths in similar triangles. What do you notice?



Let's Try It:

Let's practice demonstrating our understanding that quotients of pairs of side lengths in similar triangles are equal.



Name: ____________________ G8 U2 Lesson 8 - Let's Try It

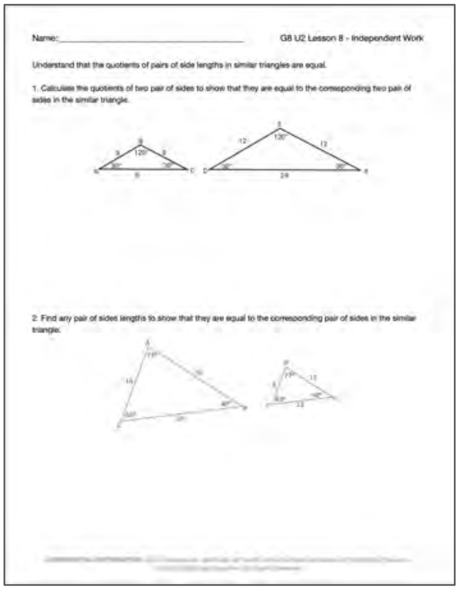
Understand that the quotients of pairs of side lengths in similar triangles are equal.

1. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pair of sides in the similar triangle.

2. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pair of sides in the similar triangle.

On your Own:

Now it's time to demonstrate our your understanding that hte quotients of pairs of side lengths in similar triangles are equal on your own.



Name:______________________________ G8 U2 Lesson 8 - Independent Work

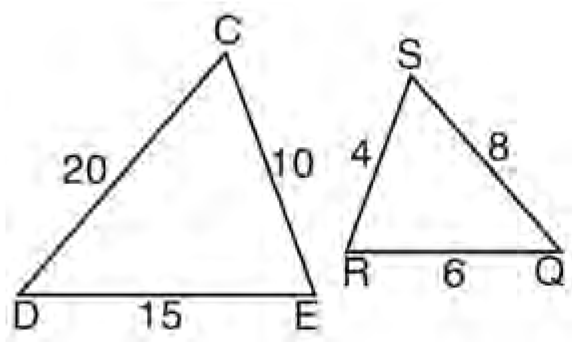
Understand that the quotients of pairs of side lengths in similar triangles are equal.

1. Calculate the quotients of two pair of sides to show that they are equal to the corresponding two pair of sides in the similar triangle.

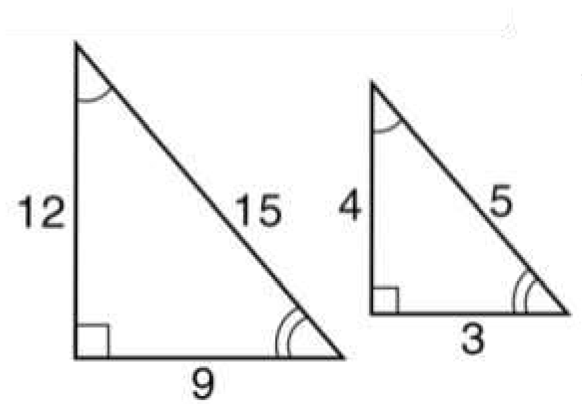
2. Find any pair of sides lengths to show that they are equal to the corresponding pair of sides in the similar triangle.

Understand that the quotients of pairs of side lengths in similar triangles are equal.

1. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.



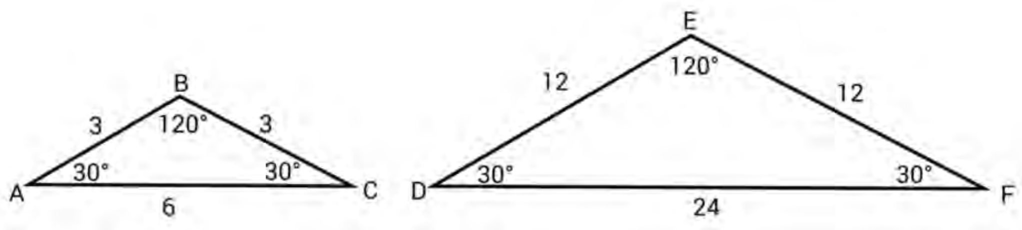
2. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.



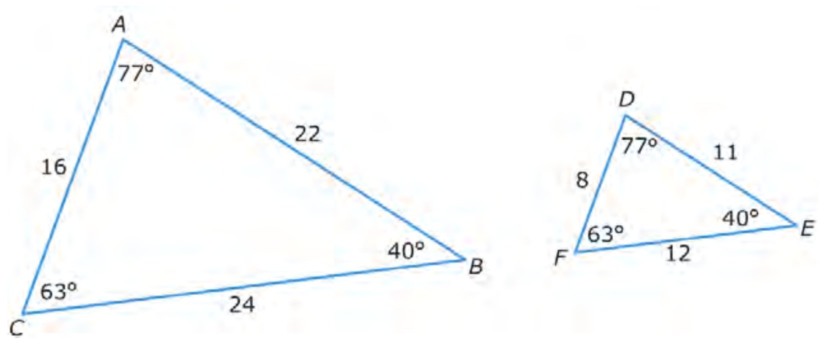
Name: ____ G8 U2 Lesson 7 - Independent Work

Understand that the quotients of pairs of side lengths in similar triangles are equal.

1. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.



2. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.



Name: Answer Key

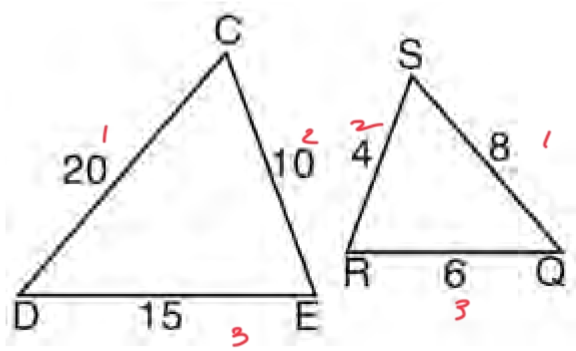
G8 U2 Lesson 7 - Let's Try It

Understand that the quotients of pairs of side lengths in similar triangles are equal.

1. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.

$\frac{CD}{CE} = \frac{20}{10} = 2$ ✓

$\frac{SQ}{SR} = \frac{8}{4} = 2$ ✓



$\frac{DE}{CE} = \frac{15}{10} = \frac{3}{2}$ ✓

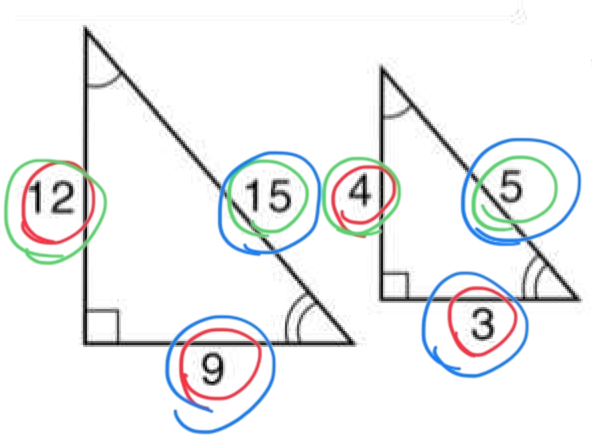
$\frac{QR}{SR} = \frac{6}{4} = \frac{3}{2}$ ✓

*Third Set

$\frac{DE}{CD} = \frac{15}{20} = \frac{3}{4}$ ✓

$\frac{QR}{SQ} = \frac{6}{8} = \frac{3}{4}$ ✓

2. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.



$\frac{12}{9} = \frac{4}{3}$ ✓

$\frac{4}{3}$ ✓

$\frac{12}{15} = \frac{4}{5}$ ✓

$\frac{4}{5}$ ✓

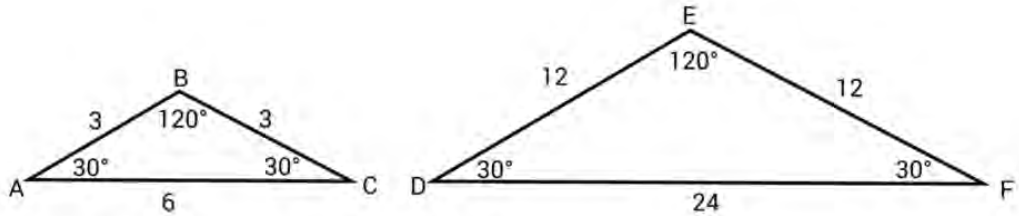
$\frac{9}{15} = \frac{3}{5}$ ✓

$\frac{3}{5}$ ✓

Name: Answer Key

Understand that the quotients of pairs of side lengths in similar triangles are equal.

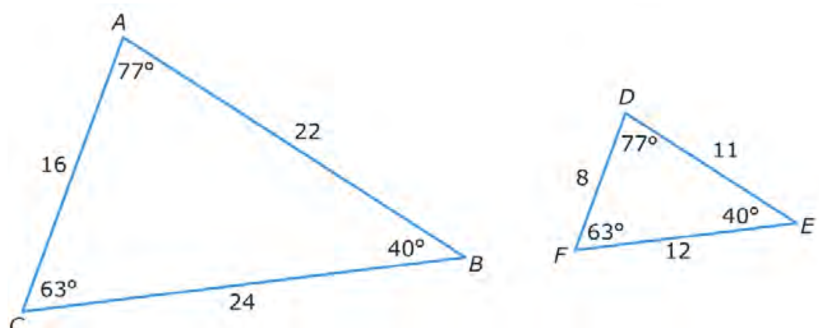
1. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.



$\frac{AB}{BC} = \frac{3}{3} = 1$ ✓ | $\frac{BC}{AC} = \frac{3}{6} = \frac{1}{2}$ ✓ | $\frac{AC}{AB} = \frac{6}{3} = 2$ ✓

$\frac{DE}{EF} = \frac{12}{12} = 1$ ✓ | $\frac{EF}{DF} = \frac{12}{24} = \frac{1}{2}$ ✓ | $\frac{DF}{DE} = \frac{24}{12} = 2$ ✓

2. Calculate the quotients of two pairs of sides to show that they are equal to the corresponding two pairs of sides in the similar triangle.



$\frac{AC}{AB} = \frac{16}{22} = \frac{8}{11}$ ✓ | $\frac{AB}{BC} = \frac{22}{24} = \frac{11}{12}$ ✓ | $\frac{BC}{AC} = \frac{24}{16} = \frac{3}{2}$ ✓

$\frac{DF}{DE} = \frac{8}{11}$ ✓ | $\frac{DE}{EF} = \frac{11}{12}$ ✓ | $\frac{EF}{DF} = \frac{12}{8} = \frac{3}{2}$ ✓

G8 U2 Lesson 8
Find the slope of a line on a grid using properties of slope triangles.

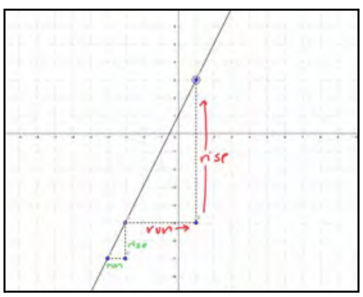
G8 U2 Lesson 9 - Find the slope of a line on a grid using properties of slope triangles.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will use what we know about triangles to help us to calculate the slope of a line. First, think about when you've heard about the word slope in real life. If you've ever gone snowboarding, you've likely heard someone say, "go up the hill" or "go down the hill." At many ski resorts where they teach you to ski, they'll instead say, "let's hit the slopes."

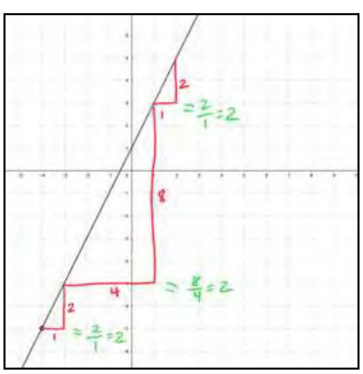
A slope is another word to identify how much something rises over a particular distance or how much something declines over a particular distance, like any hill you might use to have your winter fun. *(Draw the run/distance, decline arrow, and slope to show students the relationship between skiing and the concept of slope.)*

Let's Talk (Slide 4): Let's consider the slope of a line on a grid. How do you think we can calculate the slope?
Possible Students Answers, Key Points:
- We can use rise over run.



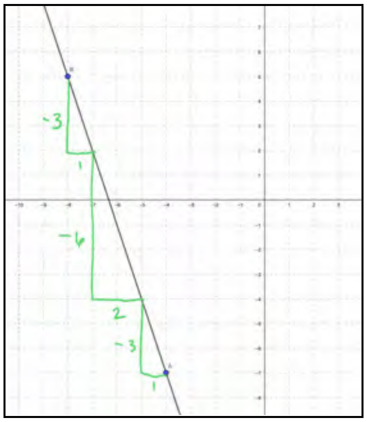
Yes. Rise over run is a trick we often learn to remember how to calculate the slope of a line on a grid. But did you know that the reason that works is because the rise and the run create multiple similar triangles. And what we know about similar triangles is that their corresponding side lengths are proportional. Remember that the quotients of pairs of sides on similar triangles are equal. *(Draw some similar triangles that you can connect to the line and show the quotient of the sides so students can see that they are similar.)*

Let's Think (Slide 5): Now, take the line of $y = 2x + 1$ for example. Let's use slope triangles to calculate the slope of the line.

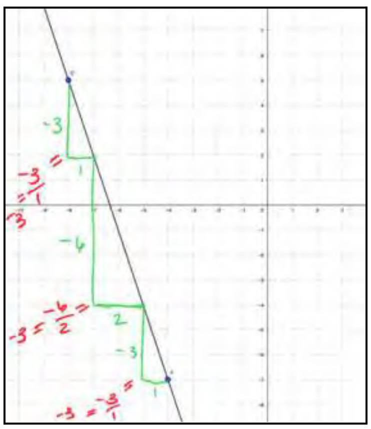


Let's first identify the rise and run in each triangle. In this case, the slope is increasing so we know it's a rise instead of a decline. *(Draw the slope triangles - try to vary the sizes - and label the number of units across and up.)* That's it - it's that simple. The slope of this line is 2 and we use quotients of similar triangles and slope triangles to figure it out.

Let's Think (Slide 6): Let's try again but with a line that is declining.



Like before, let's start by drawing triangles to identify the decline. In this case, because there is a decline rather than a rise, I am writing the numbers as negative to show that direction. *(Draw the triangles and label the sides.)*



Now, like in the previous example, we'll calculate the quotient of the sides of our similar triangles to determine the quotient. That quotient is the slope/

Let's Try it (Slides 7): Let's work on using what we know about slope triangles to calculate the slope of lines. Remember, you can either rise or decline but the run or distance across will always be positive and go from left to right.

WARM WELCOME

Find the slope of a line on a grid using properties of slope triangles.

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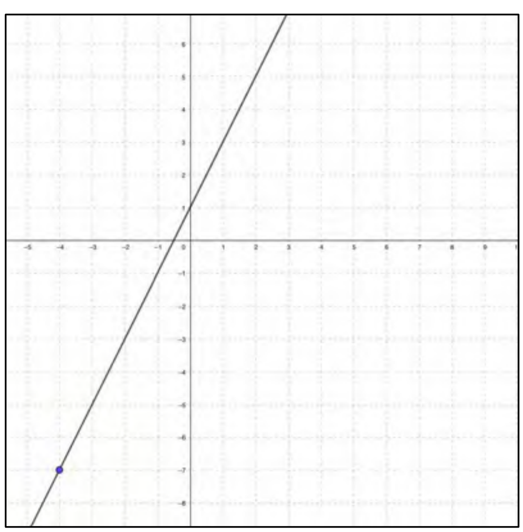
Let's Review:

A slope is another word to identify how much something rises over a particular distance or how much something declines over a particular distance.

Let's Talk:

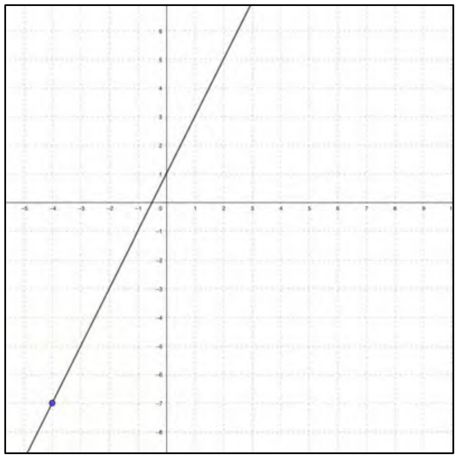
How do slope triangles help us to calculate the slope of a line?

Identify the rise and run of the line by creating multiple similar triangles.



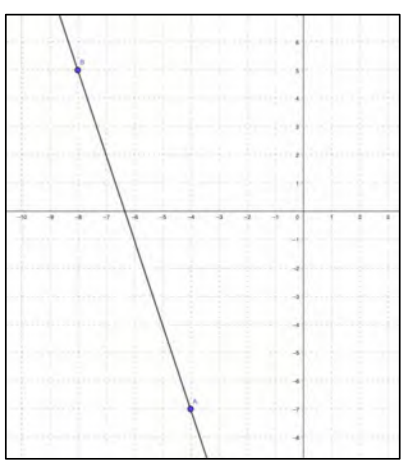
Let's Think: **How do slope triangles help us to calculate the slope of a line?**

Use the similar triangles and the quotients of their side lengths to calculate the slope of each line.



Let's Think: **How do slope triangles help us to calculate the slope of a line?**

Use the similar triangles and the quotients of their side lengths to calculate the slope of each line.



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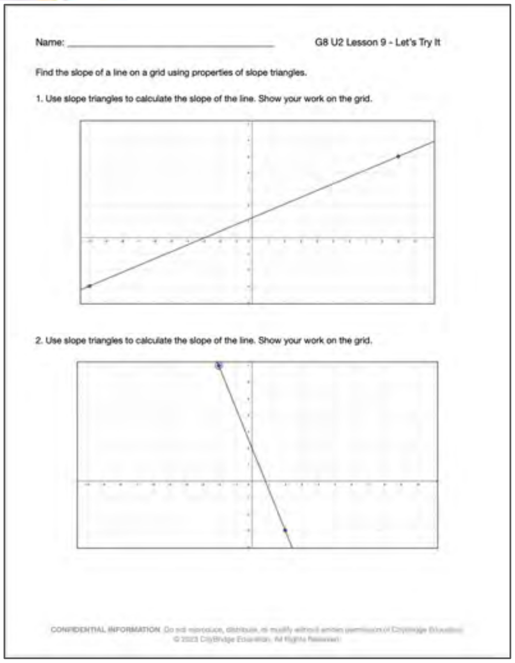
Let's Try It:

Let's practice using slope triangles to calculate the slope of lines.


Name: ______________________ G8 U2 Lesson 9 - Let's Try It

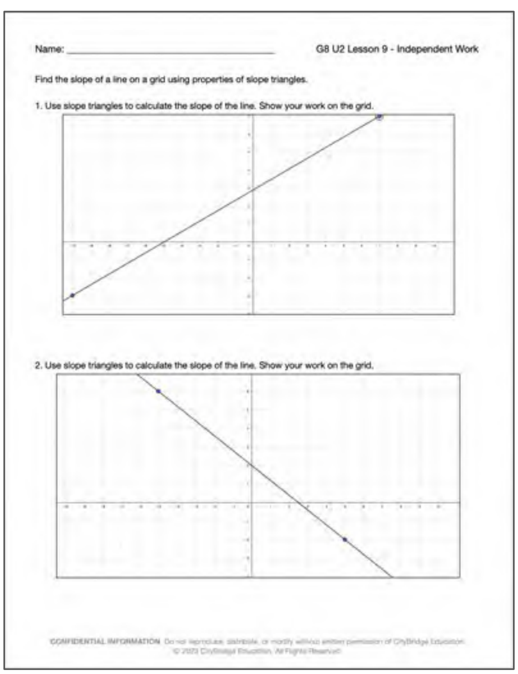
Find the slope of a line on a grid using properties of slope triangles.

1. Use slope triangles to calculate the slope of the line. Show your work on the grid.

2. Use slope triangles to calculate the slope of the line. Show your work on the grid.

On your Own:

Now it's time to practice using slope triangles to calculate the slope of a line on your own.


Name: ______________________ G8 U2 Lesson 9 - Independent Work

Find the slope of a line on a grid using properties of slope triangles.

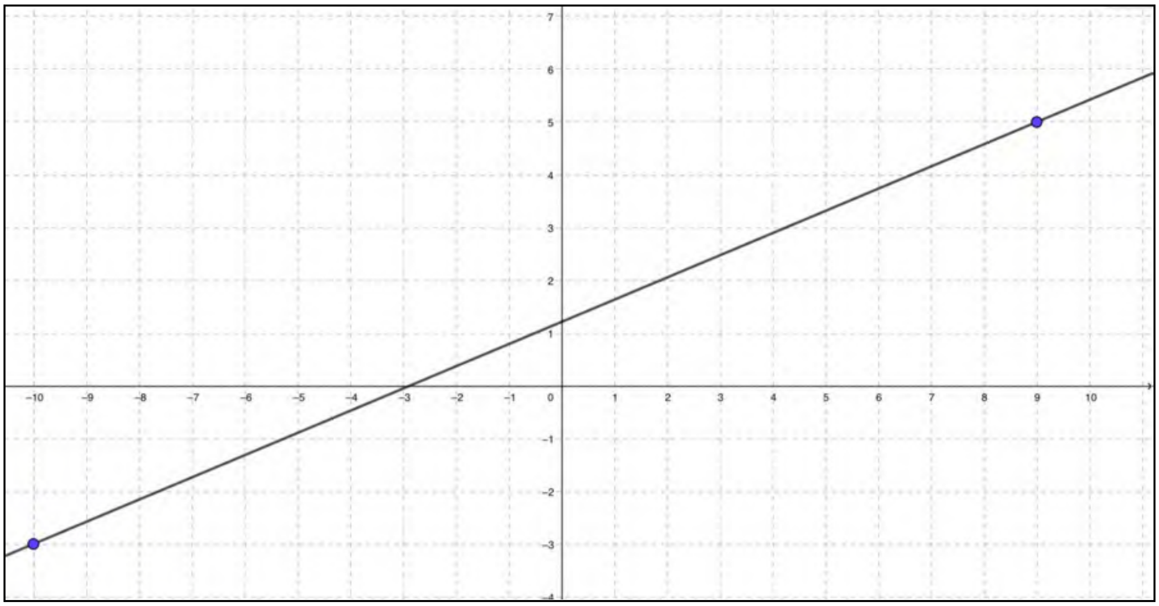
1. Use slope triangles to calculate the slope of the line. Show your work on the grid.

2. Use slope triangles to calculate the slope of the line. Show your work on the grid.

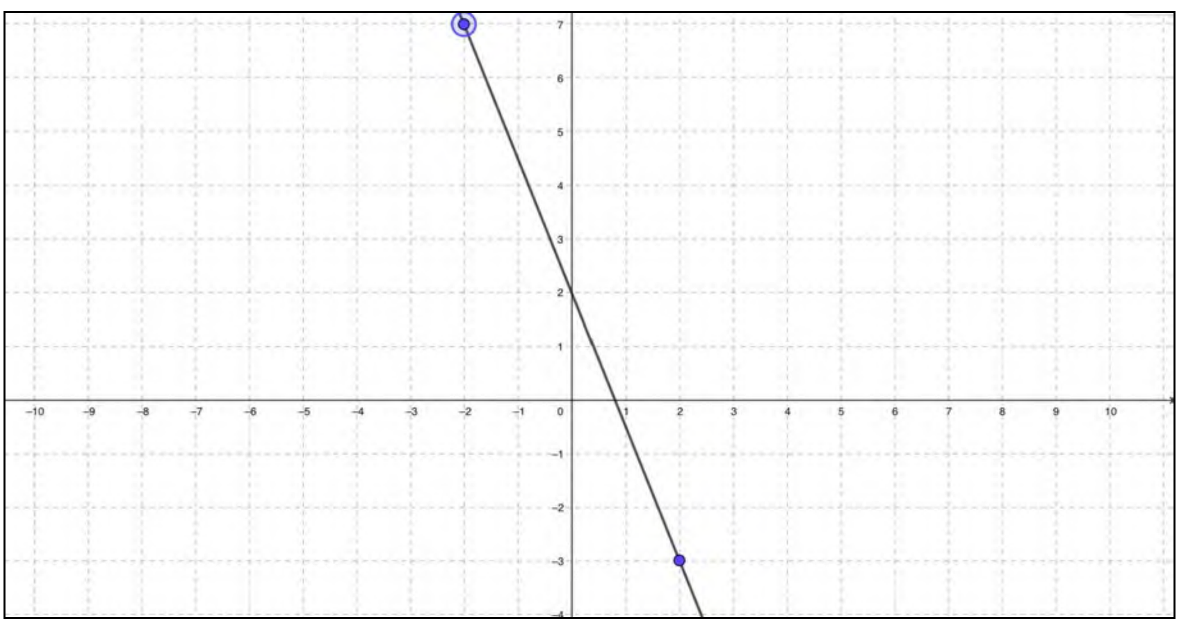
Name: ____ G8 U2 Lesson 8 - Let's Try It

Find the slope of a line on a grid using properties of slope triangles.

1. Use slope triangles to calculate the slope of the line. Show your work on the grid.



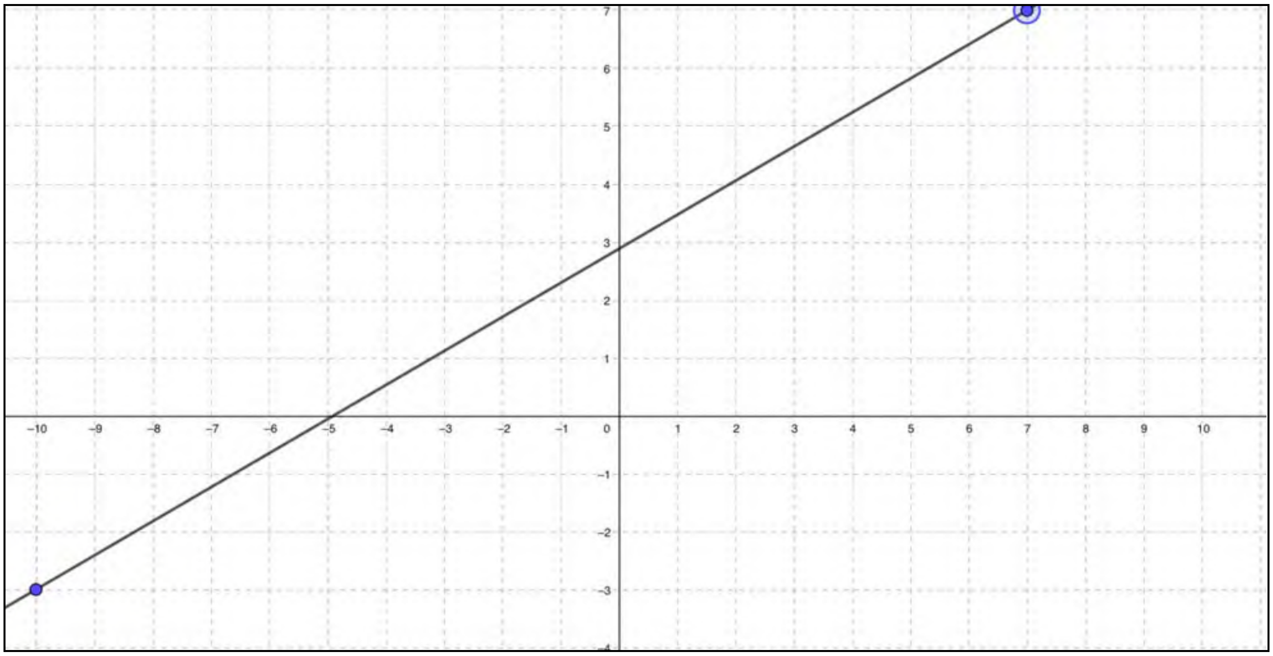
2. Use slope triangles to calculate the slope of the line. Show your work on the grid.



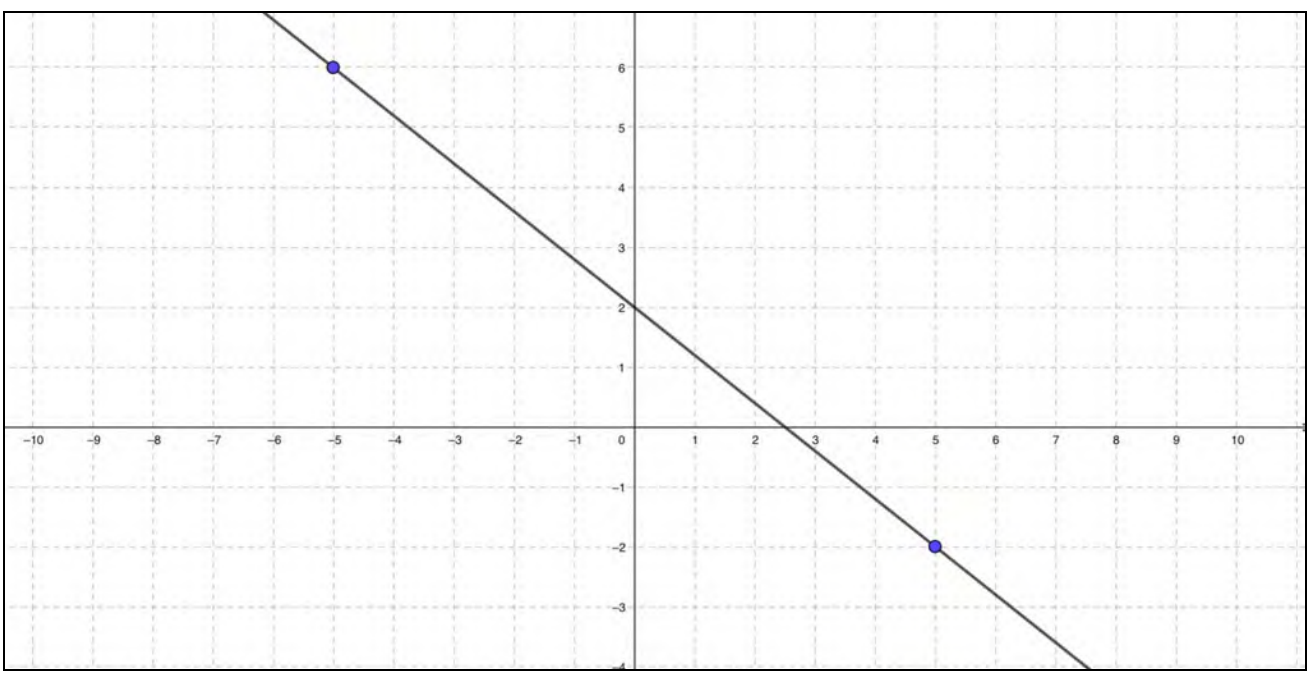
Name: ______________________________ G8 U2 Lesson 8 - Independent Work

Find the slope of a line on a grid using properties of slope triangles.

1. Use slope triangles to calculate the slope of the line. Show your work on the grid.



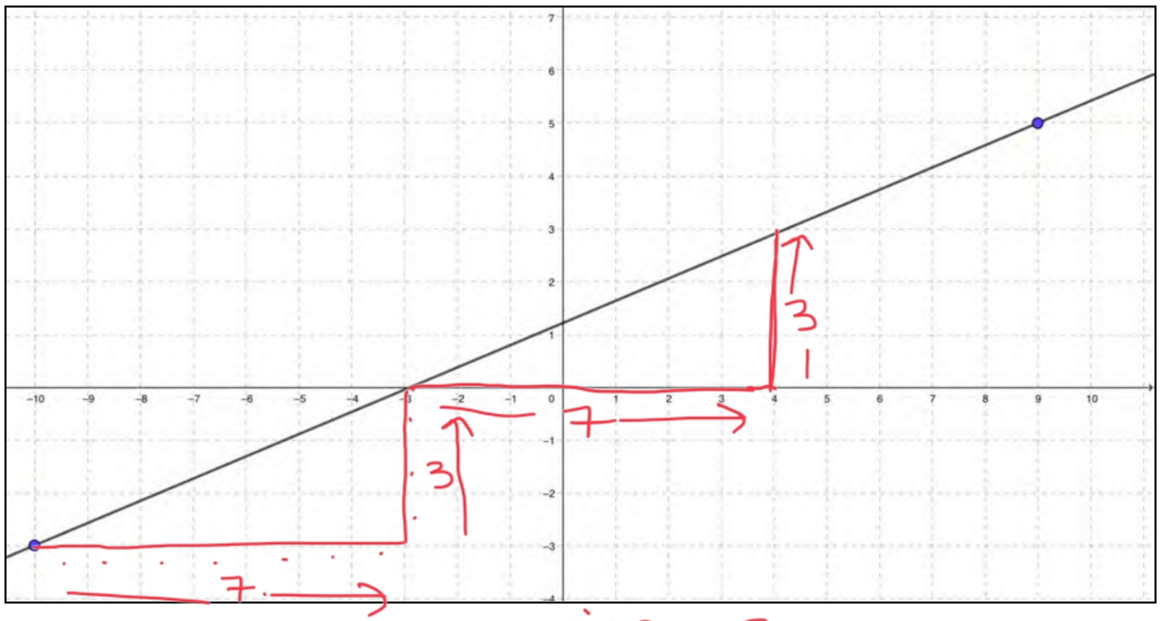
2. Use slope triangles to calculate the slope of the line. Show your work on the grid.



Name: Answer Key

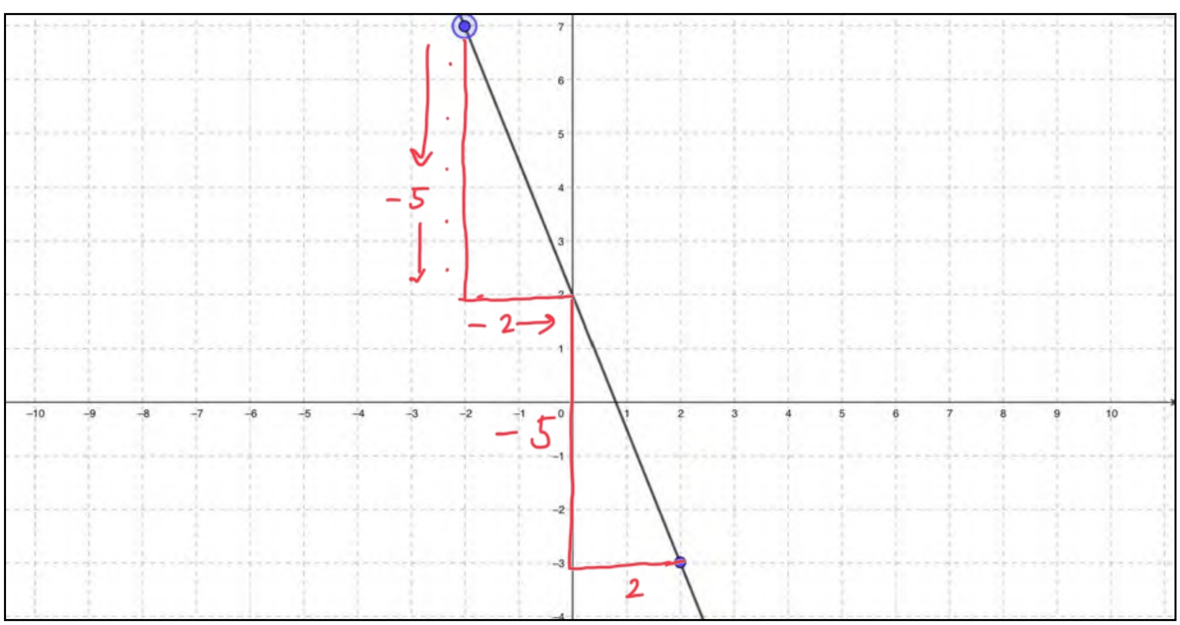
Find the slope of a line on a grid using properties of slope triangles.

1. Use slope triangles to calculate the slope of the line. Show your work on the grid.



slope $= \frac{\text{rise}}{\text{run}} = \frac{3}{7}$

2. Use slope triangles to calculate the slope of the line. Show your work on the grid.

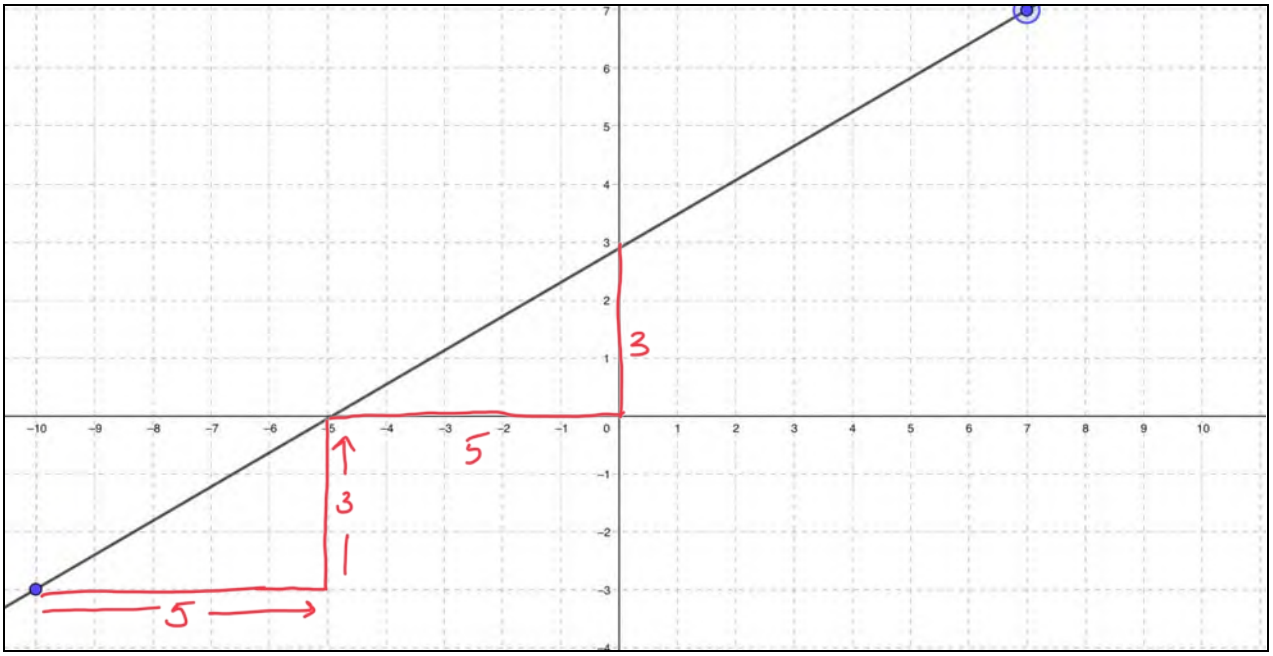


slope $= \frac{\text{rise}}{\text{run}} = -\frac{5}{2}$

Name: Answer Key

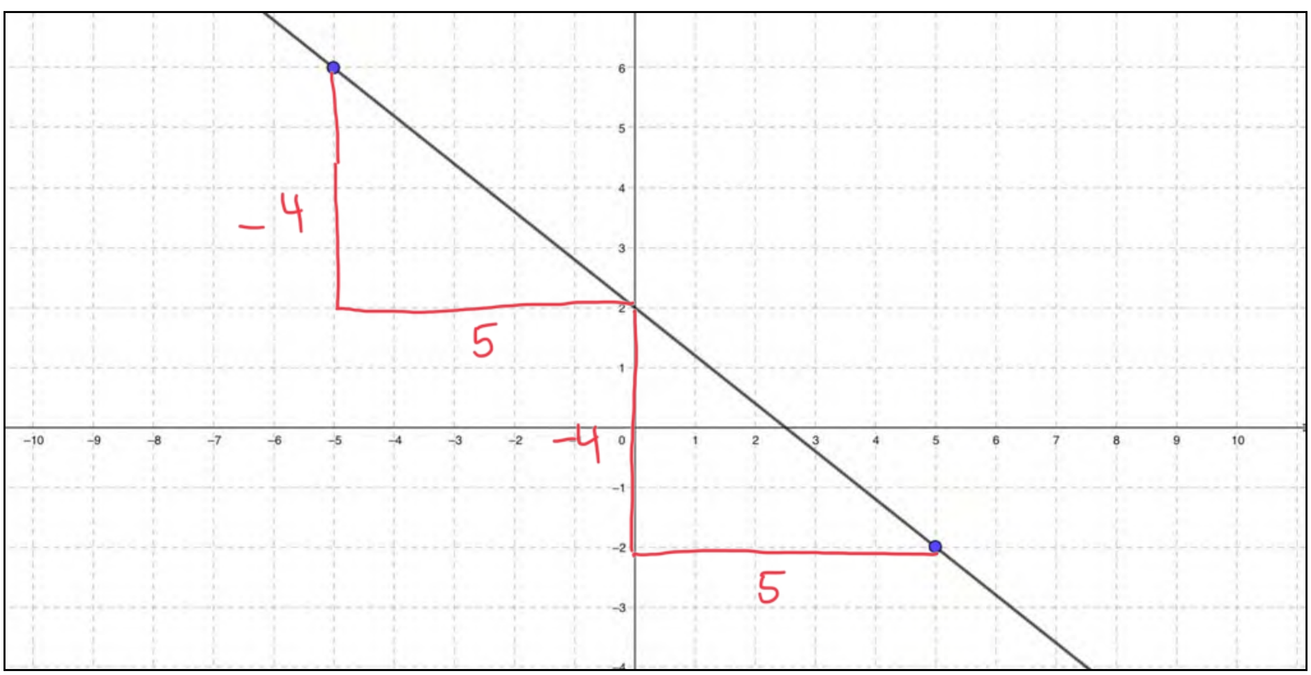
Find the slope of a line on a grid using properties of slope triangles.

1. Use slope triangles to calculate the slope of the line. Show your work on the grid.



slope = $\frac{3}{5}$

2. Use slope triangles to calculate the slope of the line. Show your work on the grid.



slope = $\frac{-4}{5}$

G8 U2 Lesson 9
Write an equation for a line.

CITYTUTOR DC

G8 U2 Lesson 9 - Write an equation for a line.

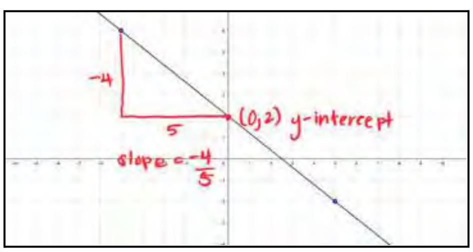
Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will write equations of lines.

Remember from the last time, a slope is another word to identify how much something rises over a particular distance or how much something declines over a particular distance, like any hill you might use to have your winter fun. *(Draw the run/distance, decline arrow, and slope to show students the relationship between skiing and the concept of slope.)*

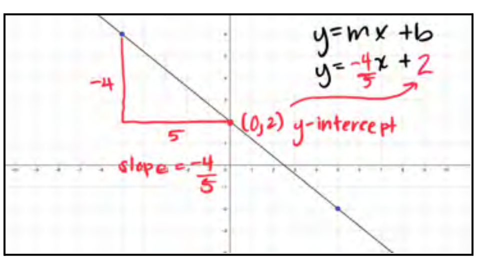
Let's Talk (Slide 4): In order to write an equation for a line, we need to know its slope and its *y*-intercept. Doe you know what a *y*-intercept is? Even if you don't, think about what the word intercept might mean and think about what that might mean in the case of a line? Possible Students Answers, Key Points:

- The *y*-intercept is where the line crosses the *y*-axis.
- When you intercept something you cross it or block it.



That's right. So, in this case. We want to find the place where the line crosses the *y*-axis. We will also use our slope triangles to find the slope so we can write a final equation. *(Put a point on the y-intercept and label its coordinates. Use slop triangles to identify the slope.)*

Let's Think (Slide 5): The formula to find the equation of a line from a graph is $y = mx + b$ where m is the slope and b is the y-intercept. So just by knowing those two things, we can simply write the equation of a line. Let's try it.



We already identified the slope is $-4/5$ and we know the *y*-intercept is 2. I know it's 2 because the 0 in the coordinate set represents the *x*-coordinate. *(Write the formula to write the equation for the line and substitute the slope and the y-intercept.)* Success!!

Let's Try it (Slides 7): Let's work on using what we know about slope triangles to calculate the slope of lines. Remember, you can either rise or decline but the run or distance across will always be positive and go from left to right.

WARM WELCOME

Write an equation for a line.

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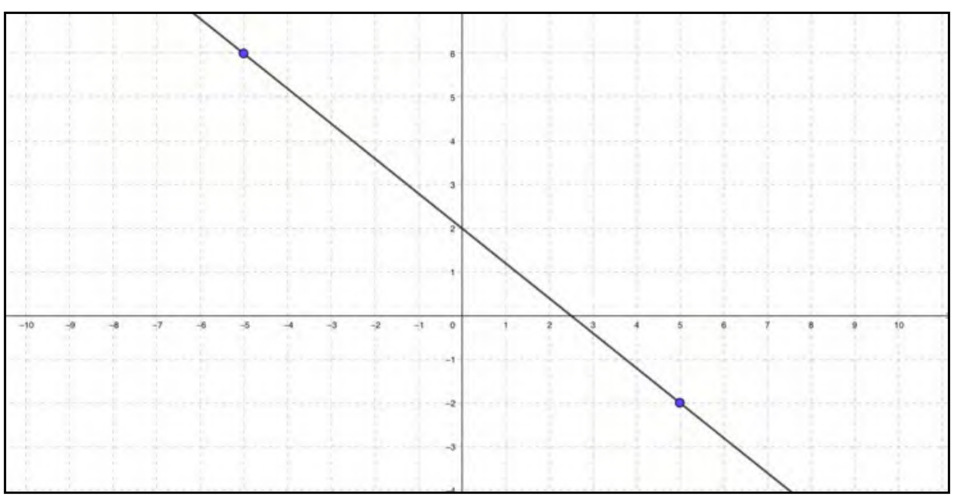
Let's Review:

A slope is another word to identify how much something rises over a particular distance or how much something declines over a particular distance.

Let's Talk:

How do you use slope and y-intercepts to write the equation for a line?

Identify the slope and *y*-intercept.



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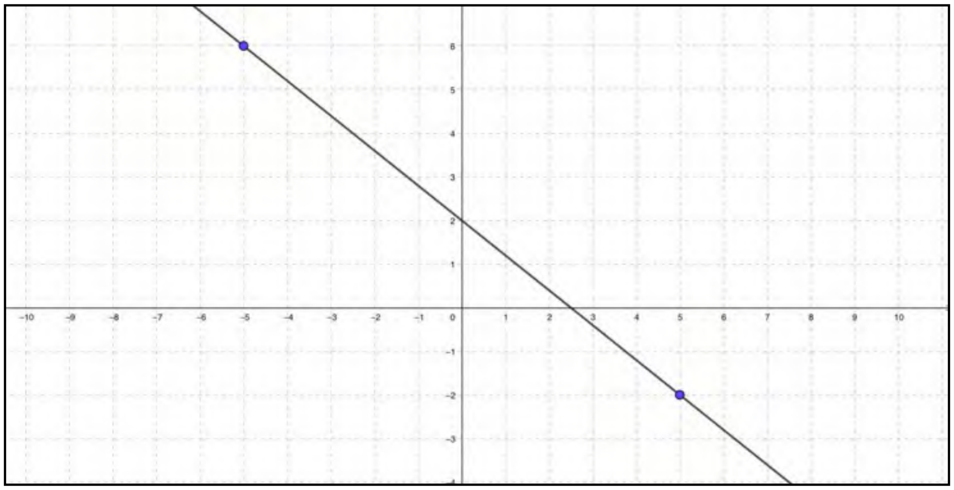
Let's Think:

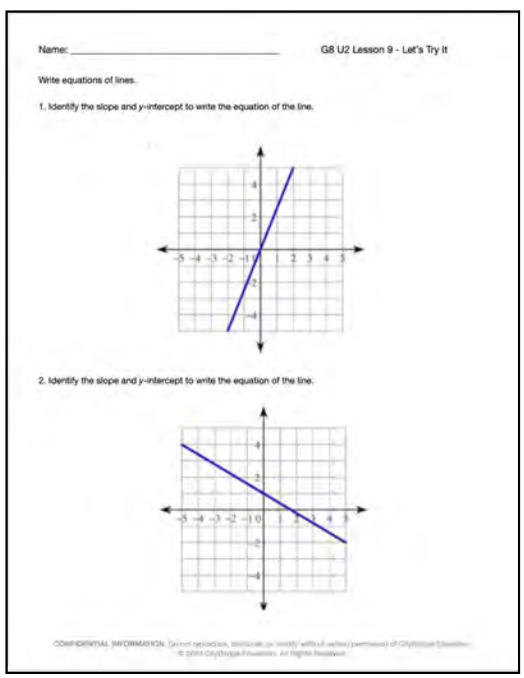
How do you use slope and y-intercepts to write the equation for a line?

Write the equation of a line using slope-intercept form, *y*=m*x* + b.



Let's Try It:

Let's practice writing equations of lines.



Name: ______________________ G8 U2 Lesson 9 - Let's Try It

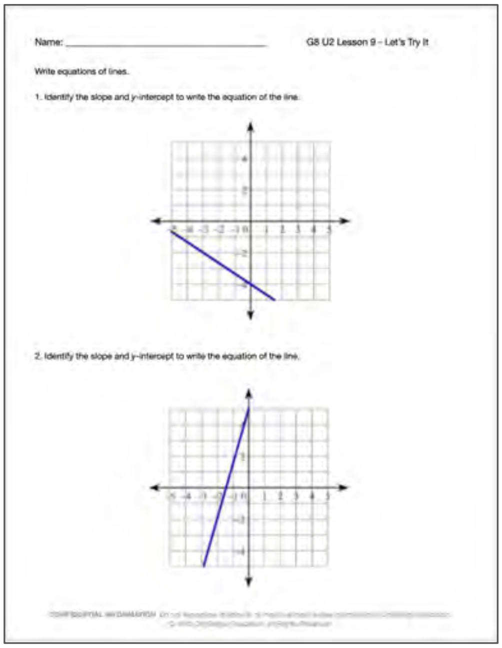
Write equations of lines.

1. Identify the slope and y-intercept to write the equation of the line.

2. Identify the slope and y-intercept to write the equation of the line.

On your Own:

Now it’s time to practice writing equations of lines on your own.


Name: ______________________ G8 U2 Lesson 9 – Let’s Try It

Write equations of lines.

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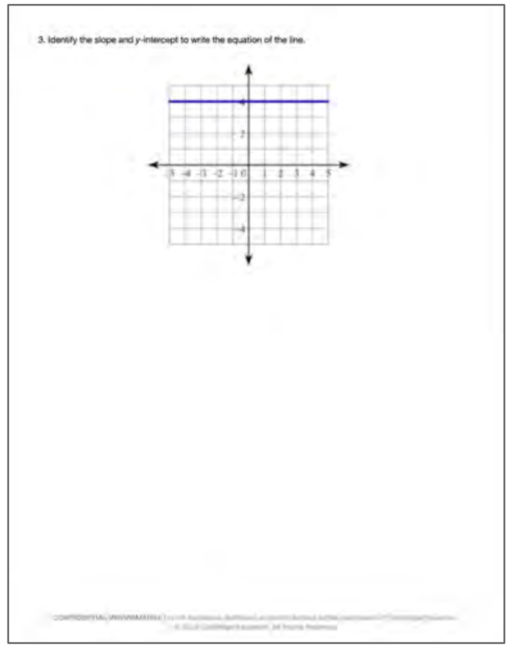
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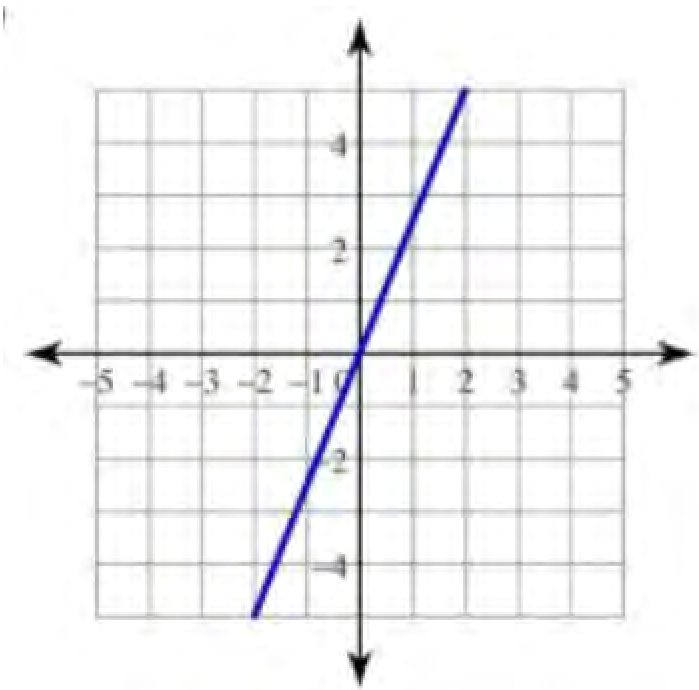
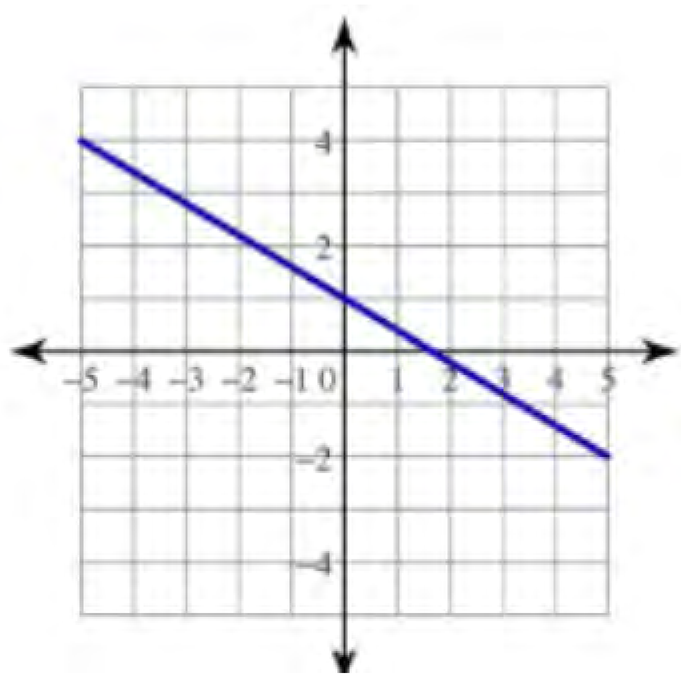
Name: ____ G8 U2 Lesson 9 - Let's Try It

Write equations of lines.

1. Identify the slope and y-intercept to write the equation of the line.

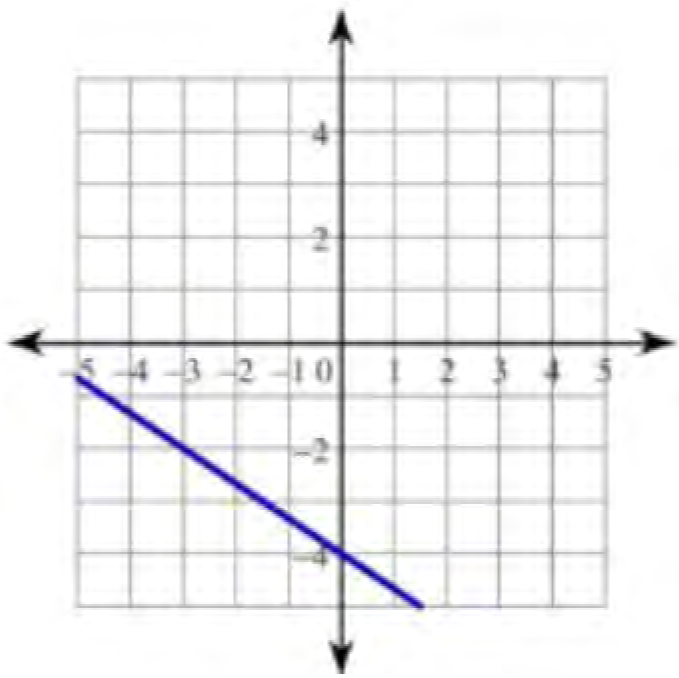


2. Identify the slope and y-intercept to write the equation of the line.

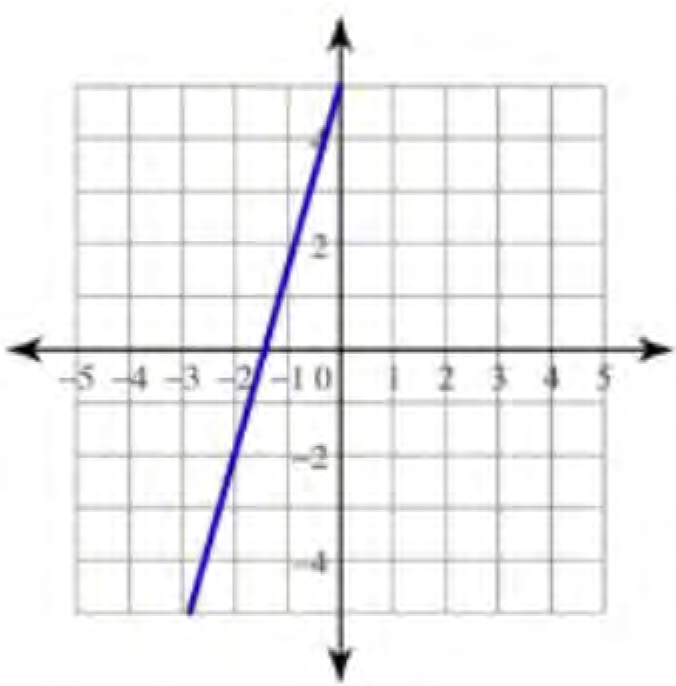


Write equations of lines.

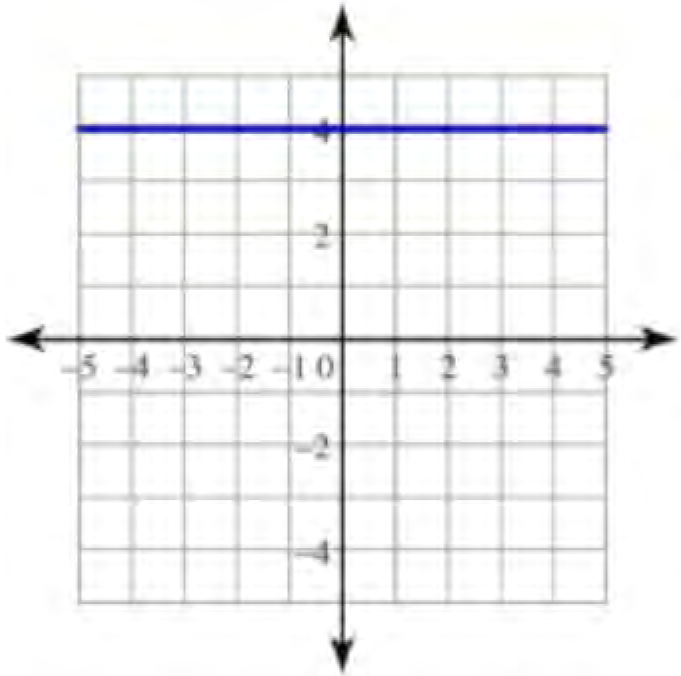
1. Identify the slope and *y*-intercept to write the equation of the line.



2. Identify the *y*-intercept to write the equation of the line.



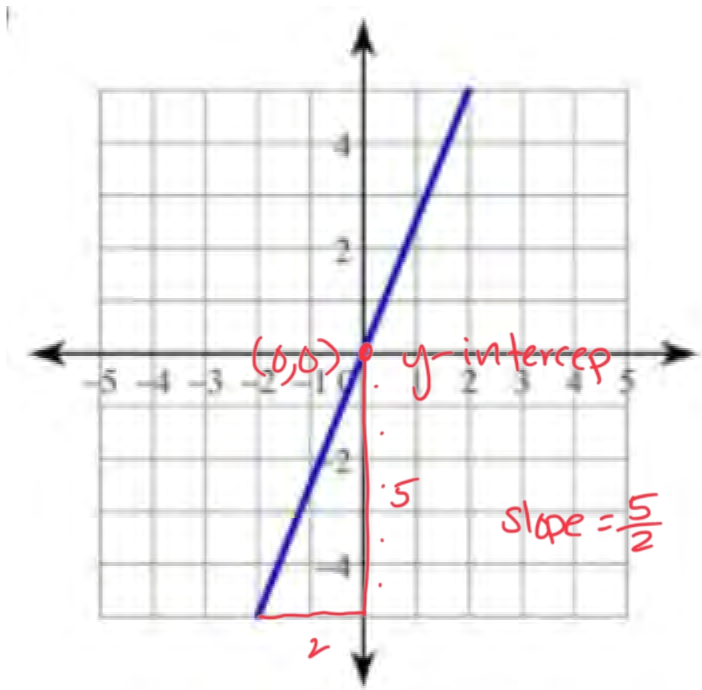
3. Identify the slope and y-intercept to write the equation of the line.



Name: Answer Key

Write equations of lines.

1. Identify the slope and y-intercept to write the equation of the line.

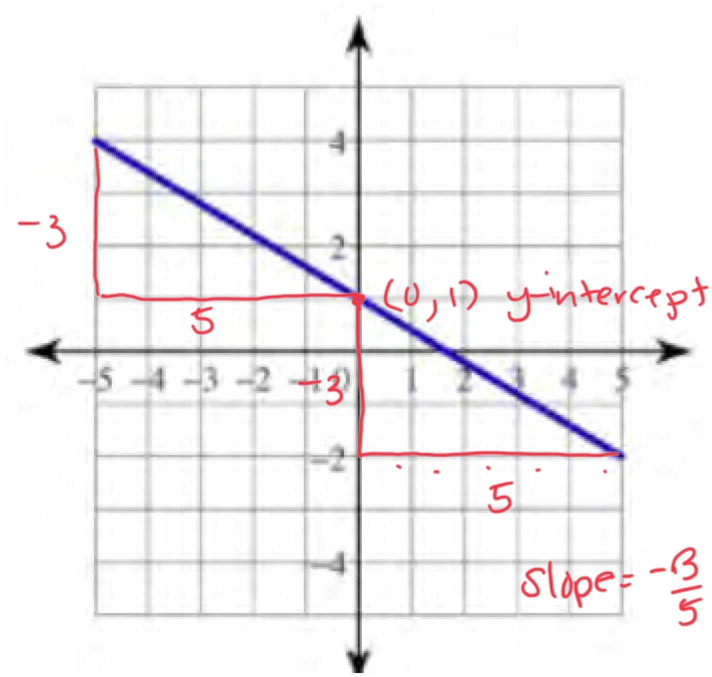


$y = mx + b$

$y = \frac{5}{2}x + 0$

$y = \frac{5}{2}x$

2. Identify the slope and y-intercept to write the equation of the line.



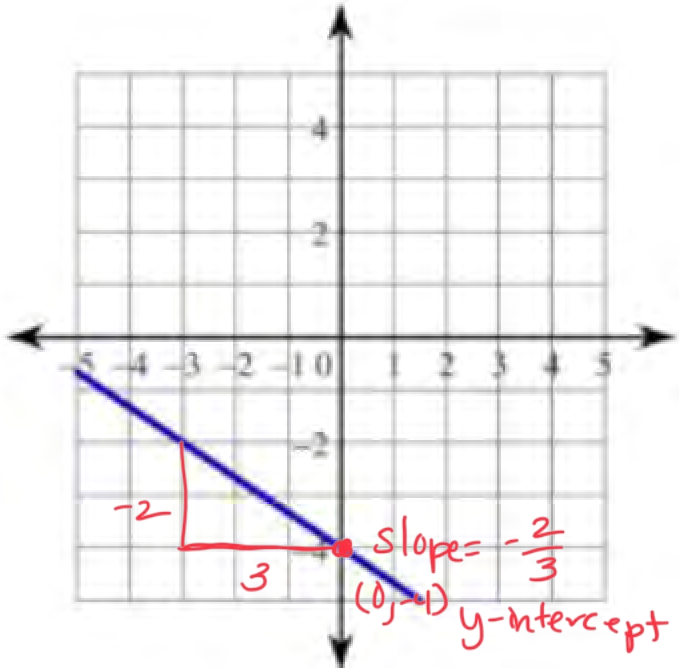
$y = mx + b$

$y = -\frac{3}{5}x + 1$

Name: Answer Key

Write equations of lines.

1. Identify the slope and *y*-intercept to write the equation of the line.

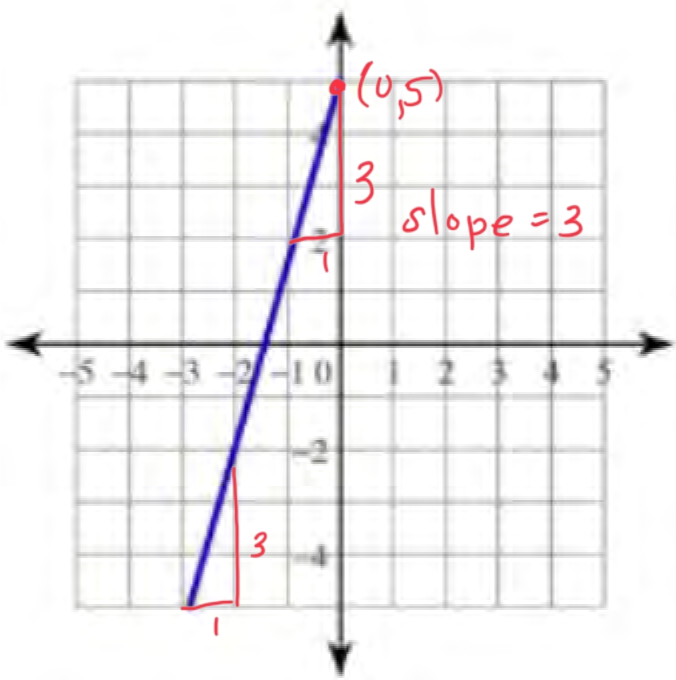


$y = mx + b$

$y = \frac{-2}{3}x + (-4)$

$y = \frac{-2}{3}x - 4$

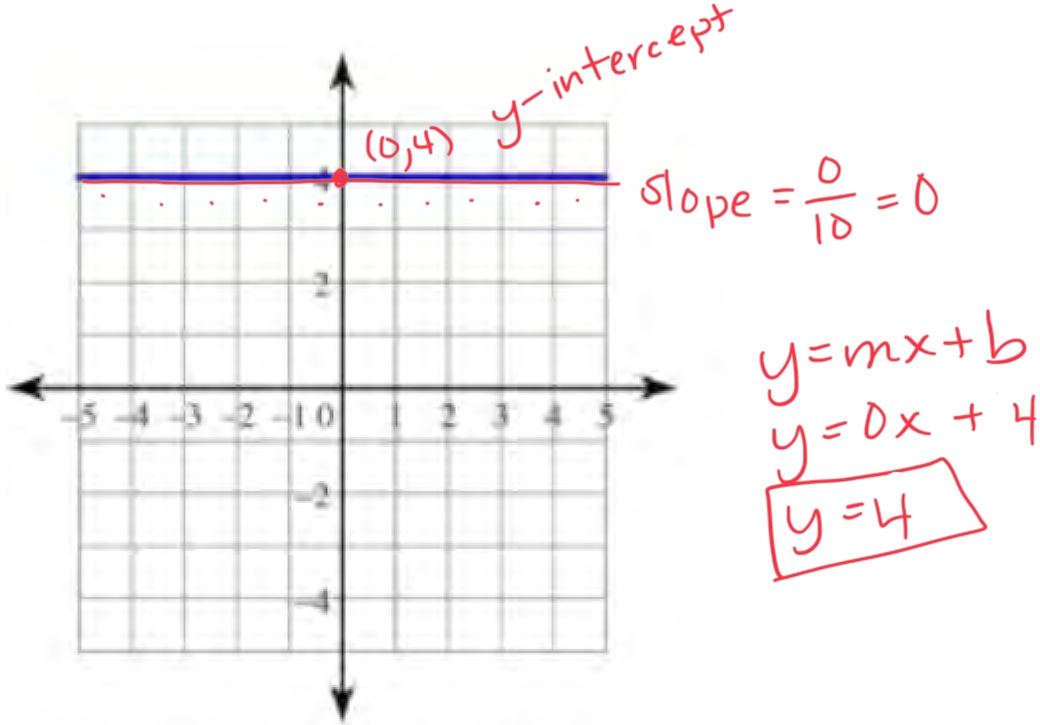
2. Identify the slope and *y*-intercept to write the equation of the line.



$y = mx + b$

$y = 3x + 5$

3. Identify the slope and *y*-intercept to write the equation of the line.



G8 U2 Lesson 10
Use an equation of a line to determine if a point is on the line.

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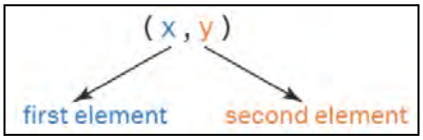
G8 U2 Lesson 10 - Use an equation of a line to determine if a point is on the line.

Warm Welcome (Slide 1): Tutor Choice

Frame the Learning/Connection to Prior Learning (Slides 2 - 3): Today we will use equations of lines to determine if a point is on the line. In order to do that, we'll rely on our algebra skills to balance equations.



Remember, the *x*-coordinate is the first number in an ordered pair and the *y*-coordinate is the 2nd number.

Let's Talk (Slide 4): How do you think we might check to see if a point is on a line? Possible Students Answers, Key Points:

- Graph the line and plot the point to see if it is there.
- Plug in the coordinates to the equation to see if it checks out.

Let's Think (Slide 5): That's right. Let's consider the equation $y = -3x + 4$. Is the point (3, -5) on the line? Rather than graphing the line and looking for the point, let's plug it in and see what happens.

$y = -3x + 4$, $(\overset{x}{3}, \overset{y}{-5})$

First, let's identify which coordinate is which. *(Write x and y above the appropriate coordinate.)*

$y = -3x + 4$, $(\overset{x}{3}, \overset{y}{-5})$

$-5 = -3(3) + 4$

Next, let's substitute the values into the equation. *(Substitute the values of x and y and draw arrows to show the students.)*

$y = -3x + 4$, $(\overset{x}{3}, \overset{y}{-5})$

$-5 = -3(3) + 4$

$-5 = -9 + 4$

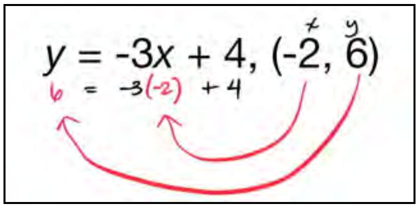
$\boxed{-5 = -5}$ ✓

Now, let's solve the equation to see if it keeps the equation balanced. *(Simplify the expression.)*

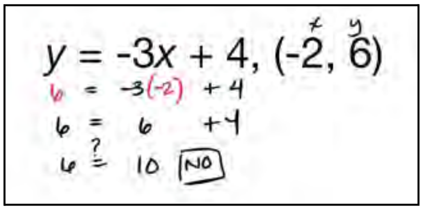
In this case, the answer is yes! Since -5 is in fact equal to -5, this point is on the line. Now, let's try one that does not check out.

$y = -3x + 4$, $(\overset{x}{-2}, \overset{y}{6})$

Again, let's identify which coordinate is which. *(Write x and y above the appropriate coordinate.)*



Next, let's substitute the values into the equation. *(Substitute the values of x and y and draw arrows to show the students.)*



Now, let's solve the equation to see if it keeps the equation balanced. *(Simplify the expression.)*

In this case, the answer is no because the coordinates do not create a true statement.

Let's Try it (Slides 7): Let's work on determining if points lie on a line. Remember, be sure to identify your coordinates and substitute them for the correct variable. If the coordinates make the equation balance, the point is on the line. If not, the point is not on the line.

WARM WELCOME

Use an equation of a line to determine if a point is on the line.

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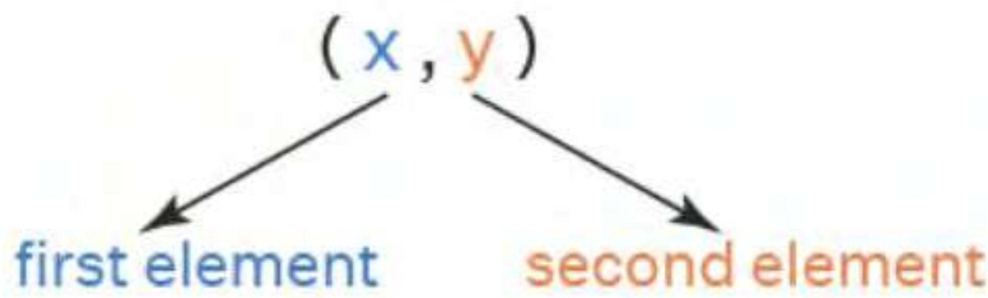
Let's Review:

The *x*-coordinate is the first number in a an ordered pair and the *y*-coordinate is the second number.



Let's Talk:

How do you determine if a point is on a line?

What might we do to determine if the point is on the line?

$$y = -3x + 4, (3, -5)$$

Let's Think: **How do you determine if a point is on a line?**

Determine if the point is on the line.

$$y = -3x + 4, (3, -5)$$

Let's Think: **How do you determine if a point is on a line?**

Determine if the point is on the line.

$$y = -3x + 4, (-2, 6)$$

Let's Try It:

Let's practice determining if a point is on a line.

Name: ______________________ G8 U2 Lesson 10 - Let's Try It

Determine if the point is on the line. Explain why or why not for each problem.

1. $y = -\frac{1}{5}x + 2$, (-15, 5)

2. $y = 2x - 2$, (0, 4)

On your Own:

Now it's time to practice determining is a point is on a line on your own.

Name: ______________________ G8 U2 Lesson 10 - Independent Work

Determine if the point is on the line. Explain why or why not for each problem.

1. $y = -\frac{2}{7}x + 2$, (-7, 0)

2. $y = 4x - 6$, (1, -2)

3. $y = -2x + 5$, (2, 1)

Determine if the point is on the line. Explain why or why not for each problem.

1. $y = -\frac{1}{5}x + 2$, (-15, 5)

2. $y = 2x - 2$, (0, 4)

Name: ____ G8 U2 Lesson 10 - Independent Work

Determine if the point is on the line. Explain why or why not for each problem.

1. $y = -\frac{2}{7}x + 2$, (-7, 0)

2. $y = 4x - 6$, (1, -2)

3. $y = -2x + 5$, (2, 1)

Name: Answer Key

Determine if the point is on the line. Explain why or why not for each problem.

1. $y = -\frac{1}{5}x + 2$, (-15, 5)

$5 = -\frac{1}{5}(-15) + 2$

$5 = 3 + 2$

$5 = 5$ ✓

Yes. Since (-15, 5) makes the equation true, it is a point on the line.

2. $y = 2x - 2$, (0, 4)

$4 = 2(0) - 2$

$4 = 0 - 2$

$4 \stackrel{?}{=} -2$ NO

No. (0, 4) does not make the equation true.

Name: Answer Key

Determine if the point is on the line. Explain why or why not for each problem.

1. $y = -\frac{2}{7}x + 2$, (-7, 0)

x y

$0 = -\frac{2}{7}(-7) + 2$

$0 = 2 + 2$

$0 \stackrel{?}{=} 4$ NO

NO. (-7,0) does not make the equation true.

2. $y = 4x - 6$, (1, -2)

x y

$-2 = 4(1) - 6$

$-2 = 4 - 6$

$-2 = -2$ ✓

Yes. (1,-2) makes the equation true so it is a point on the line.

3. $y = -2x + 5$, (2, 1)

x y

$1 = -2(2) + 5$

$1 = -4 + 5$

$1 = 1$ ✓

Yes. (2,1) makes the equation true so it is a point on the line.

Eighth Grade Math Lesson Materials

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G8 Unit 3:

Linear Relationships

G8 U3 Lesson 1
Represent proportions with graphs, equations, tables and stories.

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G8 U3 Lesson 1 - Today we will represent proportions with graphs, equations, tables and stories.

Warm Welcome (Slide 1): Tutor choice

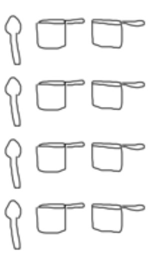
Frame the Learning/Connect to Prior Learning (Slide 2): The next 15 lessons are going to be about linear relationships. You've actually already done one kind of linear relationship in 6th and 7th grade called proportions. And really, you've been working with linear relationships your whole life without even realizing it. So I know you're going to do great!

Let's Review (Slide 3): Today we will represent proportions with graphs, equations, tables and stories. This is review. For instance, what even is a proportion? Let's read this story together as an example. Follow along with your eyes and read silently in your head while I read out loud. *For the remainder of this lesson and the following lessons in this unit, the prior sentence is the cue that you can always use when you are reading. Before to point to each word as you read to help kids follow along.* "To make Kool-Aid, you need 1 tablespoon of powder for every 2 cups of water. Let's imagine making different amounts of Kool-Aid." First, I am going to draw 1 spoon and 2 cups. That's the recipe so you don't have to do much there. We're just making sense of what the problem is saying.

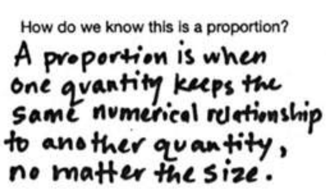
Now let's imagine we want to make more Kool-Aid. What might I draw next? Possible Student Answers, Key Points:

- You can keep drawing spoons and cups.
- You can draw 1 more spoon and 2 more cups.
- You can repeat the picture you just drew.

This is a recipe that tells us how much powder for how much water. Let me draw another spoon. If I add more powder without adding anymore water. My drink will be super super sweet like it will not even taste good. So I need to add more water too. And I'm going to add 2 cups because that was the recipe: 1 tablespoon of powder for 2 cups of water.



But let's say I'm having a party and I want a really huge amount of Kool-Aid. I could keep going with as many spoons as I like. But I'd also need to keep going with 2 cups for every tablespoon. So I can draw that exact same picture again and again!



This is an example of a proportion! In the picture, we can see that still have that same basic ratio or relationship that we had from the very beginning. I can see the 1 tablespoon and the 2 cups together. It's kind of like a building block and I just keep using that same basic block to keep building and building. I will write that as "A proportion is when one quantity keeps the same numerical relationship to another quantity no matter the size."

In this case, because one quantity, the powder, kept the same numerical relationship to the other quantity, the water, the Kool-aid is always going to taste the same no matter how much we make. But let's write down some specific numbers on the next slide so we can see this exactly.

t	c
0	0

Let's Talk (Slide 4): Today the main thing we need to review is the answer to this question: "How do we know when a relationship is a proportion?" We have the same story and we have our same picture. Now I am going to represent what we drew on a table. First, I need to label each column. I will write t for tablespoons and c for cups. Now, you might want to jump straight to filling in the recipe. But we actually didn't start with 1 tablespoon of powder and 2 cups of water. We started with nothing. I am going to write 0 for tablespoons and 0 for cups. That is secretly the start. Or sometimes we call it the initial amount - even though it wasn't spelled out in the story.

t	c
0	0
1	2

Now we can write in 1 for tablespoons and 2 for cups.

t	c
0	0
1	2
2	4

Now let's think about what happened as we drew the next picture. *Cover up the bottom two rows in the picture and point as you count up the first two rows.* I see 2 tablespoons and 4 cups.

t	c
0	0
1	2
2	4
3	6

Now let's think about what happened as we drew the next picture. *Cover up the bottom row in the picture and point as you count up the first three rows.* I see 3 tablespoons and 6 cups.

t	c
0	0
1	2
2	4
3	6
4	8

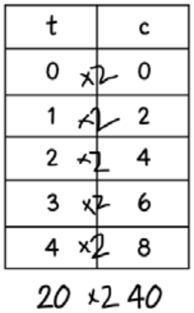
And the next picture! I see 4 tablespoons and 8 cups. Look carefully at our table. What do you notice? Possible Student Answers, Key Points:

- The left column keeps adding 1.
- The right column keeps adding 2.
- If you look across, it is always "times 2."

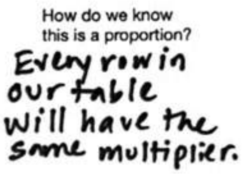
There are several patterns to notice. Going down, we see that the left column keeps adding 1. 0 plus 1 is 1. 1 plus 1 is 2. 2 plus 1 is 3. 3 plus 1 is 4. And the right column keeps adding 2. 0 plus 2 is 2. 2 plus 2 is 4. 4 plus 2 is 6. 6 plus 2 is 8. All that makes sense because, of course, every time we drew, we drew 1 more tablespoon and 2 more cups.

t	c
0	0
1	2
2	4
3	6
4	8
20	40

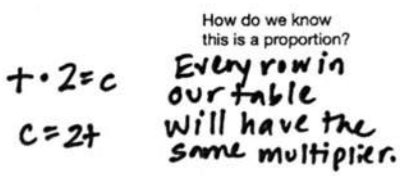
But we might not always see the adding patterns going down if we jumped to a super huge amount of Kool-Aid like 20 tablespoons and 40 cups of water. *Write those numbers under the table, lined up with the correct columns.*



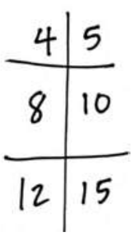
So it is really the pattern going across that is super super important. I can see in this example that it is always times 2. 0 times 2 is 2. 1 times 2 is 2. 2 times 2 is 4. 3 times 2 is 6. 4 times 2 is 8 and even here, 20 times 2 is 40. This number is so important; it has its own name. It is called the "constant of proportionality." Constant means always the same. So for a proportion, the multiplier will be constant or the same.



We wanted to figure out how to tell if something was a proportion. I will write it down, "Every row in our table will have the same multiplier." It is called the "constant of proportionality. Or another word you will in the next few lessons is "constant rate of change." It's the same idea. These amounts change together in the same way, in a way that is constant.



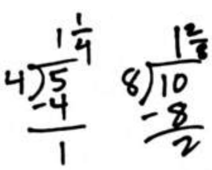
And since it's always going to have the same multiplier, we can write it as an equation: t times 2 = c or c = 2t. These mean the same thing just written in a different order.



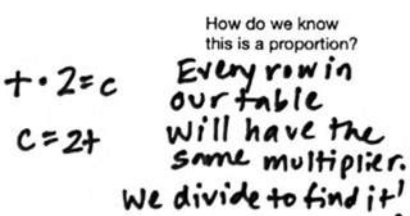
This one is pretty easy because the "times 2" really jumps out at us. What would we do if we had trickier number like these, for example, and we didn't know what the multiplier was. *Jot the example table on the board.* Do any of you remember from last year?

Possible Student Answers, Key Points:

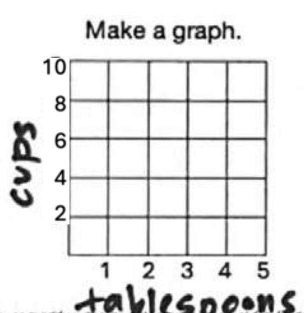
- We can use guess and check.
- We can divide.



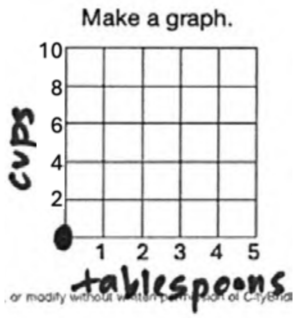
We know it is a multiplier so we can use the opposite operation to work backwards from multiplication: division! I can do 5 divided by 4, which is 1 and 1 fourth. 10 divided by 8 is 1 and 2 eighths, which if I simplify is 1 and 1 fourth. And I could keep going. *You can erase the example table if you want to.*



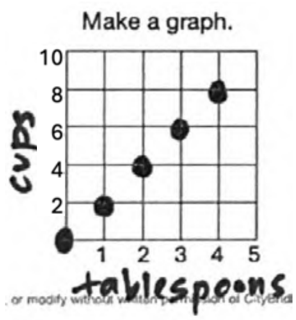
Let's return to our original table. We don't have to do it because it's so easy but we can see division works here too. *Point from the right hand number to the left hand number as you go through each row. 2 divided by 1 is 2. 4 divided by 2 is 2. 6 divided by 3 is 2. 8 divided by 4 is 2.* Division works to find the multiplier! I'm going to write that down, "We divide to find it."



Let's Think (Slide 5): So we reviewed tables. Now let's review graphs. We still have the same story and the same picture and now the same table. Let's graph it to answer this top question, "How do we know when a relationship is a proportion?" I am going to start by labeling my axes here just like we labeled the top of our columns. The x-axis is tablespoons and the y-axis is cups.

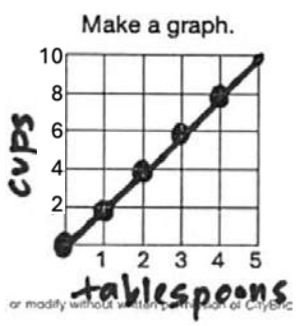


Remember how we had nothing as our initial value? This first row of my table is 0 and 0. I am going to graph that like a coordinate, (0,0), and put a point here.

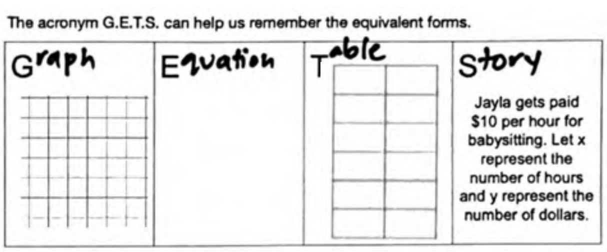


Now it's time for my next point which is the next row on my graph, (1, 2). I find the 1 on the x-axis, which is tablespoons. *Point to the 1 on the x-axis, the horizontal axis.* I find the 2 on the y-axis, which is cups. *Point to the 2 on the y-axis, the vertical axis.* Then I see where they meet up and draw a point. *Drag your fingers from each number to show how they meet and draw the point.* Let's keep going! The next row is (2,4). The next row is (3,6). The next row is (4,8). What do you notice? Possible Student Answers, Key Points:

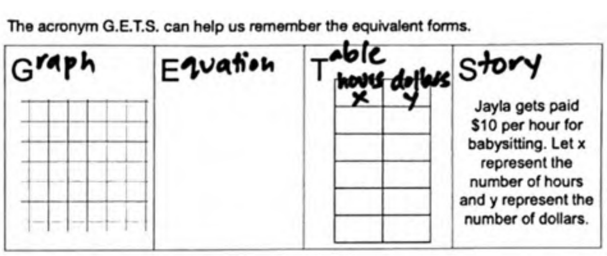
- The points keeping going up diagonally.
- The points go up like a staircase.
- The points go up 2 and over 1.
- The points make a diagonal line.



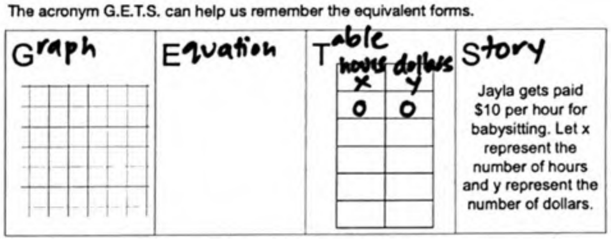
We might see two things. First of all, I can connect the dots and make a straight line. That is always going to be true about proportions. Also, the whole reason they make a straight line is because they go up like a nice steady staircase up 2 and over 1, up 2 and over 1, up 2 and over 1, up 2 and over 1, up 2 and over 1. We're going to talk more about that tomorrow. Because all those equal stairs are important. They are constant just like our CONSTANT of proportionality! But for now, I just want to point out one more thing, and that is that our graph goes through this special point, (0,0), which has a special name, "the origin." This is always going to be true about proportions. so I am going to write all that down. "The graph is always a straight line through the origin, (0,0)."



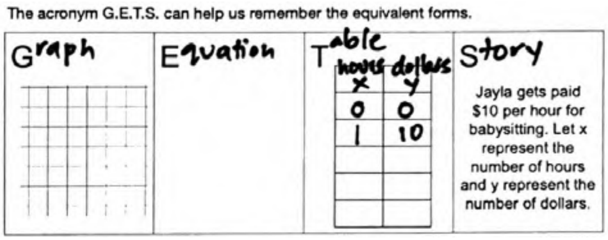
Let's Think (Slide 6): Let's put all this together. "We can use a graph, equation, table and story to represent a proportion. They are all equivalent forms that are just different ways to show the same relationship. Kind of like how "bro" is another word for "brother." We can remember this with the acronym, GETS. G stands for Graph. E stands for Equation. T stands for Table and S stands for Story.



Let's fill this in. We'll start by reading the story. I will read it out loud while you follow along silently. *Read the story in the far right column.* I am going to use this story to fill in the table. We have x and y. I will write hours about the x and dollars above the y to help me remember which is which.

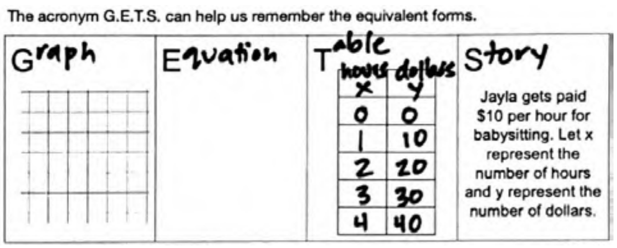


Now, the story doesn't say anything about Jayla having money when she starts so we're going to think of her initial starting point as 0 hours and 0 dollars. I'm going to write that in my table.

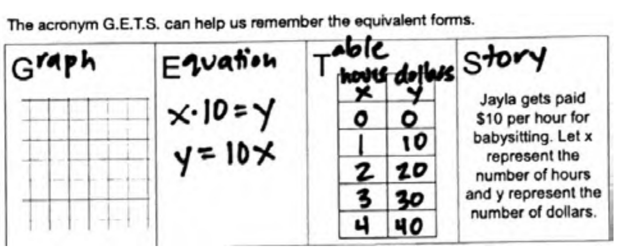


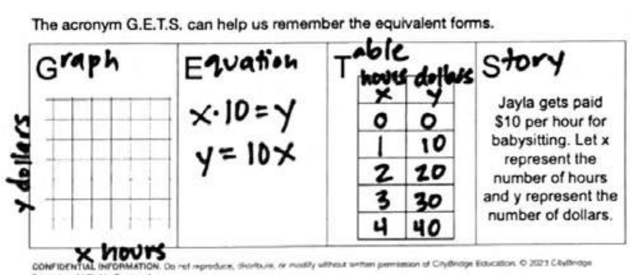
She gets paid $10 per hour. So when she works 1 hour, she gets paid $10. I am going to use that to fill in my next row.

Let's think about if she keeps working. If she works another hour, that will be another $10. We want that same, constant relationship, which is the whole idea of what a proportion is. Another hour makes 2 hours. Another $10 makes $20. So now we have 2 hours gets $20 so I am going to use that to fill in another row. You see where I'm going here, right? Another hour and another $10 means 3 hours gets $30. I'll write that in. And another hour and another $10 means 4 hours gets $40.

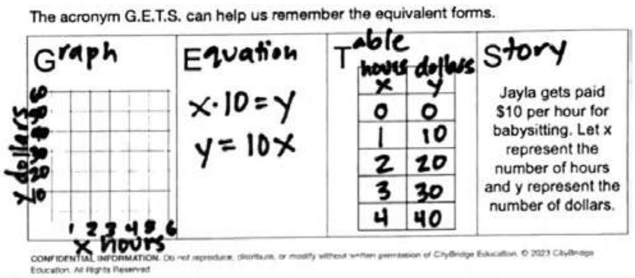


Now here is where we use the ideas from earlier. We said, "Every row in our table will have the same multiplier called a constant of proportionality." We can divide to find it, like 10 divided by 1 is 10 and 20 divided by 2 is 10 and 30 divided by 3 is 10 and 40 divided by 4 is 10. But we don't have to do that in this case because the "times 10" is kind of obvious. That helps me write the equation. It is the left side times 10 makes the right side. We can write x times 10 equals y. But usually it is written the opposite way but it still all equals up to y. I write, y = 10x. The 10x means the same as 10 times x or x times 10. We just write it that way for short.

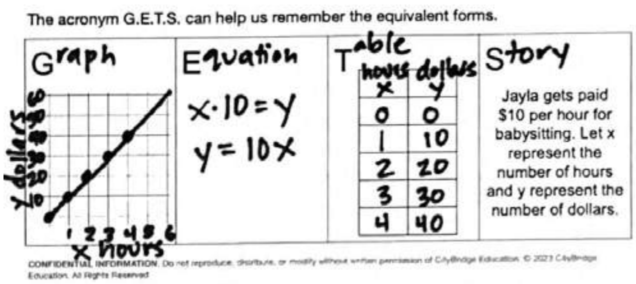




And finally, we get to our graph. We said, "The graph of a proportion is always a straight line that goes through the origin, (0,0)." Let's see if that's true. I going to label the horizontal line, x and hours. I am going to label the vertical lines, y and dollars.



For numbers, I will put 1, 2, 3, 4, 5, 6 on the x-axis. The numbers have to go right under the little tick marks to keep me organized. On the y-axis, I will put 10, 20, 30, 40, 50, 60. The numbers have to go right next to the little tick marks to keep me organized. If I am not exactly lined up, I will start to confuse myself.



And finally, we can put points. *Point to each row on the table as you talk it through and graph it.* I see 0 and 0 so I will make a point at (0,0). I see 1 and 10 so I will make a point at (1,10). I see 2 and 20 so I will make a point at (2,20). I will see 3 and 30 so I will make a point at (3,30). I see 4 and 40 so I will make a point at (4,40). Does it make a straight line? YES! Does it go through the origin? YES! So all these equivalent forms all show that our story is a proportion in their own special way! I love the way all of math works together like that. It is very cool!

Let's Try It (Slide 7): Let's practice filling out GETS together now. I will walk you through each step.

WARM WELCOME

Today we will represent proportions with graphs, equations, tables and stories.

Let's Review: What is a proportion?

To make Kool-Aid, you need 1 tablespoon of powder for every 2 cups of water. Let's imagine making different amounts of Kool-Aid.

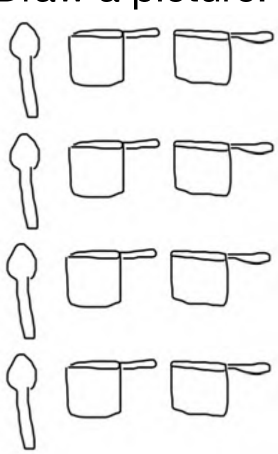
Draw a picture:

How do we know this is a proportion?

Let's Talk: How do we know when a relationship is a proportion?

To make Kool-Aid, you need 1 tablespoon of powder for every 2 cups of water. Let's imagine making different amounts of Kool-Aid.

Draw a picture:



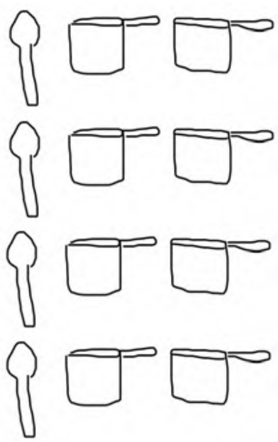
Make a table.

How do we know this is a proportion?

Let's Think: How do we know when a relationship is a proportion?

To make Kool-Aid, you need 1 tablespoon of powder for every 2 cups of water. Let's imagine making different amounts of Kool-Aid.

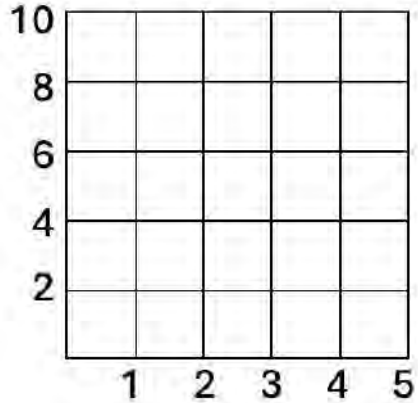
Draw a picture:



Make a table.

t	c
0	0
1	2
2	4
3	6
4	8

Make a graph.

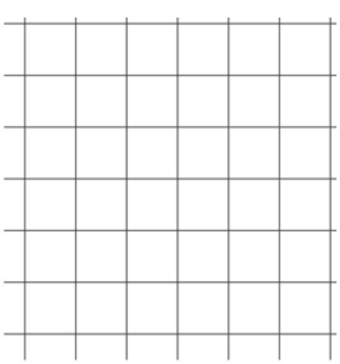


How do we know this is a proportion?

Let's Think: We can use a graph, equation, table and story to represent a proportion.

The acronym G.E.T.S. can help us remember the equivalent forms.

G



E

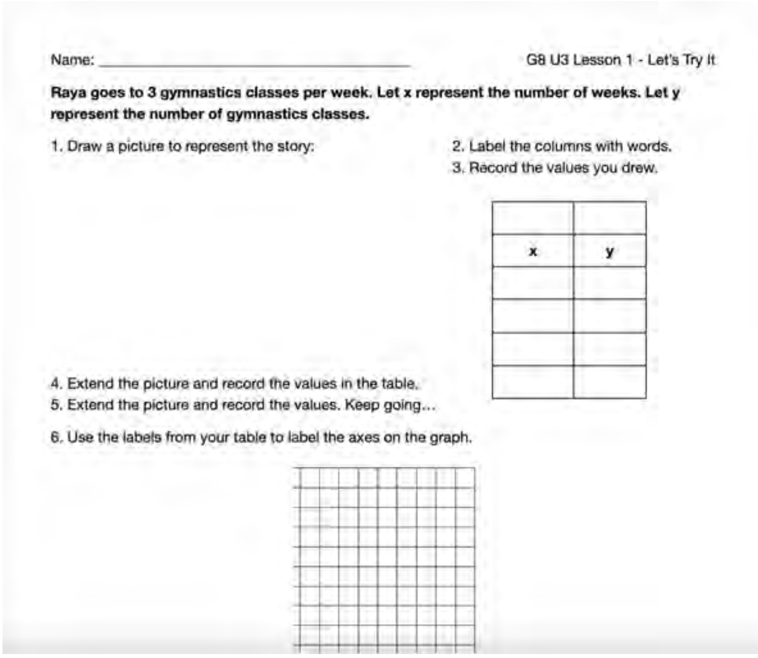
T

S

Jayla gets paid $10 per hour for babysitting. Let x represent the number of hours and y represent the number of dollars.

Let's Try It:

Let's practice making pictures, tables and graphs for proportions together!



Name: ______________________ G8 U3 Lesson 1 - Let's Try It

Raya goes to 3 gymnastics classes per week. Let x represent the number of weeks. Let y represent the number of gymnastics classes.

1. Draw a picture to represent the story:

2. Label the columns with words.
3. Record the values you drew.

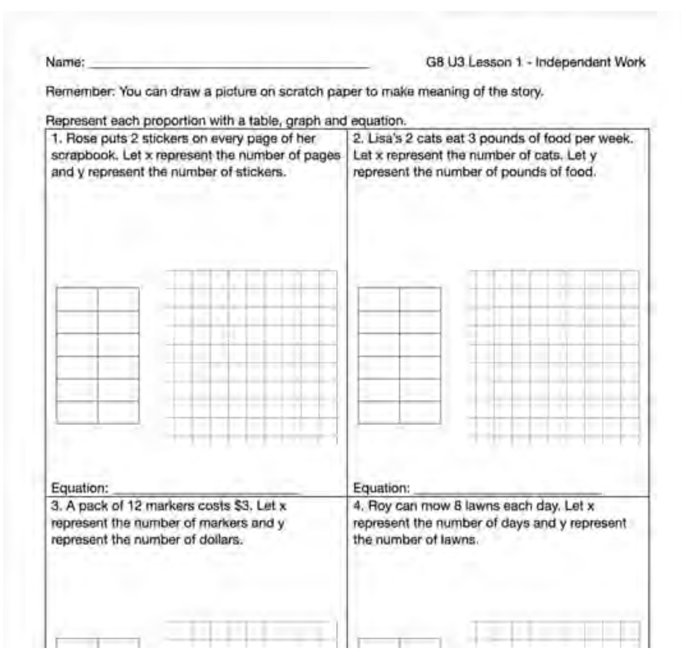
x	y

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going...

6. Use the labels from your table to label the axes on the graph.

On your Own:

Now it's time for you to do it on your own.



Name: ______________________ G8 U3 Lesson 1 - Independent Work

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each proportion with a table, graph and equation.

1. Rose puts 2 stickers on every page of her scrapbook. Let x represent the number of pages and y represent the number of stickers.

Equation: ______________

2. Lisa's 2 cats eat 3 pounds of food per week. Let x represent the number of cats. Let y represent the number of pounds of food.

Equation: ______________

3. A pack of 12 markers costs $3. Let x represent the number of markers and y represent the number of dollars.

4. Roy can mow 8 lawns each day. Let x represent the number of days and y represent the number of lawns.

Raya goes to 3 gymnastics classes per week. Let x represent the number of weeks. Let y represent the number of gymnastics classes.

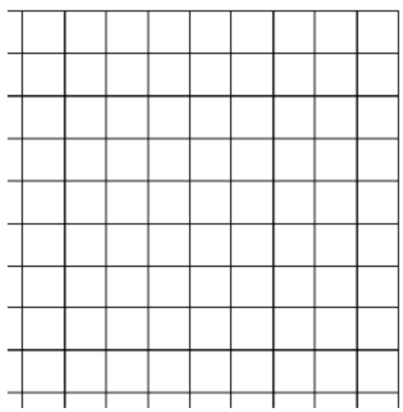
1. Draw a picture to represent the story:

2. Label the columns with words.
3. Record the values you drew.

x	**y**

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going…

6. Use the labels from your table to label the axes on the graph.



7. Fill in the numbers on each axis. You might have to skip count in order to reach the highest number on your table.
8. Use each row of the table as a set of coordinates.
9. Look for a pattern going across the table OR divide y by x for each row to find the multiplier.

10. Write an equation in $y = kx$ form. ____________________

Michael does 7 laps around the field every 2 minutes. Let x represent the number of minutes. Let y represent the number of laps.

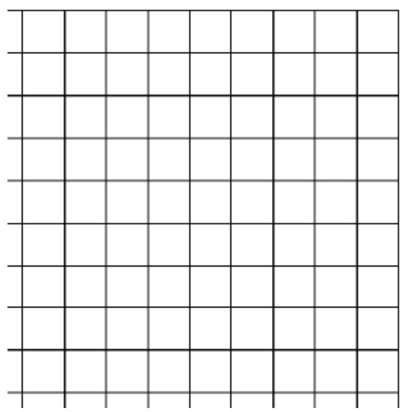
11. Draw a picture to represent the story:

12. Label the columns with words.
13. Record the values you drew.

x	**y**

14. Extend the picture and record the values in the table.
15. Extend the picture and record the values. Keep going…

16. Use the labels from your table to label the axes on the graph.



17. Fill in the numbers on each axes. You might have to skip count in order to reach the highest number on your table.
18. Use each row of the table as a set of coordinates.
19. Look for a pattern going across the table OR divide y by x for each row to find the multiplier.

20. Write an equation in y = kx form. ___________________

Name: ______________________________________

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each proportion with a table, graph and equation.

1. Rose puts 2 stickers on every page of her scrapbook. Let x represent the number of pages and y represent the number of stickers.

Equation: ____________________________

2. Lisa's 2 cats eat 3 pounds of food per week. Let x represent the number of cats. Let y represent the number of pounds of food.

Equation: ____________________________

3. A pack of 12 markers costs $3. Let x represent the number of markers and y represent the number of dollars.

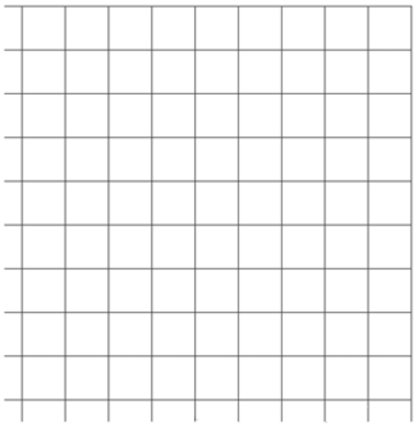
Equation: ____________________________

4. Roy can mow 8 lawns each day. Let x represent the number of days and y represent the number of lawns.

Equation: ____________________________

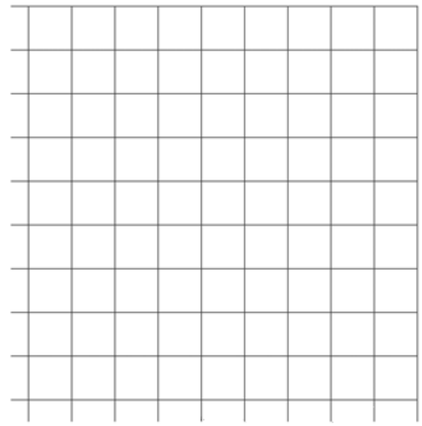
Represent each proportion with a table and a graph.

5. Jane is going to average about 45 miles per hour the whole way from New York to California. Let x represent the number of hours and y represent the number of miles.



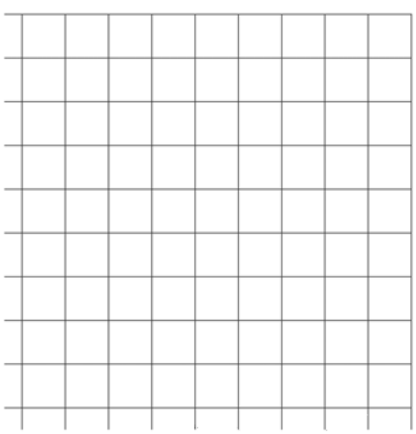
Equation: ______________________________

6. Francie hired 2 helpers to work with 10 children. Let x represent the number of children. Let y represent the number of helpers.



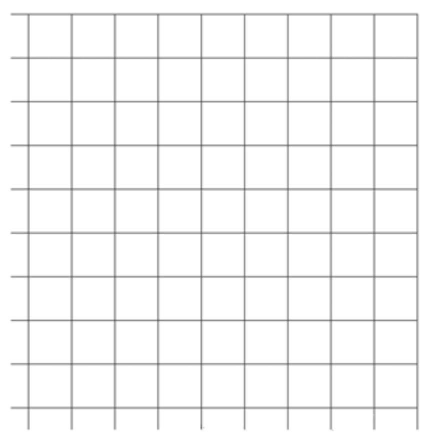
Equation: ______________________________

7. Each teacher at Parkland Middle School spends 3 hours grading for the 4 classes she teaches. Let x represent the number of classes. Let y represent the number of hours.



Equation: ______________________________

8. It takes 6 inches of ribbon per bow that Lisa makes. Let x represent the number of bows and y represent the number of inches of ribbon.

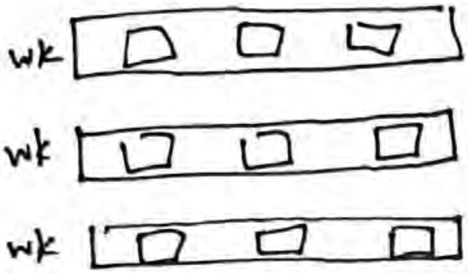


Equation: ______________________________

Name: ANSWER KEY

Raya goes to 3 gymnastics classes per week. Let x represent the number of weeks. Let y represent the number of gymnastics classes.

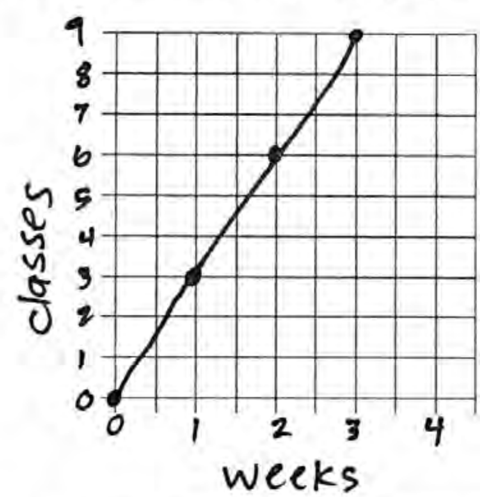
1. Draw a picture to represent the story:



2. Label the columns with words.
3. Record the values you drew.

weeks	classes
x	y
0	0
1	3
2	6
3	9

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going...
6. Use the labels from your table to label the axes on the graph.



7. Fill in the numbers on each axes. You might have to skip count in order to reach the highest number on your table.
8. Use each row of the table as a set of coordinates.
9. Look for a pattern going across the table OR divide y by x for each row to find the multiplier.

$1 \times 3 = 3$

$2 \times 3 = 6$

$3 \times 3 = 9$

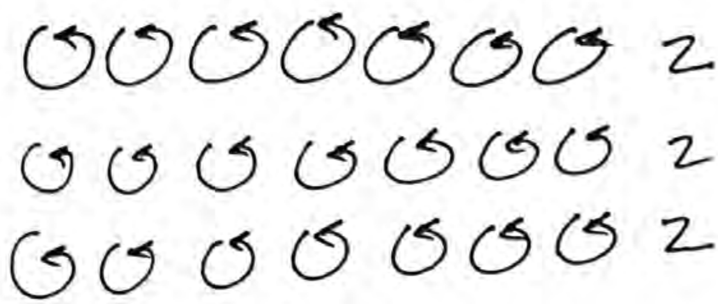
$x \cdot 3 = y$

$3 \div 1 = 3$, $6 \div 2 = 3$, $9 \div 3 = 3$

10. Write an equation in y = kx form. $y = 3x$

Michael does 7 laps around the field every 2 minutes. Let x represent the number of minutes. Let y represent the number of laps.

11. Draw a picture to represent the story:



12. Label the columns with words.

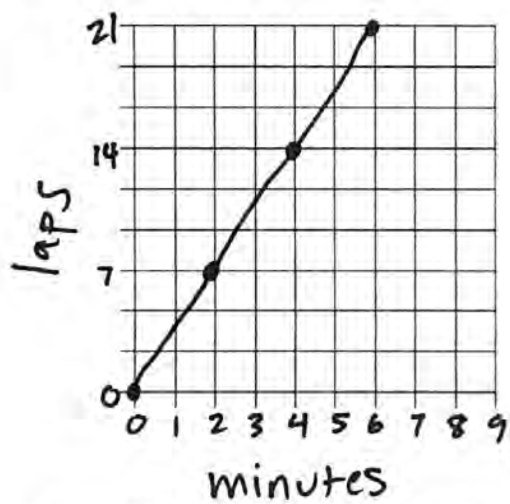
13. Record the values you drew.

minutes	laps
x	y
0	0
2	7
4	14
6	21

14. Extend the picture and record the values in the table.

15. Extend the picture and record the values. Keep going...

16. Use the labels from your table to label the axes on the graph.



17. Fill in the numbers on each axes. You might have to skip count in order to reach the highest number on your table.

18. Use each row of the table as a set of coordinates.

19. Look for a pattern going across the table OR divide y by x for each row to find the multiplier.

$7 \div 2 = 3\frac{1}{2}$ $\quad 14 \div 4 = 3\frac{2}{4}$ $\quad 21 \div 6 = 3\frac{3}{6}$

20. Write an equation in y = kx form. $y = 3\frac{1}{2}x$

Name: ANSWER KEY

G8 U3 Lesson 1 - Independent Work

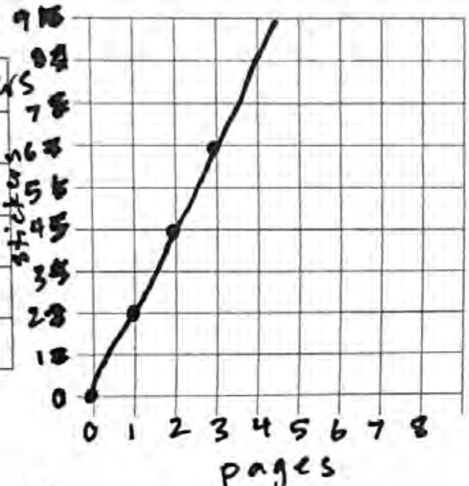
Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each proportion with a table, graph and equation.

1. Rose puts 2 stickers on every page of her scrapbook. Let x represent the number of pages and y represent the number of stickers.

$2 \div 1 = 2$, $4 \div 2 = 2$

pages	stickers
x	y
0	0
1	2
2	4
3	6

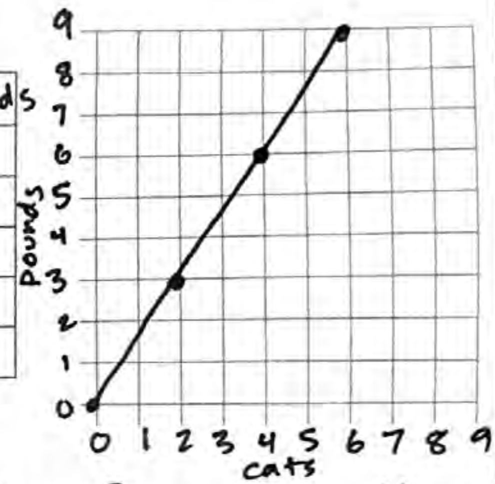


Equation: $y = 2x$

2. Lisa's 2 cats eat 3 pounds of food per week. Let x represent the number of cats. Let y represent the number of pounds of food.

$3 \div 2 = 1\frac{1}{2}$, $6 \div 4 = 1\frac{2}{4}$

cats	pounds
x	y
0	0
2	3
4	6
6	9

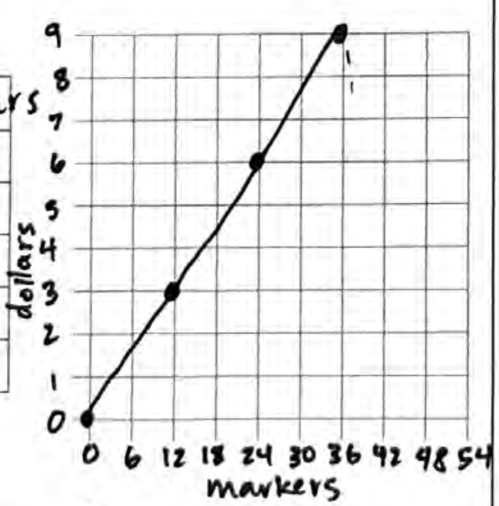


Equation: $y = \frac{3}{2}x$ or $y = 1\frac{1}{2}x$

3. A pack of 12 markers costs $3. Let x represent the number of markers and y represent the number of dollars.

$3 \div 12 = 0.\frac{3}{12}$, $6 \div 24 = 0.\frac{6}{24}$

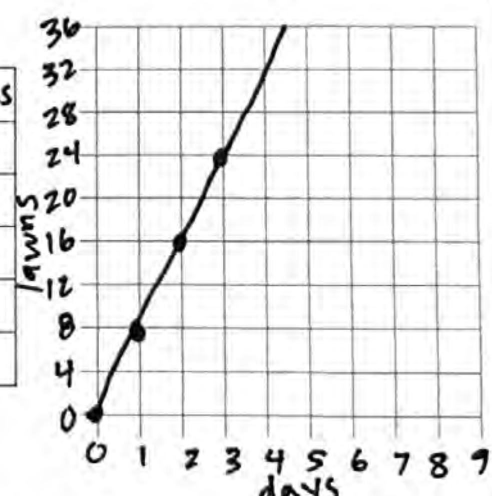
markers	dollars
x	y
0	0
12	3
24	6
36	9



Equation: $y = \frac{1}{4}x$

4. Roy can mow 8 lawns each day. Let x represent the number of days and y represent the number of lawns.

days	lawns
x	y
0	0
1	8
2	16
3	24

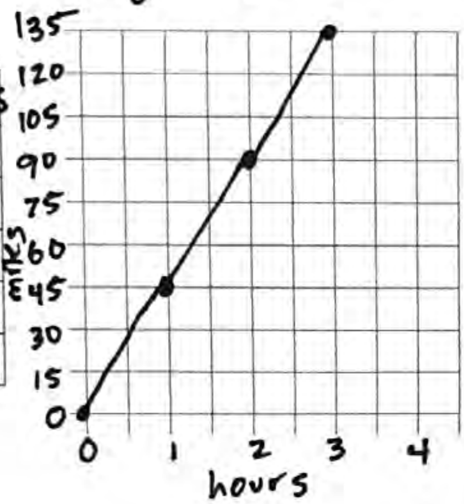


Equation: $y = 8x$

Represent each proportion with a table and a graph.

5. Jane is going to average about 45 miles per hour the whole way from New York to California. Let x represent the number of hours and y represent the number of miles.

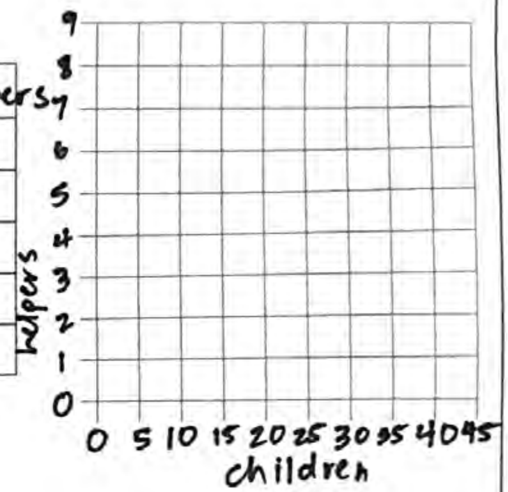
hours	miles
x	y
0	0
1	45
2	90
3	135



Equation: $y = 45x$

6. Francie hired 2 helpers to work with 10 children. Let x represent the number of children. Let y represent the number of helpers.

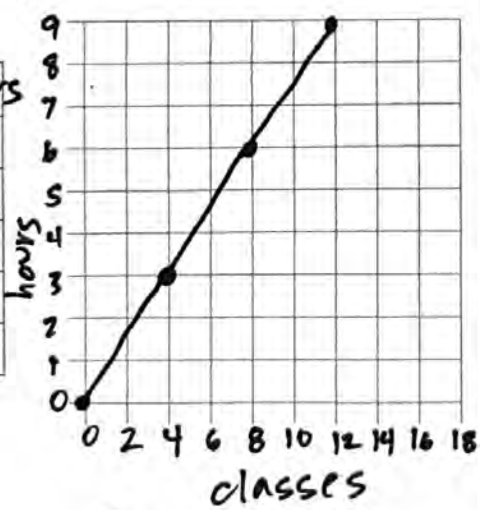
children	helpers
x	y
0	0
10	2
20	4
30	6



Equation: $y = \frac{1}{5}x$

7. Each teacher at Parkland Middle School spends 3 hours grading for the 4 classes she teaches. Let x represent the number of classes. Let y represent the number of hours.

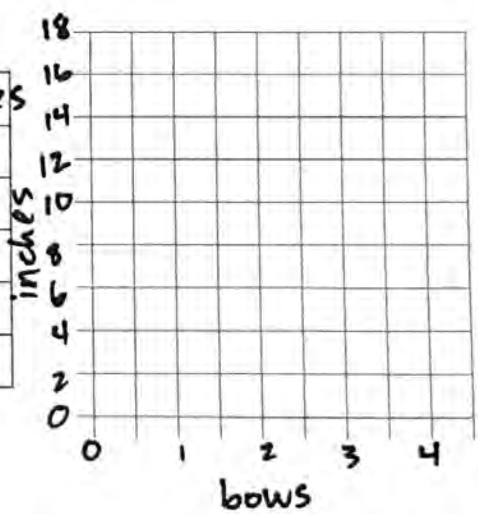
classes	hours
x	y
0	0
4	3
8	6
12	9



Equation: $y = \frac{3}{4}x$

8. It takes 6 inches of ribbon per bow that Lisa makes. Let x represent the number of bows and y represent the number of inches of ribbon.

bows	inches
x	y
0	0
1	6
2	12
3	18



Equation: $y = 6x$

G8 U3 Lesson 2
Connect the meaning of the unit rate, the constant of proportionality and the slope.

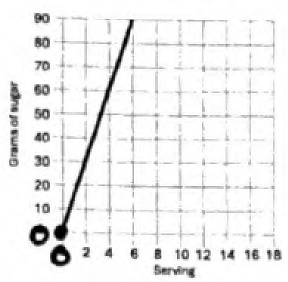
G8 U3 Lesson 2 - Today we will make meaning of the y-intercept and slope on a graph of a proportion.

Warm Welcome (Slide 1): Tutor choice

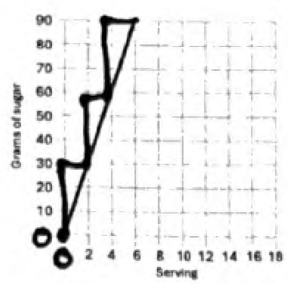
Frame the Learning/Connect to Prior Learning (Slide 2): Today we will make meaning of the y-intercept and slope on a graph of a proportion. Yesterday we reviewed how to represent a proportion with a graph, equation, table and story. Today we are going to focus a little more on graphs to make meaning of the y-intercept and the slope.

Let's Review (Slide 3): The graph of a proportion always has two special features. These two features help us know that a graph is a proportion so we can answer this question, "How do we know if the graph below shows a proportion?" First, let's read the description. I will read it out loud while you read it silently in your head. *Read the problem.* Now we want to know if this shows a proportion. Do you remember this from previous grades? What two parts of this graph are important for us to pay attention to? Possible Student Answers, Key Points:

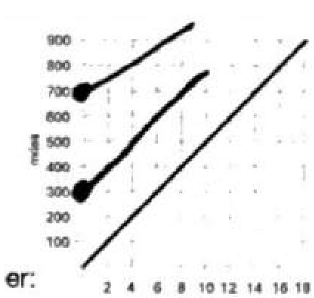
- The graph intercepts the origin.
- The graph goes through the point (0,0).
- The graph is a straight line.
- The graph has a constant slope.



The purpose of this question is to see what children already know. It is important to accept many different versions of correct explanations. However, if children say something that is not totally correct (for example, the line always goes up) be sure to say, "Not exactly." There are two key things to notice about this graph. First, it goes through the point (0,0), which is called the origin. *Draw a point at (0,0).*

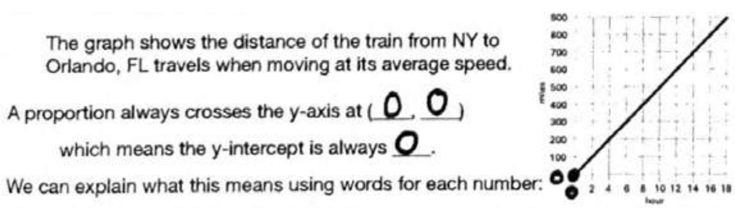


Second, the graph is a straight line. The line can go up or down or straight across. But the point is that it is a straight line. This matter because it means it has a constant slope which we also call a constant rate of change. You can see this if I draw a staircase from point to point. This graph is up 30, over 2 then up 30, over 2 then up 30, over 2 again. It is always going to be the same slope no matter which part of the line I look. This is what we mean by constant. It keeps going and going, making a straight line. The important thing to remember is that if a graph is a straight line through (0,0) then it is a proportion.



Let's Talk (Slide 4): This important feature where the graph goes through (0,0) has a name. This says, "The y-intercept of a graph is where it cross the y-axis and where x = 0." The y-intercept could be any number. A graph could cross the y-axis here or here. *Draw some sample lines on the graph that have a y-intercept that doesn't equal zero and discuss their meaning.* X is equal to zero for each of these lines where it crosses the y-axis. For example, this is (0,300) so the y-intercept is 300. As another example, this is (0,700) so the y-intercept is 700. *Erase these examples when you are done.*

Now let's look at this specific example. Read the story along with me in your head while I read the story out loud. It says, "The graph shows the distance of the train from NY to Orlando, FL travels when moving at its average speed." On the graph I see hours on the x-axis and miles on the y-axis. Now, we just said that we can know this is a proportion because it crosses (0,0) and has a constant slope. *Draw a point at (0,0).* Now we can connect this to our new vocabulary word and fill in these blanks, "A proportion always crosses the y-axis at…" (0,0). "Which means the y-intercept is always…" Zero.

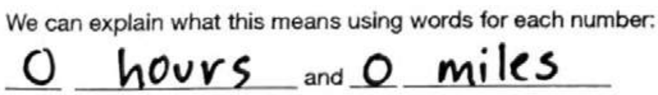

The graph shows the distance of the train from NY to Orlando, FL travels when moving at its average speed.

A proportion always crosses the y-axis at (0, 0)

which means the y-intercept is always 0.

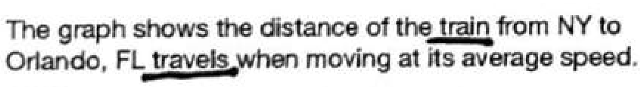
We can explain what this means using words for each number:

We can explain what this means using words for each number. The first zero is x so that represents 0 hours since hours is what the x-axis stands for. The second zero is y so that represents 0 miles since miles is what the y-axis stands for.

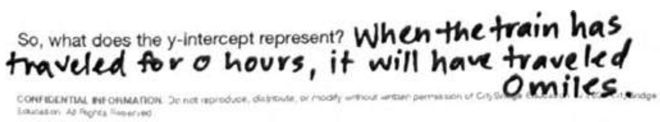

We can explain what this means using words for each number:

0 hours and 0 miles

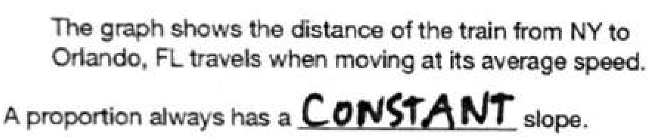
That helps us answer this question, "So, what does the y-intercept represent?" We just have to take these numbers with their words and put them in a sentence. To do this well, we want to try to use as many other words from the story as I can. For example, this story is about a train that travels. *Underline the words "train" and "travels."*


The graph shows the distance of the train from NY to Orlando, FL travels when moving at its average speed.

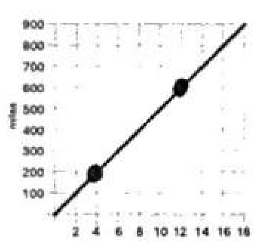
You can create a good frame for your sentence if you use the words "when" and "then." I'll do this example, "WHEN the train has traveled for 0 hours, it will have traveled 0 miles." So, let's review the big ideas. FIrst, the y-intercept is where the line crosses the y-axis. That's where x = 0. But for a proportion, y is going to equal 0 too. So we get the coordinates (0,0) and then we need to use some words with those numbers to make a when/then sentence. Now let's look at that second feature of our graph, the slope.


So, what does the y-intercept represent? When the train has traveled for 0 hours, it will have traveled 0 miles.

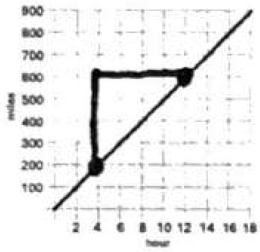
Let's Think (Slide 5): "The slope of a graph is how it increases or decreases. It is measured by the change in y divided by the change in x." You might have learned a bit about this in earlier grades but we need to make sure we all agree on what work we show and what it means today. We have the same story about the train and the same graph. We already talked about how "a proportion always has a CONSTANT slope." That means the slope will be the same no matter what part of the line you use to find it.


The graph shows the distance of the train from NY to Orlando, FL travels when moving at its average speed.

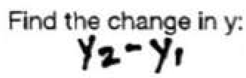
A proportion always has a CONSTANT slope.



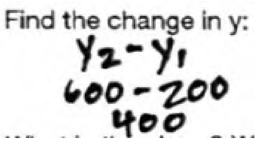
This says, "we use two points to show the change in y and the change in x." The change in y and the change in x is really just the two parts of the staircase we used earlier when we were saying "up 30 over 2." Now let's see how we get that same answer with two points because the numbers will get less obvious later on. You can pick any two points. But for now I will pick (4,200) and (12,600). *Mark the two points.*



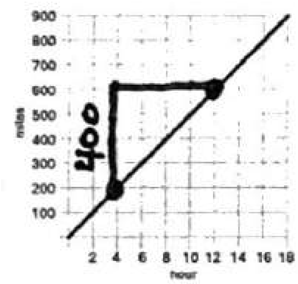
This makes a giant step on the staircase that I could count up. *Draw the "step" from one point to the other.*



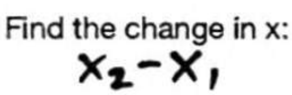
But another way to find the change or difference is subtraction. So I am going to start with y and subtract "y two" minus "y one." It is written like this. That's just like saying the second value of y minus the first value of y.



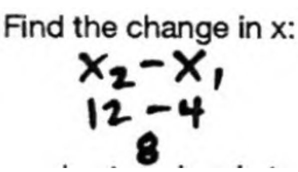
Y_2 is 600 and y_1 is 200 so this will be 600 minus 200 which is 400. That is the change in y.



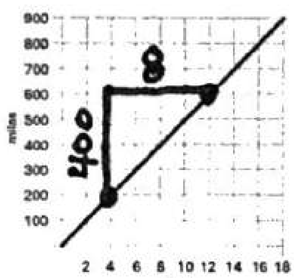
That is how much the line increased on the y axis, or in this case, it is how much that step went up. *Label the vertical line of the step you drew.*



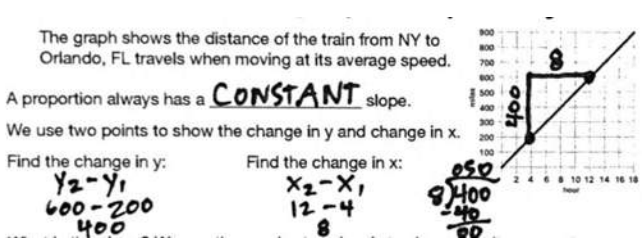
Now we need to find the change in x. Again, that is the difference so we subtract. I am going to do "x two" minus "x one" which is like saying the second value of x minus the first value of x.



X_2 is 12 and x_1 is 4 so this will be 12 minus 4 which is 8. That is the change in x.

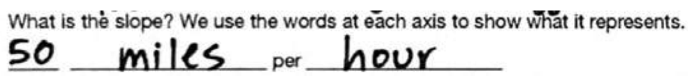


That is how much the line moved over on the x-axis, or in this case, it is how much that step went over. *Label the horizontal line of the step you drew.*



We said at the very beginning of this slide that the slope is the change in y divided by the change in x so we do 400 divided by 8. 8 doesn't go into 4 so that's 0. 8 goes into 40, 5 times. I subtract 40 and have 0. So this next place is 0.

“What is the slope?” 50! “We use the words at each axis to show what it represents.” It was y divided by x so we use the y words then the x words. This is 50 miles per hour. Because that’s what we did, right? We took the difference in miles and split them up by the hours.



Now let’s review the big ideas. The slope is found by doing the change in y divided by the change in x. A proportion always has a constant slope, which we can see because it is a straight line. That means that this would be 50 miles per hour no matter where we decided to mark our points and find the slope.

Let’s Try It (Slide 6): Let’s interpret the y-intercept and the slope of graphs together now! I will walk you through step by step.

WARM WELCOME

Today we will make meaning of the y-intercept and slope on a graph of a proportion.

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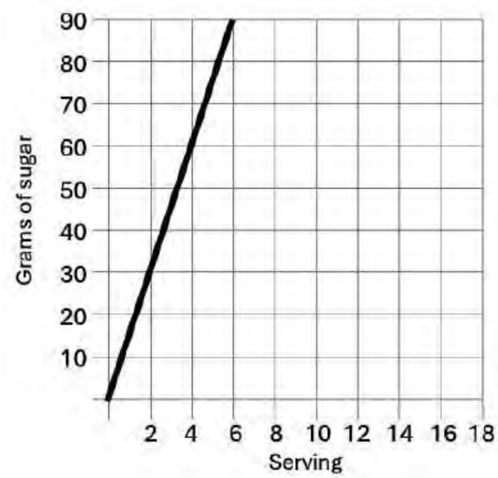
Let's Review: The graph of a proportion always has two special features.

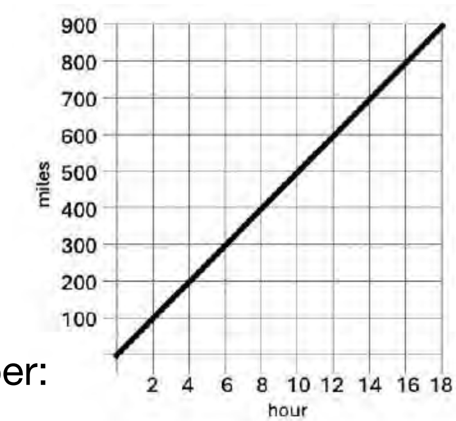
How do we know if the graph below shows a proportion?

Lea used the graph to show the amount of sugar in different amounts of Vitamin Water.



Let's Talk: The y-intercept of a graph is where it crosses the y-axis and where x = 0.

The graph shows the distance of the train from NY to Orlando, FL travels when moving at its average speed.



A proportion always crosses the y-axis at (____, ____)

which means the y-intercept is always ____.

We can explain what this means using words for each number:

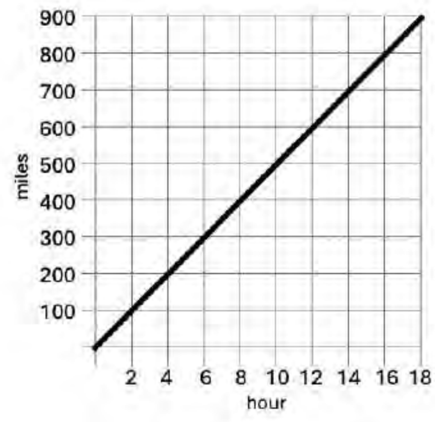
_____ ___________________ and ____ __________________

So, what does the y-intercept represent?

Let's Think:

The slope of a graph is how it increases or decreases. It is measured by the change in y divided by the change in x.

The graph shows the distance of the train from NY to Orlando, FL travels when moving at its average speed.



A proportion always has a ________________ slope.

We use two points to show the change in y and change in x.

Find the change in y:

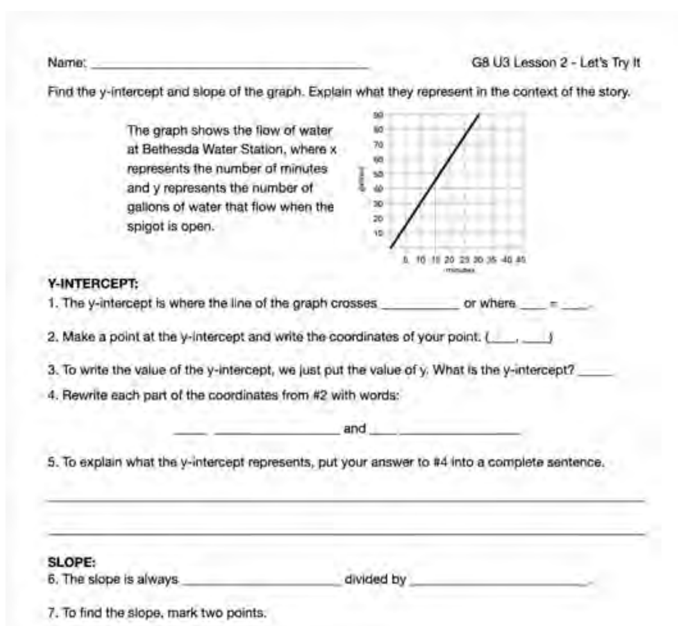
Find the change in x:

What is the slope? We use the words at each axis to show what it represents.

_____ ___________________ per ___________________

Let's Try It:

Let's interpret the y-intercept and the slope of graphs together!



Name: ______________________________ G8 U3 Lesson 2 - Let's Try It

Find the y-intercept and slope of the graph. Explain what they represent in the context of the story.

The graph shows the flow of water at Bethesda Water Station, where x represents the number of minutes and y represents the number of gallons of water that flow when the spigot is open.

Y-INTERCEPT:

1. The y-intercept is where the line of the graph crosses __________ or where ___ = ___.

2. Make a point at the y-intercept and write the coordinates of your point. (___, ___)

3. To write the value of the y-intercept, we just put the value of y. What is the y-intercept? ____

4. Rewrite each part of the coordinates from #2 with words:

____ ______________ and ____ ______________

5. To explain what the y-intercept represents, put your answer to #4 into a complete sentence.

____

____

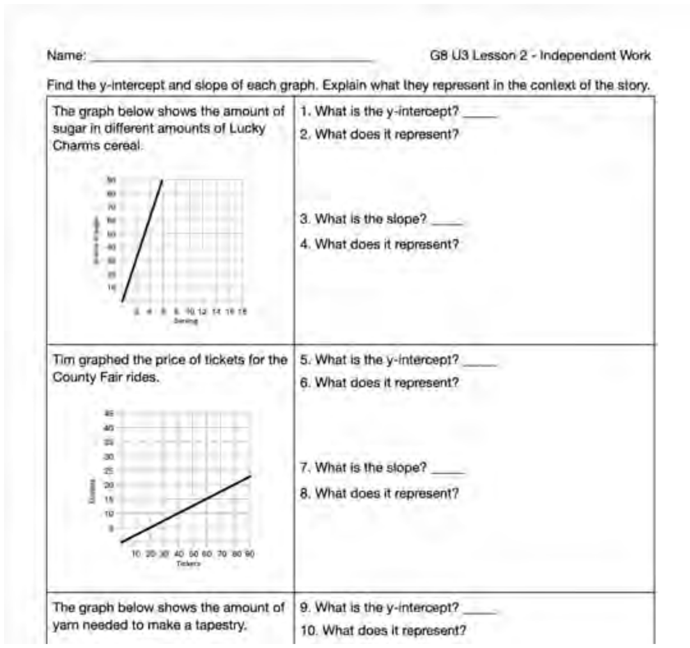
SLOPE:

6. The slope is always ________________ divided by ________________.

7. To find the slope, mark two points.

On your Own:

Now it's time for you to do it on your own.



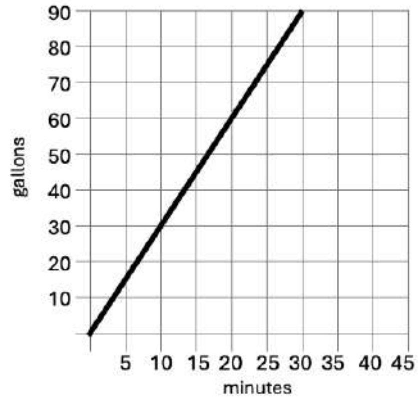
Name: ______________________ G8 U3 Lesson 2 - Independent Work

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

The graph below shows the amount of sugar in different amounts of Lucky Charms cereal.	1. What is the y-intercept? _____ 2. What does it represent? 3. What is the slope? _____ 4. What does it represent?
Tim graphed the price of tickets for the County Fair rides.	5. What is the y-intercept? _____ 6. What does it represent? 7. What is the slope? _____ 8. What does it represent?
The graph below shows the amount of yarn needed to make a tapestry.	9. What is the y-intercept? _____ 10. What does it represent?

Find the y-intercept and slope of the graph. Explain what they represent in the context of the story.

The graph shows the flow of water at Bethesda Water Station, where x represents the number of minutes and y represents the number of gallons of water that flow when the spigot is open.



Y-INTERCEPT:

1. The y-intercept is where the line of the graph crosses ___________ or where ____ = ____.

2. Make a point at the y-intercept and write the coordinates of your point. (____, ____)

3. To write the value of the y-intercept, we just put the value of y. What is the y-intercept? _____

4. Rewrite each number of the coordinates from #2 with words:

_____ __________________ and ____ _________________

5. To explain what the y-intercept represents, put your answer to #4 into a complete sentence.

____

____

SLOPE:

6. The slope is always _______________________ divided by __________________________.

7. To find the slope, mark two points.

8. To find the change in y, we must ____.

9. What is the change in y? _____-_____= _____

10. To find the change in x, we must ____.

11. What is the change in x? _____-_____= _____

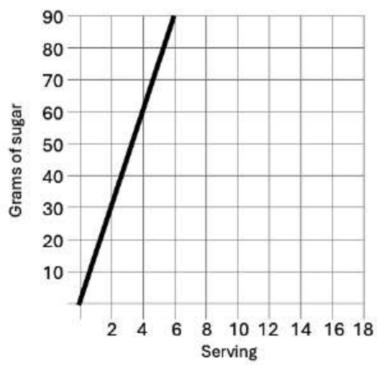
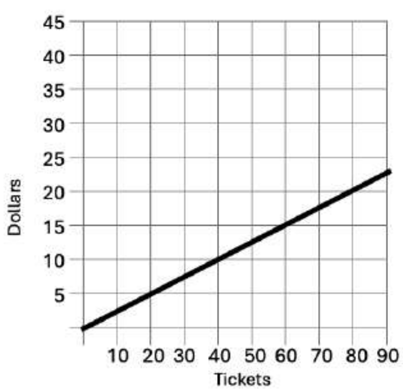
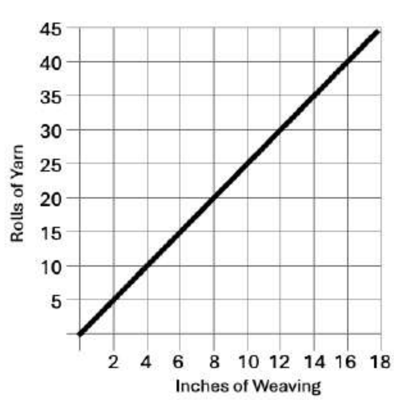
12. Use your answers to #9 and #11 to find the slope. _______

13. To explain what the slope represents, use the words at each axis, y words then x words.

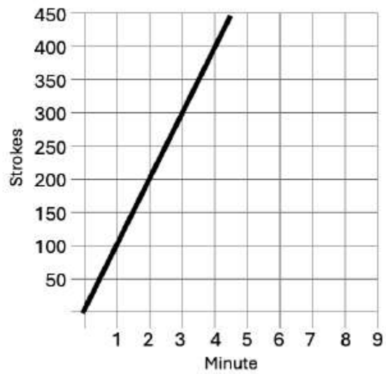
________ _____________________________ per ____________________________

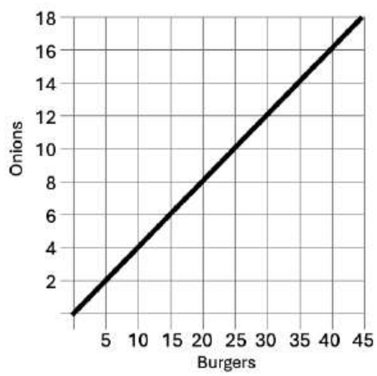
Name: ____ G8 U3 Lesson 2 - Independent Work

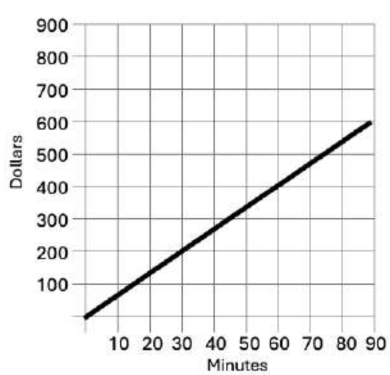
Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

The graph below shows the amount of sugar in different amounts of Lucky Charms cereal. 	1. What is the y-intercept? _____ 2. What does it represent? 3. What is the slope? _____ 4. What does it represent?
Tim graphed the price of tickets for the County Fair rides. 	5. What is the y-intercept? _____ 6. What does it represent? 7. What is the slope? _____ 8. What does it represent?
The graph below shows the amount of yarn needed to make a tapestry. 	9. What is the y-intercept? _____ 10. What does it represent? 11. What is the slope? _____ 12. What does it represent?

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

<table>
<tr>
<td>The graph below shows the amount that GW's rowing team can row at a constant rate over time.

</td>
<td>13. What is the y-intercept? _____
14. What does it represent?

15. What is the slope? _____
16. What does it represent?</td>
</tr>
<tr>
<td>Chef uses the graph to determine how many onions to buy based on the number of burgers he is going to cook.

</td>
<td>17. What is the y-intercept? _____
18. What does it represent?

19. What is the slope? _____
20. What does it represent?</td>
</tr>
<tr>
<td>Dan made a graph to show how much he bills clients based on the length of a meeting.

</td>
<td>21. What is the y-intercept? _____
22. What does it represent?

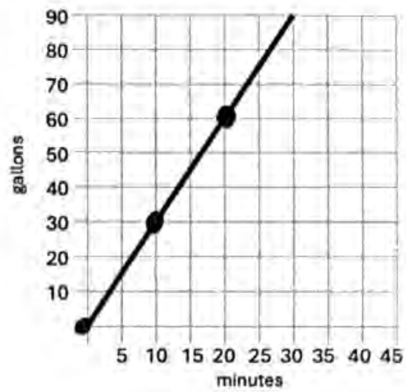
23. What is the slope? _____
24. What does it represent?</td>
</tr>
</table>

Name: ANSWER KEY

G8 U3 Lesson 2 - Let's Try It

Find the y-intercept and slope of the graph. Explain what they represent in the context of the story.

The graph shows the flow of water at Bethesda Water Station, where x represents the number of minutes and y represents the number of gallons of water that flow when the spigot is open.



Y-INTERCEPT:

1. The y-intercept is where the line of the graph crosses y-axis or where x = 0.

2. Make a point at the y-intercept and write the coordinates of your point. (0, 0)

3. To write the value of the y-intercept, we just put the value of y. What is the y-intercept? 0

4. Rewrite each number of the coordinates from #2 with words:

0 minutes and 0 gallons

5. To explain what the y-intercept represents, put your answer to #4 into a complete sentence.

When the spigot is open for 0 minutes then 0 gallons of water has flowed.

SLOPE:

6. The slope is always change in y divided by change in x.

7. To find the slope, mark two points.

8. To find the change in y, we must subtract $y_2 - y_1$.

9. What is the change in y? 60 - 30 = 30

10. To find the change in x, we must subtract $x_2 - x_1$.

11. What is the change in x? 20 - 10 = 10

12. Use your answers to #9 and #11 to find the slope. 30 ÷ 10 = 3

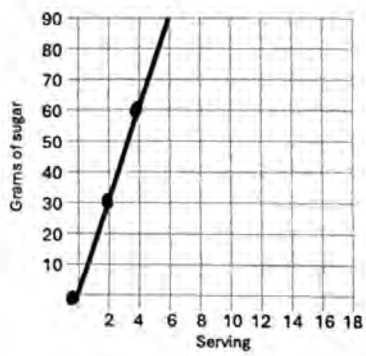
13. To explain what the slope represents, use the words at each axis, y words then x words.

3 gallons per minute

Name: ANSWER KEY

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

The graph below shows the amount of sugar in different amounts of Lucky Charms cereal.



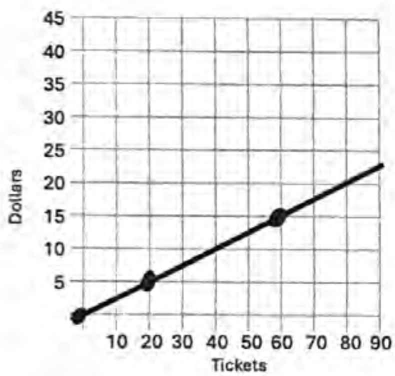
1. What is the y-intercept? 0
2. What does it represent?

When there are 0 servings then there are 0 grams of sugar.

3. What is the slope? 15 $\frac{60-30}{4-2} = \frac{30}{2} = 15$
4. What does it represent?

15 grams of sugar per serving

Tim graphed the price of tickets for the County Fair rides.



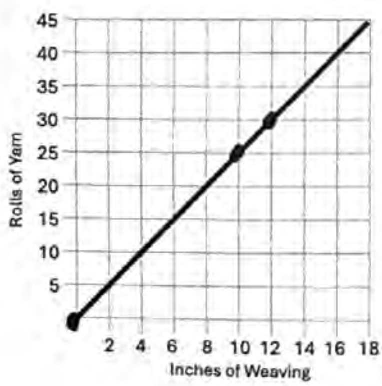
5. What is the y-intercept? 0
6. What does it represent?

When Tim buys 0 tickets, then he pays 0 dollars

7. What is the slope? $\frac{1}{4}$ $\frac{15-5}{60-20} = \frac{10}{40} = \frac{1}{4}$
8. What does it represent?

$\frac{1}{4}$ dollar per ticket

The graph below shows the amount of yarn needed to make a tapestry.



9. What is the y-intercept? 0
10. What does it represent?

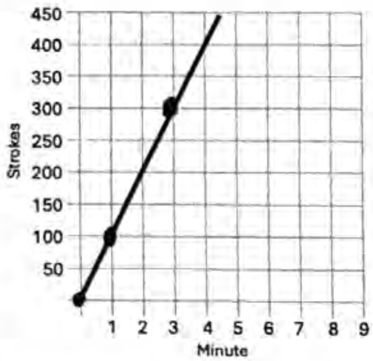
When there are 0 inches of weaving, then 0 rolls of yarn were needed.

11. What is the slope? $1\frac{1}{2}$ $\frac{30-25}{12-10} = \frac{5}{2} = 1\frac{1}{2}$
12. What does it represent?

$1\frac{1}{2}$ rolls of yarn per inch of weaving

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

The graph below shows the amount that GW's rowing team can row at a constant rate over time.



13. What is the y-intercept? 0

14. What does it represent?

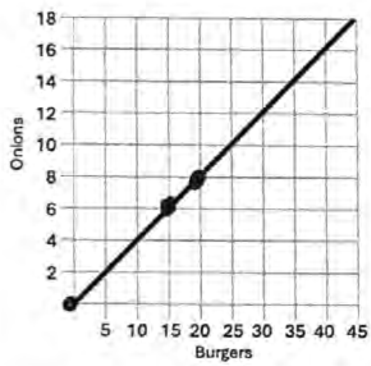
When GW's rowing team has rowed 0 min then they would have done 0 strokes.

15. What is the slope? ____ $\frac{300-100}{3-1} = \frac{200}{2} = 100$

16. What does it represent?

100 strokes per minutes

Chef uses the graph to determine how many onions to buy based on the number of burgers he is going to cook.



17. What is the y-intercept? 0

18. What does it represent?

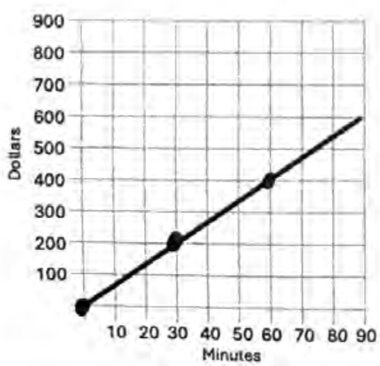
When chef is going to make 0 burgers then he needs 0 onions.

19. What is the slope? $\frac{2}{5}$ $\frac{8-6}{20-15} = \frac{2}{5}$

20. What does it represent?

$\frac{2}{5}$ onions per burger

Dan made a graph to show how much he bills clients based on the length of a meeting.



21. What is the y-intercept? 0

22. What does it represent?

When Dan has a 0 minute meeting then he bills clients 0 dollars.

23. What is the slope? $6\frac{2}{3}$ $\frac{400-200}{60-30} = \frac{200}{30}$

$30\overline{)200}$ = 006, -180, 20

24. What does it represent?

$6\frac{2}{3}$ dollars per minute

G8 U3 Lesson 3
Find the rate of change of a proportional relationship given the graph, equation, table, or situation.

CITYTUTOR DC

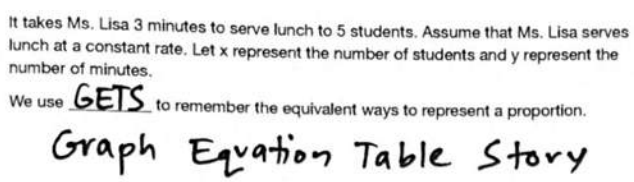
G8 U3 Lesson 3 - Today we will connect the meaning of the unit rate, the constant of proportionality and the slope.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we will connect the meaning of the unit rate, the constant of proportionality and the slope. In our last lesson we practiced finding the y-intercept and slope on a graph. Now we are going to connect those ideas to corresponding parts of a table and an equation. This is the coolest thing about math - that all the tools we use are connected and reinforce the same fundamental ideas. It's going to be awesome! Let's go!

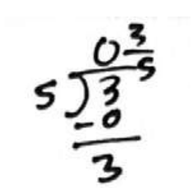
Let's Review (Slide 3): In our last lessons, we answered this question, "What ways do we have to represent a proportion?" We have a story about a proportional relationship here. Read along silently while I read it out loud. *Read the story out loud.* I know this is a proportion because it says that she serves lunch at a constant rate. That constant rate means there is a proportional relationship and when one quantity changes, the other changes proportionally. What are different ways that I could represent this proportional relationship? Possible Student Answers, Key Points:

- You can draw a picture.
- You can make a table.
- You can write an equation.
- You can make a graph.



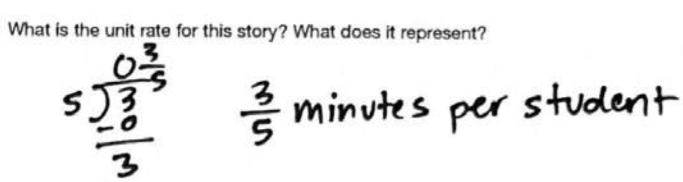
I heard so many great ideas. We can use the acronym, "GETS," to remember all the ways we represent a proportion. G stands for graph. E stands for equation. T stands for table. S stands for story. All of these can be used to show what is described in this story and what happens when the proportional relationship in this story keeps going.

Let's Talk (Slide 4): This is the main idea for today, "In all our forms of representation, there is always a form of the unit rate." I am going to show you what I mean in this example below. Read along with me silently as I read out loud. *Read the problem out loud.* Now we said that there is always a form of the unit rate so the first thing we are going to do to explore this idea is answer this question, "What is the unit rate for this story? What does it represent?" First, it might be helpful if I remind you what the unit rate is. It is the amount of something for just 1 of the other thing. Unit means one. So the unit rate is the amount of something for 1 of the other thing.

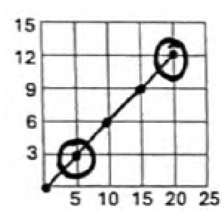


If I want to split something up equally to find the amount for our group, that would be division, right? Division tells us the amount in each group. So in this case, we're going to divide to find the amount in each unit. It will always be y divided by x which is 3 divided by 5. 5 doesn't go into 3. We get 0 and our numbers can only be represented with a fraction, 3 fifths.

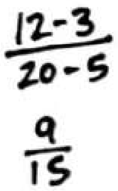
We never want to have a number without words, and that will help us know what our unit rate represents. We can figure it out by thinking about what we divided. 3 was the minutes and the 5 was the students. So we have 3 fifths minutes per student. That means it takes 3 fifth of a minute for every 1 student. Whenever we have a unit rate, it will always have two sets of words representing



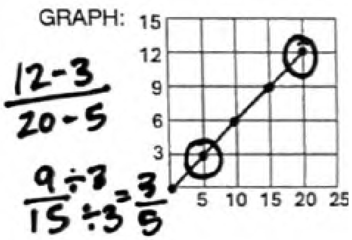
some quantity for ONE of the other quantity. In this case minutes per ONE student.



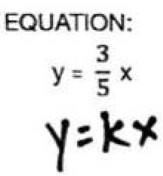
Let's Think (Slide 5): Now let's see how the unit rate shows up in the other forms. This says, "The unit rate is the slope on a graph and in an equation. It is the constant of proportionality on a table." We have the same story but now we've been given a graph, equation and table to match. Let's start by finding the slope. We practiced this in our last lesson. It is the change in y divided by the change in x. So let's select two points. I will select (5,3) and (20,12). *Circle the points.*



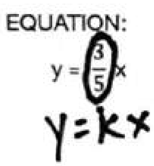
Now we do y_2 minus y_1 which is 12 minus 3. That will be divided by x_2 minus x_1 which is 20 minus 5. That's 9 divided by 15.



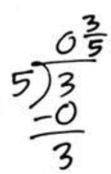
This is less than 1 whole, right? If we divide it, we'd get zero. So the only thing we can do is simplify it. I will divide the top by 3 and the bottom by 3. I get 3 fifths. This is the big idea of our lesson: the unit rate we found on the last slide is the same as the slope of the graph on this slide.



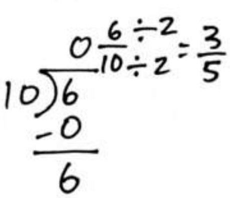
That means it is also the same as the slope in our equation. The equation for a proportion is always written in the form y = kx where k is whatever number is multiplying x. *Write y=kx underneath our equation on the slide.*



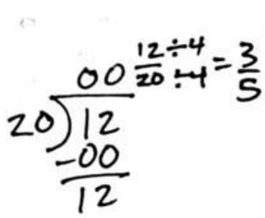
K is the slope of the equation. And we can see that in this case, it is 3 fifths. *Circle 3 fifths in the equation.* So this is the next part of our big idea: the unit rate we found on the last slide is the same as the slope of the graph on this slide AND it is the same as the slope of the equation on this slide.



We can check all this by finding the constant of proportionality on the table. Hopefully you did a bit of that back in 6th grade but we'll practice again here. It is y divided by x. So, for this first row, we have 3 divided by 5, which is 3 fifths.



Let's do the next row. It is 6 divided by 10, which is 6 tenths. We can simplify that by dividing the numerator and denominator by 2. We get 3 fifths. It is the same! That's why it's called the CONSTANT of proportionality - because it's constant.



We could keep going. Let's skip down to the bottom row for fun. We have 12 divided by 20. That is zero. We get 12 twentieths. We can simplify that by dividing the numerator and denominator by 4. We get 3 fifths. It is the same too! This is the final part of our big idea: the unit rate we found on the last slide is the same as the slope of the graph on this slide and it is the same as the slope of the equation on this slide AND it is the same as the constant of proportionality in the table on this slide.

Let's Try It (Slide 6): Let's find the unit rate, slope and constant of proportionality together! I will walk you through each step.

WARM WELCOME

Today we will connect the meaning of the unit rate, the constant of proportionality and the slope.

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Let's Review: What ways do we have to represent a proportion?

It takes Ms. Lisa 3 minutes to serve lunch to 5 students. Assume that Ms. Lisa serves lunch at a constant rate. Let x represent the number of students and y represent the number of minutes.

We use __________ to remember the equivalent ways to represent a proportion.

Let's Talk: In all our forms of representation, there is always a form of the unit rate.

It takes Ms. Lisa 3 minutes to serve lunch to 5 students. Assume that Ms. Lisa serves lunch at a constant rate. Let x represent the number of students and y represent the number of minutes.

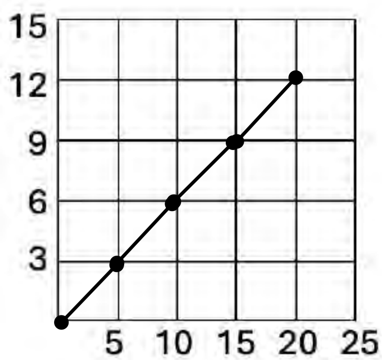
What is the unit rate for this story? What does it represent?

Let's Think:

The unit rate is the slope on a graph and in an equation. It is the constant of proportionality on a table.

It takes Ms. Lisa 3 minutes to serve lunch to 5 students. Assume that Ms. Lisa serves lunch at a constant rate. Let x represent the number of students and y represent the number of minutes.

GRAPH:



EQUATION:

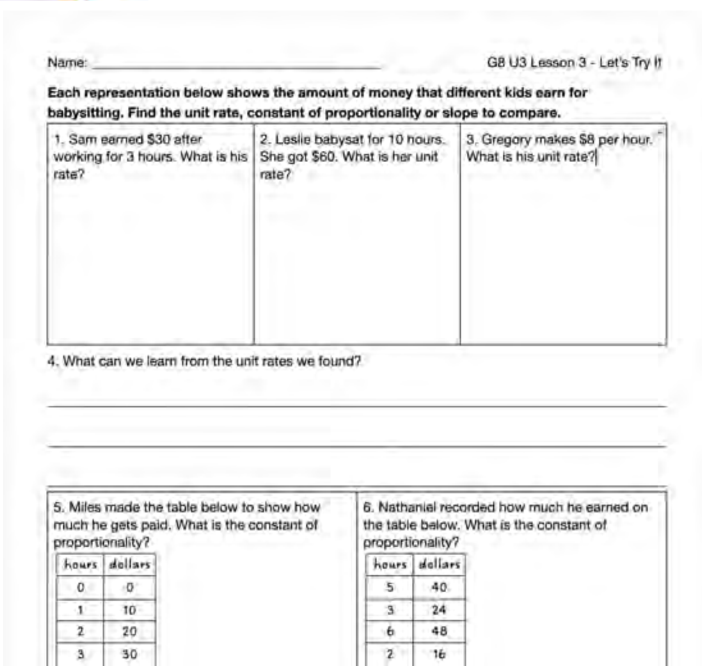
$$y = \frac{3}{5}x$$

TABLE:

x	y
0	0
5	3
10	6
15	9
20	12

Let's Try It:

Let's find the unit rate, slope and constant of proportionality together!



Name: ______________________ G8 U3 Lesson 3 - Let's Try It

Each representation below shows the amount of money that different kids earn for babysitting. Find the unit rate, constant of proportionality or slope to compare.

1. Sam earned $30 after working for 3 hours. What is his rate?	2. Leslie babysat for 10 hours. She got $60. What is her unit rate?	3. Gregory makes $8 per hour. What is his unit rate?

4. What can we learn from the unit rates we found?

5. Miles made the table below to show how much he gets paid. What is the constant of proportionality?

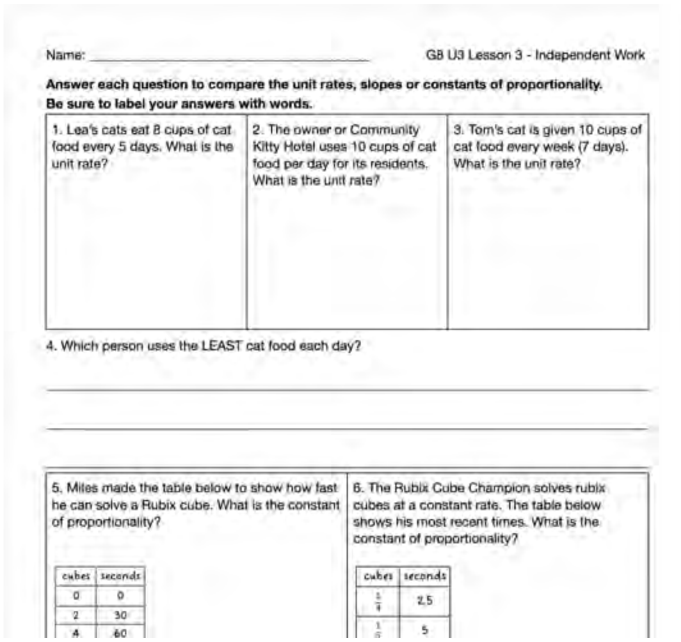
hours	dollars
0	0
1	10
2	20
3	30

6. Nathaniel recorded how much he earned on the table below. What is the constant of proportionality?

hours	dollars
5	40
3	24
6	48
2	16

On your Own:

Now it's time for you to do it on your own.



Name: ______________________ G8 U3 Lesson 3 - Independent Work

Answer each question to compare the unit rates, slopes or constants of proportionality.
Be sure to label your answers with words.

1. Lea's cats eat 8 cups of cat food every 5 days. What is the unit rate?	2. The owner or Community Kitty Hotel uses 10 cups of cat food per day for its residents. What is the unit rate?	3. Tom's cat is given 10 cups of cat food every week (7 days). What is the unit rate?

4. Which person uses the LEAST cat food each day?

____

____

5. Miles made the table below to show how fast he can solve a Rubix cube. What is the constant of proportionality?

cubes	seconds
0	0
2	30
4	60

6. The Rubix Cube Champion solves rubix cubes at a constant rate. The table below shows his most recent times. What is the constant of proportionality?

cubes	seconds
$\frac{1}{4}$	2.5
1/[illegible]	5

Each representation below shows the amount of money that different kids earn for babysitting. Find the unit rate, constant of proportionality or slope to compare.

1. Sam earned $30 after working for 3 hours. What is his rate?	2. Leslie babysat for 10 hours. She got $60. What is her unit rate?	3. Gregory makes $8 per hour. What is his unit rate?

4. What can we learn from the unit rates we found?

____

____

____

5. Miles made the table below to show how much he gets paid. What is the constant of proportionality?

hours	dollars
0	0
1	10
2	20
3	30
4	40

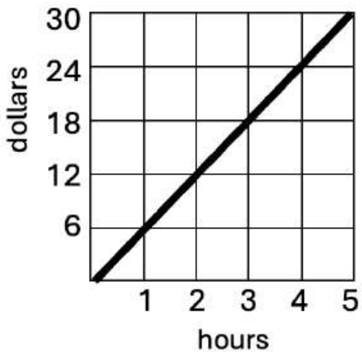
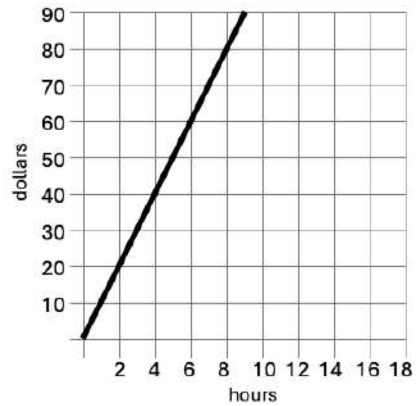
6. Nathaniel recorded how much he earned on the table below. What is the constant of proportionality?

hours	dollars
5	40
3	24
6	48
2	16

7. What can we learn from the constants of proportionality we found?

____

____

8. Rose made a graph of what she charges for different amounts of hours. What is the slope of her graph? 	9. The graph shows Matt's earnings for babysitting based on hours he works. What is the slope of the graph? 

10. What can we learn from the slopes we found?

11. Dave uses the equation y=8x to calculate his charges where x is the number of hours and y is the number of dollars. What is the slope?	12. The equation, 10x = y, could be used to find the amount that Colby earns where x represents the number of hours and y represents the number of dollars. What is slope?

13. What can we learn from the slopes we found?

Name: ____ G8 U3 Lesson 3 - Independent Work

Answer each question to compare the unit rates, slopes or constants of proportionality. Be sure to label your answers with words.

1. Lea's cats eat 8 cups of cat food every 5 days. What is the unit rate?	2. The owner or Community Kitty Hotel uses 10 cups of cat food per day for its residents. What is the unit rate?	3. Tom's cat is given 10 cups of cat food every week (7 days). What is the unit rate?

4. Which person uses the LEAST cat food each day?

____

____

____

5. Miles made the table below to show how fast he can solve a Rubix cube. What is the constant of proportionality?

cubes	seconds
0	0
2	30
4	60
5	90

6. The Rubix Cube Champion solves rubix cubes at a constant rate. The table below shows his most recent times. What is the constant of proportionality?

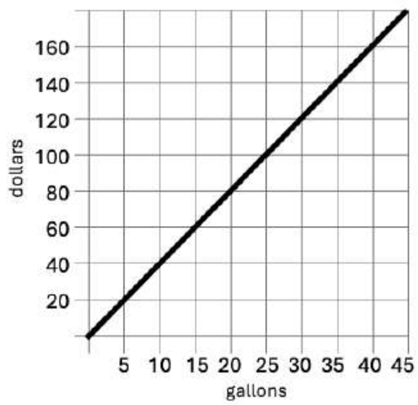
cubes	seconds
$\frac{1}{4}$	2.5
$\frac{1}{2}$	5
$\frac{3}{4}$	7.5
1	10

7. How much faster (in seconds per cube) is the Champion than Miles?

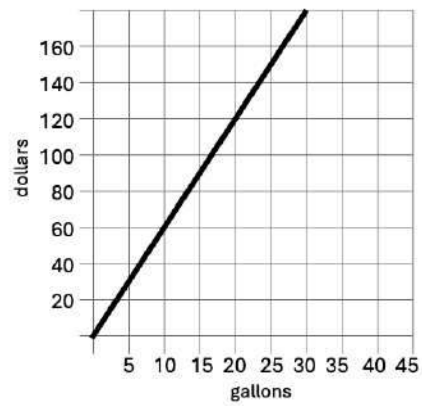
____

____

8. The prices for gas at QuickFill Gas station is shown on the graph below. What is the slope of the graph?



9. The prices for gas at SuperPump Gas station is shown on the graph below. What is the slope of the graph?



10. Which gas station is more expensive?

11. The equation y=12x shows the number of children, y, allowed at a nursery school based on the number of teachers, x. What is the slope?

12. At a school, the equation 14x = y is used to determine how many teachers must be hired depending on the number of students, where x is the number of teachers and y is the number of students. What is the slope?

13. Which school has a higher ratio of students to teachers?

Name: ANSWER KEY

Each representation below shows the amount of money that different kids earn for babysitting. Find the unit rate, constant of proportionality or slope to compare.

1. Sam earned $30 after working for 3 hours. What is his rate?

10
3)30
-30
00

10 dollars per hour

2. Leslie babysat for 10 hours. She got $60. What is her unit rate?

06
10)60
-60
00

6 dollars per hour

3. Gregory makes $8 per hour. What is his unit rate?

$8 dollars per hour

4. What can we learn from the unit rates we found?

Sam has the highest rate in dollars per hour.

5. Miles made the table below to show how much he gets paid. What is the constant of proportionality?

hours	dollars
0	0
1	10
2	20
3	30
4	40

10
1)10
-10
00

10
3)30
-3
00

10 dollars per hour

6. Nathaniel recorded how much he earned on the table below. What is the constant of proportionality?

hours	dollars
5	40
3	24
6	48
2	16

08
5)40
-40
00

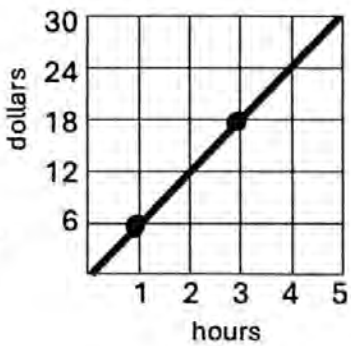
08
2)16
-16
00

8 dollars per hour

7. What can we learn from the constants of proportionality we found?

Miles has a higher rate in dollars per hour than Nathaniel.

8. Rose made a graph of what she charges for different amounts of hours. What is the slope of her graph?



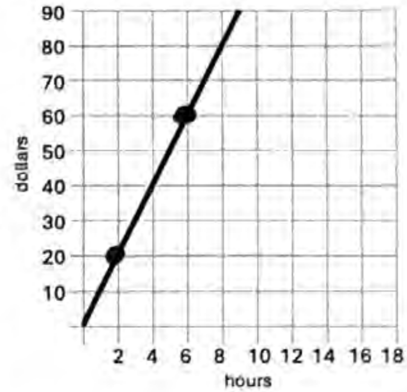
$\frac{18-6}{3-1}$

$\frac{12}{2}$

6

6 dollars per hour

9. The graph shows Matt's earnings for babysitting based on hours he works. What is the slope of the graph?



$\frac{60-20}{6-2}$

$\frac{40}{4}$

10

10 dollars per hour

10. What can we learn from the slopes we found?

Matt has a higher rate in dollars per hour than Rose.

11. Dave uses the equation y=8x to calculate his charges where x is the number of hours and y is the number of dollars. What is the slope?

8 dollars per hour

12. The equation, 10x = y, could be used to find the amount that Colby earns where x represents the number of hours and y represents the number of dollars. What is slope?

10 dollars per hour

13. What can we learn from the slopes we found?

Colby has a higher rate in dollars per hour than Dave.

Name: ANSWER KEY

Answer each question to compare the unit rates, slopes or constants of proportionality. Be sure to label your answers with words.

1. Lea's cats eat 8 cups of cat food every 5 days. What is the unit rate?	2. The owner or Community Kitty Hotel uses 10 cups of cat food per day for its residents. What is the unit rate?	3. Tom's cat is given 10 cups of cat food every week (7 days). What is the unit rate?
5) 8 = 1, −5, 3 $1\frac{3}{5}$ cups per day	10 cups per day	7) 10 = 01, −7, 3 $1\frac{3}{7}$ cups per day

4. Which person uses the LEAST cat food each day?

Tom uses the least cat food per day.

5. Miles made the table below to show how fast he can solve a Rubix cube. What is the constant of proportionality?	6. The Rubix Cube Champion solves rubix cubes at a constant rate. The table below shows his most recent times. What is the constant of proportionality?
see table A below 2) 30 = 15, −2, 10, −10, 00 4) 60 = 15, −4, 20, −20, 00 15 seconds per cube	see table B below $2.5 \div \frac{1}{4}$ $2.5 \times \frac{4}{1}$ 10 $10 \div 1$ 2.5 × 4 = 10.0 10 seconds per cube

Table A (question 5):

cubes	seconds
0	0
2	30
4	60
5	90

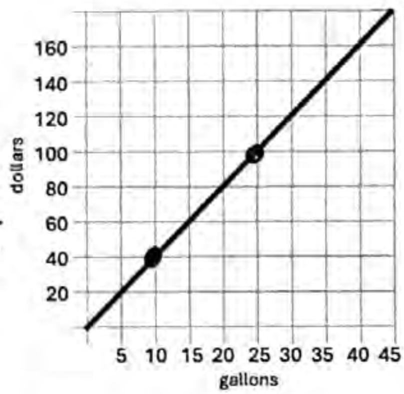
Table B (question 6):

cubes	seconds
$\frac{1}{4}$	2.5
$\frac{1}{2}$	5
$\frac{3}{4}$	7.5
1	10

7. How much faster (in seconds per cube) is the Champion than Miles?

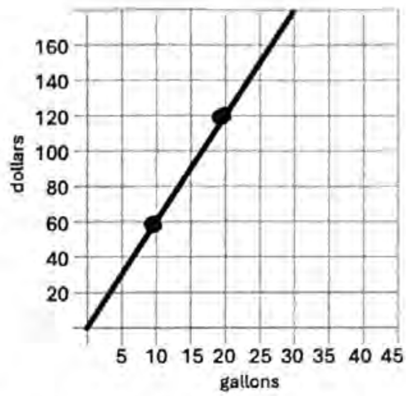
The champion is 5 seconds per cube faster.

8. The prices for gas at QuickFill Gas station is shown on the graph below. What is the slope of the graph?	9. The prices for gas at SuperPump Gas station is shown on the graph below. What is the slope of the graph?



$\frac{100-40}{25-10}$ $\frac{60}{15}$ $60 \div 15 = 04$

4 dollars per gallon



$\frac{120-60}{20-10}$ $\frac{60}{10}$ $60 \div 10 = 06$

6 dollars per gallon

10. Which gas station is more expensive?

SuperPump Gas Station is more expensive.

11. The equation y=12x shows the number of children, y, allowed at a nursery school based on the number of teachers, x. What is the slope?	12. At a school, the equation 14x = y is used to determine how many teachers must be hired depending on the number of students, where x is the number of teachers and y is the number of students. What is the slope?
12 children per teacher	14 children per teacher

13. Which school has a higher ratio of students to teachers?

The second school has a higher ratio.

G8 U3 Lesson 4
Make meaning of pairs of values that satisfy or do not satisfy a given equation or graph.

CITYTUTOR DC

G8 U3 Lesson 4 - Today we will make meaning of values that satisfy or do not satisfy a given equation or graph.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we will make meaning of values that satisfy or do not satisfy a given equation or graph. Yesterday we learned more about the rate of change for a proportion. Now we're going to see what happens when we find pairs of numbers that fit a particular proportion. Let's go!

Let's Review (Slide 3): "How do we know when an equation is true?" Possible Student Answers, Key Points:

- They have to be equal.
- The equation has to be the same on both sides.
- The equation has to be balanced.
- You can do the math and check if it works.

I heard a lot of ways of saying the same idea but I want to make sure I spell out the concept. A lot of times we think about the equal sign like it is telling us to do math from left to right. And sometimes that is correct. But we can also think of it like a balance. It says "equal" right out there, which means what is on this whole side is equal or the same as on this whole side. *Underline each side of the equation.*

Is $\underline{4(5) + 2(8)} = \underline{16}$ true?

We have to understand the equal sign meaning "the same as" or "balanced" and then we do the math from left to right like we're used to. We have to follow order of operations which you might have learned as PEMDAS. You're not going to see a lot of parentheses today. But you might see multiplication and division, and you always have to do those before you do addition and subtraction.

Is $\underline{4(5) + 2(8)} = \underline{16}$ true?

$$20 + 16 = 16$$

Let's begin. I will do 4 times 5 which is 20, and I will do 2 times 8 which is 16. *Be sure to write these numbers directly understand the corresponding place in the equation.* This had an addition sign between them so I bring that down. I bring down the equals 16 just to keep it all in line.

Is $\underline{4(5) + 2(8)} = \underline{16}$ true? NO

$$20 + 16 = 16$$
$$36 = 16$$

Now I do 20 plus 16. That's 36 and if I bring down the rest, I see 36 = 16. Nope! Unbalanced! This equation is NOT true.

$$4(4) + 2 = 10$$

Let's try the next one. It says, "Is 4x + 2 = 10 true when x = 4? I will need to plug in 4 for x. I am going to recopy it to the side. It is 4 x 4 + 2 = 10.

$$4(4) + 2 = 10$$
$$16 + 2 = 10$$

Now I do the multiplication and division first so I do 4 x 4 is 16 and I always recopy it after each step. Now it says 16 + 2 = 10.

Is $\underline{4(5) + 2(8)} = \underline{16}$ true? NO

$$20 + 16 = 16$$
$$36 = 16$$

Is 4x + 2 = 10 true when x = 4?

$$4(4) + 2 = 10$$
$$16 + 2 = 10$$
$$18 = 10$$

That becomes 18 = 10. Nope! Unbalanced! This equation is NOT true.

$4(2)+2=10$

Let's try the next one. It says, "Is 4x + 2 = 10 true when x = 2? I will need to plug in 2 for x. I am going to recopy it to the side. It is 4 x 2 + 2 = 10.

$4(2)+2=10$
$8+2=10$

Now I do the multiplication and division first so I do 4 x 2 is 8 and I always recopy it after each step. Now it says 8 + 2 = 10.

Is 4x + 2 = 10 true when x = 2? YES

$4(2)+2=10$
$8+2=10$
$10=10$

That becomes 10 = 10. Yes! It's balanced! That is TRUE.

$4(0)+2(5)=10$

Let's do the final one, 4x + 2y = 10. I need to recopy it with x as 0 and y as 5. It becomes 4 x 0 + 2 x 5 = 10.

$4(0)+2(5)=10$
$0+10=10$

I do the multiplication and division first. That becomes 0 plus 10 equals 10.

$4(0)+2(5)=10$
$0+10=10$
$10=10$

We get 10 = 10. Yes! It's balanced! That is TRUE! As we work with equations and graphs from proportions today it is going to work the same way. We plug in values and evaluate if the equation is true or if it fits the line.

Is 4x + 2y = 10 true when x = 0 and y = 5? YES

Create a table to show the different values of x and y that would satisfy the equation.

x	y

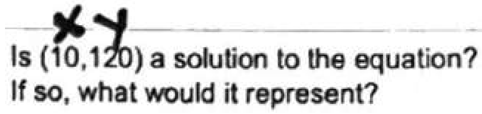
Let's Talk (Slide 4): "We say a point 'satsifies' or 'solves' the equation when the numbers can be plugged in and it stays true." Let's see what we mean. I am going to read the problem out loud and I'd like you to read along silently in your mind. *Read the problem.* This is asking us to "create a table to show the different values of x and y that would satisfy the equation." You guys already know how to do this. I am going to draw a table labeled with x and y.

Create a table to show the different values of x and y that would satisfy the equation.

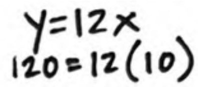
x	y
0	0
1	12
2	24
3	36

It is always a good idea to start with 0. If I plug 0 into the place of x. It is 12 times 0, which is 0. So I write (0,0). Next we can do 1. If I plug 1 into the place of x. It is 12 times 1, which is 12. So I write (1,12). Next we can do 2. If I plug 2 into the place of x. It is 12 times 2, which is 24. So I write (2,24). Next we can do 3. If I plug 3 into the place of x. It is 12 times 3, which is 36. So I write (3,36). And we could keep going but this is pretty good. Each one of these pairs is a solution that satisfies the equation.

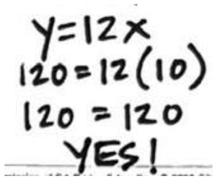
The next question asks, “Is (10,120) a solution to the equation? If so, what would it represent?” This sounds so much more complicated that it is. The only thing you really need to know is that (10,120) is



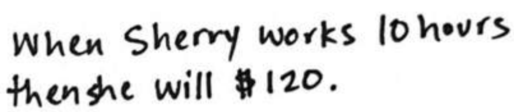
a secret way of saying what x equals and what y equals. The first number is always x and the second number is always y. I am going to label those on top so we can see it.



Now I just plug those numbers into the equation and see if it is balanced or true. I will write y = 12x and then underneath it. It would be 120 equals 12 times 10.

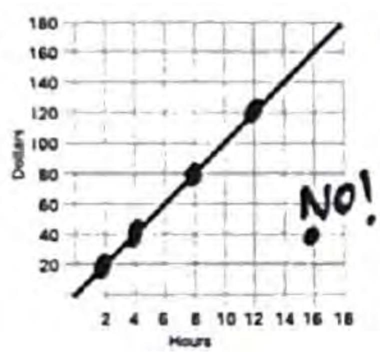


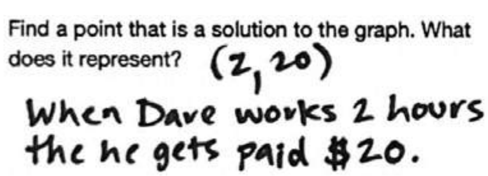
Now we have to evaluate it. 12 times 10 is 120 so we get 120 = 120. That’s balanced. So, is (10,120) a solution? Yes!



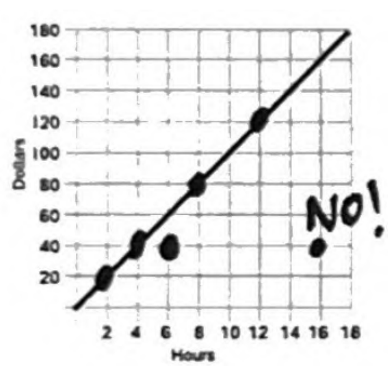
The problem asked what it would represent. To explain that, we need to use the right words with the right numbers. It says x is the number of hours. So I write, “When Sherry works 10 hours…” It says y is the number of dollars. So I write, “then she will get $120.”

Let’s Think (Slide 5): Just like it is helpful to know the numbers to fit an equation, it is helpful to know the numbers that fit a graph. “We say a point ‘satisfies’ or ‘solves’ the graph when the numbers are on the line.” I will show you what this means with this example. Read along silently while I read out loud. *Read the problem.* This says to find a point that is a solution to the graph. You know that when you make a graph, you use points and then connect them with the line. So you can look anywhere on the line and any of those points is a solution. *Mark a point on the line.* This is a solution. *Mark a point on the line.* This is a solution. *Mark a point on the line.* This is a solution. *Put a point that is NOT on the line and label it NO.* This is NOT a solution. *Put a point that is NOT on the line and label it NO.* This is NOT a solution.

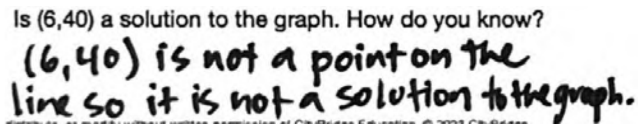




So, we could take the point (2,20) that is a dot on the line and that is one solution. To figure out what it represents, we just have to use the words. X was hours so we write, “When Dave works 2 hours…” and y was dollars so we write, “then he will get paid $20.”

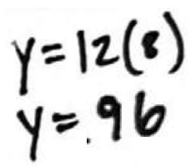


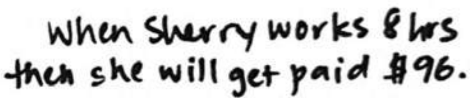
Next, this question says, “Is (6,40) a solution to the graph. How do you know?” We said the points on the line are solutions to the graph so let’s check if this is a point on the line. I find 6 on the x-axis and 40 on the y axis. Then I see where they line up. That would be a point here.



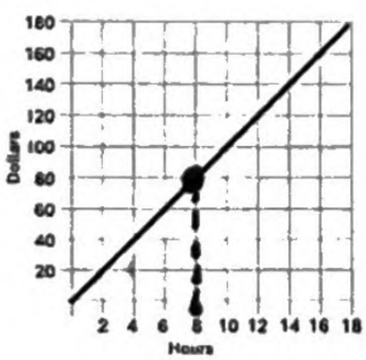
Let's write our answer. (6,40) is not a point on the line so it is not a solution to the graph.

Let's Think (Slide 6): The last thing we can do while we're looking for solutions is find x when given y or find y when given x. This says, "We can use one value to find the other value as a solution to an equation." It wants us to "Find the solution when x = 8. Explain what it means." Here's the story. *Read the story.* You can probably guess what we're going to do. I mean, we've been plugging numbers in, right? Let's plug in x = 8. I am going to rewrite the equation with 8 in place of x. It becomes y = 12 times 8. I will just do 12 times 8 off to the side here. Y = 96 so now we know the full solution is (8,96).





To explain what it means, we just use words. It told us x is hours and y is dollars. So we write, "When Sherry works 8 hours then she will get paid $96."



Let's Think (Slide 7): This is the last thing to check out. It says, "We can use one value to find the other value as a solution to a graph." The problem wants us to "Find the solution when x = 8. Explain what it means." Here's the story. *Read the story.* We know that when we were finding solutions on a graph, we are looking at points that are on the line, and points that are off the line are NOT solutions. So we need to see where x = 8 makes a point on the line. I am going to find 8 on the x-axis since x = 8. Then I go up until I hit the line. Here is the point that we are looking for.

Let's see what y is for this point. It is in line with 80 on the y-axis. Y = 80 so we get (8,80) as our solution. To explain what it means, we just use words. It told us x is hours and y is dollars. So we write, "When Dave works 8 hours then she will get paid $80." Poor Dave doesn't get paid as much as Sherry.

Let's Try It (Slide 8): Let's look at points in the equation or line together! I will walk you through each step.

WARM WELCOME

Today we will make meaning of values that satisfy or do not satisfy a given equation or graph.

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Let's Review: How do we know when an equation is true?

Is 4(5) + 2(8) = 16 true?

Is 4x + 2 = 10 true when x = 4?

Is 4x + 2 = 10 true when x = 2?

Is 4x + 2y = 10 true when x = 0 and y = 5?

Let's Talk: We say a point "satisfies" or "solves" the equation when the numbers can be plugged in and it stays true.

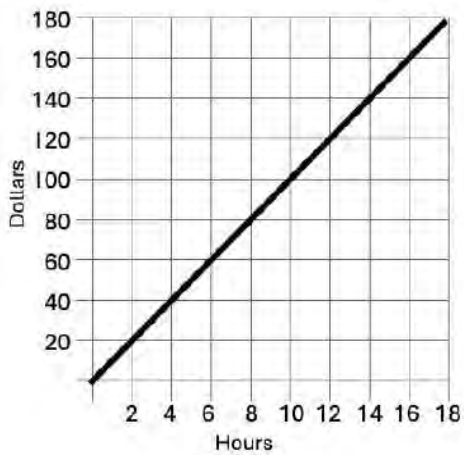
Sherry uses the equation y = 12x to find the amount of dollars she would get paid when working different amounts of time. Let x represent the number of hours and y represent the number of dollars.

Create a table to show the different values of x and y that would satisfy the equation.	Is (10,120) a solution to the equation? If so, what would it represent?

Let's Think:

We say a point "satisfies" or "solves" the graph when the numbers are on the line.

The graph below shows the amount of dollars Dave would get paid when working different amounts of time. Let x represent the number of hours and y represent the number of dollars.



Find a point that is a solution to the graph. What does it represent?

Is (6,40) a solution to the graph. How do you know?

Let's Think:

We can use one value to find the other value as a solution to an equation.

Find the solution when x = 8. Explain what it means.

Sherry uses the equation $y = 12x$ to find the amount of dollars she would get paid when working different amounts of time. Let x represent the number of hours and y represent the number of dollars.

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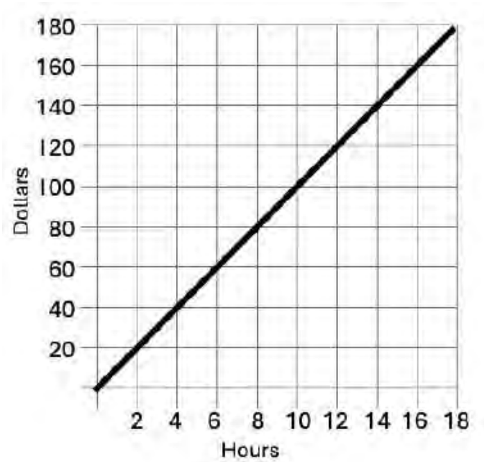
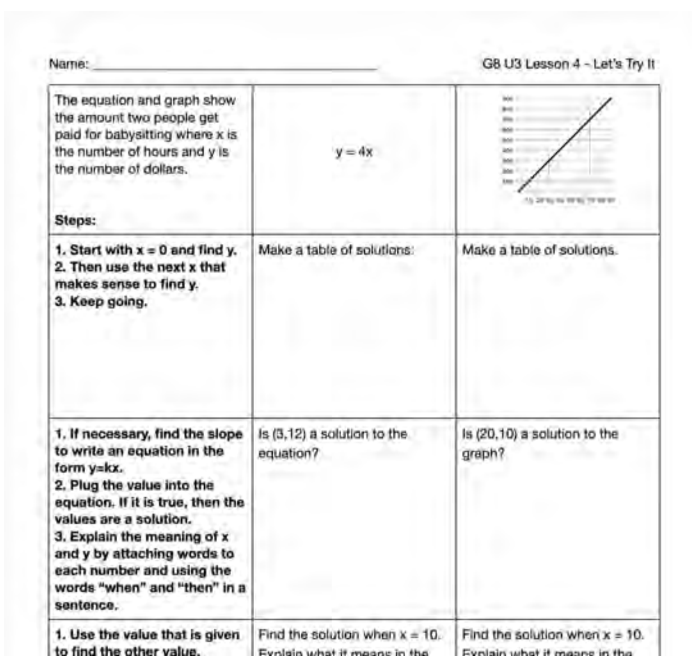
Let's Think: We can use one value to find the other value as a solution to a graph.

Find the solution when x = 8. Explain what it means.

The graph below shows the amount of dollars Dave would get paid when working different amounts of time. Let x represent the number of hours and y represent the number of dollars.



Let's Try It: Let's look at points in the equation or line together!

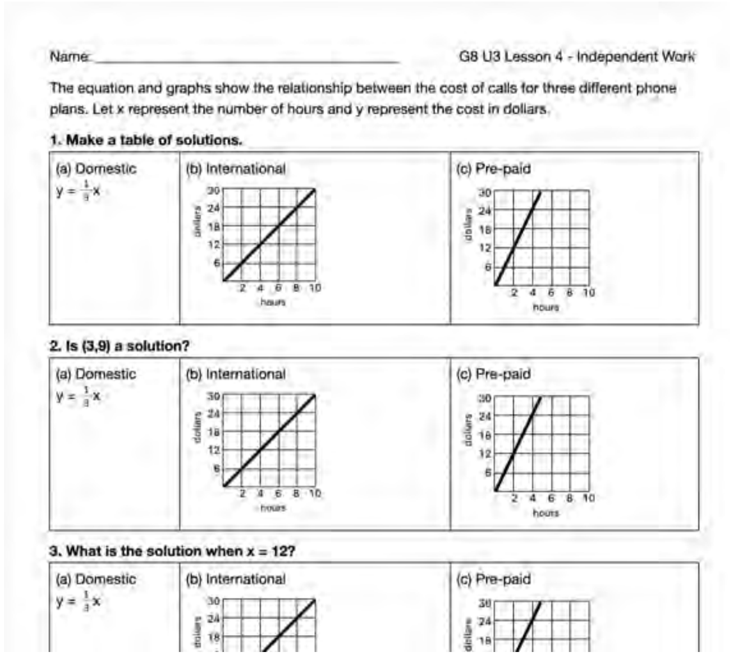


Name: ______________________ G8 U3 Lesson 4 - Let's Try It

The equation and graph show the amount two people get paid for babysitting where x is the number of hours and y is the number of dollars. **Steps:**	y = 4x	
1. Start with x = 0 and find y. **2. Then use the next x that makes sense to find y.** **3. Keep going.**	Make a table of solutions:	Make a table of solutions.
1. If necessary, find the slope to write an equation in the form y=kx. **2. Plug the value into the equation. If it is true, then the values are a solution.** **3. Explain the meaning of x and y by attaching words to each number and using the words "when" and "then" in a sentence.**	Is (3,12) a solution to the equation?	Is (20,10) a solution to the graph?
1. Use the value that is given to find the other value.	Find the solution when x = 10. Explain what it means in the	Find the solution when x = 10. Explain what it means in the

On your Own:

Now it's time for you to do it on your own!


Name: ______________________ G8 U3 Lesson 4 - Independent Work

The equation and graphs show the relationship between the cost of calls for three different phone plans. Let x represent the number of hours and y represent the cost in dollars.

1. Make a table of solutions.

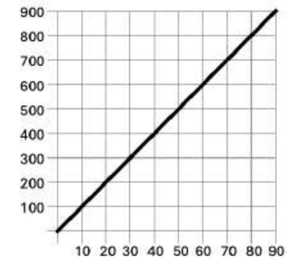
(a) Domestic	(b) International	(c) Pre-paid
$y = \frac{1}{3}x$		

2. Is (3,9) a solution?

(a) Domestic	(b) International	(c) Pre-paid
$y = \frac{1}{3}x$		

3. What is the solution when x = 12?

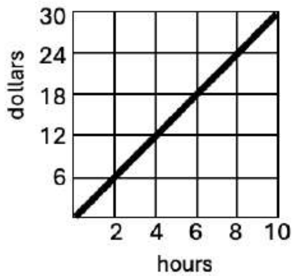
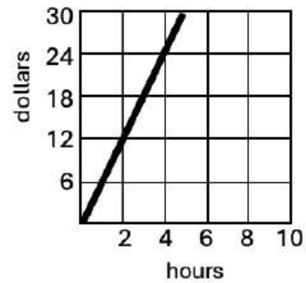
(a) Domestic	(b) International	(c) Pre-paid
$y = \frac{1}{3}x$		

The equation and graph show the amount two people get paid for babysitting where x is the number of hours and y is the number of dollars. **Steps:**	y = 4x	
1. Start with x = 0 and find y. **2. Then use the next x that makes sense to find y.** **3. Keep going.**	Make a table of solutions:	Make a table of solutions.
1. If necessary, find the slope to write an equation in the form y=kx. **2. Plug the value into the equation. If it is true, then the values are a solution.** **3. Explain the meaning of x and y by attaching words to each number and using the words "when" and "then" in a sentence.**	Is (3,12) a solution to the equation?	Is (20,10) a solution to the graph?
1. Use the value that is given to find the other value. **2. Explain the meaning of x and y by attaching words to each number and using the words "when" and "then" in a sentence.**	Find the solution when x = 10. Explain what it means in the context of the problem.	Find the solution when x = 10. Explain what it means in the context of the problem.

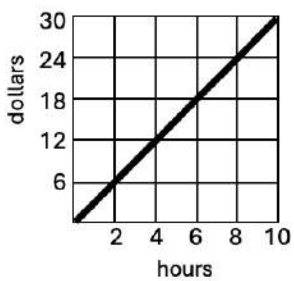
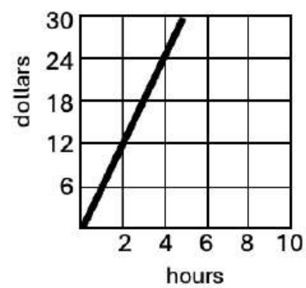
Name: ______________________________________ G8 U3 Lesson 4 - Independent Work

The equation and graphs show the relationship between the cost of calls for three different phone plans. Let x represent the number of hours and y represent the cost in dollars.

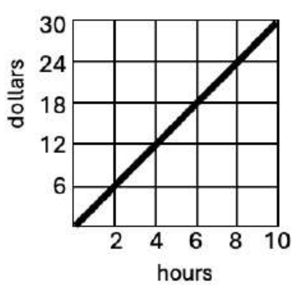
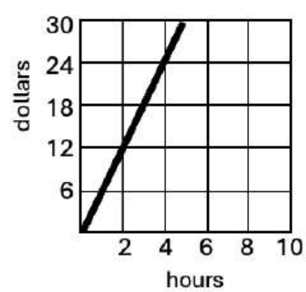
1. Make a table of solutions.

(a) Domestic	(b) International	(c) Pre-paid
$y = \frac{1}{3}x$		

2. Is (3,9) a solution?

(a) Domestic	(b) International	(c) Pre-paid
$y = \frac{1}{3}x$		

3. What is the solution when x = 12?

(a) Domestic	(b) International	(c) Pre-paid
$y = \frac{1}{3}x$		

4. Explain what each solution in #3 represents in words.

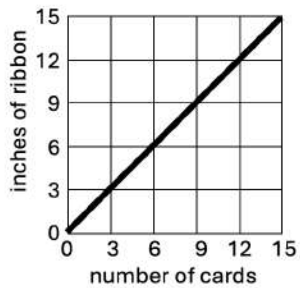
(a)	(b)	(c)

The Hallmark Card Company uses the equation and graphs below show the the amount of ribbon required for different types of cards, where x is the number of cards and y is the inches of ribbon.

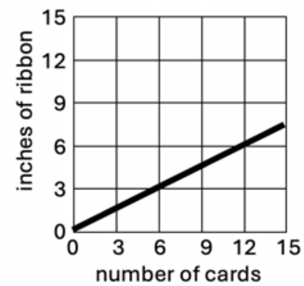
1. Make a table of solutions.

(a) Birthday
$y = 2x$

(b) Anniversary



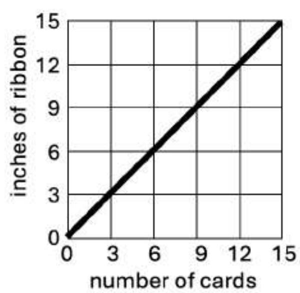
(c) Congratulations



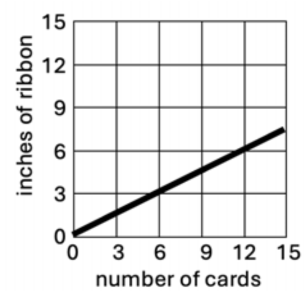
2. Is (3,9) a solution?

(a) Birthday
$y = 2x$

(b) Anniversary



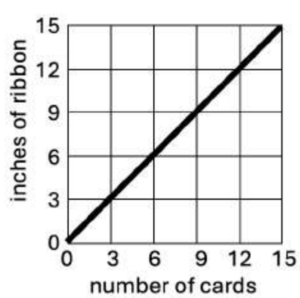
(c) Congratulations



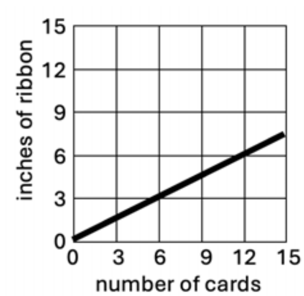
3. What is the solution when y = 100?

(a) Birthday
$y = 2x$

(b) Anniversary



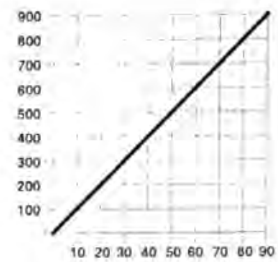
(c) Congratulations



4. Explain what each solution in #3 represents in words.

(a)	(b)	(c)

Name: ANSWER KEY G8 U3 Lesson 4 - Let's Try It

The equation and graph show the amount two people get paid for babysitting where x is the number of hours and y is the number of dollars. **Steps:**	$y = 4x$	
1. Start with x = 0 and find y. **2. Then use the next x that makes sense to find y.** **3. Keep going.**	Make a table of solutions: x | y 0 | 0 1 | 4 2 | 8 3 | 12	Make a table of solutions. x | y 0 | 0 10 | 100 20 | 200 30 | 300
1. If necessary, find the constant of proportionality to write an equation in the form y=kx. **2. Plug the value into the equation. If it is true, then the values are a solution.** **3. Explain the meaning of x and y by attaching words to each number and using the words "when" and "then" in a sentence.**	Is (3,12) a solution to the equation? $y = 4x$ $12 = 4 \cdot 3$ $12 = 12$ YES!	Is (20,10) a solution to the graph? $100 \div 10 = 10$ $200 \div 20 = 10$ $y = 10x$ $10 = 10 \cdot 20$ $10 = 200$ NO!
1. Use the value that is given to find the other value. **2. Explain the meaning of x and y by attaching words to each number and using the words "when" and "then" in a sentence.**	Find the solution when x = 10. Explain what it means in the context of the problem. $y = 4x$ $y = 4 \cdot 10$ $\boxed{y = 40}$ When the person has babysat for 10 hours, then they get paid $40.	Find the solution when x = 10. Explain what it means in the context of the problem. $y = 10x$ $y = 10 \cdot 10$ $\boxed{y = 100}$ When the person has babysat for 10 hours, then they get paid $100.

Name: ANSWER KEY

G8 U3 Lesson 4 - Independent Work

The equation and graphs show the relationship between the cost of calls for three different phone plans. Let x represent the number of hours and y represent the cost in dollars.

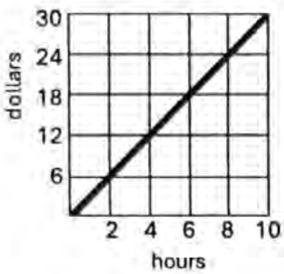
1. Make a table of solutions.

(a) Domestic

$y = \frac{1}{3}x$

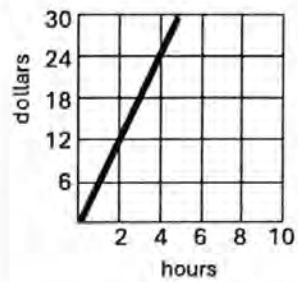
x	y
0	0
1	$\frac{1}{3}$
2	$\frac{2}{3}$
3	1

(b) International



x	y
0	0
2	6
4	12

(c) Pre-paid



x	y
0	0
2	12
4	24

2. Is (3,9) a solution?

(a) Domestic

$y = \frac{1}{3}x$

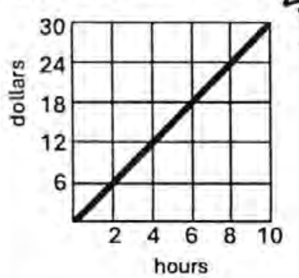
$9 = \frac{1}{3} \cdot 3$

$9 = \frac{3}{3}$

$9 = 1$

NO!

(b) International



$6 \div 2 = 3$; $12 \div 4 = 3$

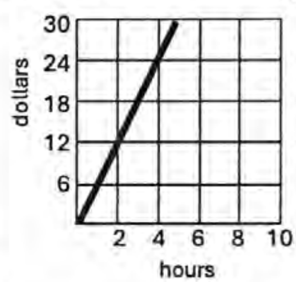
$y = 3x$

$9 = 3 \cdot 3$

$9 = 9$

YES!

(c) Pre-paid



$12 \div 2 = 6$; $24 \div 4 = 6$

$y = 6x$

$9 = 6 \cdot 3$

$9 = 18$

NO!

3. What is the solution when x = 12?

(a) Domestic

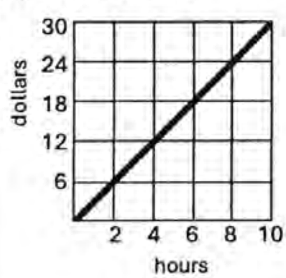
$y = \frac{1}{3}x$

$y = \frac{1}{3} \cdot 12$

$y = \frac{12}{3}$

$y = 4$

(b) International

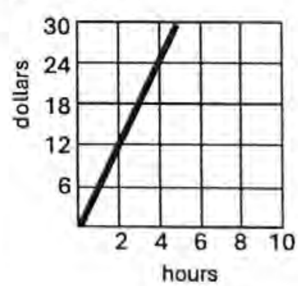


$y = 3x$

$y = 3 \cdot 12$

$y = 36$

(c) Pre-paid



$y = 6x$

$y = 6 \cdot 12$

$y = 72$

4. Explain what each solution in #3 represents in words.

(a) When a call is 12 hours then the cost is \$4.

(b) When a call is 12 hours then the cost is \$36.

(c) When a call is 12 hours then the cost is \$72.

The Hallmark Card Company uses the equation and graphs below show the the amount of ribbon required for different types of cards, where x is the number of cards and y is the inches of ribbon.

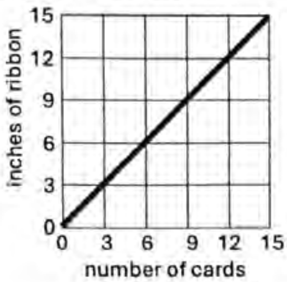
1. Make a table of solutions.

(a) Birthday

$y = 2x$

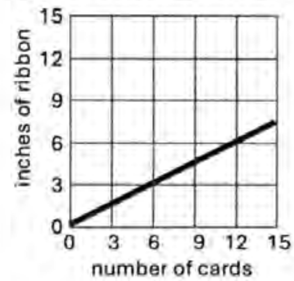
x	y
0	0
1	2
2	4
3	6

(b) Anniversary



x	y
0	0
3	3
6	6
9	9

(c) Congratulations



x	y
0	0
6	3
12	6

2. Is (3,9) a solution?

(a) Birthday

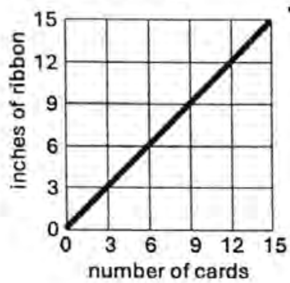
$y = 2x$

$9 = 2 \cdot 3$

$9 = 6$

NO!

(b) Anniversary



$3 \div 3 = 1$, $6 \div 6 = 1$, $9 \div 9 = 1$

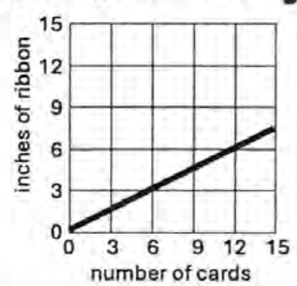
$y = 1x$

$9 = 1 \cdot 3$

$9 = 3$

NO!

(c) Congratulations



$3 \div 6 = 0.5$, $6 \div 12 = 0.5$

$y = \frac{1}{2}x$

$9 = \frac{1}{2} \cdot 3$

$9 = \frac{3}{2}$

NO!

3. What is the solution when y = 100?

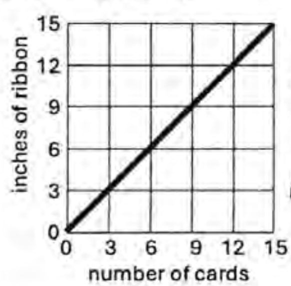
(a) Birthday

$y = 2x$

$\frac{100}{2} = \frac{2x}{2}$

$x = 50$

(b) Anniversary

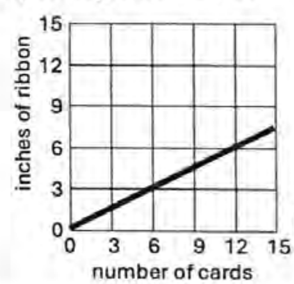


$y = 1x$

$100 = 1x$

$100 = x$

(c) Congratulations



$y = \frac{1}{2}x$

$100 \cdot 2 = \frac{1}{2}x \cdot 2$

$200 = x$

4. Explain what each solution in #3 represents in words.

(a) When there are 50 cards then 100 inches of ribbon are needed.

(b) When there are 100 cards then 100 inches of ribbon are needed.

(c) When there are 200 cards then 100 inches of ribbon are needed.

G8 U3 Lesson 5
Represent linear relationships with a graph, equation, table and story.

CITYTUTOR DC

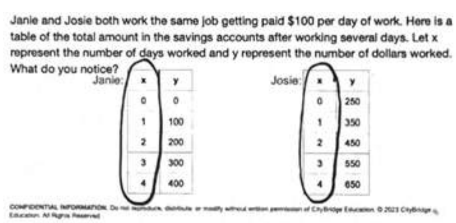
G8 U3 Lesson 5 - Today we will represent linear relationships with a graph, equation, table and story.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we will represent linear relationships with a graph, equation, table and story. In our last lessons, you did the final step that you needed with proportions. A lot of that was review from previous grades. But we wanted to make sure we had the concepts because now we are going to apply them to a larger category of relationships called linear relationships. You will see that most of what we learn is just the same. It's going to be great!

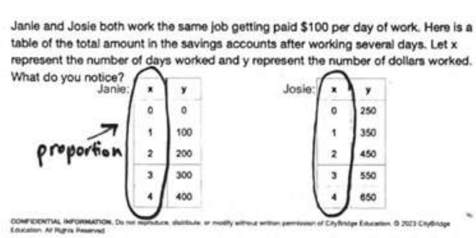
Let's Review (Slide 3): This says, "Compare the two tables to understand a relationship that is NOT a proportion. Read the problem silently with me while I read it out loud. *Read the story.* So, what do you notice? Possible Student Answers, Key Points:

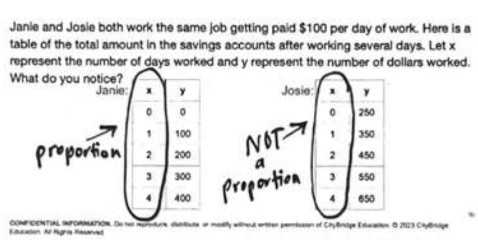
- The first column is 0, 1, 2, 3, 4 for both tables.
- Both tables have x and y.
- Janie's table counts up by 100s.
- Josie's table goes up be 100s too but it starts at 250 instead of 0.



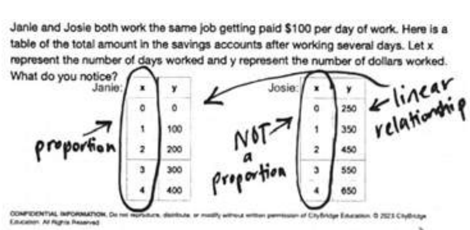
There are so many important things to notice here. The x column of both tables is the same: 0, 1, 2, 3 and 4. *Circle the x column on both tables.* The x column represents the number of days worked. *Point to that line in the text.*

So we are comparing when Janie and Josie work the same number of days. But the y column isn't exactly the same. *Point to the numbers as you say them.* Janie's starts with 0 and then it keeps going up by hundreds. 100 - 200 - 300 - 400. Josie's starts with 250 and then it actually keeps going up by hundreds. You can see 250 - 350 - 450 - 550 - 650. So they both go up by hundreds but they don't start with the same number. Let's figure out what that means using the language from the problem. We are talking about the y column so I pay attention to what y represents. This says, "y represents the number of dollars worked." *Point to the line in the text.* So if these didn't start the same that means Janie and Josie didn't start with the same amount of dollars. Maybe Josie had some money that she had already saved up. But then they each went up by $100, which makes sense because they each got paid the same amount, $100 per day. Janie's table shows a PROPORTION. We know this because it has that y-intercept of (0,0) and it has a constant of proportionality. 100 divided by 1 is 100. 200 divided by 2 is 100. 300 divided by 3 is 100. This is a proportion.



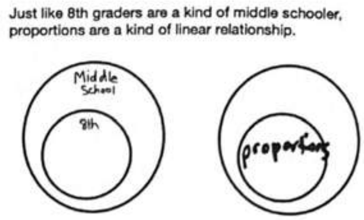


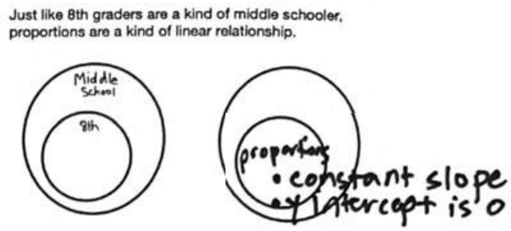
This table is NOT a proportion. It does not have that y-intercept of (0,0) and it does not have a constant of proportionality. Look, 350 divided by 1 is 350. 450 divided by 2 is 225. But we are going to see this in a few more slides - it still makes a line.



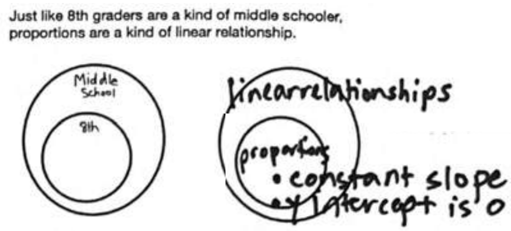
These are both called a LINEAR RELATIONSHIP, which is what we are going to learn about today.

Let's Talk (Slide 4): **"**Linear relationships have constant slope or rate of change. Proportions do too but their y-intercept is zero." One way to understand this is to think of an analogy. You all are 8th graders. You are part of a middle school. You can see that in this picture. A circle labeled 8th is inside the circle labeled middle school. Well, "just like 8th graders are a kind of middle schooler, proportions are a kind of linear relationship." You've been learning about proportions since 6th grade. I am going to label this small circle "proportions." *Label the inner circle.*



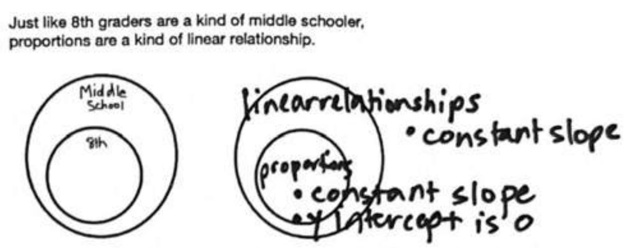


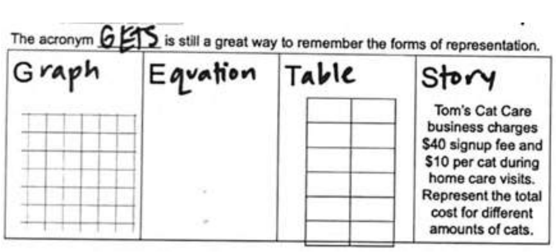
Proportins go through (0,0) and they have a constant slope. *Make notes on inner circle.*



This sentence says the analogy, "Just like 8th graders are a kind of middle schooler, proportions are a kind of linear relationship." So this bigger circle is linear relationships. *Label the outer circle.*

We are going to see throughout the rest of this unit that linear relationships also have a constant slope. *Make notes on outer circle.* Sometimes linear relationships have a y-intercept and then they are called proportions. But sometimes they don't. Just like in the Janie and Josie tables we just saw. Proportions are inside the linear relationship family just like 8th graders are part of middle school. Are all middle schoolers in 8th grade? No! Are all 8th graders in middle school? Yes! The same idea applies here. Are all linear relationships a proportion? No! Are all proportions linear relationships? Yes! The good news is that everything you have learned about proportions applies to linear relationships. You can make the exact same representations and answer questions the exact same way.





Let's Think (Slide 5): We can make a graph, equation and table to represent a linear relationship. This is going to work exactly the way we've practiced in previous lessons. Only now the y-intercept might not always be 0. The acronym GETS is still a great way to remember the forms of representation. G stands for graph. E stands for equation. T stands for table. S stands for story.

Read this story silently in your head while I read it out loud. *Read the story.* Now, it is usually useful to start with the table. And if you need to, you can always sketch a picture of what is happening to the

Table

cats	dollars
x	y

side to make sense of the problem. I am going to put x and y on my table. X will be the number of cats so I will put cats and y will be the number of dollars so I will put dollars.

Table

cats	dollars
x	y
0	40

Now, we always start with x = 0 so we'll do that here too. Except we can't just put y = 0 like we usually do. This time, we are hearing that there is a $40 signup fee. That means that even before Tom charges based on the number of cats, he charges $40. So I am going to put 40.

Table

cats	dollars
x	y
0	40
1	50
2	60
3	70

Next we think about x = 1. That means 1 cat. It says that Tom charges $10 per cat. That's just $10 for the 1 cat. But the $40 doesn't just disappear, right? So the total cost would be $50. Next we think about x = 2. That means 2 cats. We just keep going. It's still $10 for each cat but now there are two cats. That's $20 plus the $40 from before. It doesn't just disappear. So the total cost would be $60. You get the idea, right? If I add another cat, it will be another $10. That's 3 cats cost $70. If I add another cat, it will be another $10. That's 4 cats cost $80. The way I can tell this will make a straight line is that from the first row, the x column keeps going up by 1 and the y column keeps going up by 10. That's a constant rate of change, which will make a constant slope. But there isn't a constant of proportionality. If I do 50 divided by 1, it's 50. But 60 divided by 2 is 30.

Equation

$y = 10x$

Now let's think about the equation. We can't look across the table and see an obvious operation like we are used to. But we know that we are getting y each time so I'll put y equals. We also know that it is $10 per cat. That's $10 times the number of cats. That's 10x.

Equation

$y = 10x + 40$

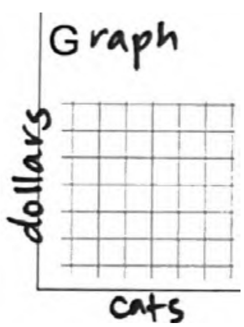
And everytime we added that starting fee of $40. Let's check in our heads if this equation works for each row. If x is 0 then it is 10 times 0 makes 0 plus 40 is 40. That works. If x is 1 then it is 10 times 1 makes 10 plus 40 is 50. That works. And we could keep going.

Equation

$y = 10x + 40$

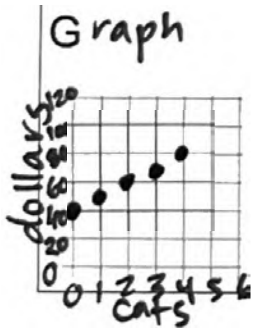
$y = mx + b$

This equation is not in y = kx form like a proportion. This is in y = mx + b form. The slope or rate of change is m. The y-intercept or initial value is b.



And finally, let's look at the graph. I am going to label the axes. X is cats and y is dollars.

I need my x-axis to go up to 4 so I will count by 1s on the x-axis, starting with 0, 1, 2, 3, 4, 5, 6. I need my y-axis to go up to 80 so it won't work to count by ones. Let's try tens. *You are modelling a Guess and Check strategy here.* 0, 10, 20, 30, 40, 50, 60. That's not enough. I guess we will skipcount by twenties. 0, 20, 40, 60, 80, 100, 120.



And now I can put my points. I am going to start with (0,40). Then (1,50). That is going right in between 40 and 60. Then (2,60). Then (3,70). Then (4,80).

Now I can connect the dots! I get a straight line. See? That's why it's a linear equation! But it doesn't cross (0,0). So it's not a proportion.

Let's Try It (Slide 6): Let's practice filling out GETS together now. I will walk you through each step.

WARM WELCOME

Today we will represent linear relationships with a graph, equation, table and story.

Let's Review: Compare the two tables to understand a relationship that is NOT a proportion.

Janie and Josie both work the same job getting paid $100 per day of work. Here is a table of the total amount in the savings accounts after working several days. Let x represent the number of days worked and y represent the number of dollars worked. What do you notice?

Janie:

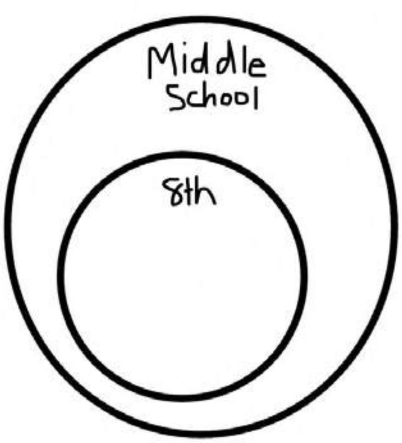
x	y
0	0
1	100
2	200
3	300
4	400

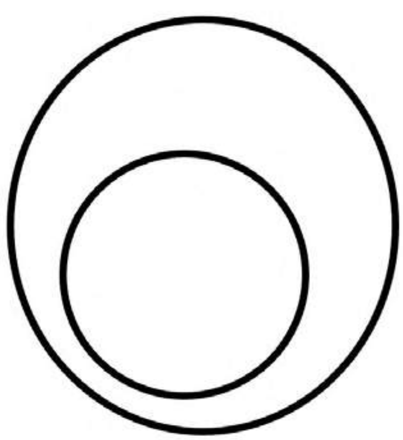
Josie:

x	y
0	250
1	350
2	450
3	550
4	650

Let's Talk: Linear relationships have a constant slope or rate of change. Proportions do too but their y-intercept is zero.

Just like 8th graders are a kind of middle schooler,
proportions are a kind of linear relationship.



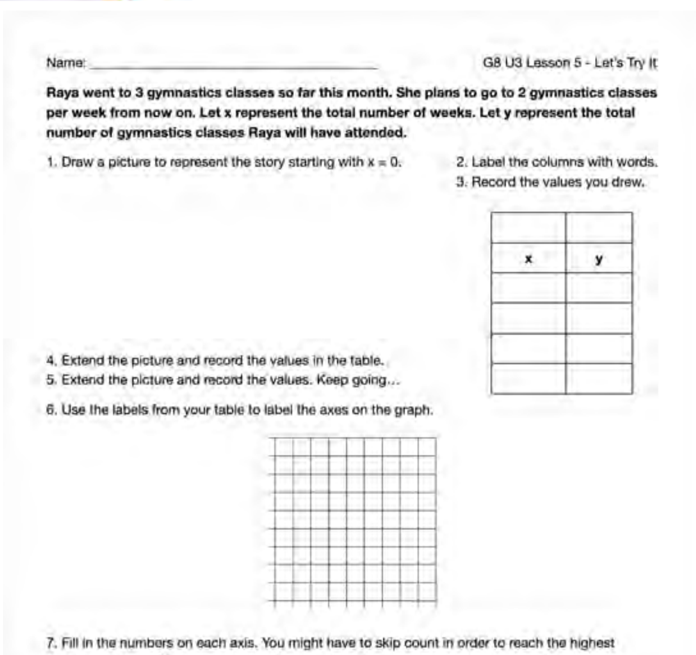


Let's Think: We can make a graph, equation and table to represent a linear relationship

The acronym _________ is still a great way to remember the forms of representation.

G	E	T	S
			Tom's Cat Care business charges $40 signup fee and $10 per cat during home care visits. Represent the total cost for different amounts of cats.

Let's Try It: Let's make a graph, equation and table for a linear relationship together!



Name: ______________________________ G8 U3 Lesson 5 - Let's Try It

Raya went to 3 gymnastics classes so far this month. She plans to go to 2 gymnastics classes per week from now on. Let x represent the total number of weeks. Let y represent the total number of gymnastics classes Raya will have attended.

1. Draw a picture to represent the story starting with x = 0.
2. Label the columns with words.
3. Record the values you drew.

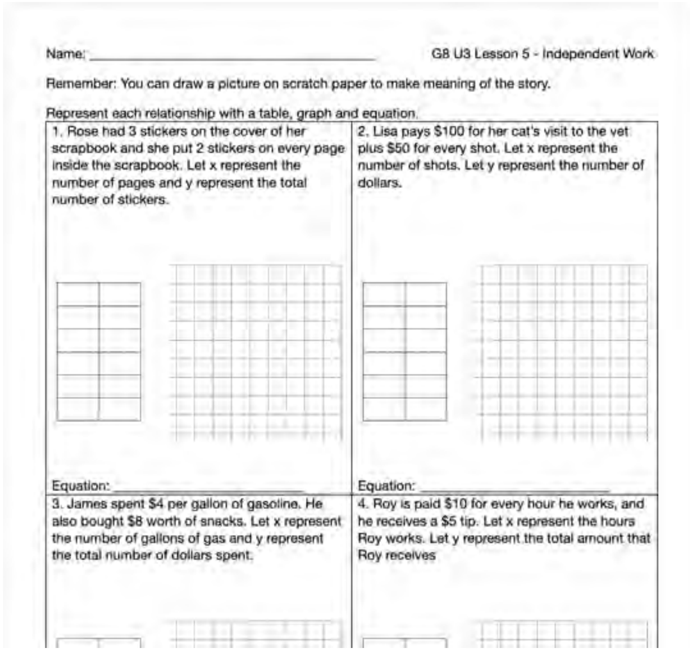
x	y

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going…
6. Use the labels from your table to label the axes on the graph.

7. Fill in the numbers on each axis. You might have to skip count in order to reach the highest

On your Own:

Now it's time for you to do it on your own!



Name: ______________________ G8 U3 Lesson 5 - Independent Work

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each relationship with a table, graph and equation.

1. Rose had 3 stickers on the cover of her scrapbook and she put 2 stickers on every page inside the scrapbook. Let x represent the number of pages and y represent the total number of stickers.

Equation: ______________

2. Lisa pays $100 for her cat's visit to the vet plus $50 for every shot. Let x represent the number of shots. Let y represent the number of dollars.

Equation: ______________

3. James spent $4 per gallon of gasoline. He also bought $8 worth of snacks. Let x represent the number of gallons of gas and y represent the total number of dollars spent.

4. Roy is paid $10 for every hour he works, and he receives a $5 tip. Let x represent the hours Roy works. Let y represent the total amount that Roy receives

Raya went to 3 gymnastics classes so far this month. She plans to go to 2 gymnastics classes per week from now on. Let x represent the total number of weeks. Let y represent the total number of gymnastics classes Raya will have attended.

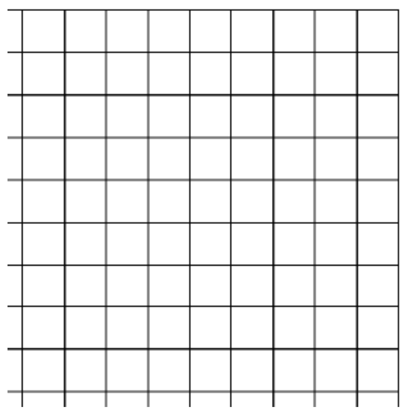
1. Draw a picture to represent the story starting with $x = 0$.

2. Label the columns with words.
3. Record the values you drew.

x	**y**

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going…

6. Use the labels from your table to label the axes on the graph.



7. Fill in the numbers on each axis. You might have to skip count in order to reach the highest number on your table.
8. Use each row of the table as a set of coordinates.
9. What was the value of y when x was 0 (also known as the y-intercept)? _______ That is b.

10. What is the rate of change (also known as the slope)? _______ That is m.

11. Write an equation in $y = mx + b$ form. ___________________

Name: ______________________________________ G8 U3 Lesson 5 - Independent Work

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each relationship with a table, graph and equation.

1. Rose had 3 stickers on the cover of her scrapbook and she put 2 stickers on every page inside the scrapbook. Let x represent the number of pages and y represent the total number of stickers.

Equation: __________________________

2. Lisa pays $100 for her cat's visit to the vet plus $50 for every shot. Let x represent the number of shots. Let y represent the number of dollars.

Equation: __________________________

3. James spent $4 per gallon of gasoline. He also bought $8 worth of snacks. Let x represent the number of gallons of gas and y represent the total number of dollars spent.

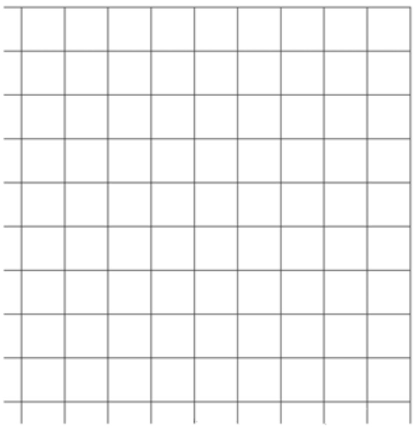
Equation: __________________________

4. Roy is paid $10 for every hour he works, and he receives a $5 tip. Let x represent the hours Roy works. Let y represent the total amount that Roy receives

Equation: __________________________

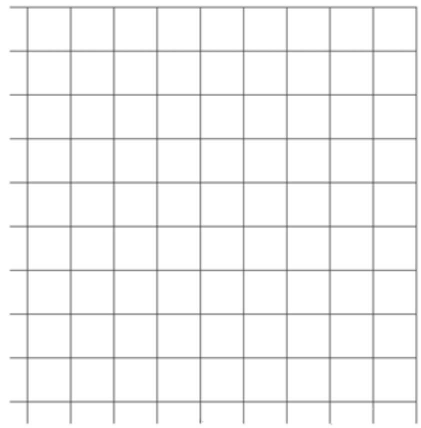
Represent each relationship with a table, graph and equation.

5. Polly bought a plant that was 5 inches tall. It grew 1 inch per week after she brought it home. Let x represent the number of weeks. Let y represent the total height of the plant in inches.



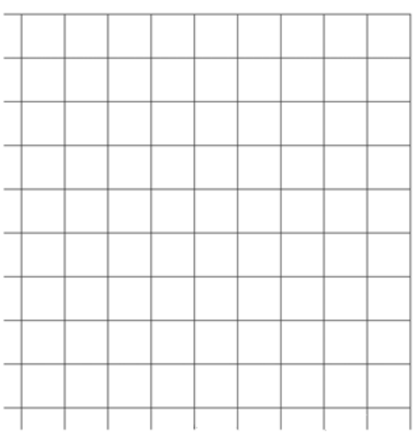
Equation: ____________________________

6. Nathaniel knew 100 Spanish words when he started his first Spanish class. Then he learned 50 words per week. Let x represent the number of weeks. Let y represent the number of words.



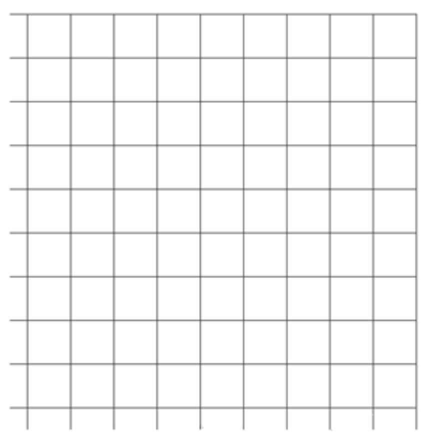
Equation: ____________________________

7. The theater department raised $500 at each of the performances they did this year. They also got a donation of $2000. Let x represent the number of performances. Let y represent the number of dollars.



Equation: ____________________________

8. Ethan's father said that he would pay him $5 every time he rakes up leaves. His grandmother gave him $20 for his birthday. Let x represent the number of times he rakes. Let y represent the number of dollars Ethan has.

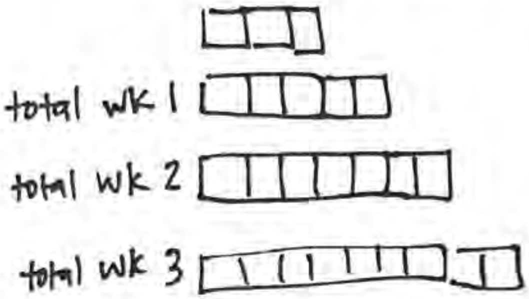


Equation: ____________________________

Name: ANSWER KEY

Raya went to 3 gymnastics classes so far this month. She plans to go to 2 gymnastics classes per week from now on. Let x represent the total number of weeks. Let y represent the total number of gymnastics classes Raya will have attended.

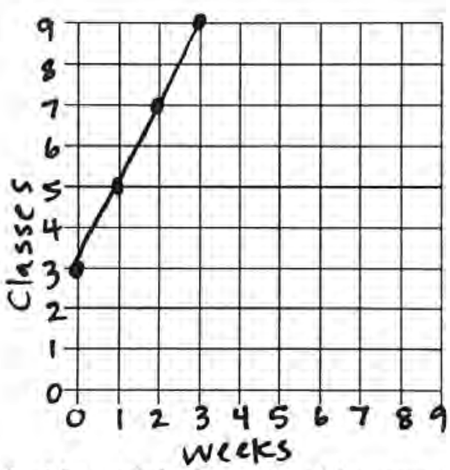
1. Draw a picture to represent the story starting with x = 0.



2. Label the columns with words.
3. Record the values you drew.

Weeks	classes
x	y
0	3
1	5
2	7
3	9

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going...
6. Use the labels from your table to label the axes on the graph.



7. Fill in the numbers on each axis. You might have to skip count in order to reach the highest number on your table.
8. Use each row of the table as a set of coordinates.
9. What was the value of y when x was 0 (also known as the y-intercept)? 3 That is b.

10. What is the rate of change (also known as the slope)? 2 That is m.

11. Write an equation in y = mx + b form. $y = 2x + 3$

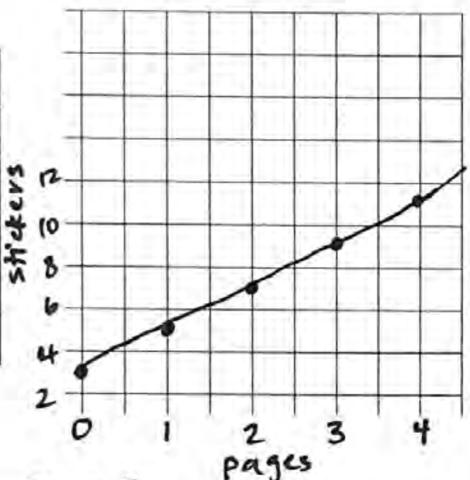
Name: ANSWER KEY

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each relationship with a table, graph and equation.

1. Rose had 3 stickers on the cover of her scrapbook and she put 2 stickers on every page inside the scrapbook. Let x represent the number of pages and y represent the total number of stickers.

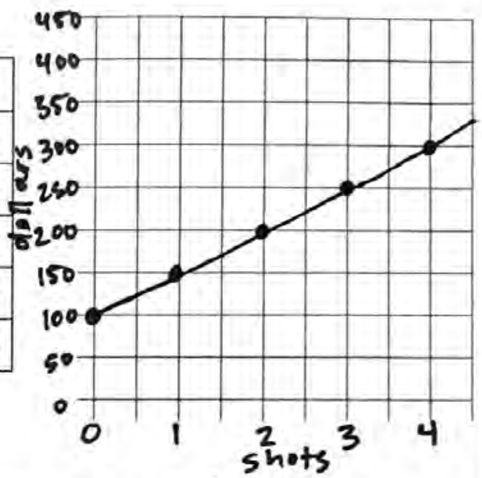
x	y
0	3
1	5
2	7
3	9
4	11



Equation: $y = 2x + 3$

2. Lisa pays $100 for her cat's visit to the vet plus $50 for every shot. Let x represent the number of shots. Let y represent the number of dollars.

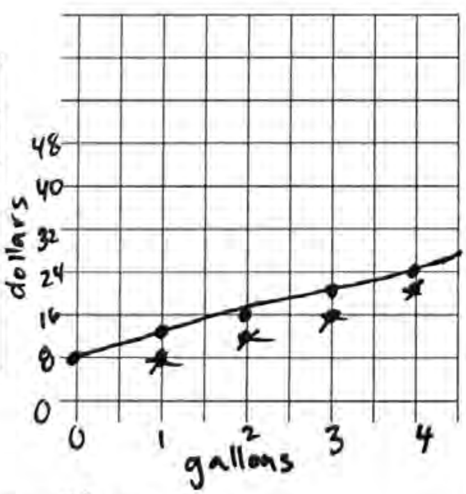
x	y
0	100
1	150
2	200
3	250
4	300



Equation: $y = 50x + 100$

3. James spent $4 per gallon of gasoline. He also bought $8 worth of snacks. Let x represent the number of gallons of gas and y represent the total number of dollars spent.

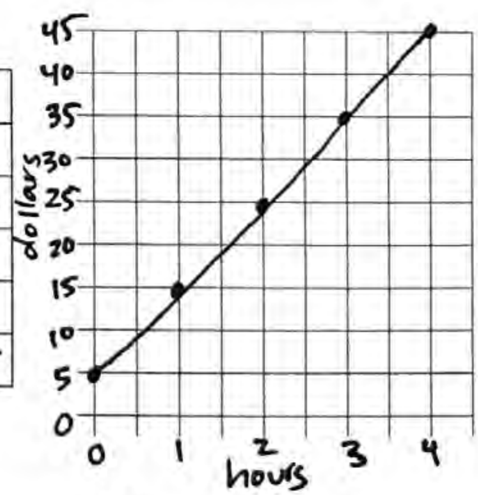
x	y
0	8
1	12
2	16
3	20
4	24



Equation: $y = 4x + 8$

4. Roy is paid $10 for every hour he works, and he receives a $5 tip. Let x represent the hours Roy works. Let y represent the total amount that Roy receives

x	y
0	5
1	15
2	25
3	35
4	45

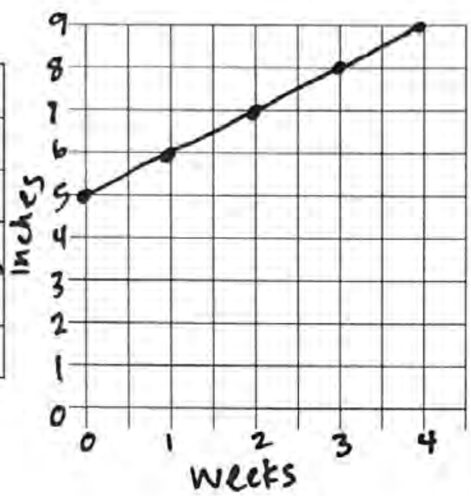


Equation: $y = 10x + 5$

Represent each relationship with a table, graph and equation.

5. Polly bought a plant that was 5 inches tall. It grew 1 inch per week after she brought it home. Let x represent the number of weeks. Let y represent the total height of the plant in inches.

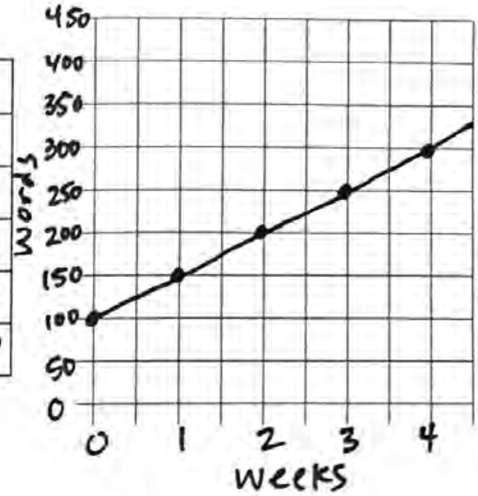
x	y
0	5
1	6
2	7
3	8
4	9



Equation: $y = 1x + 5$

6. Nathaniel knew 100 Spanish words when he started his first Spanish class. Then he learned 50 words per week. Let x represent the number of weeks. Let y represent the number of words.

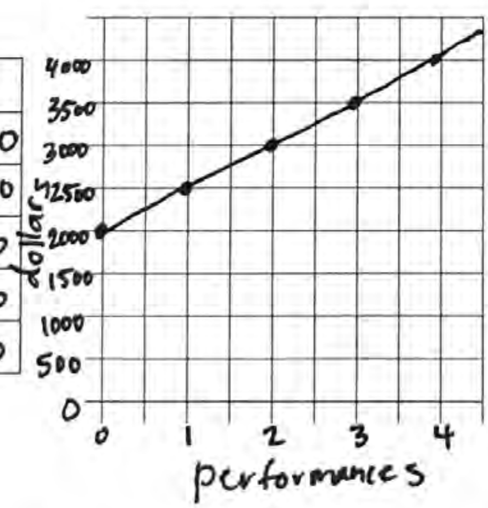
x	y
0	100
1	150
2	200
3	250
4	300



Equation: $y = 50x + 100$

7. The theater department raised $500 at each of the performances they did this year. They also got a donation of $2000. Let x represent the number of performances. Let y represent the number of dollars.

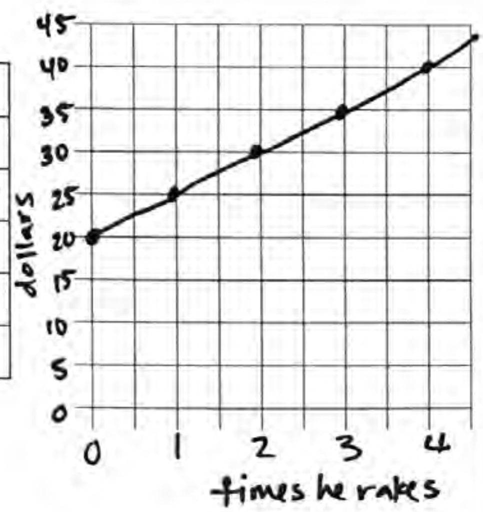
x	y
0	2000
1	2500
2	3000
3	3500
4	4000



Equation: $y = 500x + 2000$

8. Ethan's father said that he would pay him $5 every time he rakes up leaves. His grandmother gave him $20 for his birthday. Let x represent the number of times he rakes. Let y represent the number of dollars Ethan has.

x	y
0	20
1	25
2	30
3	35
4	40



Equation: $y = 5x + 20$

G8 U3 Lesson 6
Interpret the y-intercept and slope for linear graphs. Determine if they are proportions.

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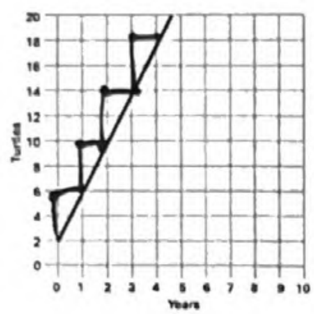
G8 U3 Lesson 6 - Today we will interpret the y-intercept and slope for linear graphs and determine if they are proportions.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we will interpret the y-intercept and slope for linear graphs and determine if they are proportions. In our last lesson we started learning about linear relationships. Today we will interpret the y-intercept and slope on graphs of linear relationships. You have already practiced doing this with proportions so you're going to be great.

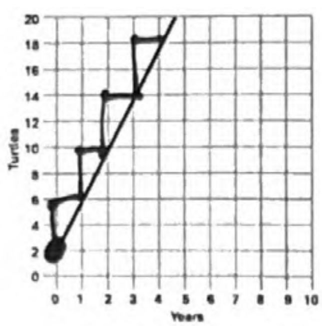
Let's Review (Slide 3): You learned this in earlier lessons: "The graph of a proportion has two special features." This helps us know when a graph is a proportion. "How do we know if the graph below shows a proportion?" I will read the story out loud while you read along in your head. *Read the story.*



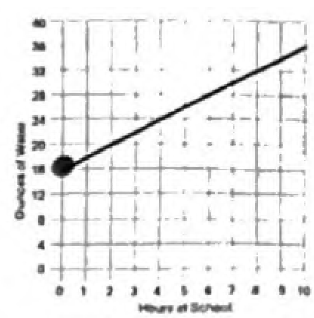
So, how do we know if the graph below shows a proportion? Possible Student Answers, Key Points:

- It is a straight line.
- It has a constant slope.
- It doesn't go through (0,0).
- It doesn't have a y-intercept of zero.

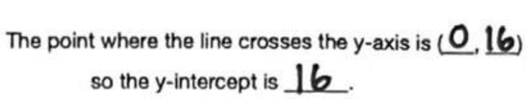
I heard a lot of important ideas. But we need to remember that the graph has to have two things together. It has to be a straight line, which means it has a constant slope. That's like a nice even staircase going up and over, up and over up and over. Does this have a constant slope? Yes!



It also needs to go through the origin, which is (0,0). I am going to make a dot at the origin. Another way of saying that is that it has to have a y-intercept of zero. Does this have a y-intercept of zero? No! So, is this a proportion? No! But it still makes a straight line so it is still a linear relationship even though it's not a proportion. So, we can still find the y-intercept and interpret it. We can still find the slope and interpret it. And that's what we're going to do today.

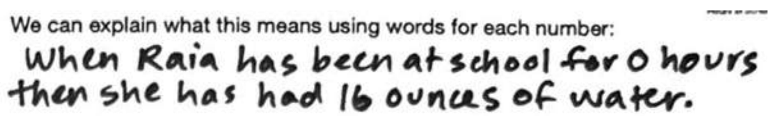


Let's Talk (Slide 4): This says, "The y-intercept of a graph is where it crosses the y-axis and where x = 0." We know that from proportions, right? It's just it was super easy with proportions because it was always zero. And now it's not. Let's figure out what it is. I am going to read the story out loud while you read along silently in your head. *Read the story.* We know the y-intercept of a graph is where it crosses the y-axis. Sometimes people think of it as where the line starts because our graphs using start at the y-axis where x = 0. I am going to put a point at that place for this line.

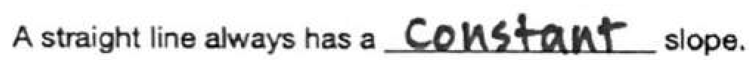


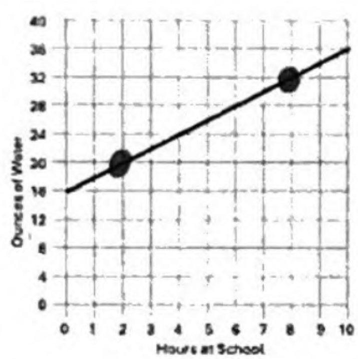
"The point where the line crosses the y-axis is…" (0,16). "So the y-intercept is…" 16. This is where we have been getting zero for the last few days. But now we're not.

But this is actually really interesting information for whatever story we are learning about because just like it shows where our line begins, it shows where our story begins. "We can explain what this means using works for each number." The 0 is x and in this case x is the hours at school. The 16 is y and in this case y is the total ounces of water. We use the “when” and “then” sentences that we used in previous lessons. When Raia has been at school for 0 hours then she has had 16 ounces of water. In other words, before Raia starts school, she’s already had 16 ounces of water. It is kind of fun to take a point on a graph and turn it into a story like that! Now let’s look at the other feature we’ve been focusing on.

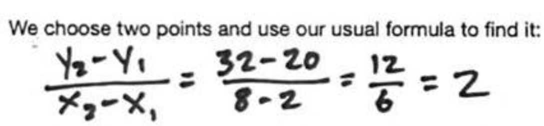


Let’s Think (Slide 5): “The slope of a graph is how it increases or decreases. It is measured by the change in y divided by the change in x.” We already knew this, right? Let’s read the story. I will say it out loud while you read along silently. *Read the story.* So we need to find the slope. First, we need to remember, a straight line always has a CONSTANT slope. That means that whichever points we use to find it, we’ll get the same answer.



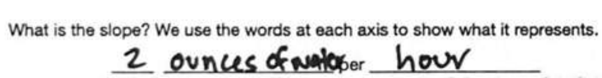


“We choose two points and use our usual formula to find it.” I am going to mark these. *Mark (2,20) and (8,32) and label them.*



The formula is y_2 - y_1 over x_2 - x_1. Now we plug the numbers. We get 32 - 20 over 8 - 2. 32 - 20 is 12 and 8 - 2 is 6. So we get 12 over 6. 12 divided by 6 is 2 so our slope is 2! Nice!

This is asking, “What is the slope? We use the words at each axis to show what it represents.” So that is 2 and then the first words we want are the y words because we started with y_2 - y_1. That’s ounces of water. Then we need the x words. That’s hours at school. So, Raia has 2 ounces of water per hour at school. Now we have a picture of her day, right? She start off with 16 ounces of water at breakfast and then when she goes to school, she has 2 ounces every hour. She must go to the water fountain. The y-intercept and the slope help us tell a story about whatever we are representing.



Let’s Try It (Slide 6): Let’s interpret the y-intercepts and slopes on graphs together. I will walk you through each step.

WARM WELCOME

Today we will interpret the y-intercept and slope for linear graphs and determine if they are proportions.

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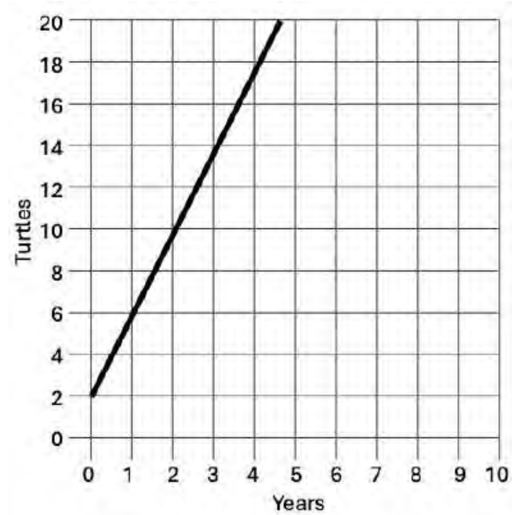
Let's Review: The graph of a proportion always has two special features.

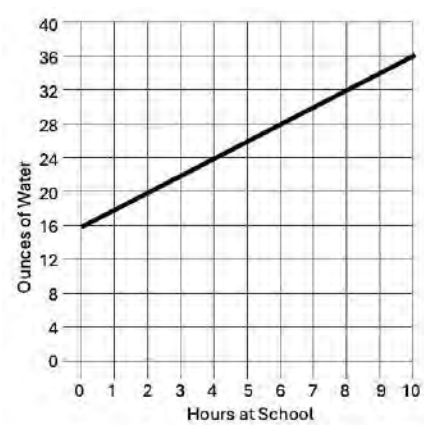
How do we know if the graph below shows a proportion?

The Baltimore Aquarium got two sea turtles. They used the graph to show the total amount of turtles after new turtles were born each year.



Let's Talk: The y-intercept of a graph is where it crosses the y-axis and where x = 0.

Raia has water with breakfast and then she takes water breaks at school. The graph shows the amount of water Raia drinks each day where x is the hours at school and y is the total ounces of water.



The point where the line crosses the y-axis is (___, ___)

so the y-intercept is ______.

We can explain what this means using words for each number:

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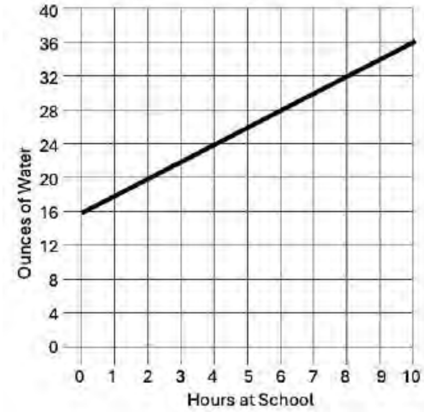
Let's Think:

The slope of a graph is how it increases or decreases. It is measured by the change in y divided by the change in x.

Raia has water with breakfast and then she takes water breaks at school. The graph shows the amount of water Raia drinks each day where x is the hours at school and y is the total ounces of water.



A straight line always has a _________________ slope.

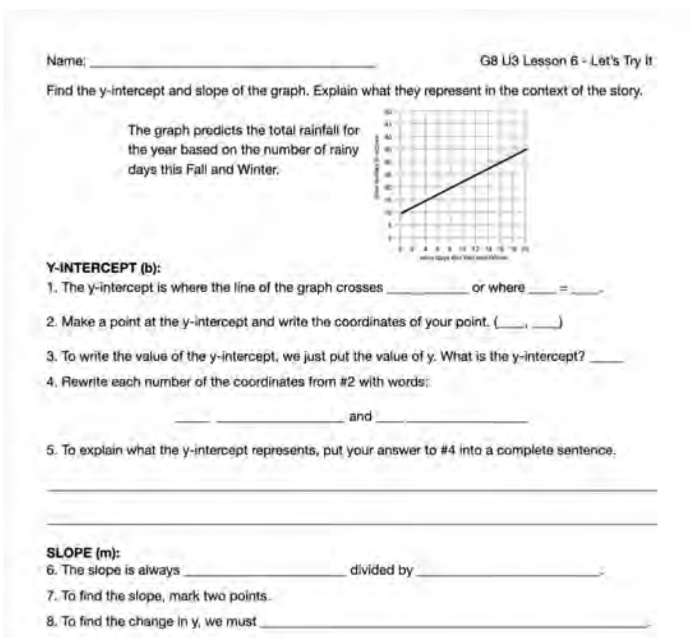
We choose two points and use our usual formula to find it:

What is the slope? We use the words at each axis to show what it represents.

_____ ___________________ per ___________________

Let's Try It:

Let's interpret the y-intercepts and slopes on graphs together!



Name: ______________________________ G8 U3 Lesson 6 - Let's Try It

Find the y-intercept and slope of the graph. Explain what they represent in the context of the story.

The graph predicts the total rainfall for the year based on the number of rainy days this Fall and Winter.

Y-INTERCEPT (b):

1. The y-intercept is where the line of the graph crosses __________ or where ____ = ____.
2. Make a point at the y-intercept and write the coordinates of your point. (____, ____)
3. To write the value of the y-intercept, we just put the value of y. What is the y-intercept? ____
4. Rewrite each number of the coordinates from #2 with words:

____ ______________ and ____________

5. To explain what the y-intercept represents, put your answer to #4 into a complete sentence.

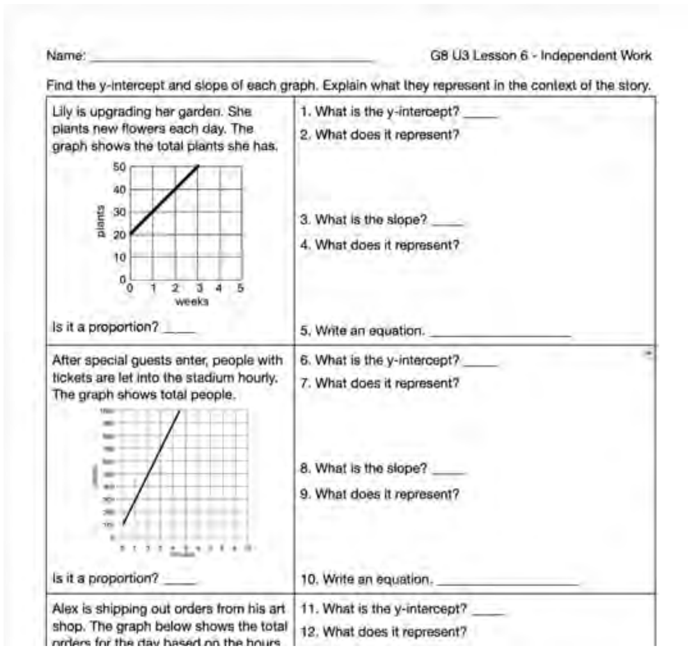
____

____

SLOPE (m):

6. The slope is always ________________ divided by ________________.
7. To find the slope, mark two points.
8. To find the change in y, we must ______________________________.

On your Own: Now it's time for you to do it on your own!



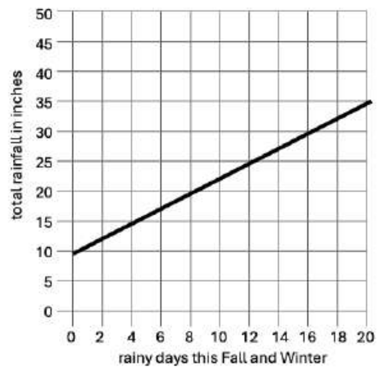
Name: ______________________ G8 U3 Lesson 6 - Independent Work

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

Lily is upgrading her garden. She plants new flowers each day. The graph shows the total plants she has. Is it a proportion? ____	1. What is the y-intercept? ____ 2. What does it represent? 3. What is the slope? ____ 4. What does it represent? 5. Write an equation. ______________
After special guests enter, people with tickets are let into the stadium hourly. The graph shows total people. Is it a proportion? ____	6. What is the y-intercept? ____ 7. What does it represent? 8. What is the slope? ____ 9. What does it represent? 10. Write an equation. ______________
Alex is shipping out orders from his art shop. The graph below shows the total orders for the day based on the hours	11. What is the y-intercept? ____ 12. What does it represent?

Find the y-intercept and slope of the graph. Explain what they represent in the context of the story.

The graph predicts the total rainfall for the year based on the number of rainy days this Fall and Winter.



Y-INTERCEPT (b):

1. The y-intercept is where the line of the graph crosses ___________ or where ____ = ____.

2. Make a point at the y-intercept and write the coordinates of your point. (____, ____)

3. To write the value of the y-intercept, we just put the value of y. What is the y-intercept? _____

4. Rewrite each number of the coordinates from #2 with words:

_____ __________________ and ____ _________________

5. To explain what the y-intercept represents, put your answer to #4 into a complete sentence.

____

____

SLOPE (m):

6. The slope is always _______________________ divided by __________________________.

7. To find the slope, mark two points.

8. To find the change in y, we must ____.

9. What is the change in y? _____-_____= _____

10. To find the change in x, we must ____.

11. What is the change in x? _____-_____= _____

12. Use your answers to #9 and #11 to find the slope. _______

13. To explain what the slope represents, use the words at each axis, y words then x words.

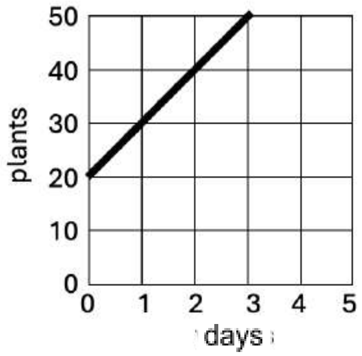
________ ______________________________ per ____________________________

14. Write an equation in y = mx + b form. ______________

Name: ____ G8 U3 Lesson 6 - Independent Work

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

Lily is upgrading her garden. She plants new flowers each day. The graph shows the total plants she has.



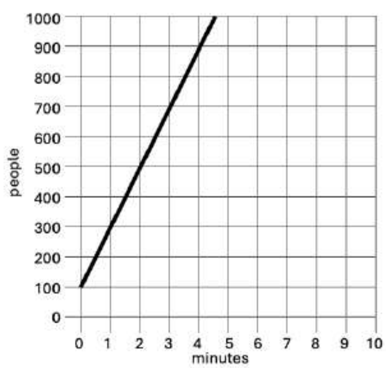
Is it a proportion? _____

1. What is the y-intercept? _____
2. What does it represent?

3. What is the slope? _____
4. What does it represent?

5. Write an equation. ____________________

After special guests enter, people with tickets are let into the stadium hourly. The graph shows total people.



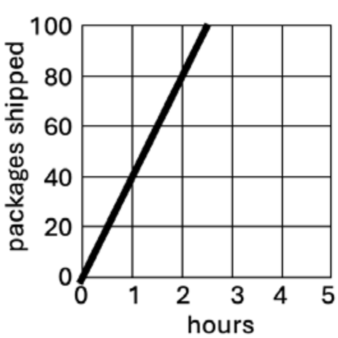
Is it a proportion? _____

6. What is the y-intercept? _____
7. What does it represent?

8. What is the slope? _____
9. What does it represent?

10. Write an equation. ____________________

Alex is shipping out orders from his art shop. The graph below shows the total orders for the day based on the hours he works this afternoon.



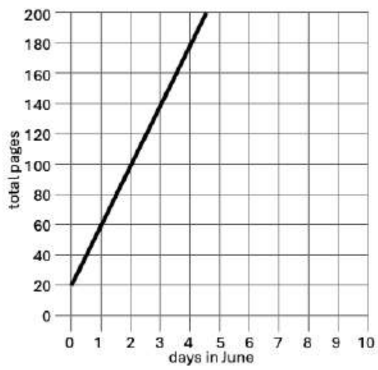
Is it a proportion? _____

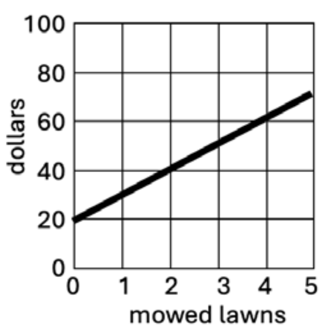
11. What is the y-intercept? _____
12. What does it represent?

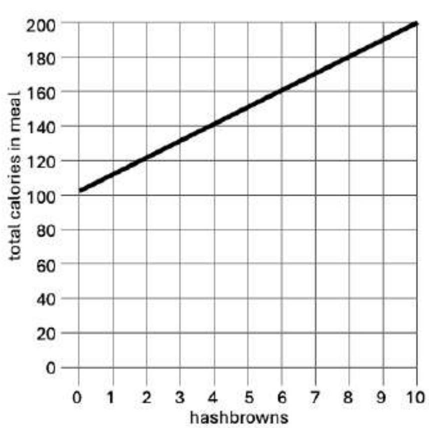
13. What is the slope? _____
14. What does it represent?

15. Write an equation. ____________________

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

<table>
<tr>
<td>Jordan started a book in May. The graph shows the total pages he'll have read based on days he reads in June.


Is it a proportion? _____</td>
<td>16. What is the y-intercept? _____
17. What does it represent?
18. What is the slope? _____
19. What does it represent?
20. Write an equation. ____________________</td>
</tr>
<tr>
<td>While saving for a new phone, Samantha gets a job mowing lawns. She uses the graph shown to calculate her total savings.


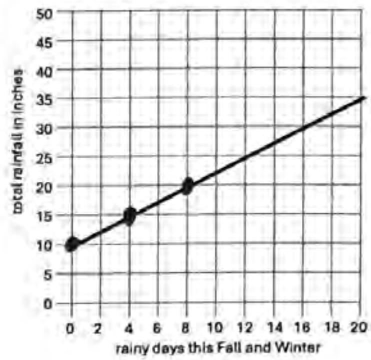
Is it a proportion? _______</td>
<td>21. What is the y-intercept? _____
22. What does it represent?
23. What is the slope? _____
24. What does it represent?
25. Write an equation. ____________________</td>
</tr>
<tr>
<td>Dunkin Donuts calculates the total calories in the egg and hashbrown meal based on the number of calories per hashbrown.


Is it a proportion? _______</td>
<td>26. What is the y-intercept? _____
27. What does it represent?
28. What is the slope? _____
29. What does it represent?
30. Write an equation. ____________________</td>
</tr>
</table>

Name: ANSWER KEY

Find the y-intercept and slope of the graph. Explain what they represent in the context of the story.

The graph predicts the total rainfall for the year based on the number of rainy days this Fall and Winter.



Y-INTERCEPT (b):

1. The y-intercept is where the line of the graph crosses y-axis or where x = 0.
2. Make a point at the y-intercept and write the coordinates of your point. (0, 10)
3. To write the value of the y-intercept, we just put the value of y. What is the y-intercept? 10
4. Rewrite each number of the coordinates from #2 with words:

   0 rainy days and 10 inches of rainfall

5. To explain what the y-intercept represents, put your answer to #4 into a complete sentence.

   When it rains 0 days in Fall and Winter then there is a total of 10 inches of rainfall predicted.

SLOPE (m):

6. The slope is always change in y divided by change in x.
7. To find the slope, mark two points.
8. To find the change in y, we must $y_2 - y_1$.
9. What is the change in y? 8 - 4 = 4
10. To find the change in x, we must $x_2 - x_1$.
11. What is the change in x? 20 - 15 = 5
12. Use your answers to #9 and #11 to find the slope. $\frac{4}{5}$
13. To explain what the slope represents, use the words at each axis, y words then x words.

    $\frac{4}{5}$ inches of rain per rainy day

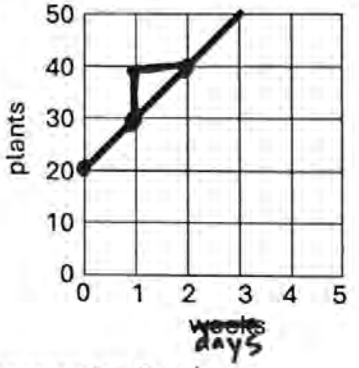
14. Write an equation in y = mx + b form. $y = \frac{4}{5}x + 10$

Name: ANSWER KEY

G8 U3 Lesson 6 - Independent Work

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

Lily is upgrading her garden. She plants new flowers each day. The graph shows the total plants she has.



Is it a proportion? NO

1. What is the y-intercept? 20

2. What does it represent?

When Lily has planted 0 days, she has 20 plants.

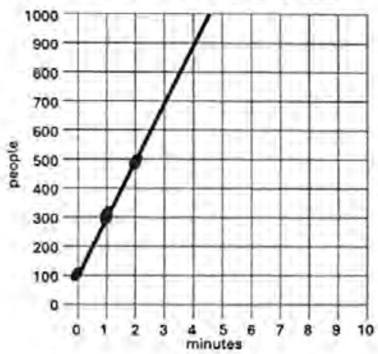
3. What is the slope? 10 $\frac{40-30}{2-1} = \frac{10}{1} = 10$

4. What does it represent?

Lily plants 10 plants per day.

5. Write an equation. $y = 10x + 20$

After special guests enter, people with tickets are let into the stadium hourly. The graph shows total people.



Is it a proportion? NO

6. What is the y-intercept? 100

7. What does it represent?

When people have been let in for 0 min, there will be 100 total people in the stadium.

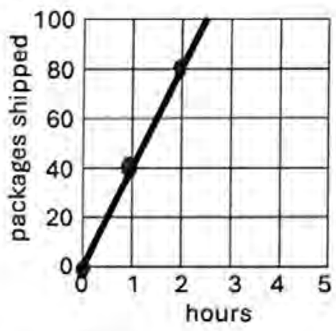
8. What is the slope? 200 $\frac{500-300}{2-1} = \frac{200}{1} = 200$

9. What does it represent?

They let in 200 people per minute.

10. Write an equation. $y = 200x + 100$

Alex is shipping out orders from his art shop. The graph below shows the total orders for the day based on the hours he works this afternoon.



Is it a proportion? YES

11. What is the y-intercept? 0

12. What does it represent?

When Alex works for 0 hours he has shipped 0 packages.

13. What is the slope? 40 $\frac{80-40}{2-1} = \frac{40}{1} = 40$

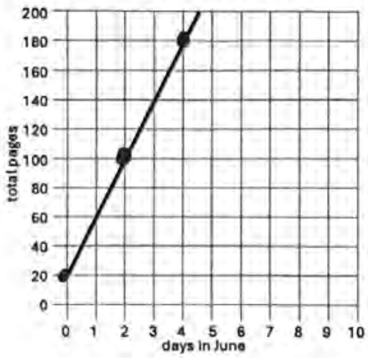
14. What does it represent?

Alex ships 40 packages per hour.

15. Write an equation. $y = 40x$

Find the y-intercept and slope of each graph. Explain what they represent in the context of the story.

Jordan started a book in May. The graph shows the total pages he'll have read based on days he reads in June.



Is it a proportion? NO

16. What is the y-intercept? 20

17. What does it represent?

When Jordan has read 0 days in June he has read 20 pages.

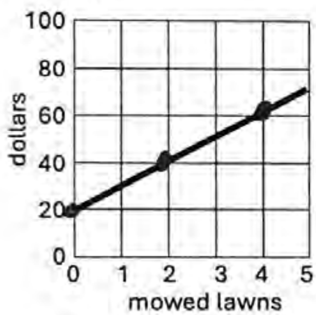
18. What is the slope? 40 $\frac{180-100}{4-2} = \frac{80}{2} = 40$

19. What does it represent?

Jordan reads 40 pages per day in June.

20. Write an equation. $y = 40x + 20$

While saving for a new phone, Samantha gets a job mowing lawns. She uses the graph shown to calculate her total savings.



Is it a proportion? NO

21. What is the y-intercept? 20

22. What does it represent?

When Samantha has mowed 0 lawns, she has $20.

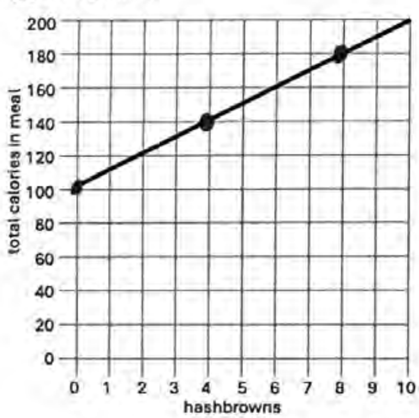
23. What is the slope? 10 $\frac{60-40}{4-2} = \frac{20}{2} = 10$

24. What does it represent?

Samantha gets $10 per lawn.

25. Write an equation. $y = 10x + 20$

Dunkin Donuts calculates the total calories in the egg and hashbrown meal based on the number of calories per hashbrown.



Is it a proportion? NO

26. What is the y-intercept? 100

27. What does it represent?

When there are 0 hashbrowns, then the meal has 100 calories.

28. What is the slope? 10 $\frac{180-140}{8-4} = \frac{40}{4} = 10$

29. What does it represent?

There are 10 calories per hash brown.

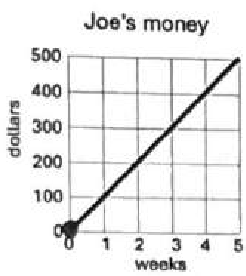
30. Write an equation. $y = 10x + 100$

G8 U3 Lesson 7
Write an equation for the line by using two points to find the y-intercept.

G8 U3 Lesson 7 - Today we will write an equation for the line by using two points to find the y-intercept.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we will write an equation for the line by using two points to find the y-intercept. In our last lesson we found the y-intercept and slope of graphs so we could write equations. We're going to do that but with one extra little challenge. You've got all the skills you need to do it so I know it will be great.

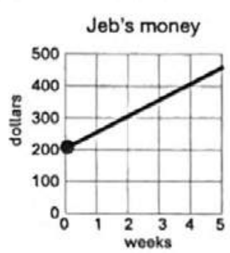


Let's Review (Slide 3): We already know this idea. It says, "The y-intercept is where the line crosses the y-axis and x = 0." We're going to find it below. This says, "The graphs below show the amount of money that three brothers saved this summer. Find the y-intercept of each." Let's start with Joe. What is the y-intercept? Possible Student Answers, Key Points:

- It is (0,0).
- It is 0.

The line crosses the y-intercept here. *Draw a dot a (0,0).*

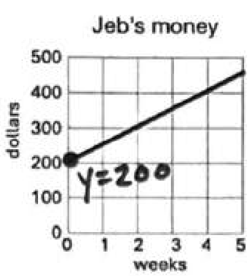
At that point, x = 0 and y = 0 too! So the y-intercept is zero. *Label the point with y = 0.* That helps us know that when the weeks are 0, meaning it's the start of the summer, Joe had $0.



What about Jeb's money? What is the y-intercept? Possible Student Answers, Key Points:

- It is (0,200).
- It is 200.

The line crosses the y-intercept here. *Draw a dot a (0,200).*



At that point, x = 0 and y = 200. So the y-intercept is 200. *Label the point with y = 200.* That helps us know that when the weeks are 0, meaning it's the start of the summer, Joe had $200. He must have had some money already saved up.

Now I want to find the y-intercept for the graph of John's money. When I look, I see that it is not exactly at 100 and it is not exactly at 200. It is in between. And it's not even exactly halfway between. I can't really tell what it is. I could estimate it. But I can't know for sure. That is what we're going to learn how to do today. And we're going to use two clear points on our graph to do it. But the big idea here is that sometimes you can see the y-intercept with your eyes and it's super obvious. But sometimes you're going to have to solve for it.

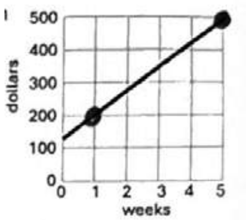
Let's Talk (Slide 4): We have the same graph on this next slide. And this says, "To find the y-intercept, we must find the slope in the formula: y = mx + b." There are a few steps in the reasoning for this.

We know that y = mx + b can be used to represent the graph where m is the slope and b is the intercept.

First, "we know that y = mx + b can be used to represent the graph" because it's a linear graph "where m is the…" SLOPE "and b is the… Y-INTERCEPT. We already knew that from our last two lessons.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

"We also know that if we only have one variable in an equation then we can solve to find it." What that means is if we can plug in a value for y and x and m then we can solve for b because it's the only one left. It's easy to figure out an x and an y. So let's figure out m. That's the slope. We know what to do. I am going to write m equals y_2 - y_1 over x_2 - x_1.



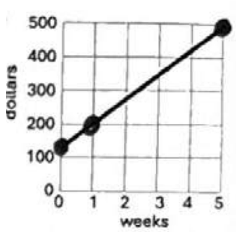
Let's mark two points that we can use to fill that in. Now, we have to be careful because there are lots of points that look kind of close to an intersection but they have to be right on a nice clear coordinate. I am going to mark this one and the only other one I can really mark is this one. *Mark a point at (1,200) and (5,500).*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{500-200}{5-1}$$
$$m = \frac{300}{4}$$
$$\boxed{m = 75}$$

Now we can fill our numbers in. I do 500 - 200 over 5 - 1. That is equal to 300 over 4. I am going to do that division over to the side, and my answer is 75! So m = 75

$$200 = 75(1) + b$$
$$200 = 75 + b$$
$$-75 \quad -75$$
$$\boxed{125 = b}$$

Now that we know the slope, m, we can substitute a value for x, y and m into our equation and solve for b, the y-intercept. You can pick any x,y pair but I will just do (1,200). So, 200 = 75 times 1 + b. Let's simplify this by doing 75 times 1 equals 75. I am going to rewrite the expression: 200 = 75 + b. I want b by itself so I will subtract 75 from both sides. That gives me 125 = b.



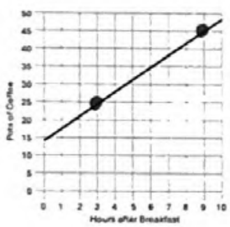
When I look to my graph, I see that it looks right. And it means that this point is (0,125) and the y-intercept is 125. But also, it means that John must have started out with $125 at the beginning of the summer.

$$y = 75x + 125$$

Now I can write the final equation in y=mx+b form. Y = 75x + 125. That's the equation. Here's the big idea - if we can't get the y-intercept by looking, then we can find it by finding the slope and then solving for it in an equation.

Let's Think (Slide 5): The awesome thing about knowing how to write an equation even when the y-intercept isn't obvious is that then you can find a value that isn't on the graph just by using the equation you come up with. This says, "If we want to find a value that's not on a graph, we have to write an equation first." That means we're going to have to figure out the slope and sometimes, if we can't see it just by looking, that means we'll have to solve for the y-intercept. I am going to read this problem out loud and I want you to read along with me silently in your head. *Read the story.* We have solved problems like this before but we always could either see it on the graph or plug it into an equation we were given. If I look for 12 hours on my graph, that's the x-axis and I don't have a 12 in my graph. And we don't have an equation so we'll have to come up with one. The first step is finding the slope. I will write m equals y_2 - y_1 over x_2 - x_1.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Let's mark two points that we can use to fill that in. Now, we have to be careful because there are lots of points that look kind of close to an intersection but they have to be right on a nice clear coordinate. I am going to mark this one and the only other one I can really mark is this one. *Mark a point at (3,25) and (9,45).*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{45 - 25}{9 - 3}$$
$$m = \frac{20}{6}$$

Now we can fill our numbers in. I do 45 - 25 over 9 - 3. That is equal to 20 over 6.

$$6\overline{)20} = 0\,3\,\frac{2}{6}\,\frac{\div 2}{\div 2}, \quad -18, \quad 2$$

I am going to do that division over to the side. 6 doesn't go into 2 so I put a zero. 6 goes into 20 three times. I subtract 18 and I have 2 leftover. I can't divide that so I turn it into a fraction - 2 sixths. Let's simplify that by dividing the top and bottom by 2. It's 3 and 1 third.

$$\boxed{m = 3\tfrac{1}{3}}$$

So m = $3\frac{1}{3}$. That's the slope.

$$25 = 3\tfrac{1}{3}(3) + b$$

Now that we know the slope, m, we can substitute a value for x, y and m into our equation and solve for b, the y-intercept. You can pick any x,y pair but I will just do (3,25). So, 25 = $3\frac{1}{3}$ times 3 + b.

$$3\tfrac{1}{3} \times 3 = 9 + 1 = 10$$

Let's simplify this by doing $3\frac{1}{3}$ times 3. It is a little tricky. I have to do 3 x 3 which is 9 and $\frac{1}{3}$ times 3, which is 1. That's 10 altogether.

$$25 = 3\tfrac{1}{3}(3) + b$$
$$25 = 10 + b$$
$$-10 \quad -10$$
$$\boxed{15 = b}$$

So I am going to rewrite the expression: 25 = 10 + b.I want b by itself so I will subtract 10 from both sides.

That gives me 15 = b. When I look to my graph, I see that this intercept is actually at 15 after all. It didn't look like it to me. But now we know for sure. And it means that this point is (0,15) and the y-intercept is 15. But also, it means that Marriot must have started out with 15 pots of coffee at breakfast.

$y = 3\frac{1}{3}x + 15$

Now I can write the final equation in y=mx+b form. $y = 3\frac{1}{3}x + 15$. That's the equation.

$y = 3\frac{1}{3}(12) + 15$

Now, can you believe that we did all that and we didn't even answer the question?!?! Finally, we can plug 12 hours into the equation and solve. That would be $y = 3\frac{1}{3}$ times 12 + 15.

$3\frac{1}{3} \times 12 = 36 + 4 = 40$

I do the math on this side 3 times 12 is 36. $\frac{1}{3}$ times 12 is 4. So that would be 40 altogether. That gives us y = 40 + 15.

$y = 3\frac{1}{3}(12) + 15$
$y = 40 + 15$
$\boxed{y = 55}$

We do that math and get y = 55. Phew! That's a lot of algebra. Do you see how I kept it in three separate sections here? And do you see how I always had my letters at the top so I knew what I was working on? That is key! And now we have this answer that we can interpret. When 12 hours have passed, the Marriot will have brewed 55 pots of coffee. Wow! That's a lot of coffee. It must be a big hotel.

Let's Try It (Slide 6): Let's find y-intercepts and equations for graphs together. I will walk you through each step.

WARM WELCOME

Today we will write an equation for the line by using two points to find the y-intercept.

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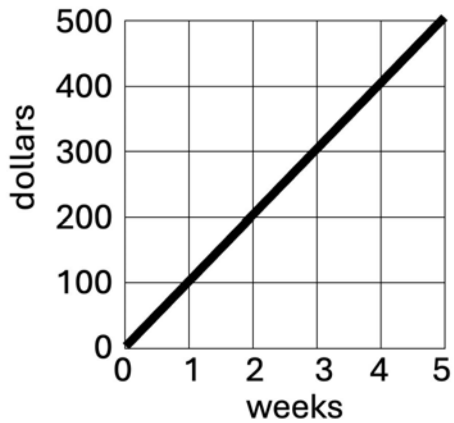
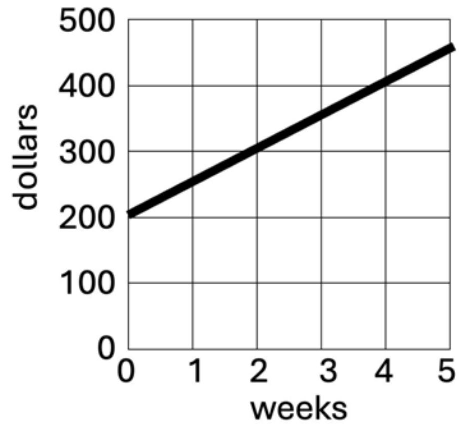
Let's Review: The y-intercept is where the line crosses the y-axis and x = 0.

The graphs below show the amount of money that three brothers saved this summer. Find the y-intercept of each.

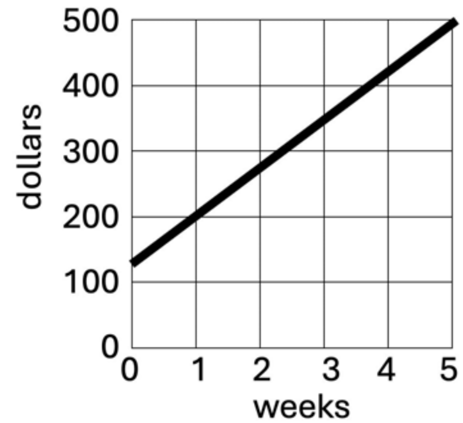
Joe's money



Jeb's money

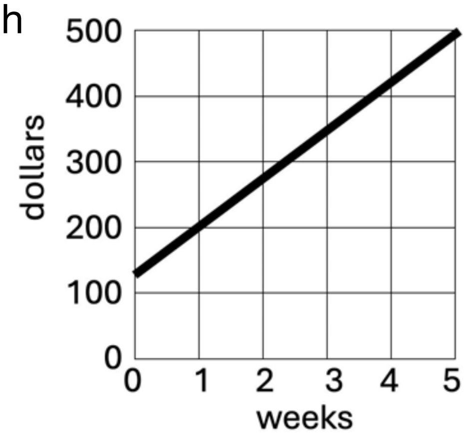


John's money



Let's Talk: To find the y-intercept, we must find the slope in the formula: y = mx + b.

We know that $y = mx + b$ can be used to represent the graph where m is the __________ and b is the _________________.



We also know that if we only have one variable in an equation then we can solve to find it. So let's figure out m.

Slope:

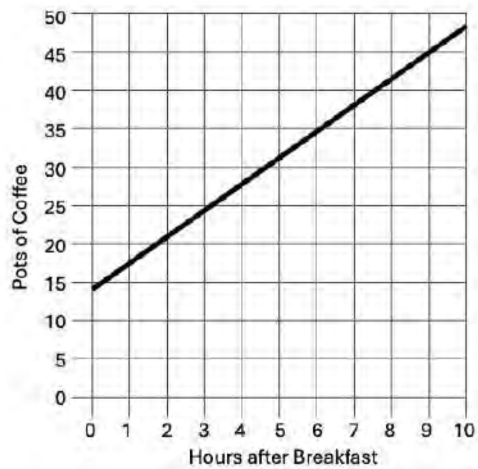
Then substitute x, y and m:

$y = mx + b$

Let's Think:

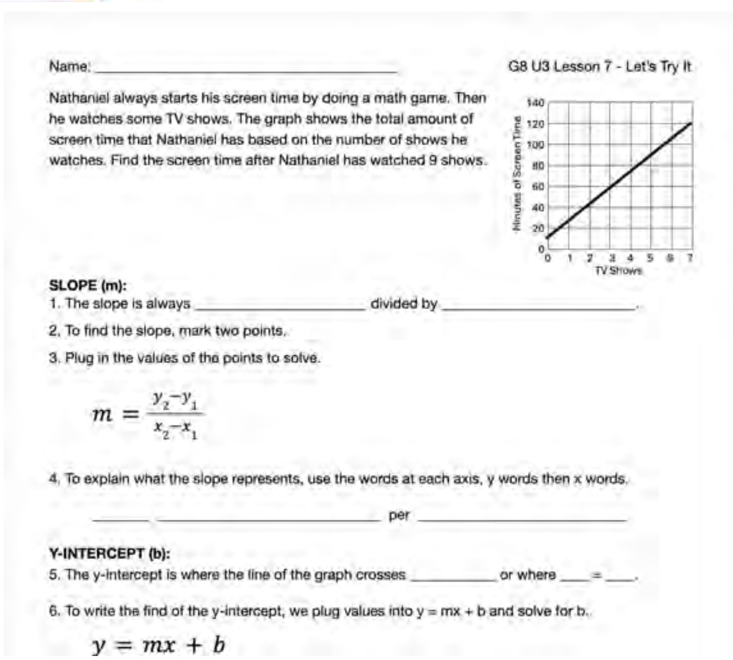
If we want to find a value that's not on a graph, we have to write an equation first.

At the Marriot Hotel, they make coffee for breakfast then they have coffee set out for people to get throughout the day. The graph below shows the total amount of coffee pots brewed, x, based on hours after the breakfast service, y. How many pots will be brewed after 12 hours?



Let's Try It:

Let's find y-intercepts and equations for graphs together!



Name: ______________________ G8 U3 Lesson 7 - Let's Try It

Nathaniel always starts his screen time by doing a math game. Then he watches some TV shows. The graph shows the total amount of screen time that Nathaniel has based on the number of shows he watches. Find the screen time after Nathaniel has watched 9 shows.

SLOPE (m):

1. The slope is always ____________ divided by ______________.
2. To find the slope, mark two points.
3. Plug in the values of the points to solve.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

4. To explain what the slope represents, use the words at each axis, y words then x words.

____ ______________ per ______________

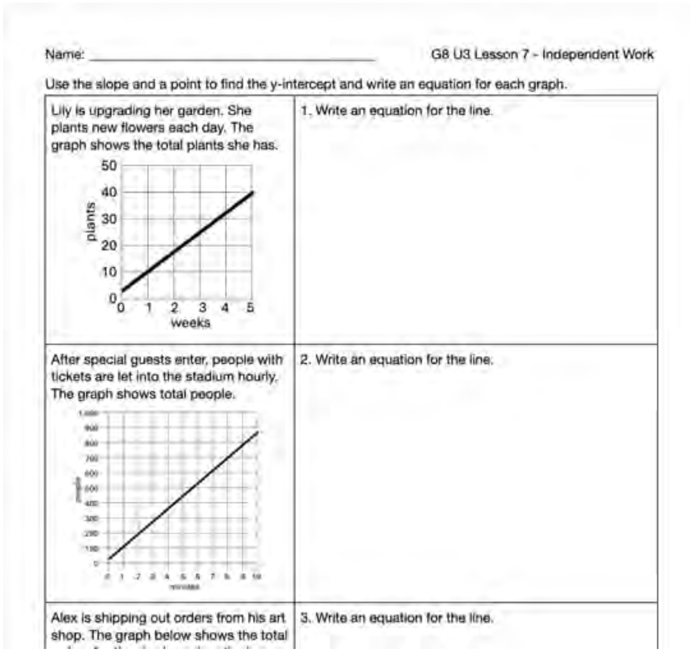
Y-INTERCEPT (b):

5. The y-intercept is where the line of the graph crosses ________ or where ___ = ___.
6. To write the find of the y-intercept, we plug values into y = mx + b and solve for b.

$$y = mx + b$$

On your Own:

Now it's time for you to do it on your own!

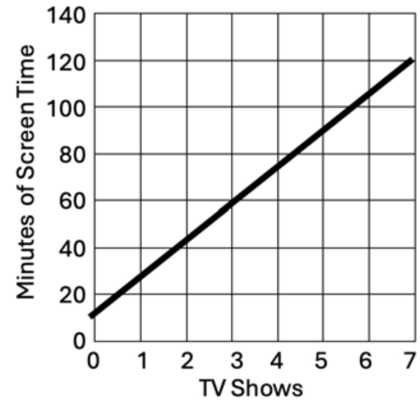


Name: ______________________________ G8 U3 Lesson 7 - Independent Work

Use the slope and a point to find the y-intercept and write an equation for each graph.

Lily is upgrading her garden. She plants new flowers each day. The graph shows the total plants she has.	1. Write an equation for the line.
After special guests enter, people with tickets are let into the stadium hourly. The graph shows total people.	2. Write an equation for the line.
Alex is shipping out orders from his art shop. The graph below shows the total	3. Write an equation for the line.

Nathaniel always starts his screen time by doing a math game. Then he watches some TV shows. The graph shows the total amount of screen time that Nathaniel has based on the number of shows he watches. Find the screen time after Nathaniel has watched 9 shows.



SLOPE (m):

1. The slope is always ______________________ divided by _________________________.
2. To find the slope, mark two points.
3. Plug in the values of the points to solve.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

4. To explain what the slope represents, use the words at each axis, y words then x words.

_______ ____________________________ per __________________________

Y-INTERCEPT (b):

5. The y-intercept is where the line of the graph crosses ___________ or where ____ = ____.
6. To write the find of the y-intercept, we plug values into y = mx + b and solve for b.

$$y = mx + b$$

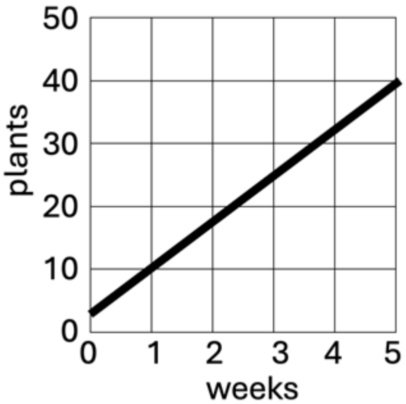
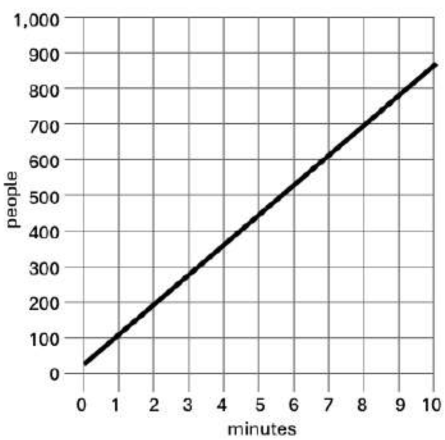
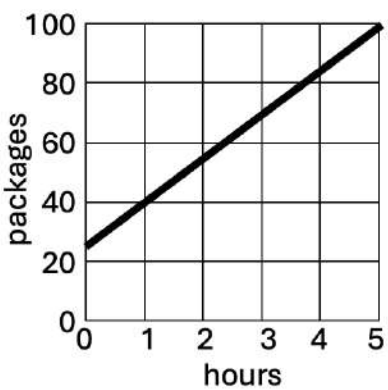
7. To explain what the y-intercept represents, put your answer to #6 into a complete sentence.

____

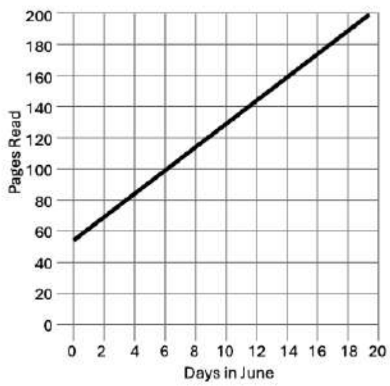
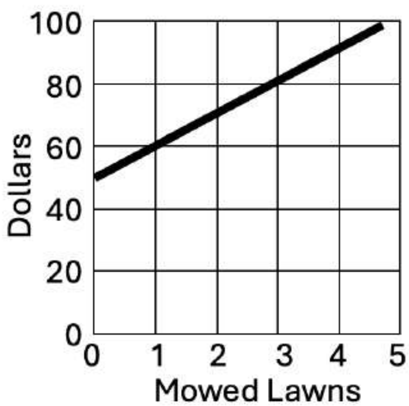
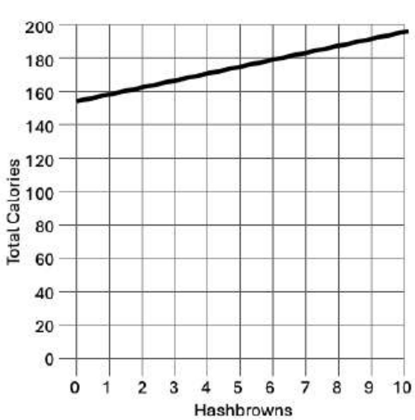
____

8. Use the answers you found to write an equation in y = mx + b form. ______________

Use the slope and a point to find the y-intercept and write an equation for each graph.

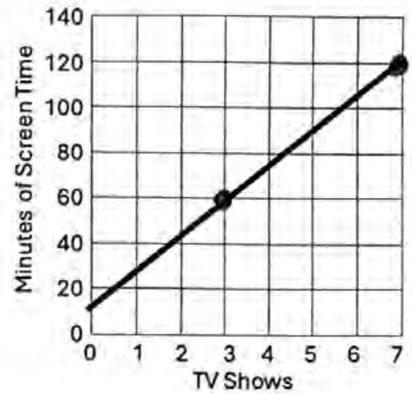
Lily is upgrading her garden. She plants new flowers each day. The graph shows the total plants she has. 	1. Write an equation for the line.
After special guests enter, people with tickets are let into the stadium hourly. The graph shows total people. 	2. Write an equation for the line.
Alex is shipping out orders from his art shop. The graph below shows the total orders for the day based on the hours he works this afternoon. 	3. Write an equation for the line.

Find the equation for the line. Then use the equation to answer the question.

Jordan started a book in May. The graph shows the total pages he'll have read based on days he reads in June.  	4. Write an equation for the line.
While saving for a new phone, Samantha gets a job mowing lawns. She uses the graph shown to calculate her total savings.  	5. Write an equation for the line.
Dunkin Donuts calculates the total calories in the egg and hashbrown meal based on the number of calories per hashbrown.  	6. Write an equation for the line.

Name: ANSWER KEY

Nathaniel always starts his screen time by doing a math game. Then he watches some TV shows. The graph shows the total amount of screen time that Nathaniel has based on the number of shows he watches. Find the screen time after Nathaniel has watched 9 shows.



SLOPE (m):

1. The slope is always $y_2 - y_1$ divided by $x_2 - x_1$.
2. To find the slope, mark two points.
3. Plug in the values of the points to solve.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{120-60}{7-3} = \frac{60}{4} = 15$$

$$\begin{array}{r} 15 \\ 4\overline{)60} \\ -4\downarrow \\ 20 \\ -20 \\ 00 \end{array}$$

4. To explain what the slope represents, use the words at each axis, y words then x words.

15 minutes per TV show

Y-INTERCEPT (b):

5. The y-intercept is where the line of the graph crosses y-axis or where x = 0.

6. To write the find of the y-intercept, we plug values into y = mx + b and solve for b.

$$y = mx + b$$

$$60 = 15(3) + b$$
$$60 = 45 + b$$
$$-45 \qquad -45$$
$$\boxed{15 = b}$$

7. To explain what the y-intercept represents, put your answer to #6 into a complete sentence.

When Nathaniel has watched 0 TV shows, he has had 15 minutes of screen time.

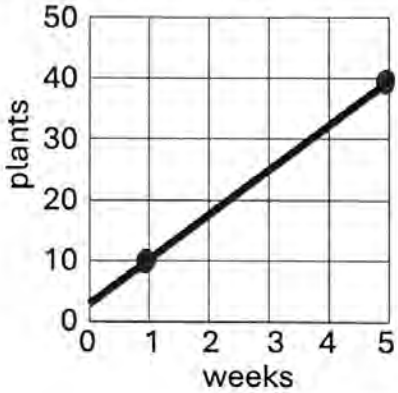
8. Use the answers you found to write an equation in $y = mx + b$ form. $y = 15x + 15$

Name: ANSWER KEY

G8 U3 Lesson 7 - Independent Work

Use the slope and a point to find the y-intercept and write an equation for each graph.

Lily is upgrading her garden. She plants new flowers each day. The graph shows the total plants she has.



1. Write an equation for the line.

$m = \frac{40-10}{5-1} = \frac{30}{4} = 7\frac{2}{4}$

$y = mx + b$

$10 = 7\frac{1}{2}(1) + b$

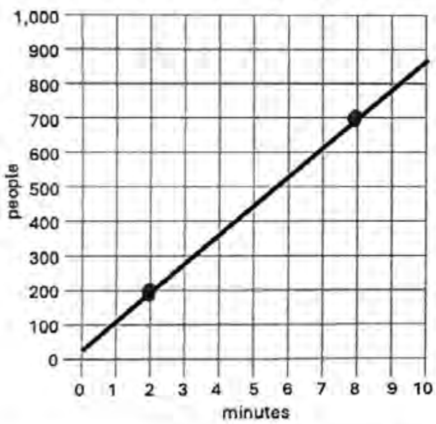
$10 = 7\frac{1}{2} + b$

$-7\frac{1}{2} \quad -7\frac{1}{2}$

$2\frac{1}{2} = b$

$y = 7\frac{1}{2}x + 2\frac{1}{2}$

After special guests enter, people with tickets are let into the stadium hourly. The graph shows total people.



2. Write an equation for the line.

$m = \frac{700-200}{8-2} = \frac{500}{6} = 83\frac{1}{3}$

$y = mx + b$

$200 = 83\frac{1}{3}(2) + b$

$200 = 166\frac{2}{3} + b$

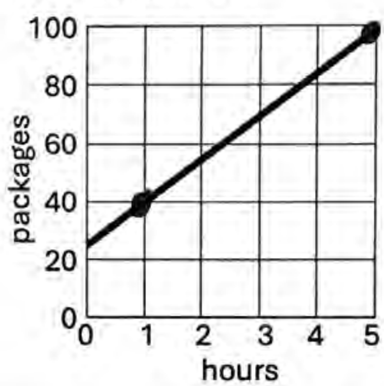
$-166\frac{2}{3} \quad -166\frac{2}{3}$

$33\frac{1}{3} = b$

$199\frac{3}{3} - 166\frac{2}{3} = 33\frac{1}{3}$

$y = 83\frac{1}{3}x + 33\frac{1}{3}$

Alex is shipping out orders from his art shop. The graph below shows the total orders for the day based on the hours he works this afternoon.



3. Write an equation for the line.

$m = \frac{100-40}{5-1} = \frac{60}{4} = 15$

$y = mx + b$

$40 = 15(1) + b$

$40 = 15 + b$

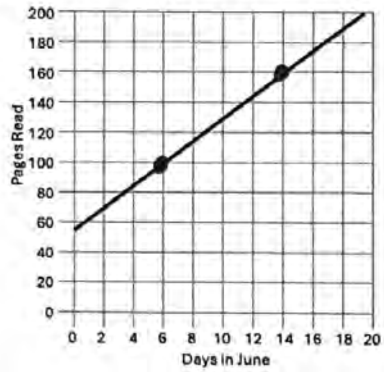
$-15 \quad -15$

$25 = b$

$y = 15x + 25$

Find the equation for the line. Then use the equation to answer the question.

<table>
<tr>
<td>
Jordan started a book in May. The graph shows the total pages he'll have read based on days he reads in June.



</td>
<td>
4. Write an equation for the line.

$m = \frac{160-100}{14-6} = \frac{60}{8} = 7\frac{1}{2}$

$8\overline{)60}$ 06, −48, 12 $8\overline{)60}$ 07 4/8, −56, 4

$7\frac{1}{2} \times 6 = 42 + 3$

$y = mx + b$
$100 = 7\frac{1}{2}(6) + b$
$100 = 45 + b$
$-45 \quad -45$
$55 = b$

$y = 7\frac{1}{2}x + 55$
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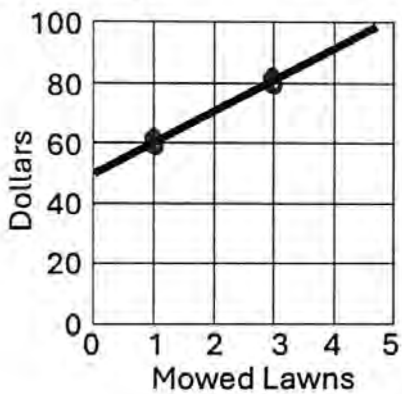
<tr>
<td>
While saving for a new phone, Samantha gets a job mowing lawns. She uses the graph shown to calculate her total savings.

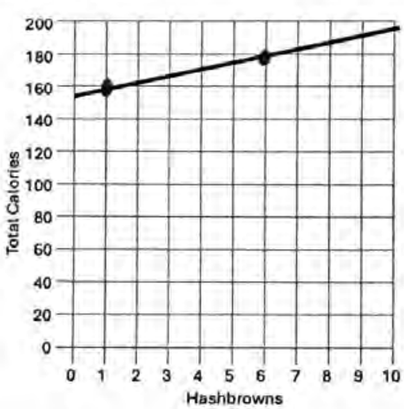


</td>
<td>
6. Write an equation for the line.

$m = \frac{80-60}{3-1} = \frac{20}{2} = 10$

$y = mx + b$
$60 = 10(1) + b$
$60 = 10 + b$
$-10 \quad -10$
$50 = b$

$y = 10x + 50$
</td>
</tr>
<tr>
<td>
Dunkin Donuts calculates the total calories in the egg and hashbrown meal based on the number of calories per hashbrown.



</td>
<td>
8. Write an equation for the line.

$m = \frac{180-160}{6-1} = \frac{20}{5} = 4$

$y = mx + b$
$160 = 4(1) + b$
$160 = 4 + b$
$-4 \quad -4$
$156 = b$

$y = 4x + 156$
</td>
</tr>
</table>

G8 U3 Lesson 8
Compare linear relationships using the given context.

CITYTUTOR DC

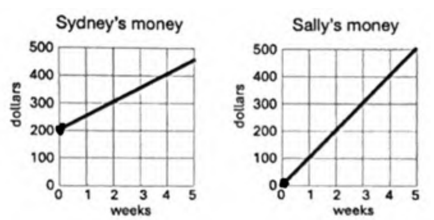
G8 U3 Lesson 8 - Today we will compare linear relationships using the given context.

Warm Welcome (Slide 1): Tutor choice

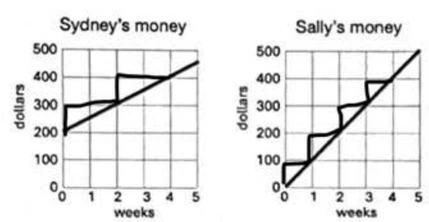
Frame the Learning/Connect to Prior Learning (Slide 2): Today we will compare linear relationships using the given context. There isn't going to be anything new that we really learn. We are just going to put it all together and use it to answer questions. This is big league stuff for 8th grade! We are going to answer the ultimate 8th grade kind of questions!

Let's Review (Slide 3): The first thing for us to know is, "There are 4 different things that we can compare in linear relationships." This will help us when we read a question because we can narrow it down to 4 possibilities of what we need to find. Let's explore this example. Read along silently in your head while I read out loud. *Read the problem.* What do you think? What different aspects of their graphs might we compare? Possible Student Answers, Key Points:

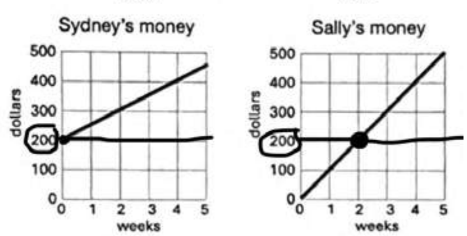
- We can compare where they start.
- We can compare their y-intercepts.
- We can compare their slopes.
- We can compare different points.
- We can compare where they stop.



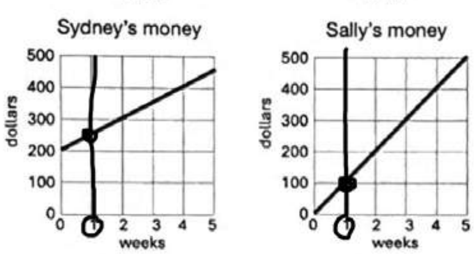
There are 4 things we can compare. The first thing is the y-intercepts. That is where the line hits the y-axis, which is where x = 0 so I can see that this line has a y-intercept here and this line has a y-intercept here. *Mark the y-intercept on both graphs. Then erase for the next part of the conversation.*



The second thing we can compare is the slopes. That is how quickly the lines go up or down. We can use y_2-y_1 over x_2-x_1. That is telling us the change in y over the change in x. *Mark the rise over run for both graphs. Then erase for the next part of the conversation.*

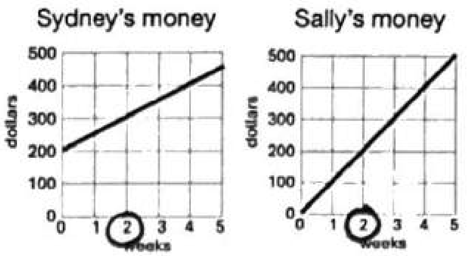


The third thing we can compare is values of x. For example, I can say, who takes longer when y = 200. And then I can see the value of x when y = 200 for each graph. *Mark a line from y = 200 on both graphs. Then erase for the next part of the conversation.*

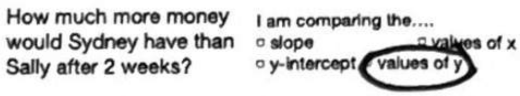


The fourth thing we can compare is value of y. For example, I can say, who is higher when x = 1. And then I can see the value of y when x = 1 for each graph. *Mark a line from x = 1 on both graphs.* So, the main idea is that we have two graphs, we can compare their y-intercepts, their slopes, their values of x or their values of y. And guess what?! That's not just for graphs! That's for tables and equations too!

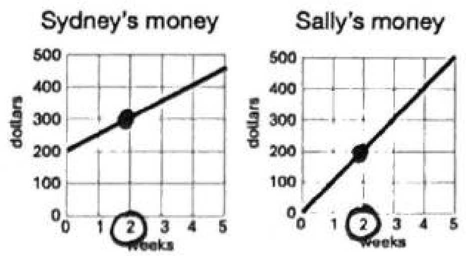
Let's Talk (Slide 4): "Everytime we have a question about a linear relationship, we want to ask ourselves, 'what am I comparing?'" So here we have the same story and graphs as before. And we have four different questions. Let's read them and decide what we're comparing. First, it says, "How



much more money would Sydney have than Sally after two weeks?" This is asking about money. That's dollars. Which is over here on the y-axis. And it says "after two weeks." I can find two weeks on the x-axis here. *Circle 2 on the x-axis of both graphs.*



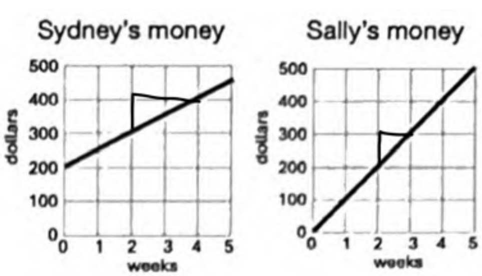
So, I know the weeks and the question is asking about the dollars. That's like I know the x values and I'm going to need to find the y values! I am comparing the values of y.



That means I am going to need to find y for each of these graphs when x = 2. In this case, Sydney has $300 at 2 weeks. *Mark a point at (2,300).* Sally has $200 at 2 weeks. *Mark a point at (2,200).* So Sydney has $100 more dollars than Sally at 2 weeks. *Erase your marks on the graphs for the next question.*

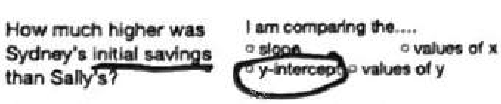
Let's do the next question. "How much higher is Sally's weekly salary than Sydney's?" This is really tricky because sometimes the thing we are looking for is disguised with a different work. This is asking

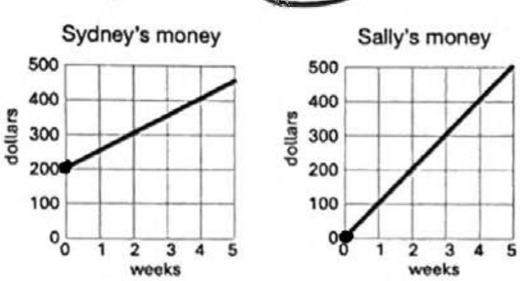
how much higher the WEEKLY SALARY is. *Underline the words "weekly salary."* Weekly salary is how much a person is paid each week. That's like dollars per week. Dollars per week is slope. So I am comparing the slope of each graph.



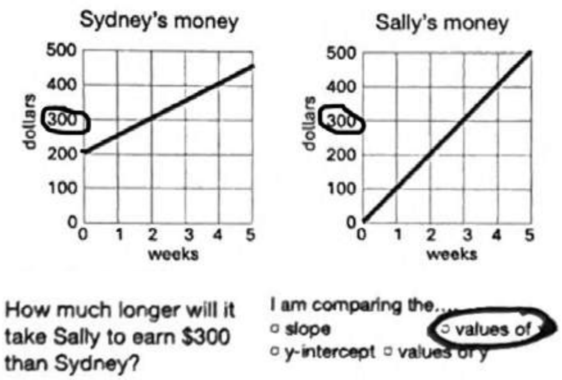
For Sydney's money, I see she goes up $100. *Mark the rise on Sydney's graph.* And that happens over 2 weeks. *Mark the rise over run on Sydney's graph.* $100 over 2 weeks is $50 per week. For Sally's money, I see she goes up $100. *Mark the rise on Sally's graph.* And that happens over 1 week. *Mark the rise over run on Sally's graph.* That's $100 per week. So Sally gets 50 more dollars per week than Sydney. We were comparing the rates of change, the rise over run, which is the slopes of the graphs. *Erase your marks on the graphs for the next question.*

Let's do the next question. "How much higher was Sydney's initial savings than Sally's?" The thing they want us to compare here is the INITIAL SAVINGS. *Underline the words "initial savings" in the question.* That's the money at the start! That's when the weeks are 0 or x = 0. So that's the y-intercept. You can see that for every question, it is almost like we are translating the story into parts of the graph.



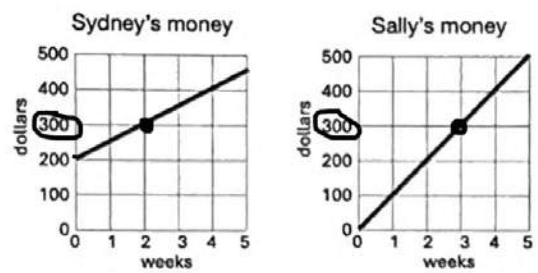


The y-intercept for Sydney is 200. *Mark a point at (0,200).* The y-intercept for Sally is 0. *Mark a point at (0,0).* So Sydney had $200 more in initial savings than Sally. *Erase your marks on the graphs for the next question.*



Let's do the next question. "How much longer will it take Sally to earn $300 than Sydney?" This is another thing we have to kind of translate. The $300 is values on the y-axis because the y-axis is dollars. *Circle 300 on each y-axis.*

I want to know how much longer. Longer is going to be measured in weeks. So I am looking for this x value when y = 300 on each graph.



In this case, I see that Sydney gets $300 at 2 weeks. *Mark a point at (2,300).* I see that Sally gets $300 at 3 weeks. *Mark a point at (3, 300).* So Sally takes 1 more week than Sydney. You can see that each of these questions is very different and this is going to take a lot of thinking about what the problem is secretly asking every time. The big idea is that every time you solve a problem about comparing linear relationships, you need to ask yourself, "What am I comparing?" And you will need to translate the story words into parts of the graph.

Let's Think (Slide 5): "The table below shows all the different ways we might find what we need." We are not going to work these out on this slide. It is just a summary of what you already know. It might seem a little overwhelming so let's look at it in pieces. We'll start with the equation column because that is the most consistent. *Circle the equation column.* For each thing, I am looking at parts of the equation. The slope is m so I have to find m in my equation. The y-intercept is b so I have to find b in my equation. I can plug in y and solve for x. Or I can plug in x and solve for y. I just use the equation and find the part I need. *Erase what you circled.*

Let's Think: **The table below shows all the different ways we might find what we need.**

To compare...	On a Graph	In an Equation	On a Table
Slopes	$m = \frac{y_2-y_1}{x_2-x_1}$	Find m in y=mx+b.	$m = \frac{y_2-y_1}{x_2-x_1}$
Y-Intercepts	Find the point that hits the y-axis OR plug x, y and m into y=mx+b and solve for b	Find b in y=mx+b.	Find the value of y when x=0 OR plug x, y and m into y=mx+b and solve for b
Values of x	Find the point on the line OR figure out y=mx+b and plug y in.	Plug y into y=mx+b.	Figure out y=mx+b and plug y in.
Values of y	Find the point on the line OR figure out y=mx+b and plug x in.	Plug x into y=mx+b.	Figure out y=mx+b and plug x in.

For slope on a graph and a table, we use the same formula we already know. *Circle the formulas.* You have already used this formula a bunch.

To compare...	On a Graph	In an Equation	On a Table
Slopes	$m = \frac{y_2-y_1}{x_2-x_1}$	Find m in y=mx+b.	$m = \frac{y_2-y_1}{x_2-x_1}$
Y-Intercepts	Find the point that hits the y-axis OR plug x, y and m into y=mx+b and solve for b	Find b in y=mx+b.	Find the value of y when x=0 OR plug x, y and m into y=mx+b and solve for b
Values of x	Find the point on the line OR figure out y=mx+b and plug y in.	Plug y into y=mx+b.	Figure out y=mx+b and plug y in.
Values of y	Find the point on the line OR figure out y=mx+b and plug x in.	Plug x into y=mx+b.	Figure out y=mx+b and plug x in.

If we look down the rest of the directions for graph, you can just look to find the point you need. *Underline "find the point" for each part of the directions.* And if that doesn't work then you'll have to make an equation and use the equation. Same goe for the table. You can either find the point you need. Or if it isn't obvious then you have to make an equation. We practiced making

To compare...	On a Graph	In an Equation	On a Table
Slopes	$m = \frac{y_2-y_1}{x_2-x_1}$	Find m in y=mx+b.	$m = \frac{y_2-y_1}{x_2-x_1}$
Y-Intercepts	Find the point that hits the y-axis OR plug x, y and m into y=mx+b and solve for b	Find b in y=mx+b.	Find the value of y when x=0 OR plug x, y and m into y=mx+b and solve for b
Values of x	Find the point on the line OR figure out y=mx+b and plug y in.	Plug y into y=mx+b.	Figure out y=mx+b and plug y in.
Values of y	Find the point on the line OR figure out y=mx+b and plug x in.	Plug x into y=mx+b.	Figure out y=mx+b and plug x in.

equations in our last lesson where you find the slope and y-intercept and write it as y=mx+b. I can leave this up later for you to look at while you're working as a reference.

Let's Think (Slide 6): This says, "We still ask ourselves the same question even when we are comparing different forms of representation." Like here we have a graph and a table. No problem! I'm going to walk you through this one so that you can see how I show my work. It's a lot of steps so the most important thing is that I use letters and labels to keep my work organized. I don't want it all over the paper. I want it nicely side by side. Read the story along with me silently in your head while I read the problem out loud. *Read the problem.* Now remember, before I just jump in, I ask myself, "What am I comparing?" I use the question to help me. It says, "How many more stickers is Maria putting on each page than Marjorie?" It's asking for stickers on each page. That's stickers per page. That's slope language. I am comparing the slope.

I am comparing the....
- slope
- y-intercept
- values of y
- values of x

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now that I know that I need to find the slope for each thing. Let's start with Marjorie. I will use m equals y two minus y one over x two minus x one.

Marjorie

x	y
0	10
1	12
2	14
3	16

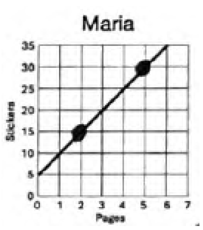
Let's use these two points. *Circle (1,12) and (3,16) on the table.*

That's m equals 16 minus 12 over 3 minus 1, which is 4 over 2. That's 2. It helps if I write the words. 2 stickers per page.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16-12}{3-1} = \frac{4}{2} = 2$$

2 stickers per page

Now let's do the graph. I will use m equals y two minus y one over x two minus x one again!



Let's use these two points. *Mark (2,15) and (5,30) on the graph.*

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30-15}{5-2} = \frac{15}{3} = 5$$

5 stickers per page

That's m equals 30 minus 15 over 5 minus 2, which is 15 over 3. That's 3. It helps if I write the words. 3 stickers per page.

Now I have the information I need to answer the question! "How many more stickers is Maria putting on each page than Marjorie?" She is doing 1 more sticker per page. I will write that as a complete sentence. Look at all of that! There are a few big ideas. First, we have said that we need to always ask ourselves, "What am I comparing?" Second, we had to keep our work super organized with labels and letters. Third, we used words as we were going along so it was easier to keep track of what the numbers we were finding represented. And it

$5-2=3$

Maria puts 3 more stickers per page than Marjorie.

made it easier to answer the final question after all those in between steps.

Let's Try It (Slide 7): Let's solve another problem together now. I will walk you through each step.

WARM WELCOME

Today we will compare linear relationships using the given context.

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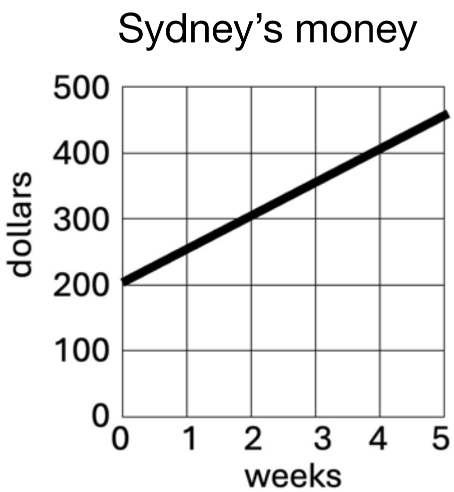
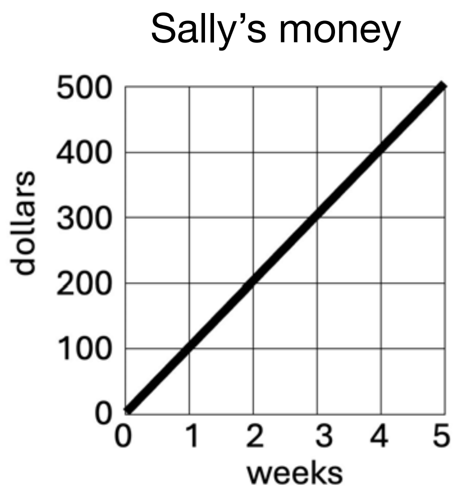
Let's Review:

There are 4 different things that we can compare in linear relationships.

The graphs below show the amount of money that two sisters saved this summer. What different aspects of their graphs might we compare?

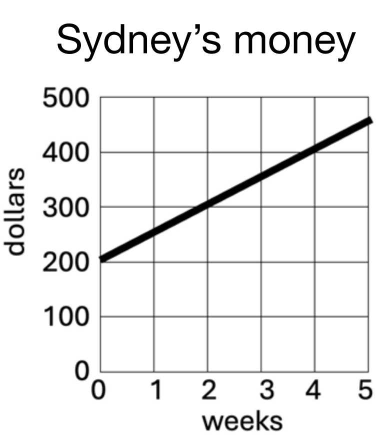


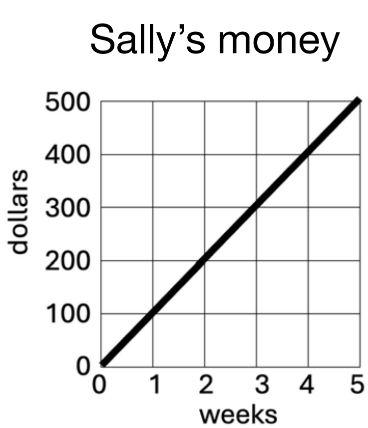


Let's Talk:

Every time we have a question about a linear relationship, we want to ask ourselves, "What am I comparing?"

The graphs below show the amount of money that two sisters saved this summer.





How much more money would Sydney have than Sally after 2 weeks?

I am comparing the….
- ▢ slope
- ▢ values of x
- ▢ y-intercept
- ▢ values of y

How much higher is Sally's weekly salary than Sydney's?

I am comparing the….
- ▢ slope
- ▢ values of x
- ▢ y-intercept
- ▢ values of y

How much higher was Sydney's initial savings than Sally's?

I am comparing the….
- ▢ slope
- ▢ values of x
- ▢ y-intercept
- ▢ values of y

How much longer will it take Sally to earn $300 than Sydney?

I am comparing the….
- ▢ slope
- ▢ values of x
- ▢ y-intercept
- ▢ values of y

Let's Think:

The table below shows all the different ways we might find what we need.

To compare…	On a Graph	In an Equation	On a Table
Slopes	$m = \frac{y_2 - y_1}{x_2 - x_1}$	Find m in y=mx+b.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Y-Intercepts	Find the point that hits the y-axis OR plug x, y and m into y=mx+b and solve for b.	Find b in y=mx+b.	Find the value of y when x=0 OR plug x, y and m into y=mx+b and solve for b
Values of x	Find the point on the line OR figure out y=mx+b and plug y in.	Plug y into y=mx+b.	Figure out y=mx+b and plug y in.
Values of y	Find the point on the line OR figure out y=mx+b and plug x in.	Plug x into y=mx+b.	Figure out y=mx+b and plug x in.

Let's Think:

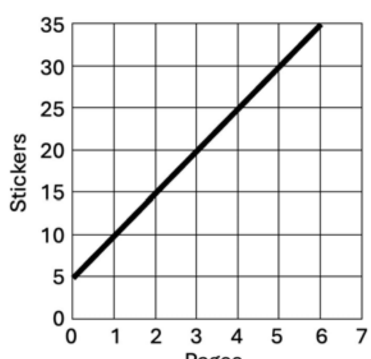
We still ask ourselves the same question even when we are comparing different forms of representation.

Marjorie is tracking the total stickers starting with the cover and then the pages of her sticker book. She uses the table below where x equals the number of pages and y equals the number of pages. Maria is using a graph to do the same. How many more stickers is Maria putting on each page than Marjorie?

Marjorie

x	*y*
0	10
1	12
2	14
3	16

Maria



I am comparing the….

- ▢ slope
- ▢ y-intercept
- ▢ values of y
- ▢ values of x

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Let's Try It:

Let's find y-intercepts and equations for graphs together!

Name: ____________________ G8 U3 Lesson 8 - Let's Try It

Jeff wants to know whether the fastest way to get to the beach from his neighborhood is on the highway or on backroads. He uses a graph to show the distance from his house that his buddy's car travels on the highway where x equals minutes and y equals miles. He uses a table to record the distance from his house that his own car travels on the backroads. How much faster is the highway car than the backroads car?

Car on Highway

Car on Backroads

x	y
0	0
2	1
4	2
6	3
8	4

I am comparing the....
- ☐ slope
- ☐ y-intercept
- ☐ values of y
- ☐ values of x

1. Start with the first representation. Find the thing you are looking for.

   In this case, I will need to do ____________________

2. Move on to the second representation. Find the thing you are looking for.

   In this case, I will need to do ____________________

On your Own:

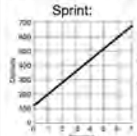
Now it's time for you to do it on your own!

Name: ____________________ G8 U3 Lesson 8 - Independent Work

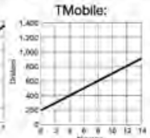
Solve the problems below. Show your work. Write your final answer in a complete sentence.

1. The graphs below show the total costs for two different cell phone carriers including the signup fee and the monthly rate. Which cell phone carrier will cost the most after 12 months?

Sprint:



TMobile:



Check one:
I am comparing the....
- ☐ slope
- ☐ y-intercept
- ☐ values of x
- ☐ values of y

Show your work:

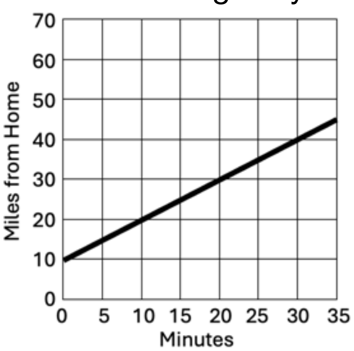
2. For November, Francisco plans to use the equation y = 18x+100 to calculate his total savings where y is the number of dollars and x is the number of hours he works. His friend, Charlie, plans to use the equation y = 20x + 50 to calculate his total savings where y is the number of dollars and x is the number of hours he works. Who is starting out with the most money leading up to November?

Check one:
I am comparing the....
- ☐ slope

Show your work:

Jeff wants to know whether the fastest way to get to the beach from his neighborhood is on the highway or on backroads. He uses a graph to show the distance from his house that his buddy's car travels on the highway where x equals minutes and y equals miles. He uses a table to record the distance from his house that his own car travels on the backroads. How much faster is the highway car than the backroads car?

Car on Highway



Car on Backroads

x	*y*
0	0
2	1
4	2
6	3
8	4

I am comparing the….
- ☐ slope
- ☐ y-intercept
- ☐ values of y
- ☐ values of x

1. Start with the first representation. Find the thing you are looking for.

   In this case, I will need to do ______________________________________

2. Move on to the second representation. Find the thing you are looking for.

   In this case, I will need to do ______________________________________

3. Now do the work to answer the question.

4. Write your answer in a complete sentence.

____

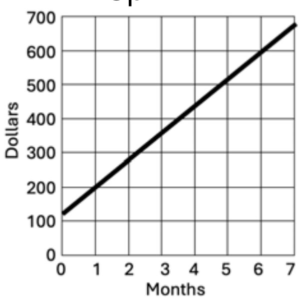
____

Name: ____

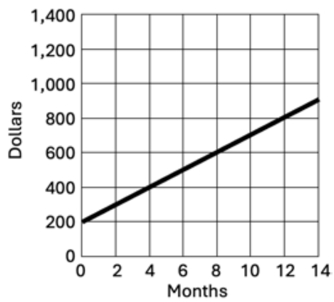
Solve the problems below. Show your work. Write your final answer in a complete sentence.

1. The graphs below show the total costs for two different cell phone carriers including the signup fee and the monthly rate. Which cell phone carrier will cost the most after 12 months?

Sprint:



TMobile:



Check one:
I am comparing the….
- ▢ slope
- ▢ y-intercept
- ▢ values of x
- ▢ values of y

Show your work:

2. For November, Francisco plans to use the equation $y = 18x+100$ to calculate his total savings where y is the number of dollars and x is the number of hours he works. His friend, Charlie, plans to use the equation $y = 20x + 50$ to calculate his total savings where y is the number of dollars and x is the number of hours he works. Who is starting out with the most money leading up to November?

Check one:
I am comparing the….
- ▢ slope
- ▢ y-intercept
- ▢ values of x
- ▢ values of y

Show your work:

Solve the problems below. Show your work. Write your final answer in a complete sentence.

3. Jeff was comparing the growth of two different plants that he bought at the store by measuring their height every week. The 1st plant was put in direct sunlight. The 2nd plant was put in indirect sunlight. He recorded their heights in a table where x is the number of weeks and y is the height in inches. How much faster did the first plant grow than the second?

1st Plant:

x	y
0	2
1	5
2	8
3	11
4	14

2nd Plant:

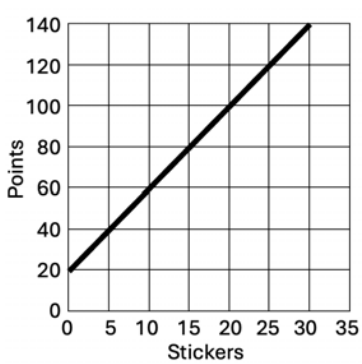
x	y
0	3
1	5
2	7
3	9
4	11

Check one:
I am comparing the….
- ▢ slope
- ▢ y-intercept
- ▢ values of x
- ▢ values of y

Show your work:

4. Customers at Mazzy's Sticker Store earn points by buying stickers. Then they can use the points for prizes at the end of the year. Mazzy uses the equation y = 4x where x represents the number of stickers and y represents the number of points. Mazzy also has a Super Savers Club where customers who join can get points for signing up and they earn more points for each sticker. The graph below shows the points based on the number of stickers for the Super Savers Club where x represents the number of stickers and y represents the number of points. The first prize requires 100 points. How many more stickers does someone need to buy to get 100 points if they are NOT in the Super Saver Club than if they are in the Super Saver Club?

Super Saver Club:



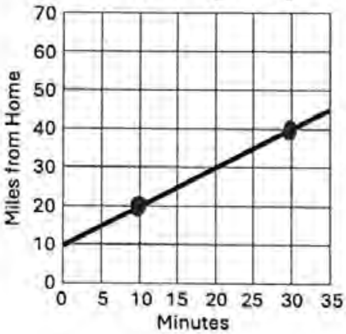
Check one:
I am comparing the….
- ▢ slope
- ▢ y-intercept
- ▢ values of x
- ▢ values of y

Show your work:

Name: ANSWER KEY

Jeff wants to know whether the fastest way to get to the beach from his neighborhood is on the highway or on backroads. He uses a graph to show the distance from his house that his buddy's car travels on the highway where x equals minutes and y equals miles. He uses a table to record the distance from his house that his own car travels on the backroads. How much faster is the highway car than the backroads car?

Car on Highway



Car on Backroads

x	y
0	0
2	1
4	2
6	3
8	4

I am comparing the....
- [x] slope
- [ ] y-intercept
- [ ] values of y
- [ ] values of x

1. Start with the first representation. Find the thing you are looking for.

In this case, I will need to do $(y_2 - y_1) / (x_2 - x_1)$

$$m = \frac{40-20}{30-10} = \frac{20}{10} = 2$$

2. Move on to the second representation. Find the thing you are looking for.

In this case, I will need to do $(y_2 - y_1) / (x_2 - x_1)$

$$m = \frac{6-2}{3-1} = \frac{4}{2} = 2$$

3. Now do the work to answer the question.

$$2 - 2 = 0$$

4. Write your answer in a complete sentence.

The highway car is 0 mph faster than the backroads car. They go the same speed.

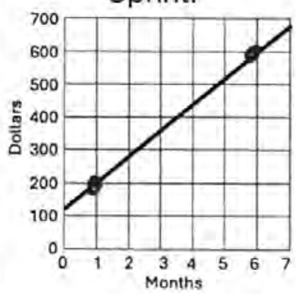
Name: ANSWER KEY

G8 U3 Lesson 8 - Independent Work

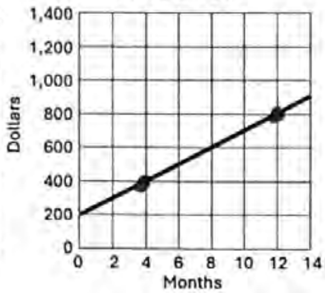
Solve the problems below. Show your work. Write your final answer in a complete sentence.

1. The graphs below show the total costs for two different cell phone carriers including the signup fee and the monthly rate. Which cell phone carrier will cost the most after 12 months?

Sprint:



TMobile:



Check one:
I am comparing the....

- ☐ slope
- ☐ y-intercept
- ☐ values of x
- ☑ values of y

Show your work:

Sprint:

$y = 80x + 120$

$y = 80(12) + 120$

$y = 960 + 120$

$y = 1080$ dollars

TMobile:

$y = 50x + 200$

$y = 50(12) + 200$

$y = 600 + 200$

$y = 800$ dollars

$m = \frac{600-200}{6-1} = \frac{400}{5} = 80$

$y = mx + b$

$200 = 80(1) + b$

$200 = 80 + b$

$-80 \quad -80$

$120 = b$

$y = 80x + 120$

$m = \frac{800-400}{12-4} = \frac{400}{8} = 50$

$y = mx + b$

$400 = 50(4) + b$

$400 = 200 + b$

$-200 \quad -200$

$200 = b$

$y = 50x + 200$

Sprint will cost the most.

2. For November, Francisco plans to use the equation y = 18x+100 to calculate his total savings where y is the number of dollars and x is the number of hours he works. His friend, Charlie, plans to use the equation y = 20x + 50 to calculate his total savings where y is the number of dollars and x is the number of hours he works. Who is starting out with the most money leading up to November?

Check one:
I am comparing the....

- ☐ slope
- ☑ y-intercept
- ☐ values of x
- ☐ values of y

Show your work:

Francisco:

$y = 18x + 100$

$b = 100$

Charlie:

$y = 20x + 50$

$b = 50$

Francisco is starting out with the most money.

Solve the problems below. Show your work. Write your final answer in a complete sentence.

3. Jeff was comparing the growth of two different plants that he bought at the store by measuring their height every week. The 1st plant was put in direct sunlight. The 2nd plant was put in indirect sunlight. He recorded their heights in a table where x is the number of weeks and y is the height in inches. How much faster did the first plant grow than the second?

1st Plant:

x	*y*
0	2
1	5
2	8
3	11
4	14

2nd Plant:

x	*y*
0	3
1	5
2	7
3	9
4	11

Check one:
I am comparing the....
- ☑ slope
- ☐ y-intercept
- ☐ values of x
- ☐ values of y

Show your work:

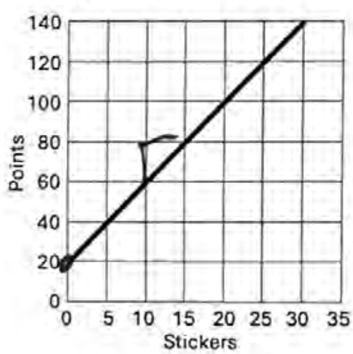
$m = \frac{11-5}{3-1} = \frac{6}{2} = 3$ $m = \frac{9-5}{3-1} = \frac{4}{2} = 2$

3 − 2 = 1

The 1st plant grew 1 inch per week faster than the 2nd plant.

4. Customers at Mazzy's Sticker Store earn points by buying stickers. Then they can use the points for prizes at the end of the year. Mazzy uses the equation y = 4x where x represents the number of stickers and y represents the number of points. Mazzy also has a Super Savers Club where customers who join can get points for signing up and they earn more points for each sticker. The graph below shows the points based on the number of stickers for the Super Savers Club where x represents the number of stickers and y represents the number of points. The first prize requires 100 points. How many more stickers does someone need to buy to get 100 points if they are NOT in the Super Saver Club than if they are in the Super Saver Club?

Super Saver Club:



Check one:
I am comparing the....
- ☐ slope
- ☐ y-intercept
- ☑ values of x
- ☐ values of y

Show your work:

$y = 4x + 20$

$100 = 4x + 20$

$-20 \quad -20$

$\frac{80}{4} = \frac{4x}{4}$

$x = 20$

NOT in club regular:

$y = 4x$

$\frac{100}{4} = \frac{4x}{4}$

$25 = x$

25 − 20 = 5

They would need to buy 5 more stickers than someone in the Super Saver Club.

G8 U3 Lesson 9
Represent and interpret linear relationships with negative rates of change.

CITYTUTOR DC

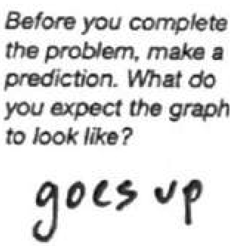
G8 U3 Lesson 9 - Today we will represent and interpret linear relationships with negative rates of change.

Warm Welcome (Slide 1): Tutor choice

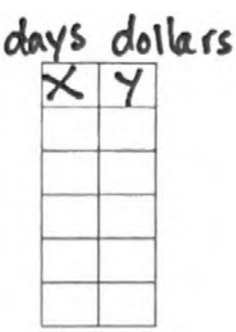
Frame the Learning/Connect to Prior Learning (Slide 2): Today we will represent and interpret linear relationships with negative rates of change. This is all going to be based on things you already know. The hardest part for kids is just if they don't know that it is possible for negative rates of change or negative slopes to exist at all. But once you know that, you can keep doing all the great math you've been doing. Let's go!

Let's Review (Slide 3): We know that linear relationships look a certain way when graphed. I am going to read this story out loud. Read silently along with me in your head. *Read the problem.* Now, there is a little extra part here. It says, "Before you complete the problem, make a prediction. What do you expect the graph to look like?" What do you think? What is your prediction? Possible Student Answers, Key Points:

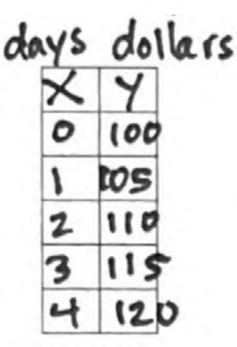
- I expect it to be a straight line.
- I expect it to be diagonal.
- I expect it to go up.
- I expect it to cross the y-axis.
- I expect it to go through points.



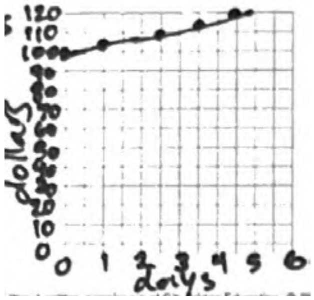
We've talked a lot about linear relationships having a y-intercept and a straight line with a constant slope. But we haven't spelled out the fact that every linear relationship we've seen so far has had a straight line that GOES UP. All of our examples so far have had a POSITIVE SLOPE. But they don't have to be that way. Let's finish this problem here and then we will do a problem with a negative slope.



First I put x and y at the top of my table. That's days and dollars, which I'll write on top just to keep myself straight.



At the very beginning of my story, it would be 0 days, and it said Graham has $100. So I will write 0 and 100 as the first row. Then it says that "every day, he gets $5 from a neighbor." So if I go to the next day, that's day 1 and he would have $105. And I can keep going, right? The next day, day 2, would be another $5 so $110 altogether. Then 3 days is $115. 4 days is $120.



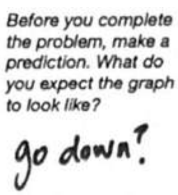
Now let's graph it. I will write days on the x-axis and dollars on the y-axis. For days, I can actually make it two lines is 1 then two lines is 2 then two lines is 3 and I keep going. For dollars, I will skip count by ten.Now I graph: (0,100) and then (1,105). I'm going to have to go halfway between to mark at 105. Then I'll do (2,110) and (3,115). I'll have to go halfway again for 115. Then I'll do (4,120). I draw a line to connect the dots, and look! It's exactly what we predicted. There is the y-intercept and it's a straight line. But also it is going up.

$m=5$

The rate or change or slope is 5 because the problem said he gets $5 per day. I'll write that as m = 5. We take for granted that this is a POSITIVE number because when numbers are positive, we just call them "numbers" and we leave off the positive sign. But next we'll look at an example with a negative slope and see that we can't leave that off.

Let's Talk (Slide 4): This says, "Imagine the story on the previous slide was reversed." I am going to read it out loud while you read it silently in your head. *Read the story and make extra emphasis on the words 'he pays his neighbor.'"* This problem is not the same. Now, Graham is not getting paid. Graham is the one doing the paying. Let's think about this extra question here, "Before you complete the problem, make a prediction. What do you expect the graph to look like?" What do you think? What is your prediction? Possible Student Answers, Key Points:

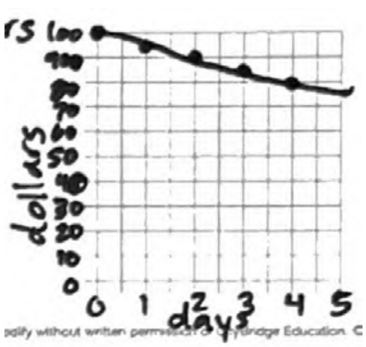
- I expect it to be a straight line.
- I expect it to be diagonal.
- I expect it to go DOWN.
- I expect it to cross the y-axis.
- I expect it to go through points.



There are a lot of things we can think about like straight lines and y-intercepts. But the important thing we might guess here is that instead of the line going up, maybe it will go down. Because now Graham is losing money instead of gaining money. Let's make a table and graph and see.

days	dollars
x	y
0	100
1	95
2	90
3	85
4	80

First I put x and y at the top of my table. That's days and dollars, which I'll write on top just to keep myself straight. At the very beginning of my story, it would be 0 days, and it said Graham has $100. So I will write 0 and 100 as the first row. Then it says that "every day, he pays his neighbor $5." So if I go to the next day, that's day 1 and he wouldn't have $105. He would have $5 less than $100, which is $95. And I can keep going, right? The next day, day 2, he would pay another $5 so that's $90 altogether. Then 3 days is $85. 4 days is $80. You get the idea.



Now let's graph it. I will write days on the x-axis and dollars on the y-axis. For days, I can actually make it two lines is 1 then two lines is 2 then two lines is 3 and I keep going. For dollars, I will skipcount by ten again. Now I graph: (0,100). That's the same. But now I graph (1,95)! I'm going to have to go halfway between 90 and 100 to mark at 95. But the most important thing is that my point went LOWER than when we started. It went down. Next I'll do (2,90). Then (3,85). I'll have to go halfway again for 85. Then I'll do (4,80). I draw a line to connect the dots, and look! It's exactly what we predicted. There is the y-intercept and it's a straight line. But it's not going up. It's going down!

The rate or change or slope is not positive 5 like on the last slide. It's the opposite. It's NEGATIVE 5 because the problem said he loses $5 per day like it is getting subtracted. I am going to write m = -5 for this one. The big idea here is that you have to read the story and picture what is happening in your mind to be clear on whether your initial amount is increasing or decreasing because that will determine if your slope should be positive or negative.

$m = -5$

Let's Think (Slide 5): We have one more representation to think about with GETS, Graph, Equation, Table and Story, right? This says, "Linear relationships with a negative slope follow the same rules as all linear relationships." That means they also have equations in the form y = mx + b. Let's work that out. We have a very similar problem but the underlined part is different. I am going to read it out loud and you should read silently in your head. *Read the problem.* This is the problem we just did with the negative slope. So, first, let's write that down: m = -5. Then we have the y-intercept which was Graham's money to start on day 0. Let's write that down: b = 100.

$m = -5$
$b = 100$

$y = mx + b$
$y = -5x + 100$

So, now we take the equation: y = mx + b and we just fill in the m and fill in the b. It would be y = -5x + 100.

And so, just as a side note, if we were to plug a value of x into this equation, such as x = 9, then it would be y = -5 times 9 plus 100. That is the same as y = -45 plus 100. And the negative number cancels 50 of the positive number, essentially subtracting. And we'd get y = 55. But the big idea here is that we really do the table and the graph and the equation the same way we always do. It's just that sometimes now the slope will be going down so it will be negative.

Let's Try It (Slide 6): Let's practice doing some problems with negative slopes together. I will walk you through each step.

WARM WELCOME

Today we will represent and interpret linear relationships with negative rates of change.

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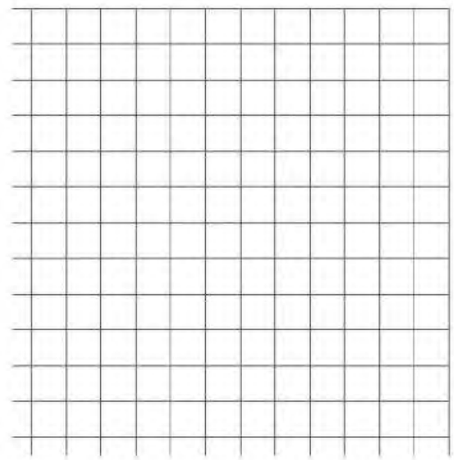
Let's Review: We know that linear relationships look a certain way when graphed.

Graham has $100. Every day, he gets $5 from the neighbor for walking her dog. Make a table and graph of Graham's money. Let x stand for days walking the dog. Let y stand for dollars.

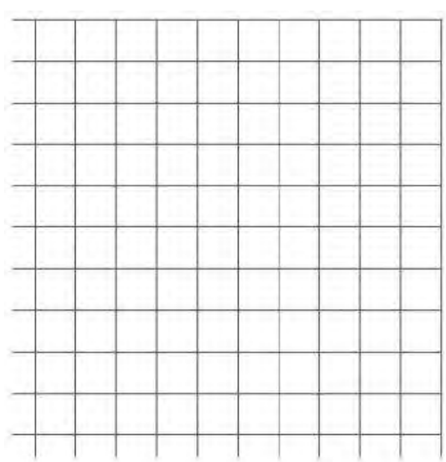
Before you complete the problem, make a prediction. What do you expect the graph to look like?



Let's Talk: Imagine the story on the previous slide was reversed.

Graham has $100. Every day, he pays his neighbor $5 for walking his dog. Make a table and graph of Graham's money. Let x stand for days walking the dog. Let y stand for dollars.

Before you complete the problem, make a prediction. What do you expect the graph to look like?



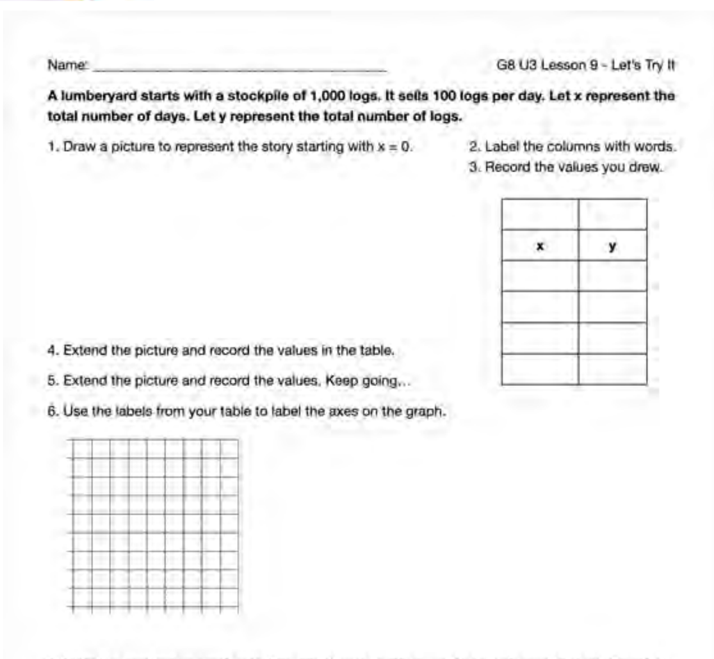
Let's Think:

Linear relationships with a negative slope follow the same rules as all linear relationships.

Graham has $100. Every day, he pays his neighbor $5 for walking his dog. Write an equation that could be used to calculate Graham's money. Let x stand for days walking the dog. Let y stand for dollars.

Let's Try It:

Let's work with negative slopes together!


Name: ______________________ G8 U3 Lesson 9 - Let's Try It

A lumberyard starts with a stockpile of 1,000 logs. It sells 100 logs per day. Let x represent the total number of days. Let y represent the total number of logs.

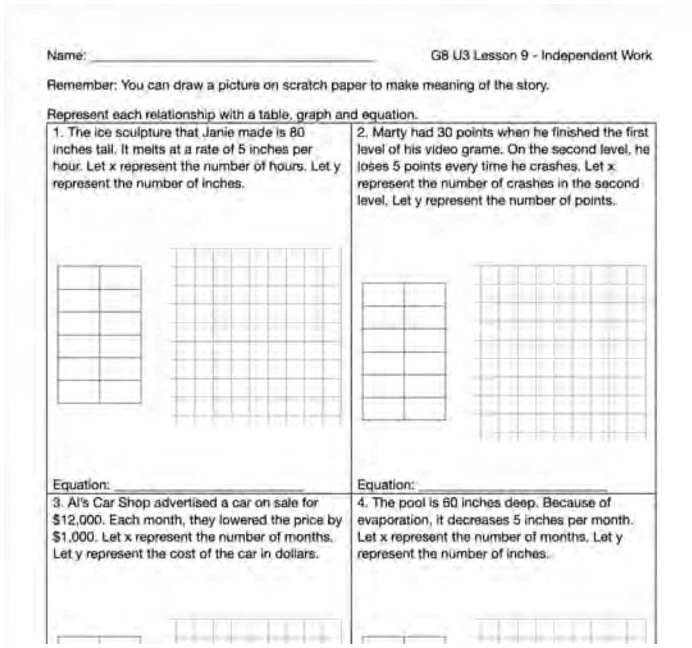
1. Draw a picture to represent the story starting with x = 0.
2. Label the columns with words.
3. Record the values you drew.

x	y

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going...
6. Use the labels from your table to label the axes on the graph.
7. Fill in the numbers on each axis. You might have to skip count in order to reach the right number

On your Own:

Now it's time for you to do it on your own!



Name: ______________________ G8 U3 Lesson 9 - Independent Work

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each relationship with a table, graph and equation.

1. The ice sculpture that Janie made is 80 inches tall. It melts at a rate of 5 inches per hour. Let x represent the number of hours. Let y represent the number of inches.

Equation: ______________

2. Marty had 30 points when he finished the first level of his video grame. On the second level, he loses 5 points every time he crashes. Let x represent the number of crashes in the second level. Let y represent the number of points.

Equation: ______________

3. Al's Car Shop advertised a car on sale for $12,000. Each month, they lowered the price by $1,000. Let x represent the number of months. Let y represent the cost of the car in dollars.

4. The pool is 60 inches deep. Because of evaporation, it decreases 5 inches per month. Let x represent the number of months. Let y represent the number of inches.

A lumberyard starts with a stockpile of 1,000 logs. It sells 100 logs per day. Let x represent the total number of days. Let y represent the total number of logs.

1. Draw a picture to represent the story starting with x = 0.

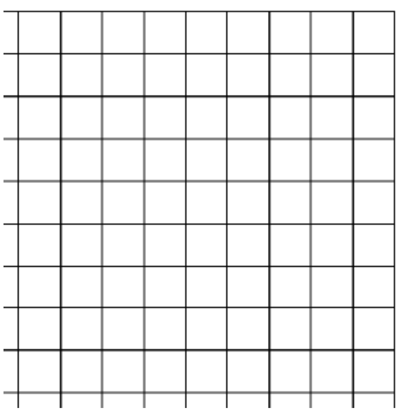
2. Label the columns with words.
3. Record the values you drew.

x	**y**

4. Extend the picture and record the values in the table.

5. Extend the picture and record the values. Keep going…

6. Use the labels from your table to label the axes on the graph.



7. Fill in the numbers on each axis. You might have to skip count in order to reach the right number.

8. Use each row of the table as a set of coordinates.

9. What was the value of y when x was 0 (also known as the y-intercept)? _______ That is b.

10. What is the rate of change (also known as the slope)? _________ That is m.

11. Write an equation in y = mx + b form. _______________________

Name: ____

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each relationship with a table, graph and equation.

1. The ice sculpture that Janie made is 80 inches tall. It melts at a rate of 5 inches per hour. Let x represent the number of hours. Let y represent the number of inches.

Equation: ___________________________

2. Marty had 30 points when he finished the first level of his video grame. On the second level, he loses 5 points every time he crashes. Let x represent the number of crashes in the second level. Let y represent the number of points.

Equation: ___________________________

3. Al's Car Shop advertised a car on sale for $12,000. Each month, they lowered the price by $1,000. Let x represent the number of months. Let y represent the cost of the car in dollars.

Equation: ___________________________

4. The pool is 60 inches deep. Because of evaporation, it decreases 5 inches per month. Let x represent the number of months. Let y represent the number of inches.

Equation: ___________________________

Represent each relationship with a table, graph and equation.

5. Sam used 100 Legos for each house he built. He started with a tub of 800 Legos. Let x represent the number of houses Sam built. Let y represent the number of Legos Sam had left.

Equation: ______________________________

6. Marvin bought 40 pounds of kitty litter. He uses 5 pounds per week. Let x represent the number of weeks. Let y represent the number of pounds of kitty litter.

Equation: ______________________________

7. The principal at Sunny Elementary has 100 stickers in her office. She gives 10 stickers to every child sent to her office for something good. Let x represent the number of students who come to the office for something good. Let y represent the number of stickers the principal has left.

Equation: ______________________________

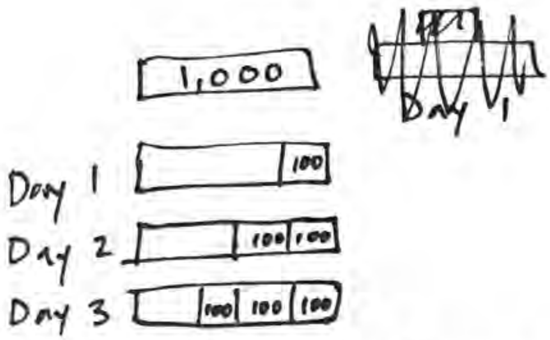
8. Lisa saved enough money to spent $50 on each day of her trip to Spain. Lisa saved $250 for the trip. Let x represent the number of days. Let y represent the number of dollars Lisa has.

Equation: ______________________________

Name: ANSWER KEY

A lumberyard starts with a stockpile of 1,000 logs. It sells 100 logs per day. Let x represent the total number of days. Let y represent the total number of logs.

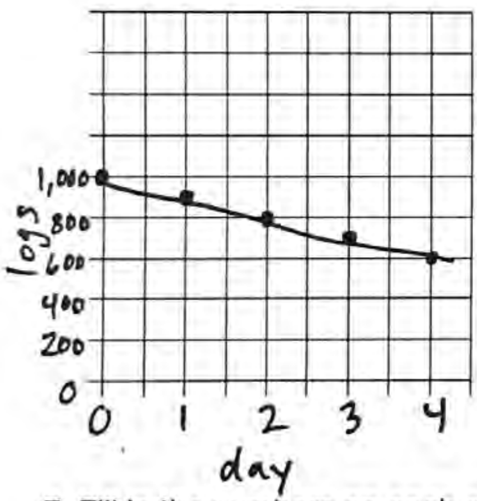
1. Draw a picture to represent the story starting with x = 0.



2. Label the columns with words.
3. Record the values you drew.

day	logs
x	y
0	1000
1	900
2	800
3	700

4. Extend the picture and record the values in the table.
5. Extend the picture and record the values. Keep going...
6. Use the labels from your table to label the axes on the graph.



7. Fill in the numbers on each axis. You might have to skip count in order to reach the right number.
8. Use each row of the table as a set of coordinates.
9. What was the value of y when x was 0 (also known as the y-intercept)? 1,000 That is b.
10. What is the rate of change (also known as the slope)? −100 That is m.

11. Write an equation in y = mx + b form. $y = -100x + 1000$

Name: ANSWER KEY

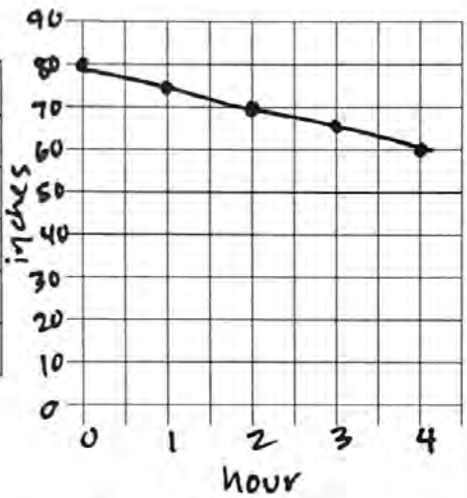
G8 U3 Lesson 9 - Independent Work

Remember: You can draw a picture on scratch paper to make meaning of the story.

Represent each relationship with a table, graph and equation.

1. The ice sculpture that Janie made is 80 inches tall. It melts at a rate of 5 inches per hour. Let x represent the number of hours. Let y represent the number of inches.

x	y
0	80
1	75
2	70
3	65
4	60



Equation: $y = -5x + 80$

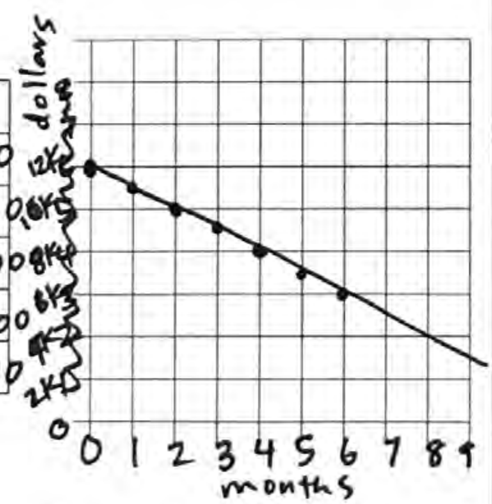
2. Marty had 30 points when he finished the first level of his video grame. On the second level, he loses 5 points every time he crashes. Let x represent the number of crashes in the second level. Let y represent the number of points.

x	y
0	30
1	25
2	20
3	15
4	10

Equation: $y = -5x + 30$

3. Al's Car Shop advertised a car on sale for $12,000. Each month, they lowered the price by $1,000. Let x represent the number of months. Let y represent the cost of the car in dollars.

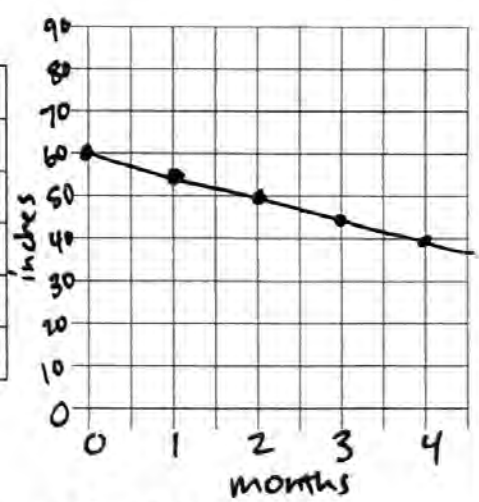
x	y
0	12000
1	11,000
2	10,000
3	9,000
4	8,000



Equation: $y = -1{,}000x + 12{,}000$

4. The pool is 60 inches deep. Because of evaporation, it decreases 5 inches per month. Let x represent the number of months. Let y represent the number of inches.

x	y
0	60
1	55
2	50
3	45
4	40

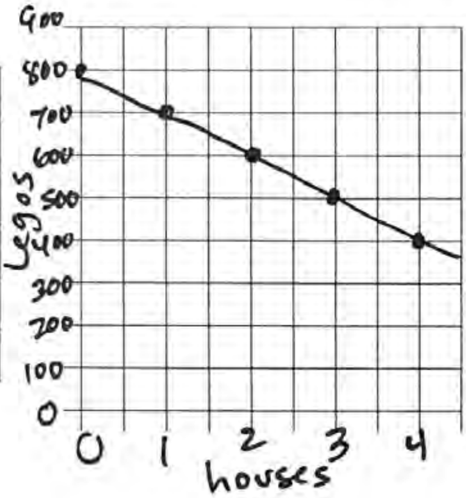


Equation: $y = -5x + 60$

Represent each relationship with a table, graph and equation.

5. Sam used 100 Legos for each house he built. He started with a tub of 800 Legos. Let x represent the number of houses Sam built. Let y represent the number of Legos Sam had left.

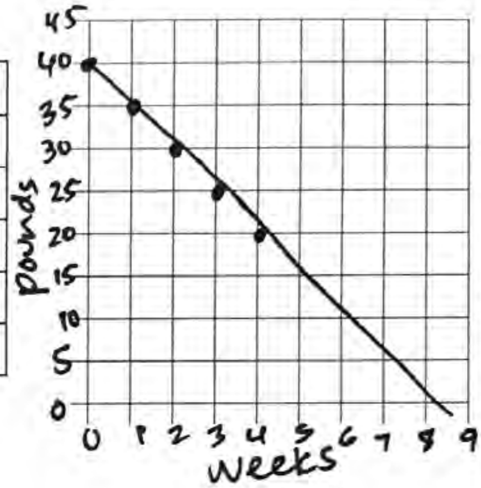
x	y
0	800
1	700
2	600
3	500
4	400



Equation: $y = -100x + 800$

6. Marvin bought 40 pounds of kitty litter. He uses 5 pounds per week. Let x represent the number of weeks. Let y represent the number of pounds of kitty litter.

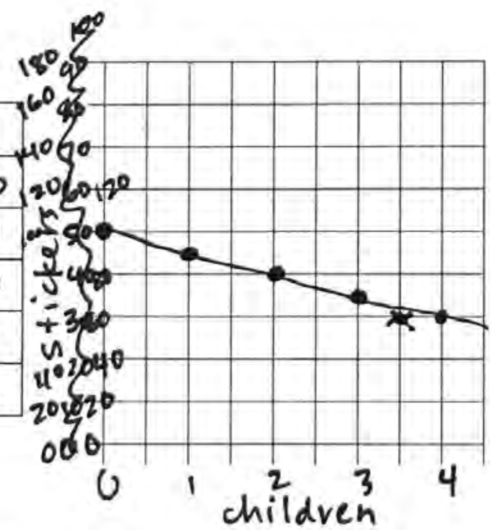
x	y
0	40
1	35
2	30
3	25
4	20



Equation: $y = -5x + 40$

7. The principal at Sunny Elementary has 100 stickers in her office. She gives 10 stickers to every child sent to her office for something good. Let x represent the number of students who come to the office for something good. Let y represent the number of stickers the principal has left.

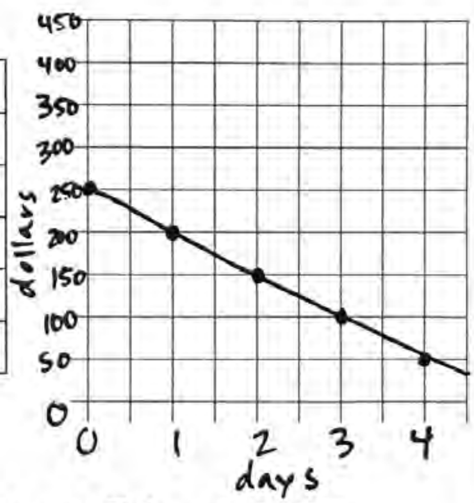
x	y
0	100
1	90
2	80
3	70
4	60



Equation: $y = -10x + 100$

8. Lisa saved enough money to spent $50 on each day of her trip to Spain. Lisa saved $250 for the trip. Let x represent the number of days. Let y represent the number of dollars Lisa has.

x	y
0	250
1	200
2	150
3	100
4	50



Equation: $y = -50x + 250$

G8 U3 Lesson 10
Write an equation and find the x-intercept for linear relationships with negative rates of change.

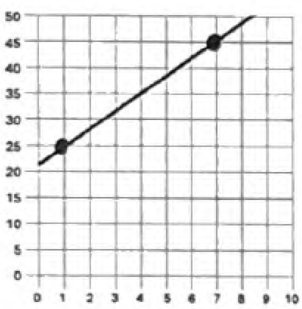
CITYTUTOR DC

G8 U3 Lesson 10 - Today we will write an equation and find the x-intercept for linear relationships with negative rates of change.

Warm Welcome (Slide 1): Tutor choice

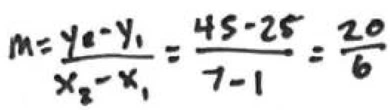
Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will write an equation and find the x-intercept for linear relationships with negative rates of change. We are going to use everything we've already learned so I know you are going to do a great job!

Let's Review (Slide 3): We know how to write an equation when the slope of a graph is positive. Sometimes it's easier than others. If I can see the y-intercept clearly, it will be easier. If I can see the rise over run clearly, it will be easier. This says, "Write an equation for the line." And when I look, I see it's not one of the easy ones. I'm going to need to do some work. What work do I need to do? Possible Student Answers, Key Points:

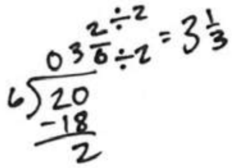


- Find the slope.
- Find the y-intercept.
- Mark two points.
- Do y_2-y_1 over x_2-x_1
- Write it in y=mx+b form.

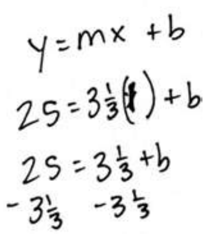
There are a few pieces we need to find here and the order really matters. If I look at my graph and I can't see the y-intercept then I have to do some other math before I can do that math so let's make a review. First, I use two points to find m. Then I use one point to find b. Let's start with m. I am going to choose this point and this point. *Mark a point at (1,25) and (7,45).*



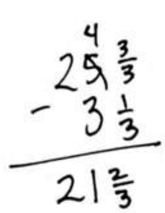
Now I plug those values into y_2-y_1 over x_2-x_1. That's 45 minus 25 over 7 minus 1, which is 20 over 6.



I divide this over to the side of my paper, and it is 3 and 2 sixths. I simplify 2 sixths and get 3 and 1 third.



Next I am going to find b using y = mx + b. I have to fill in all the other values to solve for b. I need 1 point for this. It doesn't matter which one. I'll use (1,25). So y is 25, which equals 3 and 1 third times 1 plus b. 3 and 1 third times 1 is 3 and 1 third. So this is really 25 equals 3 and 1 third plus b. To get b on its own, I have to subtract 3 and 1 third from both sides.

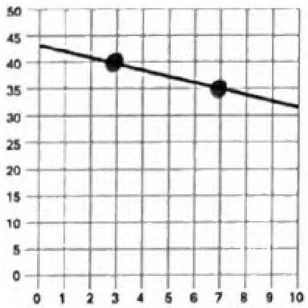


Now, this is a little tricky. Normally I could subtract this in my head. But I am subtracting a fraction. So let me write this to the side. I notice that I don't have any thirds to subtract from. I have to regroup from the 25 and write it as one less so I can get 3 thirds. It becomes 24 and 3 thirds.

Now I can subtract that 1 third. I get 2 thirds. I can subtract that 3. I get 21. I get 21 and 2 thirds as my final answer.

$$y = 3\tfrac{1}{3}x + 21\tfrac{2}{3}$$

Let's review. I did step one, which was to use two points to find m. *Point to the work for step one.* Then I did step two, which was to use one point to find b. *Point to the work for step two.* Now I can write my final equation: y = 3 and 1 third x plus 21 and 2 thirds. There are two big steps there with a lot of itty bitty steps to do the number crunching. But we did it!



Let's Talk (Slide 4): Linear relationships with a negative slope follow the same rule as all linear relationships. That means we're going to do the exact same steps we just did even though this line is going down instead of up. It is literally the exact same. There is one tricky part that we have to look out for that could trip you up, and I'll show you where there is. Let's start by writing out our steps again. First, I use two points to find m. Then I use one point to find b. Let's start with m. I am going to choose this point and this point. *Mark a point at (3,40) and (7,35).*

Now I plug those values into y_2-y_1 over x_2-x_1. So far, this is exactly like what we've done before. Now here is the tricky part. This is where you really need to pay attention. It is tempting to put 40-35 because we are used to subtracting from the bigger number. But (3,40) is x_1 and y_1. It is the first point. We want to start with y_2. That's the second point. So we need to write 35 minus 40 NOT 40 minus 35. That's 35 minus 40 over 7 minus 3. This is important because this is how we end up getting that negative answer. 35 minus 40 is NEGATIVE 5 because I don't have enough. I have to think of it as being able to subtract 35 and still having 5 left to subtract. That's negative 5. I get -5 over 4.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{35-40}{7-3} = \frac{-5}{4}$$

$$\begin{array}{r} 1\tfrac{1}{4} \\ 4\overline{)\,5} \\ -4 \\ \hline 1 \end{array}$$

I divide this over to the side of my paper, and it is NEGATIVE 1 and 1 fourth. M is negative which makes sense because m is the slope and I can see that this slope is going down; it's negative.

$$\begin{aligned} y &= mx + b \\ 40 &= -1\tfrac{1}{4}(3) + b \\ 40 &= -3\tfrac{3}{4} + b \\ +3\tfrac{3}{4} & \quad +3\tfrac{3}{4} \\ \boxed{43\tfrac{3}{4} = b} \end{aligned}$$

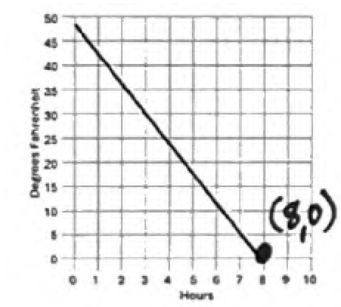
Next I am going to find b using y = mx + b. I have to fill in all the other values to solve for b. I need 1 point for this. It doesn't matter which one. I'll use (3,40). So y is 40, which equals NEGATIVE 1 and 1 fourth times 3 plus b.I have to multiply each part of my mixed number so 1 and 1 fourth times 3 is 1 times 3, which is 3, and 1 fourth times 3, which is 3 fourths. But this was negative! So this is really 40 equals NEGATIVE 3 and 3 fourths plus b. To get b on its own, I have to ADD 3 and 3 fourths to both sides. I'm not subtracting this time because I'm doing the opposite. That means adding this time. Now I have 43 and 3 fourths equals b.

$$y = -1\tfrac{1}{4}x + 43\tfrac{3}{4}$$

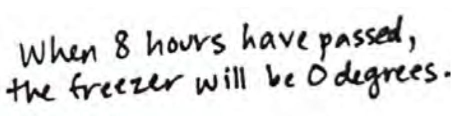
Let's review. I did step one, which was to use two points to find m. *Point to the work for step one.* Then I did step two, which was to use one point to find b. *Point to the work for step two.* Now I can write my final equation: y = NEGATIVE 1 and 1 fourth x plus 43 and 3 fourths. I still did the same two big steps there with a lot of itty bitty steps to crunch the numbers. Great job!

Let's Think (Slide 5): There's one more big idea that we need to learn for linear relationships with negative slopes. This says, "With negative slopes, sometimes our graphs will have x-intercepts." Whoa! This says x-intercepts not y-intercepts. We know that y-intercepts are where the line crosses the y-axis and x = 0. So what do you think x-intercepts are? Possible Student Answers, Key Points:

- X-intercepts are where the line crosses the x-axis.
- X-intercepts are where y = 0.



X-intercepts are where the line crosses the x-axis so they are where y = 0. I am going to mark a point at the x-intercept on this graph. You can see that it is all the way down here, y isn't there at all. The y is zero. *Put a point at (8,0).* I can see that it is super close to 8. I'm not sure that it is exactly 8 but it is pretty close. Let's think about what this means. I am going to write the coordinates so we can add words and use "when" and "then" in our sentences. It is (8,0).



I can write, "When 8 hours have passed, the freezer will be 0 degrees." It will be frozen! This is helpful. If the y-intercept helps us understand when the graph starts. Then it's kind of like the x-intercept helps us understand when the graph stops.

Let's Try It (Slide 6): Let's write equations for graphs with negative slopes together. I will walk you through each step.

WARM WELCOME

Today we will write an equation and find the x-intercept for linear relationships with negative rates of change.

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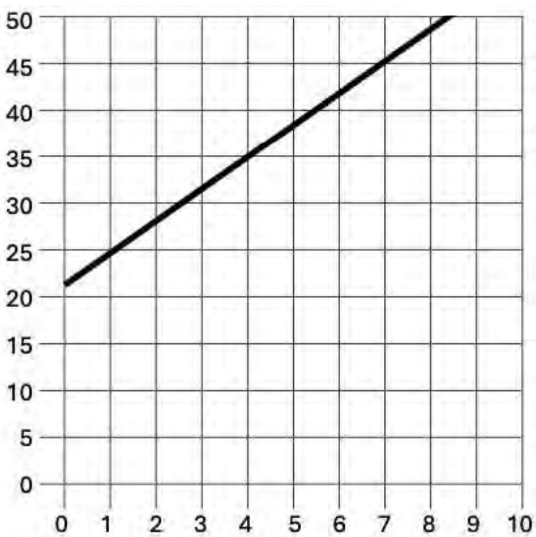
Let's Review:

We know how to write an equation when the slope of a graph is positive.

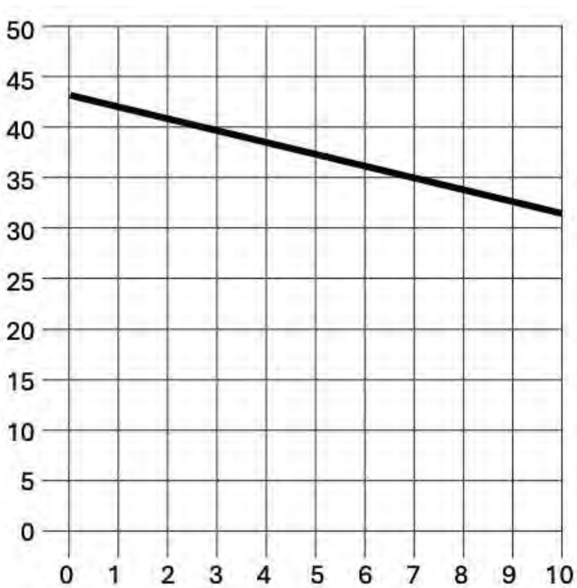
Write an equation for the line.



Let's Talk:

Linear relationships with a negative slope follow the same rule as all linear relationships.

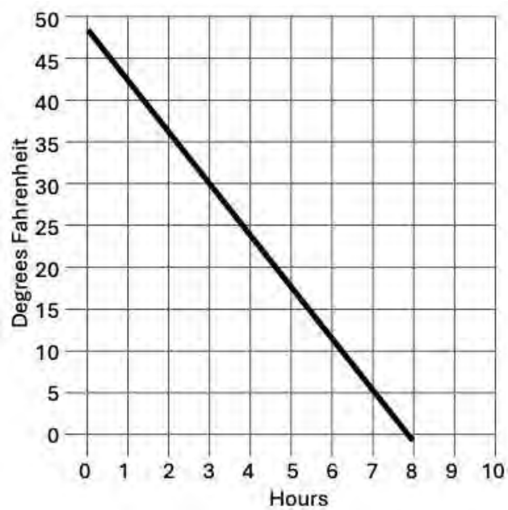
Write an equation for the line.



Let's Think:

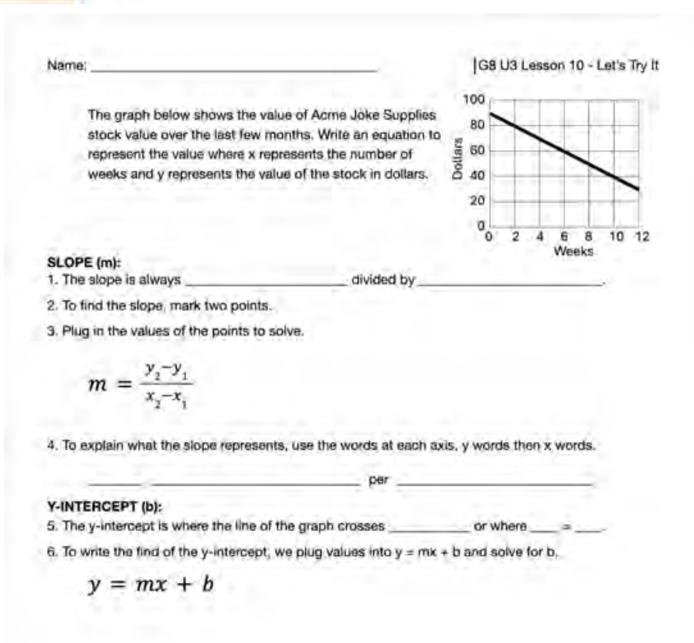
With negative slopes, sometimes our graphs will have x-intercepts.

Chef Harvey graphed the temperature each minute after he fixed his freezer. Find the x-intercept and explain what it represents in the context of the story.



Let's Try It:

Let's find y-intercepts and equations for graphs together!



Name: ______________________ | G8 U3 Lesson 10 - Let's Try It

The graph below shows the value of Acme Joke Supplies stock value over the last few months. Write an equation to represent the value where x represents the number of weeks and y represents the value of the stock in dollars.

SLOPE (m):

1. The slope is always ______________ divided by ______________.
2. To find the slope, mark two points.
3. Plug in the values of the points to solve.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

4. To explain what the slope represents, use the words at each axis, y words then x words.

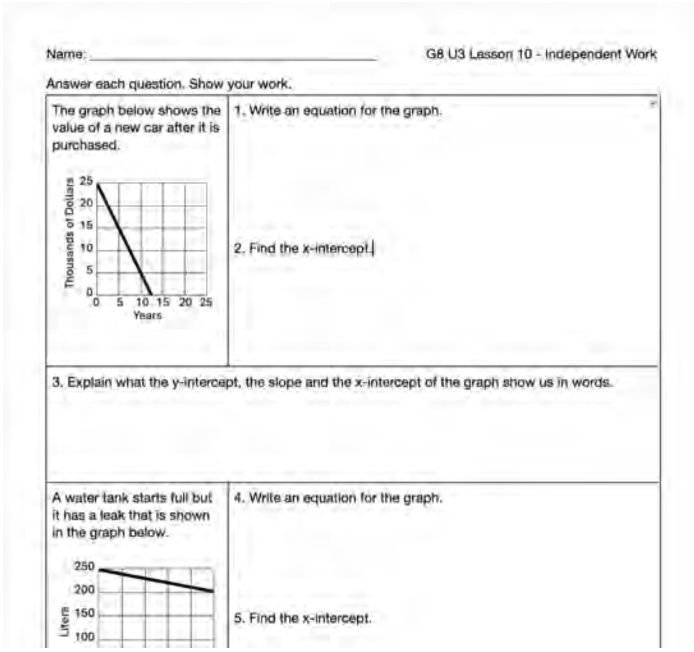
______________ per ______________

Y-INTERCEPT (b):

5. The y-intercept is where the line of the graph crosses __________ or where ___ = ___.
6. To write the find of the y-intercept, we plug values into y = mx + b and solve for b.

$$y = mx + b$$

On your Own: Now it's time for you to do it on your own!

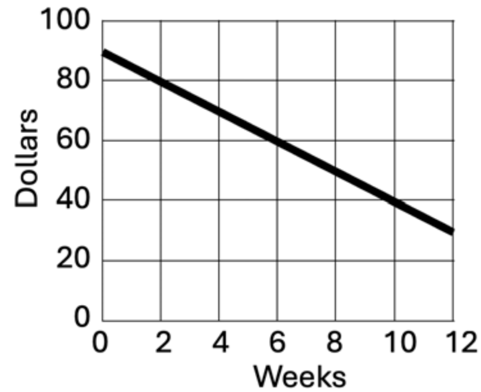


Name: ______________________ G8 U3 Lesson 10 - Independent Work

Answer each question. Show your work.

The graph below shows the value of a new car after it is purchased.	1. Write an equation for the graph. 2. Find the x-intercept.
3. Explain what the y-intercept, the slope and the x-intercept of the graph show us in words.	
A water tank starts full but it has a leak that is shown in the graph below.	4. Write an equation for the graph. 5. Find the x-intercept.

The graph below shows the value of Acme Joke Supplies stock value over the last few months. Write an equation to represent the value where x represents the number of weeks and y represents the value of the stock in dollars.



SLOPE (m):

1. The slope is always ______________________ divided by _________________________.
2. To find the slope, mark two points.
3. Plug in the values of the points to solve.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

4. To explain what the slope represents, use the words at each axis, y words then x words.

_______ _____________________________ per ___________________________

Y-INTERCEPT (b):

5. The y-intercept is where the line of the graph crosses ___________ or where ____ = ____.
6. To write the find of the y-intercept, we plug values into y = mx + b and solve for b.

$$y = mx + b$$

7. To explain what the y-intercept represents, put your answer to #6 into a complete sentence.

____

____

8. Use the answers you found to write an equation in y = mx + b form. _________________________

X-INTERCEPT:

9. The x-intercept is where the line of the graph crosses ____________ or where ____ = ____.

10. To write the find of the x-intercept, we plug y = 0 into the equation we made and solve for x.

$$y = mx + b$$

11. To explain what the x-intercept represents, put your answer to #10 into a complete sentence.

____

____

12. Explain what the y-intercept, the slope and the x-intercept of the graph show us in words. Put all the ideas from #4, #7 and #11 into one answer.

____

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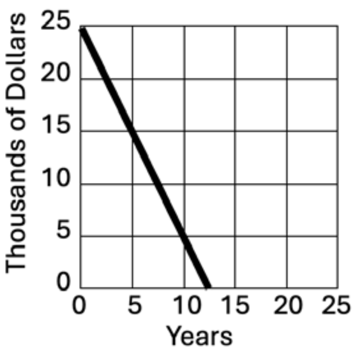
Name: ____ G8 U3 Lesson 10 - Independent Work

Answer each question. Show your work.

The graph below shows the value of a new car after it is purchased.

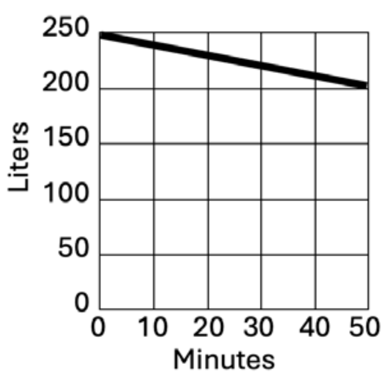


1. Write an equation for the graph.

2. Find the x-intercept.

3. Explain what the y-intercept, the slope and the x-intercept of the graph show us in words.

A water tank starts full but it has a leak that is shown in the graph below.



4. Write an equation for the graph.

5. Find the x-intercept.

6. Explain what the y-intercept, the slope and the x-intercept of the graph show us in words.

Solve.

Sara used the table below to keep track of her money over the summer weeks. Let x equal the number of weeks and y equal the number of dollars.

x	*y*
2	9
4	6
6	3
8	0

7. Write an equation for the table.

8. Find the x-intercept.

9. Explain what the y-intercept, the slope and the x-intercept of the table show us in words.

The amount of gas in Nolan's car during his trip to Florida is shown below.

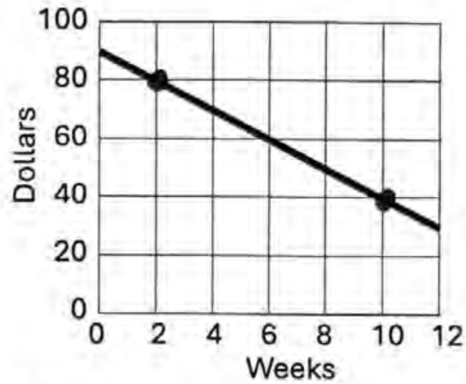
10. Write an equation for the graph.

11. Find the x-intercept.

12. Explain what the y-intercept, the slope and the x-intercept of the table show us in words.

Name: ANSWER KEY

The graph below shows the value of Acme Joke Supplies stock value over the last few months. Write an equation to represent the value where x represents the number of weeks and y represents the value of the stock in dollars.



SLOPE (m):

1. The slope is always $y_2 - y_1$ divided by $x_2 - x_1$.
2. To find the slope, mark two points.
3. Plug in the values of the points to solve.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{40-80}{10-2} = \frac{-40}{8} = -5$$

4. To explain what the slope represents, use the words at each axis, y words then x words.

-5 dollars per week

Y-INTERCEPT (b):

5. The y-intercept is where the line of the graph crosses y-axis or where x = 0.
6. To write the find of the y-intercept, we plug values into y = mx + b and solve for b.

$$y = mx + b$$
$$80 = -5(2) + b$$
$$80 = -10 + b$$
$$+10 \quad +10$$
$$\boxed{90 = b}$$

7. To explain what the y-intercept represents, put your answer to #6 into a complete sentence.

When 0 weeks have passed, the value is $90.

8. Use the answers you found to write an equation in y = mx + b form. $y = -5x + 90$

X-INTERCEPT:

9. The x-intercept is where the line of the graph crosses x-axis or where y = 0.

10. To write the find of the x-intercept, we plug y = 0 into the equation we made and solve for x.

$$y = mx + b$$

$$0 = -5(x) + 90$$

$$-90 \qquad -90$$

$$\frac{-90}{-5} = \frac{-5x}{-5}$$

$$18 = x$$

11. To explain what the x-intercept represents, put your answer to #10 into a complete sentence.

When 18 weeks have passed, the stock will be worth $0.

12. Explain what the y-intercept, the slope and the x-intercept of the graph show us in words. Put all the ideas from #4, #7 and #11 into one answer.

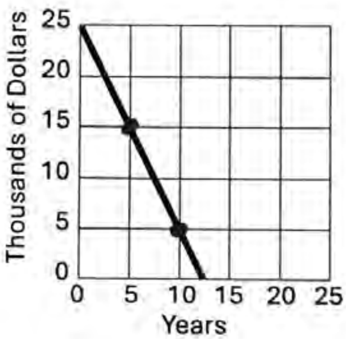
The stock will start with a value of $90. It will decrease $5 per week. Then after 18 weeks, it will be worth $0.

Name: ANSWER KEY

G8 U3 Lesson 10 - Independent Work

Answer each question. Show your work.

The graph below shows the value of a new car after it is purchased.



1. Write an equation for the graph.

$m = \frac{5-15}{10-5} = \frac{-10}{5} = \boxed{-2}$

$y = -2x + 25$

$y = mx + b$

$15 = -2(5) + b$

$15 = -10 + b$

$+10 \quad +10$

$\boxed{25 = b}$

2. Find the x-intercept.

$y = -2x + 25$

$0 = -2x + 25$

$-25 \quad -25$

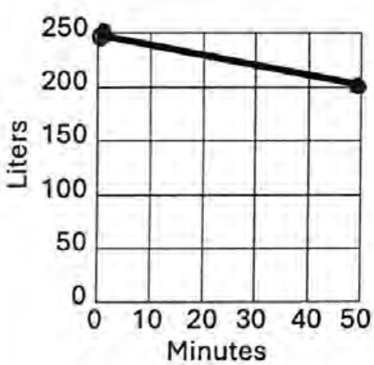
$\frac{-25}{-2} = \frac{-2x}{-2}$

$x = \boxed{12\frac{1}{2}}$

3. Explain what the y-intercept, the slope and the x-intercept of the graph show us in words.

The car starts at a value of 25 thousand dollars. Its value decreases 2 thousand per years. After $12\frac{1}{2}$ years, it will be worth 0 dollars.

A water tank starts full but it has a leak that is shown in the graph below.



4. Write an equation for the graph.

$m = \frac{200-250}{50-0} = \frac{-50}{50} = \boxed{-1}$

$y = -1x + 250$

$y = mx + b$

$250 = -1(0) + b$

$\boxed{250 = b}$

5. Find the x-intercept.

$y = -1x + 250$

$0 = -1x + 250$

$-250 \quad -250$

$\frac{-250}{-1} = \frac{-1x}{-1}$

$x = \boxed{250}$

6. Explain what the y-intercept, the slope and the x-intercept of the graph show us in words.

The tank holds 250 Liters. It leaks out 1 Liter per minute. After 250 minutes, it will have 0 Liters.

Solve.

Sara used the table below to keep track of her money over the summer weeks. Let x equal the number of weeks and y equal the number of dollars.

x	y
2	9
4	6
6	3
8	0

7. Write an equation for the table.

$m = \frac{6-9}{4-2} = \frac{-3}{2} = -1\frac{1}{2}$

$y = mx + b$

$9 = -\frac{3}{2}(2) + b$

$9 = -3 + b$

$+3 \quad +3$

$12 = b$

$y = -1\frac{1}{2}x + 12$

8. Find the x-intercept.

$y = -1\frac{1}{2}x + 12$

$0 = -1\frac{1}{2}x + 12$

$-12 \quad -12$

$-12 = -\frac{3}{2}x \quad \div -\frac{2}{3}$

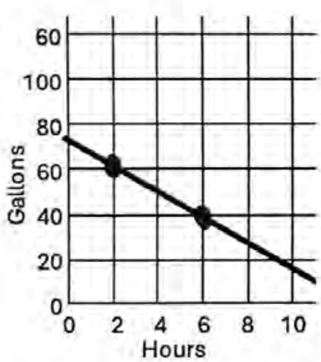
$\frac{24}{3} = x$

$8 = x$

9. Explain what the y-intercept, the slope and the x-intercept of the table show us in words.

Sara started with $12. Each week she lost $1½ dollars. After 8 weeks, she had $0.

The amount of gas in Nolan's car during his trip to Florida is shown below.



10. Write an equation for the graph.

$m = \frac{40-60}{6-2} = \frac{-20}{4} = -5$

$y = mx + b$

$60 = -5(2) + b$

$60 = -10 + b$

$+10 \quad +10$

$70 = b$

$y = -5x + 70$

11. Find the x-intercept.

$y = -5x + 70$

$0 = -5x + 70$

$-70 \quad -70$

$\frac{-70}{-5} = \frac{-5x}{-5}$

$70 \div 5 = 14$

$x = +14$

12. Explain what the y-intercept, the slope and the x-intercept of the table show us in words.

The car started with 70 gallons. It uses 5 gallons per hour. At 14 hours, it will have 0 gallons.

G8 U3 Lesson 11
Write equations for horizontal and vertical lines.

CITYTUTOR DC

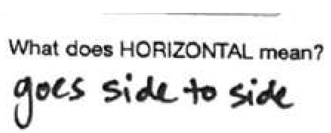
G8 U3 Lesson 11 - Today we will write equations for horizontal and vertical lines.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will write equations for horizontal and vertical lines. We are going to use the ideas we've built over the whole unit to notice a pattern, and then you will able to easily use graphs, equations and tables for these sorts of lines.

Let's Review (Slide 3): This says, "Diagonal straight lines show linear relationships. Horizontal and vertical lines do not show linear relationships." We have been working on linear relationships shown by diagonal straight lines. But today we're going to talk about these other kind of lines. What does HORIZONTAL mean? Possible Student Answers, Key Points:

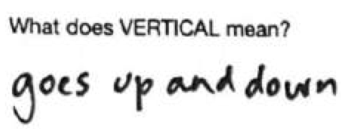
- It means going side to side.
- It is a line that goes across.



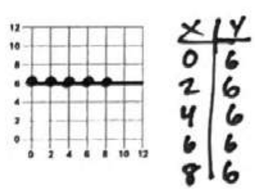
A horizontal line goes side to side. We can remember it by thinking of the "horizon." That's the line of the earth when we look far away. It goes side to side.

What does VERTICAL mean? Possible Student Answers, Key Points:

- It means going up and down.



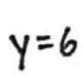
A vertical line goes up and down. One trick that people have used to remember is thinking of the v as pointing down. There used to be a wresting move called the "vertical supplex" which involved lifting someone vertically. These are the lines that we are going to represent today.



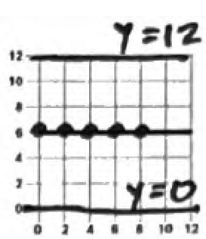
Let's Talk (Slide 4): A table helps us see how to write an equation for horizontal or vertical lines. This says, "Make a table for each graph. Then write an equation." I will draw a table and write the labels x and y. I am going to mark some points along the line. Let's fill them in. I see (0,6) and (2,6) and (4,6) and (6,6) and (8,6) and we can see how it will keep going.

What do you notice about this table? Possible Student Answers, Key Points:

- The right column is always 6.
- There is the same number for y every time.

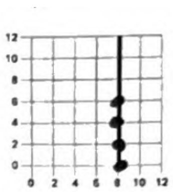


Every single number in this column is a 6. That means y is always a 6. Think to yourself how you might write an equation if y is always 6. *Give the kids 15 seconds of think time without letting anyone call out.* It is almost too easy. If y is always 6 then we write y = 6. This means that x can be anything but y is always 6 which is exactly what we saw in the table.



And guess what? If I drew other horizontal lines, they would always be y equals something. Here I am drawing a line of (0,12), (2,12), (4,12) and y = 12. Or here I am drawing a line of (0,0), (2,0), (4,0) and y = 0. You can memorize that horizontal lines will always be y equals something. But you can also always make a table to remind yourself.

You can probably guess what the equation for a vertical line might always be. But let's use that table first. I am going to draw the table with x and y. Then I will mark some points on the line. I will record (8,0) then (8,2) then (8,4) then (8,6) and this would keep going all the way to 8 and infinity! What do you notice about this table? Possible Student Answers, Key Points:



x	y
8	0
8	2
8	4
8	6

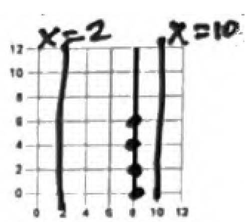
- The left column is always 8.
- There is the same number for every time.

Every single number in this column is a 8. That means x is always a 8.

x=8

Think to yourself how you might write an equation if x is always 8. *Give the kids 5 seconds of think time without letting anyone call out.* It is almost too easy. If x is always 8 then we write x = 8. This means that y can be anything but x is always 8 which is exactly what we saw in the table.

And once again, if I draw similar lines, they would always be x equals something. Here I am drawing a line of (2,0), (2,2), (2,4) and x = 2. Or here I am drawing a line of (10,0), (10,2), (10,4) and x = 10. You can memorize that vertical lines will always be x equals something. But you can also always make a table to remind yourself. One thing to notice is that the "x equals" lines don't go the same way as the x-axis. They intersect the x axis right at the number it equals. The "y equals" lines also don't go the same way as the y-axis. Instead, they intersect the y axis right at the number it equals. That's just another way to help you remember which equation goes which way. But you can always make a table.

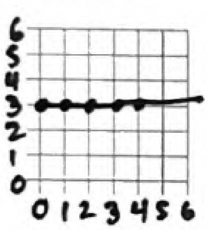


x	y
	3
	3
	3
	3
	3

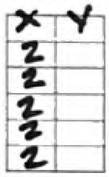
Let's Think (Slide 5): We can also draw a graph from a table or equation. Now, we just talked about ways of memorizing which ways the lines go depending on whether they are "x equals" or "y equals" but it is easy to forget that they go the other way as their axis. So, I like to just do a quick table and then it becomes super obvious. This says, "Make a table and graph for y = 3." I put x and y at the top of the table, and since it says that y = 3 I am just going to put a 3 all the way down the y column.

x	y
0	3
1	3
2	3
3	3
4	3

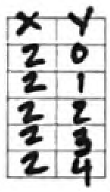
It doesn't matter what x equals. X could be 0 or x could be 1 or x could be 2 or 3 or 4.



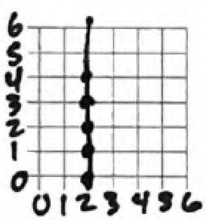
I am going to quickly put numbers on these axes. When I graph (0,3), (1,3), (2,3), (3,3) it becomes obvious that I am getting a horizontal line.



Let’s do this next one, x = 2. I put x and y at the top of the table, and since it says that x = 2 I am just going to put a 3 all the way down the x column.



It doesn’t matter what y equals. Y could be 0 or y could be 1 or y could be 2 or 3 or 4.



I am going to quickly put numbers on these axes. When I graph (2,0), (2,1), (2,2), (2,3) it becomes obvious that I am getting a vertical line. The big idea here is the same one as the whole units. We learned the acronym GETS because a graph, equation, table and story can all be equivalent ways to see the same relationship. For these special lines, it is especially useful to do a quick table to make sure you see the trend.

Let’s Try It (Slide 6): Let’s represent more horizontal and vertical lines together. I will walk you through each step.

WARM WELCOME

Today we will write equations for horizontal and vertical lines.

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Let's Review:

Diagonal straight lines show linear relationships. Horizontal and vertical lines do not show linear relationships.

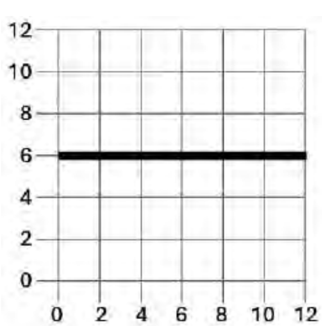
What does HORIZONTAL mean?

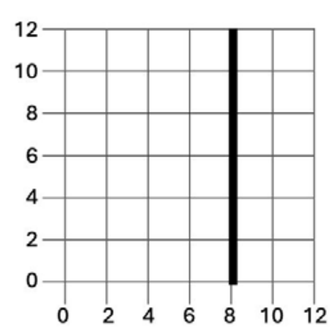
What does VERTICAL mean?

Let's Talk:

A table helps us see how to write an equation for horizontal or vertical lines.

Make a table for each graph. Then write an equation.

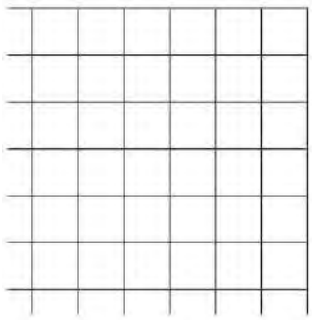




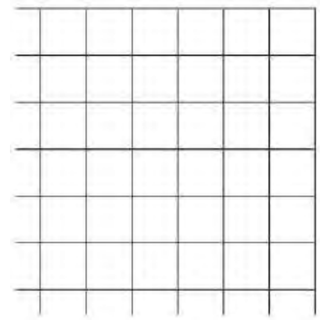
Let's Think:

We can also draw a graph from a table or equation.

Make a table and graph for y = 3.

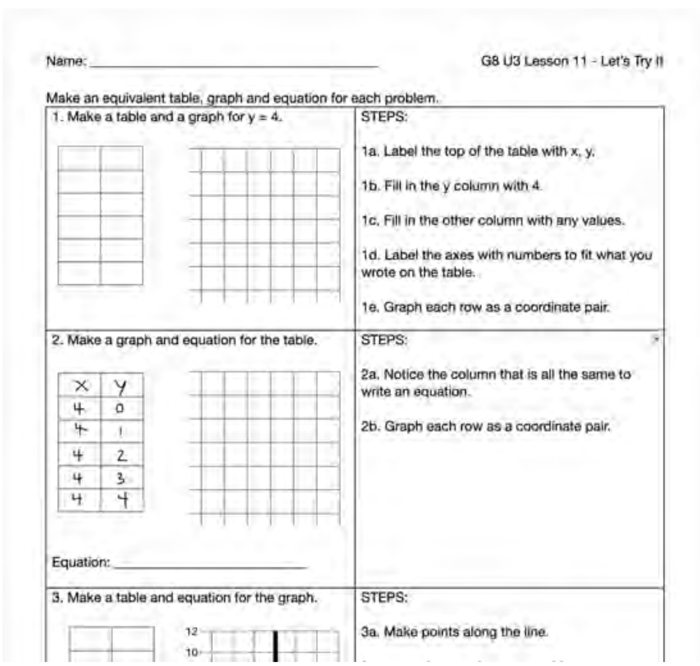


Make a table and graph for x = 2.



Let's Try It:

Let's find y-intercepts and equations for graphs together!



Name: ____________________ G8 U3 Lesson 11 - Let's Try It

Make an equivalent table, graph and equation for each problem.

1. Make a table and a graph for y = 4.

STEPS:

1a. Label the top of the table with x, y.

1b. Fill in the y column with 4.

1c. Fill in the other column with any values.

1d. Label the axes with numbers to fit what you wrote on the table.

1e. Graph each row as a coordinate pair.

2. Make a graph and equation for the table.

x	y
4	0
4	1
4	2
4	3
4	4

Equation: ________________

STEPS:

2a. Notice the column that is all the same to write an equation.

2b. Graph each row as a coordinate pair.

3. Make a table and equation for the graph.

12

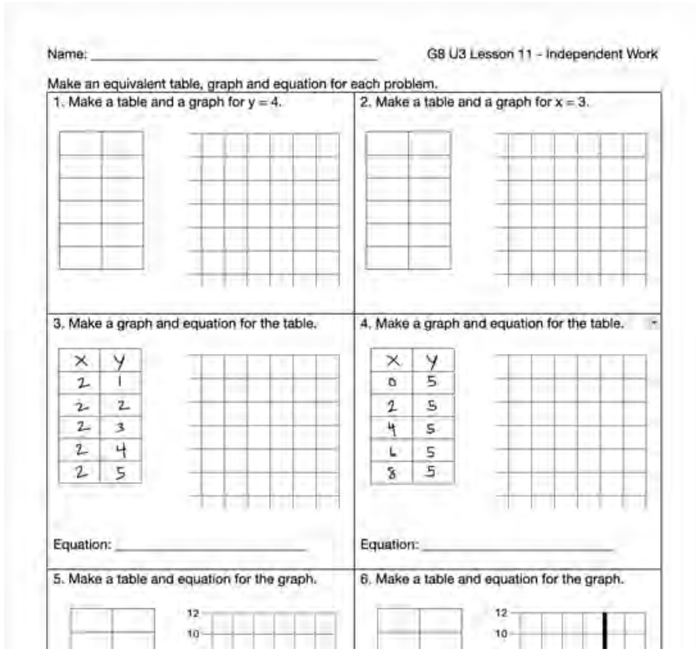
10

STEPS:

3a. Make points along the line.

On your Own:

Now it's time for you to do it on your own!



Name: ______________________ G8 U3 Lesson 11 - Independent Work

Make an equivalent table, graph and equation for each problem.

1. Make a table and a graph for y = 4.

2. Make a table and a graph for x = 3.

3. Make a graph and equation for the table.

x	y
2	1
2	2
2	3
2	4
2	5

Equation: ______________________

4. Make a graph and equation for the table.

x	y
0	5
2	5
4	5
6	5
8	5

Equation: ______________________

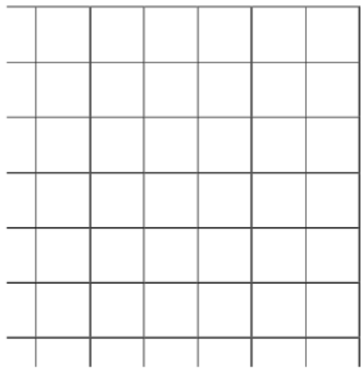
5. Make a table and equation for the graph.

6. Make a table and equation for the graph.

Name: ____ G8 U3 Lesson 11 - Let’s Try It

Make an equivalent table, graph and equation for each problem.

1. Make a table and a graph for y = 4.



STEPS:

1a. Label the top of the table with x, y.

1b. Fill in the y column with 4.

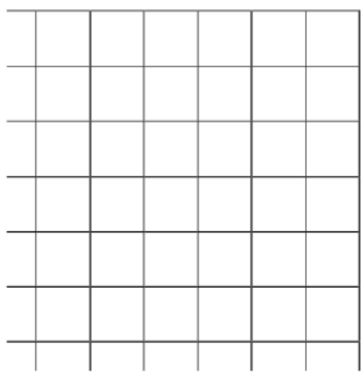
1c. Fill in the other column with any values.

1d. Label the axes with numbers to fit what you wrote on the table.

1e. Graph each row as a coordinate pair.

2. Make a graph and equation for the table.

x	y
4	0
4	1
4	2
4	3
4	4



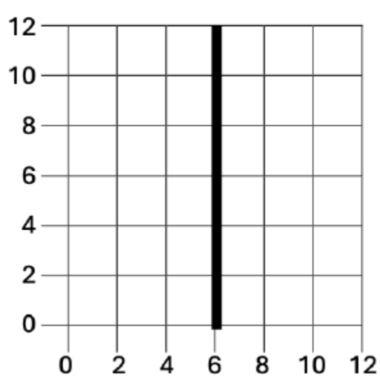
Equation: ____________________________

STEPS:

2a. Notice the column that is all the same to write an equation.

2b. Graph each row as a coordinate pair.

3. Make a table and equation for the graph.



Equation: ____________________________

STEPS:

3a. Make points along the line.

3b. Label the top of the table with x, y.

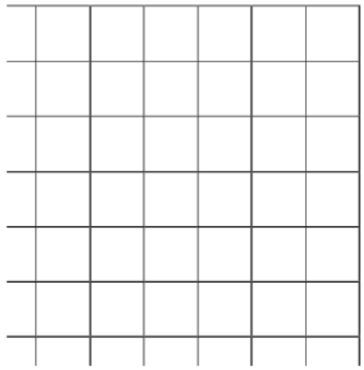
3c. Use the points you marked to fill in the table.

3d. Notice the column that is all the same to write an equation.

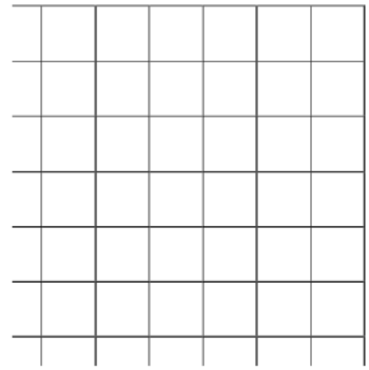
Name: ____

Make an equivalent table, graph and equation for each problem.

1. Make a table and a graph for y = 4.

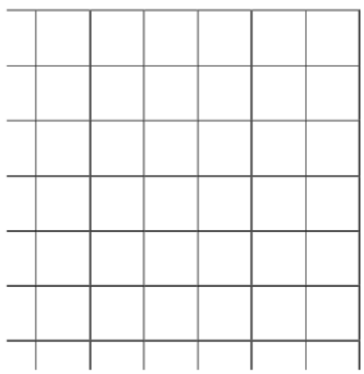


2. Make a table and a graph for x = 3.



3. Make a graph and equation for the table.

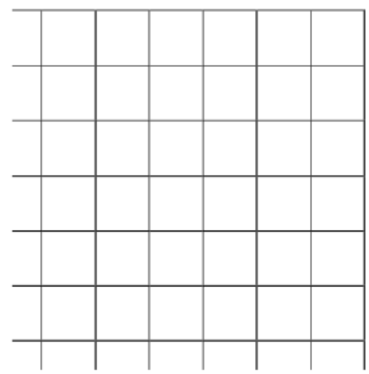
x	y
2	1
2	2
2	3
2	4
2	5



Equation: ____________________________

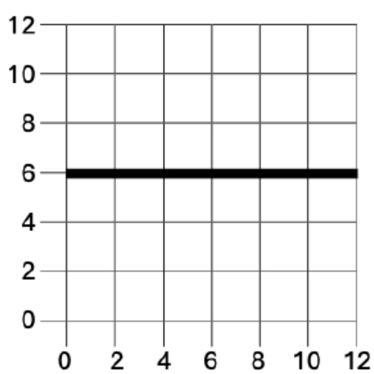
4. Make a graph and equation for the table.

x	y
0	5
2	5
4	5
6	5
8	5



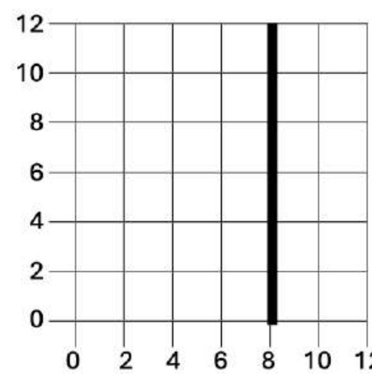
Equation: ____________________________

5. Make a table and equation for the graph.



Equation: ____________________________

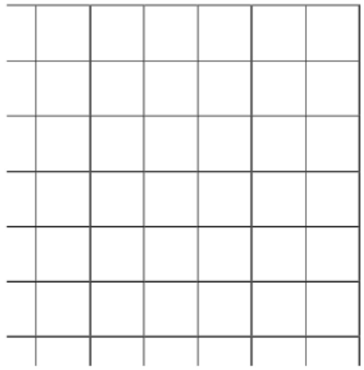
6. Make a table and equation for the graph.



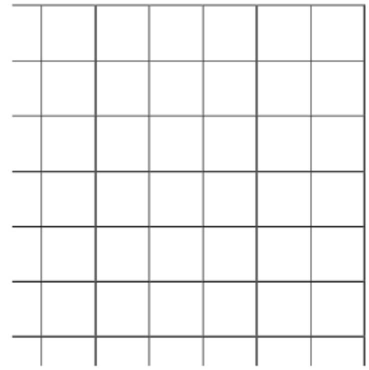
Equation: ____________________________

Make an equivalent table, graph and equation for each problem.

7. Make a table and a graph for y = 3.

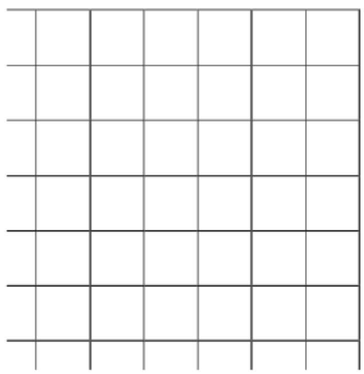


8. Make a table and a graph for x = y.



9. Make a graph and equation for the table.

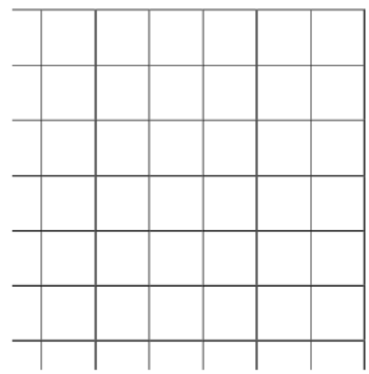
x	y
2	4
3	4
4	4
5	4
6	4



Equation: ______________________________

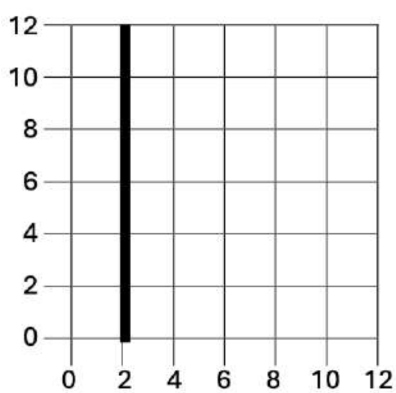
10. Make a graph and equation for the table.

x	y
1	0
1	1
1	2
1	3
1	4



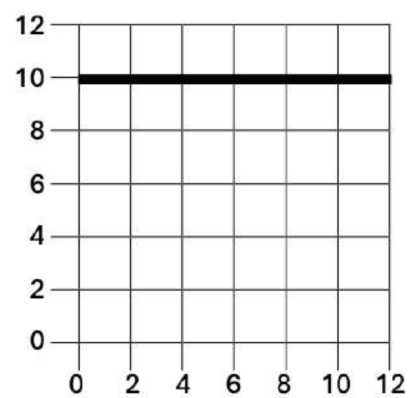
Equation: ______________________________

11. Make a table and equation for the graph.



Equation: ______________________________

12. Make a table and equation for the graph.



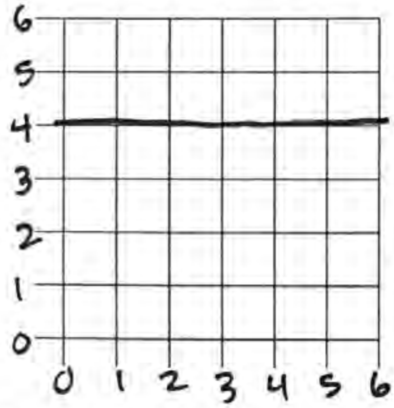
Equation: ______________________________

Name: ANSWER KEY

Make an equivalent table, graph and equation for each problem.

1. Make a table and a graph for $y = 4$.

x	y
0	4
1	4
2	4
3	4
4	4



STEPS:

1a. Label the top of the table with x, y.

1b. Fill in the y column with 4.

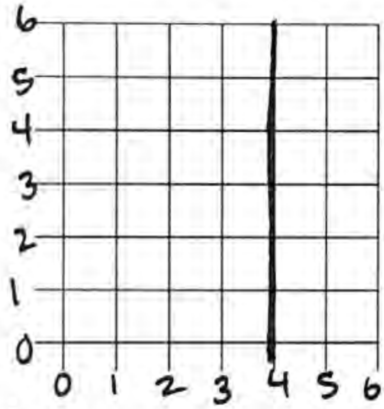
1c. Fill in the other column with any values.

1d. Label the axes with numbers to fit what you wrote on the table.

1e. Graph each row as a coordinate pair.

2. Make a graph and equation for the table.

x	y
4	0
4	1
4	2
4	3
4	4



Equation: $x = 4$

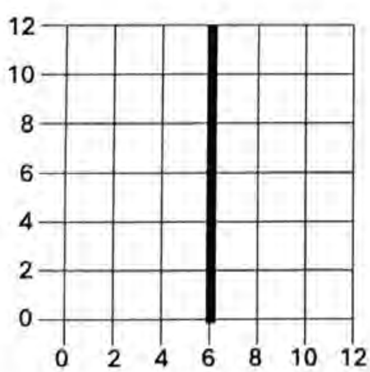
STEPS:

2a. Notice the column that is all the same to write an equation.

2b. Graph each row as a coordinate pair.

3. Make a table and equation for the graph.

x	y
6	0
6	2
6	4
6	6
6	8



Equation: $x = 6$

STEPS:

3a. Make points along the line.

3b. Label the top of the table with x, y.

3c. Use the points you marked to fill in the table.

3d. Notice the column that is all the same to write an equation.

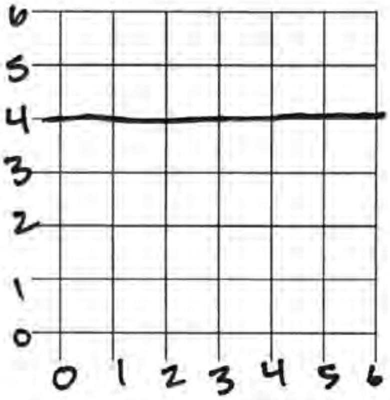
Name: ANSWER KEY

G8 U3 Lesson 11 - Independent Work

Make an equivalent table, graph and equation for each problem.

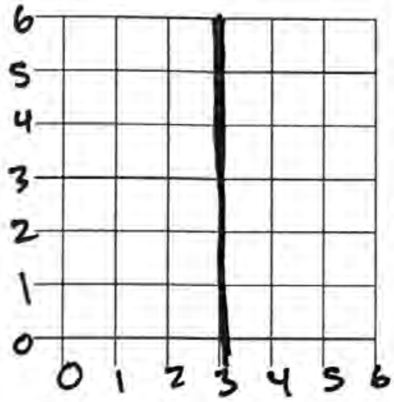
1. Make a table and a graph for y = 4.

x	y
0	4
1	4
2	4
3	4
4	4



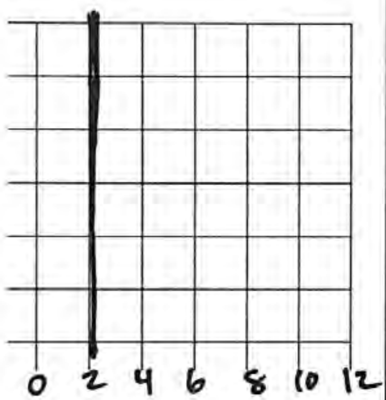
2. Make a table and a graph for x = 3.

x	y
3	0
3	1
3	2
3	3
3	4



3. Make a graph and equation for the table.

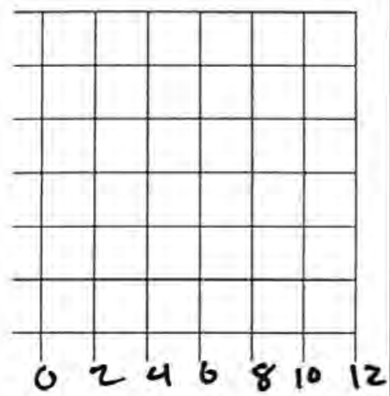
x	y
2	1
2	2
2	3
2	4
2	5



Equation: x = 2

4. Make a graph and equation for the table.

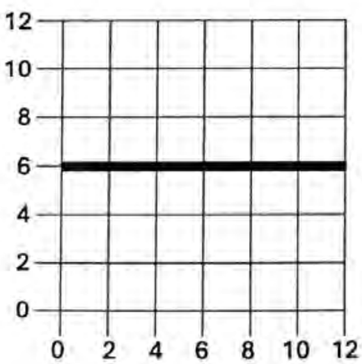
x	y
0	5
2	5
4	5
6	5
8	5



Equation: y = 5

5. Make a table and equation for the graph.

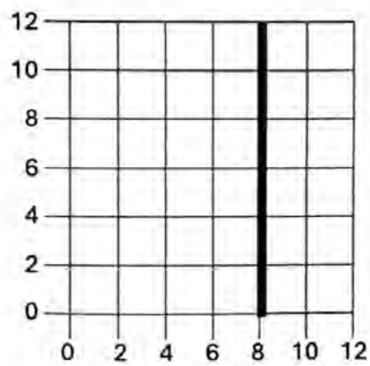
x	y
0	6
2	6
4	6
6	6
8	6



Equation: y = 6

6. Make a table and equation for the graph.

x	y
8	0
8	2
8	4
8	6
8	8

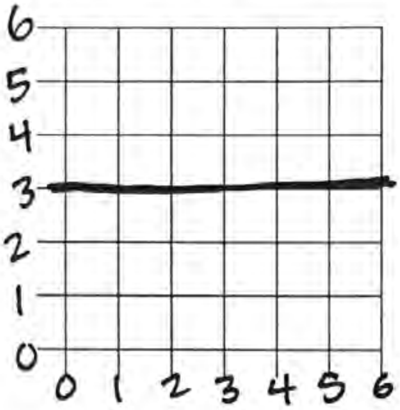


Equation: x = 8

Make an equivalent table, graph and equation for each problem.

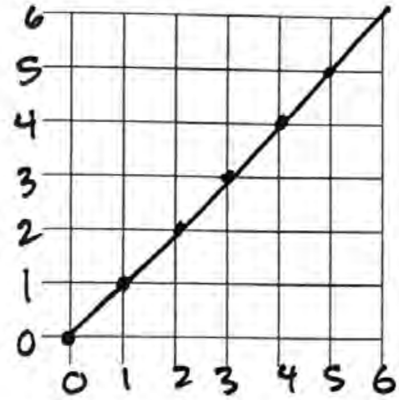
7. Make a table and a graph for y = 3.

x	y
0	3
1	3
2	3
3	3
4	3



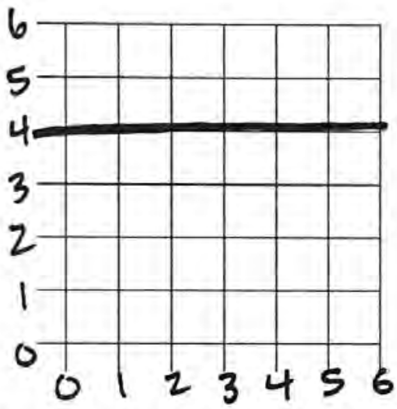
8. Make a table and a graph for x = y.

x	y
0	0
1	1
2	2
3	3
4	4



9. Make a graph and equation for the table.

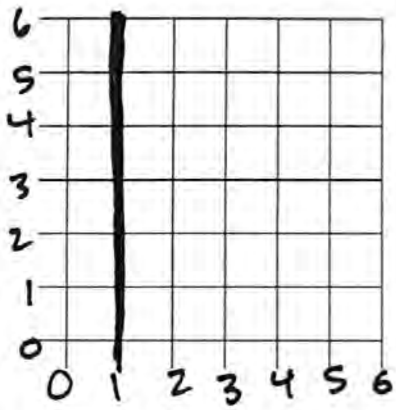
x	y
2	4
3	4
4	4
5	4
6	4



Equation: y = 4

10. Make a graph and equation for the table.

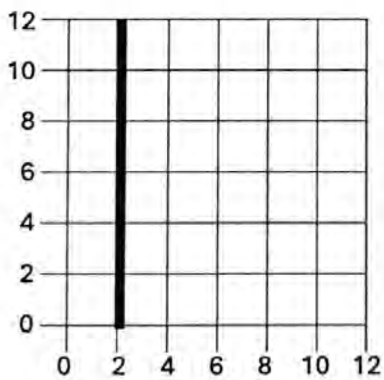
x	y
1	0
1	1
1	2
1	3
1	4



Equation: x = 1

11. Make a table and equation for the graph.

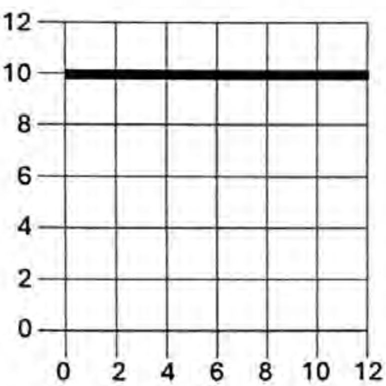
x	y
2	0
2	2
2	4
2	6
2	8



Equation: x = 2

12. Make a table and equation for the graph.

x	y
0	10
2	10
4	10
6	10
8	10



Equation: y = 10

Eighth Grade Math Lesson Materials

G8 Unit 4:

Rational Number Arithmetic

G8 U4 Lesson 1
Solve linear equations by performing balanced moves on both sides.

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G8 U4 Lesson 1 - Today we will solve linear equations by performing balanced moves on both sides.

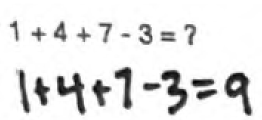
Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will solve linear equations by performing balanced moves on both sides. We've already been doing some of this in our last unit. You did some of this in previous grades. But today we need to review the main concept of why we do it the way we do. Because in the next lessons, it gets really sophisticated. So, let's make sure we pay close attention to not just getting the right answer but also understanding why our process works so that we have a strong foundation for the rest of this unit.

Let's Review (Slide 3): It is helpful to notice that we think of the equal sign in two different ways. I'll show you what this means with the equations below. This says, "Simplify each equation to find the value of the question mark." How do you simplify this first equation? Possible Student Answers, Key Points:

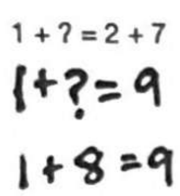
- You just do the math.
- You do 1 plus 4 is 5 plus 7 is 12 minus 3 is 9.
- You add then subtract.

For an equation like this, where all the operations are on one side and the question mark is on the other, we can just crunch the number. 1 plus 4 is 5. 5 plus 7 is 12. 12 minus 3 is 9. So, 1 + 4 + 7 - 3 equals 9. When we see the equal sign here, we almost think of it as a direction to "DO THE MATH!" In elementary school, this is the main way we think about the equal sign because there is so much pure number crunching. There's nothing wrong with it. But it is only one perspective. There is another perspective.



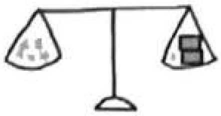
How might you think about simplifying this next equation? Possible Student Answers, Key Points:

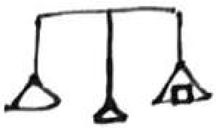
- You add 2 + 7 and get 9.
- You have to think about how to make 9 since 2 plus 7 is 9.
- The answer is 8 because 1 plus 8 is 9, and 2 + 7 is 9.



For an equation like this, it is not as simple as "DO THE MATH!" For an equation like this, we SHIFT OUR PERSPECTIVE and think about the equal sign as a balance. It says, "This side is the same as this side." So, I might still number crunch but I keep the two sides separate, and I am thinking how to keep them the same or equal to each other. I could rewrite the equation as 1 plus question mark equals 9. And now it is obvious that the question mark is 8 because 1 plus 8 is also 9.

So, how did we shift our perspective of the equal sign in the two different equations? In one equation, we thought of the equal sign as something telling us to "DO THE MATH!" and in the other equation , we thought of the equal sign as something telling us, "This side is the same as this side." Both of these ways of thinking of the equal sign are correct. Sometimes one way is more helpful than the other. And for solving equations, it is the second perspective that is going to be helpful.

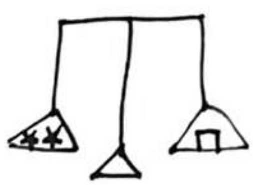




Let's Talk (Slide 4): Thinking of the equal sign as a balance helps us understand how to solve equations. So, this is the "this side is the same as this side" perspective. The question says, "If the scale below is balanced, how many stars must be hidden in one of the two equal boxes?" It would be really helpful if I had a balance with just one box, right? If it looked like this, I would know exactly how many stars were hidden in the box because the left side would equal the right side.

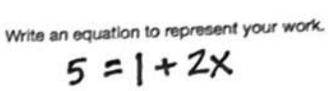
So, let's think about how we could get this scale to have just one box. But - and this is the key - we need it to stay balanced the whole time, right? That's how we know one side equals the other. I want to get one box by itself so I am going to start by taking away the star on the right. But if I cross out one star on this side then I have to cross out one star on the other side to keep it balanced. Whatever I do to one side I have to do to the other side so that it stays equal.

Now I have 4 stars is the same as 2 boxes. I want just one box. So I will split this right side of the graph into two parts. But if I split the right side into two parts then I need to split the left side into two parts to keep it balanced. Then I can see that there would be 2 stars in each box.

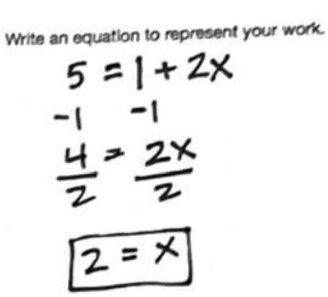


My scale in the bottom would be 2 stars equals the mystery box.

Now we aren't going to spend 8th grade drawing scales. This just shows us the idea of what happens when we solve equations because solving for a variable is like finding the amount in the mystery box. This says, "Write an equation to represent your work." This will help us see how what we did with the balance teaches us what to do with numbers. On the left side, I had 5 stars so I'm just going to write 5. And that equals whatever is on the other side. On the other side, I have 1 star so I'll write 1 plus I have these two mystery boxes. The mystery boxes are the unknown and I have two of them so I'm going to write that as 2x. X stands for an unknown amount and there are two unknown amounts so 2x.

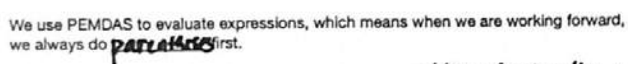


Now, when we tried to find the amount of stars in one box, that was like solving for x. And just like we'd like to get one box by itself on the balance, we want to get x by itself on one side of the equation. The first thing we did was take a star from both sides. I will write that as minus 1 on this side and minus 1 on this side. That left us with 4 stars balanced with 2 boxes, which in this case is 5 minus 1 makes 4 on the left and 1 minus 1 is zero so 2x is alone on the right. Then we split what was left by 2 so we'd just have 1 box. In our equation, that is the same as dividing by two on both sides. Again, whatever I do to one side, I do to the other to keep the equation balanced. That leaves us with 4 divided by 2, which is 2, on the left. On the right, 2 divided by 2 is 1 so we just have x. 2 equals x.

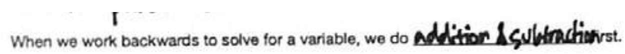


Let's take a moment to notice what we did here. The doing the same to both sides is obvious. The other thing we ended up doing is WORKING BACKWARDS from the operations that written in the expression. In the original expression, it was x times 2 plus 1. And then we worked backwards: minus 1 divided by 2. I kind of think of it like a zipper. The equation is all zipped up and then we pull the zipper down. *Make a motion like zipping a hoodie up then down.* Zip and unzip. The equation is zipped and we unzip, working backwards. And of course, as we unzip we do it to both sides to keep the equation balanced. This is the main idea of our work today.

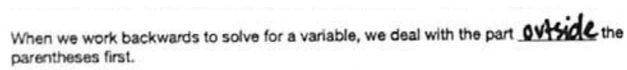
Let's Think (Slide 5): Here is the big key idea of the day, "To solve for a variable, we must work backwards from the order of operations while keeping the equation balanced." There are a few ideas that will help us. This is something we already know. *Read the first sentence.* "We used PEMDAS to evaluate expressions, which means when we are working forward, we always do…" PARENTHESES first.



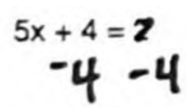
PEMDAS is the acronym for order of operations. P stands for parentheses. E stands for exponents. M stands for multiplication. D stands for division. Multiplication and division go together.*Circle the MD in PEMDAS.* A stands for addition. S stands for subtraction. Addition and subtraction go together. *Circle the AS in PEMDAS.* Now we just talked about zipping and unzipping. We just said that to solve for a variable, we're going to have to work backwards. "When we work backwards to solve for a variable, we do…" ADDITION AND SUBTRACTION first.



This also means "When we work backwards to solve for a variable, we deal with the part OUTSIDE the parentheses first." I'll do two examples and then we'll practice together.

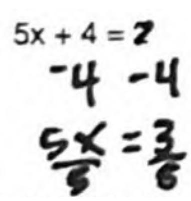


I have 5 times x plus 4 equals 7. I am going to start by working backwards from the addition and subtraction. So here is plus 4 and to cancel that out. I work backwards and do minus 4. I have to do that on both sides. The most important thing about this unit is that I actually show my work and keep it really carefully lined up and organized so that we can understand all the steps we did.



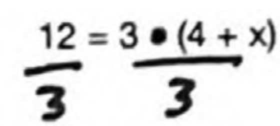
That gives me 5x = 3.

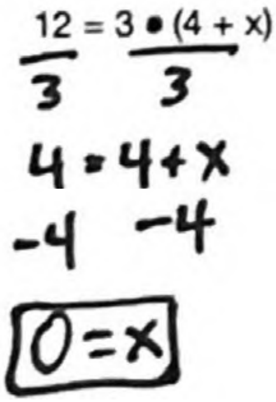
I want x by itself, and right now it is getting multiplied by 5. So I am going to cancel that out. I need to do the opposite operation like I am working backwards, I divide by 5 and I do that on both sides.



3 divided by 5 doesn't really make a nice even number so I am just going to leave that as a fraction. I will write x equals 3 fifths. I put a rectangle around it so I can follow the work on my paper.

Let's do the next one. Now, if we were evaluating this expression on the right, we would do the stuff in the parentheses first and then multiply it by 3. But we are trying to work backwards. So I will first divide by 3. That will cancel that 3 out and leave me with a simpler expression. I do that to both sides.





That gives me 4 equals 4 + x.

Now I subtract 4 from both sides and I am left with zero equals x.

Let's Try It (Slide 6): Let's solve more equations together. I will walk you through step by step.

WARM WELCOME

Today we will solve linear equations by performing balanced moves on both sides.

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Let's Review: It is helpful to notice that we think of the equal sign in two different ways.

Simplify each equation to find the value of the question mark.

$1 + 4 + 7 - 3 = ?$

$1 + ? = 2 + 7$

How did we shift our perspective of the equal sign in the two different equations?

Let's Talk: Thinking of the equal sign as a balance helps us understand how to solve equations.

If the scale below is balanced, how many stars must be hidden in one of the two equal boxes?

Write an equation to represent your work.

Let’s Think:

To solve for a variable, we must work backwards from the order of operations while keeping the equation balanced.

We use PEMDAS to evaluate expressions, which means when we are working forward, we always do ___________ first.

When we work backwards to solve for a variable, we do _____________________ first.

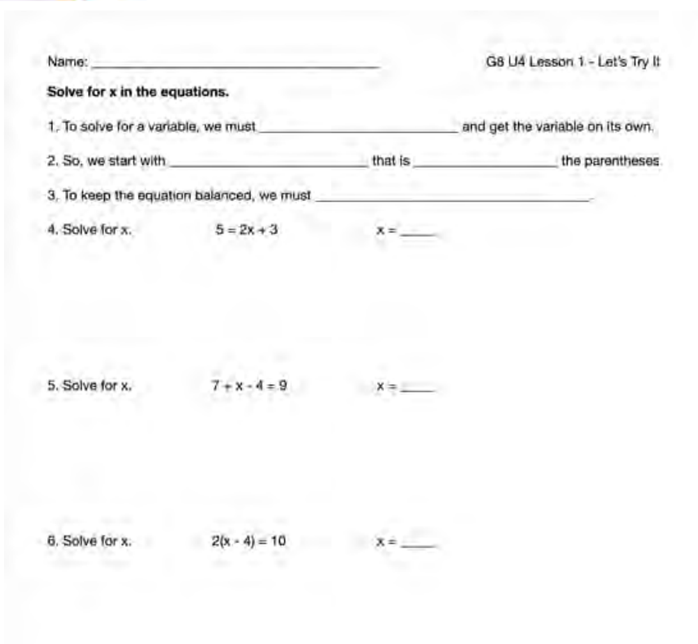
When we work backwards to solve for a variable, we deal with the part _________ the parentheses first.

$5x + 4 = 2$

$12 = 3 \cdot (4 + x)$

Let’s Try It:

Let’s solve some more equations together!



Name: ______________________________ G8 U4 Lesson 1 - Let's Try It

Solve for x in the equations.

1. To solve for a variable, we must ____________________ and get the variable on its own.

2. So, we start with ____________________ that is ______________ the parentheses.

3. To keep the equation balanced, we must ____________________________.

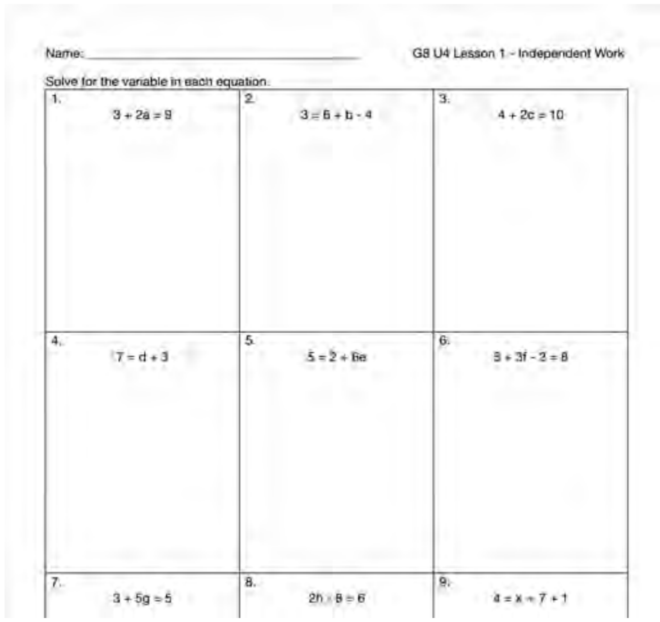
4. Solve for x. $5 = 2x + 3$ x = ____

5. Solve for x. $7 + x - 4 = 9$ x = ____

6. Solve for x. $2(x - 4) = 10$ x = ____

On your Own:

Now it's time for you to do it on your own!



Name: ____________________ G8 U4 Lesson 1 - Independent Work

Solve for the variable in each equation.

1. $3 + 2a = 9$	2. $3 = 6 + b - 4$	3. $4 + 2c = 10$
4. $7 = d + 3$	5. $5 = 2 + 6e$	6. $3 + 3f - 2 = 8$
7. $3 + 5g = 5$	8. $2h - 8 = 6$	9. $4 = x + 7 + 1$

Name: ____

Solve for x in the equations.

1. To solve for a variable, we must ____________________________ and get the variable on its own.

2. So, we start with ____________________________ that is ____________________ the parentheses.

3. To keep the equation balanced, we must ______________________________________.

4. Solve for x. $5 = 2x + 3$ x = _____

5. Solve for x. $7 + x - 4 = 9$ x = _____

6. Solve for x. $2(x - 4) = 10$ x = _____

7. Solve for x. $3 + x \div 8 = 1$ x = _____

Name: ____ G8 U4 Lesson 1 - Independent Work

Solve for the variable in each equation.

1. $3 + 2a = 9$	2. $3 = 6 + b - 4$	3. $4 + 2c = 10$
4. $7 = d + 3$	5. $5 = 2 + 6e$	6. $5 + 3f - 2 = 8$
7. $3 + 5g = 5$	8. $2h - 8 = 6$	9. $4 = x \div 7 + 1$

Solve for the variable in each equation.

10. $7 = 2j - 3$	11. $6 = 1 + 4k - 7$	12. $4(m - 2) = 4$
13. $9 + 4n = 10$	14. $(6 + x) \div 7 = 11$	15. $6 + x \div 7 = 11$
16. $3 + 2g = 5$	17. $3 + 2h - 8 = 10$	18. $x \div 7 + 1 = 4$

Name: ANSWER KEY

Solve for x in the equations.

1. To solve for a variable, we must work backwards and get the variable on its own.

2. So, we start with addition and subtraction that is outside the parentheses.

3. To keep the equation balanced, we must do the same to both sides.

4. Solve for x. $5 = 2x + 3$ x = 1

$-3 \quad -3$

$\frac{2}{2} = \frac{2x}{2}$

$1 = x$

5. Solve for x. $7 + x - 4 = 9$ x = 6

$+4 \quad +4$

$7 + x = 13$

$-7 \quad -7$

$x = 6$

6. Solve for x. $\frac{2(x - 4)}{2} = \frac{10}{2}$ x = 9

$x - 4 = 5$

$+4 \quad +4$

$x = 9$

7. Solve for x. $3 + x \div 8 = 1$ x = $-\frac{1}{4}$

$-3 \quad -3$

$\frac{x \div 8}{8} = \frac{-2}{8}$

$x = -\frac{2 \div 2}{8 \div 2} = -\frac{1}{4}$

Name: ANSWER KEY

G8 U4 Lesson 1 - Independent Work

Solve for the variable in each equation.

1. $3 + 2a = 9$

   $-3 \quad -3$

   $\frac{2a}{2} = \frac{6}{2}$

   $a = 3$

2. $3 = 6 + b - 4$

   $+4 \quad +4$

   $7 = 6 + b$

   $-6 \quad -6$

   $1 = b$

3. $4 + 2c = 10$

   $-4 \quad -4$

   $\frac{2c}{2} = \frac{6}{2}$

   $c = 3$

4. $7 = d + 3$

   $-3 \quad -3$

   $4 = d$

5. $5 = 2 + 6e$

   $-2 \quad -2$

   $\frac{3}{6} = \frac{6e}{6}$

   $\frac{3 \div 3}{6 \div 3} = e$

   $\frac{1}{2} = e$

6. $5 + 3f - 2 = 8$

   $-5 \quad -5$

   $3f - 2 = 3$

   $+2 \quad +2$

   $\frac{3f}{3} = \frac{5}{3}$

   ($5 \div 3 = 1\frac{2}{3}$: $5 - 3 = 2$)

   $f = 1\frac{2}{3}$

7. $3 + 5g = 5$

   $-3 \quad -3$

   $\frac{5g}{5} = \frac{2}{5}$

   $g = \frac{2}{5}$

8. $2h - 8 = 6$

   $+8 \quad +8$

   $\frac{2h}{2} = \frac{14}{2}$

   $h = 7$

9. $4 = x \div 7 + 1$

   $-1 \quad -1$

   $3 = x \div 7$

   $\times 7 \quad \times 7$

   $21 = x$

Solve for the variable in each equation.

10. $7 = 2j - 3$ $+3 \quad +3$ $\frac{10}{2} = \frac{2j}{2}$ $\boxed{5 = j}$	11. $6 = 1 + 4k - 7$ $+7 \quad +7$ $13 = 1 + 4k$ $-1 \quad -1$ $\frac{12}{4} = \frac{4k}{4}$ $\boxed{3 = k}$	12. $\frac{4(m - 2)}{4} = \frac{4}{4}$ $m - 2 = 1$ $+2 \quad +2$ $\boxed{m = 3}$
13. $9 + 4n = 10$ $-9 \quad -9$ $\frac{4n}{4} = \frac{1}{4}$ $\boxed{n = \frac{1}{4}}$	14. $(6 + x) \div 7 = 11$ $\times 7 \quad \times 7$ $6 + x = 77$ $-6 \quad -6$ $\boxed{x = 71}$	15. $6 + x \div 7 = 11$ $-6 \quad -6$ $x \div 7 = 5$ $\times 7 \quad \times 7$ $\boxed{x = 35}$
16. $3 + 2g = 5$ $-3 \quad -3$ $\frac{2g}{2} = \frac{2}{2}$ $\boxed{g = 1}$	17. $3 + 2h - 8 = 10$ $+8 \quad +8$ $3 + 2h = 18$ $-3 \quad -3$ $\frac{2h}{2} = \frac{15}{2}$ $2\overline{)15}$ = $07\frac{1}{2}$, -14, 1 $\boxed{h = 7\frac{1}{2}}$	18. $x \div 7 + 1 = 4$ $-1 \quad -1$ $x \div 7 = 3$ $\times 7 \quad \times 7$ $\boxed{x = 21}$

G8 U4 Lesson 2
Solve linear equations by thinking about the subtraction symbol in two different ways.

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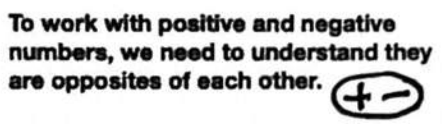
G8 U4 Lesson 2 - Today we will solve linear equations by thinking about the subtraction symbol in two different ways.

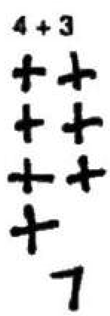
Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will solve linear equations by thinking about the subtraction symbol in two different ways. The reason we'll need to do this is because sometimes the minus sign is used for subtraction but sometimes it is used to show that a number is negative. We need to practice moving between those two ideas as we work with equations. Let's go!

Let's Review (Slide 3): To work with positive and negative numbers, we need to understand that they are opposites of each other. I think of positive numbers like going up the stairs and negative numbers like going down the stairs. If I go up a step and then down a step, it cancels out and I am back where I started. So if I have 1 positive and 1 negative, they cancel out and I have nothing, zero. I am going to write that here just to record the idea. And I'll draw a little picture underneath. I want a plus sign for the positive one and a minus sign for the negative one. And I'll circle this pair to show they cancel out. There's nothing left outside this circle.



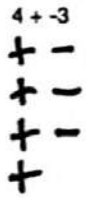
This says, "use the idea of opposites to add the integers below." You may have learned how to do this in a previous grade. But one thing that sometimes happens is that our teacher teaches us so many rules that the rules start to get mixed up in our head and we forget which is which. But if we can bring ourselves back to an idea of what is happening it will help us remember the rule. An easy way to do that is to draw a picture. As soon as you start to draw even a little bit, you will see what is happening.



Let's start with a super easy problem. You all know the answer. But just to get our drawing strategy going, let me draw it out. We have a positive 4 so I'll draw 4 plus signs. We have a positive 3 so I'll draw 3 plus signs. There's not any negative to cancel the positive. It's just a bunch of positives. It's like we went up some stairs and then we went up some more stairs. We just end up going up a lot of stairs. Anyway, we count these up and we get seven positives, which is just seven. You all knew 4 plus 3 was 7. But this picture is showing us how positives and negatives work, and we get our first rule - a positive plus a positive is positive. You don't even need to think about it as a rule because it makes so much sense. That's not such a big deal because you already knew that one. But let's see if we can get the other problems to make sense too.

It is negative 4 plus negative 3. For negative 4, I will draw 4 negatives. For negative 3, I will draw 3 negatives. That's not any positive to cancel the negative. It's just a bunch of negative. It's like we went down some stairs and then we went down some more stairs. We just end up going down a lot of stairs. So we count these up and we get seven negatives. Again, this picture is showing us a rule you might have heard before - a negative plus a negative is a negative. Or your teacher might have said, "When the signs are the same, you add." But without understanding why, it is easy to forget. So we draw a quick picture and it makes more sense to us.

Now we know if we have -435 + -3,256, we can just add those up too. We don't need to draw the picture because we saw how it worked with small numbers.



Let’s look at the next one. I have a positive 4 so I draw 4 positives. I have a negative 3 so I draw 3 negatives. *Be sure to draw these so that the amounts are lined up side by side.*

Here, I do have things that cancel. A positive and a negative together cancel eachother out. Another positive and negative cancel eachother out. Another positive and negative cancel eachother out. When I look at what is left after the cancellations, there is just one positive there, which we write at 1. And that makes sense because I had a bigger positive number than negative number. I had more positives than negatives so of course I will end up with overall positive after they cancel out.

Again, a quick picture helps us understand that canceling is happening. A lot of times teachers will say, “When signs are different, we subtract.” That’s technically true. But I like to use the word “cancel” because this is an addition problem and thinking of just randomly changing the operation is confusing. But we know positives and negatives cancel each other out so we’ll have to cancel. Then I have 4 thirds plus negative 1 half, this is too complicated to draw a picture. But we know these are going to cancel eachother out a little bit and we’ll have positives leftover.

Last one! We’re going to see the same idea as the last problem because we have positives and negatives, which will cancel eachother out. I draw 4 negatives and 3 positives.

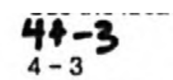
These cancel and these cancel and these cancel. I have one negative left. My answer is negative 1.

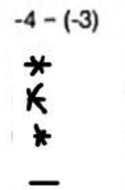
We have the same big idea as always - a quick sketch me how the numbers will interact and I either count them all up or cancel some out. In all of these cases, the minus sign means “negative.” So now let’s think about what happens when we see the minus sign meaning “subtract.”

Let’s Talk (Slide 4): When we subtract integers, we will need to shift between two different ways of thinking about the minus sign. We still know that positive and negative together cancel eachother out. That’s not going to change. Let’s see how that is going to work with these problems. It says, “use the idea of opposites to subtract the integers below.” We have 4 - 3. So first of all, you know 4 - 3. It’s 1. Easy. You knew that in Kindergarten. And in Kindergarten, you were probably taught that in order to draw subtraction you cross things out. Like this.

I know it is going to feel lame to continue thinking about this problem but stick with me because it will teach us something about subtracting integers for harder problems. And this is where we are going to learn the most important idea for today. We can draw 4 and cross 3 out. But another way to get rid of 3 would be to ADD 3 NEGATIVES. Look at this picture. I draw 4 positives. If I add 3 negatives. Then I cancel this one and this one and this one. And I'm left with 1 just like before.

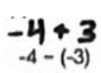
Now, why would we do this? We wouldn't. But it shows us one important idea: subtracting is the same as adding the opposite. Say it after me, "Subtracting is the same as adding the opposite." That is called a SHIFT IN PERSPECTIVE. It means we can look at the minus sign and think, "Oh, subtraction! That means crossing out." Or we can look at that same minus sign and think, "Oh, a minus sign! I will write it as adding the opposite." Let me rewrite the problem just so we start to see these as equivalent. Another way to write 4 minus 3 is to write 4 plus negative 3.



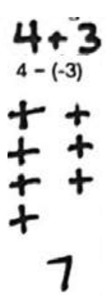


We will see this same idea on the next problem. I start with negative four. So let me draw 4 negatives. Now I need to take away 3 negatives. I can cross them out and I have one negative leftover.

I could also draw it another way. I start with 4 negatives. Now I want to subtract or get rid of 3 negatives. I get rid of negatives by adding the opposite. So I will add 3 positives. This cancels. This cancels. This cancels. I am left with negative 1. That's the exact same answer we got before. So, once again, we can think of the minus sign as "takeaway" or "cross out." But we can also think of it as a negative and we just add the opposite to find the answer.

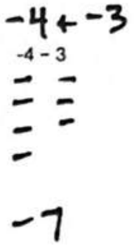


That's a shift in perspective, a shift in how we think about things. That would be negative 4 plus 3. And this problem actually looks simpler than the original, doesn't it?



Now here's where we're going to see shifting our perspective becoming actually important. We start with 4. I will draw 4 positive. Then it wants us to subtract negative 3. That would mean crossing out 3 negatives. But there aren't any negatives to cross out! I am suddenly pretty stuck if I only think of this one perspective that the minus sign means subtract. Instead I'm going to think of it as adding the opposite. If I add 3 positive, that is the same as if I were crossing out 3 negatives. They aren't there but positives cancel negatives, right? So I will add 3 positives. I see 4 positives and 3 positives. Nothing gets circled. My answer is 7.

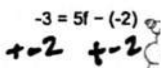
I can see all these minuses and get really panicky. Or I can see this minus and SHIFT MY PERSPECTIVE. I will change it to 4 plus the opposite, 3. You might have heard this before called "KEEP CHANGE FLIP." That is a trick that people sometimes say. But that gets really confusing - when do you do it? Why do you do it? It is better to actually understand that adding the opposite cancels something out so it is the same as subtraction.

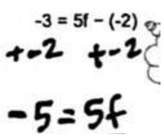


Let's do one more! We start with 4 negatives so I will draw those. Now if I look at this minus sign as subtraction, I need to cross out 3 positives. I don't have any positives. I can't cross them out. Instead, I have to SHIFT MY PERSPECTIVE. I will think of the minus sign as canceling and I will add the opposite to cancel them. I will add 3 negatives. I can see this is 7 negatives.

This problem turned into negative 4 plus negative 3. And then it's easy to see, "Oh, I have negatives and more negatives. My answer is going to be a lot of negatives." The big idea here is that subtracting is the same as adding the opposite.

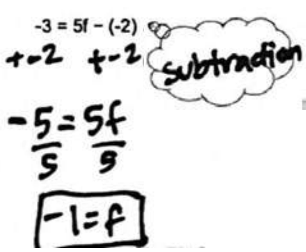
Let's Think (Slide 5): "We will need to shift how we think of the minus sign as we solve equations." This says, "Solve for the variable in the equations. Think about when you might have to think about the minus sign as negative instead of subtraction." Now, hopefully you remember from previous grades that we solve equations by working backwards and doing the same thing to both sides to keep the equation balanced. Now, this first one is really tricky because we were just looking at minus a negative on the last slide and we said it was easier to switch it to adding the opposite. But remember, we're working backwards to get to f anyway so I can just add negative 2 and I'll be working backwards. I have to do that on both sides.

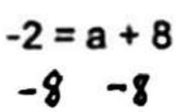




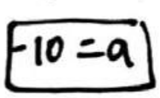
Now I can think to myself, negative 3 and negative 2 is a lot of negatives put together. I get negative five equals 5f.

I work backwards on the 5f by dividing by 5 on both sides. Negative 5 divided by 5 is negative 1, which equals f. Now, I'll mention that we haven't spent any time talking about multiplying and dividing positive and negatives today. That's because we can use our old idea of multiplication and division as groups to understand the rules there. If I have positive and negatives getting multiplied then I either have negative groups or groups of negatives. And either way my answer will be negative. If I have a negative times a negatives that's like negative groups of negatives and now we're talking about the opposite of negatives, which is positives. The point is that in this case, I thought of the minus sign as subtraction and I just worked backwards from subtraction.





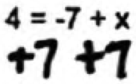
Let's look at the next one. At first, there isn't even a minus sign to worry about. I want to get a by itself so I will subtract 8 from both sides.

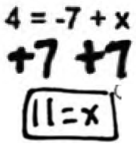


Alert! Alert! I have negative 2 minus 8! NOW we're in a tricky spot! I can't cross out 8 when there isn't even 8 there! I have to think of this as adding negative 8. Some people won't even rewrite this. They will just do a quick invisible shift in their mind where they turn minus 8 into negative 8. And that is totally acceptable. If the shift doesn't happen for you as quickly yet, it's no big deal. Put a little plus sign and think of this as adding negative 8 instead. Now we can think 2 negatives and 8 negatives is a lot of negatives altogether. That's 10 negatives or negative 10 equals a.

In this case, we thought of the minus sign as negative.

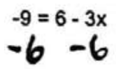
Let's do one more. We have 4 equals negative 7 plus x. We want x by itself so we have to get rid of this negative 7. You can do a whole lot of thinking about how we get rid of negatives. But if we think of the minus sign as subtraction then it's obvious that to get rid of it, we add 7. And I'm going to do that to both sides.



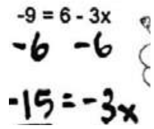


Then I get 11 equals x. In this case, again, someone might have this quick invisible moment in their head where they change the negative sign to the idea of subtracting. Great. Let's spell it out just because it's still kind of new for us.

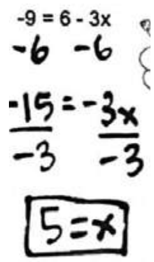
We thought of the minus sign as subtraction. The big idea here is that you will have to stop and think. You might try to think of the minus sign as subtraction and feel stuck so then you can try thinking of it as a negative. You will have to do a little bit of trial and error to solve these equations.



Let's Think (Slide 6): Let's do two more examples that are a little trickier. Here we have negative 9 equals 6 minus 3x. I will start by getting rid of this 6 by subtracting 6 on both sides.

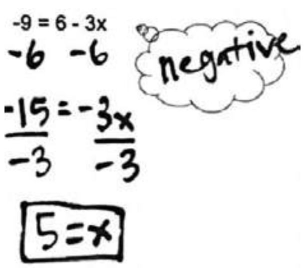


When I write it over on the left, I think of it as negative 6 so that becomes negatives and more negatives. I get lots of negatives. It's negative 15. That equals minus 3x.

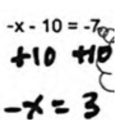


Now this is really interesting. When I first looked at the problem, it seemed like 3x was being subtracted. But now it looks more like negative 3 times x. No problem! I shift my perspective. To work backward, I will divide each side by negative 3.

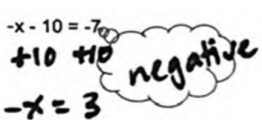
Negative 15 divided by negative 3 is positive 5 because it's like I have the negative groups of negative. In other words, the opposite of negative is positive. 5 equals x.



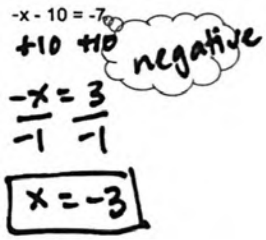
I thought of the minus sign as a negative.



One more! I will start by adding 10 to both sides to work backwards from the subtracting happening here. The positive 10 cancels the negative 7 so I get negative x equals positive 3.



So far, I saw the minus sign as subtraction. But the next step is a super big deal! When I see a negative x and I need the x by itself, it is easiest if I think of it as negative 1 times x. That is a really important shift in perspective.



Then I divide by negative 1 on both sides. That cancels out on the left and I get x equals something. 3 divided by negative 1 is negative 3 because I have different signs. X equals negative 3.

Let's Try It (Slide 7): Let's solve equations with minus signs together. I will walk you through each step.

WARM WELCOME

Today we will solve linear equations by thinking about the subtraction symbol in two different ways.

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Let's Review:

To work with positive and negative numbers, we need to understand they are opposites of each other.

Use the idea of opposites to add the integers below.

$4 + 3$ $\quad$ $-4 + -3$ $\quad$ $4 + -3$ $\quad$ $-4 + 3$

Let's Talk:

When we subtract integers, we will need to shift between two different ways of thinking about minus sign.

Use the idea of opposites to subtract the integers below.

$4 - 3$ $\quad$ $-4 - (-3)$ $\quad$ $4 - (-3)$ $\quad$ $-4 - 3$

Let's Think:

We will need to shift how we think of the minus sign as we solve equations.

Solve for the variable in the equations. Think about when you might have to think about the minus sign as negative instead of subtraction.

$-3 = 5f - (-2)$

$-2 = a + 8$

$4 = -7 + x$

Let's Think:

We will need to shift how we think of the minus sign as we solve equations.

Solve for the variable in the equations. Think about when you might have to think about the minus sign as negative instead of subtraction.

$-9 = 6 - 3x$

$-x - 10 = -7$

Let's Try It:

Let's solve equations with minus signs together!

Name: ______________________ G8 U4 Lesson 2 - Let's Try It

Solve for x in the equations. Think about when you might have to think about the minus sign as negative instead of subtraction.

1. To solve for a variable, we must ______________________ and get the variable on its own.

2. To keep the equation balanced, we must ______________________.

3. Solve for x. $-5 = x + 3$ x = ____

4. Solve for x. $-7 + x = -9$ x = ____

5. Solve for x. $2 = -2 - 9x$ x = ____

6. Solve for x. $2x - (-4) = 10$ x = ____

On your Own:

Now it's time for you to do it on your own!

Name: ______________________ G8 U4 Lesson 2 - Independent Work

Solve for the variable in each equation.

1. $-9 + a = 4$	2. $-3 = b + 4$	3. $8 + c = -10$
4. $7 = d - (-3)$	5. $-3 = -2 + e$	6. $i - 2 = -8$
7. $3 - g = -2$	8. $h + (-8) = -3$	9. $4 = -8 - i$

Name: ______________________________________ G8 U4 Lesson 2 - Let's Try It

Solve for x in the equations. Think about when you might have to think about the minus sign as negative instead of subtraction.

1. To solve for a variable, we must __________________________ and get the variable on its own.

2. To keep the equation balanced, we must ____________________________________.

3. Solve for x. $-5 = x + 3$ x = _____

4. Solve for x. $-7 + x = -9$ x = _____

5. Solve for x. $2 = -2 - 9x$ x = _____

6. Solve for x. $2x - (-4) = 10$ x = _____

7. Solve for x. $-6 = 5 - x$ x = _____

Name: ____ G8 U4 Lesson 2 - Independent Work

Solve for the variable in each equation.

1. $-9 + a = 4$	2. $-3 = b + 4$	3. $8 + c = -10$
4. $7 = d - (-3)$	5. $-3 = -2 + e$	6. $f - 2 = -8$
7. $3 - g = -2$	8. $h + (-8) = -3$	9. $4 = -8 - i$

Solve for the variable in each equation.

10. $-7 = 2j - (-3)$	11. $-6 = -1 + 6k$	12. $-m - 2 = -4$
13. $-9 - 4n = 3$	14. $-6 = -2b + 4$	15. $8 - c = 10$
16. $3 + 2g = -5$	17. $-h + (-8) = -10$	18. $1 = 7 - i$

Name: ANSWER KEY

Solve for x in the equations. Think about when you might have to think about the minus sign as negative instead of subtraction.

1. To solve for a variable, we must work backwards and get the variable on its own.

2. To keep the equation balanced, we must do the same to both sides.

3. Solve for x. $-5 = x + 3$ $x = -8$

$-3 \quad -3$

$-8 = x$

4. Solve for x. $-7 + x = -9$ $x = -2$

$+7 \quad +7$

$x = -2$

5. Solve for x. $2 = -2 - 9x$ $x = -\frac{4}{9}$

$+2 \quad +2$

$\frac{4}{-9} = \frac{-9x}{-9}$

6. Solve for x. $2x - (-4) = 10$ $x = 3$

$2x + 4 = 10$

$-4 \quad -4$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

7. Solve for x. $-6 = 5 - x$ $x = 11$

$-5 \quad -5$

$\frac{-11}{-1} = \frac{-x}{-1}$

$11 = x$

Name: ANSWER KEY

Solve for the variable in each equation.

1. $-9 + a = 4$ $+9 \quad +9$ $a = 13$	2. $-3 = b + 4$ $-4 \quad -4$ $-7 = b$	3. $8 + c = -10$ $-8 \quad -8$ $c = -18$
4. $7 = d - (-3)$ $7 = d + 3$ $-3 \quad -3$ $4 = d$	5. $-3 = -2 + e$ $+2 \quad +2$ $-1 = e$	6. $f - 2 = -8$ $+2 \quad +2$ $f = -6$
7. $3 - g = -2$ $-3 \quad -3$ $\frac{-g}{-1} = \frac{-5}{-1}$ $g = 5$	8. $h + (-8) = -3$ $+8 \quad +8$ $h = 5$	9. $4 = -8 - i$ $+8 \quad +8$ $\frac{12}{-1} = \frac{-i}{-1}$ $-12 = i$

Solve for the variable in each equation.

10.

$-7 = 2j - (-3)$

$-7 = 2j + 3$

$-3 \quad -3$

$\frac{-10}{2} = \frac{2j}{2}$

$\boxed{-5 = j}$

11.

$-6 = -1 + 6k$

$+1 \quad +1$

$\frac{-5}{6} = \frac{6k}{6}$

$\boxed{-\frac{5}{6} = k}$

12.

$-m - 2 = -4$

$+2 \quad +2$

$\frac{-m}{-1} = \frac{-2}{-1}$

$\boxed{m = 2}$

13.

$-9 - 4n = 3$

$+9 \quad +9$

$\frac{-4n}{-4} = \frac{12}{-4}$

$\boxed{n = -3}$

14.

$-6 = -2b + 4$

$-4 \quad -4$

$\frac{-10}{-2} = \frac{-2b}{-2}$

$\boxed{5 = b}$

15.

$8 - c = 10$

$-8 \quad -8$

$\frac{-c}{-1} = \frac{2}{-1}$

$\boxed{c = -2}$

16.

$3 + 2g = -5$

$-3 \quad -3$

$\frac{2g}{2} = \frac{-8}{2}$

$\boxed{g = -4}$

17.

$-h + (-8) = -10$

$+8 \quad +8$

$\frac{-h}{-1} = \frac{-2}{-1}$

$\boxed{h = 2}$

18.

$1 = 7 - i$

$-7 \quad -7$

$\frac{-6}{-1} = \frac{-i}{-1}$

$\boxed{6 = i}$

G8 U4 Lesson 3
Solve linear equations with the distributive property.

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G8 U4 Lesson 3 - Today we will solve linear equations with the distributive property.

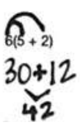
Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will solve linear equations with the distributive property. This is the next level of complexity in what we've already been doing. You are ready for it! Let's go!

Let's Review (Slide 3): This says, "The distributive property is an important part of solving equations." Raise your hand if you ever remember hearing about the distributive property in an earlier grade. What do you remember? Possible Student Answers, Key Points:

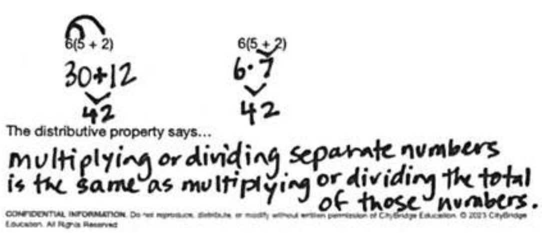
- It means that you can multiply or divide a whole amount by something and you'll get the same answer as if you multiply or divide it in pieces instead.
- It means you take that 6 and multiply it by 5. Then you multiply it by 2 and you get 30 plus 12.

You might hear a whole lot of correct and incorrect responses. The purpose of the question is just to get a sense of what kids know. Do not confirm or correct student answers. Here's what I would say. I would say the distributive property says that we can distribute a multiplier or divisor to each piece of a whole and then we would multiply or divide each piece. Or we can put all the pieces together into a whole amount and then multiply or divide them. And we will get the same answer either way.



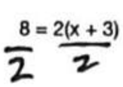
Let me show you with this example. If I have the stuff in this parentheses times 6. I will multiply each part of my parentheses. Let me draw arrows just to show you what I'm thinking. I will do 6 times 5 and 6 times 2. 6 times 5 is 30. That gets added to 6 times 2, which is 12. 30 plus 12 is 42.

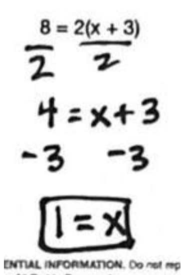
Now let's do the same problem a different way. This time, I will follow order of operations and do the parentheses first. 5 plus 2 is 7 so now I have 6 x 7. 6 x 7 is 42. I get the same answer.



So, the distributive property says… multiplying or dividing separate numbers is the same as multiplying or dividing the total of those numbers. This is going to be very important because we are going to need to solve equations that have the distributive property. That's what today's lesson is about.

Let's Talk (Slide 4): There are two ways to solve an equation when we have the distributive property. We can cancel out the part outside the parentheses with the opposite operation. Or we can distribute the factor and then solve. The first one usually has fewer steps. Let me show you what I mean. I see that I have 8 equals 2 times this amount in parentheses, x plus 3. I will need to work backwards and divide by 2 on each side.





Then I get 4 equals x plus 3. I want to get x by itself so I will subtract 3 on both sides. I get 1 equals x. When we do the problem like this, we are thinking about which steps we would solve and then working backwards from those steps. Like normally we would do whatever is in the parentheses and then multiply it. But we're going to work backwards from that so we'll get rid of the multiplication first.

$$8 = 2(x + 3)$$
$$8 = 2x + 6$$

The other way to do it is to distribute the factor. That means multiply the factor times everything in the parentheses. Then we'll get a simpler equation. I will draw arrows to show everything that has to get multiplied.So I will do 8 equals 2 times x, which is 2x. Then I do 2 x 3, which is 6. Now I have 8 equals 2x plus 6.

$$8 = 2(x + 3)$$
$$8 = 2x + 6$$
$$-6 \qquad -6$$
$$\frac{2}{2} = \frac{2x}{2}$$
$$\boxed{1 = x}$$

I can subtract 6 from each side, which leaves me with 2 equals 2x. Now I divide by 2 on each side. I get 1 equals x. The same answer! Yay! That means we're doing it right!What I can't do is just attach this 2 to the x and rewrite the expression as if the parentheses were never there. The parentheses are there because everything inside these parentheses has to get multiplied by 2. I can't write 8 = 2x =3. *Write the equation and then cross it out.* I either have to cancel out that multiplier like we did in the first problem or distribute that multiplier like we did in the second problem.

Let's Think (Slide 5): There is one additional thing that makes this a bit trickier. This says, "If we are subtracting a factor, we must shift our perspective and think of it as a negative number as we multiply and divide." Let's solve for the variables and I'll show you what I mean. The first problem is straightforward. I want to start by canceling the addition and subtraction outside the parentheses so I will subtract 8 from both sides. That leaves me with 9 times 2x minus 10 equals 54.

$$8 + 9(2x - 10) = 62$$
$$-8 \qquad -8$$
$$9(2x-10) = 54$$

$$8 + 9(2x - 10) = 62$$
$$-8 \qquad -8$$
$$\frac{9(2x-10)}{9} = \frac{54}{9}$$
$$2x - 10 = 6$$
$$+10 \quad +10$$
$$\frac{2x}{2} = \frac{16}{2}$$
$$\boxed{x = 8}$$

Now I will divide by 9 on each side. That gives me 2x plus 10 equals 6.

We keep working backwards. I will add 10 to both sides. That gives me 2x equals 16.

I divide by 2 on each side and get x equals 8.

The next problem is exactly the same except that not it is 8 MINUS 9 instead of 8 plus 9. It starts the same way. I subtract 8 from both sides.

$$8 - 9(2x - 10) = 62$$
$$-8 \qquad -8$$
$$-9(2x-10) = 54$$

The key thing is that as the eight is canceled out, I still have this minus sign before the 9. It doesn't disappear because everything after it is supposed to be subtracted or taken away. So, I will leave it in front of the 9 and now it's like NEGATIVE nine times what it's in the parentheses. I have negative nine times 2x - 10 equals 54.

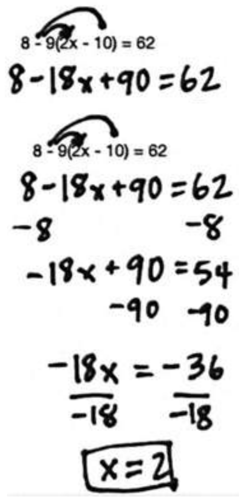
$$8 - 9(2x - 10) = 62$$
$$-8 \qquad -8$$
$$\frac{-9(2x-10)}{-9} = \frac{54}{-9}$$
$$2x - 10 = -6$$
$$+10 \quad +10$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$\boxed{x = 2}$$

I keep going but now when I divide, I divide by NEGATIVE 9 on both sides. That gives me 2x minus 10 equals NEGATIVE 6. Because I have different signs, a positive divided by a negative.

I want to add 10 to both sides. That leaves me with 2x on the left. On the right, I have negative 6 plus positive 10. They cancel and I'm only left with positive 4.

I divide by 2 on each side. I get x equals 2. So you see, I get a different answer with that minus sign there. I can't just let it disappear.

Let’s do this one more time to see what happens if I wanted to distribute first. I can’t think of it as distributing 9 because it is 8 minus 9. And to distribute it, I would need to think of it as 8 plus negative 9. And in fact, I would think of this minus 10 as plus negative 10 too. Then I would distribute NEGATIVE 9. It would be NEGATIVE 9 times 2x and NEGATIVE 9 times negative 10. I would get 8 plus negative 18x plus 90 equals 62.



Let’s keep solving. I will subtract 8 from both sides. I get negative 18x plus 90 equals 54. Then I subtract 90 from each side. That’s kind of hard. I might have to do 90 minus 54 on the side of my paper to see what is left when these cancel eachother out. I can do it in my head, and it leaves negative 36. So we have negative 18x equals negative 36. I divide by negative 18 on both sides, and I get x equals 2.

I want you to see that it is very tricky to distribute something that is being subtracted. It is easier to cancel it out with an opposite operation. But if you have to then you can. And you need to look out for whether it is necessary to change it to adding a negative like we did in our last lesson.

Let’s Try It (Slide 6): Let’s solve equations with the distributive property together. I will walk you through each step.

WARM WELCOME

Today we will solve linear equations with the distributive property.

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Let's Review: The distributive property is an important part of solving equations.

Evaluate the expression below two ways.

6(5 + 2)   6(5 + 2)

The distributive property says…

Let's Talk: There are two ways to solve an equation when we have the distributive property.

We can cancel out the part outside the parentheses with the opposite operation. Or we can distribute the factor and then solve.

8 = 2(x + 3)   8 = 2(x + 3)

Let's Think:

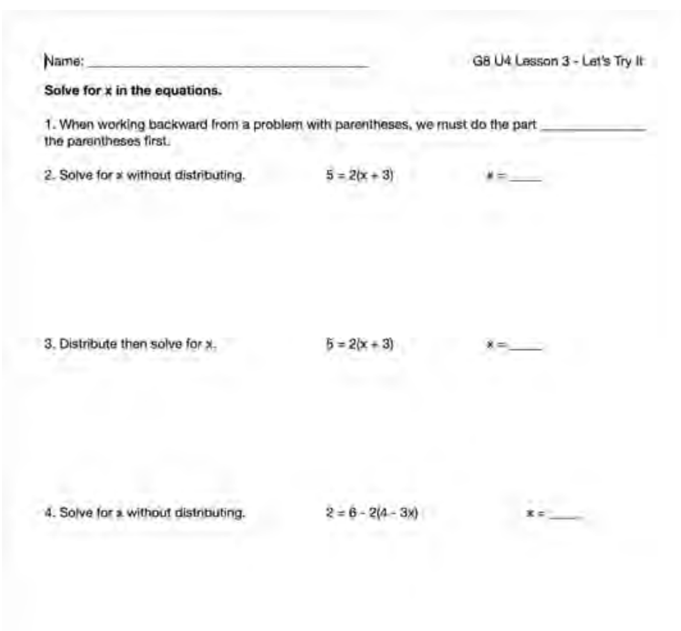
If we are subtracting a factor, we must shift our perspective and think of it as a negative number as we multiply and divide.

Solve for the variables.

$8 + 9(2x - 10) = 62$ $\quad$ $8 - 9(2x - 10) = 62$ $\quad$ $8 - 9(2x - 10) = 62$

Let's Try It:

Let's solve some equations together!



Name: ______________________ G8 U4 Lesson 3 - Let's Try It

Solve for x in the equations.

1. When working backward from a problem with parentheses, we must do the part __________ the parentheses first.

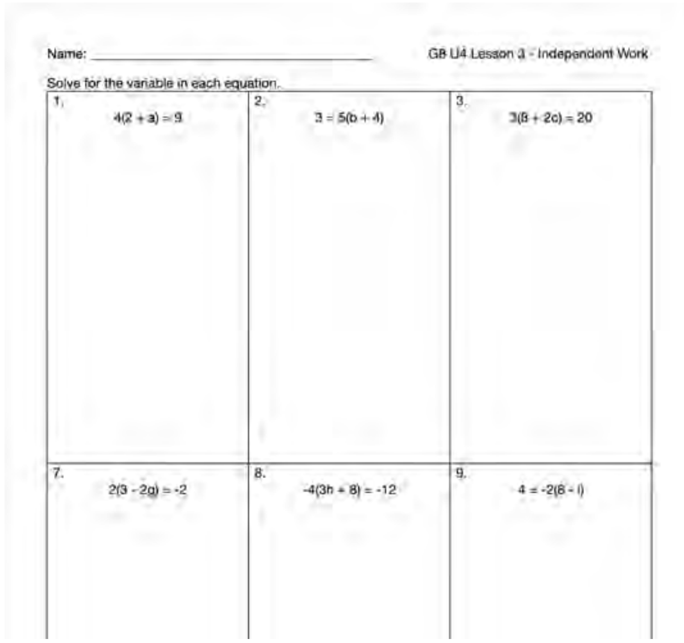
2. Solve for x without distributing. $5 = 2(x + 3)$ $x =$ ____

3. Distribute then solve for x. $5 = 2(x + 3)$ $x =$ ____

4. Solve for x without distributing. $2 = 6 - 2(4 - 3x)$ $x =$ ____

On your Own:

Now it's time for you to do it on your own!



Name: ______________________ G8 U4 Lesson 3 - Independent Work

Solve for the variable in each equation.

1. $4(2 + a) = 9$	2. $3 = 5(b + 4)$	3. $3(8 + 2c) = 20$
7. $2(3 - 2g) = -2$	8. $-4(3h + 8) = -12$	9. $4 = -2(8 - i)$

Name: ____

Solve for x in the equations.

1. When working backward from a problem with parentheses, we must do the part _______________ the parentheses first.

2. Solve for x without distributing. $5 = 2(x + 3)$ x = _____

3. Distribute then solve for x. $5 = 2(x + 3)$ x = _____

4. Solve for x without distributing. $2 = 6 - 2(4 - 3x)$ x = _____

5. Distribute then solve for x. $2 = 6 - 2(4 - 3x)$ x = _____

Name: ________________________________ G8 U4 Lesson 3 - Independent Work

Solve for the variable in each equation.

1. $4(2 + a) = 9$	2. $3 = 5(b + 4)$	3. $3(8 + 2c) = 20$
7. $2(3 - 2g) = -2$	8. $-4(3h + 8) = -12$	9. $4 = -2(8 - i)$

Solve for the variable in each equation.

10. $6 = 4(3j - 3) + 12$	11. $5 = 2(-1 + 6k) - 1$	12. $3 + 6(2m + 3) = 57$
16. $-2(3 + 2g) = -5$	17. $3(-h - 8) + 5 = -10$	18. $-6 = 9 - 3(7i + 3)$

Name: ANSWER KEY

Solve for x in the equations.

1. When working backward from a problem with parentheses, we must do the part Outside the parentheses first.

2. Solve for x without distributing. $5 = 2(x + 3)$ x = $-\frac{1}{2}$

$\frac{5}{2} = \frac{2(x+3)}{2}$

$2\overline{)5}$ = $2\frac{1}{2}$; -4; 1

$2\frac{1}{2} = x + 3$

$-3 \quad -3$

$-\frac{1}{2} = x$

3. Distribute then solve for x. $5 = 2(x + 3)$ x = $-\frac{1}{2}$

$5 = 2x + 6$

$-6 \quad -6$

$\frac{-1}{2} = \frac{2x}{2}$

$-\frac{1}{2} = x$

4. Solve for x without distributing. $2 = 6 - 2(4 - 3x)$ x = $\frac{2}{3}$

$-6 \quad -6$

$\frac{-4}{-2} = \frac{-2(4-3x)}{-2}$

$2 = 4 - 3x$

$-4 \quad -4$

$\frac{-2}{-3} = \frac{-3x}{-3}$ $\frac{2}{3} = x$

5. Distribute then solve for x. $2 = 6 - 2(4 - 3x)$ x = $\frac{2}{3}$

$2 = 6 - 8 + 6x$

$+8 \quad +8$

$10 = 6 + 6x$

$-6 \quad -6$

$\frac{4}{6} = \frac{6x}{6}$ $\frac{4 \div 2}{6 \div 2} = x$ $\frac{2}{3} = x$

Name: ANSWER KEY

G8 U4 Lesson 3 - Independent Work

Solve for the variable in each equation.

1.	2.	3.
$4(2 + a) = 9$ $8 + 4a = 9$ $-8 \quad -8$ $\frac{4a}{4} = \frac{1}{4}$ $a = \frac{1}{4}$	$3 = 5(b + 4)$ $3 = 5b + 20$ $-20 \quad -20$ $\frac{-17}{5} = \frac{5b}{5}$ $-3\frac{2}{5} = b$ $17 \div 5 = 3$ R 2 ($17 - 15 = 2$)	$3(8 + 2c) = 20$ $24 + 6c = 20$ $-24 \quad -24$ $\frac{6c}{6} = \frac{-4}{6}$ $c = -\frac{4 \div 2}{6 \div 2}$ $c = -\frac{2}{3}$
7.	8.	9.
$\frac{2(3 - 2g)}{2} = \frac{-2}{2}$ $3 - 2g = -1$ $-3 \quad -3$ $\frac{-2g}{-2} = \frac{-4}{-2}$ $g = 2$	$\frac{-4(3h + 8)}{-4} = \frac{-12}{-4}$ $3h + 8 = 3$ $-8 \quad -8$ $\frac{3h}{3} = \frac{-5}{3}$ $h = -1\frac{2}{3}$ $5 \div 3 = 1\frac{2}{3}$ ($5 - 3 = 2$)	$\frac{4}{-2} = \frac{-2(8 - i)}{-2}$ $-2 = 8 - i$ $-8 \quad -8$ $\frac{-10}{-1} = \frac{-i}{-1}$ $10 = i$

Solve for the variable in each equation.

10.	11.	12.
$6 = 4(3j - 3) + 12$ $-12 \quad -12$ $-6 = 4(3j - 3)$ $-6 = 12j - 12$ $+12 \quad +12$ $\frac{6}{12} = \frac{12j}{12}$ $\frac{6 \div 6}{12 \div 6} = j$ $\boxed{\frac{1}{2} = j}$	$5 = 2(-1 + 6k) - 1$ $+1 \quad +1$ $6 = -2 + 12k$ $+2 \quad +2$ $\frac{8}{12} = \frac{12k}{12}$ $\frac{8 \div 4}{12 \div 4} = k$ $\boxed{\frac{2}{3} = k}$	$3 + 6(2m + 3) = 57$ $-3 \quad -3$ $\frac{6(2m+3)}{6} = \frac{54}{6}$ $2m + 3 = 9$ $-3 \quad -3$ $\frac{2m}{2} = \frac{6}{2}$ $\boxed{m = 3}$
16.	17.	18.
$-2(3 + 2g) = -5$ $-6 + -4g = -5$ $+6 \quad +6$ $\frac{-4g}{-4} = \frac{1}{-4}$ $\boxed{g = -\frac{1}{4}}$	$3(-h - 8) + 5 = -10$ $-5 \quad -5$ $\frac{3(-h-8)}{3} = \frac{-15}{3}$ $-h - 8 = -5$ $+8 \quad +8$ $\frac{-h}{-1} = \frac{3}{-1}$ $\boxed{h = -3}$	$-6 = 9 - 3(7i + 3)$ $-9 \quad -9$ $\frac{-15}{-3} = \frac{-3(7i + 3)}{-3}$ $5 = 7i + 3$ $-3 \quad -3$ $\frac{2}{7} = \frac{7i}{7}$ $\boxed{\frac{2}{7} = i}$

G8 U4 Lesson 4
Solve linear equations with variables on both sides.

G8 U4 Lesson 4 - Today we will solve linear equations with variables on both sides.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will solve linear equations with variables on both sides. It is all of the ideas we've already been working with one extra twist. You are going to do great!

Let's Review (Slide 3): We know that we need to keep any equation we are solving balanced by doing the same operation to both sides. This says, "Solve for x." What steps might I do first? Possible Student Answers, Key Points:

- Distribute the 7.
- Add 5 to both sides.

=There are actually two different ways I could start. As our equations get more sophisticated, that is going to happen more and more often. Sometimes I think about what numbers I would get with each step in my head before I write anything down. Then I can think what would be friendlier. For example, in this problem, I could do the distribution. I'm thinking in my head that would give me 14x plus 21 minus 5. I could work backwards from that but it's a lot of numbers. The other idea would be to add 5 to both sides. I'm thinking in my head that gets rid of the 5 on the left and makes things a little simpler. So let's do that. I am going to add 5 to both sides.

$$7(2x + 3) - 5 = 16$$
$$+5 \quad +5$$

$$7(2x + 3) - 5 = 16$$
$$+5 \quad +5$$
$$\frac{7(2x+3)}{7} = \frac{21}{7}$$

Now I have 7 times 2x + 3 equals 21. There are two different ways I could do the next step too! I'm kind of always doing some pre-thinking in my head about what would make my life easier. What steps might I do next? Possible Student Answers, Key Points:

- Distribute the 7.
- Divide by 7 on both sides.

Once again, I think about what numbers I would get with each step in my head before I write anything down. Then I can think what would be friendlier. For example, a lot of time it's easier to distribute the factor outside the parentheses. Then I won't have parentheses anymore. That's nice. Or I could divide by 7 on both sides. Sometimes that gives me some really tricky numbers. Sometimes that gives me fractions. But in this case, it's actually a nice friendly number so I'll do that and save myself some steps. I divide by 7 on both sides.

$$7(2x + 3) - 5 = 16$$
$$+5 \quad +5$$
$$\frac{7(2x+3)}{7} = \frac{21}{7}$$
$$2x + 3 = 3$$
$$-3 \quad -3$$

I get 2x + 3 equals 3. What step should I do next? Possible Student Answers, Key Points:

- Subtract 3 on both sides.

Now in this case, I don't have two options that I can choose between. The order of operations, which is PEMDAS, tells us that multiplication and division come before addition and subtraction. To work backwards from that, I have to work on the addition and subtraction first. I am going to subtract 3 from both sides.

$$2x = 0$$

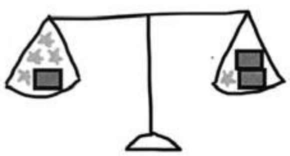
Now this might seem a little scary because I am going to get 2x equals ZERO. But it is totally okay to get zero as an answer. But that doesn't mean we can just drop one whole side of the equation. We write down zero and keep going.

$$7(2x + 3) - 5 = 16$$
$$\quad +5 \quad +5$$
$$\frac{7(2x+3)}{7} = \frac{21}{7}$$
$$2x + 3 = 3$$
$$\quad -3 \quad -3$$
$$\frac{2x}{2} = \frac{0}{2} \qquad \boxed{x = 0}$$

Now I can divide by 2 on each side.

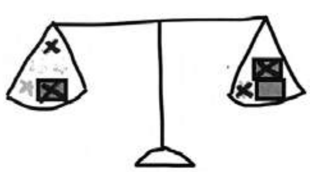
That gives me x on this side and 2 divided by zero is still zero. So x equals zero. Just because zero means nothing doesn't mean you didn't get an answer. Zero can be an answer, and that's what we got here. Great work!

Let's Talk (Slide 4): For the big idea of today's lesson, we are back to a model we used a few lessons ago. This says, "The balance can help us think about what to do when there is a variable on both sides of the equal sign." So you can see in our picture that now our balance has boxes on both sides. That is totally fine. We still do the same thing to both sides so that it stays balanced. This says, "If the scale below is balanced, how many stars must be hidden in one of the equal boxes?" So we know that we are trying to figure out what the scale would look like if there were only 1 box on one side. Then we would know what it is balanced with. I am going to draw that underneath just so we can think about where we're trying to end up.

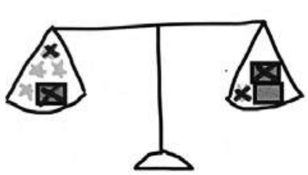


Now, one big idea that is going to keep coming up is that before I just dive in and start writing down numbers, I am going to think in my head, "What could make this simpler?" or "What could make this friendlier?" In this case, I am going to cross out a star on each side because then the right side will only have boxes and that's a bit easier to think about.

Now I want to get just one box on one side and right now I have boxes on both sides. That is not very simple or friendly. So I am going to take off this box on the left. And if I do that, I have to take off a box on the right to keep it balanced.



Look what we have! There are 3 stars on the left and 1 box on the right. So now I know 3 stars equals 1 box.



Next this says to write an equation to represent your work. Remember the stars we know and the boxes are the unknowns like a question mark or a variable. On the left side we have 4 stars so I'll write 4 plus x, which stands for the unknown amount in the box. That equals what we have on the right side. The right side has 1 star so I'll write 1 plus 2x for the 2 boxes, which is 2 unknowns.

Write an equation to represent your work.

$$4 + x = 1 + 2x$$

Write an equation to represent your work.

$$\begin{array}{ccc} 4 + x & = & 1 + 2x \\ -1 & & -1 \end{array}$$

The first thing we did was take away a star from each side. That's the same as doing minus one on the left and minus one on the right. We can see that we are working backwards from the addition that is written on the right.

Write an equation to represent your work.

$$\begin{array}{ccc} 4 + x & = & 1 + 2x \\ -1 & & -1 \\ 3 + x & = & 2x \end{array}$$

That gives us 3 + x equals 2x.

Write an equation to represent your work.

$$\begin{array}{ccc} 4 + x & = & 1 + 2x \\ -1 & & -1 \\ 3 + x & = & 2x \\ -x & & -x \end{array}$$

Next we wanted to get one x by itself. Sometimes people call that "isolating the variable." So we crossed out a box on each side. That's the same as subtracting x on the side and subtracting x on this side.

Then we just have 3 on the left. That equals x on the right. That's because we're thinking of it like 2 boxes minus 1 box or like 2 exes minus 1 ex. That leaves 1 ex but we just write x. And that's our answer. The big idea here is that even if we have variables on both sides, it works the same way as equations we've done before. We still are working backwards. We still are doing the same thing to both sides to keep the equation balanced. The only new thing is that we want to isolate the variable so we might need to add or subtract a variable from one side so that all the variables are on the other side.

Write an equation to represent your work.

$$\begin{array}{ccc} 4 + x & = & 1 + 2x \\ -1 & & -1 \\ 3 + x & = & 2x \\ -x & & -x \\ 3 & = & x \end{array}$$

Let's Think (Slide 5): Just like we can add or subtract numbers, we can add or subtract a variable and its coefficient. Coefficient is just the word for the number before a variable. So for this 2x, for example, 2 is the coefficient. *Point to the 2x.* For this 10x, 10 is the coefficient. *Point to the 10x.* On the last slide, we had 2 boxes and 1 box and that's just like 2 exes and 1 ex. Those are variables with coefficients. Now, here is one really important tip: "It is generally easier to subtract the number with the smaller coefficient so you don't get negative numbers if you don't have to." That means in this example, it is going to be easier to subtract 2x than 10x. This is where that thing we talked about earlier, where we do a little bit of thinking in our mind before we start number crunching, can really come in handy. Let me show you what I mean. We have the same equation written twice here and I am going to solve it two different ways. Maybe first I see this plus 10x and I decide to subtract 10x from both sides.

$$\begin{array}{ccc} 6 + 2x & = & 3 + 10x \\ -10x & & -10x \end{array}$$

$$\begin{array}{l} 6 + 2x = 3 + 10x \\ \quad -10x \qquad -10x \\ 6 - 8x = 3 \end{array}$$

Then on the left side, I get 6 and we have to think about this 2x, which is positive, and the minus 10x, which is like a negative. You can already see how subtracting the variable with the bigger coefficient is
making things a little tricky. I have to think about how positive cancels negative so I get negative 8x. And on the right side, it equals 3.

$$\begin{array}{l} 6 + 2x = 3 + 10x \\ \quad -10x \qquad -10x \\ 6 - 8x = 3 \\ -6 \qquad -6 \\ \quad -8x = -3 \end{array}$$

Now in order to keep going, I will subtract 6 from both sides.

The left side 6 and minus 6 cancel eachother out. So it's just negative 8x. On the right side, I have positive 3 and minus 6 or I can think of it as negative 6. Positive 3 and negative 6 cancel eachother out. We are left with negative 3.

$$\begin{array}{l} 6 + 2x = 3 + 10x \\ \quad -10x \qquad -10x \\ 6 - 8x = 3 \\ -6 \qquad -6 \\ \quad \dfrac{-8x}{-8} = \dfrac{-3}{-8} \\ \quad \boxed{x = \dfrac{3}{8}} \end{array}$$

Now I divide by negative 8 on both sides.

I get x equals something. I have to think a negative divided by a negative is positive because they have the same sign as each other. So x equals 3 eighths.

$$\begin{array}{l} 6 + 2x = 3 + 10x \\ \quad -2x \qquad -2x \end{array}$$

Now, I want you to notice how much easier it is if I subtract the number with the smaller coefficient instead. This is the same problem. But I am going to subtract 2x from both sides.

$$\begin{array}{l} 6 + 2x = 3 + 10x \\ \quad -2x \qquad -2x \\ \quad 6 \quad = 3 + 8x \end{array}$$

Then I get 6 equals 3 plus 8x. It is really to figure out 10x minus 2x is 8x. No negatives are involved.

$$\begin{array}{l} 6 + 2x = 3 + 10x \\ \quad -2x \qquad -2x \\ \quad 6 \quad = 3 + 8x \\ \ -3 \qquad -3 \\ \quad 3 = 8x \end{array}$$

Next I will subtract 3 from each side.

This is easy too because it's just 6 minus 3. I get 3 equals 8x.

$$\begin{array}{l} 6 + 2x = 3 + 10x \\ \quad -2x \qquad -2x \\ \quad 6 \quad = 3 + 8x \\ \ -3 \qquad -3 \\ \quad \dfrac{3}{8} = \dfrac{8x}{8} \\ \quad \boxed{\dfrac{3}{8} = x} \end{array}$$

I divide by 8 on both sides.

And I get 3 eighths equals x. It is the same answer as before but I didn't have to think about negative numbers at all. So, it is going to be a good idea to think and do some number crunching in my mind before I start writing. And then I will try to do the numbers that will make things simpler and easier.

Let's Think (Slide 6): I want us to explore one more tricky spot. This says, "Sometimes we might begin with one strategy and realize there is a simpler strategy. Sometimes we might get zero." We've already talked about how we might do some thinking in our head before we just jump to a first step. The other thing to realize is that it's okay to get zero. Just as long as we don't totally drop it, we can keep working and making sure the equation stays balanced. Here is a good example. When I look at this problem, I might think that it would be easiest if I divided each side by 3. That would cancel out the 3 on the right side. But then I would end up with a fraction on the left side of my equation. It would be like 1 divided by 3, and that isn't a really friendly number to work with. So that is invisible thinking that I am doing before I just start writing things now. And now I might decide to distribute the 3 instead. I would do the 3 times the 2x and the 3 times the 6.

$x = 3(2x - 6)$

$x = 3(2x - 6)$
$x = 6x - 18$

My new equation is x equals 6x minus 18.

$x = 3(2x - 6)$
$x = 6x - 18$
$-x \quad -x$

Now let's say I remember that I want to subtract the variable with the smaller coefficient so I do minus x on both sides.

$x = 3(2x - 6)$
$x = 6x - 18$
$0 = 5x - 18$
$+18 \quad +18$

When I rewrite this, I will get x minus x, which is zero. Even though zero is nothing, I can't just leave this side empty. I write down the zero because there is still more math to do and I will have to add, subtract, multiply or divide that zero. I get zero equals 5x minus 18. I want to get x alone so I will add 18 to both sides.

$x = 3(2x - 6)$
$x = 6x - 18$
$-x \quad -x$
$0 = 5x - 18$
$+18 \quad +18$
$\frac{18}{5} = \frac{5x}{5}$

Now we can see why it was so important to keep that zero. I have zero plus 18 which is 18. That equals 5x.

I divide by 5 on both sides.

$5 \overline{)18}$ → $03\frac{3}{5}$, -15, 3

And I am going to have to go off to the side of my paper and do that division. 5 goes into 18 three times. I subtract 15 and have 3 leftover, which becomes 3 fifths.

$x = 3(2x - 6)$
$x = 6x - 18$
$-x \quad -x$
$0 = 5x - 18$
$+18 \quad +18$
$\frac{18}{5} = \frac{5x}{5}$
$\boxed{3\frac{3}{5} = x}$

I get 3 and 3 fifths equals x. That is my answer. So again, we have to do some pre-thinking. Sometimes we might begin with one strategy and realize there is a simpler strategy. And sometimes we might get zero. That's fine. We write that zero down and keep doing the math.

Let's Try It (Slide 7): Let's solve some equations with variables on both sides together. I will walk you through each step.

WARM WELCOME

Today we will solve linear equations with variables on both sides.

Let's Review:

We know that we need to keep any equation we are solving balanced by doing the same operation to both sides.

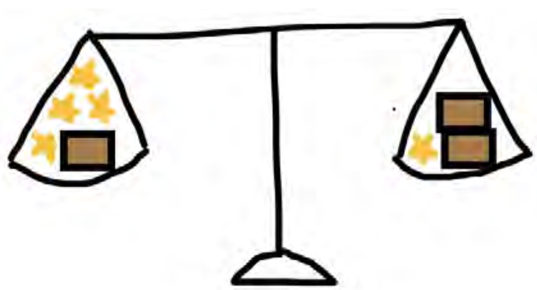
Solve for x.

$7(2x + 3) - 5 = 16$

Let's Talk:

The balance can help us think about what to do when there is a variable on both sides of the equal sign.

If the scale below is balanced, how many stars must be hidden in one of the equal boxes?



Write an equation to represent your work.

Let's Think: **Just like we can add or subtract numbers, we can add or subtract a variable and its coefficient.**

It is generally easier to subtract the number with the smaller coefficient so you don't get negative numbers if you don't have to.

$6 + 2x = 3 + 10x$

$6 + 2x = 3 + 10x$

Let's Think: **Sometimes we might begin with one strategy and realize there is a simpler strategy. Sometimes we might get zero.**

Solve for x.

$x = 3(2x - 6)$

Let's Try It:

Let's solve some equations together!

Name: ______________________________ G8 U4 Lesson 4 - Let's Try It

Solve for x in the equations.

1. To solve for a variable, we must ____________________ and get the variable on its own.

2. To keep the equation balanced, we must ______________________________.

3. Solve for x. $-7x = 3(2x + 1)$ $x =$ ____

4. Solve for x. $-9x - 6 = -7x + 6$ $x =$ ____

5. Solve for x. $2(6 - 3x) = -18 + 9x$ $x =$ ____

On your Own:

Now it's time for you to do it on your own!

Name: ______________________________ G8 U4 Lesson 4 - Independent Work

Solve for the variable in each equation.

1. $2 + 4a = 9 - 3a$	2. $3b = 5(b + 4)$	3. $3(8 + 2c) = 5c + 10$
7. $3(-g + 5) = 2(3 - 3g)$	8. $-4(4h + 8) = 8h$	9. $-5i = -2(1 - 2i) - 1$

Name: ____

Solve for x in the equations.

1. To solve for a variable, we must ____________________________ and get the variable on its own.

2. To keep the equation balanced, we must ______________________________________.

3. Solve for x. $-7x = 3(2x + 1)$ x = _____

4. Solve for x. $-9x - 6 = -7x + 6$ x = _____

5. Solve for x. $2(6 - 3x) = -18 + 9x$ x = _____

Name: ____________________________________

Solve for the variable in each equation.

1. $2 + 4a = 9 - 3a$	2. $3b = 5(b + 4)$	3. $3(8 + 2c) = 5c + 10$
7. $3(-g + 5) = 2(3 - 3g)$	8. $-4(4h + 8) = 8h$	9. $-5i = -2(1 -2i) - 1$

Solve for the variable in each equation.

10. $4j + 3 = 20 - 3j - 3$	11. $3k + 13 = 2(-1 - 6k)$	12. $6(2m + 3) = 17 - m$
16. $-20 + 7g = -5 + g - 15$	17. $3(h - 8) + 5 = 2h$	18. $-3a = 10 + 2a$

Name: ANSWER KEY

Solve for x in the equations.

1. To solve for a variable, we must work backwards and get the variable on its own.

2. To keep the equation balanced, we must do the same to both sides.

3. Solve for x. $-7x = 3(2x + 1)$ $x = -\frac{3}{13}$

$-7x = 6x + 3$

$-6x \quad -6x$

$\frac{-13x}{-13} = \frac{3}{-13}$

$x = -\frac{3}{13}$

4. Solve for x. $-9x - 6 = -7x + 6$ $x = -6$

$+9x \quad +9x$

$-6 = 2x + 6$

$-6 \quad -6$

$\frac{-12}{2} = \frac{2x}{2}$

$-6 = x$

5. Solve for x. $2(6 - 3x) = -18 + 9x$ $x = 2$

$12 - 6x = -18 + 9x$

$+6x \quad +6x$

$12 = -18 + 15x$

$+18 \quad +18$

$\frac{30}{15} = \frac{15x}{15}$

$2 = x$

Name: ANSWER KEY G8 U4 Lesson 4 - Independent Work

Solve for the variable in each equation.

1. $2 + 4a = 9 - 3a$ $+3a \quad +3a$ $2 + 7a = 9$ $-2 \quad -2$ $\frac{7a}{7} = \frac{7}{7}$ $\boxed{a = 1}$	2. $3b = 5(b + 4)$ $3b = 5b + 20$ $-3b \quad -3b$ $0 = 2b + 20$ $-20 \quad -20$ $\frac{-20}{2} = \frac{2b}{2}$ $\boxed{-10 = b}$	3. $3(8 + 2c) = 5c + 10$ $24 + 6c = 5c + 10$ $-5c \quad -5c$ $24 + c = 10$ $-24 \quad -24$ $\boxed{c = -14}$
7. $3(-g + 5) = 2(3 - 3g)$ $-3g + 15 = 6 - 6g$ $+3g \quad +3g$ $15 = 6 - 3g$ $-6 \quad -6$ $\frac{9}{-3} = \frac{-3g}{-3}$ $\boxed{-3 = g}$	8. $-4(4h + 8) = 8h$ $-16h - 32 = 8h$ $+16h \quad +16h$ $\frac{-32}{24} = \frac{24h}{24}$ $\boxed{-1\frac{1}{3} = h}$ $24\overline{)32}$: 01, -24, 8 $1\frac{8 \div 8}{24 \div 8} = 1\frac{1}{3}$	9. $-5i = -2(1 - 2i) - 1$ $-5i = -2 + 4i - 1$ $-4i \quad -4i$ $-9i = -2 - 1$ $\frac{-9i}{-9} = \frac{-3}{-9}$ $i = \frac{3 \div 3}{9 \div 3}$ $\boxed{i = \frac{1}{3}}$

Solve for the variable in each equation.

10.

$4j + 3 = 20 - 3j - 3$

$+3j \quad +3j$

$7j + 3 = 20 - 3$

$7j + 3 = 17$

$-3 \quad -3$

$\frac{7j}{7} = \frac{14}{7}$

$j = 2$

11.

$3k + 13 = 2(-1 - 6k)$

$3k + 13 = -2 - 12k$

$+12k \quad +12k$

$15k + 13 = -2$

$-13 \quad -13$

$\frac{15k}{15} = \frac{-15}{15}$

$k = -1$

12.

$6(2m + 3) = 17 - m$

$12m + 18 = 17 - m$

$+m \quad +m$

$13m + 18 = 17$

$-18 \quad -18$

$\frac{13m}{13} = \frac{-1}{13}$

$m = -\frac{1}{13}$

16.

$-20 + 7g = -5 + g - 15$

$-g \quad -g$

$-20 + 6g = -5 - 15$

$-20 + 6g = -20$

$+20 \quad +20$

$\frac{6g}{6} = \frac{0}{6}$

$g = 0$

17.

$3(h - 8) + 5 = 2h$

$3h - 24 + 5 = 2h$

$-2h \quad -2h$

$h - 24 + 5 = 0$

$+24 \quad +24$

$h + 5 = 24$

$-5 \quad -5$

$h = 19$

18.

$-3a = 10 + 2a$

$-2a \quad -2a$

$\frac{-5a}{-5} = \frac{10}{-5}$

$a = -2$

G8 U4 Lesson 5
Solve linear equations by combining like terms.

CITYTUTOR DC

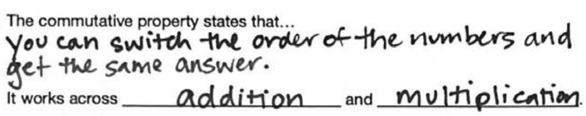
G8 U4 Lesson 5 - Today we will solve linear equations by combining like terms.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will solve linear equations by combining like terms. This is the final lesson in this series on solving equations, and then in our next lesson we get to move onto applications on this important skill. Let's go!

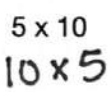
Let's Review (Slide 3): Today we are starting by asking, "What is the commutative property?" That is going to really help us simplify the equations we have to solve. So, what do you remember from earlier grades. What is the commutative property? Possible Student Answers, Key Points:

- The commutative property is the turnaround rule.
- The commutative property means you can switch the order of the numbers and get the same answer.
- The commutative property means you can do 3 x 4 or 4 x 3.

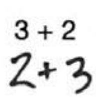


The commutative property states that… you can switch the order of the numbers and get the same answer. But here's the key thing to remember - it only work across ADDITION and MULTIPLICATION. 5 takeaway 1 is not the same as 1 takeaway 5. You can't switch the order of subtraction or division.

Let's use the commutative property to rearrange the expressions. 5 x 10 is easy. It can be rewritten as

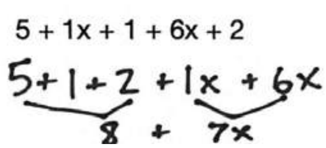


10 x 5. We would get the same answer either way. If I could 5 ten times. *Put up your fingers as you skip count by 5.* That is 5 - 10 - 15 - 20 - 25 - 30 - 35- 40 - 45 - 50 - 55 - 60 - 65 - 70 - 75 - 80 - 85 - 90 - 95 - 100! Or I could count 10 five times. *Put up your fingers as you skip count by 10.* That is 10 - 20 - 30 - 40 - 50! We get the same answer even if we switch the order.



3 + 2 is easy! We get the same answer if we do 2 + 3. I can count up from 3 and get 3 - 4 - 5! Or I can count up from 2 and get 2 - 3 - 4 - 5!

This last one might look scary but really it's easy too. As long as I stick to using the commutative property across addition, I can just move all of these separate addends around in a different order. So I

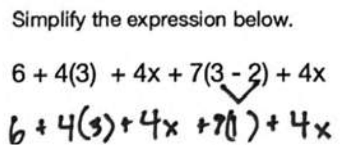


could do 5 + 1 + 2 + 1x + 6x. It's still the same numbers just getting added in a different order. This is the expression that is really interesting because it helps us see that this expression can be simplified. 5 plus 1 plus 2 is 8 and 1x plus 6x is 7x. So when I use the commutative property, it helps me see that this expression is the same as 8 + 7x. Let's do this again on the next slide so we're clear on how the commutative property is so helpful here.

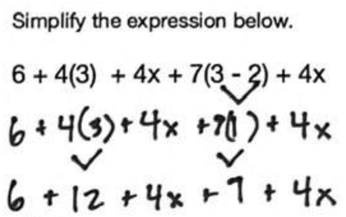
Let's Talk (Slide 4): You probably already know how to simplify an expression from previous grades. How would you go about simplifying this expression? *The purpose of this question is just to hear what students think. It is not to arrive at a right answer. So you can collect responses and just say, "Interesting!" after each one.* Possible Student Answers, Key Points:

- I would do what is in the parentheses first.
- I would use PEMDAS.
- I would do 4 times 3.

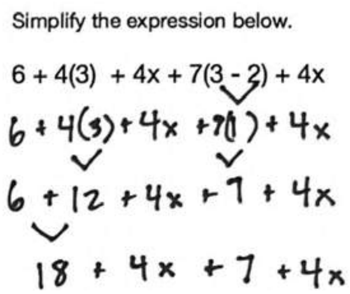
- I would do 7 times 2 and 7 times 3.
- I would put 4x and 4x together.



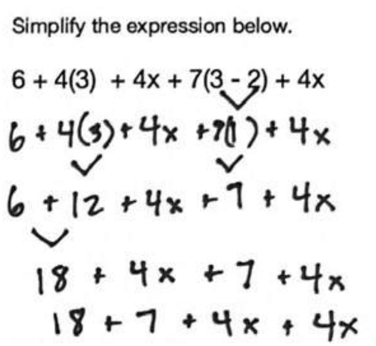
There's actually many different right ways to simplify the expression. But in the past, you probably focused a lot on PEMDAS. You might start with the parentheses, and do 3 minus 2 is 1. Then rewrite the expression as 6 + 4(3) + 4x + 7(1) + 4x.



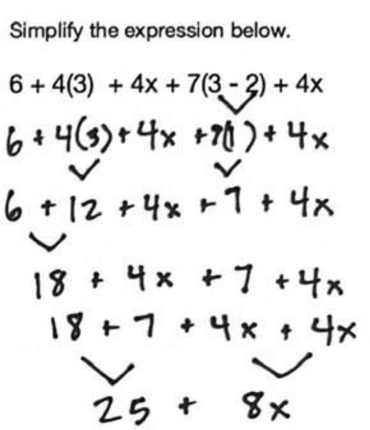
Then you would do the multiplication and division. So that would be 4 times 3 and 7 times 1 in this case. You would end up with 6 + 12 + 4x + 7 + 4x. You can't do the multiplication and division for the variables because you don't know what they are.



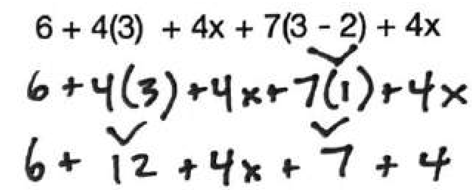
So next up is the addition and subtraction, and you would probably want to start working from left to right. You would do 6 plus 12 is 18 plus 4x. You still have this 7 and then another 4x. It feels tricky to do that without knowing what the variables are.



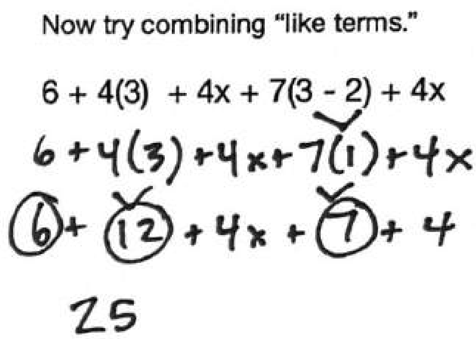
This is where the commutative property can be really helpful. I can change the order of the addends and rewrite this as 18 plus 7 plus 4x plus 4x.



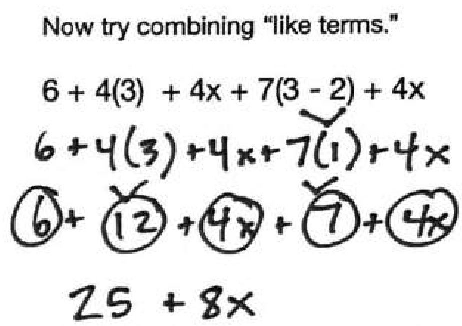
And now I could go back to adding. 18 plus 7 is 25 and 4x plus 4x is 8x. My new expression is 25 + 8x. This is the most important big idea of today's lesson. We ended up putting all the plain numbers together - the 6 and the 12 and the 7. And we ended up putting all the same variables together. This is called "combining like terms." We are combining or putting together the parts of the equations that are "like" or similar. Numbers with numbers and variables with variables.



From now on, if you understand the commutative property, you can go ahead and do that without rewriting the whole expression. In the part of PEMDAS when it is time to add and subtract, you can circle all the “like terms” and put them together. Let me show you what I mean. First, I am going to do the parentheses, just like before. I get 6 + 4(3) + 4x + 7(1) + 4x. Then I do the multiplication and division and get 6 + 12 + 4x + 7 + 4x.

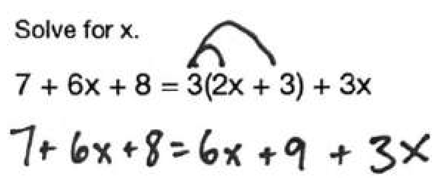


Now I am going to circle all the numbers - 6 and 12 and 7 - and I am going to put them together. 6 and 12 is 18. 18 plus 7 is 25.

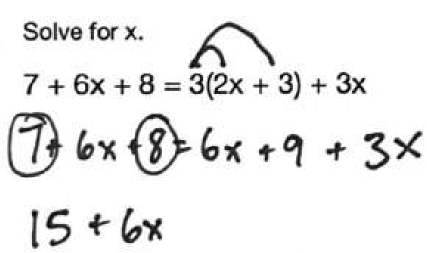


Then I am going to circle all the variables with their coefficients - 4x and 4x. That makes 8x. My final expression is 25 + 8x. This final expression is much simpler than the original. So now imagine if it were part of an equation you had to solve! It would be way easier to simplify it and combine like terms before you start working backwards to isolate x. We’re going to do that on the next slide.

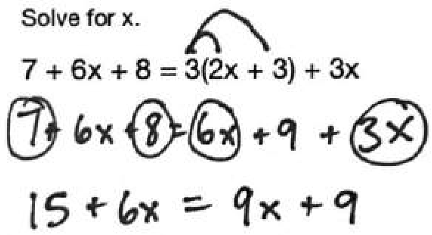
Let’s Think (Slide 5): We will use the commutative property across addition to combine like terms.



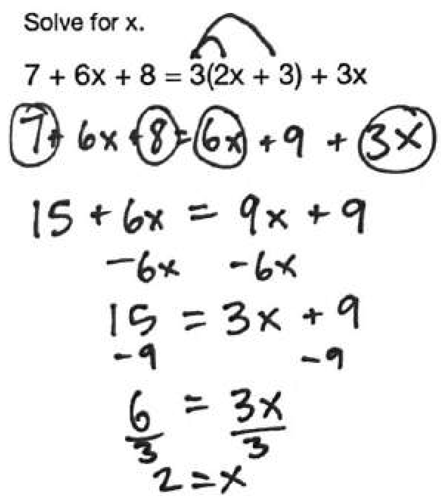
This problem wants us to solve for x. Now, BEFORE I start working backwards to solve, I can try to simplify the expression on each side and make this into an easier problem. I can’t do the math inside the parentheses because there is a variable there. But I can distribute the factor outside the parentheses. I don’t have to but let’s do that for now. I will draw my arrows to show how I’m multiplying each part. I get 7 + 6x + 8 = 6x + 9 + 3x.



Now it would be time to add or subtract so that means it is time to combine like terms. Here’s the key. I have NOT even started solving my problem yet. So I’m not working on both sides of the equation and keeping it balanced YET. I am just simplifying the left side to make my life easier and simplifying the right side to make my life easier. They are separate sides. I look over at the left and I collect the numbers 7 and 8. Let’s circle those. They make 15 so this side is really 15 + 6x.

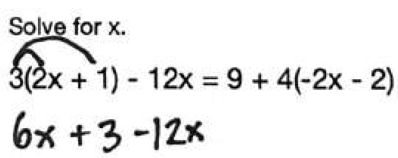


On the other side, I don't have any plain numbers to collect. But I can collect 6x and 3x. Let me circle them. That makes 9x plus 9. Look at this new equation. 15 + 6x equals 9x + 9. This is WAY WAY easier to solve. You can all work backwards and get this. The thing that made it so easy is that we combined like terms.

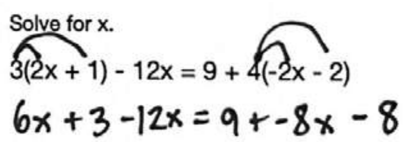


Let's finish this up. I can subtract 6x from both sides. I get 15 = 3x + 9. I can subtract 9 from both sides. I get 6 = 3x. I can divide by 3 on both sides. I get 2 = x. Nice! If I hadn't collected like terms, I still could have solved this. But it would have taken a lot of extra steps. So from now on, we will collect like terms, keeping each side of the equation separate, to simplify our equation. Then we can solve.

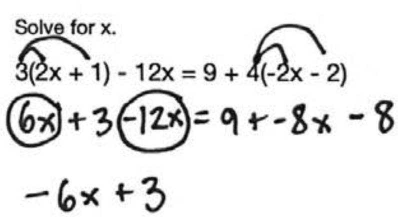
Let's Think (Slide 6): We have one extra complication that we need to discuss, and that is minus signs! This says, "Since we are using the commutative property across addition, it can help to turn our subtraction into adding the opposite." This might not always be true. But sometimes it can be true. I will show you what I mean. First off, I will do some distribution - since multiplication and division comes before addition and subtraction. I'm not solving yet. I'm just doing some work on the left and doing some work on the right. 3 times 2x is 6x and 3 times 1 is 3. I have all of that minus 12x.



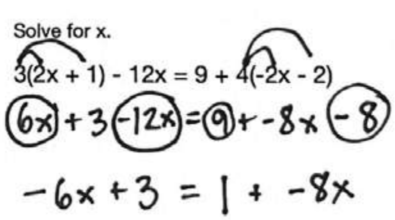
On the other side, I have 9 plus this stuff I have to distribute. 4 times negative 2x is negative 8x. Then there is a minus sign and 4 times 2 is 8.

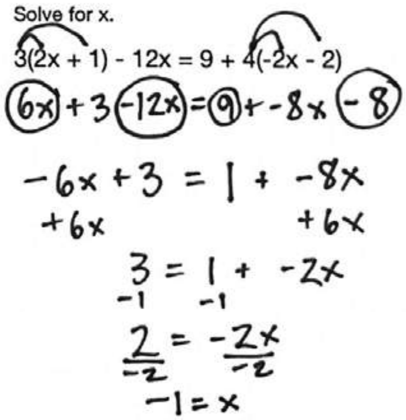


Now I can collect like terms. And here is where thinking of the minus sign as adding the opposite or in other words, negative, can be helpful. For example, on the left, I can't just circle 6x and 12x because the problem doesn't have us adding 6x and 12x. It has us subtracting 12x. Instead, I am going to circle the whole minus 12x. Now I'm really thinking of it like negative 12x. And I can combine 6x and negative 12x, which leaves me with -6x + 3.



On this other side, I want to circle like terms. I can't just circle 9 and 8 and ignore that this 8 is being subtracted. I am going to circle the whole minus 8. Now I'm really thinking of it like negative 8. Positive 9 and negative 8 cancel so I am left with 1 plus negative 8x.





Now this looks like something we know how to solve pretty easily. I will add 6x to both sides. That gives me 3 = 1 + (-2x). Then I subtract 1 from both sides. That gives me 2 = -2x. I divide by negative 2 on both sides. I get -1 = x.

So, from now, we are going to combine like terms before we start solving for the variable. We are going to include the minus sign and think of a number as negative if we're trying to combine it.

Let's Try It (Slide 7): Let's do some combining like terms to solve equations together. I will walk you through each step.

WARM WELCOME

Today we will solve linear equations by combining like terms.

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Let's Review: What is the commutative property?

The commutative property states that…

It works across ___________________________ and _________________________.

Use the commutative property to rearrange the expressions.

5 x 10　　　　3 + 2　　　　5 + 1x + 1 + 6x + 2

Let's Talk: We can use the commutative property across addition to help us simplify our expressions.

Simplify the expression below.

6 + 4(3) + 4x + 7(3 - 2) + 4x

Now try combining "like terms."

6 + 4(3) + 4x + 7(3 - 2) + 4x

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Let's Think: **We will use the commutative property across addition to combine like terms.**

Solve for x.

$7 + 6x + 8 = 3(2x + 3) + 3x$

Let's Think: **Since we are using the commutative property across addition, it can help to turn our subtraction into adding the opposite.**

Solve for x.

$3(2x + 1) - 12x = 9 + 4(-2x - 2)$

Let's Try It:

Let's combine some like terms together!

Name: ______________________ G8 U4 Lesson 5 - Let's Try It

Solve for x in the equations.

1. Our first step will be to ______________________
2. Then we will ______________________
3. Then we can ______________________ to solve.
4. Solve for x. $6 + 3(1x + 4) = 17 + 4x + 10$ x = ____
5. Solve for x. $3 - 7x + 5 = 2x + 3(2x + 1)$ x = ____

On your Own:

Now it's time for you to do it on your own!

Name: ______________________ G8 U4 Lesson 5 - Independent Work

Solve for the variable in each equation.

1. $3(4x-1)-9 = -8 + 8x - 9$	2. $6w + 1 - w = 7(2w+10)-6$	3. $6 + 3(8 + 2c) = 8 + 5c + 10$
7. $3(-g + 5) - 10 = g - 2(3 - 3g)$	8. $6 - 4(4h + 8) = 8h - h$	9. $6(2i - 1) - 5i = -2(1 - 2i) - 1$

Name: ______________________________________

Solve for x in the equations.

1. Our first step will be to ____________________________________.

2. Then we will ____________________________________.

3. Then we can ____________________________________ to solve.

4. Solve for x. $3(2x-2)-9 = -6 + 4x - 5$ x = _____

5. Solve for x. $3(h - 8) + 5 = 2h(3 + 4)$ x = _____

6. Solve for x. $9 - 2(6 + x) = 9 + 2x + 6$ x = _____

Name: ____

Solve for the variable in each equation.

1. $3(4x–1)–9 = -8 + 8x - 9$	2. $6w +1 - w = 7(2w+10)–6$	3. $6 + 3(8 + 2c) = 8 + 5c + 10$
4. $3(-g + 5) - 10 = g - 2(3 - 3g)$	5. $6 - 4(4h + 8) = -8h - h$	6. $6(2i - 1) - 5i = -2(1- 2i) - 1$

Solve for the variable in each equation.

7. $4j + 3 - 2j = 19 - 3(j - 3)$	8. $2k - 4k + 13 = 2(-1 - 6k) + 5$	9. $3(2x-1)-4 = -4 + 4x - 3$
10. $4 + 6w + 1 - w = 7w + 10$	11. $4(-6x + 2)=-4(-5x - x) + 8$	12. $-3a + 3(a+4)= 10 + 2a - 2$

Name: ANSWER KEY

Solve for x in the equations.

1. Our first step will be to distribute.

2. Then we will collect like terms.

3. Then we can work backwards to solve.

4. Solve for x. $3(2x-2)-9 = -6 + 4x - 5$ x = 2

$6x-6-9=-6+4x-5$

$6x-15=-11+4x$

$+15 \quad +15$

$6x = 4+4x$

$-4x \quad -4x$

$\frac{2x}{2}=\frac{4}{2}$

$x=2$

5. Solve for x. $3(h - 8) + 5 = 2h(3 + 4)$ x = $-1\frac{8}{11}$

$3h-24+5=6h+8h$

$3h-19=14h$

$-3h \quad -3h$

$\frac{-19}{11}=\frac{11h}{11}$

$-\frac{19}{11}=h$

$-1\frac{8}{11}=h$

6. Solve for x. $9 - 2(6 + x) = 9 + 2x + 6$ x = $-1\frac{1}{2}$

$9-12+-2x=9+2x+6$

$-3+-2x=3+2x$

$+2x \quad +2x$

$-3=3+4x$

$-3 \quad -3$

$-\frac{6}{4}=\frac{4x}{4} \rightarrow -1\frac{2}{4}=x \rightarrow -1\frac{1}{2}=x$

Name: ANSWER KEY

Solve for the variable in each equation.

1. $3(4x-1)-9 = -8 + 8x - 9$ $12x-3-9=-8+8x-9$ $12x-12=-17+8x$ $+12 \quad +12$ $12x=5+8x$ $-8x \quad -8x$ $\frac{4x}{4}=\frac{5}{4}$ $x=1\frac{1}{4}$	2. $6w + 1 - w = 7(2w+10)-6$ $6w+1-w=14w+70-6$ $6w+1-w=14w+64$ $5w+1=14w+64$ $-1 \quad -1$ $5w=14w+63$ $-14w \quad -14w$ $\frac{-9w}{-9}=\frac{63}{-9}$ $w=-7$	3. $6 + 3(8 + 2c) = 8 + 5c + 10$ $6+24+6c=8+5c+10$ $30+6c=18+5c$ $-6c \quad -6c$ $30=18-1c$ $-18 \quad -18$ $\frac{22}{-1}=\frac{-1c}{-1}$ $22=c$
3. $3(-g + 5) - 10 = g - 2(3 - 3g)$ $-3g+15-10=g-6+6g$ $-3g+5=-6+7g$ $+3g \quad +3g$ $5=-6+10g$ $+6 \quad +6$ $\frac{11}{10}=\frac{10g}{10}$ $10\overline{)11}$, -10, 1 → $01\frac{1}{10}$ $1\frac{1}{10}=g$	4. $6 - 4(4h + 8) = 8h - h$ $6-16h-32=8h-h$ $-16h-26=8h-h$ $-16h-26=9h$ $+16h \quad +16h$ $\frac{-26}{7}=\frac{7h}{7}$ $7\overline{)26}$, -21, 5 → $03\frac{5}{7}$ $-3\frac{5}{7}=h$	5. $6(2i - 1) - 5i = -2(1- 2i) - 1$ $12i-6-5i=-2+4i-1$ $7i-6=-3+4i$ $+3 \quad +3$ $7i-3=4i$ $-7i \quad -7i$ $\frac{-3}{-3}=\frac{-3i}{-3}$ $1=i$

Solve for the variable in each equation.

7. $4j + 3 - 2j = 19 - 3(j - 3)$ $4j+3-2j=19-3j+9$ $2j+3=28-3j$ $-2j \quad -2j$ $3=28-5j$ $-28 \quad -28$ $\frac{-25}{-5}=\frac{-5j}{-5}$ $5=j$	8. $2k - 4k + 13 = 2(-1 - 6k) + 5$ $2k-4k+13=-2-12k+5$ $-2k+13=-12k+3$ $+2k \quad +2k$ $13=-10k+3$ $-3 \quad -3$ $\frac{10}{-10}=\frac{-10k}{-10}$ $-1=k$	9. $3(2x-1)-4 = -4 + 4x - 3$ $6x-3-4=-4+4x-3$ $6x-7=-7+4x$ $+7 \quad +7$ $6x=4x$ $-4x \quad -4x$ $\frac{2x}{2}=\frac{0}{2}$ $x=0$
10. $4 + 6w + 1 - w = 7w + 10$ $4+5w+1=7w+10$ $5+5w=7w+10$ $-5w \quad -5w$ $5=2w+10$ $-10 \quad -10$ $\frac{-5}{2}=\frac{2w}{2}$ $-1\frac{1}{2}=w$	11. $4(-6x + 2)=-4(-5x - x) + 8$ $-24x+8=20x+4x+8$ $-24x+8=24x+8$ $-8 \quad -8$ $-24x=24x$ $-24x \quad -24x$ $\frac{-48x}{-48}=\frac{0}{-48}$ $x=0$	12. $-3a + 3(a+4)= 10 + 2a - 2$ $-3a+3a+12=10+2a-2$ $0+12=8+2a$ $-8 \quad -8$ $\frac{4}{2}=\frac{2a}{2}$ $2=a$

G8 U4 Lesson 6
Determine whether a linear equation has one solution, no solutions, or infinitely many solutions.

CITYTUTOR DC

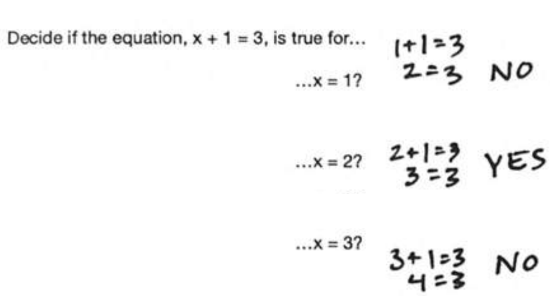
G8 U4 Lesson 6 - Today we will determine whether a linear equation has one solution, no solution, or infinitely many solutions.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will determine whether a linear equation has one solution, no solution, or infinitely many solutions. This is going to be the last lesson before we move into systems of equations, which is really the biggest thing in 8th grade algebra. And we need this lesson to be able to do so let's go.

Let's Review (Slide 3): We know how to plug in values to see if they make the equation true. This is asking us to "decide if the equation, x+1=3, is true for" these different values. What do we do to check if x = 1 is a solution for the equation? Possible Student Answers, Key Points:

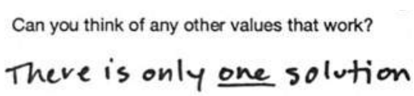
- Plug in 1 for x.



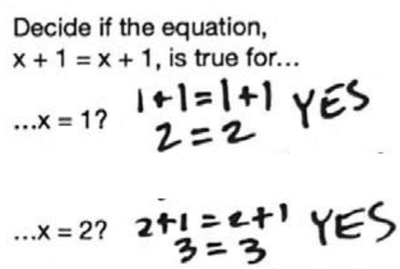
We put in 1 where we see x. So it is 1 + 1 = 3, which becomes 2 = 3. That is not true so is it a solution? No!

Let's try x = 2. That becomes 2 + 1 = 3. That becomes 3 = 3. That IS true so is it a solution? Yes!

Let's try x = 3. That becomes 3 + 1 = 3. That becomes 4 = 3. That is not true so is it a solution? No!



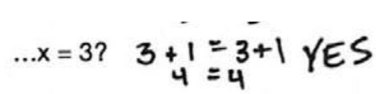
Can you think of any other values that work? X = 4? X = 5? That would just get us bigger and bigger. It would still not be equal or true. This might seem really obvious but this equation only 1 solution. Our variable equals a number like we're used to. We get one answer or one solution.

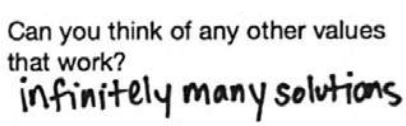


Let's Talk (Slide 4): Some equations have more than one solution, and some equations have no solution. Let's try two other equations and see what happens when we plug in numbers like we did on the last slide. First, x = 1. It would be 1 + 1 = 1 + 1, which is simplified to 2 = 2. That IS true so it's a solution.

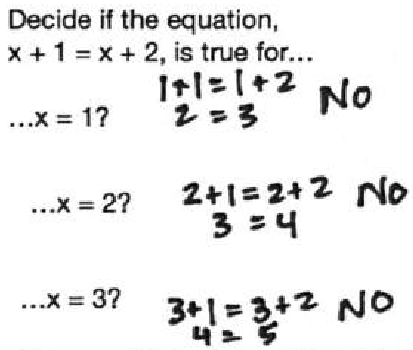
Let's do x = 2 next. It would be 2 + 1 = 2 + 1, which is simplified to 3 = 3. That IS true so it's also solution! Interesting!

Let's do x = 3 next. It would be 3 + 1 = 3 + 1, which is simplified to 4 = 4. That IS true so it's also solution! "Can you think of any other values that work?" Well, the left side of the equation is the same as the right side of the equation. So it seem likes every value would work, doesn't it? Of course 5 plus 1 would be the same as 5 plus 1 and 6 plus 1 would be the same as 6 plus 1 and we could keep going on and on forever.





When one side of the equation matches the other side exactly, it will ALWAYS be true so ALL the numbers are solutions. We say it has "infinitely many solutions.

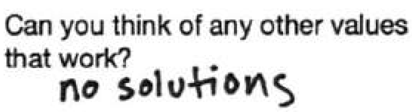


Let's try the next equation and see how that one works. First, x = 1. It would be 1 + 1 = 1 + 2, which is simplified to 2 = 3. That is NOT true so it is NOT a solution.

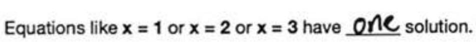
Next up is x = 2. It would be 2 + 1 = 2 + 2, which is simplified to 3 = 4. That is NOT true so it is NOT a solution.

Let's try x = 3. It would be 3 + 1 = 3 + 2, which is simplified to 4 = 5. That is NOT true so it is NOT a solution.

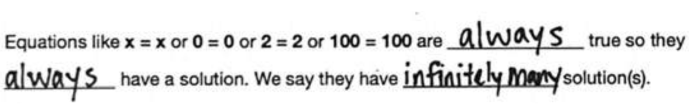
Can you think of any other values that work? *Let the students suggest values and substitute them into the equation to see if they are solutions. None of them will work.* There aren't any values that work because of course x plus one number would never equal x plus another number. Another way to see this is that if we try to solve for x, I would subtract x on both sides. That gives me 1 equals 2, which is impossible. So, x + 1 = x + 2 is NEVER true so it NEVER has any solutions. We say it has "no solutions."



Let's Think (Slide 5): Based on these examples we can notice types. "We can tell how many solutions an equation will have depending on how it looks when it is simplified." Here is our summary.

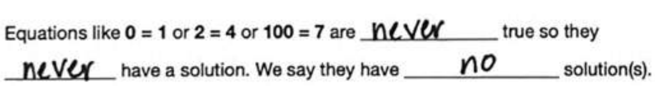


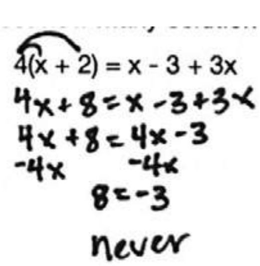
Equations like x = 1 or x = 2 or x = 3 have ONE solution. These are the usual solutions that we have been finding for the last 5 lessons.



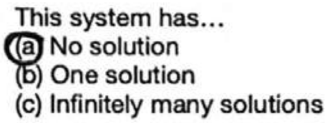
Equations like x = x or 0 = 0 or 2 = 2 or 100 = 100 are ALWAYS true so they ALWAYS have a solution. We say they have INFINITELY MANY solution(s).

Equations like 0 = 1 or 2 = 4 or 100 = 7 are NEVER true so they NEVER have a solution. We say they have NO solution(s). One of the tricks we can use to help ourselves know the solutions is to think about when the simplified equation is true. If it just true for that one number, it has one solution. If it is ALWAYS true that every number is ALWAYS a solution. That's infinitely many. If it is NEVER true that there is NEVER a solution. We say they have NO solutions.

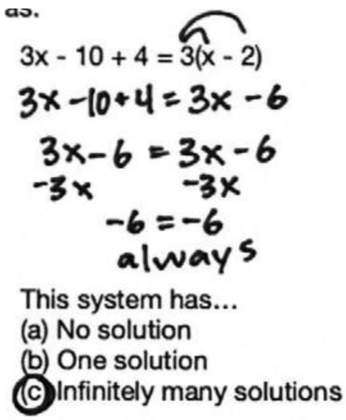




Let's Think (Slide 6): I will model two examples and then we can do some together. This says, "We must try to solve the equations to get it in a form where we can tell how many solutions it has." So for example, this wants us to "select how many solutions each equation has." But I can't just look at these and tell. These look like the same complicated equations we've been working on for a while now. So, I am going to start to try and solve them. And as I get towards the end, they will start to look like one of the types we discussed on the last slide. Let's start with this first equation. I am going to distribute the 4. That gives me 4x + 8 = x - 3 + 3x. Now I am going to collect like terms on the right side. I add x and 3x, which gives me 4x. That's 4x + 8 = 4x - 3. Now this is starting to look like something I can make sense of but let's keep going. I will subtract 4x from both sides. Now I have 8 equals -3.



That will NEVER be true, which means that there is NEVER a solution. I will circle choice (a).



Let's do the next one. I will distribute the 3. My new equation is 3x - 10 + 4 = 3x - 6. Next, I will collect like terms. Negative 10 and 4 make negative 6. So I get 3x minus 6 equals 3x minus 6. We can see where this is going.I subtract 3x from both sides. I get negative 6 equals negative 6. Now, that will ALWAYS be true, which means that there is ALWAYS a solution.

I will circle choice (c). So, sometimes we will need to do some solving in order to see how many solutions an equation has.

Let's Try It (Slide 7): Let's explore more equations together now. I will walk you through each step.

WARM WELCOME

Today we will determine whether a linear equation has one solution, no solution, or infinitely many solutions.

Let's Review: We know how to plug in values to see if they make the equation true.

Decide if the equation, $x + 1 = 3$, is true for…

…$x = 1$?

…$x = 2$?

…$x = 3$?

Can you think of any other values that work?

Let's Talk: Some equations have more than one solution, and some equations have no solution.

Decide if the equation, $x + 1 = x + 1$, is true for…

…$x = 1$?

…$x = 2$?

…$x = 3$?

Can you think of any other values that work?

Decide if the equation, $x + 1 = x + 2$, is true for…

…$x = 1$?

…$x = 2$?

…$x = 3$?

Can you think of any other values that work?

Let's Think: **We can tell how many solutions an equation will have depending on how it looks when it is simplified.**

Equations like **x = 1** or **x = 2** or **x = 3** have ______ solution.

Equations like **x = x** or **0 = 0** or **2 = 2** or **100 = 100** are _____________ true so they ___________ have a solution. We say they have ________________ solution(s).

Equations like **0 = 1** or **2 = 4** or **100 = 7** are _____________ true so they ___________ have a solution. We say they have ________________ solution(s).

Let's Think: **We must try to solve the equation to get it in a form where we can tell how many solutions it has.**

Select how many solutions each equation has.

$4(x + 2) = x - 3 + 3x$

This system has…
(a) No solution
(b) One solution
(c) Infinitely many solutions

$3x - 10 + 4 = 3(x - 2)$

This system has…
(a) No solution
(b) One solution
(c) Infinitely many solutions

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Let's Try It:

Let's explore more equations together!

Name: ____________________ G8 U4 Lesson 6 - Let's Try It

Use the equations to fill in the sentences.

$2 = 2$ $-8 = -8$ $0 = 0$

1. Equations like the ones below are __________ true which means there is ________ solution.
2. In these cases, we say there are ____________ solution(s).

$2 = 1$ $4 = -4$ $0 = 1$

3. Equations like the ones below are __________ true which means there is ________ solution.
4. In these cases, we say there are ____________ solution(s).

Determine if the equations below have no solution, one solution or infinitely many solutions.

1. $3x + 7 + 3x = 1 + 2(x + 3)$	2. $3(x + x) = -2 + x - 8$	3. $-4x + 9 - 3x + 1 = 10 - 1(6x + 1)$

On your Own:

Now it's time for you to do it on your own!

Name: ____________________ G8 U4 Lesson 6 - Independent Work

Solve for the variable in each equation.

1. $x = x$ This system has... (a) No solutions (b) One solution (c) Infinitely many solutions	2. $x + x = 2$ This system has... (a) No solutions (b) One solution (c) Infinitely many solutions	3. $x = x + 1$ This system has... (a) No solutions (b) One solution (c) Infinitely many solutions
4. $3x + 9 = 12 + 3x$	5. $4 - 2x = -2x + 4$	6. $9 - 3x = 4x - 5$

Name: ____

Use the equations to fill in the sentences.

2 = 2 -8 = -8 0 = 0

1. Equations like the ones below are ______________ true which means there is _________ solution.

2. In these cases, we say there are ________________ solution(s).

2 = 1 4 = -4 0 = 1

3. Equations like the ones below are ______________ true which means there is _________ solution.

4. In these cases, we say there are ________________ solution(s).

Determine if the equations below have no solution, one solution or infinitely many solutions.

1.	2.	3.
3x + 7 + 3x = 1 + 2(x +3)	3(x + x) = -2 + x -8	-4x + 9 - 3x + 1 = 10 - 1(6x + 1)
This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	This system has… (a) No solutions (b) One solution (c) Infinitely many solutions

Name: ____

Solve for the variable in each equation.

1.	2.	3.
$x = x$	$x + x = 2$	$x = x + 1$
This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	This system has… (a) No solutions (b) One solution (c) Infinitely many solutions
4.	5.	6.
$3x + 9 = 12 + 3x$	$4 - 2x = -2x + 4$	$9 - 3x = 4x - 5$
This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	This system has… (a) No solutions (b) One solution (c) Infinitely many solutions

Solve for the variable in each equation.

7. $\frac{x+1}{7} - 5 = -2$ This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	8. $7(2 + x) = 7x - 2$ This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	8. $4x + 5 + 2x = 2(3x + 4)$ This system has… (a) No solutions (b) One solution (c) Infinitely many solutions
10. $3x + 10 - x = 2(x + 5) - 1$ This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	11. $20 + 19x = 4x + 8(2x - 1)$ This system has… (a) No solutions (b) One solution (c) Infinitely many solutions	12. $17 + 2(x - 9) = x -1 + x$ This system has… (a) No solutions (b) One solution (c) Infinitely many solutions

Name: ANSWER KEY

Use the equations to fill in the sentences.

$2 = 2$ $-8 = -8$ $0 = 0$

1. Equations like the ones below are always true which means there is always a solution.
2. In these cases, we say there are infinitely many solution(s).

$2 = 1$ $4 = -4$ $0 = 1$

3. Equations like the ones below are never true which means there is never a solution.
4. In these cases, we say there are no solution(s).

Determine if the equations below have no solution, one solution or infinitely many solutions.

1.	2.	3.
$3x + 7 + 3x = 1 + 2(x + 3)$	$3(x + x) = -2 + x - 8$	$-4x + 9 - 3x + 1 = 10 - 1(6x + 1)$
$6x + 7 = 1 + 2x + 6$ $6x + 7 = 2x + 7$ $-2x \quad -2x$ $4x + 7 = 7$ $-7 \quad -7$ $\frac{4x}{4} = \frac{0}{4}$ $x = 0$	$3x + 3x = -2 + x - 8$ $6x = -10 + x$ $-x \quad -x$ $\frac{5x}{5} = \frac{-10}{5}$ $x = -2$	$-7x + 10 = 10 - 7x - 1$ $-7x + 10 = 9 - 7x$ $+7x \quad +7x$ $10 = 9$ never
This system has... (a) No solutions (b) One solution (circled) (c) Infinitely many solutions	This system has... (a) No solutions (b) One solution (circled) (c) Infinitely many solutions	This system has... (a) No solutions (circled) (b) One solution (c) Infinitely many solutions

Name: ANSWER KEY

Solve for the variable in each equation.

1. $x = x$ $-x \quad -x$ $0 = 0$ always This system has... (a) No solutions (b) One solution (c) Infinitely many solutions [circled]	2. $x + x = 2$ $\frac{2x}{2} = \frac{2}{2}$ $x = 1$ This system has... (a) No solutions (b) One solution [circled] (c) Infinitely many solutions	3. $x = x + 1$ $-x \quad -x$ $0 = 1$ never This system has... (a) No solutions [circled] (b) One solution (c) Infinitely many solutions
4. $3x + 9 = 12 + 3x$ $-3x \quad -3x$ $9 = 12$ never This system has... (a) No solutions [circled] (b) One solution (c) Infinitely many solutions	5. $4 - 2x = -2x + 4$ $+2x \quad +2x$ $4 = 4$ always This system has... (a) No solutions (b) One solution (c) Infinitely many solutions [circled]	6. $9 - 3x = 4x - 5$ $+3x \quad +3x$ $9 = 7x - 5$ $-5 \quad -5$ $\frac{4}{7} = \frac{7x}{7}$ $\frac{4}{7} = x$ This system has... (a) No solutions (b) One solution [circled] (c) Infinitely many solutions

Solve for the variable in each equation.

7. $\frac{x+1}{7} - 5 = -2$ $+5 \quad +5$ $\frac{x+1}{7} = 3$ $\times 7 \quad \times 7$ $x+1 = 21$ $-1 \quad -1$ $x = 20$ This system has… (a) No solutions (b) One solution (circled) (c) Infinitely many solutions	8. $7(2 + x) = 7x - 2$ $14 + 7x = 7x - 2$ $-7x \quad -7x$ $14 = -2$ never This system has… (a) No solutions (circled) (b) One solution (c) Infinitely many solutions	8. $4x + 5 + 2x = 2(3x + 4)$ $2x + 5 = 6x + 8$ $-5 \quad -5$ $2x = 6x + 3$ $-6x \quad -6x$ $\frac{-4x}{-4} = \frac{3}{-4}$ $x = -\frac{3}{4}$ This system has… (a) No solutions (b) One solution (circled) (c) Infinitely many solutions
10. $3x + 10 - x = 2(x + 5) - 1$ $3x + 10 - x = 2x + 10 - 1$ $2x + 10 = 2x + 9$ $-2x \quad -2x$ $10 = 9$ never This system has… (a) No solutions (circled) (b) One solution (c) Infinitely many solutions	11. $20 + 19x = 4x + 8(2x - 1)$ $20 + 19x = 4x + 16x - 8$ $20 + 19x = 20x - 8$ $+8 \quad +8$ $28 + 19x = 20x$ $-19x \quad -19x$ $28 = x$ This system has… (a) No solutions (b) One solution (circled) (c) Infinitely many solutions	12. $17 + 2(x - 9) = x - 1 + x$ $17 + 2x - 18 = x - 1 + x$ $2x - 1 = 2x - 1$ $-2x \quad -2x$ $-1 = -1$ always This system has… (a) No solutions (b) One solution (c) Infinitely many solutions (circled)

G8 U4 Lesson 7
Understand what a system of equations is. Determine whether a system of equations will have one solution, no solution or infinitely many solutions using graphs.

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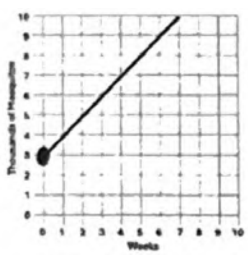
G8 U4 Lesson 7 - Today we will understand what a system of equations is and determine whether a system of equations will have one solution, no solutions or infinitely many solutions using graphs.

Warm Welcome (Slide 1): Tutor choice

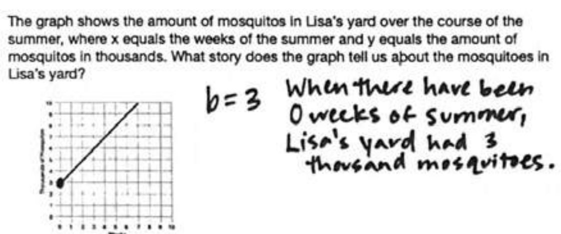
Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will understand what a system of equations is and determine whether a system of equations will have one solution, no solutions or infinitely many solutions using graphs. This is very exciting because systems of equations is really the height of algebra. This is as complicated as it gets. There will be variations on this concept throughout the year. But we're exploring the big idea of algebra for this year now!

Let's Review (Slide 3): We know that graphs can be used to represent real life relationships. In our last unit, we spent many lessons on both proportions and linear relationships. Let's review one. Read along silently with me while I read out loud. *Read the problem silently.* Before we answer the question. Let's ask ourselves this - what parts of the graph could we find to make up the story? What elements should we look for on our graph? Possible Student Answers, Key Points:

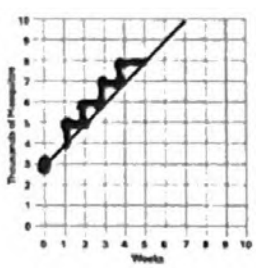


- We can find the slope.
- We should look for the y-intercept.

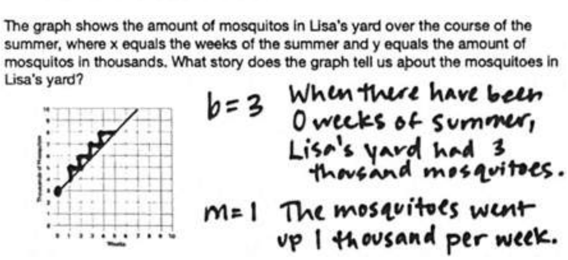
There are two main things we learned to look for in linear relationship: the y-intercept and the slope. The y-intercept is where the line crosses the y-axis. It is also where x = 0. It often tells us where things begin because the x-axis often begins at zero. The y-intercept on this graph is here. *Mark a point at (0,3).* So in this case, it is 3.



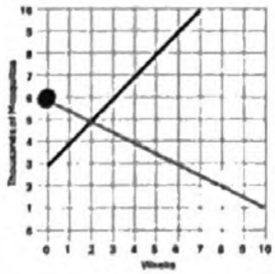
Equations for linear relationships are usually written in the form y=mx+b and b in the y-intercept so I am going to write b = 3. But 3 what? I turn this into a sentence with words from the story. When there have been 0 weeks of summer, Lisa's yard had 3 thousand mosquitoes.



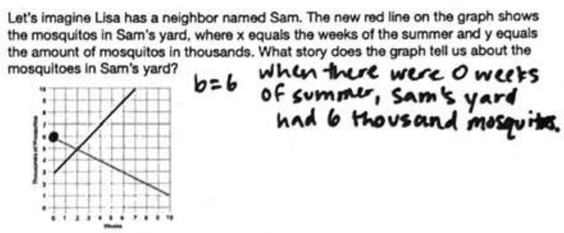
The other part of the graph is the slope. That is measure in rise over run or change in y over change in x. We can use the equation: y_2 minus y_1 over x_2 minus x_1. But I am just going to mark it like a staircase for now because this line has a constant slope which means it has goes up and goes over, goes up and goes over, goes up and goes over.



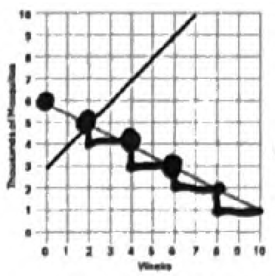
In this case, it goes up 1 thousand mosquitos for every 1 week. We write that as m equals 1. But 1 what? I turn this into a sentence with words from the story. The mosquitos went up 1 thousand mosquitoes per week. It is so cool that we can show phenomenon in the real world with both numbers and a picture!



Let's Talk (Slide 4): Now we are going to take this to the next level. Because that last slide only had one relationship or what would be one equation. A SYSTEM of equations is two or more equations that use the same set of variables. Read along silently with me while I read out loud. *Read the problem silently.* Before we worry about the "system" part of all this. Let's first just name that we can do all the same analysis for the red line that we did for the black line. So, we can still find the y-intercept. It is here. *Mark a point at (0,6).*

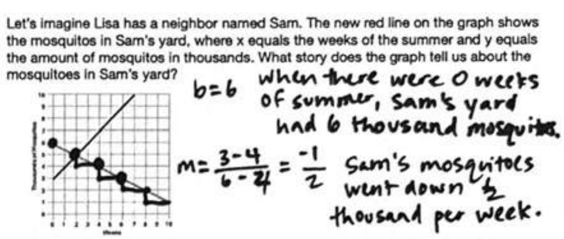


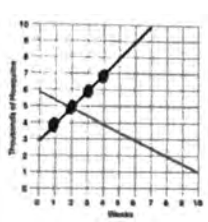
So, b = 6 and I know when there were 0 weeks of summer, Sam had 6 thousand mosquitos in his yard.



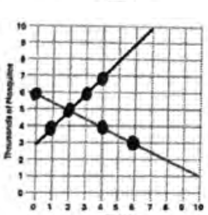
The slope in this case is going down. I'm going to mark 2 points to see it. *Mark (4,4) and (6,3).*

Then we can do m equals y_2 minus y_1 over x_2 minus x_1, which is 3 minus 4 over 6 minus 4. That's negative 1 over 2. So, the mosquito population at Sam's is going down. We can see that in the picture. Maybe he's using some sort of bug spray or something. But it is going down one half thousand mosquitoes per week. The other thing we could calculate for this one is an x-intercept. We don't do that now. But you get the idea - we can have a whole additional line, which would be its own table and its own equation and its own story. But it shares the same variables, which means it shares the same context. In this case, both lines are about thousands of mosquitos each week of summer. It's very cool!

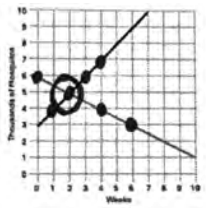




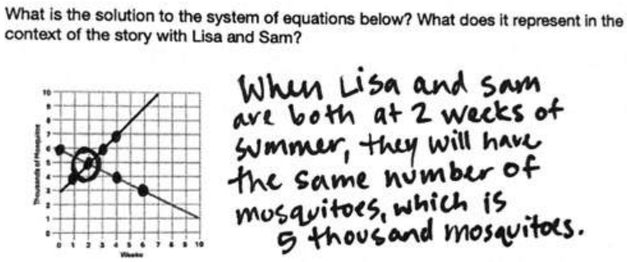
Let's Think (Slide 5): The solution to a system of equations is the point that satisfies all the equations. It is where all the lines meet. It helps to remember that in past lessons, a line could have lots of solutions, right? Every point on the black line is a solution to the equation for the black line. *Draw points along the black line. Be sure to include (2,5) in your points.*



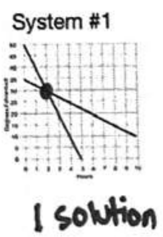
The red line also has tons of solutions. Every point on the red line is a solution to the equation for the red line. *Draw points along the red line. Be sure to include (2,5) in your points.* So, it makes sense that the solution to the whole system is the point that satisfies all the equations. In other words, it has to be a point that goes on all the lines. And that is going to be where all the lines meet.



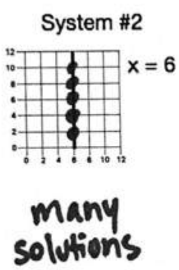
This says, “What is the solution to the system of equations below? What does it represent in the contest of the story with Lisa and Sam?” The point that is on both lines is where these lines meet. Right here. *Circle (2,5).*



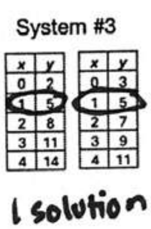
That is when the black line and the red line have the same x and the same y. In the context of this story, that is when there is the same number of mosquitos at the same number of weeks. *Point to the labels for axes as you are using those words.* We can write, “When Lisa and Sam are both at 2 weeks of summer, they will have the same number of mosquitoes, which is 5 thousand mosquitoes.”



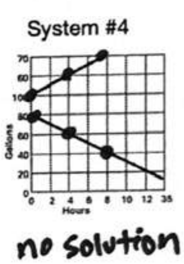
Let’s Think (Slide 6): Now, there are situations where the system won’t have a nice point like that. “We can think about where lines might meet to see if the system has no solution, one solution or many solutions.” This is asking us to “determine how many solutions each system has.” Let’s look. In system #1, this looks like the graph we already saw. I see one point where they meet right here. *Mark a point at (2,30).* So this system has 1 solution.



Okay, this system has a graph and an equation. Let’s think about where they might meet. If I graph x = 6, I draw a line where x is always 6. That is the same as this line already drawn, right? Every point on this line matches every point in this equation. This could be a solution. *Mark a point on the line.* This could be a solution and so on. *Mark many points on the line.* So this system has infinitely many points that are the same. It has infinitely many solutions.



Let’s look at the next one. These are tables but it works the same way. All of these are points and we want the point that is the same. Let me give you 10 seconds to look for it. *Wait for 10 seconds.* It is (1,5). That is the point where they will meet because it is the same point for both of them. This table is all the solutions for one equation. This table is all the solutions for one equation. But this one point is the solution to the whole system. It has one solution.



Last one! Now we are looking for a point that works for both lines. But notice that none of the points on the top line… *Mark points on the top line.* …are the same as points on the bottom line. *Mark points on the bottom line.* The lines don’t meet and they aren’t going to meet if we keep going on in time. So this system has no solution.

Let’s Try It (Slide 7): Let’s identify more solutions to systems together. I will walk you through each step.

WARM WELCOME

Today we will understand what a system of equations is and determine whether a system of equations will have one solution, no solutions or infinitely many solutions using graphs.

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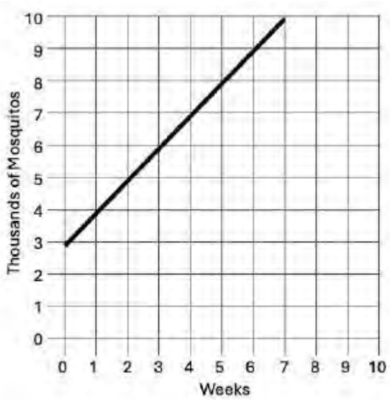
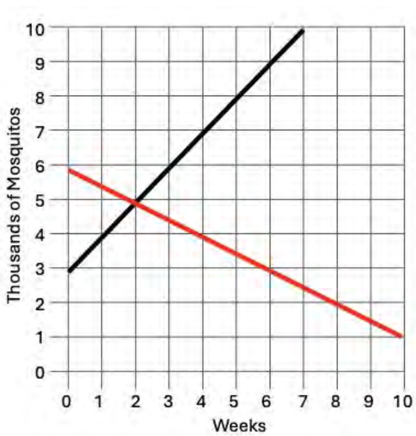
Let's Review: We know that graphs can be used to represent real life relationships.

The graph shows the amount of mosquitos in Lisa's yard over the course of the summer, where x equals the weeks of the summer and y equals the amount of mosquitos in thousands. What story does the graph tell us about the mosquitoes in Lisa's yard?



Let's Talk: A system of equations is two or more equations that use the same set of variables.

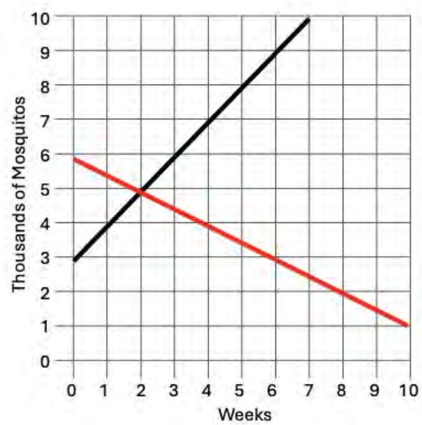
Let's imagine Lisa has a neighbor named Sam. The new red line on the graph shows the mosquitos in Sam's yard, where x equals the weeks of the summer and y equals the amount of mosquitos in thousands. What story does the graph tell us about the mosquitoes in Sam's yard?



Let's Talk:

The solution to a system of equations is the point that satisfies all the equations. It is where all the lines meet.

What is the solution to the system of equations below? What does it represent in the context of the story with Lisa and Sam?

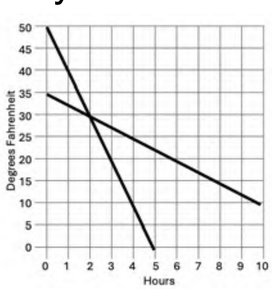


Let's Think:

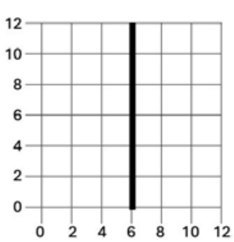
We can think about where lines might meet to see if the system has no solution, one solution or many solutions.

Determine how many solutions each system has.

System #1



System #2



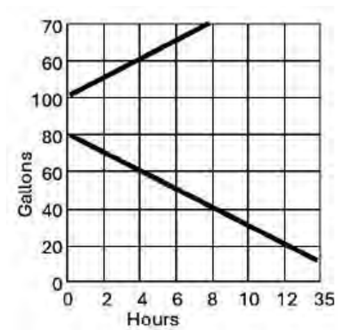
x = 6

System #3

x	*y*
0	2
1	5
2	8
3	11
4	14

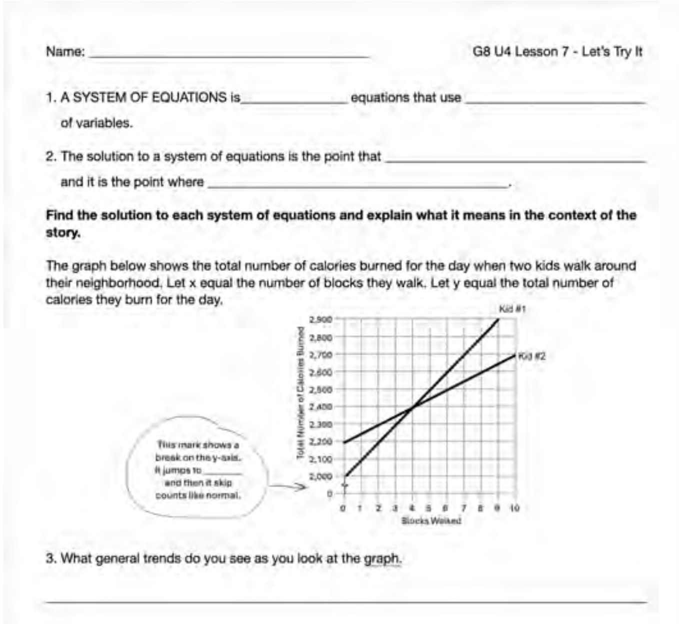
x	*y*
0	3
1	5
2	7
3	9
4	11

System #4



Let's Try It:

Let's identify more solutions to systems together!



Name: ______________________ G8 U4 Lesson 7 - Let's Try It

1. A SYSTEM OF EQUATIONS is__________ equations that use ______________ of variables.

2. The solution to a system of equations is the point that ______________________ and it is the point where ________________________.

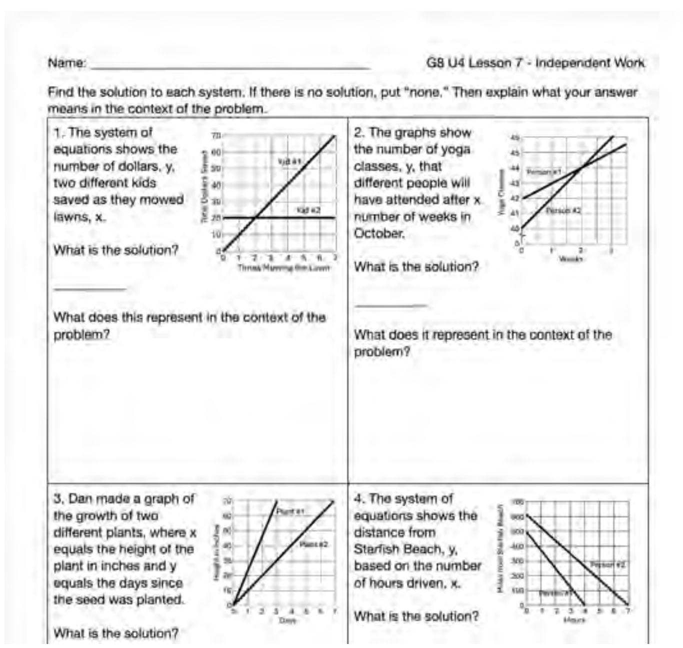
Find the solution to each system of equations and explain what it means in the context of the story.

The graph below shows the total number of calories burned for the day when two kids walk around their neighborhood. Let x equal the number of blocks they walk. Let y equal the total number of calories they burn for the day.

3. What general trends do you see as you look at the graph.

On your Own:

Now it's time for you to do it on your own!



Name: ______________________ G8 U4 Lesson 7 - Independent Work

Find the solution to each system. If there is no solution, put "none." Then explain what your answer means in the context of the problem.

1. The system of equations shows the number of dollars, y, two different kids saved as they mowed lawns, x.

What is the solution?

What does this represent in the context of the problem?

2. The graphs show the number of yoga classes, y, that different people will have attended after x number of weeks in October.

What is the solution?

What does it represent in the context of the problem?

3. Dan made a graph of the growth of two different plants, where x equals the height of the plant in inches and y equals the days since the seed was planted.

What is the solution?

4. The system of equations shows the distance from Starfish Beach, y, based on the number of hours driven, x.

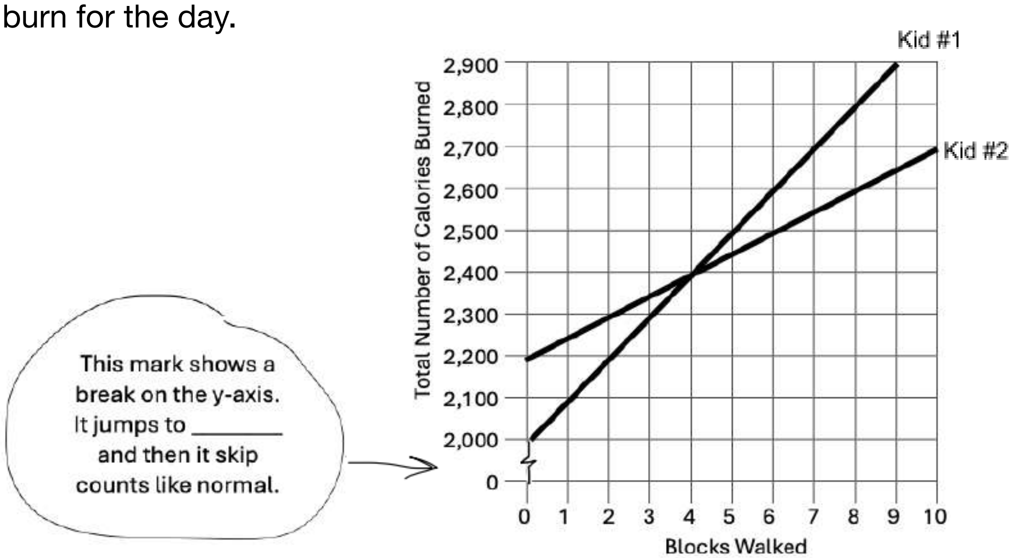
What is the solution?

1. A SYSTEM OF EQUATIONS is_______________ equations that use ___________________________ of variables.

2. The solution to a system of equations is the point that ______________________________________ and it is the point where ____.

Find the solution to each system of equations and explain what it means in the context of the story.

The graph below shows the total number of calories burned for the day when two kids walk around their neighborhood. Let x equal the number of blocks they walk. Let y equal the total number of calories they burn for the day.



3. What general trends do you see as you look at the graph.

____

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4. What is the solution to the system? (____, ____)

5. What does the solution represent in the context of the story?

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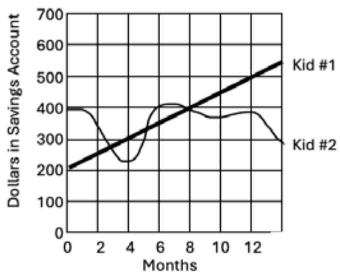
Determine whether each system has no solutions, one solution or multiple solutions.

Each of the graphs below shows the amount of money that different kids had in their savings accounts after a number of months in 2023. Let x equal the number of months. Let y equal the number of dollars in their savings accounts.

6.



This system has ________ solution(s).

What does that mean in the context of the graph?

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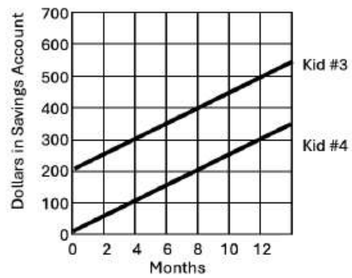
7.



This system has ________ solution(s).

What does that mean in the context of the graph?

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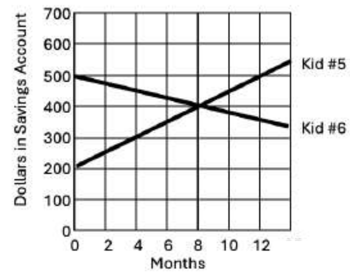
8.



This system has ________ solution(s).

What does that mean in the context of the graph?

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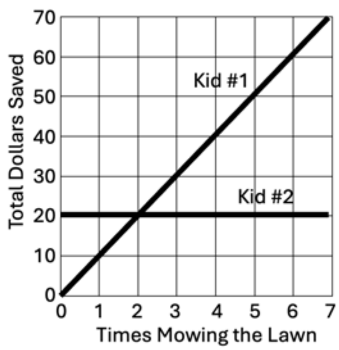
Name: ____ G8 U4 Lesson 7 - Independent Work

Find the solution to each system. If there is no solution, put "none." Then explain what your answer means in the context of the problem.

1. The system of equations shows the number of dollars, y, two different kids saved as they mowed lawns, x.

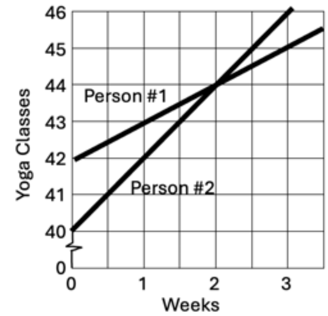


What is the solution?

____

What does this represent in the context of the problem?

2. The graphs show the number of yoga classes, y, that different people will have attended after x number of weeks in October.

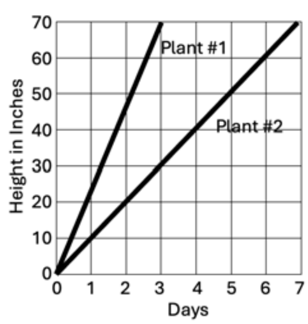


What is the solution?

____

What does it represent in the context of the problem?

3. Dan made a graph of the growth of two different plants, where x equals the height of the plant in inches and y equals the days since the seed was planted.

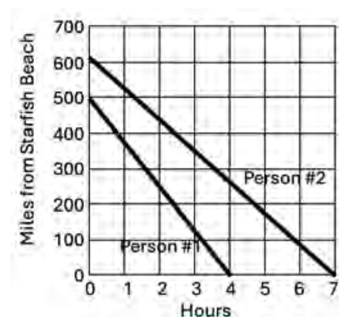


What is the solution?

____

What does this represent in the context of the problem?

4. The system of equations shows the distance from Starfish Beach, y, based on the number of hours driven, x.



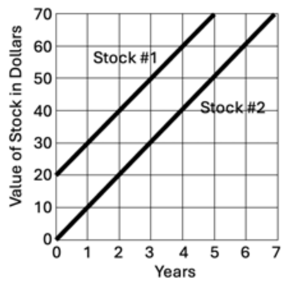
What is the solution?

____

What does this represent in the context of the problem?

Circle from the multiple choice to indicate if the system of equations has no solution, one solution or many solutions.

5.



This system has…
(a) No solution
(b) One solution
(c) Many solutions

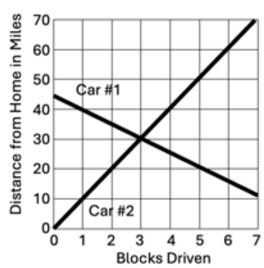
6.

x	*y*
0	0
1	5
2	10
3	15
4	20

x	*y*
0	5
1	7.5
2	10
3	12.5
4	15

This system has…
(a) No solution
(b) One solution
(c) Many solutions

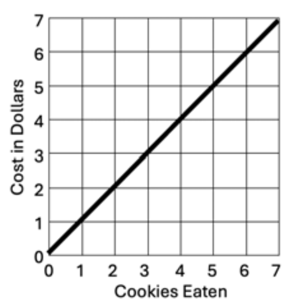
7.



This system has…
(a) No solution
(b) One solution
(c) Many solutions

8.

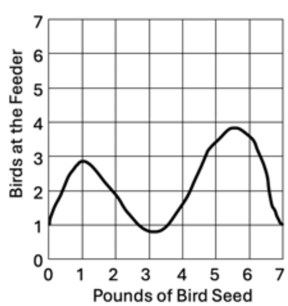
x = 3 and



This system has…
(a) No solution
(b) One solution
(c) Many solutions

9.

y = 2 and



This system has…
(a) No solution
(b) One solution
(c) Many solutions

Name: ANSWER KEY

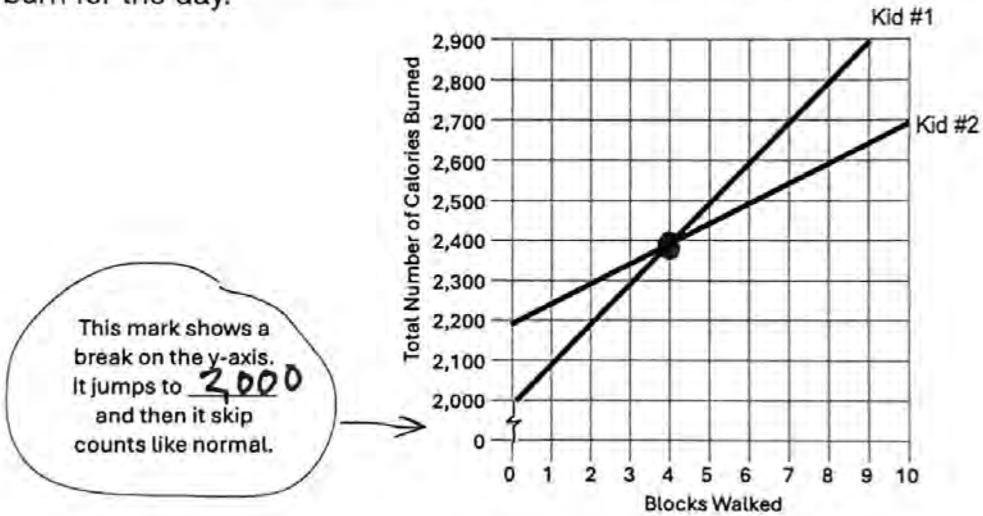
G8 U4 Lesson 7 - Let's Try It

1. A SYSTEM OF EQUATIONS is 2 or more equations that use the same of variables.

2. The solution to a system of equations is the point that satisfies all equations and it is the point where all the lines meet.

Find the solution to each system of equations and explain what it means in the context of the story.

The graph below shows the total number of calories burned for the day when two kids walk around their neighborhood. Let x equal the number of blocks they walk. Let y equal the total number of calories they burn for the day.



3. What general trends do you see as you look at the graph.

Kid #1 starts at 2,000 and increases a lot.

Kid #2 starts at 2,200 and increases a little more slowly.

4. What is the solution to the system? (4, 2400)

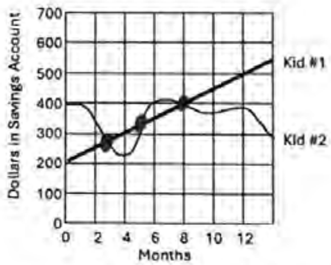
5. What does the solution represent in the context of the story?

Both kids will have burned the same number of calories, which is 2,400, when they have walked the same amount of 4 blocks.

Determine whether each system has no solutions, one solution or multiple solutions.

Each of the graphs below shows the amount of money that different kids had in their savings accounts after a number of months in 2023. Let x equal the number of months. Let y equal the number of dollars in their savings accounts.

6.

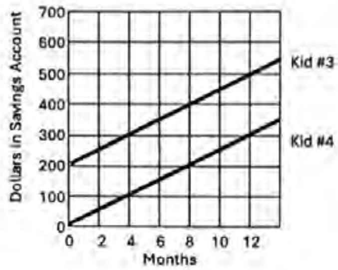


This system has many solution(s).

What does that mean in the context of the graph?

There are many times when kid #1 and kid #2 have the same amount of money at the same time.

7.

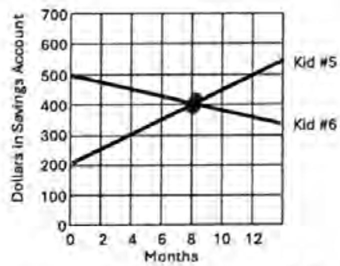


This system has no solution(s).

What does that mean in the context of the graph?

There is no time when both kid #3 and kid #4 have the same amount of money as each other.

8.



This system has one solution(s).

What does that mean in the context of the graph?

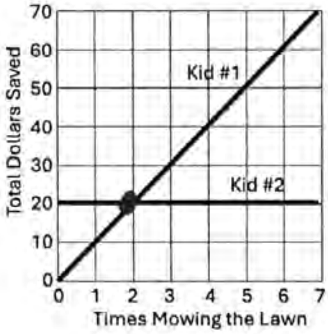
There is one time, which is at 8 months, when kid #5 and #6 both have the same amount of money, which is $400.

Name: ANSWER KEY

G8 U4 Lesson 7 - Independent Work

Find the solution to each system. If there is no solution, put "none." Then explain what your answer means in the context of the problem.

1. The system of equations shows the number of dollars, y, two different kids saved as they mowed lawns, x.



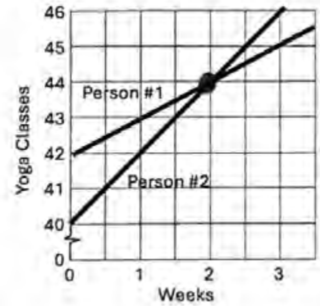
What is the solution?

(2, 20)

What does this represent in the context of the problem?

When both kids have mowed 2 lawns, they will have the same amount of money saved, which is $20.

2. The graphs show the number of yoga classes, y, that different people will have attended after x number of weeks in October.



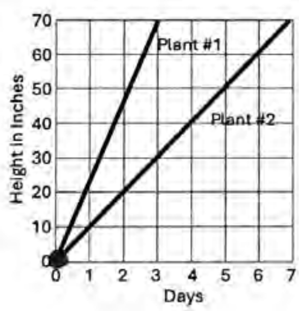
What is the solution?

(2, 44)

What does it represent in the context of the problem?

When both people have done yoga for 2 weeks in October, they will both have done 44 classes total.

3. Dan made a graph of the growth of two different plants, where x equals the height of the plant in inches and y equals the days since the seed was planted.



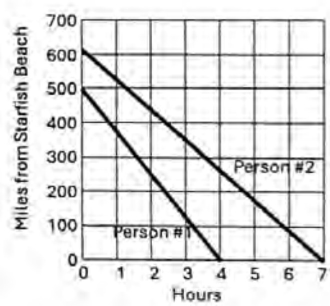
What is the solution?

(0, 0)

What does this represent in the context of the problem?

When both plants have grown for 0 days, they will be at the same height, which is 0 inches.

4. The system of equations shows the distance from Starfish Beach, y, based on the number of hours driven, x.



What is the solution?

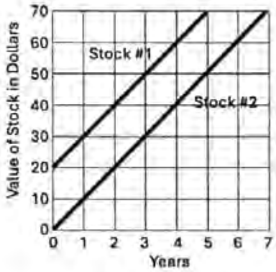
none

What does this represent in the context of the problem?

There is no point at which person #1 and #2 will be at the same distance from Starfish Beach at the same time.

Circle from the multiple choice to indicate if the system of equations has no solution, one solution or many solutions.

5.



This system has...
(a) No solution
(b) One solution
(c) Many solutions

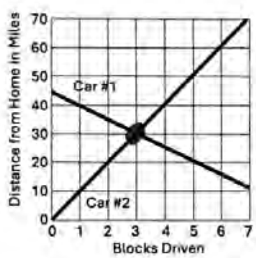
6.

x	y
0	0
1	5
2	10
3	15
4	20

x	y
0	5
1	7.5
2	10
3	12.5
4	15

This system has...
(a) No solution
(b) One solution
(c) Many solutions

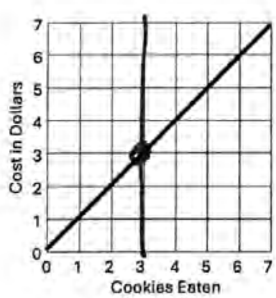
7.



This system has...
(a) No solution
(b) One solution
(c) Many solutions

8.

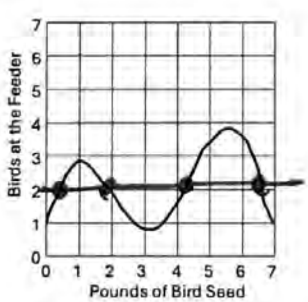
x = 3 and



This system has...
(a) No solution
(b) One solution
(c) Many solutions

9.

y = 2 and



This system has...
(a) No solution
(b) One solution
(c) Many solutions

G8 U4 Lesson 8
Determine if a point is a solution to a system of equations and explain its meaning.

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G8 U4 Lesson 8 - Today we will determine if a point is a solution to a system of equations and explain its meaning.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will determine if a point is a solution to a system of equations and explain its meaning. You already know how to determine if a point is a solution to one equation so you're not going to have any trouble with two or more equations. Let's go!

Let's Review (Slide 3): We know to plug coordinates into an equation to see if it is a solution. We can do these very quickly. It is asking, "Is (5,21) a solution to the equation: y = 3x + 6?" We put 21 in place of y. So it is 21 equals 3 times 5 for x plus 6. Now we do the multiplication 21 = 15 + 6. 15 plus 6 is 21. 21 equals 21! Yay! It works. The equation is true so (5,21) is a solution.

Is (5,21) a solution to the equation: y = 3x + 6?

$21 = 3(5) + 6$
$21 = 15 + 6$
$21 = 21$
YES

Let's do the next one. It's still asking, "Is (5,21) a solution to the equation?" But the equation is different. We will write 21 equals 4 times 5 plus 1. I do the multiplication and I get 21 equals 20 plus 1. That's 21 equals 21!

Is (5,21) a solution to the equation: y = 4x + 1

$21 = 4(5) + 1$
$21 = 20 + 1$
$21 = 21$
NO

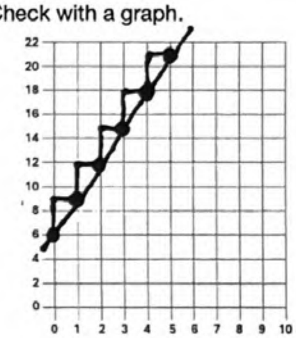
Another true equation! So, is it a solution? Yes! Now, we treated them as separate equations. But on the next slide, we are going to list them together. They will still be the same equations. But now we're going to put them together and call them a system.

Let's Talk (Slide 4): This says, "We can plug coordinates into each equation in a system to see if it is a solution." That's actually what we did on the last slide. We plugged the coordinates into each equation. If they were a solution to each equation then they are a solution to the whole system. "Is (5,21) a solution to the system below? Yes.

Is (5,21) a solution to the system below?

$y = 3x + 6$
$y = 4x + 1$
YES

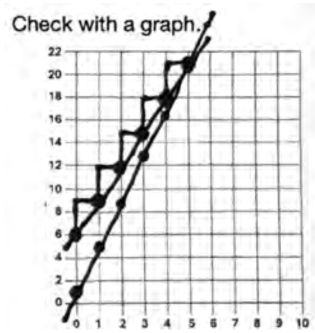
Check with a graph.



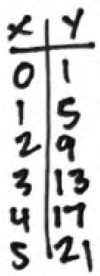
But let's graph it just to be sure. The first equation is in the form, y=mx+b, so I know this has a y - intercept of 6. *Put a dot at (0,6).* And then the slope is 3. So it goes up 3 over 1 and up 3 over 1 and up 3 over one. *Keep putting dots at this slope until the end of the graph. Then draw a line through them.* That's y = 3x +6. I am going to write that on the line so I know what it is.

x	y
0	6
1	9
2	12
3	15
4	18
5	21

Another way I could have done this is with a table. When I plug in 0, I get 6. When I plug in 1, I get 9. When I plug in 2, I get 12. When I plug in 3, I get 15. When I plug in 4, I get 18. When I plug in 5, I get 21.

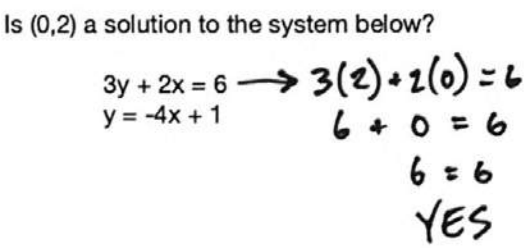


Next, let's graph y = 4x + 1. This equation is also in the form, y=mx+b, so I know the y-intercept is 1. *Put a dot at (0,4).* Then the slope is 4. So it goes up 4 over 1 and up 4 over 1 and up 4 over 1. *Keep putting dots at this slope until the end of the graph. Then draw a line through them.* That's y = 4x + 1. I am going to write that on the line so I know what it is.

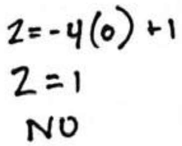


If I had made a table, I would have started it the same way as the last table. I plug in 0, and I get 1. I plug in 1, and I get 5. I plug in 2, and I get 9. I plug in 3, and I get 13. I plug in 4, and I get 17. I plug in 5, and I get 21.

Either way, we have this common point, (5,21). It worked when we plugged it into both equations separately. It worked when we looked for it on the graph for both equations in the same system.



Let's Think (Slide 5): Equations can be written in any form, and we can still plug in the coordinate to check if it is a solution. This is asking, "Is (0,2) a solution to the system below?" We just plug in the coordinates to both equations. 3 times 2 plus 2 times 0 equals 6. Next step, we'll multiply. We get 6 plus 0 equals 6, which is 6 equals 6. So, it is a solution to that equation.



Let's do the next equation. 2 = -4(0) + 1. We do the multiplication first. 2 = 0 + 1, which is 2 equals 1. Uh-oh. It doesn't give us a true equation.

So (0,2) is a solution to the first equation but not the second equation. That means it's NOT a solution to the whole system. But really to do this work, you are not doing any math that you haven't done before, right? You are going to be great at this!

Let's Try It (Slide 6): Let's check some solutions to systems of equations together. I will walk you through each step.

WARM WELCOME

Today we will determine if a point is a solution to a system of equations and explain its meaning.

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Let's Review: **We know to plug coordinates into an equation to see if it is a solution.**

Is (5,21) a solution to the equation: y = 3x + 6?

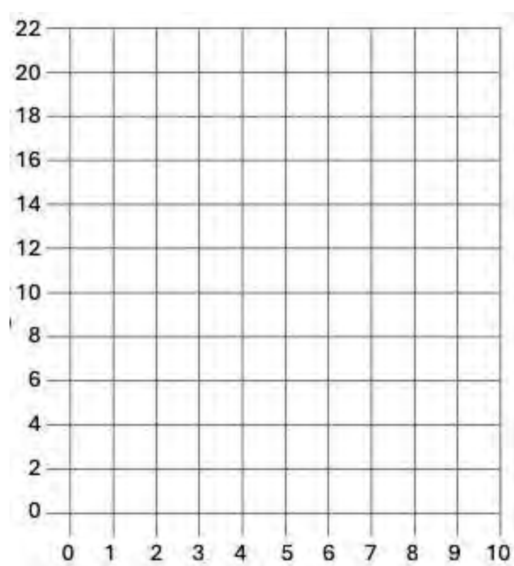
Is (5,21) a solution to the equation: y = 4x + 1

Let's Talk: **We can plug coordinates into each equation in a system to see if it is a solution.**

Is (5,21) a solution to the system below?

y = 3x + 6
y = 4x + 1

Check with a graph.



Let's Think:

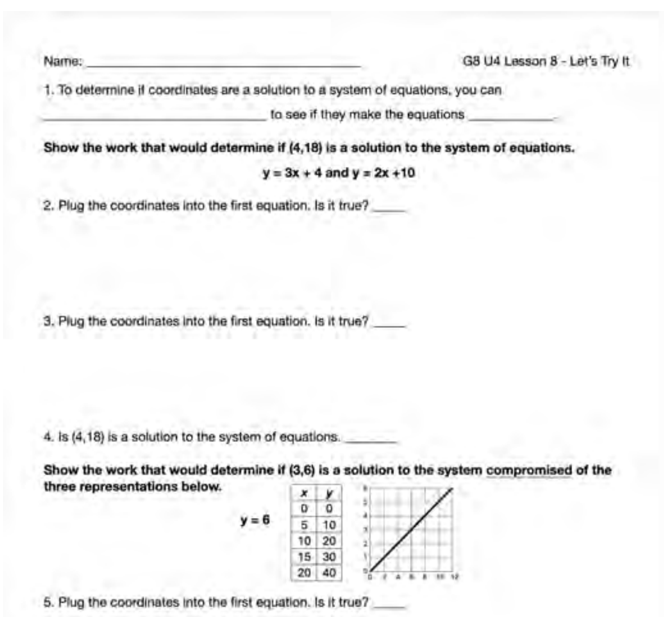
Equations can be written in any form, and we can still plug in the coordinate to check if it is a solution.

Is (0,2) a solution to the system below?

$3y + 2x = 6$
$y = -4x + 1$

Let's Try It:

Let's check some solutions to systems of equations together!



Name: ______________________ G8 U4 Lesson 8 - Let's Try It

1. To determine if coordinates are a solution to a system of equations, you can ______________________ to see if they make the equations __________.

Show the work that would determine if (4,18) is a solution to the system of equations.

y = 3x + 4 and y = 2x +10

2. Plug the coordinates into the first equation. Is it true? _____

3. Plug the coordinates into the first equation. Is it true? _____

4. Is (4,18) is a solution to the system of equations. _______

Show the work that would determine if (3,6) is a solution to the system compromised of the three representations below.

y = 6

x	y
0	0
5	10
10	20
15	30
20	40

5. Plug the coordinates into the first equation. Is it true? _____

On your Own:

Now it's time for you to do it on your own!

Name: ______________________ G8 U4 Lesson 8 - Independent Work

Determine if the coordinates are a solution to the system of equations given.

1. Is (4,18) a solution? $y = 3x + 4$ $y = 2x + 10$	2. Is (2,5) a solution? $3x + 4y = 26$ $8x - 2y = 6$
3. Is (3,0) a solution? $y + 9 = 3x$ $3x + 2y = 9$	4. Is (1,6) a solution? $y = 3x + 3$ $4y = 2x + 22$

Name: ___________________________________

1. To determine if coordinates are a solution to a system of equations, you can

____________________________ to see if they make the equations __________.

Show the work that would determine if (4,18) is a solution to the system of equations.

y = 3x + 4 and y = 2x +10

2. Plug the coordinates into the first equation. Is it true? _____

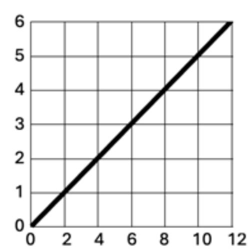
3. Plug the coordinates into the first equation. Is it true? _____

4. Is (4,18) is a solution to the system of equations. ________

Show the work that would determine if (3,6) is a solution to the system compromised of the three representations below.

y = 6

x	*y*
0	0
5	10
10	20
15	30
20	40



5. Plug the coordinates into the first equation. Is it true? _____

6. Write an equation for the table. Plug the coordinates into the equation. Is it true? _____

7. Look for the point on the line that is graphed. Is it a solution? ______

8. Is (3,6) is a solution to the system of equations? ________

Name: ____ G8 U4 Lesson 8 - Independent Work

Determine if the coordinates are a solution to the system of equations given.

1. Is (4,18) a solution? $y = 3x + 4$ $y = 2x + 10$	2. Is (2,5) a solution? $3x + 4y = 26$ $8x - 2y = 6$
3. Is (3,0) a solution? $y + 9 = 3x$ $3x + 2y = 9$	4. Is (1,6) a solution? $y = 3x + 3$ $4y = 2x + 22$

Determine if the coordinates are a solution to the system of equations given.

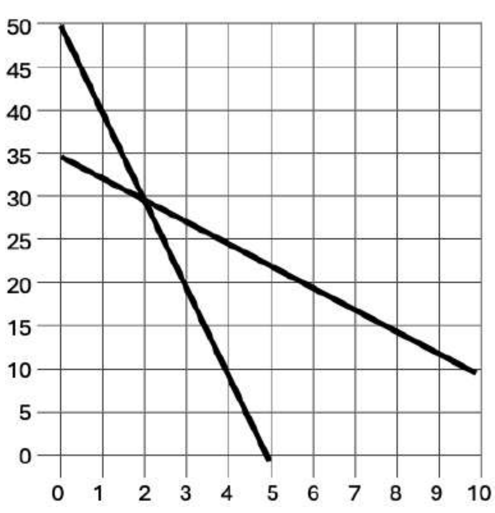
5. Is (2,10) a solution?

$$x = 2$$
$$y = 3x + 4$$

6. Is (1,6) a solution?

$$y = 6x$$
$$y = 2x + 4$$
$$3x - 3y = 15$$

7. Is (2,30) a solution?



8. Is (10,40) a solution?

x	y
0	0
1	4
2	8
3	12

and y = 3x + 1x

Name: ANSWER KEY

1. To determine if coordinates are a solution to a system of equations, you can substitute to see if they make the equations true.

Show the work that would determine if (4,18) is a solution to the system of equations.

y = 3x + 4 and y = 2x +10

2. Plug the coordinates into the first equation. Is it true? NO

$y = 3x + 4$
$18 = 3(4) + 4$
$18 = 12 + 4$
$18 \neq 16$

3. Plug the coordinates into the first equation. Is it true? YES

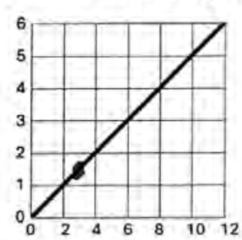
$y = 2x + 10$
$18 = 2(4) + 10$
$18 = 8 + 10$
$18 = 18$

4. Is (4,18) is a solution to the system of equations. NO

Show the work that would determine if (3,6) is a solution to the system compromised of the three representations below.

y = 6

x	y
0	0
5	10
10	20
15	30
20	40



5. Plug the coordinates into the first equation. Is it true? YES

$y = 6$
$6 = 6$

6. Write an equation for the table. Plug the coordinates into the equation. Is it true? YES

$x \cdot 2 = y$
$3 \cdot 2 = 6$
$6 = 6$

7. Look for the point on the line that is graphed. Is it a solution? NO

8. Is (3,6) is a solution to the system of equations? NO

Name: ANSWER KEY

G8 U4 Lesson 8 - Independent Work

Determine if the coordinates are a solution to the system of equations given.

1. Is (4,18) a solution? NO

$$y = 3x + 4$$
$$y = 2x + 10$$

① $18 = 3(4) + 4$
$18 = 12 + 4$
$18 = 16$ NO

2. Is (2,5) a solution? YES

$$3x + 4y = 26$$
$$8x - 2y = 6$$

① $3(2) + 4(5) = 26$
$6 + 20 = 26$
$26 = 26$ YES

② $8(2) - 2(5) = 6$
$16 - 10 = 6$
$6 = 6$ YES

3. Is (3,0) a solution? YES

$$y + 9 = 3x$$
$$3x + 2y = 9$$

① $y + 9 = 3x$
$0 + 9 = 3(3)$
$9 = 9$ YES

② $3x + 2y = 9$
$3(3) + 2(0) = 9$
$9 + 0 = 9$
$9 = 9$ YES

4. Is (1,6) a solution? YES

$$y = 3x + 3$$
$$4y = 2x + 22$$

① $y = 3x + 3$
$6 = 3(1) + 3$
$6 = 3 + 3$
$6 = 6$ YES

② $4y = 2x + 22$
$4(6) = 2(1) + 22$
$24 = 2 + 22$
$24 = 24$ YES

Determine if the coordinates are a solution to the system of equations given.

5. Is (2,10) a solution? YES

$$x = 2$$
$$y = 3x + 4$$

① $x = 2$
$2 = 2$ YES

② $y = 3x + 4$
$10 = 3(2) + 4$
$10 = 6 + 4$
$10 = 10$ YES

6. Is (1,6) a solution? NO

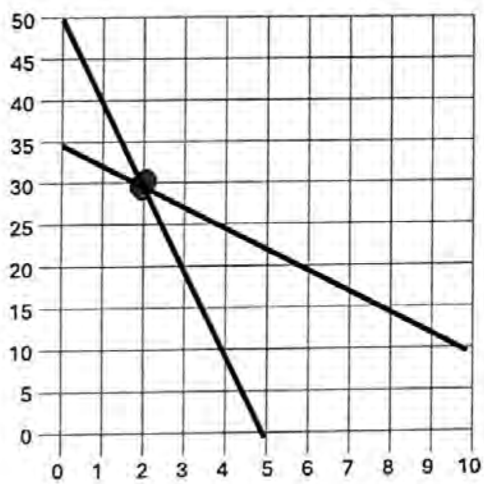
$$y = 6x$$
$$y = 2x + 4$$
$$3x - 3y = 15$$

① $6 = 6(1)$
$6 = 6$ YES

② $6 = 2(1) + 4$
$6 = 2 + 4$
$6 = 6$ YES

③ $3(1) - 3(6) = 15$
$3 - 18 = 15$
$-15 = 15$ NO

7. Is (2,30) a solution? YES



8. Is (10,40) a solution? YES

x	y
0	0
1	4
2	8
3	12

$y = 4x$
$40 = 4(10)$
$40 = 40$ YES

and y = 3x + 1x

$40 = 3(10) + 1(10)$
$40 = 30 + 10$
$40 = 40$ YES

G8 U4 Lesson 9
Understand how to find a solution to a system of equations by setting two expressions equal to each other.

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G8 U4 Lesson 9 - Today we will understand how to find a solution to a system of equations by setting two expressions equal to each other.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will understand how to find a solution to a system of equations by setting two expressions equal to each other. This is going to reach back to our last unit on linear equations. We know how to find the solution to one equation, and now we're going to find the solutions to two or more. Let's go!

x	y
0	100
1	110
2	120
3	130
4	140
5	150
6	160

Let's Review (Slide 3): We can see the solution to a system of equations on a table. Read the problem silently along with me while I read it out loud. *Read the problem.* We already have tables set up for us here. Let's start plugging in numbers. We'll start with John. At 0 weeks, John already has $100 in his savings account. So I am going to write (0,100). Each week he gets $10. So in week 1, he'll have $110. Then in week 2, he'll have $120. In week 3, he'll have $130. In week 4, he'll have $140. In week 5, he'll have $130. In week 6, he'll have $160. *Be sure to leave the last row of the table for the expression we will add at the end of the slide.*

x	y
0	110
1	115
2	120
3	125
4	130
5	135
6	140

Now let's do Roy. At 0 weeks, Roy already has $110 in his savings account. So I am going to write (0,110). Each week he gets $5. So in week 1, he'll have $115. Then in week 2, he'll have $120. In week 3, he'll have $125. In week 4, he'll have $130. In week 5, he'll have $135. In week 6, he'll have $140. *Be sure to leave the last row of the table for the expression we will add at the end of the slide.*

x	y
0	100
1	110
2	120
3	130
4	140
5	150
6	160

x	y
0	110
1	115
2	120
3	125
4	130
5	135
6	140

Now look at the table! We can see where John will catch up to Roy. When they are both at week 2, they will both have $120. That's our solution! This is review of work we've done before. We're just doing it with two stories in the same context instead of one. But let's think about what we've done here and use it to come up with a strategy for trickier problems.

x	y
0	100
1	110
2	120
3	130
4	140
5	150
6	160
x	10x+10

Let's Talk (Slide 4): This says, "When expressions equal the same variable, we can set them equal to each other and solve." We have the exact same story with the table filled in the same way as on our last slide. Here's what I want to point out. For John, every y in the column was essentially multiplying the number of weeks times 10 plus the $100 he started out with at the beginning. So the equation for John is y = 10x + 100. And I could even write that in as a row on my table. We always had an x. And then the y was always 10x + 100.

x	y
0	110
1	115
2	120
3	125
4	130
5	135
6	140
x	5x+110

We could think of Roy's the same way. We always had an x. And then the y for Roy was always 5x + 110. The equation is y = 5x + 110. And I could put that on my table. We always had an x. And then the y was always 5x + 110.

In the past, when we only had one equation, it wasn't super useful to put an expression on our table. And we generally won't do it for systems either. But it does help us see that with systems we are really looking for where this y expression equals this y expression. Then the x will equal the x. That's just like we were looking for 120 equal to 120 and 2 equal to 2. Let's see what happens when I set the expressions equal to each other and solve. I'll write 10x + 100 = 5x + 110. I subtract 100 from both sides. That gives me 10x = 5x + 10. I subtract 5x from both sides. That gives me 5x = 10. I divide by 5 on both sides. That gives me x = 2. That is the same value for x that we came up with on the table, right?!?!? Awesome! So next time, we don't have to make a table! We can solve algebraically by setting the expressions equal to each other.

$$10x + 100 = 5x + 110$$
$$-100 \quad -100$$
$$10x = 5x + 10$$
$$-5x \quad -5x$$
$$\frac{5x}{5} = \frac{10}{5}$$
$$\boxed{x = 2}$$

$$y = 10x + 100$$
$$y = 10(2) + 100$$
$$y = 20 + 100$$
$$\boxed{y = 120}$$

That gave us x so let's plug x into one of the equations to find y. That would be y = 10(2) + 100. That's y = 20 + 100 or y = 120. That's the same answer.

$$y = 5x + 110$$
$$y = 5(2) + 110$$
$$y = 10 + 110$$
$$\boxed{y = 120}$$

And let's check our other equation just to be sure. That would be y = 5(2) + 110. That gives us y = 10 + 110. That's y = 120. Same answer again! So, we can set up tables and fill in lots of values like we did on the last slide. Or if the expressions equal the same variable, we can set them equal to each other and solve them algebraically. In this case, we had both expressions equal to y. So we set them equal to each other and then we crunched those numbers.

$$4x - 6 = 2x + 10$$
$$+6 \quad +6$$
$$4x = 2x + 16$$
$$-2x \quad -2x$$
$$\frac{2x}{2} = \frac{16}{2}$$
$$\boxed{x = 8}$$

Let's Think (Slide 5): We just said this, "If we can solve for one variable algebraically, we can use it to solve for the other variable." This says, "What is the solution to the system?" We have two equations. They are both set equal to y. So we know that the solution is when they are both equal to each other. I will write 4x - 6 = 2x + 10. I add 6 to both sides. That's 4x = 2x + 16. I subtract 2x from both sides. That's 2x = 16. I divide by 2 on both sides. I get x = 8. The idea is that if I were to make tables for both of these and keep plugging in numbers, I get the same y when x was 8.

$$y = 4x - 6$$
$$y = 4(8) - 6$$
$$y = 32 - 6$$
$$\boxed{y = 26}$$

Now we have one more step to find the y. We take an equation and plug in x equals 8. It doesn't matter which equation we do. Let's do y = 4x - 6. That would be y = 4(8) - 6. I do the multiplication first. It is y = 32 - 6 then y = 26.

What is the solution to the system? (8 , 26) So, the solution to this system is (8,26).

$$y = 2x + 10$$
$$y = 2(8) + 10$$
$$y = 16 + 10$$
$$\boxed{y = 26}$$

Let's plug that into the second equation and check if it's right. That would be 26 = 2(8) + 10. 2 times 8 is 16 so I have 26 = 16 + 10. Then 26 = 26. It is a solution! This is way easier than putting all my numbers on a table!

Let's Try It (Slide 6): Let's find solutions to systems of equations together. I will walk you through each step.

WARM WELCOME

Today we will understand how to find a solution to a system of equations by setting two expressions equal to each other.

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Let's Review: We can see the solution to a system of equations on a table.

John started out with $100 in his savings account. His parents pay him $10 every week he does chores. Roy started out with $110 in his savings account. But his parents pay him $5 every time week he does chores. Let x be the number of weeks and y be the number of dollars. At which point will John and Roy have the same amount of money at the same time?

x	y

x	y

Let's Talk: When expressions equal the same variable, we can set them equal to each other and solve algebraically.

John started out with $100 in his savings account. His parents pay him $10 every week he does chores. Roy started out with $110 in his savings account. But his parents pay him $5 every time week he does chores. Let x be the number of weeks and y be the number of dollars. At which point will John and Roy have the same amount of money at the same time?

x	y
0	100
1	110
2	120
3	130
4	140
5	150
6	160

x	y
0	110
1	115
2	120
3	125
4	130
5	135
6	140

Let's Think:

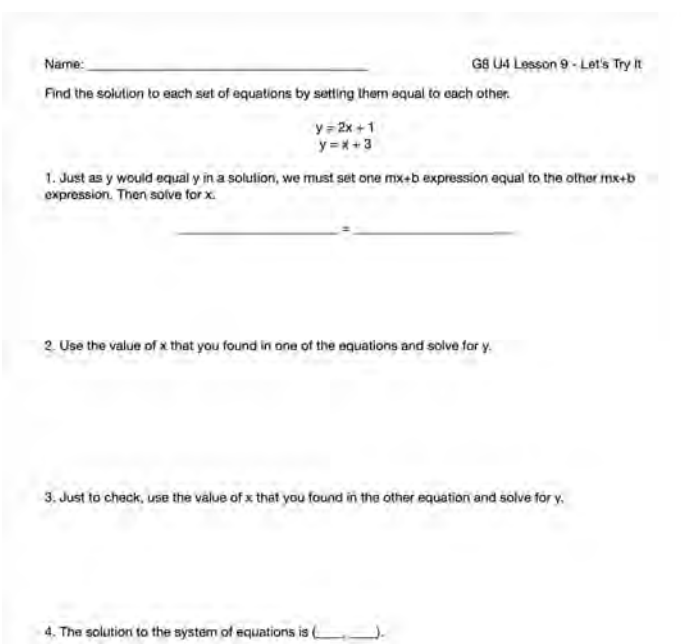
If we can solve for one variable algebraically, we can use it to solve for the other variable.

What is the solution to the system? (____, ____)

$y = 4x - 6$
$y = 2x + 10$

Let's Try It:

Let's find solutions to systems of equations together!



Name: ______________________ G8 U4 Lesson 9 - Let's Try It

Find the solution to each set of equations by setting them equal to each other.

$y = 2x + 1$
$y = x + 3$

1. Just as y would equal y in a solution, we must set one mx+b expression equal to the other mx+b expression. Then solve for x.

______________ = ______________

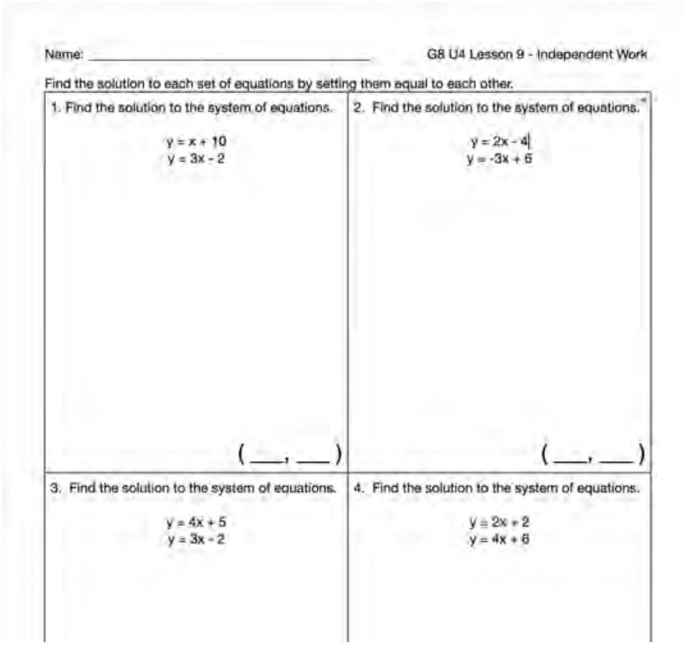
2. Use the value of x that you found in one of the equations and solve for y.

3. Just to check, use the value of x that you found in the other equation and solve for y.

4. The solution to the system of equations is (____, ____).

On your Own:

Now it's time for you to do it on your own!



Name: ______________________ G8 U4 Lesson 9 - Independent Work

Find the solution to each set of equations by setting them equal to each other.

1. Find the solution to the system of equations.

$y = x + 10$
$y = 3x - 2$

(___ , ___)

2. Find the solution to the system of equations.

$y = 2x - 4$
$y = -3x + 6$

(___ , ___)

3. Find the solution to the system of equations.

$y = 4x + 5$
$y = 3x - 2$

4. Find the solution to the system of equations.

$y = 2x + 2$
$y = 4x + 6$

Find the solution to each set of equations by setting them equal to each other.

$$y = 2x + 1$$
$$y = x + 3$$

1. Just as y would equal y in a solution, we must set one mx+b expression equal to the other mx+b expression. Then solve for x.

______________________ = ______________________

2. Use the value of x that you found in one of the equations and solve for y.

3. Just to check, use the value of x that you found in the other equation and solve for y.

4. The solution to the system of equations is (____, ____).

Name: ____

Find the solution to each set of equations by setting them equal to each other.

1. Find the solution to the system of equations.

$y = x + 10$
$y = 3x - 2$

(___, ___)

2. Find the solution to the system of equations.

$y = 2x - 4$
$y = -3x + 6$

(___, ___)

3. Find the solution to the system of equations.

$y = 4x + 5$
$y = 3x - 2$

(___, ___)

4. Find the solution to the system of equations.

$y = 2x + 2$
$y = 4x + 6$

(___, ___)

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations. $y = 2x + 4$ $y = 3x - 2$ (___, ___)	6. Find the solution to the system of equations. $y = -2x + 3$ $y = 3x - 2$ (___, ___)
7. Find the solution to the system of equations. $y = x + 11$ $y = 12x$ (___, ___)	8. Find the solution to the system of equations. $y = x + 1$ $y = 2x - 7$ (___, ___)

Name: ANSWER KEY

Find the solution to each set of equations by setting them equal to each other.

$$y = 2x + 1$$
$$y = x + 3$$

1. Just as y would equal y in a solution, we must set one mx+b expression equal to the other mx+b expression. Then solve for x.

$2x + 1 = x + 3$

$-x \qquad -x$

$x + 1 = 3$

$-1 \qquad -1$

$x = 2$

2. Use the value of x that you found in one of the equations and solve for y.

$y = 2x + 1$

$y = 2(2) + 1$

$y = 4 + 1$

$y = 5$

3. Just to check, use the value of x that you found in the other equation and solve for y.

$y = x + 3$

$5 = 2 + 3$

$5 = 5$

4. The solution to the system of equations is (2, 5).

Name: ANSWER KEY

G8 U4 Lesson 9 - Independent Work

Find the solution to each set of equations by setting them equal to each other.

1. Find the solution to the system of equations.

$y = x + 10$
$y = 3x - 2$

$x + 10 = 3x - 2$
$+2 \quad +2$
$x + 12 = 3x$
$-x \quad -x$
$\frac{12}{2} = \frac{2x}{2}$
$6 = x$
$y = x + 10$
$y = 6 + 10$
$y = 16$

(6 , 16)

2. Find the solution to the system of equations.

$y = 2x - 4$
$y = -3x + 6$

$2x - 4 = -3x + 6$
$+4 \quad +4$
$2x = -3x + 10$
$+3x \quad +3x$
$\frac{5x}{5} = \frac{10}{5}$
$x = 2$
$y = 2(2) - 4$
$y = 4 - 4$
$y = 0$

(2 , 0)

3. Find the solution to the system of equations.

$y = 4x + 5$
$y = 3x - 2$

$4x + 5 = 3x - 2$
$+2 \quad +2$
$4x + 7 = 3x$
$-4x \quad -4x$
$\frac{7}{-1} = \frac{-1x}{-1}$
$-7 = x$
$y = 3(-7) - 2$
$y = -21 - 2$
$y = -23$

(-7 , -23)

4. Find the solution to the system of equations.

$y = 2x + 2$
$y = 4x + 6$

$2x + 2 = 4x + 6$
$-2 \quad -2$
$2x = 4x + 4$
$-2x \quad -2x$
$0 = 2x + 4$
$-4 \quad -4$
$\frac{-4}{2} = \frac{2x}{2}$
$-2 = x$
$y = 2(-2) + 2$
$y = -4 + 2$
$y = -2$

(-2 , -2)

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations. $y = 2x + 4$ $y = 3x - 2$ $2x+4=3x-2$ $+2 \quad +2$ $2x+6=3x$ $-2x \quad -2x$ $6=x$ $y=3(6)-2$ $y=18-2$ $y=16$ (6 , 16)	6. Find the solution to the system of equations. $y = -2x + 3$ $y = 3x - 2$ $-2x+3=3x-2$ $+2x \quad +2x$ $3=5x-2$ $+2 \quad +2$ $\frac{5}{5}=\frac{5x}{5}$ $1=x$ $y=3(1)-2$ $y=3-2$ $y=1$ (1 , 1)
7. Find the solution to the system of equations. $y = x + 11$ $y = 12x$ $x+11=12x$ $-x \quad -x$ $\frac{11}{11}=\frac{11x}{11}$ $1=x$ $y=12(1)$ $y=12$ (1 , 12)	8. Find the solution to the system of equations. $y = x + 1$ $y = 2x - 7$ $x+1=2x-7$ $+7 \quad +7$ $x+8=2x$ $-x \quad -x$ $8=x$ $y=2(8)-7$ $y=16-7$ $y=9$ (8 , 9)

G8 U4 Lesson 10
Solve equations with fractions.

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G8 U4 Lesson 10 - Today we will solve equations with fractions.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will solve equations with fractions. This is a little pause from our systems of equations work because our next lesson on systems of equations is going to have fractions come up. Today we're going to review some fraction ideas that hopefully you have seen before so that we are ready for the next lesson.

Let's Review (Slide 3): Order of operations is the foundation for solving expressions. What is order of operations? Possible Student Answers, Key Points:

- It is PEMDAS.
- It is the order that you have to do your math in.
- It means do parentheses then exponents then multiplication and division then addition and subtraction.

The acronym that we use for order of operations is PEMDAS, where P stands for parentheses, E stands for exponents, MD stands for multiplication and division and AS stands for addition and subtraction. We use order of operations to evaluate expressions, working forward. But on our next slide, when we are working backwards to solve for a variable, we still use order of operations - just in the opposite direction. So we have to be clear on how PEMDAS words. And it gets especially confusing when fractions start to show up. So, let's look at these two problems.

How are the equations below different? How do we solve them differently?

P
E
MD
AS

$\frac{8}{2} + 1 = ?$ $\frac{8+1}{2} = ?$

This says, "How are the equations below different? How do we solve them differently?" The first thing that I hope you notice is that in the first equation we have 8 over 2 and then there is addition. But in the second equation we also have the addition over 2. These two problems are not the same. We can think of the 8 over 2 like 8 halves. But we can also think of it like 8 over 2. And with PEMDAS, that means we will do that part first. We will do the division before the addition. 8 divided by 2 is 4. So we have 4 + 1 = ?. Our answer is 5 = ?.

$\frac{8}{2} + 1 = ?$

$4 + 1 = ?$

$5 = ?$

But in the next problem, even though multiplication and division always come before addition and subtraction, we aren't going to divide by 2. We have a whole amount over the 2, which means we can almost think of it like there are parentheses holding this whole amount over the 2. We would need to do this part first and then divide by 2. 8 + 1 is 9 so this is 9 over 2 = ?. Now we can divide. I do that off to the side 9 divided by 2. My answer is 4 and a half equals question mark. There are two big ideas here. First, we are thinking of the denominator of our fraction as division. Second, if there is more than one thing in the numerator then it is like that whole amount is in parentheses together.

$\frac{(8+1)}{2} = ?$

$\frac{9}{2} = ?$

$4\frac{1}{2} = ?$

$2\overline{)9}$ = $4\frac{1}{2}$, -8, 1

Let's Talk (Slide 4): Now, this has already come up, and we've discussed it in earlier lessons too. But it is worth reviewing. "We work backwards from Order of Operations to solve equations." So, instead of doing what is inside the parentheses first, we work backwards from what is outside the parentheses first. Instead of doing multiplication and division first then addition and subtraction, we work backwards from the addition and subtraction first then multiplication and division. Now, this question is asking, "How are the equations below different? How do we solve them differently?" And that's super important to think about because they look kind of the same. They have all the same digits. But in the first equation, we just have x over 2 then we add 1.

$$\frac{x}{2} + 1 = 12 \qquad \frac{(x+1)}{2} = 12$$

But in the second equation, we have all of this, x + 1, over 2. To help myself, I am going to put parentheses around that part. That helps me see that it is together over 2.

$$\frac{x}{2} + 1 = 12$$
$$-1 \quad -1$$
$$\frac{x}{2} = 11$$
$$\cdot 2 \quad \cdot 2$$
$$\boxed{x = 22}$$

Now, let's work backwards from PEMDAS to solve. When we are working backwards, we look at the addition and subtraction first so I will do minus 1 on both sides. That gives me x over 2 equals 11. It looks like we have this denominator of 2 left. But if we think of it as dividing by 2 then it is easy to do the opposite operation. I am going to multiply by 2 on this side so I have to multiply by 2 on this side. I get x = 22. Great!

$$\frac{(x+1)}{2} = 12$$
$$\cdot 2 \quad \cdot 2$$
$$x + 1 = 24$$
$$-1 \quad -1$$
$$\boxed{x = 23}$$

This next problem is different. Since I have x + 1 together over the 2, I can't just subtract 1 away. I have to deal with this denominator first. That's why putting parentheses is nice. It helps me see that I have to deal with this part outside the parentheses first when I'm working backwards. So, I will multiply each side times 2. That leaves me with x + 1 = 24.

Now I subtract 1 from both sides and I get x = 23. Parentheses were really helpful here. And if we are working backwards from Order of Operations to solve then we have to work on the part outside the parentheses first.

Let's Think (Slide 5): Let's take this one step further with some other fractional representations. This says, "To cancel out multiplication by a fraction, we can multiply by the reciprocal." Now, that is when we are thinking of the fraction as a fraction instead of a dividing number. But let's see where this comes up. In this first equation, the fraction is going to be multiplied times x and x only. So we just work backwards starting with the addition. I will subtract 4 from each side. We get 2 thirds x equals 2. This is where the reciprocal comes in. To cancel out that 2 thirds, we will multiply by 3 over 2 on both sides. Just to convince you this works, 3 x 2 makes 6 in the numerator and 2 x 3 makes 6 in the denominator. That's 6 over 6 which is just 1. In other words we just have x on this side. 2 x 3 is 6 divided by 2 is 3. Our answer is x = 3.

$$\frac{2}{3}x + 4 = 6$$
$$-4 \quad -4$$
$$\frac{3}{2} \cdot \frac{2}{3}x = 2 \cdot \frac{3}{2}$$
$$x = \frac{6}{2}$$
$$\boxed{x = 3}$$

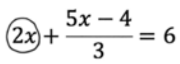
$$\frac{3}{2} \cdot \frac{2}{3}(x+4) = 6 \cdot \frac{3}{2}$$
$$x + 4 = \frac{18}{2}$$
$$x + 4 = 9$$
$$-4 \quad -4$$
$$\boxed{x = 5}$$

Now, for the next problem, x + 4 is together in parentheses. When we work backwards, we have to take care of this 2 thirds first. That is the part outside the parentheses. I will multiply this whole thing by the reciprocal 3 over 2 on both sides. We know that cancels out on the left and we are left with x + 4. On the right, 6 x 3 is 18 divided by 2 is 9. We get x + 4 = 9. I subtract 4 from both sides. I get x = 5. We don't get the same answer because with parentheses, this is a totally different problem.

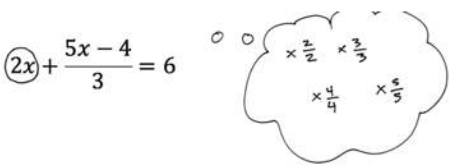
$$\frac{(2x+4)}{3} = 6$$
$$\cdot 3 \quad \cdot 3$$
$$2x + 4 = 18$$
$$-4 \quad -4$$
$$\frac{2x}{2} = \frac{14}{2}$$
$$\boxed{x = 7}$$

Final problem! We still have a 2 and a 3 kind of. But now they are broken up. Remember that we said we'd think of this whole amount in the numerator as if it is in parentheses. So to work backwards, I will multiply by 3 on each side. I get 2x + 4 = 18. I will subtract 4 on both sides. I get 2x = 14. I divide by 2 on both sides and I get x = 7. Again, it is a totally different problem because the parentheses were in a different spot and sometimes we think of the denominator as part of a fraction and we cancel with the reciprocal. Sometimes we think of the denominator as a dividing number and we cancel by multiplying.

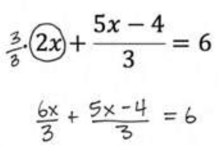
Let's Think (Slide 6): Now we have one more fraction complication we will need to master to be super systems of equations solvers. This says, "Sometimes we will need to create common denominators to combine like terms." So for example, in this problem. We have a denominator of 3 for just a section of the expression. I can't get rid of it by multiplying the whole thing by 3. At least not until I get rid of the 2x that is outside all that over here. *Circle the 2x.*



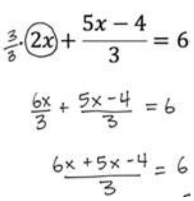
This is going to get a bit complicated! First, I want to remind you of something that you probably first started hearing about in like 3rd grade. *Draw a thought bubble to the side.* We know we can multiply the top and the bottom of a fraction by the same number and get an equivalent form. That means it is the same value just written a different way. 1 half times 2 over 2 is 2 fourths, which is still the same. We can multiply any number times 2 over 2 or times 3 over 3 or times 4 over 4. *Write those examples in the thought bubble.*



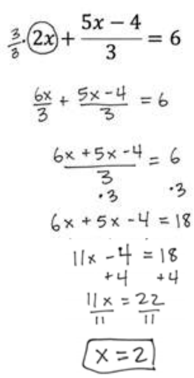
For this problem, it would be super nice if this 2x had a denominator of 3. Because I can add things with like denominators. So, I am going to multiply 2x times 3 over 3. Now, I'm not going to do that to both sides of the equation because I'm not working backwards yet. I'm just writing the fraction in an equivalent form. It's like if I were changing 1 + 4 into 5. Or 2 x 1 into 2. When I do this I get 6x over 3 plus the rest of the expression equals 6.



And now I can finally combine like terms and make the equation look like something that we know how to solve. First, I will add these fractions. I get 6x + 5x - 4 over 3 equals 6.



Now, finally, I can multiply by 3 on each side. I get 6x + 5x - 4 equals 18. I will combine 6x and 5x. I get 11x - 4 equals 18. I will add 4 to both sides. 11x = 22. I divide by 11 on both sides. I get x = 2.



Let's Try It (Slide 7): Let's solve some equations with fractions together. I will walk you through each step.

WARM WELCOME

Today we will solve equations with fractions.

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Let's Review: Order of operations is the foundation for solving expressions.

How are the equations below different? How do we solve them differently?

$$\frac{8}{2} + 1 = ? \qquad \frac{8+1}{2} = ?$$

Let's Talk: We work backwards from Order of Operations to solve equations.

How are the equations below different? How do we solve them differently?

$$\frac{x}{2} + 1 = 12 \qquad \frac{x+1}{2} = 12$$

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Let's Think:

To cancel out multiplication by a fraction, we can multiply by the reciprocal.

Solve for x.

$$\frac{2}{3}x + 4 = 6 \qquad \frac{2}{3}(x + 4) = 6 \qquad \frac{2x + 4}{3} = 6$$

Let's Think:

Sometimes we will need to create common denominators to combine like terms.

Solve for x.

$$2x + \frac{5x - 4}{3} = 6$$

Let's Try It:

Let's solve some equations with fractions together!

Name: ______________________________ G8 U4 Lesson 10 - Let's Try It

Write equivalent fractions.

We'll start with some quick algebra work that we'll need for this lesson. Fractions are equivalent as long as we multiply the top and bottom by the same number such as: ____ or ____ or ____.

1. Turn 6x into a fraction with a denominator of 2:
2. Turn 2x - 3 into a fraction with a denominator of 5:
3. Turn -5x into a fraction with a denominator of 3:

Circle the problems that will need a common denominator to simplify.

4. Find numbers where we can't combine like terms with an x in them because of a denominator.

   a. $7 + \frac{x+1}{5} = 10$ b. $7x + \frac{x+1}{5} = 10$ c. $5 + \frac{1}{4}x - 1 = 6x$

Show your work to solve for x.

5. $\frac{3}{4}(x + 4) = 15$
6. $\frac{3}{4}x + 4 = 15$
7. $3x + \frac{x+4}{4} = 15$
8. $\frac{3x+4}{4} = 15$

On your Own:

Now it's time for you to do it on your own!

Name: ______________________________ G8 U4 Lesson 10 - Independent Work

Solve for x.

1. $3 + \frac{3x}{4} = 8$	2. $\frac{3}{4}(x + 7) = 21$
3. $\frac{1}{4}x + 7 = 21$	4. $\frac{2}{3} + \frac{x}{4} = 1$

Write equivalent fractions.

We'll start with some quick algebra work that we'll need for this lesson. Fractions are equivalent as long as we multiply the top and bottom by the same number such as: ____ or ____ or ____.

1. Turn 6x into a fraction with a denominator of 2:

2. Turn 2x - 3 into a fraction with a denominator of 5:

3. Turn -5x into a fraction with a denominator of 3:

Find the problems that will need a common denominator to simplify.

4. Circle the equations where we can't combine like terms with an x in them because of a denominator.

   a. $7 + \frac{x+1}{5} = 10$ b. $7x + \frac{x+1}{5} = 10$ c. $5 + \frac{1}{4}x - 1 = 6x$

Show your work to solve for x.

5. $\frac{3}{4}(x + 4) = 15$

6. $\frac{3}{4}x + 4 = 15$

7. $3x + \frac{x+4}{4} = 15$

8. $\frac{3x+4}{4} = 15$

Name: ______________________________________ G8 U4 Lesson 10 - Independent Work

Solve for x.

1. $3 + \frac{2x}{4} = 8$	2. $\frac{3}{4}(x + 7) = 21$
3. $\frac{3}{4}x + 7 = 21$	4. $\frac{2}{3} + \frac{x}{4} = 1$
5. $\frac{2x+3}{6} = 5$	6. $(4 - 2x)\frac{4}{5} = 8$

Solve for x.

7. $3x + \frac{2x - 2}{4} = 8$	8. $3 + \frac{2x-2}{4} = 8$
9. $\frac{x + 1}{4} + 2x = 7$	10. $4\left(\frac{2x-2}{4} + x\right) = 8$
11. $\frac{3}{4}(4x + 7) + x = 2$	12. $\frac{1}{4}x + 6 + 4x = 5x$

Name: ANSWER KEY

Write equivalent fractions.

We'll start with some quick algebra work that we'll need for this lesson. Fractions are equivalent as long as we multiply the top and bottom by the same number such as: $\frac{2}{2}$ or $\frac{3}{3}$ or $\frac{4}{4}$.

1. Turn 6x into a fraction with a denominator of 2: $(6x) \times \frac{2}{2} = \frac{12x}{2}$
2. Turn 2x - 3 into a fraction with a denominator of 5: $(2x-3) \cdot \frac{5}{5} = \frac{10x-15}{5}$
3. Turn -5x into a fraction with a denominator of 3: $(-5x) \cdot \frac{3}{3} = \frac{-15x}{3}$

Find the problems that will need a common denominator to simplify.

4. Circle the equations where we can't combine like terms with an x in them because of a denominator.

a. $7 + \frac{x+1}{5} = 10$ b. $7x + \frac{x+1}{5} = 10$ (circled) c. $5 + \frac{1}{4}x - 1 = 6x$

Show your work to solve for x.

5.

$\frac{4}{3} \cdot \frac{3}{4}(x + 4) = 15 \cdot \frac{4}{3}$

$x + 4 = \frac{60}{3}$

$x + 4 = 20$

$-4 \quad -4$

$x = 16$

$3\overline{)44}$ → $14\frac{2}{3}$: -3, 14, -12, 2

6.

$\frac{3}{4}x + 4 = 15$

$-4 \quad -4$

$\frac{4}{3} \cdot \frac{3}{4}x = 11 \cdot \frac{4}{3}$

$x = \frac{44}{3}$

$x = 16\frac{2}{3}$

7.

$\frac{4}{4} \cdot 3x + \frac{x+4}{4} = 15$

$\frac{12x}{4} + \frac{x+4}{4} = 15$

$\frac{12x + x + 4}{4} = 15$

$\cdot 4 \quad \cdot 4$

$12x + x + 4 = 60$

$13x + 4 = 60$

$-4 \quad -4$

$13x = 56$

$x = 4\frac{4}{13}$

$13\overline{)56}$ → 04: 52, 4

8.

$\frac{3x+4}{4} = 15$

$\cdot 4 \quad \cdot 4$

$3x + 4 = 60$

$-4 \quad -4$

$3x = 54$

$x = 18$

$3\overline{)54}$ → 18: -3, 24, -24, 00

Name: ANSWER KEY

Solve for x.

1. $3 + \frac{2x}{4} = 8$ $-3 \quad -3$ $\frac{2x}{4} = 5$ $\cdot 4 \quad \cdot 4$ $\frac{2x}{2} = \frac{20}{2}$ $x = 10$	2. $\frac{4}{3} \cdot \frac{3}{4}(x + 7) = 21 \cdot \frac{4}{3}$ $x + 7 = \frac{84}{3}$ $x + 7 = 28$ $-7 \quad -7$ $x = 21$ $84 \div 3 = 28$: -6, 24, -24, 00
3. $\frac{3}{4}x + 7 = 21$ $-7 \quad -7$ $\frac{4}{3} \cdot \frac{3}{4}x = 14 \cdot \frac{4}{3}$ $x = \frac{56}{3}$ $x = 18\frac{2}{3}$ $56 \div 3 = 18$: -3, 26, -24, 2	4. $\frac{2}{3} + \frac{x}{4} = 1$ $-\frac{2}{3} \quad -\frac{2}{3}$ $4 \cdot \frac{x}{4} = \frac{1}{3} \cdot 4$ $x = \frac{4}{3}$ $x = 1\frac{1}{3}$
5. $\frac{2x+3}{6} = 5$ $\cdot 6 \quad \cdot 6$ $2x + 3 = 30$ $-3 \quad -3$ $\frac{2x}{2} = \frac{27}{2}$ $x = 13\frac{1}{2}$	6. $(4 - 2x)\frac{4}{5} = 8$ $\cdot \frac{5}{4} \quad \cdot \frac{5}{4}$ $4 - 2x = \frac{40}{4}$ $4 - 2x = 10$ $-4 \quad -4$ $\frac{-2x}{-2} = \frac{6}{-2}$ $x = -3$

Solve for x.

7. $\frac{4}{4} \cdot 3x + \frac{2x-2}{4} = 8$ $\frac{12x}{4} + \frac{2x-2}{4} = 8$ $\frac{12x+2x-2}{4} = 8$ $\cdot 4 \quad \cdot 4$ $12x + 2x - 2 = 32$ $14x - 2 = 32$ $+2 \quad +2$ $\frac{14x}{14} = \frac{34}{14}$ $x = 2\frac{6}{14}$	8. $3 + \frac{2x-2}{4} = 8$ $-3 \quad -3$ $\frac{2x-2}{4} = 5$ $\cdot 4 \quad \cdot 4$ $2x - 2 = 20$ $+2 \quad +2$ $\frac{2x}{2} = \frac{22}{2}$ $x = 11$
9. $\frac{x+1}{4} + 2x \cdot \frac{4}{4} = 7$ $\frac{x+1}{4} + \frac{8x}{4} = 7$ $\frac{9x+1}{4} = 7$ $\cdot 4 \quad \cdot 4$ $9x + 1 = 28$ $-1 \quad -1$ $\frac{9x}{9} = \frac{27}{9}$ $x = 3$	10. $4\left(\frac{2x-2}{4} + x\right) = 8$ $\frac{8x-8}{4} + 4x \cdot \frac{4}{4} = 8$ $\frac{8x-8}{4} + \frac{16x}{4} = 8$ $4 \cdot \frac{24x-8}{4} = 8 \cdot 4$ $24x - 8 = 32$ $+8 \quad +8$ $\frac{24x}{24} = \frac{40}{24}$ $x = 1\frac{16}{24}$ $1\frac{2}{3} = x$
11. $\frac{3}{4}(4x + 7) + x = 2$ $\frac{12x+21}{4} + x \cdot \frac{4}{4} = 2$ $\frac{12x+21}{4} + \frac{4x}{4} = 2$ $4 \cdot \frac{16x+21}{4} = 2 \cdot 4$ $16x + 21 = 8$ $-21 \quad -21$ $\frac{16x}{16} = \frac{-13}{16}$ $x = -\frac{13}{16}$	12. $\frac{1}{4}x + 6 + 4x = 5x$ $4\frac{1}{4}x + 6 = 5x$ $-4\frac{1}{4}x \quad -4\frac{1}{4}x$ $\frac{4}{3} \cdot 6 = \frac{3}{4}x \cdot \frac{4}{3}$ $\frac{24}{3} = x$ $8 = x$

G8 U4 Lesson 11
Find the solution for a system of equations using substitution.

CITYTUTOR DC

G8 U4 Lesson 11 - Today we will find the solution for a system of equations using substitution.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will find the solution for a system of equations using substitution. There's one big idea we will need to understand and then we're just using the skills we already have. So you're going to do great! Let's get started!

Let's Review (Slide 3): In our last lesson, we learned that we can set expressions equal to each other to solve a system. Read the problem silently along with me in your head while I read the problem out loud. *Read the problem.* How can I set up an equation to solve this system? Possible Student Answers, Key Points:

- Put one expression equal to the other.
- Write 15x + 90 = 10x + 105

$$15x + 90 = 10x + 105$$
$$-90 \qquad -90$$
$$15x = 10x + 15$$
$$-10x \qquad -10x$$
$$\frac{5x}{5} = \frac{15}{5}$$
$$\boxed{x = 3}$$

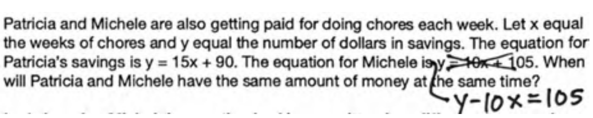
Both of these equations are already written equal to y. So we can just put one y expression to the other y expression. I will write 15x + 90 = 10x + 105. Now I can solve. We can subtract 90 from both sides. 15x = 10x + 15. Now I will subtract 10x from both sides. I get 5x = 15. I divide by 5 on both sides. I get x = 3.

$$y = 15x + 90$$
$$y = 15(3) + 90$$
$$y = 45 + 90$$
$$\boxed{y = 135} \qquad (3,135)$$

That's only part of my solution. Now I will use x = 3 to find the y. I will take this first equation and plug in x = 3. It is y = 15(3) + 90. 15 times 3 is 45 so it's y = 45 + 90. I can do that off to the side and I get y = 135.

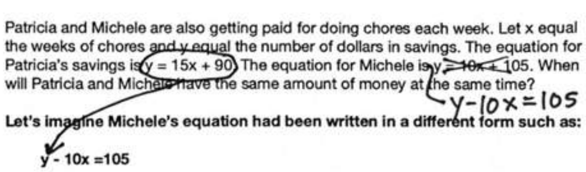
My solution is (3,135), which means that when Patricia and Michele have both done 3 weeks of chores then they will both have 145 dollars. Now we are going to take this exact same problem and tweak it a little on the next slide.

Let's Talk (Slide 4): This says, "We can also use substitution to solve algebraically." I will show you what this means. We have the exact same story as before. But then it says, "Let's imagine Michele's equation had been written in a different form such as: y - 10x =100." Imagine this other equation for Michele wasn't given to us. *Cross out y =10x + 105 in the story problem.*


Patricia and Michele are also getting paid for doing chores each week. Let x equal the weeks of chores and y equal the number of dollars in savings. The equation for Patricia's savings is y = 15x + 90. The equation for Michele is y = 10x + 105. When will Patricia and Michele have the same amount of money at the same time?

y-10x=105

Even though it's not obvious that we can just set these expressions equal to each other, we still know that whatever we got for y in the equation could be plugged in for y in the other equation. For example, if y = 2 for this equation… *Point to Patricia's equation.* …then we can plug in y = 2 for this equation. *Point to Michele's equation.* Well, we don't have anything nice and simple like y = 2. But we can still plug in what we know y equals from one equation into the other equation. That is called substitution because we are going to substitute the y. Let me show you what I mean. I am going to put in what I know y equals, which is right here. *Circle the expression 15x + 90 and draw an error to the y in the other equation.* I am going to substitute this y right here.


Patricia and Michele are also getting paid for doing chores each week. Let x equal the weeks of chores and y equal the number of dollars in savings. The equation for Patricia's savings is y = 15x + 90. The equation for Michele is y = 10x + 105. When will Patricia and Michele have the same amount of money at the same time?

y-10x=105

Let's imagine Michele's equation had been written in a different form such as:

y - 10x =105

$$y - 10x = 105$$
$$15x + 90 - 10 = 105$$
$$15x + 80 = 105$$
$$-80 \quad -80$$
$$\frac{15x}{15} = \frac{45}{15}$$
$$\boxed{x = 3}$$

Let’s rewrite this. Now y is 15x + 90 so we get 15x + 90 - 10x = 105. No problem! I combine like terms which are the 15x minus 10x. That’s 5x + 90 = 105. I subtract 90 from both sides. That’s 5x = 15. I divide by 5 on both sides and I get x = 3! That’s the same answer we got on the last slide.

$$y - 10(3) = 105$$
$$y - 30 = 105$$
$$+30 \quad +30$$
$$\boxed{y = 135}$$

I can plug it back into an equation like we did last time to find y. Let’s use this second equation. It would be y - 10(3) = 105. That’s the same as y - 30 = 105. I add 30 to both sides and I get y = 135. It is the same solution! It would have the same meaning! So, the big idea here is that if our equations aren’t written in y = mx +b form and we can’t set them equal to each other, we can use substitution instead.

Let’s Think (Slide 5): Sometimes we will need to find an equivalent form of one equation before we can do substitution. Now, I will warn you - once we get to this level of Mathematics, our numbers can get really unfriendly. That is totally fine. The real secret is just to write clearly and in an organized way. I’ll show you. This says, “What is the solution to the system?” I see that neither of my equations are written in y=mx+b form. And in fact, this is a really common way that equations are written. It is called standard form, where we have a multiplier of x and a multiplier of y. If we want to substitute a value of x or y, we are going to have to do some math first to find an equivalent form of one of the equations. I am going to solve this first equation for y. Let me write it over to the side of my paper: 4x + 3y = 7. Let’s solve for y. I will subtract 4x from both sides. Then 3y = 7 - 4x. Now I will divide by 3 on both sides. I get y equals 7 - 4x over 3. Here are the messy numbers I was talking about. But we’re not going to crunch these out. Let’s leave them as they are and see how our equations work out.

$$4x + 3y = 7$$
$$-4x \quad -4x$$
$$\frac{3y}{3} = \frac{7-4x}{3}$$
$$\boxed{y = \frac{7-4x}{3}}$$

$$3x + 5y = 8$$
$$3x + 5\left(\frac{7-4x}{3}\right) = 8$$

Now that we have a “y equals equation” we can substitute. Let me write the other equation to the side to give myself a guide. I will keep 3x plus. But now when I substitute y, I am going to need parentheses so that it is 5 times the whole amount. I will write 5 times “7 - 4x over 3” in parentheses equals 8.

$$3x + 5y = 8$$
$$3x + 5\left(\frac{7-4x}{3}\right) = 8$$
$$3x + \frac{35-20x}{3} = 8$$

Again, this is not that friendly-looking. First, let’s distribute the 5. I get 3x + (35 - 20x) over 3 equals 8. You know distribution so this is not a big deal.

$$3x + 5y = 8$$
$$3x + 5\left(\frac{7-4x}{3}\right) = 8$$
$$\frac{3}{3} \cdot 3x + \frac{35-20x}{3} = 8$$
$$\frac{9x}{3} + \frac{35-20x}{3} = 8$$

Now normally, our next step would be to combine like terms. But that is hard because we have this fraction with a denominator of 3. This is the only really new algebra we are going to do that you haven’t done before. I am going to turn 3x into a fraction with a denominator of 3 so I can add it to this other fraction. It might look complicated by it is actually really simple. For fractions, we always multiply the top and bottom by the same number. So I am going to multiply 3x times 3 over 3. That gives me 9x over 3.

$$3x+5y=8$$
$$3x+5\left(\frac{7-4x}{3}\right)=8$$
$$\frac{3}{3}\cdot 3x+\frac{35-20x}{3}=8$$
$$\frac{9x}{3}+\frac{35-20x}{3}=8$$

If I put that in my equation then I have 9x over 3 plus 35 minus 20x over 3 equals 8. I can put that altogether now that they have the same denominators. So it is all of 9x plus 35 minus 20x over 3 equals 8.

$$3x+5y=8$$
$$3x+5\left(\frac{7-4x}{3}\right)=8$$
$$\frac{3}{3}\cdot 3x+\frac{35-20x}{3}=8$$
$$\frac{9x+35-20x}{3}=8$$
$$\cdot 3 \qquad \cdot 3$$
$$9x+35-20x=24$$
$$-11x+35=24$$
$$-35 \qquad -35$$
$$\frac{-11x}{-11}=\frac{-11}{-11}$$
$$\boxed{x=1}$$

It is easy from here on out, I promise. We just work backwards. First, to work backwards from dividing by 3, I will multiply by 3 on each side. That gives me 9x plus 35 minus 20x equals 24. Finally I can combine like terms! 9x and negative 20x is negative 11x plus 35 equals 24. I subtract 35 from both sides and get negative 11x equals negative 11. I divide by negative 11 on both sides and get x equals 1. Isn't that crazy? We did all that algebra and eventually landed at 1. I told you that we could make this friendly!

$$4x+3y=7$$
$$4(1)+3y=7$$
$$4+3y=7$$
$$-4 \qquad -4$$
$$\frac{3y}{3}=\frac{3}{3}$$
$$\boxed{y=1}$$

The final step is figuring out y. I am going to plug x = 1 into an equation. I'll start with 4x + 3y = 7. That would be 4(1) + 3y = 7. That's 4 + 3y = 7. I subtract 4 from both sides. I get 3y = 3. I divide by 3 on both sides. I get y = 1. The solution to this system is (1,1).

Let's review the steps we did here. First, I rewrote one equation so it was equal to a variable. *Point to that step. You could even label it "Step #1."* Second, I substituted that into the other equation. *Point to that step. You could even label it "Step #2."* Third, I plugged that answer into an equation to find the other variable. *Point to that step. You could even label it "Step #3."* That is what you're going to do every time you want to use substitution. Sometimes you can skip the 1st step if it is already written in a friendly way. If it is not written in a friendly way then you might have to use a little bit of equivalent fractions to make it easier to work with.

Let's Try It (Slide 6): Let's find solutions to systems of equations together. I will walk you through each step.

WARM WELCOME

Today we will find the solution for a system of equations using substitution.

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Let's Review: We can set expressions equal to each other to solve a system.

Patricia and Michele are also getting paid for doing chores each week. Let x equal the weeks of chores and y equal the number of dollars in savings. The equation for Patricia's savings is $y = 15x + 90$. The equation for Michele is $y = 10x + 105$. When will Patricia and Michele have the same amount of money at the same time?

Let's Talk: We can also use substitution to solve algebraically.

Patricia and Michele are also getting paid for doing chores each week. Let x equal the weeks of chores and y equal the number of dollars in savings. The equation for Patricia's savings is $y = 15x + 90$. The equation for Michele is $y = 10x + 105$. When will Patricia and Michele have the same amount of money at the same time?

Let's imagine Michele's equation had been written in a different form such as:

$y - 10x = 105$

Let's Think:

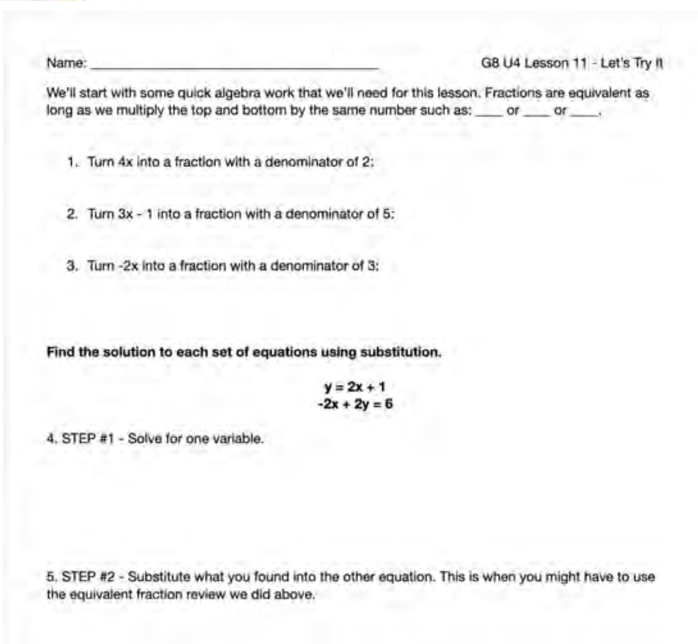
Sometimes we will need to find an equivalent form of one equation before we do substitution.

What is the solution to the system? (____, ____)

$4x + 3y = 7$
$3x + 5y = 8$

Let's Try It:

Let's find solutions to systems of equations together!



Name: ______________________________ G8 U4 Lesson 11 - Let's Try It

We'll start with some quick algebra work that we'll need for this lesson. Fractions are equivalent as long as we multiply the top and bottom by the same number such as: ____ or ____ or ____.

1. Turn 4x into a fraction with a denominator of 2:
2. Turn 3x - 1 into a fraction with a denominator of 5:
3. Turn -2x into a fraction with a denominator of 3:

Find the solution to each set of equations using substitution.

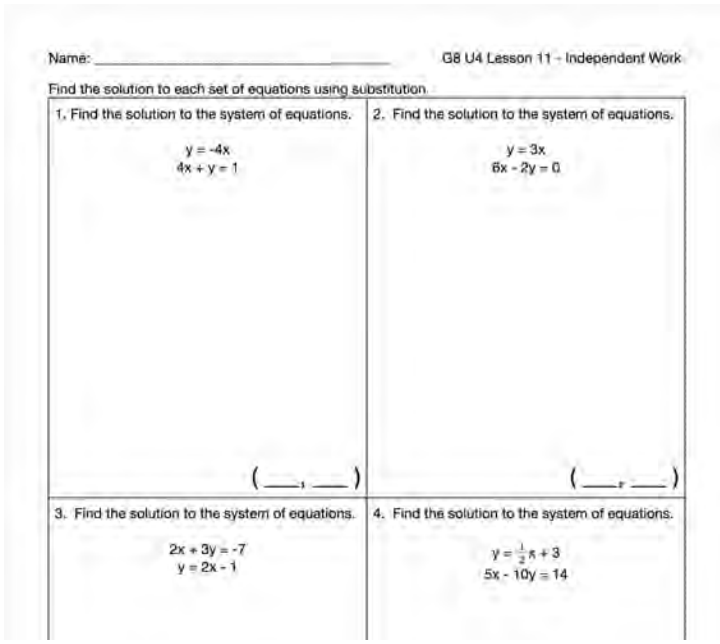
$y = 2x + 1$
$-2x + 2y = 6$

4. STEP #1 - Solve for one variable.

5. STEP #2 - Substitute what you found into the other equation. This is when you might have to use the equivalent fraction review we did above.

On your Own:

Now it’s time for you to do it on your own!



Name: ______________________ G8 U4 Lesson 11 - Independent Work

Find the solution to each set of equations using substitution.

1. Find the solution to the system of equations. $y = -4x$ $4x + y = 1$ (___, ___)	2. Find the solution to the system of equations. $y = 3x$ $6x - 2y = 0$ (___, ___)
3. Find the solution to the system of equations. $2x + 3y = -7$ $y = 2x - 1$	4. Find the solution to the system of equations. $y = \frac{1}{2}x + 3$ $5x - 10y = 14$

We'll start with some quick algebra work that we'll need for this lesson. Fractions are equivalent as long as we multiply the top and bottom by the same number such as: ____ or ____ or ____.

1. Turn 4x into a fraction with a denominator of 2:

2. Turn 3x - 1 into a fraction with a denominator of 5:

3. Turn -2x into a fraction with a denominator of 3:

Find the solution to each set of equations using substitution.

$$y = 2x + 1$$
$$-2x + 2y = 6$$

4. STEP #1 - Solve for one variable.

5. STEP #2 - Substitute what you found into the other equation. This is when you might have to use the equivalent fraction review we did above.

6. STEP #3 - Use what you got in #2 to find the other variable by putting it into an equation.

Find the solution to each set of equations using substitution.

$$7x - 8y = -12$$
$$-4x + 2y = 3$$

4. STEP #1 - Solve for one variable.

5. STEP #2 - Substitute what you found into the other equation. This is when you might have to use the equivalent fraction review we did above.

6. STEP #3 - Use what you got in #2 to find the other variable by putting it into an equation.

Find the solution to each set of equations using substitution.

1. Find the solution to the system of equations. $y = 4x$ $4x + y = 1$ (___, ___)	2. Find the solution to the system of equations. $y = 3x$ $6x + 2y = 0$ (___, ___)
3. Find the solution to the system of equations. $2x + 3y = -7$ $y = 2x - 1$ (___, ___)	4. Find the solution to the system of equations. $y = \frac{1}{2}x + 3$ $11x + 10y = 14$ (___, ___)

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations.

$$x - 3y = -9$$
$$2x + 7y = 8$$

(___, ___)

6. Find the solution to the system of equations.

$$4x + 2y = 8$$
$$8x - y = 1$$

(___, ___)

7. Find the solution to the system of equations.

$$2x + y = -4$$
$$3x - 2y = -6$$

(___, ___)

8. Find the solution to the system of equations.

$$6x - 8y = 5$$
$$12x + 10y = 23$$

(___, ___)

Name: ANSWER KEY

We'll start with some quick algebra work that we'll need for this lesson. Fractions are equivalent as long as we multiply the top and bottom by the same number such as: $\frac{2}{2}$ or $\frac{5}{5}$ or $\frac{3}{3}$.

1. Turn 4x into a fraction with a denominator of 2: $4x \times \frac{2}{2} = \frac{8x}{2}$

2. Turn 3x - 1 into a fraction with a denominator of 5: $3x-1 \times \frac{5}{5} = \frac{15x-5}{5}$

3. Turn -2x into a fraction with a denominator of 3: $-2x \times \frac{3}{3} = \frac{-6x}{3}$

Find the solution to each set of equations using substitution.

y = 2x + 1
-2x + 2y = 6

4. STEP #1 - Solve for one variable.

We can skip this because we have $y=2x+1$.

5. STEP #2 - Substitute what you found into the other equation. This is when you might have to use the equivalent fraction review we did above.

$-2x + 2y = 6$
$-2x + 2(2x+1) = 6$
$-2x + 4x + 2 = 6$
$2x + 2 = 6$
$ -2 \quad -2$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

6. STEP #3 - Use what you got in #2 to find the other variable by putting it into an equation.

$-2x + 2y = 6$
$-2(2) + 2y = 6$
$-4 + 2y = 6$
$+4 \qquad +4$
$\frac{2y}{2} = \frac{10}{2}$

$y = 5$

Find the solution to each set of equations using substitution.

$$7x - 8y = -12$$
$$-4x + 2y = 3$$

4. STEP #1 - Solve for one variable.

$7x - 8y = -12$
$-7x \quad\quad -7x$

$\frac{-8y}{-8} = \frac{-12-7x}{-8}$

$y = \frac{-12-7x}{-8}$

5. STEP #2 - Substitute what you found into the other equation. This is when you might have to use the equivalent fraction review we did above.

$-4x + 2\left(\frac{-12-7x}{-8}\right) = 3$

$\frac{-8}{-8} \times -4x + \frac{-24-14x}{-8} = 3$

$\frac{32x}{-8} + \frac{-24-14x}{-8} = 3$

$\frac{32x - 24 - 14x}{-8} = 3$

$\times -8 \quad\quad \times -8$

$32x - 24 - 14x = -24$
$+24 \quad\quad +24$

$32x - 14x = 0$

$\frac{18x}{18} = \frac{0}{18}$

$\boxed{x = 0}$

6. STEP #3 - Use what you got in #2 to find the other variable by putting it into an equation.

$-4x + 2y = 3$

$-4(0) + 2y = 3$

$0 + \frac{2y}{2} = \frac{3}{2}$

$\boxed{y = 1\frac{1}{2}}$

Name: ANSWER KEY

G8 U4 Lesson 11 - Independent Work

Find the solution to each set of equations using substitution.

1. Find the solution to the system of equations.

$y = 4x$
$4x + y = 1$

#2
$4x + 4x = 1$
$\frac{8x}{8} = \frac{1}{8}$
$x = \frac{1}{8}$

#3
$y = 4x$
$y = 4(\frac{1}{8})$
$y = \frac{4 \div 4}{8 \div 4}$
$y = \frac{1}{2}$

$(\frac{1}{8}, \frac{1}{2})$

2. Find the solution to the system of equations.

$y = 3x$
$6x + 2y = 0$

#2
$6x + 2(3x) = 0$
$6x + 6x = 0$
$\frac{12x}{12} = \frac{0}{12}$
$x = 0$

#3
$y = 3(0)$
$y = 0$

$(0, 0)$

3. Find the solution to the system of equations.

$2x + 3y = -7$
$y = 2x - 1$

#2
$2x + 3(2x - 1) = -7$
$2x + 6x - 3 = -7$
$8x - 3 = -7$
$+3 \quad +3$
$\frac{8x}{8} = \frac{-4 \div 4}{8 \div 4}$
$x = -\frac{1}{2}$

#3
$y = 2(-\frac{1}{2}) - 1$
$y = -1 - 1$
$y = -2$

$(-\frac{1}{2}, -2)$

4. Find the solution to the system of equations.

$y = \frac{1}{2}x + 3$
11+ 10y = 14

#2
$11x + 10(\frac{1}{2}x + 3) = 14$
$11x + 5x + 30 = 14$

$16x + 30 = 14$
$-30 \quad -30$
$\frac{16x}{16} = \frac{-16}{16}$
$x = -1$

#3
$y = \frac{1}{2}(-1) + 3$
$y = -\frac{1}{2} + 3$
$y = 2\frac{1}{2}$

$(-1, 2\frac{1}{2})$

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations. x - 3y = -9 2x + 7y = 8 #1 x - 3y = -9 -x -x -3y = -9 - x -3 -3 y = (-9 - x)/-3 #2 2x + 7y = 8 2x + 7((-9-x)/-3) = 8 2x + (-63 - 7x)/-3 = 8 -3x/-3 -6x/-3 + (-63-7x)/-3 = 8 (-6x-63-7x)/-3 = 8 ·-3 ·-3 -13x-63 = -24 +63 +63 -13x/-13 = 39/-13 x=-3 #3 x - 3y = -9 -3 - 3y = -9 +3 +3 -3y/-3 = -6/-3 y=2 (-3, 2)	6. Find the solution to the system of equations. 4x + 2y = 8 8x - y = 1 #1 8x - y = 1 -8x -8x -y = 1 - 8x -1 -1 y = -1 + 8x #2 4x+2y = 8 4x+2(-1+8x) = 8 4x - 2 + 16x = 8 +2 +2 20x/20 = 10/20 x = 1/2 #3 8(1/2) - y = 1 4 - y = 1 -4 -4 -y/-1 = -3/-1 y = 3 (1/2, 3)
7. Find the solution to the system of equations. 2x + y = -4 3x - 2y = -6 #1 2x + y = -4 -2x -2x y = -4 - 2x #2 3x - 2y = -6 3x - 2(-4-2x) = -6 3x + 8 + 4x = -6 7x + 8 = -6 -8 -8 7x/7 = -14/7 x = -2 #3 2(-2) + y = -4 -4 + y = -4 +4 +4 y = 0 (-2, 0)	8. Find the solution to the system of equations. 6x - 8y = 5 12x + 10y = 23 #1 6x - 8y = 5 -6x -6x -8y/-8 = (5 - 6x)/-8 y = (5-6x)/-8 #3 12x + 10((5-6x)/-8) = 23 -8/-8 · 12x + (50 - 60x)/-8 = 23 -96x/-8 + (50-60x)/-8 = 23 (-96x+50-60x)/-8 = 23 ·-8 ·-8 -96x+50-60x = -184 -156x + 50 = -184 -50 -50 -156x/-156 = -234/-156 156)234 = 001, -156, 78 1 78/156 = x #3 6(1 1/2) - 8y = 5 9 - 8y = 5 -9 -9 -8y/-8 = -4/-8 y = 1/2 (1 1/2, 1/2)

G8 U4 Lesson 12
Find the solution for a system of equations using elimination.

CITYTUTOR DC

G8 U4 Lesson 12 - Today we will find the solution for a system of equations using elimination.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will find the solution for a system of equations using elimination. We already know how to find a solution with substitution. But sometimes this other method will be easier. Let's go!

$$x - y = 11$$
$$+y \quad +y$$
$$x = 11 + y$$

Let's Review (Slide 3): We already know we can solve a system of equations by substituting the variable in one equation with information from the other equation. Read the problem silently along with me in your head while I read it out loud. *Read the problem.* Let's review substitution so we know the answer when we go to the next slide and explore the next strategy. For my first step, I solve for one of the variables. The easiest one to solve for is x - y = 11. I add y to both sides, and I get x equals 11 + y.

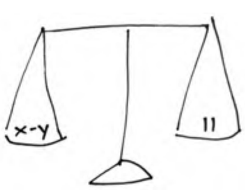
$$2x + y = 19$$
$$2(11 + y) + y = 19$$
$$22 + 2y + y = 19$$
$$22 + 3y = 19$$
$$-22 \qquad -22$$
$$\frac{3y}{3} = \frac{-3}{3}$$
$$\boxed{y = -1}$$

Now I can plug that value in for x. I get 2 times the whole amount of 11 + y in parentheses. That equals 19. I will need to distribute the 2. I get 22 + 2y + y =19. I can combine like terms and get 22 + 3y = 19. Then I subtract 22 from each side. I get 3y = -3. I divide by 3 on each side. That gives me y = -1.

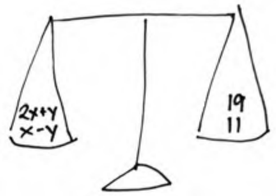
$$2x + y = 19$$
$$2x + (-1) = 19$$
$$+1 \qquad +1$$
$$\frac{2x}{2} = \frac{20}{2}$$
$$\boxed{x = 10}$$

The final step is that I will have to plug that into an original equation. I will take 2x + y = 19. I will put -1 in place of y. To solve this problem I will add 1 to both sides. I get 2x equals 20. I divide by 2 on each side. Then I get x = 10. So the solution with substitution is (10, -1). Now I'm going to teach you elimination and we should get the same answer.

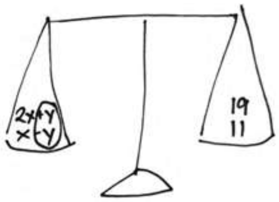
Let's Talk (Slide 4): This is what I was saying when we started, "We can also solve a system of equation using a strategy called elimination." We have the exact same problem about Rachel and John with the exact same equations that we did on our last slide. But before I show you what elimination is, I want us to agree on one idea. Over all of our units, we have talked about how the equal sign means both sides are the same. We can think of it like a balance.



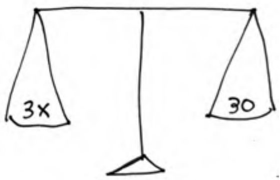
We can put one equation on the balance and the equal sign tells us it is the same on both sides.



My other equation also has an equal sign. It is also balanced. So if I put it on my balance then it will stay balanced because I am putting the same amount on each side.



What this shows us is we can add up all the left sides of the equations and all the right sides of the equations and we'll get one balanced equation. But - and this is the coolest part - sometimes parts of those equations will cancel eachother out. Like in this example, plus y and minus y cancel. So when I add these up, I get 2x plus x, which is 3x on the left. I am going to draw a new balance to show how that works. I get 3x on the left. On the right, I add 19 and 11 and I get 20.



Now I have a brand new equation, 3x = 30 and I can solve it and find x and use that to find y and so on.

$$\begin{array}{r} 2x + y = 19 \\ + \quad x - y = 11 \\ \hline \frac{3x}{3} = \frac{30}{3} \\ \boxed{x = 10} \end{array}$$

Let me show you how this would look numerically. I will write down both equations with the x's and the y's and the equal signs lined up. Then I circle where it is canceling. Then I add up the rest. 2x plus x is 3x. 19 plus 11 is 30. I get 3x = 30.

Now I divide by 3 on each side. I get x = 10.

$$\begin{array}{l} 2x + y = 19 \\ 2(10) + y = 19 \\ 20 + y = 19 \\ -20 \qquad -20 \\ \boxed{y = -1} \end{array}$$

My last step is the same step we did in our last lesson with substitution. Once I know one variable, I use it to find y. Let's use 2x + y = 19. I plug in x = 10. I get 2 times 10 plus y equals 19. That's the same as 20 plus y equals 19. I subtract 20 from both sides. I get x = -1. The solution to this system is (10, -1). That the same as we got on the last slide! So it works!

Let's Think (Slide 5): The greatest thing about this new strategy is that it's simple addition. But we have one more thing to understand written here: "Sometimes we will need to multiply one or more equations to set up the cancellation." I'll show you what this means. Here we have a system of equations. If I just add these up, there isn't going to be anything that cancels. I'll get 15x plus 4y equals 54. That is impossible to solve. So I have to multiply one of these equations to turn it into something that WILL cancel. It would be nice if one of these 2y was negative, right? Then positive 2y and negative 2y will cancel. So, I am going to multiply each side of this bottom equation times -1.

$$\begin{array}{l} 7x + 2y = 24 \\ -1(8x + 2y) = (30)(-1) \end{array}$$

$$\begin{array}{l} 7x + 2y = 24 \\ -8x - 2y = -30 \end{array}$$

Then I keep 7x + 27 = 24. But my next equation becomes -8x-2y=-30.

$$\begin{array}{l} 7x + 2y = 24 \\ -8x - 2y = -30 \end{array}$$

Now I can cancel! Let me circle that part that cancels.

$$\begin{array}{r} 7x + 2y = 24 \\ -8x - 2y = -30 \\ \hline -1x = -6 \\ \frac{-1x}{-1} = \frac{-6}{-1} \\ \boxed{x = 6} \end{array}$$

I get -1x = -6. I will divide by -1 on both sides. I get x = 6.

$$\begin{array}{l} 7x + 2y = 24 \\ 7(6) + 2y = 24 \\ 42 + 2y = 24 \\ -42 \qquad\quad -42 \\ \frac{2y}{2} = \frac{-18}{2} \\ \boxed{y = -9} \end{array}$$

Just like always, I plug that x into one of my original equations, and I'll get y. Let's use 7x+2y=24. I rewrite it as 7 times 6 plus 2y equals 24. That's 42 plus 2y equals 24. I will subtract 42 from each side. I get 2y equals -18. I divide by 2 on each side. I get y = -9. The solution to this system is (6,-9).

We're going to do one more. And I promise this one is as complicated as it can get. Again, I have a system of equations. If I just add these up, there isn't going to be anything that cancels. I'll get 8x minus 7y equals 12. That's impossible to solve. So I have to multiply to make this into something that will cancel. But I can't really think of anything to multiply just one equation to turn it into something that would cancel with the other. I can't multiply 3x to make negative 5x. I can't multiply -2y to make positive 5y. So, I am going to have to multiply BOTH equations. I do this by thinking of a number that I could easily make with what I have. That's called a common multiple. If I multiply each side of my top equation times negative 5, I will get negative 15x. And then I can multiply each side of my bottom equation times 3, and I will get a positive 15x. Those will cancel. I'm not writing this down yet because I just want to make sure you understand the idea. I decided to get negative 15x on top and positive 15x on the bottom so I'll have something that cancels.

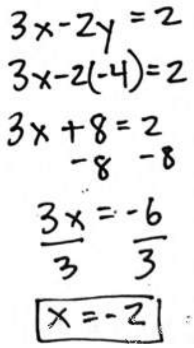
$$\begin{array}{r} -5(3x - 2y) = 2(-5) \\ 3(5x - 5y) = 10(3) \end{array}$$

Let's write this out. I multiply each side of the top equation by -5. I multiply each side of the bottom equation by 3.

$$\begin{array}{r} -15x + 10y = -10 \\ 15x - 15y = 30 \\ \hline -5y = 20 \\ \frac{-5y}{-5} = \frac{20}{-5} \\ \boxed{y = -4} \end{array}$$

For the top equation, that gives me -15x+10y=-10. For the bottom equation, that gives me 15x -15y = 30. Now I have something that will cancel. Let me circle -15x and 15x to show they cancel out.

That leaves me with -5y = 20. I divide by -5 on each side. I get y = –4.



The final step is plugging that value in. Let's use 3x-2y=2. I would get 3x - 2 times -4 equals 2. That simplifies to 3x + 8 = 2. I subtract 8 on both sides. I get 3x = -6. I divide by 3 on both sides. That gives me x = -2. My solution is (-2,-4).

The big idea is that we might need to multiply one or more equation to set up the cancellation we want to do to solve the system.

Let's Try It (Slide 6): Let's find more solutions to systems of equations together. I will walk you through each step.

WARM WELCOME

Today we will find the solution for a system of equations using elimination.

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Let's Review: We can solve a system of equations by substituting the variable in one equation with information from the other equation.

Two people use equation to represent the amount they spent on clothes in dollars, y, compared to the amount they spend on toys in dollars, x, using the equations below. Rachel uses the equation, x - y = 11. Jacob uses the equation, 2x + y = 19. What would the quantities be when Rachel and Jacob spend an equal amount on both clothes and toys?

Let's Talk: We can also solve a system of equations using a strategy called elimination.

Two people use equation to represent the amount they spent on clothes in dollars, y, compared to the amount they spend on toys in dollars, x, using the equations below. Rachel uses the equation, x - y = 11. Jacob uses the equation, 2x + y = 19. What would the quantities be when Rachel and Jacob spend an equal amount on both clothes and toys?

Let's Think:

Sometimes we will need to multiply one or more equations to set up the cancellation.

What is the solution to the system? (____, ____)

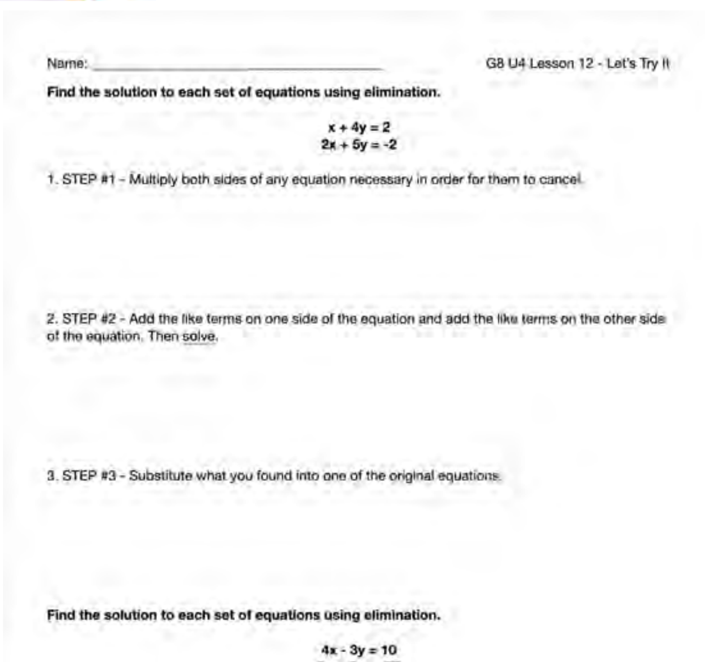
$7x + 2y = 24$
$8x + 2y = 30$

What is the solution to the system? (____, ____)

$3x - 2y = 2$
$5x - 5y = 10$

Let's Try It:

Let's find solutions to systems of equations together!



Name: ______________________ G8 U4 Lesson 12 - Let's Try It

Find the solution to each set of equations using elimination.

$x + 4y = 2$
$2x + 5y = -2$

1. STEP #1 - Multiply both sides of any equation necessary in order for them to cancel.

2. STEP #2 - Add the like terms on one side of the equation and add the like terms on the other side of the equation. Then solve.

3. STEP #3 - Substitute what you found into one of the original equations.

Find the solution to each set of equations using elimination.

$4x - 3y = 10$

On your Own:

Now it's time for you to do it on your own!

Name: ______________________ G8 U4 Lesson 12 - Independent Work

Find the solution to each set of equations using elimination.

1. Find the solution to the system of equations.

$$5x-y=12$$
$$-5x+3y=-6$$

(___ , ___)

2. Find the solution to the system of equations.

$$3y+2x=6$$
$$5y-2x=10$$

(___ , ___)

3. Find the solution to the system of equations.

$$x+4y=2$$
$$2x+5y=-2$$

4. Find the solution to the system of equations.

$$3x+y=9$$
$$5x+4y=22$$

Find the solution to each set of equations using elimination.

$$x + 4y = 2$$
$$2x + 5y = -2$$

1. STEP #1 - Multiply both sides of any equation necessary in order for them to cancel.

2. STEP #2 - Add the like terms on one side of the equation and add the like terms on the other side of the equation. Then solve.

3. STEP #3 - Substitute what you found into one of the original equations.

Find the solution to each set of equations using elimination.

$$4x - 3y = 10$$
$$3x + 5y = -7$$

4. STEP #1 - Multiply both sides of any equation necessary in order for them to cancel.

5. STEP #2 - Add the like terms on one side of the equation and add the like terms on the other side of the equation. Then solve.

6. STEP #3 - Substitute what you found into one of the original equations.

Find the solution to each set of equations using elimination.

1. Find the solution to the system of equations.	2. Find the solution to the system of equations.
$5x-y=12$ $-5x+3y=-6$ (___, ___)	$3y+2x=6$ $5y-2x=10$ (___, ___)
3. Find the solution to the system of equations. $x+4y=2$ $2x+5y=-2$ (___, ___)	4. Find the solution to the system of equations. $3x+y=9$ $5x+4y=22$ (___, ___)

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations.

$$2x+y=7$$
$$x-2y=6$$

(___, ___)

6. Find the solution to the system of equations.

$$4x + 2y = 8$$
$$8x - y = 1$$

(___, ___)

7. Find the solution to the system of equations.

$$3x-9y=6$$
$$2x-2y=8$$

(___, ___)

8. Find the solution to the system of equations.

$$3x+2y=12$$
$$-4x+3y=1$$

(___, ___)

Name: ANSWER KEY

Find the solution to each set of equations using elimination.

$$-2(x + 4y) = 2(-2)$$
$$2x + 5y = -2$$

1. STEP #1 - Multiply both sides of any equation necessary in order for them to cancel.

$$-2x + -8y = -4$$
$$2x + 5y = -2$$

2. STEP #2 - Add the like terms on one side of the equation and add the like terms on the other side of the equation. Then solve.

$$\frac{-3y}{-3} = \frac{-6}{-3}$$
$$\boxed{y = 2}$$

3. STEP #3 - Substitute what you found into one of the original equations.

$$x + 4y = 2$$
$$x + 4(2) = 2$$
$$x + 8 = 2$$
$$-8 \quad -8$$
$$\boxed{x = -6}$$

Find the solution to each set of equations using elimination.

$$3(4x - 3y) = 10(3)$$
$$-4(3x + 5y) = -7(-4)$$

4. STEP #1 - Multiply both sides of any equation necessary in order for them to cancel.

$$12x - 9y = 30$$
$$-12x - 20y = 28$$

5. STEP #2 - Add the like terms on one side of the equation and add the like terms on the other side of the equation. Then solve.

$$\frac{-29y}{-29} = \frac{58}{-29}$$

$$\boxed{y = -2}$$

6. STEP #3 - Substitute what you found into one of the original equations.

$$4x - 3y = 10$$

$$4(x) - 3(-2) = 10$$

$$4x + 6 = 10$$

$$+6 \quad +6$$

$$\frac{4x}{4} = \frac{16}{4}$$

$$\boxed{x = 4}$$

Find the solution to each set of equations using elimination.

1. Find the solution to the system of equations.

$5x-y=12$
$-5x+3y=-6$

$+2y=6$
$\frac{2y}{2}=\frac{6}{2}$

$y=3$

$5x-y=12$
$5x-3=12$
$+3\quad +3$
$\frac{5x}{5}=\frac{15}{5}$
$x=3$

(3, 3)

2. Find the solution to the system of equations.

$3y+2x=6$
$5y-2x=10$

$\frac{8y}{8}=\frac{16}{8}$

$y=2$

$3y+2x=6$
$3(2)+2x=6$
$6+2x=6$
$-6\quad -6$
$\frac{2x}{2}=\frac{0}{2}$
$x=0$

(0, 2)

3. Find the solution to the system of equations.

$-2(x+4y)=2(-2)$
$2x+5y=-2$

$-2x-8y=-4$
$2x+5y=-2$

$\frac{-3y}{-3}=\frac{-6}{-3}$

$y=2$

$x+4y=2$
$x+4(2)=2$
$x+8=2$
$-8\quad -8$
$x=-6$

(-6, 2)

4. Find the solution to the system of equations.

$-4(3x+y)=9(-4)$
$5x+4y=22$

$-12x-4y=-36$
$5x+4y=22$

$\frac{-7x}{-7}=\frac{-14}{-7}$

$x=2$

$3x+y=9$
$3(2)+y=9$
$6+y=9$
$-6\quad -6$
$y=3$

(2, 3)

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations.

$2x+y=7$
$-2(x-2y=6)(-2)$

$2x + y = 7$
$-2x + 4y = -12$

$\frac{5y}{-5} = \frac{-5}{-5}$

$y=1$

$2x + y = 7$
$2x + 1 = 7$
$-1 \quad -1$
$\frac{2x}{2} = \frac{6}{2}$
$x = 3$

(3 , 1)

6. Find the solution to the system of equations.

$4x + 2y = 8$
$2(8x - y = 1)2$
$16x - 2y = 2$

$\frac{20x}{20} = \frac{10}{20}$

$x = \frac{1}{2}$

$4x + 2y = 8$
$4(\frac{1}{2}) + 2y = 8$
$2 + 2y = 8$
$-2 \quad -2$
$\frac{2y}{2} = \frac{6}{2}$
$y=3$

($\frac{1}{2}$, 3)

7. Find the solution to the system of equations.

$-2(3x-9y=6)(-2)$
$3(2x-2y=8)(3)$

$-6x + 18y = -12$
$6x - 6y = 24$

$\frac{12y}{12} = \frac{12}{12}$

$y=1$

$3x - 9y = 6$
$3x - 9(1) = 6$
$3x - 9 = 6$
$+9 \quad +9$
$\frac{3x}{3} = \frac{15}{3}$ $x=5$

(5 , 1)

8. Find the solution to the system of equations.

$-3(3x+2y=12)-3$
$2(-4x+3y=1)(2)$

$-9x - 6y = -36$
$-8x + 6y = 2$

$\frac{-17x}{-17} = \frac{-34}{-17}$

$x=2$

$3x + 2y = 12$
$3(2) + 2y = 12$
$6 + 2y = 12$
$-6 \quad -6$
$\frac{2y}{2} = \frac{6}{2}$ $y=3$

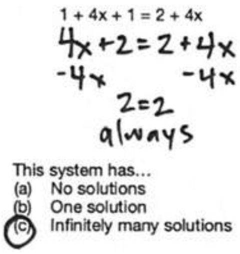
(2 , 3)

G8 U4 Lesson 13
Determine whether a system of equations will have one solution, no solutions or infinitely many solutions using equations.

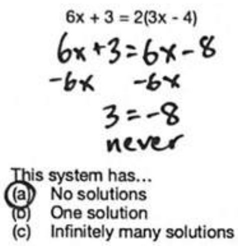
G8 U4 Lesson 13 - Today we will determine whether a system of equations will have one solution, no solutions or infinitely many solutions using equations.

Warm Welcome (Slide 1): Tutor choice

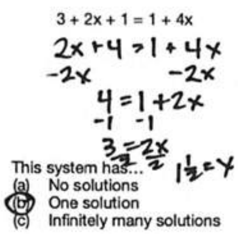
Frame the Learning/Connect to Prior Learning (Slide 2): In today's lesson, we will determine whether a system of equations will have one solution, no solutions or infinitely many solutions using equations. This is basically a chance to practice what we've already learned. You're going to do great!



Let's Review (Slide 3): We know how to determine how many solutions an equation has. Let's work through these examples to refresh our memory. First, I will combine like terms. 1 and 1 is 2. So I get 4x + 2 = 2 + 4x. I will subtract 4x from both sides. I get 2 = 2. That is always true, which means this will always have a number that works. And that is infinitely many solutions.

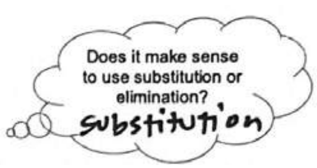


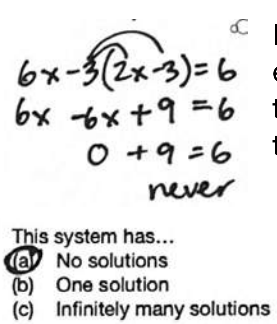
Let's do the next one. Here we have to distribute. We'll get 6x + 3 equals 6x - 8. I am going to subtract 6x from both sides. That gives us 3 = -8. That is never possible, right? So there will never be a solution. That means NO solutions.



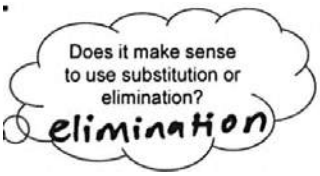
Last one. Let's combine like terms. We get 2x + 4 = 1 + 4x. Let's subtract 2x from both sides. We get 4 = 1 + 2x. Then we subtract 1 from each side. We get 3 = 2x. We divide by 2 on each side. That's 1 and 1 half equals x. That is one solution.

Let's Talk (Slide 4): We can determine the number of solutions for a system of equations just by trying to solve it with substitution. Let's try out this set. We are going to try to solve it just like we always do. Now that we've learned two strategies, we will have to ask ourselves this question, "Does it make sense to use substitution or elimination?" Now, we see that one equation is already set equal to y. So that would make it very easy to substitute. Let's do that.

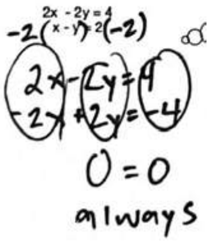




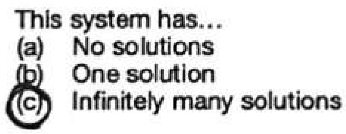
I am going to rewrite 6x - 3 times the whole amount of 2x - 3 in parentheses equals 6. Let's distribute the 3. We get 6x - 6x + 9 = 6. When we combine like terms, the 6x - 6x just cancels. We get 9 = 6. That's impossible. That's NEVER true. So there's never a solution. There's no solution. The big idea is we go about solving the system of equations like we've learned. And just like we've learned, if it looks a certain way then we will know if it doesn't have a solution.



Let's Think (Slide 5): We can determine the number of solutions for a system of equations just by trying to solve it with elimination. Once again, we will have to ask ourselves this question, "Does it make sense to use substitution or elimination?" In this case, both equations are written in standard form with the x's and y's all on one side. And it would be easy to multiply one equation so that part of it can cancel. Let's do that.



I am going to multiply the bottom equation by -2 on each side. Then we'd have 2x - 2y = 4 and we'd have -2x + 2y = -4. We are going to add these together. But let's circle the parts that cancel. 2x and -2x cancel. -2y and 2y cancel. But actually, 4 and -4 cancel too, don't they. Or we could think of it as 0 + 0 = 0.



0 = 0 is always true. So this always has a solution. It has infinitely many solutions.

So, we are going to solve all our systems of equations like normal, choosing between substitution or elimination. And sometimes we'll get equations that are always true with infinitely many solutions and sometimes we'll get equations that are never true with no solutions. And then sometimes we'll just get our usual one solution.

Let's Try It (Slide 6): Let's determine the number of solutions for systems of equations together. I will walk you through each step.

WARM WELCOME

Today we will determine whether a system of equations will have one solution, no solutions or infinitely many solutions using equations.

Let's Review: We know how to determine how many solutions an equation has.

Determine the number of solutions.

$1 + 4x + 1 = 2 + 4x$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

Determine the number of solutions.

$6x + 3 = 2(3x - 4)$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

Determine the number of solutions.

$3 + 2x + 1 = 1 + 4x$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

Let's Talk: We can determine the number of solutions for a system of equations just by trying to solve it with substitution.

Determine the number of solutions.

$y = 2x - 3$
$6x - 3y = 6$

Does it make sense to use substitution or elimination?

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

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Let's Think:

We can determine the number of solutions for a system of equations just by trying to solve it with elimination.

Determine the number of solutions.

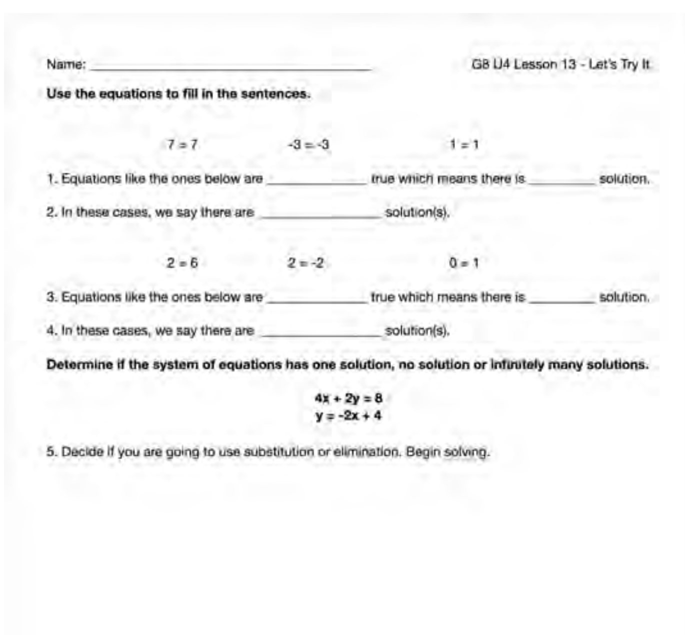
Does it make sense to use substitution or elimination?

$$2x - 2y = 4$$
$$-x + y = -2$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

Let's Try It:

Let's determine the number of solutions for systems of equations together!



Name: ________________________ G8 U4 Lesson 13 - Let's Try It

Use the equations to fill in the sentences.

7 = 7 -3 = -3 1 = 1

1. Equations like the ones below are ________ true which means there is ______ solution.
2. In these cases, we say there are __________ solution(s).

2 = 6 2 = -2 0 = 1

3. Equations like the ones below are ________ true which means there is ______ solution.
4. In these cases, we say there are __________ solution(s).

Determine if the system of equations has one solution, no solution or infinitely many solutions.

$$4x + 2y = 8$$
$$y = -2x + 4$$

5. Decide if you are going to use substitution or elimination. Begin solving.

On your Own:

Now it's time for you to do it on your own!

Name: ______________________________ G8 U4 Lesson 13 - Independent Work

Determine if the system of equations has one solution, no solution or infinitely many solutions.

1. Find the solution to the system of equations.

$$y = 4x$$
$$4x - y = 1$$

This system has...
(a) No solutions
(b) One solution
(c) Infinitely many solutions

2. Find the solution to the system of equations.

$$3x + 3y = 1$$
$$6x - 6y = 2$$

This system has...
(a) No solutions
(b) One solution
(c) Infinitely many solutions

3. Find the solution to the system of equations.

$$2x + 3y = -7$$
$$2x - y = 1$$

4. Find the solution to the system of equations.

$$y=3-2x$$
$$6x+3y=9$$

Use the equations to fill in the sentences.

7 = 7 -3 = -3 1 = 1

1. Equations like the ones below are ____________ true which means there is ________ solution.

2. In these cases, we say there are ______________ solution(s).

2 = 6 2 = -2 0 = 1

3. Equations like the ones below are ____________ true which means there is ________ solution.

4. In these cases, we say there are ______________ solution(s).

Determine if the system of equations has one solution, no solution or infinitely many solutions.

$$4x + 2y = 8$$
$$y = -2x + 4$$

5. Decide if you are going to use substitution or elimination. Begin solving.

This system has…

(a) No solutions
(b) One solution
(c) Infinitely many solutions

Determine if the system of equations has one solution, no solution or infinitely many solutions.

$$2x + y = -4$$
$$3x - 2y = -6$$

This system has…

(a) No solutions
(b) One solution
(c) Infinitely many solutions

Determine if the system of equations has one solution, no solution or infinitely many solutions.

1. Find the solution to the system of equations.

$$y = 4x$$
$$4x - y = 1$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

2. Find the solution to the system of equations.

$$3x + 3y = 1$$
$$-6x - 6y = -2$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

3. Find the solution to the system of equations.

$$2x + 3y = -7$$
$$2x - y = 1$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

4. Find the solution to the system of equations.

$$y=3-2x$$
$$6x+3y=9$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations.

$$y = 3x$$
$$6x - 2y = 0$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

6. Find the solution to the system of equations.

$$4x+2y=6$$
$$6x+3y=9$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

7. Find the solution to the system of equations.

$$-6x + 3y = -7$$
$$y = 2x - 1$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

8. Find the solution to the system of equations.

$$3x - 9y = -27$$
$$2x + 7y = 8$$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

Name: ANSWER KEY

Use the equations to fill in the sentences.

$7 = 7$ $-3 = -3$ $1 = 1$

1. Equations like the ones below are always true which means there is always solution.
2. In these cases, we say there are infinitely many solution(s).

$2 = 6$ $2 = -2$ $0 = 1$

3. Equations like the ones below are never true which means there is never a solution.
4. In these cases, we say there are no solution(s).

Determine if the system of equations has one solution, no solution or infinitely many solutions.

$4x + 2y = 8$
$y = -2x + 4$

5. Decide if you are going to use (substitution) or elimination. Begin solving.

$4x + 2(-2x + 4) = 8$

$4x - 4x + 8 = 8$

$0 + 8 = 8$

$8 = 8$

always

This system has…
- (a) No solutions
- (b) One solution
- ((c)) Infinitely many solutions

Determine if the system of equations has one solution, no solution or infinitely many solutions.

$$2(2x + y) = -4(2)$$
$$3x - 2y = -6$$

$$4x + 2y = -8$$
$$3x - 2y = -6$$

$$\frac{7x}{7} = \frac{-14}{7}$$

$$\boxed{x = -2}$$

$$3x - 2y = -6$$
$$3(-2) - 2y = -6$$
$$-6 - 2y = -6$$
$$+6 \quad\quad +6$$
$$\frac{-2y}{-2} = \frac{0}{-2}$$

$$\boxed{y = 0}$$

This system has…

(a) No solutions

(b) One solution

(c) Infinitely many solutions

Name: ANSWER KEY

Determine if the system of equations has one solution, no solution or infinitely many solutions.

1. Find the solution to the system of equations.

$y = 4x$
$4x - y = 1$

$4x - 4x = 1$

$0 = 1$

never

This system has…
- (a) No solutions ← circled
- (b) One solution
- (c) Infinitely many solutions

2. Find the solution to the system of equations.

$2(3x + 3y) = 1(2)$
$-6x - 6y = -2$

$6x + 6y = 2$
$-6x - 6y = -2$

$0 = 0$

always

This system has…
- (a) No solutions ← circled
- (b) One solution
- (c) Infinitely many solutions

3. Find the solution to the system of equations.

$2x + 3y = -7$
$3(2x - y) = 1(3)$

$2x + 3y = -7$
$6x - 3y = 3$

$\frac{8x}{8} = \frac{-4}{8}$

$x = -\frac{1}{2}$

$2x + 3y = -7$
$2(-\frac{1}{2}) + 3y = -7$
$-1 + 3y = -7$
$+1 \quad\quad +1$
$\frac{3y}{3} = \frac{-6}{3}$
$y = -2$

This system has…
- (a) No solutions
- (b) One solution ← circled
- (c) Infinitely many solutions

4. Find the solution to the system of equations.

$y = 3 - 2x$
$6x + 3y = 9$

$6x + 3(3 - 2x) = 9$

$6x + 9 - 6x = 9$

$0 + 9 = 9$

always

This system has…
- (a) No solutions
- (b) One solution
- (c) Infinitely many solutions ← circled

Determine if the coordinates are a solution to the system of equations given.

5. Find the solution to the system of equations.

$y = 3x$
$6x - 2y = 0$

$6x - 2(3x) = 0$
$6x - 6x = 0$
$0 = 0$
always

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

6. Find the solution to the system of equations.

$-3(4x+2y=6)(-3)$
$2(6x+3y=9)(2)$

$-12x - 6y = -18$
$12x + 6y = 18$

$0 = 0$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

7. Find the solution to the system of equations.

$-6x + 3y = -7$
$y = 2x - 1$

$-6x + 3(2x - 1) = -7$
$-6x + 6x - 3 = -7$
$-3 = -7$
never

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

8. Find the solution to the system of equations.

$-2(3x - 9y) = -27(-2)$
$3(2x + 7y) = 8(3)$

$-6x + 18y = 54$
$6x + 21y = 24$

$\frac{39y}{39} = \frac{78}{39}$

$y = 2$

$3x - 9y = -27$
$3x - 9(2) = -27$
$3x - 18 = -27$
$+18 \quad +18$
$\frac{3x}{3} = \frac{-9}{3}$
$x = -3$

This system has…
(a) No solutions
(b) One solution
(c) Infinitely many solutions

CITYTUTOR DC

Eighth Grade Math Lesson Materials

G8 Unit 5:

Exponents and Scientific Notation

G8 U5 Lesson 1
Compare quantities using expressions that represent repeated multiplication, and comprehend that repeated division of a number is equivalent to repeated multiplication by its reciprocal.

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G8 U5 Lesson 1 - Students will compare quantities using expressions that represent repeated multiplication, and comprehend that repeated division of a number is equivalent to repeated multiplication by its reciprocal.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today is our first lesson in our unit that's all about exponents and scientific notation. We'll learn more about scientific notation in later lessons, but I imagine you're already somewhat familiar with the idea of exponents from earlier grades. What do you already know about exponents? Possible Student Answers, Key Points:

- I know they are represented by a little number at the top right corner of a larger number.
- I know the larger number is called the base.
- They tell us how many times to multiply the base by itself.

Let's explore more about exponents. Today, we'll compare quantities using expressions that represent repeated multiplication, and we'll see how we can use exponents to help us think about repeated division.

Let's Talk (Slide 3): Before we jump into some problems, take a look at the four cards here. *(pause)* After looking at them, try to think of a reason why one of the cards doesn't belong and the other three do belong. Depending on how you think about the cards, you might have more than one idea. Possible Student Answers, Key Points:

- The yellow card doesn't belong because it is the only one that shows an exponential expression in expanded form.
- The red card doesn't belong because it is the only one that operates with a value of 3.
- The blue card doesn't belong because it is the only one that deals with addition.
- The green card doesn't belong because it is the only one that includes an exponent.

Great ideas! Let's look closely at the yellow and the green cards, since those most directly relate to our upcoming work with exponents. The green card shows 2 to the third power. We call 2 the base, and 3 is the exponent. We know that when we have 2 to the third power, that means we multiply 2 by itself three times. So the yellow card is equivalent to the green card, just in expanded form. What is the value of 2 to the third power? How do you know? Possible Student Answers, Key Points:

- The value of 2 to the third power is 8.
- I know 2 x 2 is 4, and I know 4 times another 2 is 8.

Nice work. Let's work on some problems involving exponents together.

Let's Think (Slide 4): This problem wants us to think about how exponents show up in a real-world scenario. *(read the problem aloud)* When we double something, we multiply it by 2. When we triple something, we multiply it by 3. When we quadruple something, like in this problem, we multiply it by 4. Let's work together to fill in the table.

WEEK	EXPANDED	EXPONENT	VALUE
1	4		
2	4 • 4		
3	4 • 4 • 4		
4	4 • 4 • 4 • 4		

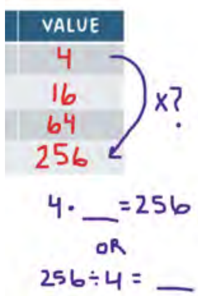
In week 1, they only earned 4 dollars. *(write 4 in the expanded column)* In week 2, they want to quadruple that amount, or multiply it by 4. *(write 4 • 4 in expanded column)* In week 3, they want to quadruple their money from week 2. I can write that by writing 4 • 4 then times 4 again. *(fill the expression in the expanded column)* These expressions are in expanded form, because we can see every factor being multiplied. How could I write the expression for week 4 in expanded form? (4 • 4 • 4 • 4) *(fill in table)*

WEEK	EXPANDED	EXPONENT	VALUE
1	4	4^1	4
2	4 • 4	4^2	16
3	4 • 4 • 4	4^3	64
4	4 • 4 • 4 • 4	4^4	256

We can write each of these expanded values using an exponent, since each expression involves repeated multiplication. Week 1 is just one factor of 4, so we can write that as 4 to the first power. *(write that in the exponent column)* Week 2 is two factors of 4, so we can write that as 4 to the second power. *(write that in the exponent column)* How could we write the expanded expressions from the other weeks using an exponent? How do you know? *(complete exponent column as student shares)* Possible Student Answers, Key Points:

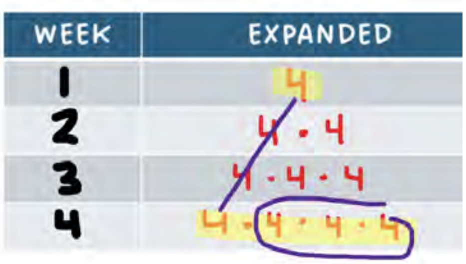
- Week 3 would be 4 to the third power, because there are 3 factors of 4.
- Week 4 would be 4 to the fourth power, because there are 4 factors of 4.

Each of our exponential expressions has the same base, 4. This makes sense, because we were multiplying factors of 4 every week. The exponent increased by one each time, since we were multiplying by an extra factor of 4 in each consecutive week. Let's fill in the values. *(fill in last column as you narrate and as student shares)* Week 1 is just 4. Week 2 is 16, because 4 times 4 is 16. What are the last two values? Use scratch paper to help you if necessary. (64 and 256)



We've completed the table. Let's look at our values closely to answer the last part that asks us how many times more money plan to earn in Week 4 than in Week 1. *(draw arrow from 4 to 256 with "x ?")* We can think of this a couple ways.

We can think 4, the value from week 1, times what number would give us 256, the value from week 4. *(write 4 • _____ = 256)* I could also think about that relationship in terms of division. Instead of 4 times something equals 256, I can think of it as 256 divided by 4 equals something. *(write 256 divided by 4 = _____)* Use either equation to figure out the value. I encourage you to use scratch paper. (64)



The value of week 4 is 64 times greater than the value of week 1. There is another way we can think of this. Let's look at the expanded form from each week. *(highlight week 1 and week 4 in expanded form)* If I want to know how many times greater week 4 is than week 1, I can look at their factors. I notice they both have one factor of 4. *(draw line to connect one factor of 4 in each expression)* How many more factors of 4 does week 4 have? (3 factors of 4) Week 4 has 3 more factors of 4, and I know 3 factors of 4 is 64. That's the same answer we got when we used the equations a minute ago. Either strategy can help us determine how many times greater one exponential value is than another.

Let's Think (Slide 5): Let's look at one more problem before we get some practice. This problem wants us to evaluate each expression. Notice, the base of each expression is a fraction.

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$$

$$\frac{1}{2\cdot 2\cdot 2}=\frac{1}{8}$$

Let's find ½ to the third power. I can write this as ½ times ½ times ½ , since the exponent tells me I need 3 factors of the base. *(write it out)* I know ½ times ½ is ¼, and I know ¼ times another ½ is ⅛. *(write = ⅛)*

If multiplying with fractions isn't your favorite, you may have noticed that multiplying by ½ is the same as dividing by 2. You can think of multiplying a fraction repeatedly as dividing repeatedly by the denominator. That might look like this. *(write 1 in numerator and 2 • 2 • 2 in denominator as shown)* Either representation is acceptable.

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{16}$$

$$\frac{1}{2\cdot 2\cdot 2\cdot 2}=\frac{1}{16}$$

Help me with the other expression. How could I expand the expression to help me find the value? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- We need 4 factors of ½ since the exponent is 4. That would look like ½ times ½ times ½ times ½.
- I know ½ times ½ is ¼. I know ¼ times ½ is ⅛. I know ⅛ times ½ is 1/16.

We can think of this expression as ½ times itself four times. We could also think of it as 1 divided by 2 four times. In either case, we end up with a value of 1/16. Repeated multiplication of a unit fraction, like ½, can be thought of as repeated division of the reciprocal.

Let's Try it (Slides 6 - 7): Now let's try out a few more practice problems together before you get a chance to independently show what you know. We'll use the exponent to help us determine how many factors of the base we need to write in expanded form. We also know that if we see a fractional base, we can think of it as dividing by the reciprocal if we prefer. Let's use everything we've learned so far to tackle these next problems.

WARM WELCOME

Today we will compare quantities using expressions that represent repeated multiplication, and comprehend that repeated division of a number is equivalent to repeated multiplication by its reciprocal.

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Let's Talk:

Which one doesn't belong? Why?

2 x 2 x 2	2 x 3	2 + 2 + 2	2^3

Let's Think:

A drama club earned $4 the first week of their fundraiser. They want to quadruple their earnings each week for the next several weeks. Complete the table to show how much they'll earn in Week 4.

WEEK	EXPANDED	EXPONENT	VALUE
1			
2			
3			
4			

They plan to earn __________ times as much money in Week 4 as they earned in Week 1.

Let's Think:

Evaluate each expression.

$(\frac{1}{2})^3$	$(\frac{1}{2})^4$

Let's Try It:

Let's explore comparing quantities using expressions that represent repeated multiplication together.

Name: ______________________________ G8 U5 Lesson 1 - Let's Try It

The expanded expression 5 · 5 · 5 can be written as 5^3.

1. The ____________ is the factor being multiplied. What is the base in the expression 5^3?
2. The ____________ tells us how many factors of the base are being multiplied. What is the exponent in the expression 5^3?
3. Find the value of the expression.

A performance artist has 2 large spheres of clay. Each hour, the performance artist splits each sphere into identical, smaller spheres. This doubles the number of spheres each hour.

1. Fill in the table to show how many spheres there will be each hour in expanded form.

HOUR	EXPANDED	EXPONENT	VALUE
1	2		
2	2 · ____		
3	2 · ____ · ____		

2. Fill in the table to show each expanded expression written using a single exponent.
3. What is the same about each exponent expression? What is different?
4. Fill in the column labeled "value" to show how many spheres the artist would have each hour.
5. How many spheres would the performance artist have in Hour 4? Hour 5? Show how you know.

Let's compare the number of spheres in the 3rd hour to the number of spheres in the 5th hour.

6. How many factors of 2 are in the 3rd hour? ________
7. How many factors of 2 are in the 5th hour? ________
8. How many more factors of 2 are in 2^5 than 2^3? ________
9. The number of spheres in the 5th hour is ________ times the number of spheres in the 3rd hour.

Now compare the number of spheres in the hours named below. Show how you know.

10. The number of spheres in the 4th hour is ________ times the number of spheres in the 1st hour.
11. The number of spheres in the 5th hour is ________ times the number of spheres in the 4th hour.

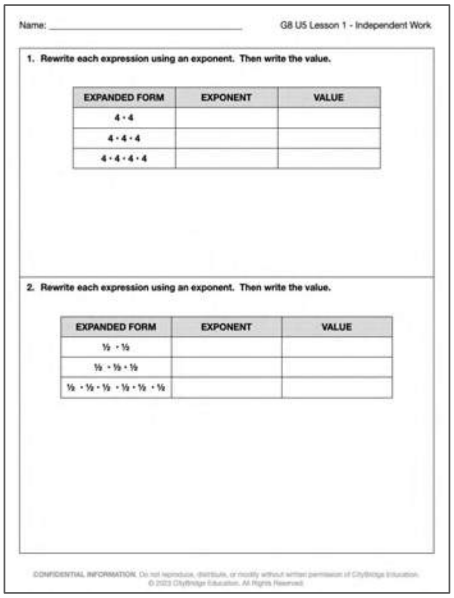
We can work with fractional bases the same way we work with whole number bases. The first two rows of the table are completed.

12. Fill in the rest of the table.

EXPANDED	EXPONENT	FRACTION	VALUE
$\frac{1}{3}$	$\frac{1}{3}^1$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{3}^2$	$\frac{1}{3 \cdot 3}$	$\frac{1}{9}$

On your Own:

Now it's time to compare quantities using expressions that represent repeated multiplication on your own.



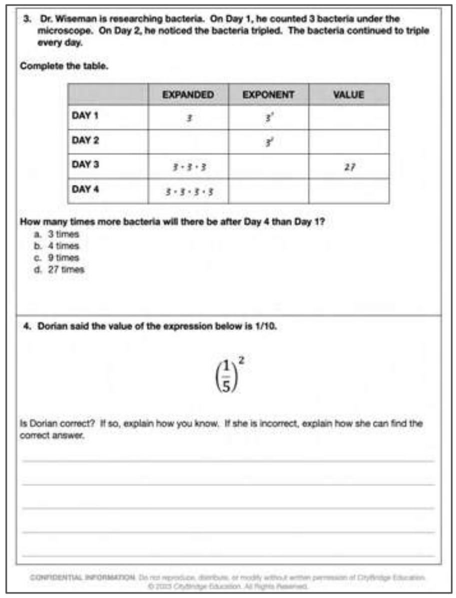
Name: ______________________ G8 U5 Lesson 1 - Independent Work

1. Rewrite each expression using an exponent. Then write the value.

EXPANDED FORM	EXPONENT	VALUE
4 • 4		
4 • 4 • 4		
4 • 4 • 4 • 4		

2. Rewrite each expression using an exponent. Then write the value.

EXPANDED FORM	EXPONENT	VALUE
½ • ½		
½ • ½ • ½		
½ • ½ • ½ • ½ • ½ • ½		



3. Dr. Wiseman is researching bacteria. On Day 1, he counted 3 bacteria under the microscope. On Day 2, he noticed the bacteria tripled. The bacteria continued to triple every day.

Complete the table.

	EXPANDED	EXPONENT	VALUE
DAY 1	3	3^1	
DAY 2		3^2	
DAY 3	3 • 3 • 3		27
DAY 4	3 • 3 • 3 • 3		

How many times more bacteria will there be after Day 4 than Day 1?

a. 3 times
b. 4 times
c. 9 times
d. 27 times

4. Dorian said the value of the expression below is 1/10.

$$\left(\frac{1}{5}\right)^2$$

Is Dorian correct? If so, explain how you know. If she is incorrect, explain how she can find the correct answer.

The expanded expression 5 • 5 • 5 can be written as 5^3.

1. The ______________ is the factor being multiplied. What is the base in the expression 5^3?

2. The ______________ tells us how many factors of the base are being multiplied. What is the exponent in the expression 5^3?

3. Find the value of the expression.

A performance artist has 2 large spheres of clay. Each hour, the performance artist splits each sphere into identical, smaller spheres. This doubles the number of spheres each hour.

1. Fill in the table to show how many spheres there will be each hour in expanded form.

HOUR	EXPANDED	EXPONENT	VALUE
1	2		
2	2 • _____		
3	2 • _____ • _____		

2. Fill in the table to show each expanded expression written using a single exponent.

3. What is the same about each exponent expression? What is different?

____

____

____

4. Fill in the column labeled "value" to show how many spheres the artist would have each hour.

5. How many spheres would the performance artist have in Hour 4? Hour 5? Show how you know.

Let's compare the number of spheres in the 3rd hour to the number of spheres in the 5th hour.

6. How many factors of 2 are in the 3rd hour? __________

7. How many factors of 2 are in the 5th hour? __________

8. How many more factors of 2 are in 2^3 than 2^5? __________

9. The number of spheres in the 5th hour is __________ times the number of spheres in the 3rd hour.

Now compare the number of spheres in the hours named below. Show how you know.

10. The number of spheres in the 4th hour is __________ times the number of spheres in the 1st hour.

11. The number of spheres in the 5th hour is __________ times the number of spheres in the 4th hour.

We can work with fractional bases the same way we work with whole number bases. The first two rows of the table are completed.

12. Fill in the rest of the table.

EXPANDED	EXPONENT	FRACTION	VALUE
$\frac{1}{3}$	$\frac{1}{3}^{1}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3} \bullet \frac{1}{3}$	$\frac{1}{3}^{2}$	$\frac{1}{3 \bullet 3}$	$\frac{1}{9}$

1. Rewrite each expression using an exponent. Then write the value.

EXPANDED FORM	EXPONENT	VALUE
4 • 4		
4 • 4 • 4		
4 • 4 • 4 • 4		

2. Rewrite each expression using an exponent. Then write the value.

EXPANDED FORM	EXPONENT	VALUE
½ • ½		
½ • ½ • ½		
½ • ½ • ½ • ½ • ½ • ½		

3. Dr. Wiseman is researching bacteria. On Day 1, he counted 3 bacteria under the microscope. On Day 2, he noticed the bacteria tripled. The bacteria continued to triple every day.

Complete the table.

	EXPANDED	EXPONENT	VALUE
DAY 1	3	3^1	
DAY 2		3^2	
DAY 3	$3 \cdot 3 \cdot 3$		27
DAY 4	$3 \cdot 3 \cdot 3 \cdot 3$		

How many times more bacteria will there be after Day 4 than Day 1?

a. 3 times
b. 4 times
c. 9 times
d. 27 times

4. Dorian said the value of the expression below is 1/10.

$$\left(\frac{1}{5}\right)^2$$

Is Dorian correct? If so, explain how you know. If she is incorrect, explain how she can find the correct answer.

The expanded expression 5 · 5 · 5 can be written as 5^3.

1. The base is the factor being multiplied. What is the base in the expression 5^3?
2. The exponent tells us how many factors of the base are being multiplied. What is the exponent in the expression 5^3?
3. Find the value of the expression.

   5 · 5 · 5
   25 · 5 = 125

A performance artist has 2 large spheres of clay. Each hour, the performance artist splits each sphere into identical, smaller spheres. This doubles the number of spheres each hour.

1. Fill in the table to show how many spheres there will be each hour in expanded form.

HOUR	EXPANDED	EXPONENT	VALUE
1	2	2^1	2
2	2 · 2	2^2	4
3	2 · 2 · 2	2^3	8

2. Fill in the table to show each expanded expression written using a single exponent.
3. What is the same about each exponent expression? What is different?

   The base is the same in each expression. The exponents increase by 1.

4. Fill in the column labeled "value" to show how many spheres the artist would have each hour.
5. How many spheres would the performance artist have in Hour 4? Hour 5? Show how you know.

   Hour 4: 2·2·2·2 → 4 · 4 → 16

   Hour 5: 2·2·2·2·2 → 16 · 2 → 32

Let's compare the number of spheres in the 3rd hour to the number of spheres in the 5th hour.

6. How many factors of 2 are in the 3rd hour? 3

7. How many factors of 2 are in the 5th hour? 5

8. How many more factors of 2 are in 2^3 than 2^5? 2

9. The number of spheres in the 5th hour is 4 (2×2) times the number of spheres in the 3rd hour.

Now compare the number of spheres in the hours named below. Show how you know.

10. The number of spheres in the 4th hour is 8 times the number of spheres in the 1st hour.

4th: $2 \cdot 2 \cdot 2 \cdot 2$
1st: 2

11. The number of spheres in the 5th hour is 2 times the number of spheres in the 4th hour.

5th: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
4th: $2 \cdot 2 \cdot 2 \cdot 2$

We can work with fractional bases the same way we work with whole number bases. The first two rows of the table are completed.

12. Fill in the rest of the table.

EXPANDED	EXPONENT	FRACTION	VALUE
$\frac{1}{3}$	$\frac{1}{3}^1$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{3}^2$	$\frac{1}{3 \cdot 3}$	$\frac{1}{9}$
$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{3}^3$	$\frac{1}{3 \cdot 3 \cdot 3}$	$\frac{1}{27}$
$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{3}^4$	$\frac{1}{3 \cdot 3 \cdot 3 \cdot 3}$	$\frac{1}{81}$
$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$	$\frac{1}{3}^5$	$\frac{1}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$	$\frac{1}{243}$

1. **Rewrite each expression using an exponent. Then write the value.**

EXPANDED FORM	EXPONENT	VALUE
4 • 4	4^2	16
4 • 4 • 4	4^3	64
4 • 4 • 4 • 4	4^4	256

2. **Rewrite each expression using an exponent. Then write the value.**

EXPANDED FORM	EXPONENT	VALUE
½ • ½	$\frac{1}{2}^2$	1/4
½ • ½ • ½	$\frac{1}{2}^3$	1/8
½ • ½ • ½ • ½ • ½ • ½	$\frac{1}{2}^6$	1/64

$\frac{1}{8}$ $\frac{1}{8}$

3. **Dr. Wiseman is researching bacteria. On Day 1, he counted 3 bacteria under the microscope. On Day 2, he noticed the bacteria tripled. The bacteria continued to triple every day.**

Complete the table.

	EXPANDED	EXPONENT	VALUE
DAY 1	3	3^1	3
DAY 2	$3 \cdot 3$	3^2	9
DAY 3	$3 \cdot 3 \cdot 3$	3^3	27
DAY 4	$3 \cdot 3 \cdot 3 \cdot 3$	3^4	81

How many times more bacteria will there be after Day 4 than Day 1?

a. 3 times
b. 4 times
c. 9 times
d. 27 times

$81 = \_\_\_ \times 3$

4. **Dorian said the value of the expression below is 1/10.**

$$\left(\frac{1}{5}\right)^2$$

Is Dorian correct? If so, explain how you know. If she is incorrect, explain how she can find the correct answer.

Dorian is incorrect. $\frac{1}{5}^2$ means $\frac{1}{5} \times \frac{1}{5}$. She can multiply across to get a numerator of 1 and a denominator of 25. The correct answer is $\frac{1}{25}$.

G8 U5 Lesson 2
Generalize the exponent rule 10n · 10m = 10n+m and write equivalent exponential expressions of multiplication expressions with a base of 10.

CITYTUTOR DC

G8 U5 Lesson 2 - Students will generalize the exponent rule $10^n \bullet 10^m = 10^{n+m}$ and write equivalent exponential expressions of multiplication expressions with a base of 10.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our last lesson, we kickstarted our unit covering all things exponents. Today, we'll continue exploring exponents, specifically thinking about what happens when we multiply exponential expressions. As we work today, I want you to see if you can predict or discover patterns that could help make our work efficient.

Let's Talk (Slide 3): Take a second to look at the expression shown here. What do you notice about it? What do you wonder? Possible Student Answers, Key Points:

- I notice there are a lot of tens being multiplied.
- I notice the first group has 4 tens and the second group has 7 tens. There are 11 tens altogether.
- I wonder if there is an easier way to write this. Maybe exponents could help us.
- I wonder what the total value is.

This expression shows two groups of tens being multiplied together. It's a pretty cumbersome expression to look at with all these tens. We can use exponents to help us think about this expression. Think back to our previous lesson. How could I write 4 factors of 10 and 6 factors of ten using exponents? Possible Student Answers, Key Points:

- Four factors of 10 can be 10 to the fourth power.
- Six factors of 10 can be 10 to the sixth power.

$$10^4 \bullet 10^7$$

(write 10 to the fourth power times 10 to the seventh power) Instead of writing out all the tens in expanded form like we were originally shown, we could write this equivalent expression using exponents.

$$10^{11}$$

We could even take it a step further. We saw that there were 11 tens being multiplied together. Instead of writing them all out, and instead of writing them as two exponential factors, we could simply write this expression as 10 to the eleventh power. *(write 10 to the 11th power)* That makes sense, because we have 11 factors of ten being multiplied. The expanded form or either of the two exponential expressions can be used to represent the same thing. We'll use this thinking today to help us multiply expressions involving exponents. Let's dive in!

Let's Think (Slide 4): Our first problem wants us to rewrite each of the expressions below using a single exponent. Before we begin, take a second to look at each problem. What do you notice about the three exponential expressions? Possible Student Answers, Key Points:

- They all have a base of 10. They each have different exponents.
- The second expression involves 3 factors.
- The last expression has really big exponents.

$$10^4 \cdot 10^2$$

$$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10)$$

$$10^6$$

Let's look at the first expression. To multiply these, it can help to write each factor in expanded form. *(write expression as you narrate)* I can write 10 to the fourth power as four factors of 10. I can write 10 to the second power as two factors of 10. How many factors of 10 do we see in all? (six factors of 10) There are six factors of 10, so we can think of the product of 10 to the fourth power and 10 to the second power as being 10 to the sixth power. *(write 10 to the sixth power)*

We wrote each factor in expanded form, then we could easily see how many total factors of 10 compose the product.

$$10^3 \cdot 10^1 \cdot 10^5$$
$$(10 \cdot 10 \cdot 10) \cdot (10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$$

Let’s try the next one. How could I write this expression using expanded form? Possible Student Answers, Key Points:

- 10 to the third power is 3 factors of 10. 10 to the first power is 1 factor of 10. 10 to the fifth power is 5 factors of 10.

$$10^9$$

(write expanded form as student shares) We can expand each factor to see how many total factors of 10 are in the expression. We see there are nine factors of 10 being multiplied, so we can write the product as 10 to the ninth power. *(write 10 to the ninth power)*

We noticed that the third expression has exponents that are pretty big. It would take us a really long time to write this expression in expanded form. Thinking about our last couple problems, can we think of a pattern or a rule that could help us tackle this problem more efficiently?

The first problem had 4 factors of 10 times 2 factors of 10, and we ended up with 6 factors of 10. The second problem had 3 factors of 10 times 1 factor of 10 times 5 factors of 10, and we ended up with 9 factors of 10. In a sense, we were just combining all the factors of 10. Instead of expanding the expression, is there another pattern or rule we could use to help us multiply by powers of 10? (We can add the exponents, since we’re just combining the factors of 10 together.)

$$10^{4+2}$$

We can just add the exponents. In the first example, instead of expanding to see all 6 factors of 10, we could have just added the exponents of 4 and 2 to show 10 to the sixth power. *(write expression as shown here)*

$$10^{3+1+5}$$

In the second example, instead of expanding to see all 9 factors of 10, we could have just added the exponents of 3, 1, and 5 to show 10 to the 9th power. *(Write expression as shown here)* When multiplying by powers of 10, we can add the exponents. That can often be more efficient than expanding every expression.

$$10^{42} \cdot 10^{19}$$
$$10^{42+19}$$
$$10^{61}$$

How can we use this rule to find the value of the third expression? *(write as student shares their thinking)* Possible Student Answers, Key Points:

- We can add 42 and 19 to see how many factors of 10 there would be.
- 42 + 19 is 61, so we can think of the product as 10 to the 61st power.

42 factors of 10 times 19 more more factors of 10 would be 61 factors of 10. We can write that as 10 to the 61st power. We simply added the exponents to show all the factors of 10 being multiplied.

Let’s Think (Slide 5): Let’s try one more together before we move into some practice.

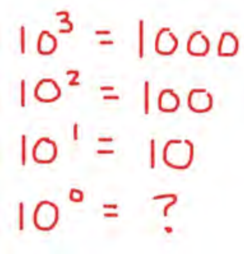
$$10^{7+0}$$
$$10^7$$

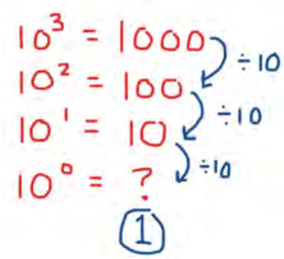
This problem wants us to find the product of 10 to the seventh power and 10 to the zero power. Our answer should be written as an expression with a single exponent. How could we use the rule we just learned to find the product? Possible Student Answers, Key Points:

- We can add the exponents. 7 + 0 = 7, so our answer would be 10 to the 7th power.

We can add the exponents. *(write as you narrate)* 7 plus 0 is 7, so our answer would be 10 to the 7th power.

10 to the seventh power is our answer. The rule worked! Let’s pause for a second to think about any number to the 0 power, because this can be a common area for confusion if we’re not careful.





(write an organized list of powers of 10 as you narrate) I know 10 to the 3rd power is 10 times 10 times 10, which is 1,000. I know 10 to the 2nd power is 10 times 10, which is 100. I know 10 to the first power is just one factor of 10, so it's 10. I can use the patterns I see here to help me figure out the value of 10 to the zero power.

Each value is 1/10 of the value above it. Or we can think of dividing by 10 each time to find the next value. *(draw arrows that show the dividing by 10 pattern)* If the pattern continues, we can see that 10 divided by 10 is 1. So 10 to the 0 power is 1.

If we go back to the original problem, we were asked to find 10 to the seventh power times 10 to the zero power. That means we're finding 10 the seventh power times 1. Our answer of 10 to the seventh power, makes sense!

We'll see more work with 0 exponents in future lessons. It can be helpful to know that anything to the zero power is always just 1.

Let's Try it (Slides 6 - 7): Now we'll work on a few more examples together. As we saw today, we can multiply expressions with exponents a couple different ways. What are some strategies we saw to help us?
Possible Student Answers, Key Points:

- We can write the expressions in expanded form to see how many factors of ten there are in all.
- We can add the exponents. This is helpful any time, but particularly when it would be inefficient to write the expressions in standard form.

We can use expanded form or our rule to help us as we work through the next problems. Let's go for it!

WARM WELCOME

Today we will generalize the exponent rule $10^n \bullet 10^m = 10^{n+m}$ and write equivalent exponential expressions of multiplication expressions with a base of 10.

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Let's Talk:

What do you notice? What do you wonder?

$$(10 \bullet 10 \bullet 10 \bullet 10) \bullet (10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10)$$

Let's Think:

Rewrite each as an expression with a single exponent.

$10^4 \bullet 10^2$ $10^3 \bullet 10^1 \bullet 10^5$ $10^{42} \bullet 10^{19}$

Can we figure out a rule?

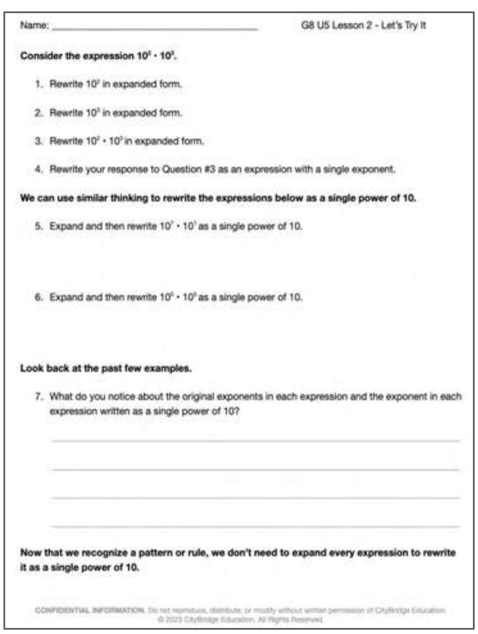
Let's Think:

Rewrite as an expression with a single exponent.

$$10^7 \cdot 10^0$$

Let's Try It:

Let's explore generalizing the exponent rule $10^n \cdot 10^m = 10^{n+m}$ and writing equivalent exponential expressions of multiplication expressions with a base of 10 together.



Name: ______________________ G8 U5 Lesson 2 - Let's Try It

Consider the expression $10^2 \cdot 10^3$.

1. Rewrite 10^2 in expanded form.
2. Rewrite 10^3 in expanded form.
3. Rewrite $10^2 \cdot 10^3$ in expanded form.
4. Rewrite your response to Question #3 as an expression with a single exponent.

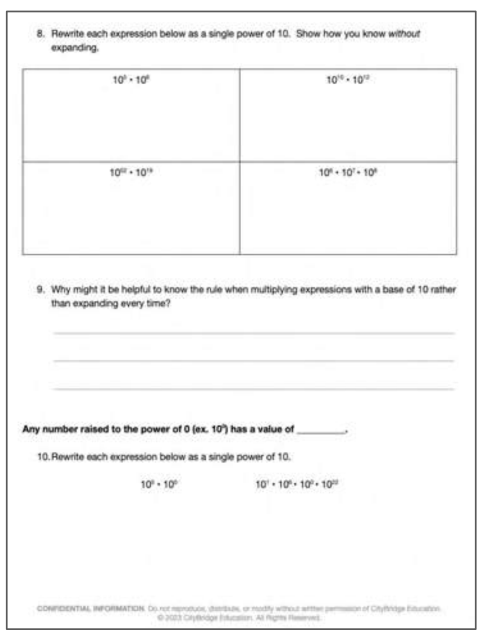
We can use similar thinking to rewrite the expressions below as a single power of 10.

5. Expand and then rewrite $10^7 \cdot 10^1$ as a single power of 10.
6. Expand and then rewrite $10^{[illegible]} \cdot 10^{[illegible]}$ as a single power of 10.

Look back at the past few examples.

7. What do you notice about the original exponents in each expression and the exponent in each expression written as a single power of 10?

Now that we recognize a pattern or rule, we don't need to expand every expression to rewrite it as a single power of 10.



8. Rewrite each expression below as a single power of 10. Show how you know *without* expanding.

$10^{[illegible]} \cdot 10^{[illegible]}$	$10^{[illegible]} \cdot 10^{[illegible]}$
$10^{[illegible]} \cdot 10^{[illegible]}$	$10^{[illegible]} \cdot 10^{[illegible]} \cdot 10^{[illegible]}$

9. Why might it be helpful to know the rule when multiplying expressions with a base of 10 rather than expanding every time?

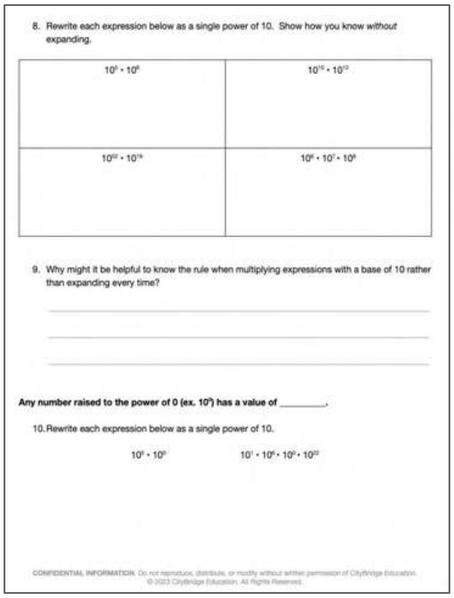
Any number raised to the power of 0 (ex. 10^0) has a value of _________.

10. Rewrite each expression below as a single power of 10.

$10^{[illegible]} \cdot 10^0$ $10^{[illegible]} \cdot 10^{[illegible]} \cdot 10^0 \cdot 10^{[illegible]}$

On your Own:

Now it's time to generalize the exponent rule $10^n \cdot 10^m = 10^{n+m}$ and write equivalent exponential expressions of multiplication expressions with a base of 10 on your own.



8. Rewrite each expression below as a single power of 10. Show how you know without expanding.

$10^3 \cdot 10^6$	$10^{15} \cdot 10^{12}$
$10^{22} \cdot 10^{19}$	$10^6 \cdot 10^7 \cdot 10^8$

9. Why might it be helpful to know the rule when multiplying expressions with a base of 10 rather than expanding every time?

____

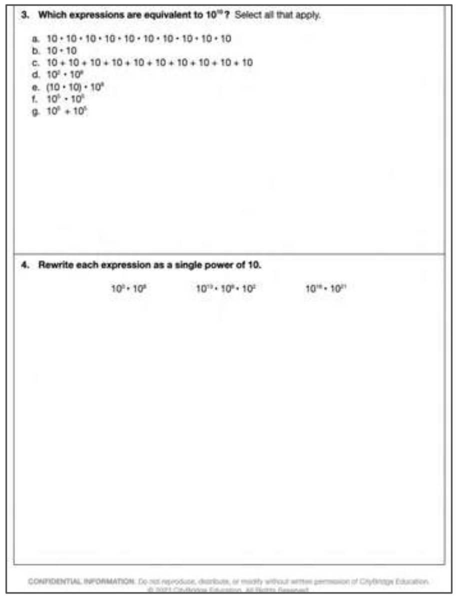
____

____

Any number raised to the power of 0 (ex. 10^0) has a value of ________.

10. Rewrite each expression below as a single power of 10.

$10^8 \cdot 10^0$ $\qquad$ $10^1 \cdot 10^6 \cdot 10^0 \cdot 10^{22}$



3. **Which expressions are equivalent to 10^{10}?** Select all that apply.

a. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
b. $10 \cdot 10$
c. $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10$
d. $10^2 \cdot 10^8$
e. $(10 \cdot 10) \cdot 10^8$
f. $10^5 \cdot 10^5$
g. $10^5 + 10^5$

4. **Rewrite each expression as a single power of 10.**

$10^3 \cdot 10^8$ $\qquad$ $10^{13} \cdot 10^8 \cdot 10^2$ $\qquad$ $10^{16} \cdot 10^{21}$

Consider the expression $10^2 \cdot 10^3$.

1. Rewrite 10^2 in expanded form.

2. Rewrite 10^3 in expanded form.

3. Rewrite $10^2 \cdot 10^3$ in expanded form.

4. Rewrite your response to Question #3 as an expression with a single exponent.

We can use similar thinking to rewrite the expressions below as a single power of 10.

5. Expand and then rewrite $10^7 \cdot 10^1$ as a single power of 10.

6. Expand and then rewrite $10^5 \cdot 10^5$ as a single power of 10.

Look back at the past few examples.

7. What do you notice about the original exponents in each expression and the exponent in each expression written as a single power of 10?

____

____

____

____

Now that we recognize a pattern or rule, we don't need to expand every expression to rewrite it as a single power of 10.

8. Rewrite each expression below as a single power of 10. Show how you know *without* expanding.

$10^5 \bullet 10^6$	$10^{10} \bullet 10^{12}$
$10^{52} \bullet 10^{19}$	$10^6 \bullet 10^7 \bullet 10^8$

9. Why might it be helpful to know the rule when multiplying expressions with a base of 10 rather than expanding every time?

____

____

____

Any number raised to the power of 0 (ex. 10^0) has a value of _________.

10. Rewrite each expression below as a single power of 10.

$10^5 \bullet 10^0$ $\qquad$ $10^1 \bullet 10^6 \bullet 10^0 \bullet 10^{22}$

1. **Complete the table.** The first example is done for you.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^5 \bullet 10^2$	10 • 10 • 10 • 10 • 10 • 10 • 10	10^7
$10^2 \bullet 10^2$		
$10^5 \bullet 10^3$		
$10^4 \bullet 10$		

2. **Maria grouped factors of ten in expanded form as shown below.**
   a. Rewrite each grouping using exponents.
   b. Then rewrite each using a single exponent.

$(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10)$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$

3. Which expressions are equivalent to 10^{10}? Select all that apply.

a. $10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$
b. $10 \bullet 10$
c. $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10$
d. $10^2 \bullet 10^8$
e. $(10 \bullet 10) \bullet 10^8$
f. $10^5 \bullet 10^5$
g. $10^5 + 10^5$

4. Rewrite each expression as a single power of 10.

$10^0 \bullet 10^8$ $10^{13} \bullet 10^9 \bullet 10^2$ $10^{16} \bullet 10^{21}$

Name: KEY

Consider the expression $10^2 \cdot 10^3$.

1. Rewrite 10^2 in expanded form. $10 \cdot 10$
2. Rewrite 10^3 in expanded form. $10 \cdot 10 \cdot 10$
3. Rewrite $10^2 \cdot 10^3$ in expanded form. $(10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$
4. Rewrite your response to Question #3 as an expression with a single exponent. 10^5

We can use similar thinking to rewrite the expressions below as a single power of 10.

5. Expand and then rewrite $10^7 \cdot 10^1$ as a single power of 10.

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10)$

10^8

6. Expand and then rewrite $10^5 \cdot 10^5$ as a single power of 10.

$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$

10^{10}

Look back at the past few examples.

7. What do you notice about the original exponents in each expression and the exponent in each expression written as a single power of 10?

The exponent in the answer is the sum of the exponents in the original expression.

Now that we recognize a pattern or rule, we don't need to expand every expression to rewrite it as a single power of 10.

8. Rewrite each expression below as a single power of 10. Show how you know *without* expanding.

$10^5 \cdot 10^6$	$10^{10} \cdot 10^{12}$
$10^{5+6} = 10^{11}$	$10^{10+12} = 10^{22}$
$10^{52} \cdot 10^{19}$	$10^6 \cdot 10^7 \cdot 10^8$
$10^{52+19} = 10^{71}$	$10^{6+7+8} = 10^{21}$

Like this one!

9. Why might it be helpful to know the rule when multiplying expressions with a base of 10 rather than expanding every time?

If the exponent is a big number it would be tedious to write out in expanded form.

Any number raised to the power of 0 (ex. 10^0) has a value of 1.

10. Rewrite each expression below as a single power of 10.

$10^5 \cdot 10^0$

10^{6+0}

10^6

$10^1 \cdot 10^6 \cdot 10^0 \cdot 10^{22}$

$10^{1+6+0+22}$

10^{29}

Name: KEY

1. Complete the table. The first example is done for you.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^5 \cdot 10^2$	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	10^7
$10^2 \cdot 10^2$	$10 \cdot 10 \cdot 10 \cdot 10$	10^4
$10^5 \cdot 10^3$	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	10^8
$10^4 \cdot 10$	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	10^5

2. Maria grouped factors of ten in expanded form as shown below.

a. Rewrite each grouping using exponents.

b. Then rewrite each using a single exponent.

$(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10)$

$10^3 \cdot 10^2$

10^5

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$

$10^4 \cdot 10^5$

10^9

3. **Which expressions are equivalent to 10^{10}?** Select all that apply.

(a.) $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ ←
~~b. $10 \cdot 10$~~
~~c. $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10$~~
(d.) $10^2 \cdot 10^8$
(e.) $(10 \cdot 10) \cdot 10^8$ ←
(f.) $10^5 \cdot 10^5$
~~g. $10^5 + 10^5$~~

all show 10 factors of 10

4. **Rewrite each expression as a single power of 10.**

$10^0 \cdot 10^8$ $\quad$ $10^{13} \cdot 10^9 \cdot 10^2$ $\quad$ $10^{16} \cdot 10^{21}$

10^{0+8} $\quad$ 10^{13+9+2} $\quad$ 10^{16+21}

10^8 $\quad$ 10^{24} $\quad$ 10^{37}

G8 U5 Lesson 3
Explain and use a rule for raising a power of 10 to a power, that (10^n)^m = 10^(n•m)

G8 U5 Lesson 3 - Students will generalize the exponent rule $10^n \div 10^m = 10^{n-m}$ and write equivalent exponential expressions of division expressions with a base of 10.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our last lesson together, we explored different ways to multiply with powers of 10. We saw that writing the factors in expanded form could help, but we also discovered a pattern or a rule that could come in handy. Do you remember what that rule was? Possible Student Answers, Key Points:

- We don't have to use expanded form. We can just add the exponents.
- Adding the exponents works, because it tells us how many total factors of 10 we're dealing with without having to write them all out.

In today's lesson, we'll use some of this thinking. Our problems will involve taking a power of 10 to a power. If you're not sure what that means right now, don't worry. We'll see plenty of examples. As we work, try to see if you can find patterns to help us develop a rule to make our work more efficient.

Let's Talk (Slide 3): Let's look at these two expressions before we dive into our math problems. What is the same about these expressions? What is different? Possible Student Answers, Key Points:

- They both involve bases of 10. They both include exponents of 4 and 2.
- They're different colors. The first one wants us to multiply 10 to the fourth power times 10 to the second power, while the second example wants us to multiply 10 to the fourth power by itself.

$$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10)$$
$$10^6$$

We can look at these expressions in expanded form to further understand their similarities and differences. The first expression can be written as four factors of 10 multiplied by 2 factors of 10. *(write the expression in standard form)* We can see that we have six factors of 10 in all. We can simplify the expression by writing it as 10 to the sixth power. *(write 10 to the sixth power)* We also know we could have just added the exponents. 4 plus 2, means the exponent on the product is 6.

$$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$$
$$10^8$$

The second expression is different. It shows 10 to the fourth power to the second power. That means, we could represent it as 10 to the fourth power multiplied by 10 to the fourth power. How can we write that in expanded form?

(write as student shares, supporting as needed) Possible Student Answers, Key Points:

- I can think of it as 10^4 times 10^4.
- In expanded form, that's 10 x 10 x 10 x 10 times 10 x 10 x 10 x 10. There are 8 factors of 10.

In this expression, we see there are 8 factors of 10. We can simplify the expression by writing it as 10 to the eighth power. Notice, we did not add the exponents in this case. That's because we were thinking of multiplying two groups of 10 to the fourth power. That's 8 factors of 10, not 6 factors of 10.

We call this second example "taking a power to a power" because we're taking an exponential expression to another power. Let's look at a few more examples of problems involving taking a power to a power. Again, be on the lookout for patterns we can use to make our math easier.

Let's Think (Slide 4): This problem wants us to write each expression in expanded form, and then we'll write each as an expression using a single exponent. Let's start with expression A.

This expression can be read as 10 to the sixth power to the second power. We're taking a power to a power. I can think of this as 10 to the sixth power times itself. *(write 10 to the sixth power in expanded form multiplied by another 10 to the sixth power in expanded form)* That's a lot of factors of 10. How many? (12)

$$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$$

$$10^{12}$$

$$10^{2 \cdot 6}$$

10 to the sixth power to the second power resulted in 12 factors of 10. We can write that as 10 to the twelfth power. *(write that)* We multiplied 2 groups of 6 factors of ten together. We can show that by multiplying the exponents 2 and 6 since we took 2 groups of 6 factors. *(write 10 to the "2 • 6" power)*

I wonder if multiplying the exponents works every time. Let's try on the next example.

This example can be read as 10 to the third power to the fifth power. Again, we have a power to a power. How can I write this expression in expanded form? *(write as student shares)* Possible Student Answers, Key Points:

- We need to multiply 3 groups of 10 to the fifth together.
- We can show five factors of 10 times another five factors of 10 times another five factors of 10.

$$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$$

We have 15 factors of 10 in all.

$$10^{15}$$

$$10^{3 \cdot 5}$$

That's 10 to the fifteenth power. *(write 10 to the fifteenth power)* We multiplied 3 groups together. Each group had 5 factors of 10. Like the last problem, we can think of this as multiplying the exponents. 3 times 5 is 15. *(write 10 to the "3 • 5" power)* When taking a power to a power, we can use expanded form to show how many factors are being multiplied. We can also multiply the exponents together if we want to be a bit more efficient.

Let's Think (Slide 5): Let's look at one more example. This problem wants us to write the expression using a single exponent instead of repeated multiplication.

$$(10^4)^3$$

$$10^{3 \cdot 4} = 10^{12}$$

Here, I see 3 groups of 10 to the fourth power being multiplied together. We can rewrite that as 10 to the fourth power to the third power. *(write that)* Now we have the expression written as a power to a power.

Today we've seen how we can write a power to a power using a single exponent. How could I write this using a single exponent? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- You can expand to show 4 groups of 10 times 4 groups of 10 times 4 groups of 10.
- You can multiply 3 times 4 to get a single exponent of 12.

We can multiply the exponents to show that 3 groups of 4 factors would mean we have 12 factors of 10. The simplified expression is 10 to the 12th power.

Let's Try it (Slides 6 - 7): Now it's our chance to practice. We know that when we multiply powers of 10 together, we can add the exponents. We learned today that when we take powers of 10 to a power, we can multiply the exponents. Let's keep this in mind as we work. If need be, we also know that expanded form can help us think carefully about how many factors of 10 are involved. Let's get going!

WARM WELCOME

Today we will explain and use a rule for raising a power of 10 to a power, that $(10^n)^m = 10^{n \cdot m}$.

Let's Talk:

What's the same? What's different?

$10^4 \bullet 10^2$

$(10^4)^2$

Let's Think:

Write each expression in expanded form. Then write each as a single power of 10.

a. $(10^6)^2$

b. $(10^3)^5$

Can we figure out a rule?

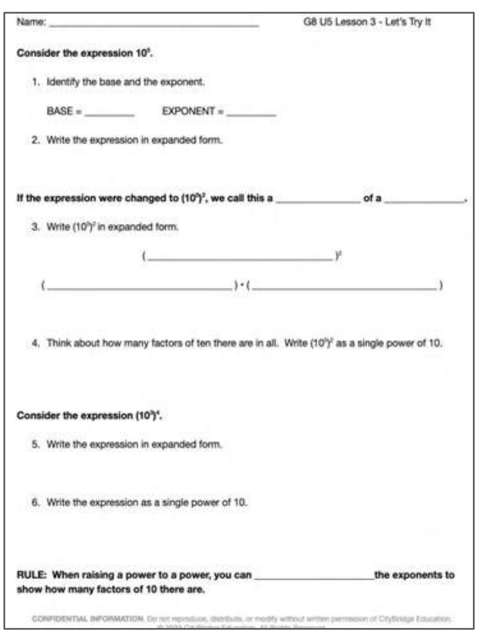
Let's Think:

Write the expression using exponents instead of repeated multiplication. Then write it using a single exponent.

$$10^4 \cdot 10^4 \cdot 10^4$$

Let's Try It:

Let's explore explaining and using a rule for raising a power to a power together.


Name: ______________________ G8 U5 Lesson 3 - Let's Try It

Consider the expression 10[illegible].

1. Identify the base and the exponent.

BASE = ________ EXPONENT = ________

2. Write the expression in expanded form.

If the expression were changed to (10[illegible])[illegible], we call this a ____________ of a ____________.

3. Write (10[illegible])[illegible] in expanded form.

(______________________)[illegible]

(______________________) • (______________________)

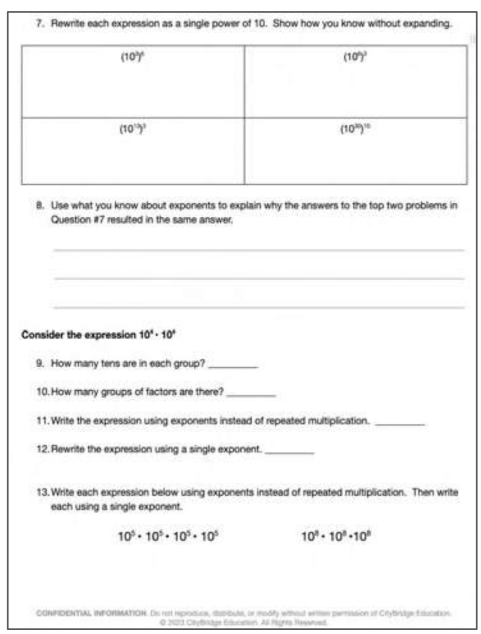
4. Think about how many factors of ten there are in all. Write (10[illegible])[illegible] as a single power of 10.

Consider the expression (10[illegible])[illegible].

5. Write the expression in expanded form.

6. Write the expression as a single power of 10.

RULE: When raising a power to a power, you can ________________ the exponents to show how many factors of 10 there are.


7. Rewrite each expression as a single power of 10. Show how you know without expanding.

(10[illegible])[illegible]	(10[illegible])[illegible]
(10[illegible])[illegible]	(10[illegible])[illegible]

8. Use what you know about exponents to explain why the answers to the top two problems in Question #7 resulted in the same answer.

Consider the expression 10[illegible] • 10[illegible]

9. How many tens are in each group? ________

10. How many groups of factors are there? ________

11. Write the expression using exponents instead of repeated multiplication. ________

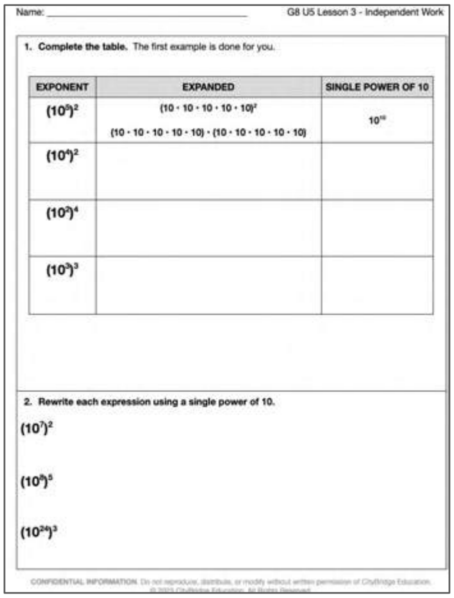
12. Rewrite the expression using a single exponent. ________

13. Write each expression below using exponents instead of repeated multiplication. Then write each using a single exponent.

$10^5 \cdot 10^5 \cdot 10^5 \cdot 10^5$ $10^8 \cdot 10^8 \cdot 10^8$

On your Own:

Now it's time to explain and use a rule for raising a power to a power on your own.



Name: ______________________ G8 U5 Lesson 3 - Independent Work

1. **Complete the table.** The first example is done for you.

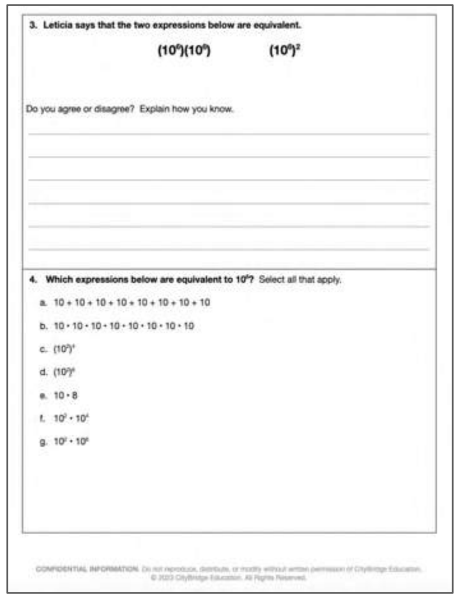
EXPONENT	EXPANDED	SINGLE POWER OF 10
$(10^5)^2$	$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)^2$ $(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	10^{10}
$(10^4)^2$		
$(10^2)^4$		
$(10^3)^3$		

2. **Rewrite each expression using a single power of 10.**

$(10^7)^2$

$(10^8)^5$

$(10^{24})^3$



3. **Leticia says that the two expressions below are equivalent.**

$(10^6)(10^6)$ $(10^6)^2$

Do you agree or disagree? Explain how you know.

4. **Which expressions below are equivalent to 10^8?** Select all that apply.

a. $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10$

b. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

c. $(10^2)^4$

d. $(10^2)^6$

e. $10 \cdot 8$

f. $10^2 \cdot 10^4$

g. $10^2 \cdot 10^6$

Consider the expression 10^5.

1. Identify the base and the exponent.

   BASE = __________ EXPONENT = __________

2. Write the expression in expanded form.

If the expression were changed to $(10^5)^2$, we call this a ________________ of a _______________.

3. Write $(10^5)^2$ in expanded form.

   (___________________________________)^2

   (___________________________________) • (___________________________________)

4. Think about how many factors of ten there are in all. Write $(10^5)^2$ as a single power of 10.

Consider the expression $(10^3)^4$.

5. Write the expression in expanded form.

6. Write the expression as a single power of 10.

RULE: When raising a power to a power, you can _______________________the exponents to show how many factors of 10 there are.

7. Rewrite each expression as a single power of 10. Show how you know without expanding.

$(10^3)^6$	$(10^6)^3$
$(10^{13})^3$	$(10^{30})^{10}$

8. Use what you know about exponents to explain why the answers to the top two problems in Question #7 resulted in the same answer.

____

____

____

Consider the expression $10^4 \bullet 10^4$

9. How many tens are in each group? __________

10. How many groups of factors are there? __________

11. Write the expression using exponents instead of repeated multiplication. __________

12. Rewrite the expression using a single exponent. __________

13. Write each expression below using exponents instead of repeated multiplication. Then write each using a single exponent.

$10^5 \bullet 10^5 \bullet 10^5 \bullet 10^5$

$10^8 \bullet 10^8 \bullet 10^8$

1. **Complete the table.** The first example is done for you.

EXPONENT	EXPANDED	SINGLE POWER OF 10
$(10^5)^2$	$(10 \bullet 10 \bullet 10 \bullet 10 \bullet 10)^2$ $(10 \bullet 10 \bullet 10 \bullet 10 \bullet 10) \bullet (10 \bullet 10 \bullet 10 \bullet 10 \bullet 10)$	10^{10}
$(10^4)^2$		
$(10^2)^4$		
$(10^3)^3$		

2. **Rewrite each expression using a single power of 10.**

$(10^7)^2$

$(10^8)^5$

$(10^{24})^3$

3. Leticia says that the two expressions below are equivalent.

$$(10^6)(10^6) \qquad\qquad (10^6)^2$$

Do you agree or disagree? Explain how you know.

____

____

____

____

____

____

4. Which expressions below are equivalent to 10^8? Select all that apply.

a. $10 + 10 + 10 + 10 + 10 + 10 + 10 + 10$

b. $10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$

c. $(10^2)^4$

d. $(10^2)^6$

e. $10 \bullet 8$

f. $10^2 \bullet 10^4$

g. $10^2 \bullet 10^6$

Name: KEY

Consider the expression 10^5.

1. Identify the base and the exponent.

   BASE = 10 EXPONENT = 5

2. Write the expression in expanded form.

   $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

If the expression were changed to $(10^5)^2$, we call this a power **of a** power**.**

3. Write $(10^5)^2$ in expanded form.

   $(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)^2$

   $(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$

4. Think about how many factors of ten there are in all. Write $(10^5)^2$ as a single power of 10.

   10^{10}

Consider the expression $(10^3)^4$.

5. Write the expression in expanded form.

   $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

6. Write the expression as a single power of 10.

   10^{12}

RULE: When raising a power to a power, you can multiply **the exponents to show how many factors of 10 there are.**

7. Rewrite each expression as a single power of 10. Show how you know without expanding.

$(10^3)^6$ $10^{3\cdot 6} = 10^{18}$	$(10^6)^3$ $10^{6\cdot 3} = 10^{18}$
$(10^{13})^3$ $10^{13\cdot 3} = 10^{39}$	$(10^{30})^{10}$ $10^{30\cdot 10} = 10^{300}$

8. Use what you know about exponents to explain why the answers to the top two problems in Question #7 resulted in the same answer.

6 groups of 3 factors of 10 and 3 groups of 6 factors of 10 will both result in 18 factors of 10.

Consider the expression $10^4 \cdot 10^4$

9. How many tens are in each group? 4

10. How many groups of factors are there? 2

11. Write the expression using exponents instead of repeated multiplication. $(10^4)^2$

12. Rewrite the expression using a single exponent. 10^8

13. Write each expression below using exponents instead of repeated multiplication. Then write each using a single exponent.

$10^5 \cdot 10^5 \cdot 10^5 \cdot 10^5$

$(10^5)^4 = 10^{20}$

$10^8 \cdot 10^8 \cdot 10^8$

$(10^8)^3 = 10^{24}$

Name: KEY

1. **Complete the table.** The first example is done for you.

EXPONENT	EXPANDED	SINGLE POWER OF 10
$(10^5)^2$	$(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)^2$ $(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	10^{10}
$(10^4)^2$	$(10 \cdot 10 \cdot 10 \cdot 10)^2$ $(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$(10^2)^4$	$(10 \cdot 10)^4$ $(10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10)$	10^8
$(10^3)^3$	$(10 \cdot 10 \cdot 10)^3$ $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$	10^9

2. **Rewrite each expression using a single power of 10.**

$(10^7)^2$ $10^{7 \cdot 2} = 10^{14}$

$(10^8)^5$ $10^{8 \cdot 5} = 10^{40}$

$(10^{24})^3$ $10^{24 \cdot 3} = 10^{72}$

3. **Leticia says that the two expressions below are equivalent.**

$$(10^6)(10^6) \qquad (10^6)^2$$

10^{6+6} $10^{6 \cdot 2}$

Do you agree or disagree? Explain how you know.

I agree. They are both equivalent to 10^{12}. The first expression shows 2 groups of 6 factors of

4. **Which expressions below are equivalent to 10^8?** Select all that apply.

~~a. 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10~~

(b.) $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ 10^8

(c.) $(10^2)^4$ $10^{2 \cdot 4}$

~~d. $(10^2)^6$~~ 10^{12}

~~e. $10 \cdot 8$~~ 80

~~f. $10^2 \cdot 10^4$~~ 10^6

(g.) $10^2 \cdot 10^6$ $10^{2+6} = 10^8$

G8 U5 Lesson 4
Generalize the exponent rule 10m 10n = 10n-m and write equivalent exponential expressions of division expressions with a base of 10.

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G8 U5 Lesson 4 - Students will generalize the exponent rule $10^n \div 10^m = 10^{n-m}$ and write equivalent exponential expressions of division expressions with a base of 10.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we'll continue working with powers of 10. In previous lessons, we've learned rules to help us multiply a power of 10 by another power of 10. We've also learned how to raise a power of 10 to another power. Our focus in this lesson will be on how we can divide expressions involving powers of 10. Similar to previous lessons, we'll use expanded form to help us think about the factors. From the expanded form, we'll work to develop more efficient rules we can apply to similar problems.

Let's Talk (Slide 3): Before we jump into work with exponents, take a second to look at the four circles here. What do you notice? What do you wonder? Possible Student Answers, Key Points:

- I notice they're all fractions. I notice they all have the same numerator and denominator. I notice the third circle involves multiplication. I notice the last circle has variables instead of numbers. I notice they're all equal to 1.
- I wonder what they equal. I wonder if they're equivalent. I wonder what this has to do with our exponent work today.

We've learned in previous years that anytime a fraction has the same numerator and denominator, it's equal to 1 whole. *(point to each circle as you describe it)* For example, 2 halves is equal to 1 whole. Or 10 tenths is equal to 1 whole. The third circle would be equal to 20 twentieths which is the same as 1 whole.

This idea will come in handy as we work with our division problems today.

Let's Think (Slide 4): Our first problems want us to write each expression using a single exponent. The first example can be read as 10 to the sixth power divided by 10 to the third power.

$$\frac{10^6}{10^3} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10}$$

Let's rewrite the division expression in fraction form. *(write 10 to the sixth power over 10 to the third power)* From here, we can expand each set of factors. How can I write the numerator and the denominator in expanded form? Possible Student Answers, Key Points:

- The numerator can be 10 x 10 x 10 x 10 x 10 x 10.
- The denominator can be 10 x 10 x 10.

$$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10}$$

Let's start by thinking about what the numerator and denominator have in common. I see they both have 3 factors of 10 in common. *(highlight three factors of 10 in the numerator and the denominator)*

$$\frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} \cdot 10 \cdot 10 \cdot 10$$
$$1 \cdot 10^3$$
$$10^3$$

Let's separate that part of the expression from the leftover factors of 10. *(rewrite the expression as shown here)* We just revisited the idea that a fraction with the same numerator and denominator is always equal to 1, so I can write the three factors of 10 over three factors of 10 as simply being 1. So the expression can now be thought of as 1 times the remaining three factors of 10, or 1 times 10 to the third power. *(write 1 x 10 to the third power)* Our simplified expression is 10 to the third power. Well done! Let's look at the other example.

How can I write this division expression in fraction form? *(write as student shares)* (10 to the tenth power over 10 to the second power) Just like last time, we can expand the numerator and denominator. What would that look like? Possible Student Answers, Key Points:

- There would be 10 factors of 10 in the numerator.
- There would be 2 factors of 10 in the denominator.

$$\frac{10^{10}}{10^2} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$$

We see that each expression has two factors of 10 in common. *(highlight two factors of 10 in the numerator and the denominator)* Let's write these separately from the remaining factors of 10.

$$\frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$
$$1 \cdot 10^8$$
$$10^8$$

What is 10 • 10 over 10 • 10 equivalent to? (1 whole) It's like 100 over 100, which would just be 1 whole. We can rewrite that part of the expression as 1. *(write as you narrate)* So, our expression now shows 1 times 8 factors of 10. We can write this as 1 times 10 to the eighth power, which is equal to 10 to the eighth power. We just rewrote the division expression as an expression with a single exponent.

Look back at both problems we just finished. Take a second to study the exponents in the problems and the exponents in each answer. Do you notice anything that could help develop an efficient rule for when we divide with powers of 10? Possible Student Answers, Key Points:

- I notice that 6 minus 3 equals 3. I notice that 10 minus 2 equals 8.
- I think we can subtract the exponents, since the matching factors of 10 cancel out.

$$10^{10-2} = 10^8$$
$$10^{6-3} = 10^3$$

When dividing with powers of 10, we can subtract the exponents. *(show subtraction of the exponents in writing as you narrate)* In our most recent problem, we can subtract 10 minus 2 to result in 8 factors of 10. In our first problem, we could subtract 6 minus 3 to result in 3 factors of 10.

This rule can help us efficiently divide with powers of 10 in any problem. Let's try out one more example.

Let's Think (Slide 5): In this problem, we see two students attempting to write the expression shown here using a single exponent. Take a moment to read each person's thinking. *(pause)* Based on the rule we just figured out, whose logic is correct? Possible Student Answers, Key Points:

- Bob is correct, because I know we can subtract the exponents when dividing with powers of 10.
- Ted is incorrect, because we don't actually divide the exponents when dealing with powers of 10.

$$\frac{10^{100}}{10^{20}} = 10^{100-20}$$
$$\downarrow$$
$$10^{80}$$

Excellent thinking. Let's show how we could arrive at the correct answer using the rule. The rule is particularly helpful for this problem, given that these numbers would take a long time to expand.

(write the expression in fraction form) We can write the division expression as a fraction. Rather than expanding both expressions out and canceling out 20 matching factors of 10, we can subtract. 100 minus 20, means we are left with an exponent of 80. The expressions would have 80 factors of 10.

Let's Try it (Slides 6 - 7): Now we'll get a chance to practice together before you work independently. What are some strategies we've seen today to help us divide powers of 10? Possible Student Answers, Key Points:

- We can write the division as a fraction.
- We can expand the expressions, and match factors of 10.
- We can subtract the exponents.

As we work, we'll want to think about which strategy best fits the given problem. Would it be an easy problem to expand? Would it maybe be easier to subtract the exponents? Let's try a few more together.

WARM WELCOME

Today we will generalize the exponent rule $10^n \div 10^m = 10^{n - m}$ and write equivalent exponential expressions of division expressions with a base of 10.

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Let's Talk:

What do you notice? What do you wonder?

$\frac{2}{2}$ $\frac{10}{10}$ $\frac{4 \cdot 5}{4 \cdot 5}$ $\frac{a}{a}$

Let's Think:

Rewrite each expression using a single exponent.

$10^6 \div 10^3$

$10^{10} \div 10^2$

Can we figure out a rule?

Let's Think:

Look at each person's thinking.
Who is correct? How do you know?

The answer is 10^{80} because 100 minus 20 is 80.

$$10^{100} \div 10^{20}$$

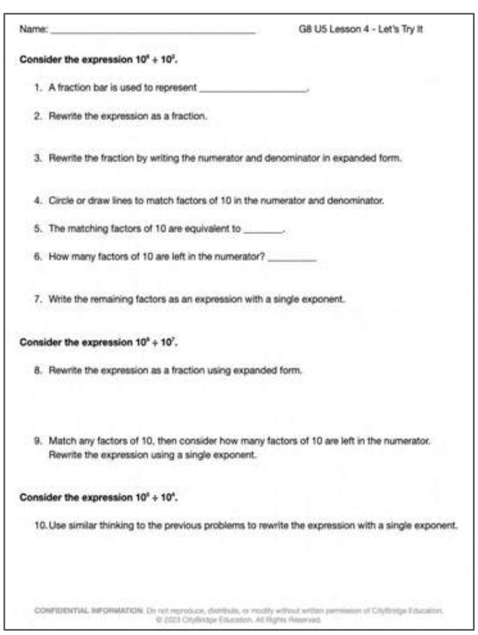
The answer is 10^{5} because 100 divided by 20 is 5.

BOB

TED

Let's Try It:

Let's explore generalizing the exponent rule $10^n \div 10^m = 10^{n-m}$ and writing equivalent exponential expressions of division expressions with a base of 10 together.



Name: ____________________ G8 U5 Lesson 4 - Let's Try It

Consider the expression $10^{[illegible]} \div 10^{[illegible]}$.

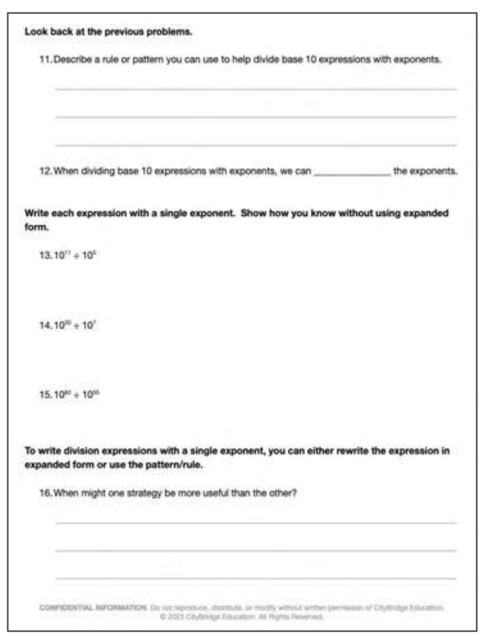
1. A fraction bar is used to represent ____________.
2. Rewrite the expression as a fraction.
3. Rewrite the fraction by writing the numerator and denominator in expanded form.
4. Circle or draw lines to match factors of 10 in the numerator and denominator.
5. The matching factors of 10 are equivalent to ______.
6. How many factors of 10 are left in the numerator? ______
7. Write the remaining factors as an expression with a single exponent.

Consider the expression $10^{[illegible]} \div 10^{7}$.

8. Rewrite the expression as a fraction using expanded form.
9. Match any factors of 10, then consider how many factors of 10 are left in the numerator. Rewrite the expression using a single exponent.

Consider the expression $10^{[illegible]} \div 10^{[illegible]}$.

10. Use similar thinking to the previous problems to rewrite the expression with a single exponent.



Look back at the previous problems.

11. Describe a rule or pattern you can use to help divide base 10 expressions with exponents.
12. When dividing base 10 expressions with exponents, we can ____________ the exponents.

Write each expression with a single exponent. Show how you know without using expanded form.

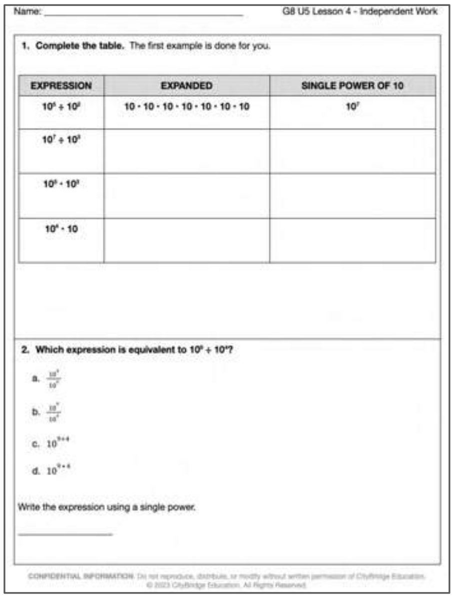
13. $10^{11} \div 10^{[illegible]}$
14. $10^{[illegible]} \div 10^{7}$
15. $10^{[illegible]} \div 10^{[illegible]}$

To write division expressions with a single exponent, you can either rewrite the expression in expanded form or use the pattern/rule.

16. When might one strategy be more useful than the other?

On your Own:

Now it's time to generalize the exponent rule $10^n \div 10^m = 10^{n-m}$ and write equivalent exponential expressions of division expressions with a base of 10 on your own.



Name: ______________________ G8 U5 Lesson 4 - Independent Work

1. **Complete the table.** The first example is done for you.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^{[illegible]} \div 10^{[illegible]}$	$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	10^7
$10^7 \div 10^{[illegible]}$		
$10^{[illegible]} \cdot 10^{[illegible]}$		
$10^{[illegible]} \cdot 10$		

2. **Which expression is equivalent to $10^{[illegible]} \div 10^{[illegible]}$?**

a. [illegible]

b. [illegible]

c. [illegible]

d. [illegible]

Write the expression using a single power.

____

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3. **Rewrite each expression as a single power of 10.** Show how you know.

$10^{[illegible]} \div 10^{27}$ $10^{[illegible]} \div 10^{[illegible]}$

4. **David says that $10^{[illegible]} \div 10^{2}$ is equivalent to $10^{[illegible]}$ because he subtracted the exponents. Trevor says that $10^{[illegible]} \div 10^{2}$ is equivalent to $10^{[illegible]}$ because he divided the exponents.**

Who is correct? Explain how you know.

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Consider the expression $10^6 \div 10^2$.

1. A fraction bar is used to represent _____________________.

2. Rewrite the expression as a fraction.

3. Rewrite the fraction by writing the numerator and denominator in expanded form.

4. Circle or draw lines to match factors of 10 in the numerator and denominator.

5. The matching factors of 10 are equivalent to ________.

6. How many factors of 10 are left in the numerator? __________

7. Write the remaining factors as an expression with a single exponent.

Consider the expression $10^9 \div 10^7$.

8. Rewrite the expression as a fraction using expanded form.

9. Match any factors of 10, then consider how many factors of 10 are left in the numerator. Rewrite the expression using a single exponent.

Consider the expression $10^5 \div 10^4$.

10. Use similar thinking to the previous problems to rewrite the expression with a single exponent.

Look back at the previous problems.

11. Describe a rule or pattern you can use to help divide base 10 expressions with exponents.

____

____

____

12. When dividing base 10 expressions with exponents, we can _______________ the exponents.

Write each expression with a single exponent. Show how you know without using expanded form.

13. $10^{11} \div 10^{5}$

14. $10^{30} \div 10^{7}$

15. $10^{82} \div 10^{55}$

To write division expressions with a single exponent, you can either rewrite the expression in expanded form or use the pattern/rule.

16. When might one strategy be more useful than the other?

____

____

____

Name: ____ G8 U5 Lesson 4 - Independent Work

1. **Complete the table.** The first example is done for you.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^5 \div 10^2$	$\frac{10 \bullet 10 \bullet 10 \bullet 10 \bullet 10}{10 \bullet 10}$	10^3
$10^7 \div 10^3$		
$10^5 \bullet 10^3$		
$10^4 \bullet 10$		

2. **Which expression is equivalent to $10^9 \div 10^4$?**

a. $\frac{10^4}{10^9}$

b. $\frac{10^9}{10^4}$

c. 10^{9+4}

d. $10^{9 \bullet 4}$

Write the expression using a single power.

____

3. **Rewrite each expression as a single power of 10.** Show how you know.

$\mathbf{10^{85} \div 10^{27}}$ $\mathbf{10^{12} \div 10^{0}}$

4. **David says that $10^8 \div 10^2$ is equivalent to 10^6 because he subtracted the exponents. Trevor says that $10^8 \div 10^2$ is equivalent to 10^4 because he divided the exponents.**

Who is correct? Explain how you know.

Name: KEY

Consider the expression $10^6 \div 10^2$.

1. A fraction bar is used to represent division.

2. Rewrite the expression as a fraction. $\frac{10^6}{10^2}$

3. Rewrite the fraction by writing the numerator and denominator in expanded form.

$\frac{10\cdot10\cdot10\cdot10\cdot10\cdot10}{10\cdot10}$

4. Circle or draw lines to match factors of 10 in the numerator and denominator.

5. The matching factors of 10 are equivalent to 1.

6. How many factors of 10 are left in the numerator? 4

7. Write the remaining factors as an expression with a single exponent.

10^4

Consider the expression $10^9 \div 10^7$.

8. Rewrite the expression as a fraction using expanded form.

$\frac{10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10}{10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10}$

9. Match any factors of 10, then consider how many factors of 10 are left in the numerator. Rewrite the expression using a single exponent.

10^2

Consider the expression $10^5 \div 10^4$.

10. Use similar thinking to the previous problems to rewrite the expression with a single exponent.

$\frac{10\cdot10\cdot10\cdot10\cdot10}{10\cdot10\cdot10\cdot10}$ 10

Look back at the previous problems.

11. Describe a rule or pattern you can use to help divide base 10 expressions with exponents.

Instead of writing each value in expanded form, we can subtract the exponents.

12. When dividing base 10 expressions with exponents, we can subtract the exponents.

Write each expression with a single exponent. Show how you know without using expanded form.

13. $10^{11} \div 10^{5}$ $10^{11-5} = 10^{6}$

14. $10^{30} \div 10^{7}$ $10^{30-7} = 10^{23}$

15. $10^{82} \div 10^{55}$ $10^{82-55} = 10^{27}$

To write division expressions with a single exponent, you can either rewrite the expression in expanded form or use the pattern/rule.

16. When might one strategy be more useful than the other?

The rule can be more helpful when dealing with exponents that would be cumbersome to expand.

like this one!

1. Complete the table. The first example is done for you.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^5 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$	~~10^7~~ 10^3
$10^7 \div 10^3$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10}$	10^4
$10^5 \cdot 10^3$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10}$	10^2
$10^4 \cdot 10$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10}$	10^3

2. Which expression is equivalent to $10^9 \div 10^4$?

a. $\frac{10^4}{10^9}$

b. $\frac{10^9}{10^4}$ (circled)

c. 10^{9+4}

d. $10^{9 \cdot 4}$

$10^9 \div 10^4 = \frac{10^9}{10^4}$

$10^{9-4} = 10^5$

Write the expression using a single power.

10^5

3. **Rewrite each expression as a single power of 10.** Show how you know.

$10^{85} \div 10^{27}$ $10^{12} \div 10^{0}$

10^{85-27}

10^{58}

10^{12-0}

10^{12}

4. **David says that $10^8 \div 10^2$ is equivalent to 10^6 because he subtracted the exponents. Trevor says that $10^8 \div 10^2$ is equivalent to 10^4 because he divided the exponents.**

$$10^8 \div 10^2 = \frac{10^8}{10^2} = 10^{8-2}$$

10^6

Who is correct? Explain how you know.

David is correct. If he used expanded form, he'd have 8 factors of 10 over 2 factors of 10. Once he cancelled out factors of 10, there would be 6 factors of 10 remaining.

G8 U5 Lesson 5
Generalize the exponent rule 10-n = 10n 1 and write equivalent exponential expressions involving negative exponents.

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G8 U5 Lesson 5 - Students will generalize the exponent rule $10^{-n} = 1/10^n$ and write equivalent exponential expressions involving negative exponents.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We've been generalizing rules about exponents for the past several lessons. What rules stand out to you from what we've learned so far? Possible Student Answers, Key Points:

- When multiplying powers of 10, we can add the exponents.
- When taking a power of power, we can multiply the exponents.
- When dividing powers of 10, we can subtract the exponents.

Today, we'll continue exploring these rules and add one more rule to our toolkit. We'll specifically look at how we can work with negative exponents. Before we do that, let's refresh on the rules we've already learned.

Let's Talk (Slide 3): Look at the exponential expressions shown here. Take a few moments independently with scratch paper, and try to rewrite each using a single, positive exponent. When you're ready, we'll check your work to review our exponent rules. *(allow student a couple minutes to work, supporting as needed)*

$10^{5+2} = 10^7$

$10^{5 \cdot 2} = 10^{10}$

(write expressions as you narrate) The first expression has two factors that are powers of 10 being multiplied. We could write each factor in expanded form, but we also know we can add the exponents to more efficiently arrive at a single-exponent expression. 5 + 2 = 7, so our answer would be 10 to the seventh power.

What's different about the second expression? How would you write it as a single-exponent expression? Possible Student Answers, Key Points:

- The second expression is asking about a power to a power.
- We can multiply the exponents, because we're taking 2 groups of 5 factors of 10. 5 x 2 = 10, so our answer is 10 to the tenth power.

$10^{5-2} = 10^3$

The last example involves dividing two powers of 10. What rule can we use to write this as an expression with a single exponent? Possible Student Answers, Key Points:

- We can subtract the exponents when we divide by powers of 10.
- 5 - 2 = 3, so our answer would be 10 to the third power.

Great work! We've learned several rules that can help us efficiently work with exponential expressions. Let's explore one more.

Let's Think (Slide 4): We're going to work with negative exponents. There's a chance you've never done this before, and that's okay! Let's start by looking at patterns with what we already know: positive exponents. We'll complete this table to see how we can use familiar patterns to make sense of negative exponents.

EXPONENT	DECIMAL	FRACTION
10^2	100.0	$\frac{100}{1}$
10^1	10.0	$\frac{10}{1}$
10^0	1.0	$\frac{1}{1}$

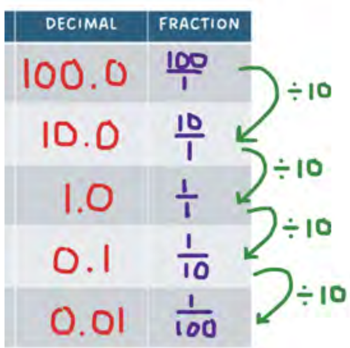
(fill in table as you narrate) We know 10 to the second power is 10 x 10, which is 100. We can write that as 100.0 in decimal form. If we want to write that as a fraction, we can write it as 100/1.

How can we write 10 to the first power as a decimal and a fraction? Possible Student Answers, Key Points:

- 10 to the first power just means one factor of 10. We can write that as 10.0 in decimal form and 10/1 in fraction form.

The next row shows 10 to the 0 power. We've learned that any base to the zero power is 1. We can write that as 1.0 for decimal form and 1/1 for fraction form.

Already, we can start to notice some place value patterns. *(trace chart with your finger as you describe the pattern)* As we move up each row, the exponent increases by 1. If we look at the values, we can see that we're multiplying each value by 10 as we move up each row. We can also think of that in reverse. As we go down each row, the exponent decreases by 1. The values are decreasing by a factor of 10. We can think of that as dividing by 10 or multiplying by 1/10 as we move down each row. These patterns can help us think about negative exponents.



(draw arrows showing the pattern of dividing by 10 for the filled-in values) Let's keep dividing by 10 or multiplying by 1/10 as we move down the table to help us fill in the other rows. The next row is 10 to the negative first power. If 10 to the zero power is 1, we can divide that by 10 to find 10 to the negative first power. What is 1 divided by 10 as a fraction? As a decimal? *(fill in values as student shares, supporting as needed)* Possible Student Answers, Key Points:

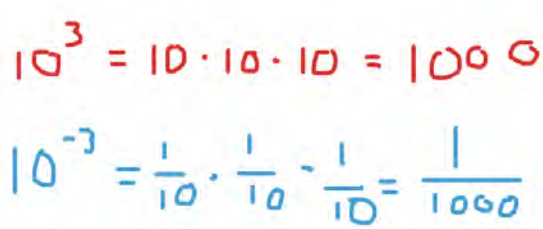
- 1 divided by 10 is one tenth.
- We can write that as 0.1 in decimal form. As a fraction, it's 1/10.

Let's keep the pattern going to find 10 to the negative second power. We can divide by 10, or multiply by 1/10 again. One tenth divided by ten is what? We'll need a fraction and a decimal. *(fill in values as student shares, supporting as needed)* Possible Student Answers, Key Points:

- One tenth divided by 10 is one hundredth
- We can write that as 0.01 in decimal form. As a fraction, it's 1/100.

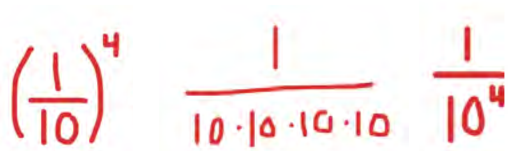
Nice work! What if the pattern were to keep going? What do you think 10 to the negative third power would be, and why? *(fill in values as student shares, supporting as needed)* Possible Student Answers, Key Points:

- We could divide 0.01 or 1/100 by 10 to find 10 to the negative third power.
- 10 to the negative third power would be one thousandth. That's 0.001 in decimal form and 1/1000 in fraction form.



Notice, we find positive powers of 10, it's like we're multiplying by 10 that many times. *(write as you narrate)* For example, 10 to the third power is 10 x 10 x 10, or 1,000. When we find negative powers of 10, it's like we're dividing by 10 that many times. Another way to think of that is that we're multiplying by 1/10 that number of times. For example, 10 to the negative third power is 1/10 x 1/10 x 1/10, or 1/1000.

Let's Think (Slide 5): Our next problem gives us two expressions. We're asked to write each using a single, positive exponent.



Let's look at the first one. Right away, I see we have 4 factors of 1/10. *(write as you narrate)* One way we can write this is as 1/10 to the fourth power. I can also think of this expression as being 1 over 10 • 10 • 10 • 10, so I can write the expression as 1 over 10 to the fourth power. Both ways show the original expression written with a single, positive exponent.

Take a look at the next expression. This can be read as ten to the negative seventh power. Look back at the table of values we completed. How can we use the patterns from the table of values to help us rewrite this as an expression using a single positive exponent? Possible Student Answers, Key Points:

- I know 10 to a negative power is like multiplying by 1/10 that many times.
- I can think of this as 1/10 times 1/10 times 1/10…seven times over.

When the power of 10 is positive, we think of that many factors of 10 being multiplied together. When the power of 10 is negative, like in this case, we think of that many factors of 1/10 being multiplied together.

$\left(\frac{1}{10}\right)^7$ OR $\frac{1}{10^7}$

That means, we could express 10 to the negative seventh power as 1/10 multiplied by itself 7 times. *(write as you narrate)* We can write that in two ways. We could write it as 1/10 to the seventh power. Or, we can also write it as 1 over 10 to the seventh power.

Let's Try it (Slides 6 - 7): Nice work assisting with those last few examples. We were able to find helpful patterns when working with negative exponents. When our base is 10, every time our exponent increases by 1, the value of the expression is multiplied by 10. When our base is 10, every time our exponent decreases by 1, the value of the expression is multiplied by 1/10 or divided by 10. Today, we saw how this place value reasoning can apply to negative exponents. When we see a negative exponent on a base of 10, we can think of the value as 1/10 being multiplied by itself that many times. We can use the table we created to help us remember patterns as we work through a few more examples. Let's dive in!

WARM WELCOME

Today we will generalize the exponent rule $10^{-n} = 1/10^{n}$ and write equivalent exponential expressions involving negative exponents.

Let's Talk:

We've learned a few rules to help us work with powers of 10.

Rewrite each expression as a single power of 10.

$10^5 \bullet 10^2$ $(10^5)^2$ $10^5 \div 10^2$

Let's Think:

Complete the table to identify patterns when working with negative exponents.

EXPONENT	DECIMAL	FRACTION
10^2		
10^1		
10^0		
10^{-1}		
10^{-2}		

Can we figure out a rule?

Let's Think:

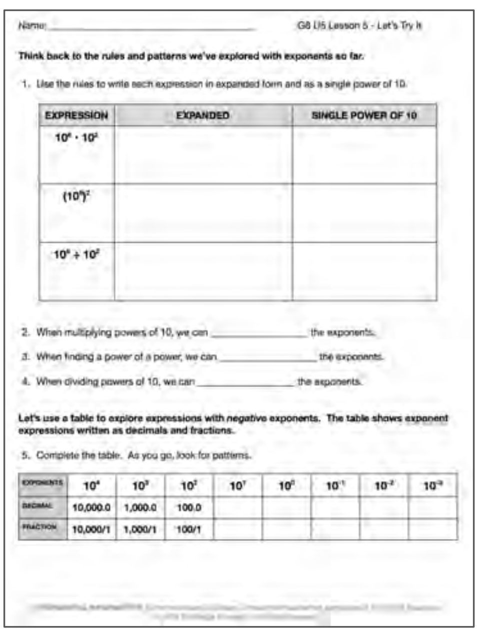
Rewrite each using a single, positive power of 10.

$$\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$$

$$10^{-7}$$

Let's Try It:

Let's explore generalizing the exponent rule $10^{-n} = 1/10^n$ and writing equivalent exponential expressions involving negative exponents together.



Name: ____________________ G8 U5 Lesson 5 - Let's Try It

Think back to the rules and patterns we've explored with exponents so far.

1. Use the rules to write each expression in expanded form and as a single power of 10.

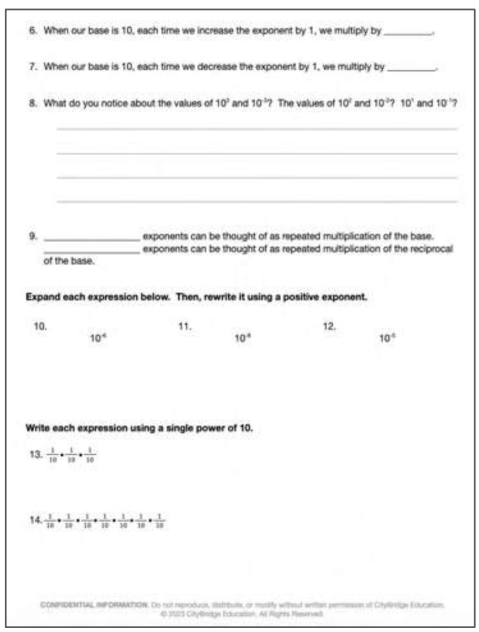
EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^{[illegible]} \cdot 10^{2}$		
$(10^{[illegible]})^{2}$		
$10^{[illegible]} \div 10^{2}$		

2. When multiplying powers of 10, we can ______________ the exponents.
3. When finding a power of a power, we can ______________ the exponents.
4. When dividing powers of 10, we can ______________ the exponents.

Let's use a table to explore expressions with *negative* exponents. The table shows exponent expressions written as decimals and fractions.

5. Complete the table. As you go, look for patterns.

EXPONENTS	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
DECIMAL	10,000.0	1,000.0	100.0					
FRACTION	10,000/1	1,000/1	100/1					



6. When our base is 10, each time we increase the exponent by 1, we multiply by ________.
7. When our base is 10, each time we decrease the exponent by 1, we multiply by ________.
8. What do you notice about the values of 10^3 and 10^{-3}? The values of 10^2 and 10^{-2}? 10^1 and 10^{-1}?

9. ______________ exponents can be thought of as repeated multiplication of the base. ______________ exponents can be thought of as repeated multiplication of the reciprocal of the base.

Expand each expression below. Then, rewrite it using a positive exponent.

10. $10^{[illegible]}$ 11. $10^{[illegible]}$ 12. $10^{[illegible]}$

Write each expression using a single power of 10.

13. $\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$

14. $\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$

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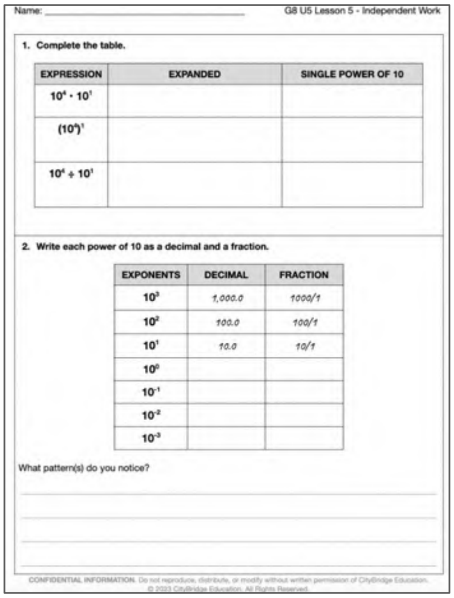
On your Own:

Now it's time to generalize the exponent rule $10^{-n} = 1/10^n$ and write equivalent exponential expressions involving negative exponents on your own.


Name: ______________________ G8 U5 Lesson 5 - Independent Work

1. Complete the table.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^4 \cdot 10^1$		
$(10^4)^1$		
$10^4 \div 10^1$		

2. Write each power of 10 as a decimal and a fraction.

EXPONENTS	DECIMAL	FRACTION
10^3	1,000.0	1000/1
10^2	100.0	100/1
10^1	10.0	10/1
10^0		
10^{-1}		
10^{-2}		
10^{-3}		

What pattern(s) do you notice?

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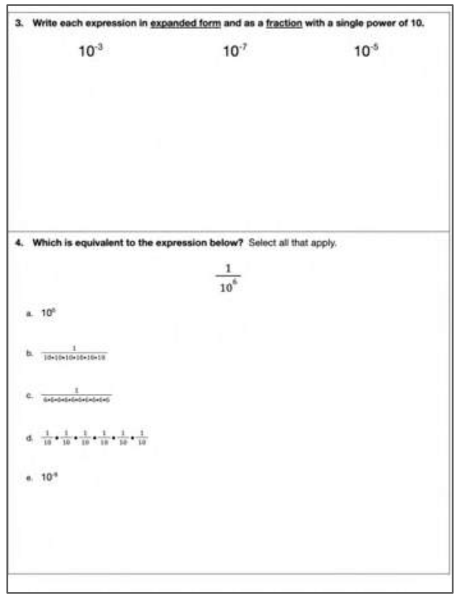
3. Write each expression in expanded form and as a fraction with a single power of 10.

10^{-3} 10^{-7} 10^{-5}

4. Which is equivalent to the expression below? Select all that apply.

$$\frac{1}{10^6}$$

a. 10^6

b. $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

c. [illegible]

d. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

e. 10^{-6}

Name: ____ G8 U5 Lesson 5 - Let's Try It

Think back to the rules and patterns we've explored with exponents so far.

1. Use the rules to write each expression in expanded form and as a single power of 10.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^6 \cdot 10^2$		
$(10^6)^2$		
$10^6 \div 10^2$		

2. When multiplying powers of 10, we can ___________________ the exponents.
3. When finding a power of a power, we can ___________________ the exponents.
4. When dividing powers of 10, we can ___________________ the exponents.

Let's use a table to explore expressions with *negative* exponents. The table shows exponent expressions written as decimals and fractions.

5. Complete the table. As you go, look for patterns.

EXPONENTS	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
DECIMAL	10,000.0	1,000.0	100.0					
FRACTION	10,000/1	1,000/1	100/1					

6. When our base is 10, each time we increase the exponent by 1, we multiply by __________.

7. When our base is 10, each time we decrease the exponent by 1, we multiply by __________.

8. What do you notice about the values of 10^3 and 10^{-3}? The values of 10^2 and 10^{-2}? 10^1 and 10^{-1}?

9. ____________________ exponents can be thought of as repeated multiplication of the base. ____________________ exponents can be thought of as repeated multiplication of the reciprocal of the base.

Expand each expression below. Then, rewrite it using a positive exponent.

10. 10^{-6}

11. 10^{-8}

12. 10^{-5}

Write each expression using a single power of 10.

13. $\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$

14. $\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$

1. **Complete the table.**

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^4 \bullet 10^1$		
$(10^4)^1$		
$10^4 \div 10^1$		

2. **Write each power of 10 as a decimal and a fraction.**

EXPONENTS	DECIMAL	FRACTION
10^3	1,000.0	1000/1
10^2	100.0	100/1
10^1	10.0	10/1
10^0		
10^{-1}		
10^{-2}		
10^{-3}		

What pattern(s) do you notice?

___

___

___

___

3. Write each expression in <u>expanded form</u> and as a <u>fraction</u> with a single power of 10.

10^{-3} 10^{-7} 10^{-5}

4. Which is equivalent to the expression below? Select all that apply.

$$\frac{1}{10^6}$$

a. 10^6

b. $\frac{1}{10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10}$

c. $\frac{1}{6 \bullet 6 \bullet 6 \bullet 6 \bullet 6 \bullet 6 \bullet 6 \bullet 6 \bullet 6 \bullet 6}$

d. $\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$

e. 10^{-6}

Name: KEY G8 U5 Lesson 5 - Let's Try It

Think back to the rules and patterns we've explored with exponents so far.

1. Use the rules to write each expression in expanded form and as a single power of 10.

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^6 \cdot 10^2$	$(10\cdot10\cdot10\cdot10\cdot10\cdot10)\cdot(10\cdot10)$	10^{6+2} 10^8
$(10^6)^2$	$(10\cdot10\cdot10\cdot10\cdot10\cdot10)\cdot(10\cdot10\cdot10\cdot10\cdot10\cdot10)$	$10^{6\cdot2}$ 10^{12}
$10^6 \div 10^2$	$\frac{10\cdot10\cdot10\cdot10\cdot10\cdot10}{10\cdot10}$	10^{6-2} 10^4

2. When multiplying powers of 10, we can add the exponents.
3. When finding a power of a power, we can multiply the exponents.
4. When dividing powers of 10, we can subtract the exponents.

Let's use a table to explore expressions with *negative* exponents. The table shows exponent expressions written as decimals and fractions.

5. Complete the table. As you go, look for patterns.

EXPONENTS	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
DECIMAL	10,000.0	1,000.0	100.0	10.0	1.0	0.1	0.01	0.001
FRACTION	10,000/1	1,000/1	100/1	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

6. When our base is 10, each time we increase the exponent by 1, we multiply by 10.

7. When our base is 10, each time we decrease the exponent by 1, we multiply by $\frac{1}{10}$.

8. What do you notice about the values of 10^3 and 10^{-3}? The values of 10^2 and 10^{-2}? 10^1 and 10^{-1}?

I notice they are multiplicative inverses.

Ex. 1000 and $\frac{1}{1000}$, 100 and $\frac{1}{100}$, 10 and $\frac{1}{10}$

9. Positive exponents can be thought of as repeated multiplication of the base.
Negative exponents can be thought of as repeated multiplication of the reciprocal of the base.

Expand each expression below. Then, rewrite it using a positive exponent.

10. 10^{-6}

$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

$\frac{1}{10^6}$ OR $\left(\frac{1}{10}\right)^6$

11. 10^{-8}

$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

$\frac{1}{10^8}$ OR $\left(\frac{1}{10}\right)^8$

12. 10^{-5}

$\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

$\frac{1}{10^5}$ OR $\left(\frac{1}{10}\right)^5$

Write each expression using a single power of 10.

13. $\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$ $\left(\frac{1}{10}\right)^3$ OR $\frac{1}{10^3}$

14. $\frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10} \bullet \frac{1}{10}$ $\left(\frac{1}{10}\right)^7$ OR $\left(\frac{1}{10^7}\right)$

Name: KEY

1. **Complete the table.**

EXPRESSION	EXPANDED	SINGLE POWER OF 10
$10^4 \cdot 10^1$	$(10 \cdot 10 \cdot 10 \cdot 10) \cdot 10$	10^5
$(10^4)^1$	$(10 \cdot 10 \cdot 10 \cdot 10)$	10^4
$10^4 \div 10^1$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10}$	10^3

2. **Write each power of 10 as a decimal and a fraction.**

EXPONENTS	DECIMAL	FRACTION
10^3	1,000.0	1000/1
10^2	100.0	100/1
10^1	10.0	10/1
10^0	1	$\frac{1}{1}$
10^{-1}	0.1	$\frac{1}{10}$
10^{-2}	0.01	$\frac{1}{100}$
10^{-3}	0.001	$\frac{1}{1000}$

What pattern(s) do you notice?

As the exponents increase by 1 we multiply each value by 10. As exponents decrease by 1, we divide each value by 10 or multiply by 1/10.

3. Write each expression in expanded form and as a fraction with a single power of 10.

10^{-3}	10^{-7}	10^{-5}
$\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}$	$\frac{1}{10\cdot10\cdot10\cdot10\cdot10\cdot10\cdot10}$	$\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}$
$\frac{1}{10^3}$	$\left(\frac{1}{10}\right)^7$	$\left(\frac{1}{10}\right)^6$
OR $\left(\frac{1}{10}\right)^3$	OR $\frac{1}{10^7}$	OR $\frac{1}{10^6}$

4. Which is equivalent to the expression below? Select all that apply.

$$\frac{1}{10^6}$$

a. ~~10^6~~

b. $\frac{1}{10\cdot10\cdot10\cdot10\cdot10\cdot10}$

c. ~~$\frac{1}{6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6\cdot6}$~~

d. $\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}$

e. 10^{-6}

G8 U5 Lesson 6
Generalize exponent rules for bases other than 10, and use exponent rules to write equivalent exponent expressions for any nonzero base.

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G8 U5 Lesson 6 - Students will generalize exponent rules for bases other than 10, and use the exponent rules to write equivalent exponent expressions for any nonzero base.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Since the beginning of this unit, we've been working with exponents. We've found helpful rules to work with negative exponents. We've explored patterns to help us multiply exponential expressions, divide exponential expressions, and even find a power of a power. We've done all of these different things, but one thing has remained constant. Our base has always been 10. Today, we'll explore whether or not the rules we've been learning about can help us work with bases other than 10. Let me show you what I mean.

Let's Talk (Slide 3): Look over these two tables. What do you notice is the same? What's different?
Possible Student Answers, Key Points:

- They both show exponent expressions and their values. They both have exponents from 2 to -2. They are both in order.
- They are different colors. The first table involves powers of 10, and the second table shows bases of 3.

EXPONENT FORM	VALUE
10^2	100
10^1	10
10^0	1
10^{-1}	1/10
10^{-2}	1/100

EXPONENT FORM	VALUE
3^2	9
3^1	3
3^0	1
3^{-1}	1/3
3^{-2}	1/9

The first table should feel familiar. We've been working with powers of 10 for some time. We even filled out a similar table during our last lesson. *(trace with your finger or draw arrows)* We notice that as the exponents increase by 1, the value is multiplied by 10 each time. Alternately, as the exponents decrease by 1, the value is divided by 10 each time, or multiplied by 1/10.

Look at the green table that shows exponential expressions with a base of 3. Do we see the same patterns? Possible Student Answers, Key Points:

- The pattern is similar, but just with a factor of 3 or ⅓ instead of 10 or /10.
- The pattern is different. The values don't increase by multiples of 10 as the exponent goes up and multiples of 1/10 as the exponent goes down.

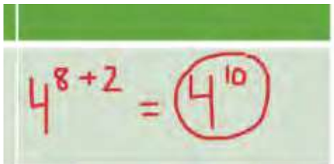
(trace with your finger or draw arrows) We notice that as the exponents increase by 1, the value is multiplied by 3 each time. Alternately, as the exponents decrease by 1, the value is divided by 3 each time, or multiplied by 1/3. The base impacts the value of the exponential expression, but the overall pattern remains similar to what we've seen with powers of 10.

So far it seems like working with powers of 10 might be similar to working with other bases. Let's explore some more.

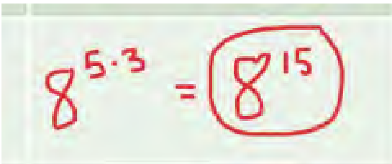
Let's Think (Slide 4): Here, we're asked to write each expression as an expression with a single, positive exponent. Notice, none of the bases are 10. Let's use what we know to help us simplify the expressions.

The first expression involves multiplying 4 to the eighth power times 4 to the second power. Mentally picture this expanded out. What are you picturing? Possible Student Answers, Key Points:

- I'm picturing 8 factors of 4 and then 2 more factors of 4.
- I'm picturing a long expression with 10 factors of 4 in all.

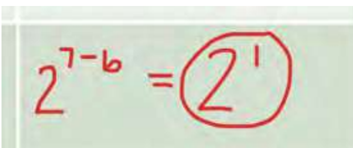


If we expanded this out, we would have 8 factors of 4 and 2 more factors of 4 being multiplied together. That means, just like we saw with powers of 10, we can add our exponents! *(write 4 to the "8+2" power)* Eight factors of 4 times 2 factors of 4 is 10 factors of 4, or 4 to the tenth power.



The next expression shows 8 to the fifth power to the third power. If I picture this in expanded form, I'm visualizing 3 groups being multiplied together. Each group has 5 factors of 8 in it. That's 15 factors of 8 in all. Even though the base is 8, the rule we learned with powers of 10 holds true. We can multiply the exponents when raising a power to a power. *(write 8 to the "5 • 3" power)* The value would be 8 to the fifteenth power.

Look at the last expression. How do you think we can write this exponential expression using a single exponent? You can use scratch paper, you can think about expanded form, and you can think about the work we've done with powers of 10. Possible Student Answers, Key Points:

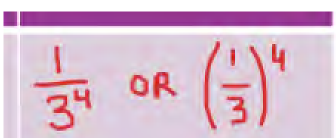


- I am picturing a fraction. I see 7 factors of 2 in the numerator and 6 factors of 2 in the denominator. If I match factors, I would be left with one factor of 2, or 2 to the first power.
- I can subtract the exponents. 7 - 6 = 1, so I know the answer is 2 to the first power.

We can think of this expression exactly how we'd think of it using powers of 10. If it helps, we can picture it expanded. Picture 7 factors of 2 in our numerator and 6 factors of 2 in our denominator. After matching up factors of 2, we'd have 1 factor of 2 remaining. Our answer is 2 to the first power. A more efficient way of tackling that would be to subtract the exponents. *(write 2 to the "7 - 6" power)* Either way we think of it, the value is 2 to the first power.

The rules we've used to work with bases of 10 are still helpful when the bases are other numbers.

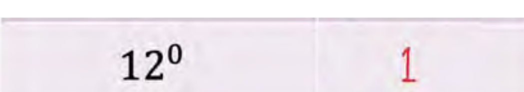
Let's Think (Slide 5): Our last problem before we practice has us consider a negative exponent and an exponent of 0.



The first expression shows 3 to the negative fourth power. We can think back to our table from the beginning of the lesson. 3 to the negative first power was ⅓. 3 to the negative second power was ⅓ • ⅓, or 1/9. If we kept that pattern going, how could we think of 3 to the negative fourth power? Possible Student Answers, Key Points:

- We can think of it as ⅓ • ⅓ • ⅓ • ⅓ or 4 factors of ⅓.
- We can think of it as 1 over 3 to the fourth power.

As our exponent decreases by 1, the value is divided by 3 or multiplied by ⅓. So if we have 3 to the negative fourth power, that means we have ⅓ times ⅓ times ⅓ times ⅓. We can write that as 1 over 3 to the fourth power or as ⅓ to the fourth power. *(fill in table)*



Lastly, we have the expression 12 to the 0 power. Look back at our table from earlier. What was 10 to the zero power? (1) What was 3 to the zero power? (1) So, 12 to the zero power is also 1. Anything to the zero power is 1. *(fill in table)*

Let's Try it (Slides 6 - 7): We explored bases other than 10 today, and we noticed that the same rules we learned in previous lessons apply to non-ten bases. This is true, assuming the bases in a single problem are consistent like in each example we've seen so far. We know we can add the exponents when multiplying exponential expressions. We know we can subtract the exponents when dividing exponential expressions. We can multiply the exponents when taking a power to a power. And we also know that working with zero and negative exponents with non-ten bases works similar to powers of 10. Let's keep these rules in mind and try out a few more problems before you get a chance to practice on your own.

WARM WELCOME

Today we will generalize exponent rules for bases other than 10, and use the exponent rules to write equivalent exponent expressions for any nonzero base.

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Let's Talk:

Look at each table of values. What's the same? What's different?

EXPONENT FORM	VALUE
10^2	100
10^1	10
10^0	1
10^{-1}	1/10
10^{-2}	1/100

EXPONENT FORM	VALUE
3^2	9
3^1	3
3^0	1
3^{-1}	1/3
3^{-2}	1/9

Let's Think:

Rewrite each expression using a single, positive exponent.

EXPONENT FORM	VALUE
$4^8 \bullet 4^2$	
$(8^5)^3$	
$2^7 \div 2^6$	

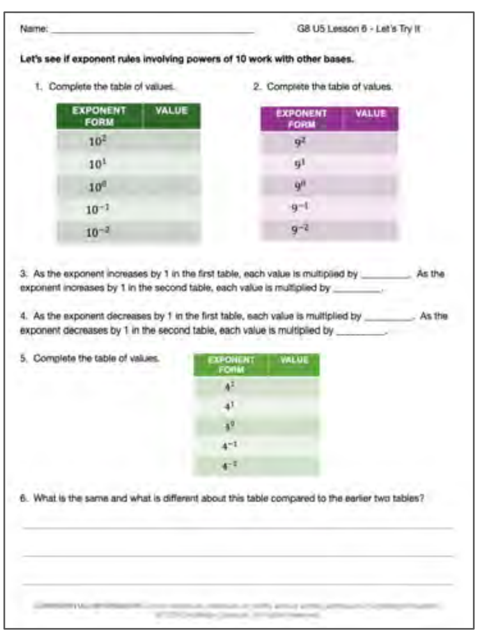
Let's Think:

Rewrite each expression using a single, positive exponent.

EXPONENT FORM	VALUE
3^{-4}	
12^{0}	

Let's Try It:

Let's explore generalizing exponent rules for bases other than 10, and using the exponent rules to write equivalent exponent expressions for any nonzero base together.


Name: ______________________ G8 U5 Lesson 6 - Let's Try It

Let's see if exponent rules involving powers of 10 work with other bases.

1. Complete the table of values.

EXPONENT FORM	VALUE
10^{2}	
10^{1}	
10^{0}	
10^{-1}	
10^{-2}	

2. Complete the table of values.

EXPONENT FORM	VALUE
9^{2}	
9^{1}	
9^{0}	
9^{-1}	
9^{-2}	

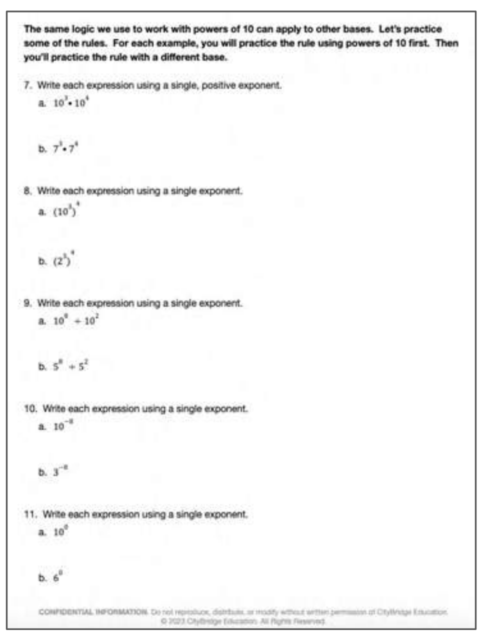
3. As the exponent increases by 1 in the first table, each value is multiplied by ________. As the exponent increases by 1 in the second table, each value is multiplied by ________.

4. As the exponent decreases by 1 in the first table, each value is multiplied by ________. As the exponent decreases by 1 in the second table, each value is multiplied by ________.

5. Complete the table of values.

EXPONENT FORM	VALUE
4^{2}	
4^{1}	
4^{0}	
4^{-1}	
4^{-2}	

6. What is the same and what is different about this table compared to the earlier two tables?


The same logic we use to work with powers of 10 can apply to other bases. Let's practice some of the rules. For each example, you will practice the rule using powers of 10 first. Then you'll practice the rule with a different base.

7. Write each expression using a single, positive exponent.
   a. $10^{3} \cdot 10^{4}$
   b. $7^{3} \cdot 7^{4}$
8. Write each expression using a single exponent.
   a. $(10^{3})^{4}$
   b. $(2^{3})^{4}$
9. Write each expression using a single exponent.
   a. $10^{8} \div 10^{2}$
   b. $5^{8} \div 5^{2}$
10. Write each expression using a single exponent.
   a. 10^{-8}
   b. 3^{-8}
11. Write each expression using a single exponent.
   a. 10^{0}
   b. 6^{0}

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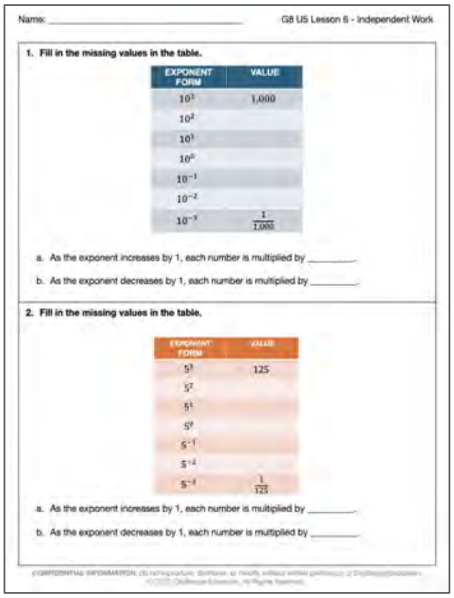
On your Own:

Now it's time to generalize exponent rules for bases other than 10, and use the exponent rules to write equivalent exponent expressions for any nonzero base on your own.



Name: ____________________ G8 U5 Lesson 6 - Independent Work

1. **Fill in the missing values in the table.**

EXPONENT FORM	VALUE
10^3	1,000
10^2	
10^1	
10^0	
10^{-1}	
10^{-2}	
10^{-3}	$\frac{1}{1,000}$

a. As the exponent increases by 1, each number is multiplied by ________.

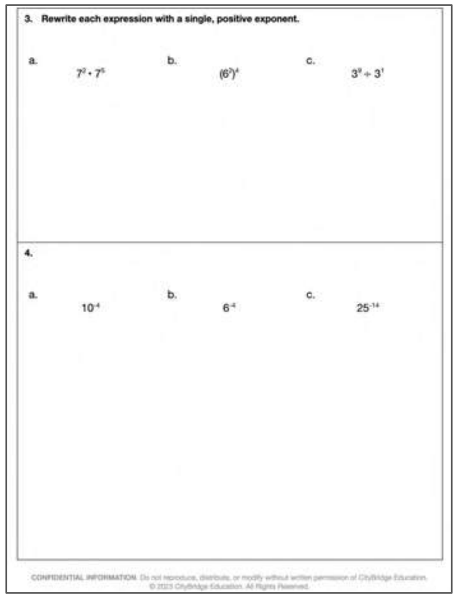
b. As the exponent decreases by 1, each number is multiplied by ________.

2. **Fill in the missing values in the table.**

EXPONENT FORM	VALUE
5^3	125
5^2	
5^1	
5^0	
5^{-1}	
5^{-2}	
5^{-3}	$\frac{1}{125}$

a. As the exponent increases by 1, each number is multiplied by ________.

b. As the exponent decreases by 1, each number is multiplied by ________.



3. **Rewrite each expression with a single, positive exponent.**

a. $7^2 \cdot 7^5$

b. $(6^2)^4$

c. $3^9 \div 3^1$

4.

a. 10^{-4}

b. 6^{-4}

c. 25^{-14}

Name: ______________________________________ G8 U5 Lesson 6 - Let’s Try It

Let’s see if exponent rules involving powers of 10 work with other bases.

1. Complete the table of values.

EXPONENT FORM	VALUE
10^2	
10^1	
10^0	
10^{-1}	
10^{-2}	

2. Complete the table of values.

EXPONENT FORM	VALUE
9^2	
9^1	
9^0	
9^{-1}	
9^{-2}	

3. As the exponent increases by 1 in the first table, each value is multiplied by __________. As the exponent increases by 1 in the second table, each value is multiplied by __________.

4. As the exponent decreases by 1 in the first table, each value is multiplied by __________. As the exponent decreases by 1 in the second table, each value is multiplied by __________.

5. Complete the table of values.

EXPONENT FORM	VALUE
4^2	
4^1	
4^0	
4^{-1}	
4^{-2}	

6. What is the same and what is different about this table compared to the earlier two tables?

____

____

____

The same logic we use to work with powers of 10 can apply to other bases. Let's practice some of the rules. For each example, you will practice the rule using powers of 10 first. Then you'll practice the rule with a different base.

7. Write each expression using a single, positive exponent.
   a. $10^3 \bullet 10^4$

   b. $7^3 \bullet 7^4$

8. Write each expression using a single exponent.
   a. $(10^3)^4$

   b. $(2^3)^4$

9. Write each expression using a single exponent.
   a. $10^8 \div 10^2$

   b. $5^8 \div 5^2$

10. Write each expression using a single exponent.
    a. 10^{-8}

    b. 3^{-8}

11. Write each expression using a single exponent.
    a. 10^0

    b. 6^0

1. Fill in the missing values in the table.

EXPONENT FORM	VALUE
10^3	1,000
10^2	
10^1	
10^0	
10^{-1}	
10^{-2}	
10^{-3}	$\frac{1}{1,000}$

a. As the exponent increases by 1, each number is multiplied by __________.

b. As the exponent decreases by 1, each number is multiplied by __________.

2. Fill in the missing values in the table.

EXPONENT FORM	VALUE
5^3	125
5^2	
5^1	
5^0	
5^{-1}	
5^{-2}	
5^{-3}	$\frac{1}{125}$

a. As the exponent increases by 1, each number is multiplied by __________.

b. As the exponent decreases by 1, each number is multiplied by __________.

3. Rewrite each expression with a single, positive exponent.

a. $7^2 \bullet 7^5$

b. $(6^2)^4$

c. $3^9 \div 3^1$

4. Rewrite each expression with a single, positive exponent.

a. 10^{-4}

b. 6^{-4}

c. 25^{-14}

Name: KEY

Let's see if exponent rules involving powers of 10 work with other bases.

1. Complete the table of values.

EXPONENT FORM	VALUE
10^2	100
10^1	10
10^0	1
10^{-1}	1/10
10^{-2}	1/100

2. Complete the table of values.

EXPONENT FORM	VALUE
9^2	81
9^1	9
9^0	1
9^{-1}	1/9
9^{-2}	1/81

3. As the exponent increases by 1 in the first table, each value is multiplied by 10. As the exponent increases by 1 in the second table, each value is multiplied by 9.

4. As the exponent decreases by 1 in the first table, each value is multiplied by 1/10. As the exponent decreases by 1 in the second table, each value is multiplied by 1/9.

5. Complete the table of values.

EXPONENT FORM	VALUE
4^2	16
4^1	4
4^0	1
4^{-1}	1/4
4^{-2}	1/16

6. What is the same and what is different about this table compared to the earlier two tables?

The base is 4. As the exponent increases the value is 4 times greater. As the exponent decreases, the value is $\frac{1}{4}$ as much as the previous.

The same logic we use to work with powers of 10 can apply to other bases. Let's practice some of the rules. For each example, you will practice the rule using powers of 10 first. Then you'll practice the rule with a different base.

7. Write each expression using a single, positive exponent.

   a. $10^3 \cdot 10^4$ 10^{3+4} 10^7

   b. $7^3 \cdot 7^4$ 7^{3+4} 7^7

8. Write each expression using a single exponent.

   a. $(10^3)^4$ $10^{3\cdot 4}$ 10^{12}

   b. $(2^3)^4$ $2^{3\cdot 4}$ 2^{12}

9. Write each expression using a single exponent.

   a. $10^8 \div 10^2$ $10^{8-2} = 10^6$

   b. $5^8 \div 5^2$ $5^{8-2} = 5^6$

10. Write each expression using a single exponent.

    a. 10^{-8} $\frac{1}{10^8}$

    b. 3^{-8} $\frac{1}{3^8}$

11. Write each expression using a single exponent.

    a. 10^0 1

    b. 6^0 1

Name: KEY

1. Fill in the missing values in the table.

EXPONENT FORM	VALUE
10^3	1,000
10^2	100
10^1	10
10^0	1
10^{-1}	1/10
10^{-2}	1/100
10^{-3}	$\frac{1}{1,000}$

a. As the exponent increases by 1, each number is multiplied by 10.

b. As the exponent decreases by 1, each number is multiplied by 1/10.

2. Fill in the missing values in the table.

EXPONENT FORM	VALUE
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	5
5^{-2}	1/25
5^{-3}	$\frac{1}{125}$

a. As the exponent increases by 1, each number is multiplied by 5.

b. As the exponent decreases by 1, each number is multiplied by 1/5.

3. **Rewrite each expression with a single, positive exponent.**

a. $7^2 \cdot 7^5$

7^{2+5}

7^7

b. $(6^2)^4$

$6^{2 \cdot 4}$

6^8

c. $3^9 \div 3^1$

3^{9-1}

3^8

4.

a. 10^{-4}

$\frac{1}{10^4}$

b. 6^{-4}

$\frac{1}{6^4}$

c. 25^{-14}

$\frac{1}{25^{14}}$

G8 U5 Lesson 7
Use an appropriate exponent rule to rewrite an expression with a single, positive exponent.

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G8 U5 Lesson 7 - Students will use an appropriate exponent rule to rewrite an expression with a single, positive exponent.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we get to pull together all the rules we've been learning about exponents. As always, it's okay to use expanded form to help simplify exponential expressions, but it can often be more efficient to use a rule. In this lesson, we'll get a chance to practice using our rules with a variety of bases. Let's get started!

Let's Talk (Slide 3): Take a look at this statement. *(read aloud)* Based on what we've learned so far, would you say this statement is always true, sometimes true, or never true? Why? Possible Student Answers, Key Points:

- This expression is always true. We saw in our last lesson that we can apply the same rules for base 10 expressions to expressions with other bases.
- This expression is sometimes true. The bases in your expression have to be the same for the rules to apply.

[NOTE: Answers may vary depending upon student experience with exponential expressions and based upon their interpretation of the statement. The goal is to stamp that if an expression has consistent bases, then we can apply the same rules/strategies we use with base 10 expressions.]

When we have expressions with a consistent base, we can apply the same rules we use when simplifying base 10 exponential expressions. Expanded form can also be helpful in some circumstances. Let's get to some practice!

Let's Think (Slide 4): This question wants us to write each expression using a single, positive exponent. What is the base for each expression? (8) Since we have a consistent base of 8, we can use our standard exponent rules to efficiently rewrite the expressions. We've practiced these rules a lot, so I'm going to ask for your support in rewriting them.

$8^{6+2} = 8^8$

How can I rewrite the expression that wants us to multiply 8 to the sixth power times 8 to the second power? Why? Possible Student Answers, Key Points:

- We can add the exponents.
- We have 6 factors of 8 times 2 more factors of 8, so we know there will be 8 factors of 8 in all.

(write as you narrate) We can add the exponents to show we are combining 6 factors of 8 times 2 more factors of 8. 6 + 2 = 8, so our rewritten expression is 8 to the eighth power.

$8^{6-4} = 8^2$

How can I rewrite the expression that wants us to divide 8 to the sixth power by 8 to the fourth power? Why? Possible Student Answers, Key Points:

- We can subtract the exponents.
- We have 6 factors of 8 over 4 factors of 8. We can match up 4 factors of 8 in the numerator and denominator, leaving us with 2 factors of 8 remaining.

(write as you narrate) We can subtract the exponents. 6 minus 4 equals 2. Picturing expanded form, we'd have 6 factors of 8 over 4 factors of 8. We could isolate 4 factors of 8 over 4 factors of 8, which would be equal to 1. That would leave us with 2 factors of 8 left. Our rewritten expression is 8 to the second power.

$$8^{-5} = \frac{1}{8^5} \text{ OR } \left(\frac{1}{8}\right)^5$$

How can I rewrite the expression that shows 8 to the negative fifth power? Why? Possible Student Answers, Key Points:

- We can think of it as dividing by 8 five times, or multiplying by ⅛ five times.
- We could write it as 1 over 8 to the fifth power or as ⅛ to the fifth power.

(write as you narrate) If this were a positive exponent, we'd think of it as 8 x 8 x 8 x 8 x 8. Since it's a negative exponent, we can think of it as ⅛ x ⅛ x ⅛ x ⅛ x ⅛. Our rewritten expression could be 1 over 8 to the fifth power. We could also write it as ⅛ to the fifth power. Either form is acceptable.

$$8^{3 \cdot 3} = 8^9$$

How can I rewrite the last expression? Why? Possible Student Answers, Key Points:

- We can multiply the exponents.
- If you picture expanded form, we'd have 3 groups being multiplied together. In each group, there would be 3 factors of 8. That's nine factors of 8.

(write as you narrate) We can multiply the exponents to show we are multiplying 3 groups of 3 factors of 8. 3 • 3 = 9, so our rewritten expression is 8 to the ninth power.

We just used exponent rules to rewrite various expressions with a non-10 base efficiently. Great work!

Let's Think (Slide 5): Our final problem wants us to look at each equation and determine whether the equation is true or false. If it's false, we'll fix it. Each of these examples involves negative exponents, so we'll want to work carefully when applying our exponent rules.

The first equation involves multiplying to factors that have bases of 5. Since they both have the same base, what rule can we apply to the exponents? What would the expression be if rewritten with a single exponent? Possible Student Answers, Key Points:

- We can add the exponents.
- 4 plus -2 is positive 2. The rewritten expression would be 5 to the second power.

$$5 \cdot 5 \cdot 5 \cdot 5 \cdot \frac{1}{5 \cdot 5}$$

(write as you narrate) Let's see if you're right. I'll start by writing each in expanded form, just to be safe. I can write 5 to the fourth power as 5 x 5 x 5 x 5. I can write 5 to the negative second power as 1 over 5 x 5.

$$\frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5^2$$

If we multiply these, that would give us a numerator of 5 x 5 x 5 x 5 and a denominator of 5 x 5. We can match up two factors of 5 in the numerator and denominator, which is equivalent to 1. That leaves us with two factors of 5. We can rewrite this as 5 to the second power.

$$5^{4+-2} = 5^2$$

That makes sense with our rule! We know we can add the exponents. 4 + -2 is equivalent to 2, so 5 to the second power makes sense. Whether we use expanded form or our exponent rule, we can see that this equation is true.

$$6^{4\cdot -2} = 6^{-8}$$

$$(6^{-2})^4 = 6^{-8}$$

$$\left(\frac{1}{6}\times\frac{1}{6}\right)^4$$

(write as you narrate) Look at the second equation. What rule can we apply to the exponents when taking a power to a power? (We can multiply the exponents.) If we multiply 4 x -2, we get -8. We could rewrite this expression as 6 to the negative eighth power. Is this equation true? (No, it's false.)

We can rewrite it using the correct rewritten expression to make it true.

Our answer also makes sense if we consider expanded form. We can think of 6 to the negative second power as being ⅙ times ⅙. We can think of the expression as being (⅙ x ⅙) to the fourth power. *(write that)* If we were to picture 4 factors that each looked like ⅙ x ⅙, we'd have eight factors of ⅙, which we know we can write as 6 to the negative eighth power.

Once again, we see that whether we think about expanded form our our exponent rules, we can arrive at an answer that makes sense.

Take a second to consider the last example on your own. When you're ready to share your thinking, let me know. Is the equation true or false? *(provide wait time)* Possible Student Answers, Key Points:

- The equation is false.
- I can subtract the exponents, since we're dividing. I can think of 4 minus -2 as 4 + 2, so the exponent should be 6.

$$8^{4--2} = 8^{4+2} = 8^6$$

$$8^4 \div 8^{-2} = 8^6$$

(write as you narrate) When dividing exponential expressions with similar bases, we know we can subtract the exponents. 4 minus 2 would result in an exponent of 2, but this problem needs us to do 4 minus *negative* 2. 4 minus -2 is the same as 4 + 2. Our exponent in the rewritten expression would be 6. 8 to the fourth power divided by 8 to the negative second power is 8 to the sixth power.

Let's Try it (Slides 6 - 7): Now we'll get a chance to do some more practice. After we complete the next few examples, you'll get a chance to show what you know independently. As we saw in the previous problems, we can use the efficient exponent rules we've already explored and apply them to non-ten bases. If necessary, it can also be helpful to think about our expressions in expanded form. Let's keep both tactics in mind as we look at the next examples.

WARM WELCOME

Today we will use an appropriate exponent rule to rewrite an expression with a single, positive exponent.

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Let's Talk:

Is the statement below ALWAYS, SOMETIMES, or NEVER true?

"The rules for evaluating exponential expressions with base 10 work for expressions with other bases."

Let's Think:

Rewrite each expression with a single, positive exponent.

$8^6 \cdot 8^2$

$\frac{8^6}{8^4}$

8^{-5}

$(8^3)^3$

Let's Think:

Is the expression below true or false? If false, rewrite to make a true statement.

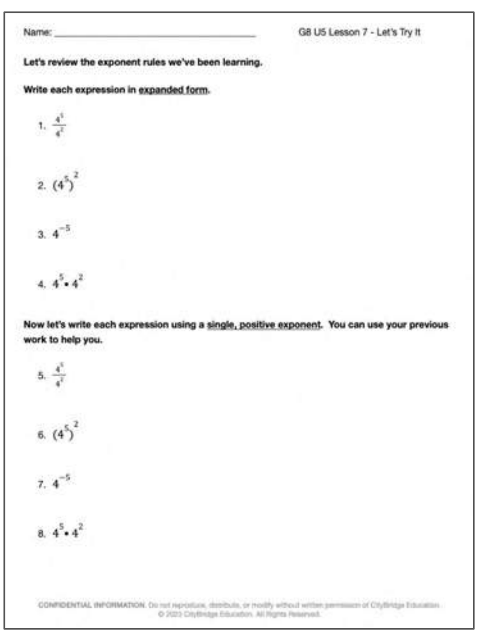
$5^4 \bullet 5^{-2} = 5^{-8}$

$(6^{-2})^4 = 6^8$

$8^4 \div 8^{-2} = 8^2$

Let's Try It:

Let's explore using an appropriate exponent rule to rewrite an expression with a single, positive exponent together.


Name: ____________________ G8 U5 Lesson 7 - Let's Try It

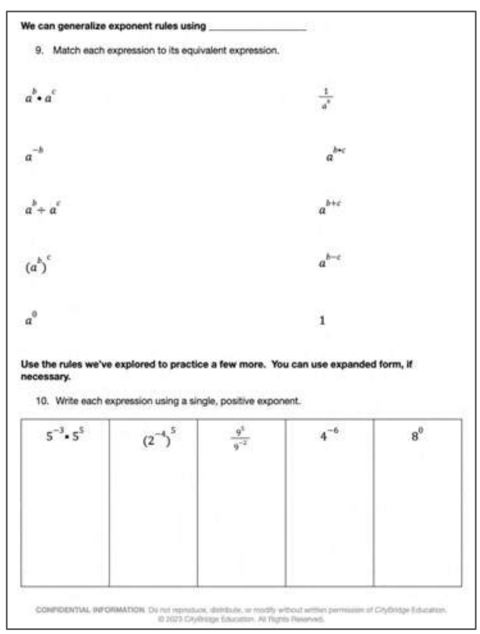
Let's review the exponent rules we've been learning.

Write each expression in expanded form.

1. $\frac{4^5}{4^2}$
2. $(4^5)^2$
3. 4^{-5}
4. $4^5 \bullet 4^2$

Now let's write each expression using a single, positive exponent. You can use your previous work to help you.

5. $\frac{4^5}{4^2}$
6. $(4^5)^2$
7. 4^{-5}
8. $4^5 \bullet 4^2$


We can generalize exponent rules using ______________

9. Match each expression to its equivalent expression.

$a^b \bullet a^c$	$\frac{1}{a^b}$
a^{-b}	$a^{b \bullet c}$
$a^b \div a^c$	a^{b+c}
$(a^b)^c$	a^{b-c}
a^0	1

Use the rules we've explored to practice a few more. You can use expanded form, if necessary.

10. Write each expression using a single, positive exponent.

$5^{-3} \bullet 5^5$	$(2^{-4})^5$	$\frac{9^3}{9^{-2}}$	4^{-6}	8^0

On your Own:

Now it's time to use an appropriate exponent rule to rewrite an expression with a single, positive exponent on your own.

Name: ______________________ G8 U5 Lesson 7 - Independent Work

1. Match an expression on the left to an equivalent expression on the right.

$4^3 \cdot 4^{-2}$	4^3
4^{-4}	$\frac{1}{4^4}$
$\frac{4^7}{4^4}$	4^1
$(4^3)^2$	4^6

2. Match an expression on the left to an equivalent expression on the right.

$m^x \cdot m^y$	m^{xy}
m^{-x}	$\frac{1}{m^x}$
$\frac{m^x}{m^y}$	m^{x-y}
$(m^x)^y$	m^{x+y}

3. Which of the following are equivalent to the expression below? Select all that apply.

$$2^4$$

a. $(2^2)^2$

b. $\frac{1}{2^4}$

c. $2^4 \cdot 2^1$

d. $2^4 \cdot 2^0$

e. $\frac{2^{[illegible]}}{2^{[illegible]}}$

f. $\frac{2^{[illegible]}}{2^{[illegible]}}$

4. Is each statement TRUE or FALSE? If it's false, rewrite it to make a true statement.

$3^4 \cdot 3^2 = 3^8$ $(7^{-2})^4 = 7^8$ $5^4 \div 5^2 = 1^2$

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Name: ____

Let's review the exponent rules we've been learning.

Write each expression in expanded form.

1. $\frac{4^5}{4^2}$

2. $(4^5)^2$

3. 4^{-5}

4. $4^5 \bullet 4^2$

Now let's write each expression using a single, positive exponent. You can use your previous work to help you.

5. $\frac{4^5}{4^2}$

6. $(4^5)^2$

7. 4^{-5}

8. $4^5 \bullet 4^2$

We can generalize exponent rules using ___________________

9. Match each expression to its equivalent expression.

$a^{b} \bullet a^{c}$	$\frac{1}{a^{b}}$
a^{-b}	$a^{b \bullet c}$
$a^{b} \div a^{c}$	a^{b+c}
$(a^{b})^{c}$	a^{b-c}
a^{0}	1

Use the rules we've explored to practice a few more. You can use expanded form, if necessary.

10. Write each expression using a single, positive exponent.

$5^{-3} \bullet 5^{5}$	$(2^{-4})^{5}$	$\frac{9^{5}}{9^{-2}}$	4^{-6}	8^{0}

Name: ______________________________

1. Match an expression on the left to an equivalent expression on the right.

$4^3 \bullet 4^{-2}$	4^3
4^{-3}	$\frac{1}{4^3}$
$\frac{4^7}{4^4}$	4^1
$(4^3)^2$	4^6

2. Match an expression on the left to an equivalent expression on the right.

$m^x \bullet m^y$	m^{xy}
m^{-x}	$\frac{1}{m^x}$
$\frac{m^x}{m^y}$	m^{x-y}
$(m^x)^y$	m^{x+y}

3. Which of the following are equivalent to the expression below? Select all that apply.

$$2^4$$

a. $(2^2)^2$

b. $\frac{1}{2^4}$

c. $2^4 \bullet 2^1$

d. $2^4 \bullet 2^0$

e. $\frac{2^8}{2^2}$

f. $\frac{2^6}{2^2}$

4. Is each statement TRUE or FALSE? If it's false, rewrite it to make a true statement.

$3^4 \bullet 3^2 = 3^8$

$(7^{-2})^4 = 7^8$

$5^4 \div 5^2 = 1^2$

Name: KEY

Let's review the exponent rules we've been learning.

Write each expression in expanded form.

1. $\frac{4^5}{4^2}$ $\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4}$

2. $(4^5)^2$ $(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4)$

3. 4^{-5} $\frac{1}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$

4. $4^5 \bullet 4^2$ $(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) \cdot (4 \cdot 4)$

Now let's write each expression using a single, positive exponent. You can use your previous work to help you.

5. $\frac{4^5}{4^2}$ $4^{5-2} = 4^3$

6. $(4^5)^2$ $4^{5 \cdot 2} = 4^{10}$

7. 4^{-5} $\frac{1}{4^5}$

8. $4^5 \bullet 4^2$ $4^{5+2} = 4^7$

We can generalize exponent rules using variables

9. Match each expression to its equivalent expression.

$a^b \bullet a^c$	$\frac{1}{a^b}$
a^{-b}	$a^{b \bullet c}$
$a^b \div a^c$	a^{b+c}
$(a^b)^c$	a^{b-c}
a^0	1

Use the rules we've explored to practice a few more. You can use expanded form, if necessary.

10. Write each expression using a single, positive exponent.

$5^{-3} \bullet 5^5$	$(2^{-4})^5$	$\frac{9^5}{9^{-2}}$	4^{-6}	8^0
5^{-3+5}	$2^{-4 \cdot 5}$	9^{5--2}	$\frac{1}{4^6}$	1
5^2	2^{-20}	9^{5+2}		
	$\frac{1}{2^{20}}$	9^7		

Name: KEY

1. Match an expression on the left to an equivalent expression on the right.

$4^3 \bullet 4^{-2} = 4^{3+-2} = 4^1$	4^3
4^{-3}	$\frac{1}{4^3}$
$\frac{4^7}{4^4}$ $4^{7-4} = 4^3$	4^1
$(4^3)^2$ $4^{3 \cdot 2}$	4^6

2. Match an expression on the left to an equivalent expression on the right.

$m^x \bullet m^y$	m^{xy}
m^{-x}	$\frac{1}{m^x}$
$\frac{m^x}{m^y}$	m^{x-y}
$(m^x)^y$	m^{x+y}

3. **Which of the following are equivalent to the expression below?** Select all that apply.

$$2^4$$

$2 \cdot 2 \cdot 2 \cdot 2$

a. $(2^2)^2$ $2^{2 \cdot 2} = 2^4$

b. $\frac{1}{2^4}$ $= 2^{-4}$

c. $2^4 \bullet 2^1$ $2^{4+1} = 2^5$

d. $2^4 \bullet 2^0$ $2^{4+0} = 2^4$

e. $\frac{2^8}{2^2}$ $2^{8-2} = 2^6$

f. $\frac{2^6}{2^2}$ $2^{6-2} = 2^4$

4. **Is each statement TRUE or FALSE? If it's false, rewrite it to make a true statement.**

FALSE

$3^4 \bullet 3^2 = 3^8$

$3^{4+2} = 3^6$

FALSE

$(7^{-2})^4 = 7^8$

$7^{-2 \cdot 4} = 7^{-8}$

FALSE

$5^4 \div 5^2 = 1^2$

$\frac{5^4}{5^2} = 5^{4-2}$

5^2

G8 U5 Lesson 8
Generalize a process for multiplying expressions with different bases having the same exponent.

CITYTUTOR DC

G8 U5 Lesson 8 - Students will generalize a process for multiplying expressions with different bases having the same exponent.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We are becoming exponent experts one lesson at a time! Up until now, we've explored rules we can apply when an exponential expression has a consistent base. But, what if the bases are different? What if the bases *and* the exponents are different? Can we still use the same rules or not? At the end of our time together today, we'll be able to start answering those questions. Let's get started.

Let's Talk (Slide 3): Look at the expressions shown here. What's the same? What's different? Possible Student Answers, Key Points:

- They all involve multiplication. They all involve bases of either 4 or 10. They all have at least one exponent that's a three.
- The first two look like problems we've solved before. The last two have different bases. The last one has different bases and different exponents.

Some of these problems look different than what we're used to. We're used to seeing exponential expressions where the base is consistent. For example, the first expression involves bases of 10. The second expression involves bases of 4. We already know how to rewrite expressions like that efficiently.

$$10^{3+3} = 10^6$$

$$4^{3+3} = 4^6$$

How could we rewrite the first two examples using a single, positive exponent? *(write as student shares, supporting as necessary)* Possible Student Answers, Key Points:

- We can add the exponents.
- The first expression could be rewritten as 10 to the 6th power, because 3 + 3 equals 6. The second expression could be rewritten as 4 to the 6th power, because 3 + 3 equals 6.

The same rules don't automatically apply to the other examples, because if you mentally picture them in expanded form, you have a number of different factors being multiplied. All is not lost, however! We can still use concepts we know about exponents to help us rewrite the expressions. It just requires a bit more strategic thinking. Let me show you what I mean.

Let's Think (Slide 4): Let's consider the third expression of 10 to the third power times 4 to the third power. We already noted that the expression involves different bases. We see a base of 10, and a base of 4. Let's expand the expression. What would this expression look like in expanded form? *(write expanded form as students shares)* Possible Student Answers, Key Points:

$$(10 \cdot 10 \cdot 10) \bullet (4 \cdot 4 \cdot 4)$$

$$(10 \cdot 4) \bullet (10 \cdot 4) \bullet (10 \cdot 4)$$

- 10 to the third power would be 10 x 10 x 10. We would multiply that by 4 x 4 x 4.
- We'd have 3 factors of 10 and 3 factors of 4.

Since we have three of each factor, we can use the commutative property to rearrange them and pair up factors. I'm going to pair up factors of 10 and 4. Watch! *(write (10 x 4) x (10 x 4) x (10 x 4))* Now I have three identical factors of 10 x 4. I can write that as 10 x 4 to the third power. *(write it)* What is 10 x 4? (40) I have three identical factors of 40, which I can write as 40 to the third power.

$$(10 \cdot 4)^3$$

$$\boxed{40^3}$$

We were able to expand each exponential factor and then rearrange and combine factors to write a simpler expression with a single exponent. Let's try another one.

Let's Think (Slide 5): What's the same about this problem compared to the previous problem? What's different? Possible Student Answers, Key Points:

- It's the same in that we have a base of 10 and a base of 4. It's also the same in that we're multiplying the exponential factors.
- It's different, because the exponents aren't identical like last time. We have an exponent of 3 and an exponent of 4.

This problem is almost the same as the last problem, but we see there are different exponents. That's not a problem. We can use similar thinking to last time.

$$(10 \cdot 10 \cdot 10) \cdot (4 \cdot 4 \cdot 4 \cdot 4)$$

Let's start by writing each factor in expanded form. *(expand both factors as shown)* I have three factors of 10 and four factors of 4. If I think about rearranging and combining factors, I know I'll be able to match up a 10 to a 4 three times, but I'll have one leftover factor of 4 that can't be matched. *(highlight or circle one factor of 4)* This 4 won't have a factor partner.

$$(10 \cdot 4) \cdot (10 \cdot 4) \cdot (10 \cdot 4) \cdot 4$$

$$40^3 \cdot 4$$

I'll rewrite the expression using the commutative property to rearrange factors so that each 10 is paired with a factor of 4. *(rewrite as shown)* We have one extra 4, as expected.

I know 10 x 4 is 40, so I have 3 equal factors of 40 and a factor of 4. If I want to simplify this expression, I can write it as 40 to the third power times 4.

Even if we don't have a matching set of factors, like 4 tens and 4 fours, we can still rearrange factors to write a simpler exponential expression.

Let's Try it (Slides 6 - 7): Now we'll practice working with different bases before you get a chance to try some on your own. If our problems have consistent bases, we can use the rules we've been practicing for several lessons. If we notice a problem has different bases, we'll want to think through the strategy we applied today. We can expand the expression, rearrange and match factors, and then rewrite the expressions using a single exponent. Let's go for it!

WARM WELCOME

Today we will generalize a process for multiplying expressions with different bases having the same exponent.

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Let's Talk: What's the same? What's different?

$10^3 \bullet 10^3$

$4^3 \bullet 4^3$

$10^3 \bullet 4^3$

$10^3 \bullet 4^4$

Let's Think: Rewrite the expression using a single, positive exponent if possible.

$10^3 \bullet 4^3$

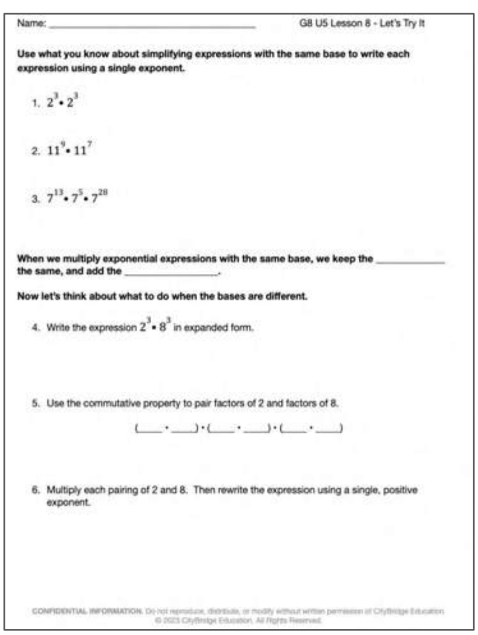
Let's Think:

Rewrite the expression using a single, positive exponent if possible.

$$10^3 \bullet 4^4$$

Let's Try It:

Let's explore generalizing a process for multiplying expressions with different bases having the same exponent together.



Name: ______________________ G8 U5 Lesson 8 - Let's Try It

Use what you know about simplifying expressions with the same base to write each expression using a single exponent.

1. $2^3 \bullet 2^3$
2. $11^9 \bullet 11^7$
3. $7^{13} \bullet 7^5 \bullet 7^{28}$

When we multiply exponential expressions with the same base, we keep the ____________ the same, and add the ______________.

Now let's think about what to do when the bases are different.

4. Write the expression $2^3 \bullet 8^3$ in expanded form.
5. Use the commutative property to pair factors of 2 and factors of 8.

(____ • ____) • (____ • ____) • (____ • ____)

6. Multiply each pairing of 2 and 8. Then rewrite the expression using a single, positive exponent.

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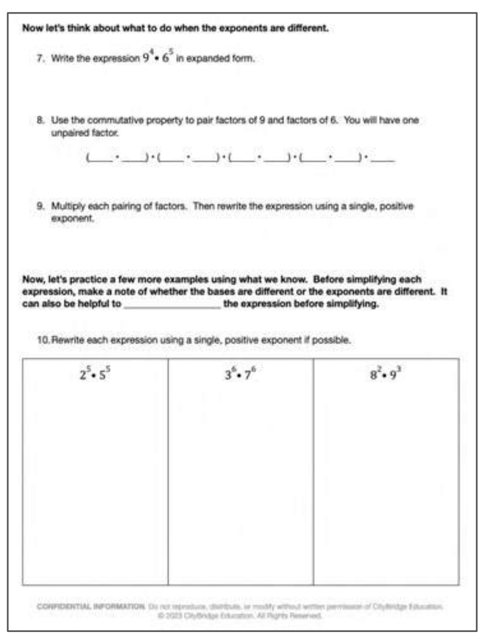

Now let's think about what to do when the exponents are different.

7. Write the expression $9^4 \bullet 6^5$ in expanded form.
8. Use the commutative property to pair factors of 9 and factors of 6. You will have one unpaired factor.

(____ • ____) • (____ • ____) • (____ • ____) • (____ • ____) • ____

9. Multiply each pairing of factors. Then rewrite the expression using a single, positive exponent.

Now, let's practice a few more examples using what we know. Before simplifying each expression, make a note of whether the bases are different or the exponents are different. It can also be helpful to ______________ the expression before simplifying.

10. Rewrite each expression using a single, positive exponent if possible.

$2^5 \bullet 5^5$	$3^6 \bullet 7^6$	$8^2 \bullet 9^3$

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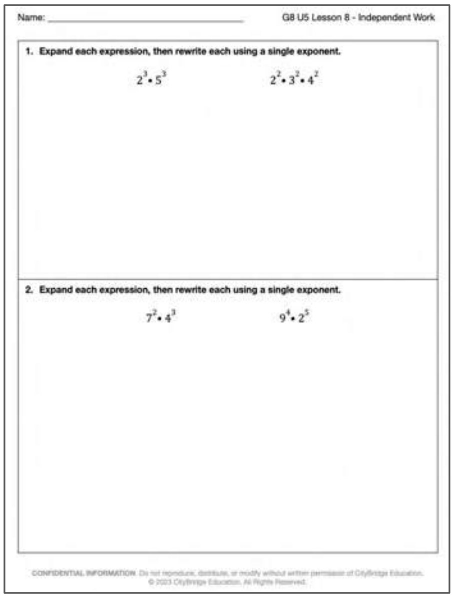
On your Own:

Now it's time to generalize a process for multiplying expressions with different bases having the same exponent on your own.



Name: ______________________ G8 U5 Lesson 8 - Independent Work

1. Expand each expression, then rewrite each using a single exponent.

$2^3 \cdot 5^3$ $2^2 \cdot 3^2 \cdot 4^2$

2. Expand each expression, then rewrite each using a single exponent.

$7^2 \cdot 4^3$ $9^4 \cdot 2^5$

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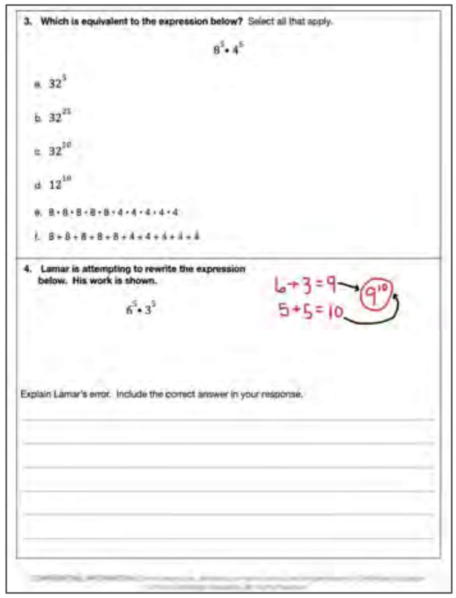


3. Which is equivalent to the expression below? Select all that apply.

$8^5 \cdot 4^5$

a. 32^5

b. 32^{25}

c. 32^{10}

d. 12^{10}

e. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

f. $8 + 8 + 8 + 8 + 8 + 4 + 4 + 4 + 4 + 4$

4. Lamar is attempting to rewrite the expression below. His work is shown.

$6^5 \cdot 3^5$

$6 + 3 = 9 \rightarrow 9^{10}$
$5 + 5 = 10$

Explain Lamar's error. Include the correct answer in your response.

Use what you know about simplifying expressions with the same base to write each expression using a single exponent.

1. $2^{3} \bullet 2^{3}$

2. $11^{9} \bullet 11^{7}$

3. $7^{13} \bullet 7^{5} \bullet 7^{28}$

When we multiply exponential expressions with the same base, we keep the _____________ the same, and add the __________________.

Now let's think about what to do when the bases are different.

4. Write the expression $2^{3} \bullet 8^{3}$ in expanded form.

5. Use the commutative property to pair factors of 2 and factors of 8.

(_____ • _____) • (_____ • _____) • (_____ • _____)

6. Multiply each pairing of 2 and 8. Then rewrite the expression using a single, positive exponent.

Now let's think about what to do when the exponents are different.

7. Write the expression $9^4 \bullet 6^5$ in expanded form.

8. Use the commutative property to pair factors of 9 and factors of 6. You will have one unpaired factor.

(_____ • _____) • (_____ • _____) • (_____ • _____) • (_____ • _____) • _____

9. Multiply each pairing of factors. Then rewrite the expression using a single, positive exponent.

Now, let's practice a few more examples using what we know. Before simplifying each expression, make a note of whether the bases are different or the exponents are different. It can also be helpful to ___________________ the expression before simplifying.

10. Rewrite each expression using a single, positive exponent if possible.

$2^5 \bullet 5^5$	$3^6 \bullet 7^6$	$8^2 \bullet 9^3$

1. Expand each expression, then rewrite each using a single exponent.

$2^3 \bullet 5^3$

$2^2 \bullet 3^2 \bullet 4^2$

2. Expand each expression, then rewrite each using a single exponent.

$7^2 \bullet 4^3$

$9^4 \bullet 2^5$

3. Which is equivalent to the expression below? Select all that apply.

$$8^5 \bullet 4^5$$

a. 32^5

b. 32^{25}

c. 32^{10}

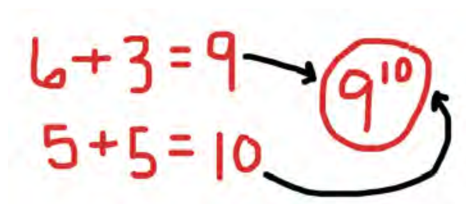
d. 12^{10}

e. 8 • 8 • 8 • 8 • 8 • 4 • 4 • 4 • 4 • 4

f. 8 + 8 + 8 + 8 + 8 + 4 + 4 + 4 + 4 + 4

4. Lamar is attempting to rewrite the expression below. His work is shown.

$$6^5 \bullet 3^5$$



Explain Lamar's error. Include the correct answer in your response.

Name: KEY

Use what you know about simplifying expressions with the same base to write each expression using a single exponent.

1. $2^3 \bullet 2^3$ $2^{3+3} = 2^6$

2. $11^9 \bullet 11^7$ $11^{9+7} = 11^{16}$

3. $7^{13} \bullet 7^5 \bullet 7^{28}$ $7^{13+5+28} = 7^{46}$

When we multiply exponential expressions with the same base, we keep the base **the same, and add the** exponents.

Now let's think about what to do when the bases are different.

4. Write the expression $2^3 \bullet 8^3$ in expanded form.

   $(2 \cdot 2 \cdot 2) \cdot (8 \cdot 8 \cdot 8)$

5. Use the commutative property to pair factors of 2 and factors of 8.

   (2 • 8) • (2 • 8) • (2 • 8)

6. Multiply each pairing of 2 and 8. Then rewrite the expression using a single, positive exponent.

   $16 \cdot 16 \cdot 16$

   16^3

Now let's think about what to do when the exponents are different.

7. Write the expression $9^4 \bullet 6^5$ in expanded form.

$(9 \cdot 9 \cdot 9 \cdot 9) \cdot (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)$

8. Use the commutative property to pair factors of 9 and factors of 6. You will have one unpaired factor.

$(\underline{9} \cdot \underline{6}) \cdot (\underline{9} \cdot \underline{6}) \cdot (\underline{9} \cdot \underline{6}) \cdot (\underline{9} \cdot \underline{6}) \cdot \underline{6}$

9. Multiply each pairing of factors. Then rewrite the expression using a single, positive exponent.

$54 \cdot 54 \cdot 54 \cdot 54 \cdot 6$

$54^4 \cdot 6$

Now, let's practice a few more examples using what we know. Before simplifying each expression, make a note of whether the bases are different or the exponents are different. It can also be helpful to expand **the expression before simplifying.**

10. Rewrite each expression using a single, positive exponent if possible.

$2^5 \bullet 5^5$	$3^6 \bullet 7^6$	$8^2 \bullet 9^3$
10^5	21^6	$(8 \cdot 8) \cdot (9 \cdot 9 \cdot 9)$ $72^2 \cdot 9$

1. **Expand each expression, then rewrite each using a single exponent.**

$2^3 \bullet 5^3$

$(2 \cdot 2 \cdot 2) \cdot (5 \cdot 5 \cdot 5)$

10^3

$2^2 \bullet 3^2 \bullet 4^2$

$2 \cdot 2 \cdot 3 \cdot 3 \cdot 4 \cdot 4$

$(2 \cdot 3 \cdot 4) \cdot (2 \cdot 3 \cdot 4)$

24^2

2. **Expand each expression, then rewrite each using a single exponent.**

$7^2 \bullet 4^3$

$7 \cdot 7 \cdot 4 \cdot 4 \cdot 4$

$28^2 \cdot 4$

$9^4 \bullet 2^5$

$9 \cdot 9 \cdot 9 \cdot 9 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$18^4 \cdot 2$

3. Which is equivalent to the expression below? Select all that apply.

$$8^5 \bullet 4^5$$

$(8\cdot8\cdot8\cdot8\cdot8)\cdot(4\cdot4\cdot4\cdot4\cdot4)$

32^5

a. 32^5

b. 32^{25}

c. 32^{10}

d. 12^{10}

e. 8 • 8 • 8 • 8 • 8 • 4 • 4 • 4 • 4 • 4

f. 8 + 8 + 8 + 8 + 8 + 4 + 4 + 4 + 4 + 4

4. Lamar is attempting to rewrite the expression below. His work is shown.

$$6^5 \bullet 3^5$$

6 + 3 = 9 → 9^{10}
5 + 5 = 10

Explain Lamar's error. Include the correct answer in your response.

Lamar added his bases and added his exponents. He has 5 factors of 6 and 3. That means he has 5 factors of 18. He can write that as 18^5.

G8 U5 Lesson 9
Describe large and small numbers as multiples of powers of 10.

CITYTUTOR DC

G8 U5 Lesson 9 - Students will describe large and small numbers as multiples of powers of 10.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We've been working with exponents since the beginning of this unit. We're getting stronger and stronger at applying exponent rules. Sometimes when we get deep in the weeds of computation, we can forget why we're even doing the things we're doing. Today's lesson is a reminder of one major reason we use exponents in everyday life, and that's to express large and small numbers efficiently. Astronomers use exponents to express the massive distance between planets. Biologists use exponents to express miniscule lengths of microscopic cells. Businesses use exponents to express large profits. Today, our goal is to describe large and small numbers as multiples of powers of 10.

Let's Talk (Slide 3): You've likely been multiplying and dividing by tens since 3rd or 4th grade. People often consider tens as being fairly friendly numbers to compute with. What rules or patterns do we know to help us multiply by tens? Feel free to use your own experience or the equations below to help explain your thinking.
Possible Student Answers, Key Points:

- Multiplying a whole number by 10 is easy, because we can just annex a zero on the right-side of the number. If we multiply by 100, we can annex two zeroes. The pattern continues.
- For every 10 we multiply by, we can shift the digits of a number left one place value. Place value patterns make multiplying by tens easy.

$24 \cdot 10 = 240$

$24 \cdot 100 = 2400$

$1.5 \cdot 10 = 15$

$1.5 \cdot 100 = 150$

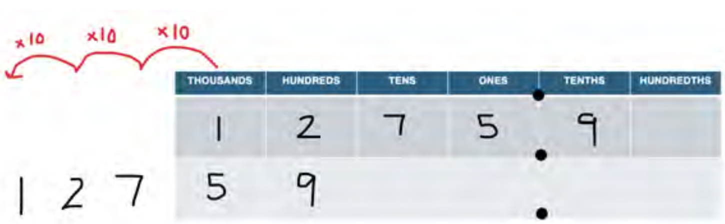
We can use patterns to help us easily find the products of multiples of 10. When dealing with whole numbers, we can annex a zero for every 10 we multiply by. We can also think about the placement of digits when we multiply by tens. Every ten we multiply shifts the digits of a number one place value left.

Let's use this thinking to find each product here. What would each product be, and how did you figure it out? *(write as student shares)* Possible Student Answers, Key Points:

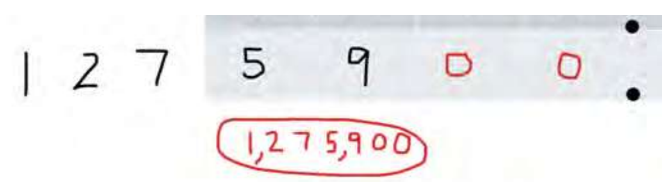
- 24 x 10 is 240, because I could just add a zero. 24 x 100 is 2400, because I could just add two zeros. Multiplying by 100 is like multiplying by 10 twice.
- 1.5 x 10 is 15, because I can just shift the digits one place value left. 1.5 x 100 is 150, because I can shift the digits two place values left.

Let's keep these efficient patterns in mind. This thinking will come in handy as we work on today's problems.

Let's Think (Slide 4): Our first set of problems asks us to find the value of two numbers multiplied by powers of 10. The first problem wants us to find the value of 1,275.9 times 10 to the third power. That means we're multiplying the number by 10 three times. As we just practiced, we know that means we can shift the digits three places values to the left. A tool we can use to help us organize our work is a place value chart.



(fill in place value chart as you narrate) I'll start by writing 1,275.9 in the place value chart. Multiplying by 10 to the third power, means each digit will shift left three place values. *(draw 3 hops with arrows, labeling each as "x10")* Now the 1 is in the millions, the 2 is in the hundred thousands, the 7 is in the ten thousands, the 5 is in the thousand, and the 9 is in the hundreds place.



I'll fill in placeholder zeros in the empty place values, and we'll have our answer. *(fill in a zero in the tens and ones places)* After filling in the place holder zeros, we can see that 1,275.9 times ten to the third power is 1,275,900.

Let's try the next problem. The problem wants us to find the value of 0.47 times 10 to the second power. How can we use what we did in the previous problem in this problem? Possible Student Answers, Key Points:

- We can put 0.47 in a place value chart. Since we're multiplying by 10 to the second power, each digit will shift left two place values.

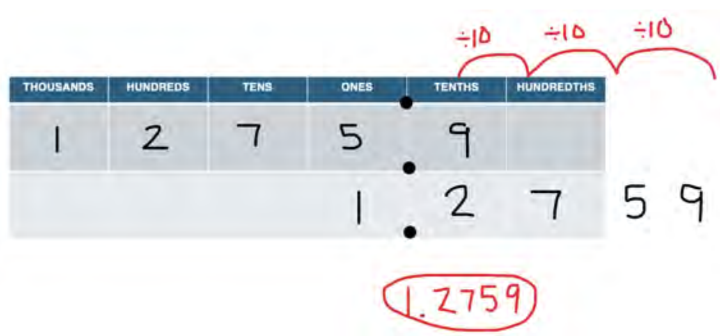
(fill in place value chart as you narrate) We know that each digit will shift two place values. We don't always *need* to use a place value chart to show this work, but it's helpful to keep everything organized. I'll start by putting 0.47 in the place value chart. Each digit will shift two place values, because each place value shift represents multiplying by 10. We'll end up with a zero in the hundreds place, a 4 in the tens place, and a 7 in the ones place. What number does this represent? (47)

The value of 0.47 times ten to the second power is 47. *(write 47)* Knowing patterns when multiplying numbers by ten really came in handy. We also see how helpful it can be to keep track of digits in a place value chart. Let's look at two more examples that have a bit of a twist.

Let's Think (Slide 5): Take a look at these two problems. What do you notice? Possible Student Answers, Key Points:

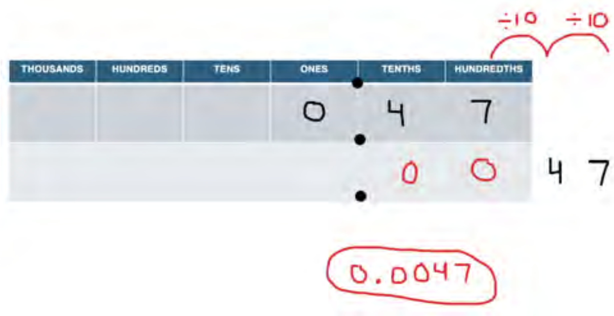
- The problems involve the same numbers as our last two examples.
- The exponents are now negative.

We can use a place value chart and patterns with tens to help us find the value of expressions involving negative exponents, too.



For the first example, let's start by writing the number in a place value chart. *(fill in place value chart as you narrate)* Multiplying by 10 to the negative third power is the same as multiplying by 1/10 x `1/10 x 1/10. We can also think of that as dividing by 10 then by 10 then by 10 again. Since division is the inverse of multiplication, the only difference in our work will be the direction our digits shift along the place value chart. I'll show three hops to the right along the place value chart to show that each digit in 1,275.9 is shifting three place values right. When I rewrite my digits, each three places over, we end up with an answer of 1.2759.

I think you can help me complete the next problem. Take a second to look it over, and then walk me through how you could use a place value chart to find the value of 0.47 times ten to the negative second power. *(fill in place value chart as student shares, supporting as needed)*



Possible Student Answers, Key Points:

- First, we can write 0.47 in the place value chart. The 4 is in the tenths place and the 7 is in the hundredths place.
- Since we're multiplying by 10 to the negative second power, that's like multiplying by 1/10 and 1/10. We can also think of that as dividing by 10 and then by 10 again.
- If each digit shifts two place values right, the value of the expression is 0.0047.

Each digit shifts two place values right, and the value of the expression is 0.0047.

Let's Try it (Slides 6 - 7): Now we'll get a chance to practice a few more problems before you work on a few of your own. Knowing that each place value is 10 times greater than the place value to its right can help us efficiently multiply by powers of 10. If the exponent is negative, we can use similar thinking, but we understand that our digits will shift a different direction. If we find it helpful, we can continue to use the place value chart as a way to organize our thinking.

WARM WELCOME

Today we will describe large and small numbers as multiples of powers of 10.

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Let's Talk:

What patterns or rules do we know to help us multiply by powers of ten?

$24 \bullet 10 = ?$

$24 \bullet 100 = ?$

$1.5 \bullet 10 = ?$

$1.5 \bullet 100 = ?$

Let's Think:

Find the value of each expression.

a. $1{,}275.9 \bullet 10^3$

b. $0.47 \bullet 10^2$

THOUSANDS	HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS

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Let's Think:

Find the value of each expression.

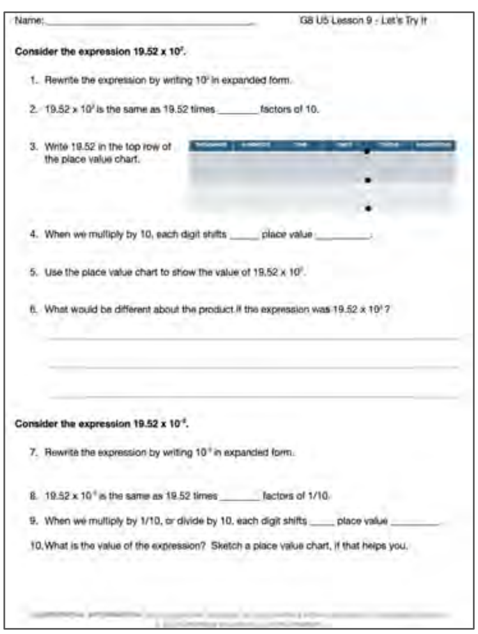
a. $1{,}275.9 \cdot 10^{-3}$

b. $0.47 \cdot 10^{-2}$

THOUSANDS	HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS

Let's Try It:

Let's explore describing large and small numbers as multiples of powers of 10 together.



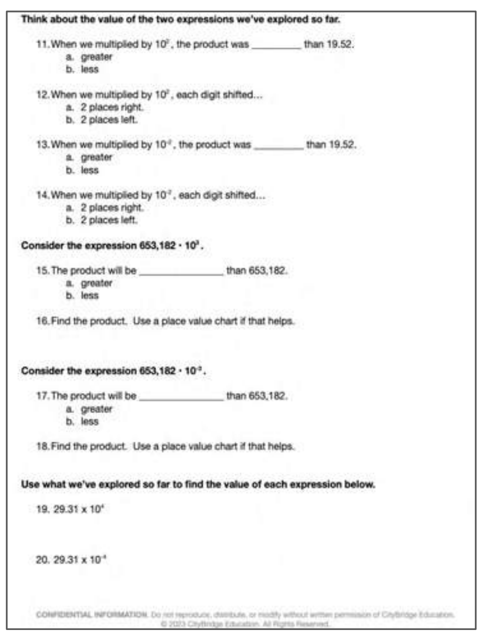
Name:________________ G8 U5 Lesson 9 - Let's Try It

Consider the expression 19.52×10^2.

1. Rewrite the expression by writing 10^2 in expanded form.
2. 19.52×10^2 is the same as 19.52 times _______ factors of 10.
3. Write 19.52 in the top row of the place value chart.
4. When we multiply by 10, each digit shifts _____ place value _________.
5. Use the place value chart to show the value of 19.52×10^2.
6. What would be different about the product if the expression was 19.52×10^3?

Consider the expression 19.52×10^{-2}.

7. Rewrite the expression by writing 10^{-2} in expanded form.
8. 19.52×10^{-2} is the same as 19.52 times _______ factors of 1/10.
9. When we multiply by 1/10, or divide by 10, each digit shifts _____ place value _________.
10. What is the value of the expression? Sketch a place value chart, if that helps you.



Think about the value of the two expressions we've explored so far.

11. When we multiplied by 10^2, the product was _________ than 19.52.
    a. greater
    b. less
12. When we multiplied by 10^2, each digit shifted...
    a. 2 places right.
    b. 2 places left.
13. When we multiplied by 10^{-2}, the product was _________ than 19.52.
    a. greater
    b. less
14. When we multiplied by 10^{-2}, each digit shifted...
    a. 2 places right.
    b. 2 places left.

Consider the expression $653{,}182 \cdot 10^3$.

15. The product will be ______________ than 653,182.
    a. greater
    b. less
16. Find the product. Use a place value chart if that helps.

Consider the expression $653{,}182 \cdot 10^{-3}$.

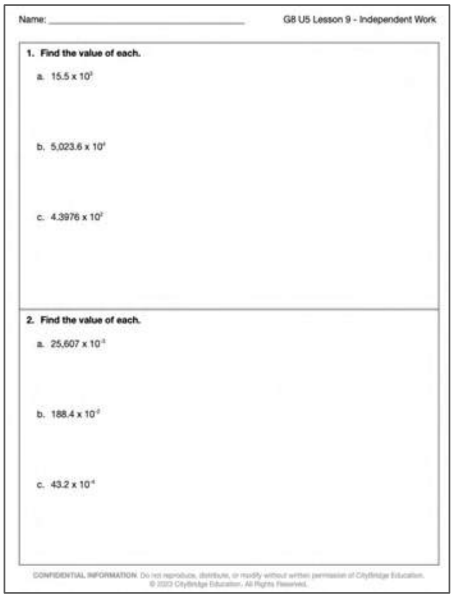
17. The product will be ______________ than 653,182.
    a. greater
    b. less
18. Find the product. Use a place value chart if that helps.

Use what we've explored so far to find the value of each expression below.

19. 29.31×10^4
20. 29.31×10^{-4}

On your Own:

Now it's time to describe large and small numbers as multiples of powers of 10 on your own.



Name: ____________________ G8 U5 Lesson 9 - Independent Work

1. **Find the value of each.**

   a. 15.5×10^3

   b. $5{,}023.6 \times 10^4$

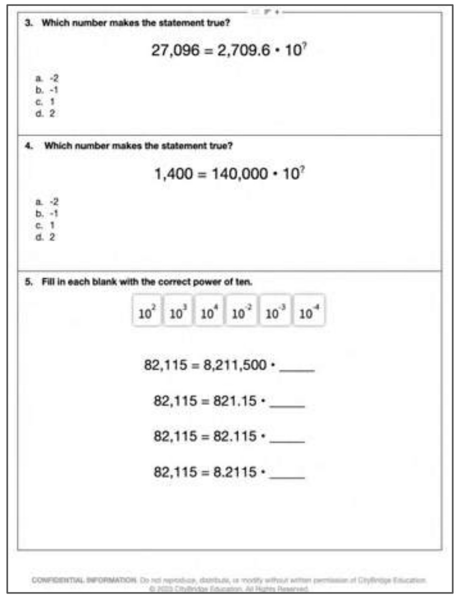
   c. 4.3976×10^2

2. **Find the value of each.**

   a. $25{,}607 \times 10^{-3}$

   b. 188.4×10^{-2}

   c. 43.2×10^{-4}



3. **Which number makes the statement true?**

$$27{,}096 = 2{,}709.6 \cdot 10^?$$

   a. -2
   b. -1
   c. 1
   d. 2

4. **Which number makes the statement true?**

$$1{,}400 = 140{,}000 \cdot 10^?$$

   a. -2
   b. -1
   c. 1
   d. 2

5. **Fill in each blank with the correct power of ten.**

10^2 10^3 10^4 10^{-2} 10^{-3} 10^{-4}

$82{,}115 = 8{,}211{,}500 \cdot$ _____

$82{,}115 = 821.15 \cdot$ _____

$82{,}115 = 82.115 \cdot$ _____

$82{,}115 = 8.2115 \cdot$ _____

Name: ____ G8 U5 Lesson 9 - Let’s Try It

Consider the expression 19.52 x 10^2.

1. Rewrite the expression by writing 10^2 in expanded form.

2. 19.52 x 10^2 is the same as 19.52 times ________ factors of 10.

3. Write 19.52 in the top row of the place value chart.

THOUSANDS	HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS

4. When we multiply by 10, each digit shifts ______ place value ___________.

5. Use the place value chart to show the value of 19.52 x 10^2.

6. What would be different about the product if the expression was 19.52 x 10^3 ?

____

____

____

Consider the expression 19.52 x 10^{-2}.

7. Rewrite the expression by writing 10^{-2} in expanded form.

8. 19.52 x 10^{-2} is the same as 19.52 times ________ factors of 1/10.

9. When we multiply by 1/10, or divide by 10, each digit shifts _____ place value __________.

10. What is the value of the expression? Sketch a place value chart, if that helps you.

Think about the value of the two expressions we've explored so far.

11. When we multiplied by 10^2, the product was __________ than 19.52.
    a. greater
    b. less

12. When we multiplied by 10^2, each digit shifted…
    a. 2 places right.
    b. 2 places left.

13. When we multiplied by 10^{-2}, the product was __________ than 19.52.
    a. greater
    b. less

14. When we multiplied by 10^{-2}, each digit shifted…
    a. 2 places right.
    b. 2 places left.

Consider the expression 653,182 • 10^3.

15. The product will be _________________ than 653,182.
    a. greater
    b. less

16. Find the product. Use a place value chart if that helps.

Consider the expression 653,182 • 10^{-3}.

17. The product will be _________________ than 653,182.
    a. greater
    b. less

18. Find the product. Use a place value chart if that helps.

Use what we've explored so far to find the value of each expression below.

19. 29.31 x 10^4

20. 29.31 x 10^{-4}

1. Find the value of each.

a. 15.5×10^{3}

b. $5{,}023.6 \times 10^{4}$

c. 4.3976×10^{2}

2. Find the value of each.

a. $25{,}607 \times 10^{-3}$

b. 188.4×10^{-2}

c. 43.2×10^{-4}

3. Which number makes the statement true?

$$27{,}096 = 2{,}709.6 \bullet 10^{?}$$

a. -2
b. -1
c. 1
d. 2

4. Which number makes the statement true?

$$1{,}400 = 140{,}000 \bullet 10^{?}$$

a. -2
b. -1
c. 1
d. 2

5. Fill in each blank with the correct power of ten.

10^{2} 10^{3} 10^{4} 10^{-2} 10^{-3} 10^{-4}

$$82{,}115 = 8{,}211{,}500 \bullet \_\_\_\_\_$$

$$82{,}115 = 821.15 \bullet \_\_\_\_\_$$

$$82{,}115 = 82.115 \bullet \_\_\_\_\_$$

$$82{,}115 = 8.2115 \bullet \_\_\_\_\_$$

Name: KEY

Consider the expression 19.52 x 10^2.

1. Rewrite the expression by writing 10^2 in expanded form. $10 \cdot 10$

2. 19.52 x 10^2 is the same as 19.52 times __2__ factors of 10.

3. Write 19.52 in the top row of the place value chart.

THOUSANDS	HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS
		1	9	5	2
1	9	5	2		

4. When we multiply by 10, each digit shifts __1__ place value __left__.

5. Use the place value chart to show the value of 19.52 x 10^2. 1,952

6. What would be different about the product if the expression was 19.52 x 10^3?

Each digit would shift one more place left. The answer would be 19,520.

Consider the expression 19.52 x 10^{-2}.

7. Rewrite the expression by writing 10^{-2} in expanded form.

$\frac{1}{10} \cdot \frac{1}{10}$ OR $\frac{1}{10 \cdot 10}$

8. 19.52 x 10^{-2} is the same as 19.52 times __2__ factors of 1/10.

9. When we multiply by 1/10, or divide by 10, each digit shifts __1__ place value __right__.

10. What is the value of the expression? Sketch a place value chart, if that helps you.

0.1952

Think about the value of the two expressions we've explored so far.

11. When we multiplied by 10^2, the product was _________ than 19.52.
    a. greater
    b. less

12. When we multiplied by 10^2, each digit shifted…
    a. 2 places right.
    b. 2 places left.

13. When we multiplied by 10^{-2}, the product was _________ than 19.52.
    a. greater
    b. less

14. When we multiplied by 10^{-2}, each digit shifted…
    a. 2 places right.
    b. 2 places left.

Consider the expression $653{,}182 \cdot 10^3$.

15. The product will be _______________ than 653,182.
    a. greater
    b. less

16. Find the product. Use a place value chart if that helps.

653,182,000

Consider the expression $653{,}182 \cdot 10^{-3}$.

17. The product will be _______________ than 653,182.
    a. greater
    b. less

18. Find the product. Use a place value chart if that helps.

653.182

Use what we've explored so far to find the value of each expression below.

19. 29.31×10^4 293,100

20. 29.31×10^{-4} 0.002931

Name: KEY

1. Find the value of each.

a. 15.5×10^3

15,500

b. $5{,}023.6 \times 10^4$

50,236,000

c. 4.3976×10^2

439.76

2. Find the value of each.

a. $25{,}607 \times 10^{-3}$

25.607

b. 188.4×10^{-2}

1.884

c. 43.2×10^{-4}

0.00432

3. **Which number makes the statement true?**

$$27{,}096 = 2{,}709.6 \cdot 10^{?}$$

×10

2709.6

27096

a. -2
b. -1
c. 1 (circled)
d. 2

4. **Which number makes the statement true?**

$$1{,}400 = 140{,}000 \cdot 10^{?}$$

140000.

1400.

$\times \frac{1}{10}$ $\times \frac{1}{10}$

a. -2 (circled)
b. -1
c. 1
d. 2

5. **Fill in each blank with the correct power of ten.**

10^2	10^3	10^4	10^{-2}	10^{-3}	10^{-4}

$82{,}115 = 8{,}211{,}500 \cdot$ 10^{-2}

$82{,}115 = 821.15 \cdot$ 10^{2}

$82{,}115 = 82.115 \cdot$ 10^{3}

$82{,}115 = 8.2115 \cdot$ 10^{4}

G8 U5 Lesson 10
Compare large numbers using powers of 10.

CITYTUTOR DC

G8 U5 Lesson 10 - Students will compare large numbers using powers of 10.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our previous lesson, we worked to represent really large numbers and really small numbers using powers of 10. That's one critical reason why we use exponents. Today, we're going to continue with that work. Our goal is to compare large numbers using powers of 10. Let's get started!

Let's Talk (Slide 3): Before we work directly with powers of ten, let's get on the same page about some ideas around comparison. Let's look at the examples of comparison here.

How many times greater is 10 than 5? How do you know? Possible Student Answers, Key Points:

- 10 is 2 times greater than 5.
- I know because 5 x 2 is 10. 10 is twice as much as 5. I know 10 divided by 5 is 2.

$10 = \_\_ \times 5$

$10 \div 5 = 2$

There are many ways to think of this. We can think "10 is how many times as much as 5?" *(write 10 = _____ x 5)* One way to solve this equation is to divide 10 by 5. That gives us 2, so we know 10 is 2 times greater than 5.

$30 = \_\_ \times 5$

$30 \div 5 = 6$

We can use similar thinking for the next example. We can think "30 is how many times as much as 5?" *(write 30 = _____ x 5)* One way to solve this equation is to divide 30 by 5. That gives us 6, so we know 30 is 6 times greater than 5.

$5000 = \_\_ \times 5$

$5000 \div 5 = 1000$

How could we use this logic with the next example? *(write equations as student shares, supporting as needed)* Possible Student Answers, Key Points:

- We can think 5,000 is how many times 5 by writing 5000 = _____ x 5.
- We can divide 5,000 by 5. 5,000 is 1,000 times greater than 5.

$42 = \_\_ \times 5$

$42 \div 5 = 8.4$

5,000 divided by 5 shows us that 5,000 is 1,000 times as much as 5. This thinking even works when the numbers aren't compatible. Look at the last example. If we want to know how many times greater is 42 than 5, we can think about what times 5 is equal to 42, and we can write 42 = ____ x 5. If we divide 42 by 5, we get 8.4 So, we can say 42 is 8.4 times greater than 5.

Let's keep this comparison thinking in mind as we look at numbers involving powers of 10.

Let's Think (Slide 4): For our first problem, it wants to know whose number is greater. Once we figure that out, they want to know how many times greater that number is than the other number.

Take a look at Maria's number. Take a look at Angel's number. Whose is greater? How do you know?
Possible Student Answers, Key Points:

- Maria's number is greater, because they both have the same power of 10, but Maria's leading factor is 32 and Angel's is only 8.
- Maria's number is greater, because it is 320,000,000 and Angel's is 80,000,000.

$32 > 8$

I notice both numbers use the same power of 10, so we can think of the leading factors 32 and 8 as being in the same place value. Since 32 is greater than 8, *(write 32 > 8)* then Maria's number is greater.

Now, let's work to determine how many times greater Maria's number is than Angel's number. Just like we saw in our earlier examples, we can divide the larger quantity by the smaller quantity to see how many times bigger it is than the smaller quantity.

$$\frac{32 \cdot 10^7}{8 \cdot 10^7} = \frac{32}{8} \cdot \frac{10^7}{10^7}$$

$$4 \cdot 1$$

$$\boxed{4}$$

Let's start by writing the division in fraction form. *(write Maria's number in the numerator and Angel's number in the denominator)* To evaluate this, we can divide the leading factors first, and then we can divide the powers of 10. *(write leading factors and powers of 10 separately as shown)*

What is 32 divided by 8? (4)

What is 10 to the seventh power divided by 10 to the seventh power? (1) It's 1, because any number over itself in fraction form is equivalent to 1 whole. *(write 4 • 1)*

4 times 1 is equal to 4. We can say Maria's number is 4 times greater than Angel's number. We used division to help us find how many times greater Maria's number is than Angel's number. Dividing the leading factors first and then the powers of 10 kept the math manageable.

Let's try another example.

Let's Think (Slide 5): *(read the problem aloud)*   This problem wants us to find how many times faster light travels through copper wire than through diamonds. I notice these values don't have the same power of 10. It can be helpful, although not 100% necessary, to compare numbers if they involve the same power of 10.

$$2.5 \times 10^8$$

$$2.5 \times 10^2 \times 10^6$$

$$250 \times 10^6$$

Let's think about how we can rewrite 2.5 x 10 to the eighth power using 10 to the sixth power instead. I know 10 to the eighth power can be written as 10 to the sixth power times 10 to the second power. I can decompose 10 to the eighth power using a number bond. *(show a number bond decomposing 10 to the eighth power into 10 to the second power and 10 to the sixth power)* I know 2.5 x 10 to the second power is the same as 250. 2.5 times 10 to the eighth power is equivalent to 250 times 10 to the sixth power. *(write 250 x 10 to the sixth power)*

$$\frac{250 \times 10^6}{160 \times 10^6} = \frac{250}{160} \times \frac{10^6}{10^6}$$

Now we can use division to find how many times faster light travels through copper wire than through diamonds. I'll write the value for the speed of light through copper wire in the numerator. I'll write the value for the speed of light through diamonds in the denominator. *(write the values in fraction form)* Think back to our last example. How can we break apart this problem to make the division more manageable? How could we write the quotient? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- We can divide the leading factors first, and then we can divide the powers of 10.
- 250 divided by 160 is 1.5625. 10 to the sixth divided by 10 to the sixth is just 1 whole.

$$1.5625 \times 1$$

$$\boxed{1.5625}$$

We're left with 1.5625 times 1, which is 1.5625. This tells us the speed light waves travel through copper wire is 1.5625 times faster than the speed at which light waves travel through diamonds.

Let's Try it (Slides 6 - 7): Now we get a chance to try a few more comparison problems together. As we've seen, we can use division to help us compare how many times greater one value is than another. When dividing with powers of 10, it can be helpful to have the same power of 10 in both quantities. We can also make dividing efficient by dividing leading factors and powers of 10 separately. Let's keep all this in mind for the next few examples. When we're done, you'll get a chance to show what you know independently.

WARM WELCOME

Today we will compare large numbers using powers of 10.

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Let's Talk:

How many times greater is 10 than 5?

How many times greater is 30 than 5?

How many times greater is 5,000 than 5?

How many times greater is 42 than 5?

Let's Think:

Whose number is greater?

How many times greater is it than the other number?

MARIA

$32 \cdot 10^7$

ANGEL

$8 \cdot 10^7$

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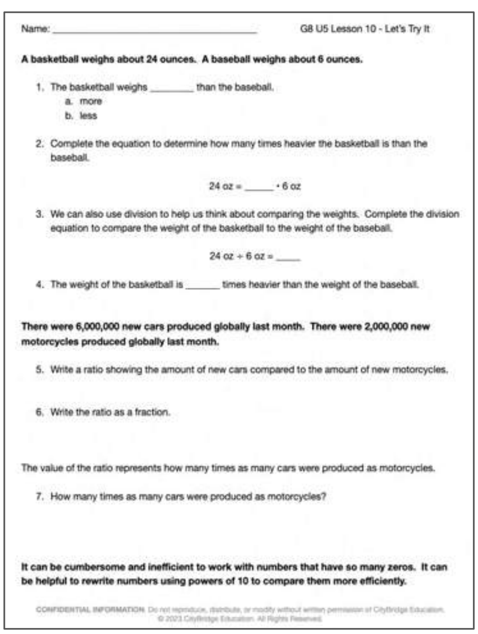
Let's Think:

Light waves travel through copper wire at a speed of $2.5 \cdot 10^8$ meters per second. Light waves travel through diamonds at $160 \cdot 10^6$ meters per second. How many times faster does light travel through copper wire than through diamonds?

Let's Try It:

Let's explore comparing large numbers using powers of 10 together.



Name: ______________________ G8 U5 Lesson 10 - Let's Try It

A basketball weighs about 24 ounces. A baseball weighs about 6 ounces.

1. The basketball weighs ________ than the baseball.
   a. more
   b. less
2. Complete the equation to determine how many times heavier the basketball is than the baseball.

   24 oz = _____ · 6 oz

3. We can also use division to help us think about comparing the weights. Complete the division equation to compare the weight of the basketball to the weight of the baseball.

   24 oz ÷ 6 oz = _____

4. The weight of the basketball is ______ times heavier than the weight of the baseball.

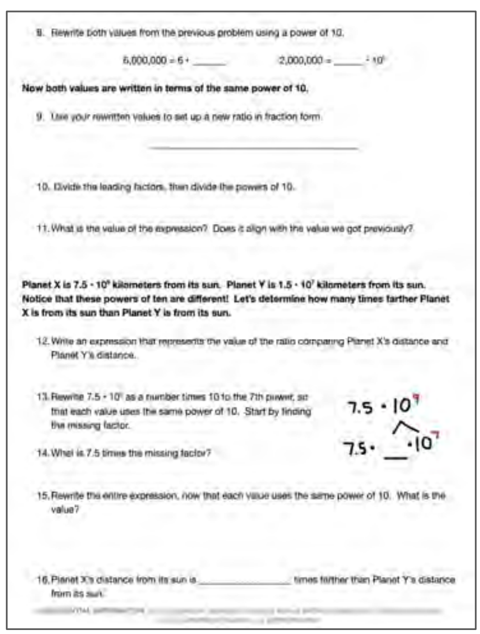
There were 6,000,000 new cars produced globally last month. There were 2,000,000 new motorcycles produced globally last month.

5. Write a ratio showing the amount of new cars compared to the amount of new motorcycles.
6. Write the ratio as a fraction.

The value of the ratio represents how many times as many cars were produced as motorcycles.

7. How many times as many cars were produced as motorcycles?

It can be cumbersome and inefficient to work with numbers that have so many zeros. It can be helpful to rewrite numbers using powers of 10 to compare them more efficiently.



8. Rewrite both values from the previous problem using a power of 10.

   6,000,000 = 6 · _____ 2,000,000 = _____ · 10[illegible]

Now both values are written in terms of the same power of 10.

9. Use your rewritten values to set up a new ratio in fraction form.

   ____

10. Divide the leading factors, then divide the powers of 10.
11. What is the value of the expression? Does it align with the value we got previously?

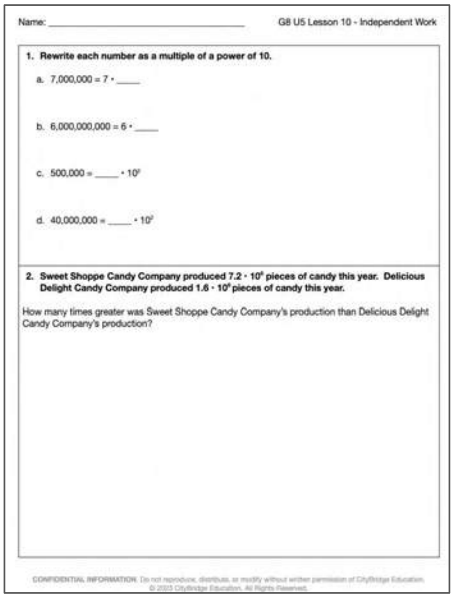
Planet X is 7.5 · 10[illegible] kilometers from its sun. Planet Y is 1.5 · 10^7 kilometers from its sun. Notice that these powers of ten are different! Let's determine how many times farther Planet X is from its sun than Planet Y is from its sun.

12. Write an expression that represents the value of the ratio comparing Planet X's distance and Planet Y's distance.
13. Rewrite 7.5 · 10[illegible] as a number times 10 to the 7th power, so that each value uses the same power of 10. Start by finding the missing factor.
14. What is 7.5 times the missing factor?

15. Rewrite the entire expression, now that each value uses the same power of 10. What is the value?
16. Planet X's distance from its sun is ______________ times farther than Planet Y's distance from its sun.

On your Own:

Now it's time to compare large numbers using powers of 10 on your own.



Name: ______________________ G8 U5 Lesson 10 - Independent Work

1. Rewrite each number as a multiple of a power of 10.

a. $7{,}000{,}000 = 7 \cdot$ _____

b. $6{,}000{,}000{,}000 = 6 \cdot$ _____

c. $500{,}000 =$ _____ $\cdot 10^5$

d. $40{,}000{,}000 =$ _____ $\cdot 10^7$

2. Sweet Shoppe Candy Company produced $7.2 \cdot 10^8$ pieces of candy this year. Delicious Delight Candy Company produced $1.6 \cdot 10^8$ pieces of candy this year.

How many times greater was Sweet Shoppe Candy Company's production than Delicious Delight Candy Company's production?

3. An employee at a company earns $7.5 \cdot 10^4$ dollars. The CEO of the company makes 9,000,000 dollars.

How many times greater is the CEO's salary than the employee's salary?

4. Comet A is $4.4 \cdot 10^6$ miles from the closest star and $8 \cdot 10^7$ miles from the closest planet.

How many times greater is the distance from Comet A to the plant than the distance from Comet A to the star?

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A basketball weighs about 24 ounces. A baseball weighs about 6 ounces.

1. The basketball weighs _________ than the baseball.
   a. more
   b. less

2. Complete the equation to determine how many times heavier the basketball is than the baseball.

   24 oz = ______ • 6 oz

3. We can also use division to help us think about comparing the weights. Complete the division equation to compare the weight of the basketball to the weight of the baseball.

   24 oz ÷ 6 oz = _____

4. The weight of the basketball is _______ times heavier than the weight of the baseball.

There were 6,000,000 new cars produced globally last month. There were 2,000,000 new motorcycles produced globally last month.

5. Write a ratio showing the amount of new cars compared to the amount of new motorcycles.

6. Write the ratio as a fraction.

The value of the ratio represents how many times as many cars were produced as motorcycles.

7. How many times as many cars were produced as motorcycles?

It can be cumbersome and inefficient to work with numbers that have so many zeros. It can be helpful to rewrite numbers using powers of 10 to compare them more efficiently.

8. Rewrite both values from the previous problem using a power of 10.

$6{,}000{,}000 = 6 \bullet$ _______ $\qquad\qquad 2{,}000{,}000 =$ ______ $\bullet\ 10^6$

Now both values are written in terms of the same power of 10.

9. Use your rewritten values to set up a new ratio in fraction form.

____

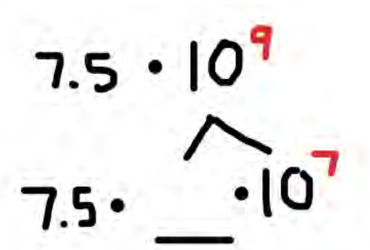
10. Divide the leading factors, then divide the powers of 10.

11. What is the value of the expression? Does it align with the value we got previously?

Planet X is $7.5 \bullet 10^9$ kilometers from its sun. Planet Y is $1.5 \bullet 10^7$ kilometers from its sun. Notice that these powers of ten are different! Let's determine how many times farther Planet X is from its sun than Planet Y is from its sun.

12. Write an expression that represents the value of the ratio comparing Planet X's distance and Planet Y's distance.

13. Rewrite $7.5 \bullet 10^9$ as a number times 10 to the 7th power, so that each value uses the same power of 10. Start by finding the missing factor.



14. What is 7.5 times the missing factor?

15. Rewrite the entire expression, now that each value uses the same power of 10. What is the value?

16. Planet X's distance from its sun is __________________ times farther than Planet Y's distance from its sun.

1. Rewrite each number as a multiple of a power of 10.

a. 7,000,000 = 7 • _____

b. 6,000,000,000 = 6 • _____

c. 500,000 = _____ • 10^2

d. 40,000,000 = _____ • 10^2

2. Sweet Shoppe Candy Company produced 7.2 • 10^6 pieces of candy this year. Delicious Delight Candy Company produced 1.6 • 10^6 pieces of candy this year.

How many times greater was Sweet Shoppe Candy Company's production than Delicious Delight Candy Company's production?

3. An employee at a company earns $7.5 \cdot 10^4$ dollars. The CEO of the company makes 9,000,000 dollars.

How many times greater is the CEO's salary than the employee's salary?

4. Comet A is $4.4 \cdot 10^8$ miles from the closest star and $8 \cdot 10^7$ miles from the closest planet.

How many times greater is the distance from Comet A to the plant than the distance from Comet A to the star?

Name: KEY

A basketball weighs about 24 ounces. A baseball weighs about 6 ounces.

1. The basketball weighs ________ than the baseball.
   a. more
   b. less

2. Complete the equation to determine how many times heavier the basketball is than the baseball.

$$24 \text{ oz} = \underline{4} \cdot 6 \text{ oz}$$

3. We can also use division to help us think about comparing the weights. Complete the division equation to compare the weight of the basketball to the weight of the baseball.

$$24 \text{ oz} \div 6 \text{ oz} = \underline{4}$$

4. The weight of the basketball is __4__ times heavier than the weight of the baseball.

There were 6,000,000 new cars produced globally last month. There were 2,000,000 new motorcycles produced globally last month.

5. Write a ratio showing the amount of new cars compared to the amount of new motorcycles.

6000 000 : 2000 000

6. Write the ratio as a fraction.

$$\frac{6000000}{2000000}$$

The value of the ratio represents how many times as many cars were produced as motorcycles.

7. How many times as many cars were produced as motorcycles?

3

It can be cumbersome and inefficient to work with numbers that have so many zeros. It can be helpful to rewrite numbers using powers of 10 to compare them more efficiently.

8. Rewrite both values from the previous problem using a power of 10.

6,000,000 = 6 • 10^6 2,000,000 = 2 • 10^6

Now both values are written in terms of the same power of 10.

9. Use your rewritten values to set up a new ratio in fraction form.

$$\frac{6 \cdot 10^6}{2 \cdot 10^6}$$

10. Divide the leading factors, then divide the powers of 10.

$6 \div 2 = 3$ $10^6 \div 10^6 = 1$

11. What is the value of the expression? Does it align with the value we got previously?

$3 \times 1 = 3$ → Yes! It's the same answer.

Planet X is 7.5 • 10^9 kilometers from its sun. Planet Y is 1.5 • 10^7 kilometers from its sun. Notice that these powers of ten are different! Let's determine how many times farther Planet X is from its sun than Planet Y is from its sun.

12. Write an expression that represents the value of the ratio comparing Planet X's distance and Planet Y's distance.

$$\frac{7.5 \times 10^9}{1.5 \times 10^7}$$

13. Rewrite 7.5 • 10^9 as a number times 10 to the 7th power, so that each value uses the same power of 10. Start by finding the missing factor.

7.5 • 10^9

7.5 • 10^2 • 10^7

14. What is 7.5 times the missing factor?

750

15. Rewrite the entire expression, now that each value uses the same power of 10. What is the value?

$$\frac{750 \times 10^7}{1.5 \times 10^7} \quad \frac{750}{1.5} \times \frac{10^7}{10^7} \quad 500 \times 1$$

16. Planet X's distance from its sun is 500 times farther than Planet Y's distance from its sun.

Name: KEY

1. Rewrite each number as a multiple of a power of 10.

a. 7,000,000 = 7 · 10^6

b. 6,000,000,000 = 6 · 10^9

c. 500,000 = 5000 · 10^2

d. 40,000,000 = 400,000 · 10^2

2. Sweet Shoppe Candy Company produced $7.2 \cdot 10^6$ pieces of candy this year. Delicious Delight Candy Company produced $1.6 \cdot 10^6$ pieces of candy this year.

How many times greater was Sweet Shoppe Candy Company's production than Delicious Delight Candy Company's production?

$$\frac{7.2 \times 10^6}{1.6 \times 10^6} \longrightarrow \frac{7.2}{1.6} \times \frac{10^6}{10^6}$$

$$4.5 \times 1$$

$$4.5$$

3. **An employee at a company earns $7.5 \cdot 10^4$ dollars. The CEO of the company makes 9,000,000 dollars.**

How many times greater is the CEO's salary than the employee's salary?

$$\frac{9 \times 10^6}{7.5 \times 10^4}$$

$$\frac{9}{7.5} \times \frac{10^6}{10^4}$$

$$1.2 \times 10^2$$

$$120$$

4. **Comet A is $4.4 \cdot 10^8$ miles from the closest star and $8 \cdot 10^7$ miles from the closest planet.**

How many times greater is the distance from Comet A to the plant than the distance from Comet A to the star?

$$\frac{4.4 \times 10^8}{8 \times 10^7}$$

$$\frac{4.4}{8} \times \frac{10^8}{10^7}$$

$$0.55 \times 10^1$$

$$5.5$$

G8 U5 Lesson 11
Represent small numbers as multiples of powers of 10, and compare numbers using powers of 10.

CITYTUTOR DC

G8 U5 Lesson 11 - Students will represent small numbers as multiples of powers of 10, and compare numbers using powers of 10.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our last lesson, we represented large numbers using powers of 10, and we spent time comparing those numbers. Today, we'll do the same thing, but we'll be working with small numbers. What scenarios can you think of that might necessitate the need to represent really small numbers? Possible Student Answers, Key Points:

- Scientists might need to use small numbers to think about really tiny things like atoms or cells under a microscope.
- Statisticians might need to use small numbers to determine probability or analyze data.
- Anyone who works with really precise measurements like an engineer or an architect or an artist might need to think about miniscule measurements.

Those are great ideas. There are many cases where representing really small numbers is necessary. Let's see how we can represent and compare small numbers using what we know about powers of 10.

Let's Talk (Slide 3): Before we jump into really small numbers, let's revisit some thinking around multiplicative comparison. Help me think about the first question. How many times as long is 10 inches compared to 5 inches? How do you know? Possible Student Answers, Key Points:

- 10 inches is 2 times as long as 5 inches, because 5 x 2 = 10.
- We can divide to compare. I know 10 divided by 5 is 2, so 10 inches is 2 times as long as 5 inches.

$$10 = \_\_ \times 5$$
$$10 \div 5 = 2$$

Like we've done before, we can think 10 is equal to what times 5? Or how many times longer than 5 is 10. *(write 10 = _____ x 5)* We've also explored how we can use division to think about comparison and find the unknown in this case. 10 divided by 5 equals 2. *(write that equation)* So 10 inches is 2 times as long as 5 inches.

What if we reversed this? The second question says how many times as long is 5 inches compared to 10 inches. Is the answer still 2? How do you know? Possible Student Answers, Key Points:

- The answer can't be 2, because 5 is not equal to 2 x 10.
- 5 inches is shorter than 10 inches, so it doesn't make sense that 5 inches would be 2 times as long as 10 inches.

$$5 = \_\_ \times 10$$
$$5 \div 10 = \frac{1}{2}$$

5 is not 2 times as long as 10 inches, because 2 times 10 is 20. So how can we think about this comparison? We can use the same structure we used in the last example. I can think: "5 is equal to what number times 10?" *(write 5 = ____ x 10)* Like before, we can use division to help find the unknown. 5 divided by 10 is 5/10 or ½ or 0.5 So we can say 5 inches is ½ times as long as 10 inches. Or 5 inches is 0.5 times as long as 10 inches. This value being less than 1 makes sense, because we already noted how 5 is shorter than 10.

Let's continue using this thinking throughout this lesson.

Let's Think (Slide 4): *(read problem aloud)* This problem wants us to determine how many times as long the diameter of the grain of salt is compared to the diameter of the plant cell. We can write the ratio comparing those two quantities as a fraction. We'll put 0.02 in the numerator, since that represents the diameter of the grain of salt. We'll put 0.00064 in the denominator, since that represents the diameter of the plant cell.

$$\frac{0.02}{0.00064} = \frac{20 \times 10^{-3}}{6.4 \times 10^{-4}}$$

(set up ratio as a fraction as stated) I'll write each of these using a power of 10 to help me divide efficiently. I can say 0.02 is 20 times 10 to the negative third power. *(rewrite the ratio with this new numerator)*

I'll rewrite 0.00064 as being 6.4 times a power of 10. What power of ten makes sense? How do you know? *(rewrite numerator as student shares, supporting as needed)* Possible Student Answers, Key Points:

- It should be 10 to the negative fourth power.
- If I shift the digits in 6.4 four place values right, I end back up with 0.00064.

$$3.125 \times 10^{1}$$
$$\boxed{31.25}$$

Now we can divide the leading factors separate from the powers of 10. 20 divided by 6.4 is 3.125. If we subtract the exponents -3 - -4, we can think of that as -3 + 4. The power of 10 when we divide is 10 to the first power. *(write 3.125 x 10 to the first power)* 3.125 times 10 is equal to 31.25.

The diameter of the grain of salt is 31.25 times as long as the diameter of the plant cell. Now let's reverse this comparison, and see if we can work it out.

Let's Think (Slide 5): Read the problem to yourself. *(pause as student reads)* What do you notice? Possible Student Answers, Key Points:

- This problem has the same information as the previous example.
- Instead of comparing how many times longer the diameter of the grain of salt is to the diameter of the plant cell, we're asked to compare the other way around. It's like the comparison is reversed.

This is like when we compared how many times as long 5 inches was compared to 10 inches. We can use the same logic.

$$\frac{0.00064}{0.02} = \frac{6.4 \times 10^{-4}}{2 \times 10^{-2}}$$

Since we're comparing how many times as long the plant cell is as the grain of salt, we'll put the plant cell's length in the numerator and the grain of salt in the denominator. *(write ratio in fraction form as stated)* We can rewrite each value using a power of 10. We'll use 6.4 times 10 to the negative fourth power to represent 0.00064. Instead of using 20 times 10 to the -3 power, I'll use 2 times 10 to the -2 power. That will just make it easier to divide the leading factors. *(rewrite ratio in fraction form using powers of 10 as described)*

Divide the leading factors. Divide the powers of 10. What's the result? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- When we divide the leading factors we can think of 6.4 divided by 2. That's 3.2.
- To divide the powers of 10, we can subtract the exponents. -4 minus -2 is the same as -4 plus 2, so the exponent is -2.
- The quotient is 3.2 x 10 to the -2 power.

$$3.2 \times 10^{-2} = \boxed{0.032}$$

The quotient is 3.2 times 10 to the -2 power. If we rewrite that in standard form, it's 0.032. That means the length of the plant cell's diameter is 0.032 times as long as the length of the grain of salt's diameter. This makes sense, because we know the plant cell's diameter is shorter than the grain of salt's diameter.

Let's Try it (Slides 6 - 7): Now let's try a few more together before you get a chance to try some out independently. When asked to compare, we saw that we can compare larger quantities to smaller quantities or smaller quantities to larger quantities multiplicatively. We'll want to pay attention to exactly what the question is asking us to compare. Once we know that, we can set up a ratio using powers of 10 to efficiently divide. Let's jump into our next examples.

WARM WELCOME

Today we will represent small numbers as multiples of powers of 10, and compare numbers using powers of 10.

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Let's Talk:

How many times as long is 10” compared to 5”?

How many times as long is 5” compared to 10”?

Let's Think:

The diameter of a plant cell is 0.00064 cm. The diameter of a grain of salt is 0.02 cm. How many times as long is the diameter of the grain of salt compared to the diameter of the plant cell?

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Let's Think:

The diameter of a plant cell is 0.00064 cm. The diameter of a grain of salt is 0.02 cm. How many times as long is the diameter of the plant cell compared to the diameter of the grain of salt?

Let's Try It:

Let's explore representing small numbers as multiples of powers of 10, and comparing numbers using powers of 10 together.

Name: ______________________ G8 U5 Lesson 11 - Let's Try It

A fast food company has sold 24,000,000,000 french fries and 400,000,000 hamburgers. Let's work to find how many times as many french fries were sold as hamburgers.

1. Represent the ratio as a fraction showing the number of french fries sold compared to the number of hamburgers sold.

It can be tricky to keep track of numbers with so many zeroes. We can use powers of 10 to help us.

2. Write each value as a multiple of a power of 10. Then rewrite the ratio as a fraction.

   24,000,000,000 = 24 • ________

   400,000 = 4 • ________

3. Divide the leading factors, then divide the powers of 10.

4. Rewrite your answer in standard form.

5. There were ________ times as many french fries sold as hamburgers.

We can use a similar process to help us compare numbers that are really small. Consider this situation: A gnat weighs 0.000008 and a ladybug weighs 0.0002. Let's work to find how many times as much is the weight of the gnat compared to the weight of the ladybug.

6. Represent the ratio as a fraction showing the weight of the gnat compared to the weight of the ladybug.

7. Write each value as a multiple of a power of 10. Then rewrite the ratio as a fraction.

   0.000008 = 8 • ________

   0.0002 = 2 • ________

8. Divide the leading factors, then divide the powers of 10.

9. The weight of the gnat is ________ times as much as the weight of the ladybug.

What if we wanted to find how many times as much is the weight of the ladybug compared to the weight of the gnat?

10. Represent the ratio as a fraction showing the weight of the ladybug compared to the weight of the gnat. Write each value as a multiple of a power of 10.

11. Divide.

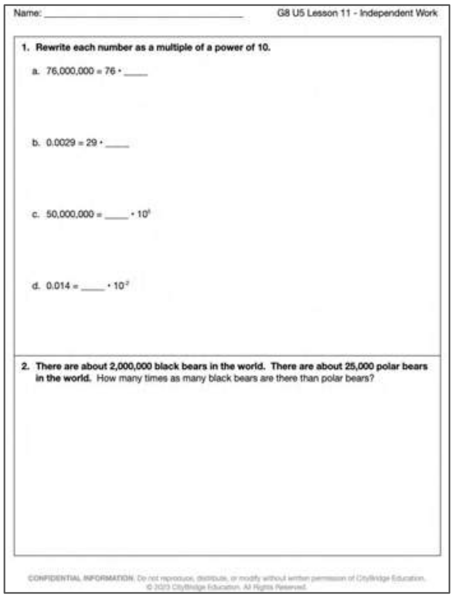
12. The weight of the ladybug is ________ times as much as the weight of the gnat.

Use what we've learned to solve one more.

13. The length of a blood cell is 0.0006 centimeters. The length of a grain of sand is 0.02 cm. How many times as long is the length of the grain of sand compared to the length of the blood cell?

On your Own:

Now it's time to represent small numbers as multiples of powers of 10, and compare numbers using powers of 10 on your own.



Name: ______________________ G8 U5 Lesson 11 - Independent Work

1. Rewrite each number as a multiple of a power of 10.

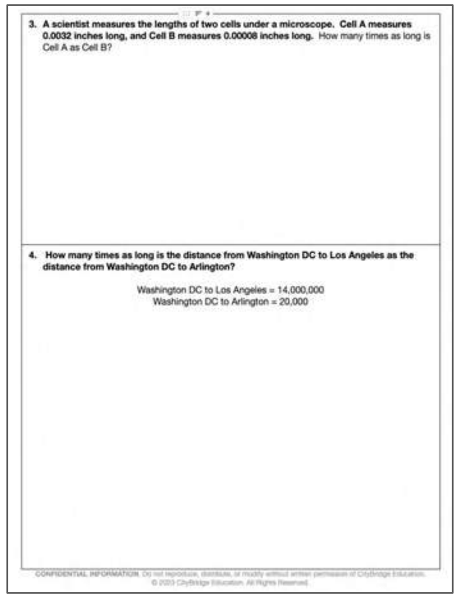
a. 76,000,000 = 76 • ____

b. 0.0029 = 29 • ____

c. 50,000,000 = ____ • 10^7

d. 0.014 = ____ • 10^{-3}

2. There are about 2,000,000 black bears in the world. There are about 25,000 polar bears in the world. How many times as many black bears are there than polar bears?



3. A scientist measures the lengths of two cells under a microscope. Cell A measures 0.0032 inches long, and Cell B measures 0.00008 inches long. How many times as long is Cell A as Cell B?

4. How many times as long is the distance from Washington DC to Los Angeles as the distance from Washington DC to Arlington?

Washington DC to Los Angeles = 14,000,000
Washington DC to Arlington = 20,000

A fast food company has sold 24,000,000,000 french fries and 400,000,000 hamburgers. Let's work to find how many times as many french fries were sold as hamburgers.

1. Represent the ratio as a fraction showing the number of french fries sold compared to the number of hamburgers sold.

It can be tricky to keep track of numbers with so many zeroes. We can use powers of 10 to help us.

2. Write each value as a multiple of a power of 10. Then rewrite the ratio as a fraction.

   24,000,000,000 = 24 • __________

   400,000,000 = 4 • __________

3. Divide the leading factors, then divide the powers of 10.

4. Rewrite your answer in standard form.

5. There were __________ times as many french fries sold as hamburgers.

We can use a similar process to help us compare numbers that are really small. Consider this situation: A gnat weighs 0.000008 and a ladybug weighs 0.0002. Let's work to find how many times as much is the weight of the gnat compared to the weight of the ladybug.

6. Represent the ratio as a fraction showing the weight of the gnat compared to the weight of the ladybug.

7. Write each value as a multiple of a power of 10. Then rewrite the ratio as a fraction.

   0.000008 = 8 • __________

   0.0002 = 2 • __________

8. Divide the leading factors, then divide the powers of 10.

9. The weight of the gnat is __________ times as much as the weight of the ladybug.

What if we wanted to find how many times as much is the weight of the ladybug compared to the weight of the gnat?

10. Represent the ratio as a fraction showing the weight of the ladybug compared to the weight of the gnat. Write each value as a multiple of a power of 10.

11. Divide.

12. The weight of the ladybug is __________ times as much as the weight of the gnat.

Use what we've learned to solve one more.

13. The length of a blood cell is 0.0006 centimeters. The length of a grain of sand is 0.02 cm. How many times as long is the length of the grain of sand compared to the length of the blood cell?

1. Rewrite each number as a multiple of a power of 10.

a. 76,000,000 = 76 • _____

b. 0.0029 = 29 • _____

c. 50,000,000 = _____ • 10^5

d. 0.014 = _____ • 10^{-2}

2. There are about 2,000,000 black bears in the world. There are about 25,000 polar bears in the world. How many times as many black bears are there than polar bears?

3. **A scientist measures the lengths of two cells under a microscope. Cell A measures 0.0032 inches long, and Cell B measures 0.00008 inches long.** How many times as long is Cell A as Cell B?

4. **How many times as long is the distance from Washington DC to Los Angeles as the distance from Washington DC to Arlington?**

Washington DC to Los Angeles = 14,000,000
Washington DC to Arlington = 20,000

Name: KEY

A fast food company has sold 24,000,000,000 french fries and 400,000,000 hamburgers. Let's work to find how many times as many french fries were sold as hamburgers.

1. Represent the ratio as a fraction showing the number of french fries sold compared to the number of hamburgers sold.

$$\frac{24{,}000{,}000{,}000}{400{,}000{,}000}$$

It can be tricky to keep track of numbers with so many zeroes. We can use powers of 10 to help us.

2. Write each value as a multiple of a power of 10. Then rewrite the ratio as a fraction.

   24,000,000,000 = 24 • 10^9

   ~~400,000~~ 400,000,000 = 4 • 10^8

3. Divide the leading factors, then divide the powers of 10.

$$\frac{24}{4} \times \frac{10^9}{10^8} \qquad 6 \times 10^1$$

4. Rewrite your answer in standard form.

   60

5. There were 60 times as many french fries sold as hamburgers.

We can use a similar process to help us compare numbers that are really small. Consider this situation: A gnat weighs 0.000008 and a ladybug weighs 0.0002. Let's work to find how many times as much is the weight of the gnat compared to the weight of the ladybug.

6. Represent the ratio as a fraction showing the weight of the gnat compared to the weight of the ladybug.

$$\frac{0.000008}{0.0002}$$

7. Write each value as a multiple of a power of 10. Then rewrite the ratio as a fraction.

0.000008 = 8 • 10^{-6}

0.0002 = 2 • 10^{-4}

8. Divide the leading factors, then divide the powers of 10.

$\frac{8}{2} \times \frac{10^{-6}}{10^{-4}}$ 4×10^{-2}

9. The weight of the gnat is 0.04 times as much as the weight of the ladybug.

What if we wanted to find how many times as much is the weight of the ladybug compared to the weight of the gnat?

10. Represent the ratio as a fraction showing the weight of the ladybug compared to the weight of the gnat. Write each value as a multiple of a power of 10.

$\frac{2 \times 10^{-4}}{8 \times 10^{-6}}$

11. Divide.

$\frac{1}{4} \times 10^{2}$ $0.25 \times 10^{2} = 25$

12. The weight of the ladybug is 25 times as much as the weight of the gnat.

Use what we've learned to solve one more.

13. The length of a blood cell is 0.0006 centimeters. The length of a grain of sand is 0.02 cm. How many times as long is the length of the grain of sand compared to the length of the blood cell?

$\frac{0.02}{0.0006} \rightarrow \frac{2 \times 10^{-2}}{6 \times 10^{-4}}$ $\frac{2}{6} \times \frac{10^{-2}}{10^{-4}}$

$\approx 0.33 \times 10^{2}$

≈ 33

Name: KEY

1. Rewrite each number as a multiple of a power of 10.

a. 76,000,000 = 76 • 10^6

b. 0.0029 = 29 • 10^{-4}

c. 50,000,000 = 500 • 10^5

d. 0.014 = 1.4 • 10^{-2}

2. There are about 2,000,000 black bears in the world. There are about 25,000 polar bears in the world. How many times as many black bears are there than polar bears?

$\frac{2\,000\,000}{25\,000} \rightarrow \frac{2 \times 10^6}{2.5 \times 10^4}$

$\frac{2}{2.5} \times \frac{10^6}{10^4}$

0.8×10^2

80

3. **A scientist measures the lengths of two cells under a microscope. Cell A measures 0.0032 inches long, and Cell B measures 0.00008 inches long.** How many times as long is Cell A as Cell B?

$$\frac{0.0032}{0.00008} \rightarrow \frac{32 \times 10^{-4}}{8 \times 10^{-5}} \qquad \frac{32}{8} \times \frac{10^{-4}}{10^{-5}}$$

$$4 \times 10^{1}$$

$$\boxed{40}$$

4. **How many times as long is the distance from Washington DC to Los Angeles as the distance from Washington DC to Arlington?**

Washington DC to Los Angeles = 14,000,000
Washington DC to Arlington = 20,000

$$\frac{140 \times 10^{5}}{20 \times 10^{3}} \rightarrow \frac{140}{20} \times \frac{10^{5}}{10^{3}}$$

$$7 \times 10^{2}$$

$$\boxed{700}$$

G8 U5 Lesson 12
Use exponent rules and powers of 10 to solve problems in context.

CITYTUTOR DC

G8 U5 Lesson 12 - Students will use exponent rules and powers of 10 to solve problems in context.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, our goal is to use exponent rules and powers of 10 to solve real-world problems. We've actually done a lot of this already in previous lessons, so today might not feel brand new. One thing we'll focus on specifically is how we can use estimation to find approximate answers when working with exponents and powers of 10. When, either in math class or the world around us, might it be useful to use an approximation or an estimate rather than an exact answer? Possible Student Answers, Key Points:

- We estimate if we need to get a quick idea. Usually when we estimate we use compatible, or friendly, numbers that are easy to calculate with.
- If I'm setting up for a party, I might use an estimation to know how many cake slices I need and how many balloons to order. The numbers don't have to be exact in situations like that.

Let's use approximations to help us think about applying exponent rules and powers of 10 to the world around us.

Let's Talk (Slide 3): Consider the following scenario. Your teacher asks you to solve the problem shown here. They want to know the approximate value. Which path would you take, and why? Possible Student Answers, Key Points:

- Since the teacher asked for an approximation, I'd choose the path on the right. The actual dividend is close to 600,000 and the actual divisor is close to 20,000. The numbers would be easy to work with in my head.
- I might choose the left hand one if I wanted to be really accurate.

Either path would get you to an answer that the teacher would likely find acceptable, but if they just asked for an approximate answer, the path on the right would be most efficient. The right hand path used approximations of the dividend and divisor that were compatible, or friendly to work with. It's much easier to think about 600,000 divided by 20,000 than to find the exact answer.

Today, we'll estimate like this to help us find approximations of answers. Let's look at our first problem.

Let's Think (Slide 4): *(read the problem aloud once all the way through)* This is asking us how many times more brain cells are in the human's brain than the mosquito's brain.

$$\frac{161{,}654{,}209{,}118}{244{,}975}$$

We've tackled many comparison problems like this before. We can start by setting up a ratio comparing the number of cells in the human brain to the number of cells in the mosquito's brain. *(write ratio with the human brain cell value in the numerator and the mosquito brain cell value in the denominator)*

Is this problem asking us for an exact answer or an estimate, and how do you know? Possible Student Answers, Key Points:

- It's asking for an estimate.
- We'll estimate, because it says "approximately how many times" in the problem.

$$\frac{160{,}000{,}000{,}000}{200{,}000}$$

Since we're being asked for an approximation, we can think of compatible numbers that are relatively close to the original numbers. I'll use 160,000,000,000 and 200,000 because they're not too far from the actual numbers and their leading digits will be easy to manipulate. *(rewrite ratio)*

Now let's compare. All these zeros can be a bit tricky to look at and work with. How can I write each of these numbers using a power of 10 that will be easy to divide with? Possible Student Answers, Key Points:

- 160,000,000,000 is 16 times 10 to the 10 power. I know because if I shift the digits in 16 ten places values left, I end up with 160,000,000,000.
- 200,000 is 2 times 10 to the 5 power. I know, because I can shift 2 to the left five place values and end up with 200,000.

$$\frac{16 \times 10^{10}}{2 \times 10^{5}}$$

(write ratio with powers of 10 as shown or as student stated if they chose another form with powers of 10 that could also easily be divided) We can now divide the leading factors and the powers of 10 in isolation. *(highlight or circle leading factors and powers of 10 as shown)*

$$8 \times 10^{5}$$

$$800{,}000$$

I quickly know that 16 divided by 2 is 8. I also can quickly determine that 10 to the 10 power divided by 10 to the 5 power is 10 to the 5 power, because I can subtract the exponents. We end up with 8 times 10 to the 5 power. *(write that)* What is the value of our answer in standard form? (800,000) So we can say that the human brain has *about* 800,000 times more cells than the mosquito's brain.

Since the teacher asked us to find an approximate answer, we were able to use compatible numbers to more efficiently arrive at an answer that's close to what the actual answer would be.

Why might we find it helpful or unhelpful to find an estimate in a given problem like this? Possible Student Answers, Key Points:

- It's helpful, because we can more efficiently arrive at a general sense of what the answer should be. We don't have to calculate as precisely.
- It might be unhelpful, because we don't get an exact answer. If the situation required precision, an estimate might be misleading or less helpful.

Let's Try it (Slides 5 - 6): Now is our chance to collaborate on a few more problems. Most problems today will ask us to approximate. We can look for that word or similar language like "about how much" as clues to tell us a question does not require an exact answer. When estimating using powers of 10, we want to find leading factors that are easy to calculate in our heads. Sometimes that means changing one value or both values, depending on the numbers in the problem. We'll keep these estimation strategies in mind as we work. After these examples, you'll get to try some out independently.

WARM WELCOME

Today we will use exponent rules and powers of 10 to solve problems in context.

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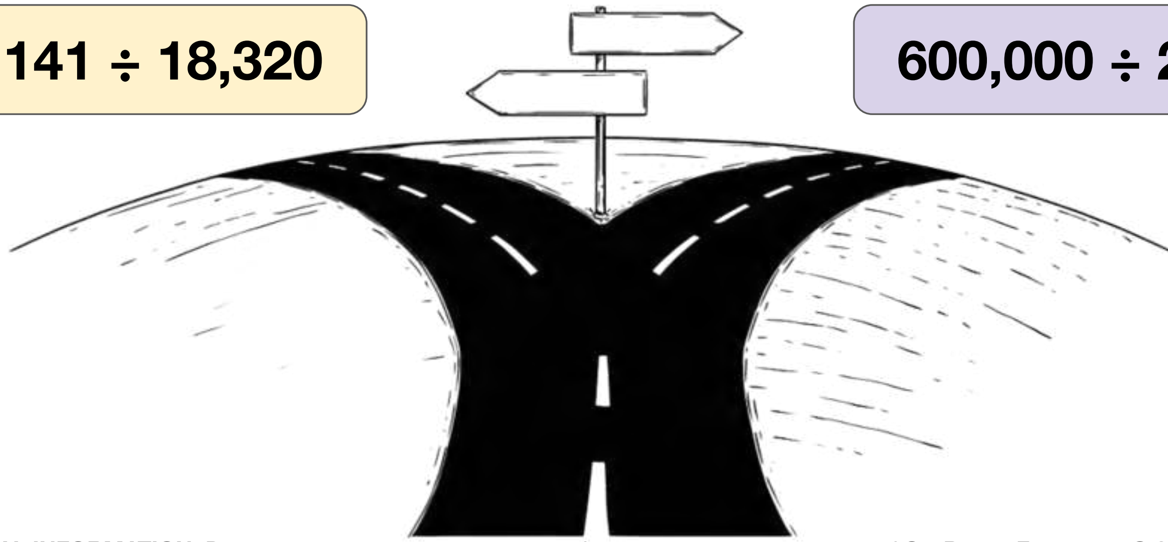
Let's Talk:

Your teacher asks to know the approximate value of 589,141 ÷ 18,320.
Which path will you take?

589,141 ÷ 18,320



600,000 ÷ 20,000

Let's Think:

A human's brain is composed of 161,654,209,118 cells. A mosquito's brain is composed of 244,975 cells.

Approximately how many times more brain cells are in the human's brain than the mosquito's brain?

Let's Try It:

Let's explore using exponent rules and powers of 10 to solve problems in context together.

Name: ______________________ G8 U5 Lesson 12 - Let's Try It

The population of Country A is 31,499. The population of Country B is 940,572. We'll work to determine *about* how many times greater is the population of Country B compared to the population of Country A.

1. The population of _______________ is greater than the population of _______________.
2. What are we trying to find out?
   a. The total population of both countries
   b. Approximately how many times greater the population of Country A is compared to the population of Country B
3. Write a ratio in fraction form comparing the population of Country B to the population of Country A.

We were asked to find an *approximation*, so our answer does not have to be exact.

4. What is a reasonable estimate for the population of Country A?
   a. 30,000
   b. 40,000
   c. 3,000
5. What is a reasonable estimate for the population of Country B?
   a. 9,000,000
   b. 900,000
   c. 90,000
6. Rewrite the reasonable estimate for the population of Country A using a power of 10.
7. Rewrite the reasonable estimate for the population of Country B using a power of 10.
8. Rewrite the ratio. Divide the leading factors, and then divide the powers of 10.
9. The population of Country B is ________ times greater than the population of Country A.

The population of two different states is shown in the table. Approximately how many times greater is the population of Florida than the population of Wyoming?

STATE	POPULATION
Florida	22,248,117
Wyoming	584,057

10. Which state has the greater population?
11. What are we trying to find out?
    a. Approximately how many times greater the population of Florida is compared to the population of Wyoming
    b. The total population of both countries
12. Write a ratio in fraction form to compare the population of Florida to the population of Wyoming.
13. Rewrite the ratio using approximations of each population to make the values easier to work with.
14. Rewrite the ratio by writing the approximations using powers of 10.
15. Divide the leading factors. Divide the powers of 10.
16. The population of Florida is approximately ________ times greater than the population of Wyoming.

On your Own:

Now it's time to use exponent rules and powers of 10 to solve problems in context on your own.

Name: ______________________ G8 U5 Lesson 12 - Independent Work

1. **Wyatt started working at his company many years ago. His starting salary was $38,976. It's many years later, and Wyatt is now CEO of the company. His CEO salary is $20,840,119.** Approximately how many times greater is Wyatt's CEO salary than his starting salary?

2. **Approximately how many times greater is the population of India than the population of Monaco?**

   India's Population: 1,428,627,663
   Monaco's Population: 36,642

3. **About how many times greater is the number of people who own a car worldwide compared to the number of people that own a car in the United States?**

   Car Owners Worldwide: 1,206,555,000
   Car Owners in United States: 283,948,000

4. **The United States grew 32,007,116 roses and 94,785,410 tulips in 2023.** About how many times greater was the number of roses produced in the United States compared to the number of tulips produced?

The population of Country A is 31,499. The population of Country B is 940,572. We'll work to determine *about* how many times greater is the population of Country B compared to the population of Country A.

1. The population of __________________ is greater than the population of ________________.

2. What are we trying to find out?
   a. The total population of both countries
   b. Approximately how many times greater the population of Country A is compared to the population of Country B

3. Write a ratio in fraction form comparing the population of Country B to the population of Country A.

We were asked to find an *approximation*, so our answer does not have to be exact.

4. What is a reasonable estimate for the population of Country A?
   a. 30,000
   b. 40,000
   c. 3,000

5. What is a reasonable estimate for the population of Country B?
   a. 9,000,000
   b. 900,000
   c. 90,000

6. Rewrite the reasonable estimate for the population of Country A using a power of 10.

7. Rewrite the reasonable estimate for the population of Country B using a power of 10.

8. Rewrite the ratio. Divide the leading factors, and then divide the powers of 10.

9. The population of Country B is _________ times greater than the population of Country A.

The population of two different states is shown in the table. Approximately how many times greater is the population of Florida than the population of Wyoming?

STATE	POPULATION
Florida	22,248,117
Wyoming	584,057

10. Which state has the greater population?

11. What are we trying to find out?
    a. Approximately how many times greater the population of Florida is compared to the population of Wyoming
    b. The total population of both countries

12. Write a ratio in fraction form to compare the population of Florida to the population of Wyoming.

13. Rewrite the ratio using approximations of each population to make the values easier to work with.

14. Rewrite the ratio by writing the approximations using powers of 10.

15. Divide the leading factors. Divide the powers of 10.

16. The population of Florida is approximately __________ times greater than the population of Wyoming.

1. **Wyatt started working at his company many years ago. His starting salary was $38,976. It's many years later, and Wyatt is now CEO of the company. His CEO salary is $20,840,119.** Approximately how many times greater is Wyatt's CEO salary than his starting salary?

2. **Approximately how many times greater is the population of India than the population of Monaco?**

India's Population: 1,428,627,663
Monaco's Population: 36,642

3. **About how many times greater is the number of people who own a car worldwide compared to the number of people that own a car in the United States?**

Car Owners Worldwide: 1,206,555,000
Car Owners in United States: 283,948,000

4. **The United States grew 32,007,116 roses and 94,785,410 tulips in 2023.** About how many times greater was the number of tulips produced in the United States compared to the number of roses produced?

Name: KEY

The population of Country A is 31,499. The population of Country B is 940,572. We'll work to determine *about* how many times greater is the population of Country B compared to the population of Country A.

1. The population of Country B is greater than the population of Country A.

2. What are we trying to find out?
   a. The total population of both countries
   (b.) Approximately how many times greater the population of Country A is compared to the population of Country B

3. Write a ratio in fraction form comparing the population of Country B to the population of Country A. $\frac{940,572}{31,499}$

We were asked to find an *approximation*, so our answer does not have to be exact.

4. What is a reasonable estimate for the population of Country A?
   (a. 30,000)
   b. 40,000
   c. 3,000

5. What is a reasonable estimate for the population of Country B?
   a. 9,000,000
   (b. 900,000)
   c. 90,000

6. Rewrite the reasonable estimate for the population of Country A using a power of 10.

   3×10^4

7. Rewrite the reasonable estimate for the population of Country B using a power of 10.

   9×10^5

8. Rewrite the ratio. Divide the leading factors, and then divide the powers of 10.

   $\frac{9 \times 10^5}{3 \times 10^4}$ $\frac{9}{3} \times \frac{10^5}{10^4}$ 3×10^1

9. The population of Country B is 30 times greater than the population of Country A.

The population of two different states is shown in the table. Approximately how many times greater is the population of Florida than the population of Wyoming?

STATE	POPULATION
Florida	22,248,117
Wyoming	584,057

10. Which state has the greater population?

Florida

11. What are we trying to find out?
   a. Approximately how many times greater the population of Florida is compared to the population of Wyoming
   b. The total population of both countries

12. Write a ratio in fraction form to compare the population of Florida to the population of Wyoming.

$$\frac{22,248,117}{584,057}$$

13. Rewrite the ratio using approximations of each population to make the values easier to work with.

$$\frac{24,000,000}{600,000}$$

14. Rewrite the ratio by writing the approximations using powers of 10.

$$\frac{24 \times 10^6}{6 \times 10^5}$$

15. Divide the leading factors. Divide the powers of 10.

4×10^1

16. The population of Florida is approximately __40__ times greater than the population of Wyoming.

Name: KEY

1. **Wyatt started working at his company many years ago. His starting salary was $38,976. It's many years later, and Wyatt is now CEO of the company. His CEO salary is $20,840,119.** Approximately how many times greater is Wyatt's CEO salary than his starting salary?

$$\frac{20,840,119}{38,976} \approx \frac{20,000,000}{40,000}$$

$$\frac{20 \times 10^6}{4 \times 10^4} \rightarrow \frac{20}{4} \times \frac{10^6}{10^4}$$

$$5 \times 10^2$$

500

2. **Approximately how many times greater is the population of India than the population of Monaco?**

India's Population: 1,428,627,663
Monaco's Population: 36,642

$$\frac{1,428,627,663}{36,642} \approx \frac{1,500,000,000}{30,000}$$

$$\frac{15 \times 10^8}{3 \times 10^4} \rightarrow \frac{15}{3} \times \frac{10^8}{10^4}$$

$$5 \times 10^4$$

50,000

3. **About how many times greater is the number of people who own a car worldwide compared to the number of people that own a car in the United States?**

Car Owners Worldwide: 1,206,555,000 → 1,200,000,000
Car Owners in United States: 283,948,000 → 300,000,000

$$\frac{12 \times 10^8}{3 \times 10^8}$$

$$\frac{12}{3} \times \frac{10^8}{10^8}$$

$4 \times 1 = 4$

4. **The United States grew 32,007,116 roses and 94,785,410 tulips in 2023.** About how many times greater was the number of ~~roses~~ tulips produced in the United States compared to the number of ~~tulips~~ roses produced?

≈ 30,000,000

≈ 90,000,000

$$\frac{9 \times 10^7}{3 \times 10^7}$$

$$\frac{9}{3} \times \frac{10^7}{10^7}$$

$3 \times 1 = 3$

G8 U5 Lesson 13
Identify numbers written in scientific notation, and rewrite numbers written in different forms in scientific notation.

CITYTUTOR DC

G8 U5 Lesson 13 - Students will identify numbers written in scientific notation, and rewrite numbers written in different forms in scientific notation.

Warm Welcome (Slide 1): Tutor choice

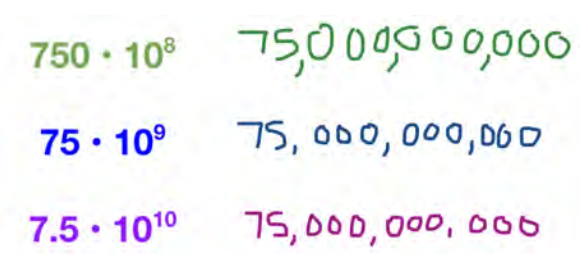
Frame the Learning/Connect to Prior Learning (Slide 2): We've been working with exponents and powers of 10 for several lessons. More recently we've been applying this thinking to everyday contexts, including using approximations of larger and small numbers. Scientists have to work with large and small numbers often, and so thousands of years ago scientists developed a uniform, agreed-upon way to represent them called scientific notation. We'll learn more about scientific notation in a few moments. Before we do, why do you think it's important for mathematicians, scientists, and other experts to have a uniform way of representing large and small numbers? Possible Student Answers, Key Points:

- It can make reading and writing the numbers simpler.
- There are many ways to represent numbers, as we've seen with exponents and powers of 10, so it might help them compare and discuss numbers if they're represented in a familiar way.

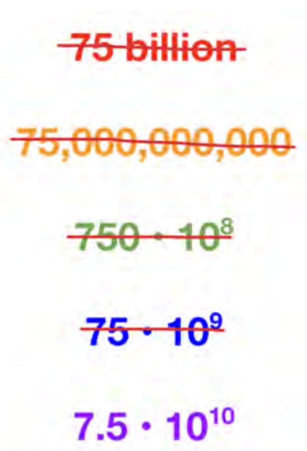
Interesting ideas. Let's keep that thinking in mind, and jump into a quick warm-up thinking exercise.

Let's Talk (Slide 3): Take a look at the numbers shown here. What's the same? What's different? Possible Student Answers, Key Points:

- They all start with 7 and 5. They all represent the same amount.
- One uses words. One is in standard form. Some have exponents. They are different colors.



These are all different ways to represent the number 75 billion or 75,000,000,000. Just to be certain, let's write the last three numbers in standard form. *(write as you narrate)* 750 times 10 the 8 power means each digit will shift 8 places left. That's 75,000,000,000. 75 times 10 to the 9 power means each digit shifts 9 places left. That's 75,000,000,000. Lastly, 7.5 times 10 to the 10 power means each digit shifts 10 places left. That's, again, 75,000,000,000.



These are all valid ways to write the number, however only one of them is written in what is called scientific notation. In order for a number to be written in scientific notation, it must be written as the product of two factors. *(cross out the first two examples)*

The second factor must be a power of 10 with an integer exponent, and the first term must be greater than or equal to 1 and less than 10. *(Cross out the green and blue expressions)* These aren't in scientific notation because the leading factors are greater than 10. The last example is the only one written in scientific notation.

As we work with numbers in scientific notation, remember the criteria. The number must be a written as a product of two factors. The first factor must be greater than or equal to 1 and less than 10, and the second factor must be an integer power of 10.

Let's Think (Slide 4): To start off, let's take a look at the four numbers shown here in various forms. We're tasked with finding which ones are in scientific notation, and which ones are not.

a. $3.1 \cdot 10^8$ ✓
b. 0.00945
c. $9 \cdot 10^{-7}$ ✓
d. $62 \cdot 10^4$

Since we just reviewed the criteria for scientific notation, see if you can identify the two numbers here that are already in scientific notation. *(circle or put a check next to A and C as student identifies them, supporting as needed)* Possible Student Answers, Key Points:

- I know A is in scientific notation. 3.1 is greater than or equal to 1, it's less than 10, and the other factor is an integer power of 10.
- I know C is in scientific notation. 9 is greater than or equal to 1, it's less than 10, and the other factor is an integer power of 10.

Answers A and C are already in scientific notation. Let's focus on the other two numbers to think about why they are not in scientific notation, and what we can do to get them there. Let's start with B. Why is B not in scientific notation? Possible Student Answers, Key Points:

- B is not in scientific notation, because it's not written as two factors. In scientific notation, the leading factor should also be greater than or equal to 1, which 0.00945 is not.
- B is not in scientific notation, because it does not use a power of 10.

To write 0.00945 in scientific notation, we'll start by shifting the digits to produce a leading factor that is 1 or more, but less than 10. If we shift the digits three places left, we end up with 9.45, which meets our criteria.

b. 0.00945 9.45×10^{-3}

(use arrows to show shifting) Some people might also think of this shifting as moving the decimal three place values right. Either way, we end up with a leading factor of 9.45. Since we shifted 3 place values left, or moved the decimal three place values right, we can write this as 9.45 times 10 to the -3 power. *(write scientific notation)*

What about D? Why is D not in scientific notation? Possible Student Answers, Key Points:

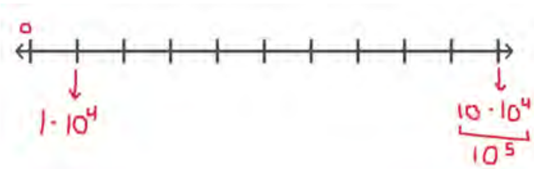
- At first glance it almost looks like it is in scientific notation, because it has two factors, one of which is an integer power of 10.
- It's not in scientific notation, because 62 is not between 1 and 10.

Let's shift digits so that the 62 becomes 6.2 This will also impact the power of 10. I can shift digits right one place value, or I can think of this change as moving the decimal left one place value. *(use arrows to show the shift)* Either way, we now have a leading factor of 6.2. Since we shifted our digits right one place, we'll increase the exponent by 1 to make sure the value of the entire number remains constant. *(write scientific notation)*

d. $62 \cdot 10^4$ 6.2×10^5

We just determined whether a set of numbers were in scientific notation or not. Those that weren't, we were able to use our understanding of powers of 10 to write them in scientific notation. We know that to be in scientific notation, the numbers must have a leading factor between 1 and 10 (inclusive of 1) and an integer power of 10.

Let's Think (Slide 5): One of the benefits of using scientific notation is that it can quickly tell us what place value the leading factor is in. This problem will help us see that. It wants us to use the number line to determine which two powers of 10 this number is between. We'll then plot the location of our number on the line.



I'll start by marking the first tick mark as 0, just to give us a starting point. *(label 0 at the first tick mark)* Our number is 8.5 times 10 to the 4 power. That means our number is in between 1 x 10 to the 4 power and 10 times 10 to the 4 power *(label the second tick mark and last tick mark accordingly)* Another way to think of 10 times 10 to the 4 power is as 10 to the 5 power. If you weren't sure about that, you could picture what 10 times 10 to the 4 power would look like expanded. *(use bracket to show that 10 times 10 to the 4 power is the same as 10 to the 5 power)*

We know that our number is somewhere between these two values. Essentially, our number is between 10,000 and 100,000. Let's count each tick mark to determine where to place 8.5 times 10 to the 4 power. The first tick mark is 1 times 10 to the 4 power. *(point to each tick mark as you count)* The next tick mark would be 2 times 10 to the 4 power, then 3 times 10 to the 4 power, then 4 times 10 to the 4 power, then 5 times 10 to the 4 power, then 6 times 10 to the 4 power, then 7 times 10 to the 4 power, then 8 times 10 to the 4 power, then 9 times 10 to the 4 power, and then our last tick mark is 10 times 10 to the 4 power.

The leading factor of 8.5 means that our number will between 8 and 9 times 10 to the 4 power. Since it's 8.5, I'll put a point directly between those two tick marks. *(mark a point as shown)* We were just able to use a number line to mark the location of a number in scientific notation between two powers of 10.

To recap a major takeaway from today, what criteria mean a number is written in scientific notation? Possible Student Answers, Key Points:

- It must be written as the product of two factors.
- One factor must be greater than or equal to 1, but less than 10. The other factor must be an integer power of 10.

Let's Try it (Slides 6 - 7): We'll practice some more together, and then you'll get a chance to show what you know about scientific notation independently. Scientific notation is helpful because it can quickly show us the place value of a large or small number. When we consider whether a number is in scientific notation or not, we'll want to identify a leading factor that is 1 or more, but less than 10. We'll also want to make sure the second factor is an integer power of 10. We've practiced a lot already, so I know you're ready to try some more scientific notation work. Let's go for it!

WARM WELCOME

Today we will identify numbers written in scientific notation, and rewrite numbers written in different forms in scientific notation.

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Let's Talk:

What's the same? What's different?

75 billion

75,000,000,000

$750 \bullet 10^8$

$75 \bullet 10^9$

$7.5 \bullet 10^{10}$

Let's Think:

Is each number in scientific notation? If not, write it in scientific notation.

a. $3.1 \bullet 10^8$

b. 0.00945

c. $9 \bullet 10^{-7}$

d. $62 \bullet 10^4$

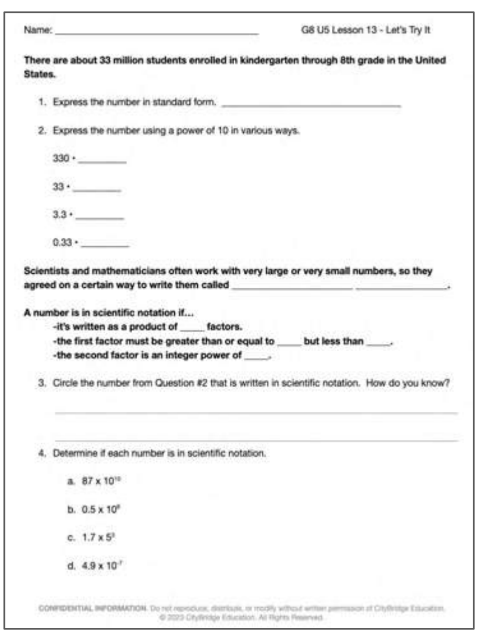
Let's Think:

Which two powers of 10 is the number between? Plot the number on the number line.

8.5×10^4

Let's Try It:

Let's explore identifying numbers written in scientific notation, and rewriting numbers written in different forms in scientific notation together.



Name: ______________________ G8 U5 Lesson 13 - Let's Try It

There are about 33 million students enrolled in kindergarten through 8th grade in the United States.

1. Express the number in standard form. ______________________
2. Express the number using a power of 10 in various ways.

   330 • ________

   33 • ________

   3.3 • ________

   0.33 • ________

Scientists and mathematicians often work with very large or very small numbers, so they agreed on a certain way to write them called ______________ ______________.

A number is in scientific notation if...
-it's written as a product of _____ factors.
-the first factor must be greater than or equal to _____ but less than _____.
-the second factor is an integer power of _____.

3. Circle the number from Question #2 that is written in scientific notation. How do you know?

4. Determine if each number is in scientific notation.

   a. $87 \times 10^{[illegible]}$

   b. $0.5 \times 10^{[illegible]}$

   c. 1.7×5^3

   d. 4.9×10^{-7}

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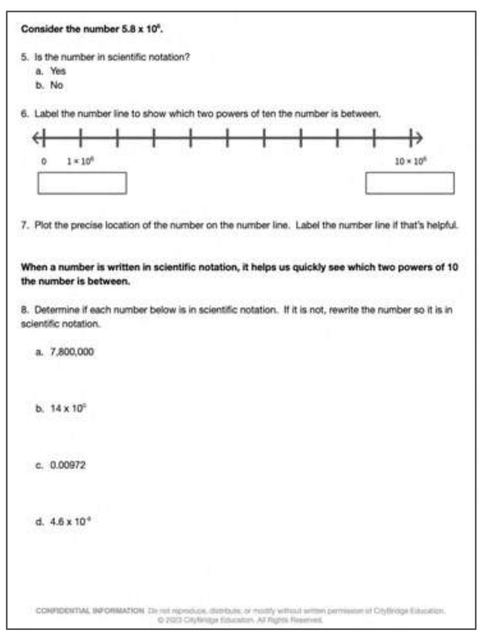

Consider the number 5.8×10^6.

5. Is the number in scientific notation?
   a. Yes
   b. No
6. Label the number line to show which two powers of ten the number is between.

   0 1×10^6 10×10^6

7. Plot the precise location of the number on the number line. Label the number line if that's helpful.

When a number is written in scientific notation, it helps us quickly see which two powers of 10 the number is between.

8. Determine if each number below is in scientific notation. If it is not, rewrite the number so it is in scientific notation.

   a. 7,800,000

   b. $14 \times 10^{[illegible]}$

   c. 0.00972

   d. $4.6 \times 10^{[illegible]}$

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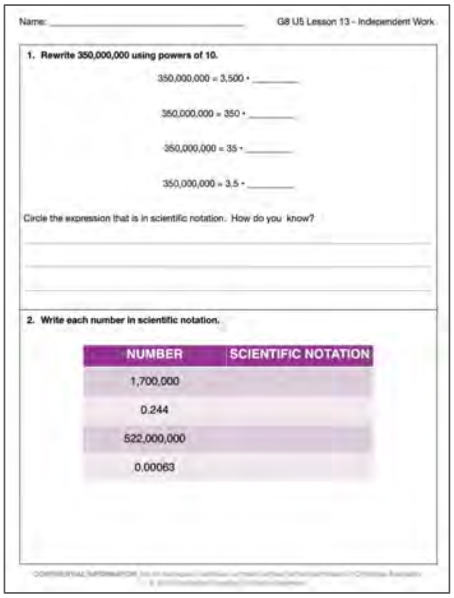
On your Own:

Now it's time to identify numbers written in scientific notation, and rewrite numbers written in different forms in scientific notation on your own.


Name: ____________________ G8 U5 Lesson 13 - Independent Work

1. Rewrite 350,000,000 using powers of 10.

350,000,000 = 3,500 • ________

350,000,000 = 350 • ________

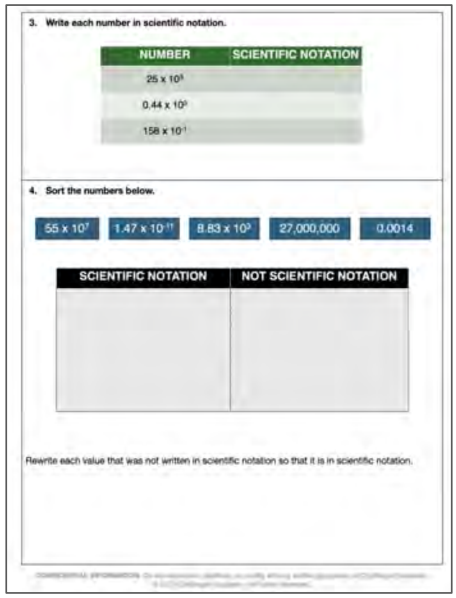
350,000,000 = 35 • ________

350,000,000 = 3.5 • ________

Circle the expression that is in scientific notation. How do you know?

2. Write each number in scientific notation.

NUMBER	SCIENTIFIC NOTATION
1,700,000	
0.244	
522,000,000	
0.00063	


3. Write each number in scientific notation.

NUMBER	SCIENTIFIC NOTATION
25×10^{5}	
0.44×10^{9}	
158×10^{-1}	

4. Sort the numbers below.

55×10^{7} 1.47×10^{-11} 8.83×10^{3} 27,000,000 0.0014

SCIENTIFIC NOTATION	NOT SCIENTIFIC NOTATION

Rewrite each value that was not written in scientific notation so that it is in scientific notation.

There are about 33 million students enrolled in kindergarten through 8th grade in the United States.

1. Express the number in standard form. ____________________________________

2. Express the number using a power of 10 in various ways.

   330 • __________

   33 • __________

   3.3 • __________

   0.33 • __________

Scientists and mathematicians often work with very large or very small numbers, so they agreed on a certain way to write them called ________________________ __________________.

A number is in scientific notation if…

-it's written as a product of _____ factors.

-the first factor must be greater than or equal to _____ but less than _____.

-the second factor is an integer power of _____.

3. Circle the number from Question #2 that is written in scientific notation. How do you know?

   ____

   ____

4. Determine if each number is in scientific notation.

   a. 87×10^{10}

   b. 0.5×10^{6}

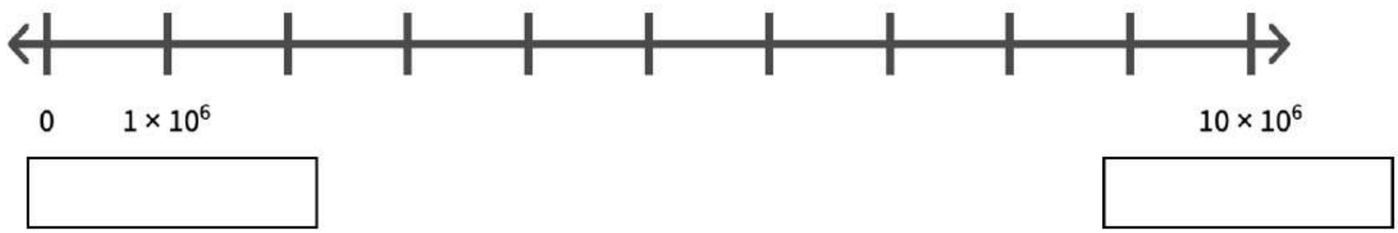
   c. 1.7×5^{3}

   d. 4.9×10^{-7}

Consider the number 5.8 x 10^6.

5. Is the number in scientific notation?
   a. Yes
   b. No

6. Label the number line to show which two powers of ten the number is between.



7. Plot the precise location of the number on the number line. Label the number line if that's helpful.

When a number is written in scientific notation, it helps us quickly see which two powers of 10 the number is between.

8. Determine if each number below is in scientific notation. If it is not, rewrite the number so it is in scientific notation.

   a. 7,800,000

   b. 14 x 10^0

   c. 0.00972

   d. 4.6 x 10^{-9}

1. Rewrite 350,000,000 using powers of 10.

350,000,000 = 3,500 • __________

350,000,000 = 350 • __________

350,000,000 = 35 • __________

350,000,000 = 3.5 • __________

Circle the expression that is in scientific notation. How do you know?

____

____

____

2. Write each number in scientific notation.

NUMBER	SCIENTIFIC NOTATION
1,700,000	
0.244	
522,000,000	
0.00063	

3. Write each number in scientific notation.

NUMBER	SCIENTIFIC NOTATION
25×10^3	
0.44×10^3	
158×10^{-1}	

4. Sort the numbers below.

55×10^7 | 1.47×10^{-11} | 8.83×10^3 | 27,000,000 | 0.0014

SCIENTIFIC NOTATION	NOT SCIENTIFIC NOTATION

Rewrite each value that was not written in scientific notation so that it is in scientific notation.

Name: KEY

There are about 33 million students enrolled in kindergarten through 8th grade in the United States.

1. Express the number in standard form. 33,000,000

2. Express the number using a power of 10 in various ways.

$330 \cdot 10^5$

$33 \cdot 10^6$

$3.3 \cdot 10^7$ (circled)

$0.33 \cdot 10^8$

Scientists and mathematicians often work with very large or very small numbers, so they agreed on a certain way to write them called scientific notation.

A number is in scientific notation if...

-it's written as a product of 2 **factors.**

-the first factor must be greater than or equal to 1 **but less than** 10**.**

-the second factor is an integer power of 10**.**

3. Circle the number from Question #2 that is written in scientific notation. How do you know?

3.3 is the only leading factor between 1 (inclusive) and 10.

4. Determine if each number is in scientific notation.

a. 87×10^{10} NO

b. 0.5×10^{6} NO

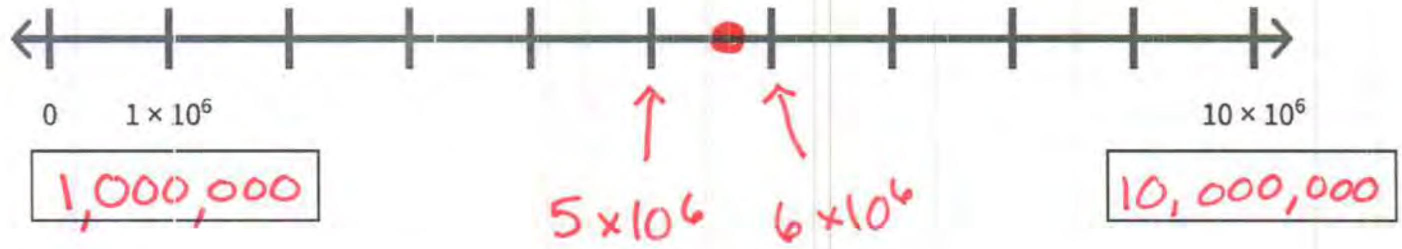
c. 1.7×5^{3} NO

d. 4.9×10^{-7} YES!

Consider the number 5.8×10^6.

5. Is the number in scientific notation?
   a. Yes
   b. No

6. Label the number line to show which two powers of ten the number is between.



7. Plot the precise location of the number on the number line. Label the number line if that's helpful.

When a number is written in scientific notation, it helps us quickly see which two powers of 10 the number is between.

8. Determine if each number below is in scientific notation. If it is not, rewrite the number so it is in scientific notation.

   a. 7,800,000
   NO 7.8×10^6

   b. 14×10^0
   NO 1.4×10^1

   c. 0.00972
   NO 9.72×10^{-3}

   d. 4.6×10^{-9}
   ↖ in scientific notation

Name: KEY

1. Rewrite 350,000,000 using powers of 10.

350,000,000 = 3,500 • 10^5

350,000,000 = 350 • 10^6

350,000,000 = 35 • 10^7

350,000,000 = 3.5 • 10^8

Circle the expression that is in scientific notation. How do you know?

3.5 is greater than or equal to 1 and less than 10. Also, 10^8 is an integer power of 10.

2. Write each number in scientific notation.

NUMBER	SCIENTIFIC NOTATION
1,700,000	1.7×10^6
0.244	2.44×10^{-1}
522,000,000	5.22×10^8
0.00063	6.3×10^{-4}

3. Write each number in scientific notation.

NUMBER	SCIENTIFIC NOTATION
25×10^3	2.5×10^4
0.44×10^3	4.4×10^2
153×10^{-1}	1.58×10^1

4. Sort the numbers below.

55×10^7 | 1.47×10^{-11} | 8.83×10^3 | 27,000,000 | 0.0014

SCIENTIFIC NOTATION	NOT SCIENTIFIC NOTATION
1.47×10^{-11}	55×10^7
8.83×10^3	27,000,000
	0.0014

Rewrite each value that was not written in scientific notation so that it is in scientific notation.

55×10^7 ↓ 5.5×10^8

27,000,000 ↓ 2.7×10^7

0.0014 ↓ 1.4×10^{-3}

G8 U5 Lesson 14
Multiply and divide numbers in scientific notation, and use scientific notation and estimation to compare quantities.

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G8 U5 Lesson 14 - Students will multiply and divide numbers in scientific notation, and use scientific notation and estimation to compare quantities.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our previous lesson, we explored scientific notation. Scientific notation is an agreed upon method for representing large or small numbers that many professionals use. What criteria must a number meet in order to be considered in scientific notation?
Possible Student Answers, Key Points:

- It must be written as the product of two factors.
- One factor must be greater than or equal to 1, but less than 10. The other factor must be an integer power of 10.

Today we will continue our work with scientific notation. Specifically, we will multiply and divide with numbers in scientific notation and see numbers in scientific notation used in comparison contexts.

Let's Talk (Slide 3): The table below shows the number of red, blue, and yellow marbles used to make an art project. After taking a quick glance at the table, why might it be challenging to compare the values? Possible Student Answers, Key Points:

- They each have different exponents.
- Some leading factors are decimals and some are whole numbers.

What do you think we could do with the numbers to make them a bit easier to compare and work with?
Possible Student Answers, Key Points:

- We could write them all in standard form.
- We could write them all in scientific notation.
- We could get them all to have the same power of 10.

Getting each number in the same form would help us more easily compare and operate with these numbers. Let's keep that in mind as we answer some questions about the marbles in this art project.

Let's Think (Slide 4): The first problem we have wants us to compare the number of blue marbles to the number of red marbles. It wants to know *approximately* how many times more blue marbles there are than red marbles. To help us compare and calculate, let's write each in scientific notation.

I'll start with the number of red marbles. *(use arrows to show shifting)* If I shift the digits over one place left, I end up with 1.24 as a leading factor. Since I shifted the digits one place left, I'll adjust the exponent by decreasing it 1. 1.24 times 10 to the 5 power is now in scientific notation. *(rewrite number in scientific notation)*

What about the blue marbles? How can I write that number in scientific notation? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- We can shift the digits to the right two place values, so the leading factor is 2.34. Since we shifted the digits two places right, we can increase the exponent by 2.
- 234 times 10 to the 3 power can be written as 2.34 times 10 to the 5 power.

Our numbers are in scientific notation. They're in the same form. As written, the leading factors still are not really compatible or easy to work with. Since this problem allows us to estimate, I'll make them a little easier by thinking of the leading factor of the blue marbles as just 2 and the leading factor of the red marbles as just 1.

(write a ratio in fraction form with the estimated values) To compare, I'll use this ratio written in fraction form using the estimated values we found. We know that from here, we can divide the leading factors and then divide the powers of 10. What would that look like mathematically? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- The leading factors are 2 and 1. 2 divided by 1 is 2.
- The powers of 10 would cancel out. The same numerator and denominator always results in a value of 1.

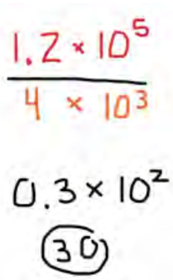
We end up with an approximate answer of 2. This means there are about 2 times as many blue marbles as red marbles. We were able to use scientific notation and estimation to help us find our approximate answer.

Let's Think (Slide 5): The next problem is similar, but now we're comparing the red marbles to the yellow marbles. Since we've already done a related example, I might ask you for a bit more support thinking through this one.

To start comparing, let's think of the yellow marbles as a number in scientific notation. How can I write 406 times 10 to the 1 power in scientific notation? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- Shift the digits in 406 two place values right. We can also think of this as shifting the decimal two places left.
- Since we shifted 406 to make it 4.06, we'll add two to the exponent to make it 10 to the 3 power.

The number of yellow marbles written in scientific notation is 4.06 times 10 to the 3. We already know that the number of red marbles in scientific notation is 1.24 times 10 to the 5 power. Since we're estimating, I'll use 4 and 1.2 as the leading factors since they're fairly compatible.



From here, we can set up a ratio to compare the red marbles to the yellow marbles. *(write as you narrate)* I'll write the approximate number of red marbles in scientific notation in the numerator and the approximate number of yellow marbles in scientific notation in the denominator. How can I divide these values? *(write as student shares)* Possible Student Answers, Key Points:

- If we divide the leading factors we can think about 1.2 divided by 4. I know 12 tenths divided by 4 is 3 tenths, or 0.3.
- We can divide the powers of 10 by subtracting the exponents. We'd end up with 10 to the 2 power.

When we divide the leading factors, we end up with 0.3. When we divide the powers of 10, we end up with 10 to the 2 power. 0.3 times 10 to the 2 power is equivalent to 30. What does that answer mean in the context of this problem? Possible Student Answers, Key Points:

- There are approximately 30 times more red marbles than yellow marbles used in the art project.

We just used estimation and scientific notation to determine that the amount of red marbles is about 30 times as much as the amount of yellow marbles. Nice work!

Let's Try it (Slides 6 - 7): It's time to collaborate on a few more problems before you have the opportunity to work independently and show what you've learned. When multiplying and dividing large or small numbers, we've seen it is helpful to write them all in scientific notation. Numbers in the same form are often easier to work with. Numbers in the same form are often easier to compare. Once we have numbers represented in scientific notation, we can efficiently use exponent rules to multiply or divide. Remember though, a number is only in scientific notation if one factor is greater than or equal to 1 and less than 10, and the other factor is an integer power of 10. Let's look at the next examples.

WARM WELCOME

Today we will multiply and divide numbers in scientific notation, and use scientific notation and estimation to compare quantities.

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Let's Talk:

Red, blue, and yellow marbles were made to create an art piece.

What makes the values difficult to compare?

COLOR	NUMBER OF MARBLES
Red	0.124×10^6
Blue	234×10^3
Yellow	406×10^1

Let's Think:

Approximately how many times more blue marbles are there than red marbles?

COLOR	NUMBER OF MARBLES
Red	0.124×10^6
Blue	234×10^3
Yellow	406×10^1

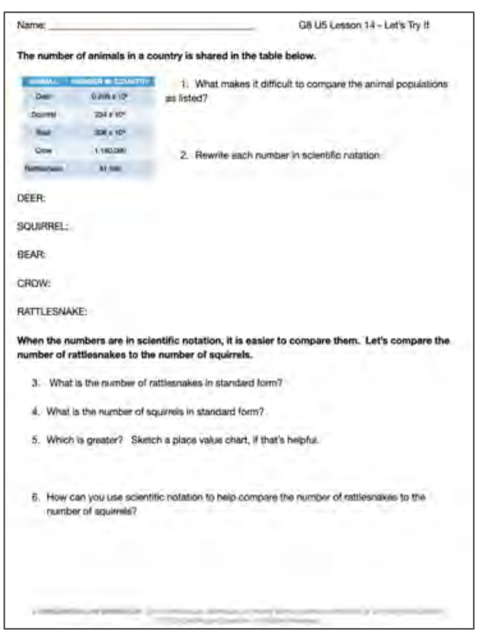
Let's Think:

Approximately how many times more red marbles are there than yellow marbles?

COLOR	NUMBER OF MARBLES
Red	0.124×10^6
Blue	234×10^3
Yellow	406×10^1

Let's Try It:

Let's explore multiplying and dividing numbers in scientific notation, and using scientific notation and estimation to compare quantities together.



Name: ____________________ G8 U5 Lesson 14 - Let's Try It

The number of animals in a country is shared in the table below.

[illegible]	[illegible]
Deer	[illegible]
Squirrel	[illegible]
Bear	[illegible]
Crow	[illegible]
Rattlesnake	[illegible]

1. What makes it difficult to compare the animal populations as listed?

2. Rewrite each number in scientific notation.

DEER:

SQUIRREL:

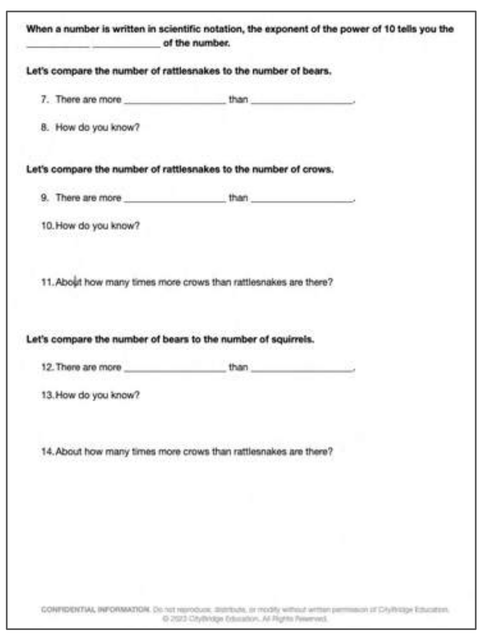
BEAR:

CROW:

RATTLESNAKE:

When the numbers are in scientific notation, it is easier to compare them. Let's compare the number of rattlesnakes to the number of squirrels.

3. What is the number of rattlesnakes in standard form?
4. What is the number of squirrels in standard form?
5. Which is greater? Sketch a place value chart, if that's helpful.
6. How can you use scientific notation to help compare the number of rattlesnakes to the number of squirrels?



When a number is written in scientific notation, the exponent of the power of 10 tells you the ____________ ____________ of the number.

Let's compare the number of rattlesnakes to the number of bears.

7. There are more ______________ than ______________.
8. How do you know?

Let's compare the number of rattlesnakes to the number of crows.

9. There are more ______________ than ______________.
10. How do you know?
11. About how many times more crows than rattlesnakes are there?

Let's compare the number of bears to the number of squirrels.

12. There are more ______________ than ______________.
13. How do you know?
14. About how many times more crows than rattlesnakes are there?

On your Own:

Now it's time to multiply and divide numbers in scientific notation, and use scientific notation and estimation to compare quantities on your own.

Name: ____________________ G8 U5 Lesson 14 - Independent Work

1. Sort the numbers below.

2.03×10^{4} | 0.00084 | $5 \times 10^{[illegible]}$ | $564.8 \times 10^{[illegible]}$

SCIENTIFIC NOTATION	NOT SCIENTIFIC NOTATION

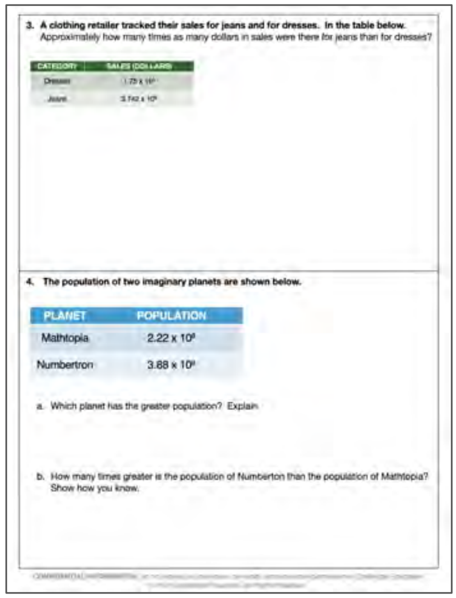
Rewrite each value that was not written in scientific notation so that it is in scientific notation.

2. Look at the table below.

a. Did Tasty Donuts or Scrumptious Donut Company sell more donuts? How do you know?

b. Did Amazing Donut Factory or Yum Yum Donuts sell more donuts? How do you know?

COMPANY	NUMBER OF DONUTS SOLD
Donut Delight	$6.1 \times 10^{[illegible]}$
Tasty Donuts	$6.3 \times 10^{[illegible]}$
Amazing Donut Factory	$3.2 \times 10^{[illegible]}$
Scrumptious Donut Company	$6.4 \times 10^{[illegible]}$
Yum Yum Donuts	$5.8 \times 10^{[illegible]}$



3. A clothing retailer tracked their sales for jeans and for dresses. In the table below. Approximately how many times as many dollars in sales were there for jeans than for dresses?

CATEGORY	SALES (DOLLARS)
Dresses	$1.75 \times 10^{[illegible]}$
Jeans	$3.742 \times 10^{[illegible]}$

4. The population of two imaginary planets are shown below.

PLANET	POPULATION
Mathtopia	$2.22 \times 10^{[illegible]}$
Numbertron	$3.88 \times 10^{[illegible]}$

a. Which planet has the greater population? Explain.

b. How many times greater is the population of Numberton than the population of Mathtopia? Show how you know.

The number of animals in a country is shared in the table below.

ANIMAL	NUMBER IN COUNTRY
Deer	0.206×10^6
Squirrel	234×10^4
Bear	506×10^3
Crow	1,160,000
Rattlesnake	41,500

1. What makes it difficult to compare the animal populations as listed?

2. Rewrite each number in scientific notation.

DEER:

SQUIRREL:

BEAR:

CROW:

RATTLESNAKE:

When the numbers are in scientific notation, it is easier to compare them. Let's compare the number of rattlesnakes to the number of squirrels.

3. What is the number of rattlesnakes in standard form?

4. What is the number of squirrels in standard form?

5. Which is greater? Sketch a place value chart, if that's helpful.

6. How can you use scientific notation to help compare the number of rattlesnakes to the number of squirrels?

When a number is written in scientific notation, the exponent of the power of 10 tells you the _____________ ______________ of the number.

Let's compare the number of rattlesnakes to the number of bears.

7. There are more _____________________ than _____________________.

8. How do you know?

Let's compare the number of rattlesnakes to the number of crows.

9. There are more _____________________ than _____________________.

10. How do you know?

11. About how many times more crows than rattlesnakes are there?

Let's compare the number of bears to the number of deer.

12. There are more _____________________ than _____________________.

13. How do you know?

14. About how many times more bear than deer are there?

1. Sort the numbers below.

2.03×10^4	0.00084	5×10^{-3}	564.8×10^8

SCIENTIFIC NOTATION	NOT SCIENTIFIC NOTATION

Rewrite each value that was not written in scientific notation so that it is in scientific notation.

2. Look at the table below.

a. Did Tasty Donuts or Scrumptious Donut Company sell more donuts? How do you know?

b. Did Amazing Donut Factory or Yum Yum Donuts sell more donuts? How do you know?

COMPANY	NUMBER OF DONUTS SOLD
Donut Delight	6.1×10^3
Tasty Donuts	8.3×10^4
Amazing Donut Factory	2.2×10^4
Scrumptious Donut Company	6.4×10^3
Yum Yum Donuts	5.9×10^4

3. A clothing retailer tracked their sales for jeans and for dresses in the table below. Approximately how many times as many dollars in sales were there for jeans than for dresses?

CATEGORY	SALES (DOLLARS)
Dresses	1.75×10^9
Jeans	3.742×10^8

4. The population of two imaginary planets are shown below.

PLANET	POPULATION
Mathtopia	2.22×10^8
Numbertron	3.88×10^9

a. Which planet has the greater population? Explain.

b. How many times greater is the population of Numberton than the population of Mathtopia? Show how you know.

Name: KEY

The number of animals in a country is shared in the table below.

ANIMAL	NUMBER IN COUNTRY
Deer	0.206×10^6
Squirrel	234×10^4
Bear	506×10^3
Crow	1,160,000
Rattlesnake	41,500

1. What makes it difficult to compare the animal populations as listed?

The numbers are in different forms.

2. Rewrite each number in scientific notation.

DEER: 2.06×10^5

SQUIRREL: 2.34×10^6

BEAR: 5.06×10^5

CROW: 1.16×10^6

RATTLESNAKE: 4.15×10^4

When the numbers are in scientific notation, it is easier to compare them. Let's compare the number of rattlesnakes to the number of squirrels.

3. What is the number of rattlesnakes in standard form? 41,500

4. What is the number of squirrels in standard form? 2,340,000

5. Which is greater? Sketch a place value chart, if that's helpful.

The number of squirrels is greater.

6. How can you use scientific notation to help compare the number of rattlesnakes to the number of squirrels?

2.34×10^6 4.15×10^4

↑ squirrels have a greater power of 10

When a number is written in scientific notation, the exponent of the power of 10 tells you the place value of the number.

Let's compare the number of rattlesnakes to the number of bears.

7. There are more bears than rattlesnakes.

8. How do you know?

$10^5 > 10^4$

Let's compare the number of rattlesnakes to the number of crows.

9. There are more crows than rattlesnakes.

10. How do you know?

$10^6 > 10^4$

11. About how many times more crows than rattlesnakes are there?

$\frac{1.2 \times 10^6}{4 \times 10^4}$ 0.3×10^2 30

Let's compare the number of bears to the number of ~~squirrels~~ deer.

12. There are more bears than deer.

13. How do you know?

5.06 > 2.06

14. About how many times more ~~crows~~ bear than ~~rattlesnakes~~ deer are there?

$\frac{5 \times 10^5}{2 \times 10^5}$

2.5×1 2.5

Name: KEY

1. Sort the numbers below.

2.03×10^4	0.00084	5×10^{-3}	564.8×10^8

SCIENTIFIC NOTATION	NOT SCIENTIFIC NOTATION
2.03×10^4	0.00084
5×10^{-3}	564.8×10^8

Rewrite each value that was not written in scientific notation so that it is in scientific notation.

0.00084 → 8.4×10^{-4}

564.8×10^8 → 5.648×10^{10}

2. Look at the table below.

a. Did Tasty Donuts or Scrumptious Donut Company sell more donuts? How do you know?

Tasty Donuts

$10^4 > 10^3$

b. Did Amazing Donut Factory or Yum Yum Donuts sell more donuts? How do you know?

Yum Yum

$5.9 > 2.2$

COMPANY	NUMBER OF DONUTS SOLD
Donut Delight	6.1×10^3
Tasty Donuts	8.3×10^4
Amazing Donut Factory	2.2×10^4
Scrumptious Donut Company	6.4×10^3
Yum Yum Donuts	5.9×10^4

3. A clothing retailer tracked their sales for jeans and for dresses. In the table below.
Approximately how many times as many dollars in sales were there for jeans than for dresses?

CATEGORY	SALES (DOLLARS)
Dresses	1.75×10^9
Jeans	3.742×10^8

$\frac{3.742 \times 10^8}{1.75 \times 10^9}$

$\frac{4 \times 10^8}{2 \times 10^9}$

2×10^{-1}

0.2

4. The population of two imaginary planets are shown below.

PLANET	POPULATION
Mathtopia	2.22×10^8
Numbertron	3.88×10^9

a. Which planet has the greater population? Explain.

Numbertron has more people.
$10^9 > 10^8$

b. How many times greater is the population of Numberton than the population of Mathtopia? Show how you know.

$\frac{4 \times 10^9}{2 \times 10^8} \rightarrow 2 \times 10^1 \rightarrow$ 20

G8 U5 Lesson 15
Add and subtract numbers in scientific notation.

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G8 U5 Lesson 15 - Students will add and subtract numbers in scientific notation.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today is our final lesson in our unit all about exponents and scientific notation. We've learned a lot. What important things stand out to you from all of our lessons so far? Possible Student Answers, Key Points:

- Writing exponential expressions in expanded form can often help us see and manipulate all the factors.
- When we multiply factors with the same base, we can add the exponents. When we divide factors with the same base, we can subtract the exponents. When we take a power to a power, we can multiply the exponents. A negative exponent is like repeated multiplication of the reciprocal of the base.
- Exponents and powers of 10 can help us write really big and really small numbers.
- A number is in scientific notation if the first factor is greater than or equal to 1, but less than 10, and the second factor is an integer power of 10.

We have really learned a lot. For our final lesson of this unit, we'll focus on scientific notation. Today, we'll add and subtract numbers in scientific notation.

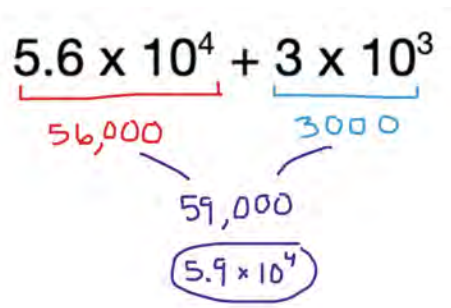
Let's Talk (Slide 3): Take a look at the two expressions here. They're both written in unit form. Which problem do you feel is more efficient to add, and why? Possible Student Answers, Key Points:

- I think the first one is more efficient to add, because it's easy to add like units.
- The first example adds thousands plus thousands. That's easy mental math for me. It's at least easier to do in my head than adding thousands and ten thousands.

The first example is easy to mentally compute. 67 thousands plus 4 more thousands is just 71 thousands.

When adding and subtracting, it's often easier to manipulate like units. This is true for fractions. It's true for decimals. It's true for whole numbers. When we have like units, it's easy to combine or take away quantities. As we work with adding and subtracting expressions in scientific notation, we'll see that this holds true If we have numbers with the same power of 10, it can make computing the sum or difference much easier.

Let's Think (Slide 4): Our first problem wants us to find the sum of the two numbers that are written in scientific notation. One arguably less efficient way to find the sum is just to rewrite the numbers in standard form. Let's start by doing that, and then I'll show you another way that you might find easier.



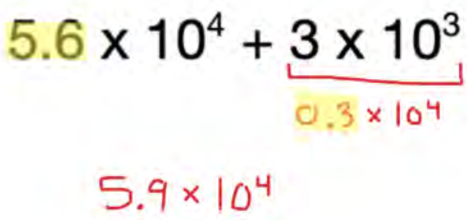
(write as you narrate) I can think of 5.6 times 10 to the 4 power as 56,000 in standard form. I know, because I can shift each digit four place values left. I can think of 3 times 10 to the 3 power as being 3,000 in standard form. I know, because I can shift each digit 3 place values left. If I add 56,000 and 3,000, I end up with 59,000.

The prompt requires me to write my answer in scientific notation. If I shift the digits in 59,000 to the right 4 place values, I end up with 5.9 times 10 to the 4 power.

This process wasn't overly challenging. We know how to do each part of the process from previous lessons. The downside is that I had to convert both numbers *out* of scientific notation, and then I had to convert the sum back *into* scientific notation. Let's explore a way we can do this without having to change the form of our addends nearly as much.

Instead of rewriting everything, I can make this problem easier by changing one of the addends so that the powers of 10 are consistent throughout the problem. Let's change 3 times 10 to the 3 power so it has a matching power of 10 to the first addend. How can I do that? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

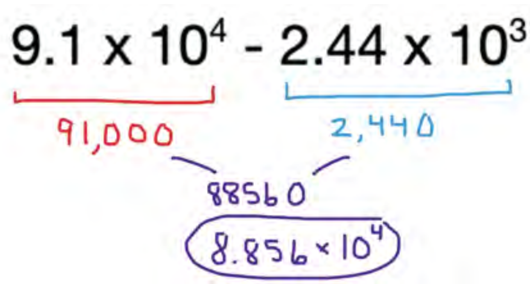
- We can shift 3 one place value right, and increase the exponent by 1.
- I know 3 times 10 to the 3 power is equivalent to 0.3 times 10 to the 4 power.



We now are combining addends with the same power of 10. Essentially, we're combining addends with the same unit. The unit is 10 to the 4 power or ten thousands. All we have to do now is add the leading factors. 5.6 ten thousands plus 0.3 ten thousands would be 5.9 ten thousands. We can add the leading factors, and the power of 10 remains the same. We end up with a sum of 5.9 times 10 to the 4 power. *(write answer)*

To add expressions in scientific notation, we can either rewrite them in standard form and then convert them back to scientific notation, or we can make our addends have a consistent power of 10 so we can add the leading factors easily. Either way works, and you might find you switch up your strategy depending on the numbers in a given problem. Let's look at one more example.

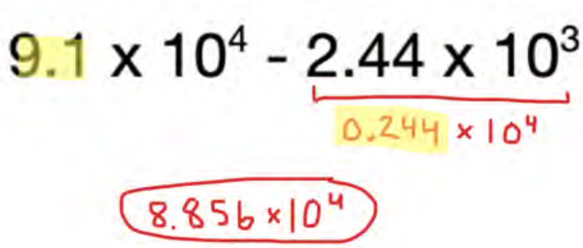
Let's Think (Slide 5): This problem is similar, but now we're being asked to subtract. Again, we can always write our numbers in standard form to add or subtract. Let's try that out, and then we'll try the strategy where we rewrite the powers of 10.



How can I write each number in this problem in standard form? *(write as student shares, supporting as needed)* Possible Student Answers, Key Points:

- 9.1 times 10 to the 4 power is equivalent to 91,000.
- 2.44 times 10 to the 3 power is 2,440.

When we subtract 91,000 minus 2,440, we get a difference of 88,560. One downside to this strategy is that our answer doesn't end up in scientific notation like was asked. If we shift each digit 4 place values right, we end up with an answer of 8.856 times 10 to the 4 power. *(write answer)*



Let's try the same problem, but now we'll adjust the powers of 10 rather than rewrite the numbers in standard form. I'll change 2.44 times 10 to the 3 power so it is written to the power of 4. I can shift the digits one place value right, then rewrite it as 0.244 times 10 to the 4 power. *(rewrite subtrahend as stated)* Now we have the same power of 10, which is like having the same unit. If I subtract the leading factors, 9.1 minus 0.244 is 8.856, and the power of 10 remains constant. 9.1 ten thousands minus 0.244 ten thousands is 8.856 ten thousands. We can write that as 8.856 times 10 to the 4. *(write answer)*

Of the two strategies we've explored to add or subtract numbers in scientific notation, which do you prefer and why? Possible Student Answers, Key Points:

- I like changing the powers of 10 to be the same, because then all I have to do is focus primarily on the leading factors.

- I like changing the powers of 10 to be the same, because sometimes writing really big or really small numbers in standard form can be tedious.
- I like converting the numbers to standard form, because I feel more comfortable adding and subtracting in standard from.

As we practice more problems, you're welcome to choose either strategy.

Let's Try it (Slides 6 - 7): Let's work on a few more problems involving addition and subtraction with quantities in scientific notation. As we saw from our previous examples, we'll want to make sure our values use the same power of 10 if we are working with exponents. We can also rewrite numbers in standard form if that's helpful, but it does leave room for error and can be a cumbersome process in certain problems. Let's keep these previous examples in mind as we tackle a few more. As always, you'll get a chance to show what you know independently once we're finished.

WARM WELCOME

Today we will add and subtract in scientific notation.

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Let's Talk:

Which is more efficient to add? Why?

67 thousands + 4 thousands

67 thousands + 4 ten thousands

Let's Think:

Determine the sum in scientific notation.

$$5.6 \times 10^4 + 3 \times 10^3$$

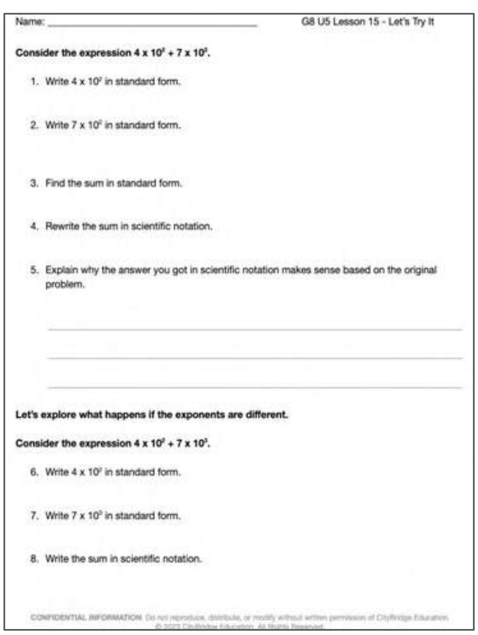
Let's Think:

Determine the difference in scientific notation.

$$9.1 \times 10^4 - 2.44 \times 10^3$$

Let's Try It:

Let's explore adding and subtracting in scientific notation together.



Name: ____________________ G8 U5 Lesson 15 - Let's Try It

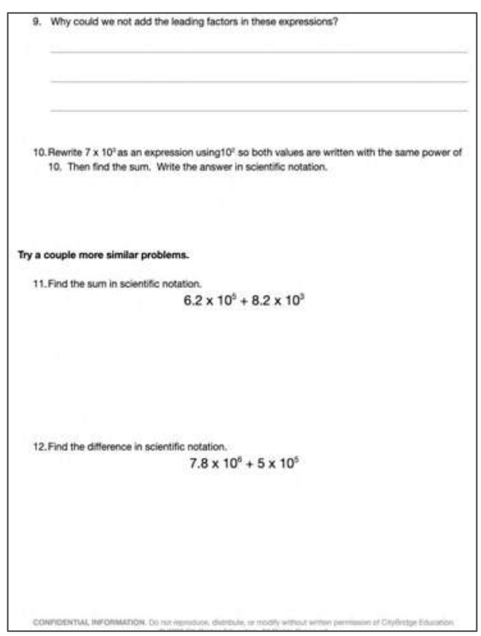
Consider the expression $4 \times 10^2 + 7 \times 10^2$.

1. Write 4×10^2 in standard form.
2. Write 7×10^2 in standard form.
3. Find the sum in standard form.
4. Rewrite the sum in scientific notation.
5. Explain why the answer you got in scientific notation makes sense based on the original problem.

Let's explore what happens if the exponents are different.

Consider the expression $4 \times 10^2 + 7 \times 10^3$.

6. Write 4×10^2 in standard form.
7. Write 7×10^3 in standard form.
8. Write the sum in scientific notation.



9. Why could we not add the leading factors in these expressions?
10. Rewrite 7×10^3 as an expression using 10^2 so both values are written with the same power of 10. Then find the sum. Write the answer in scientific notation.

Try a couple more similar problems.

11. Find the sum in scientific notation.

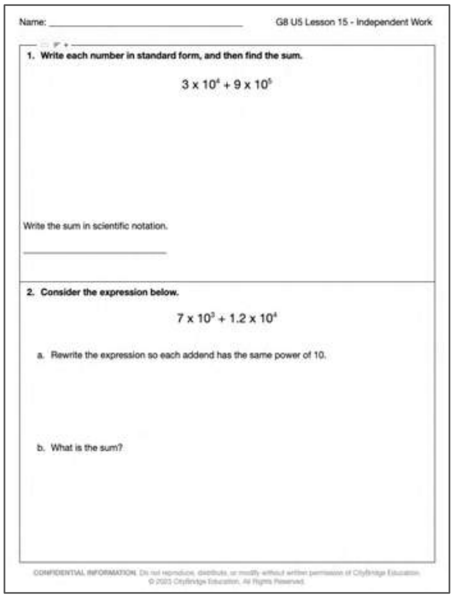
$$6.2 \times 10^5 + 8.2 \times 10^3$$

12. Find the difference in scientific notation.

$$7.8 \times 10^6 + 5 \times 10^5$$

On your Own:

Now it's time to add and subtract in scientific notation on your own.



Name: ______________________ G8 U5 Lesson 15 - Independent Work

1. Write each number in standard form, and then find the sum.

$$3 \times 10^4 + 9 \times 10^5$$

Write the sum in scientific notation.

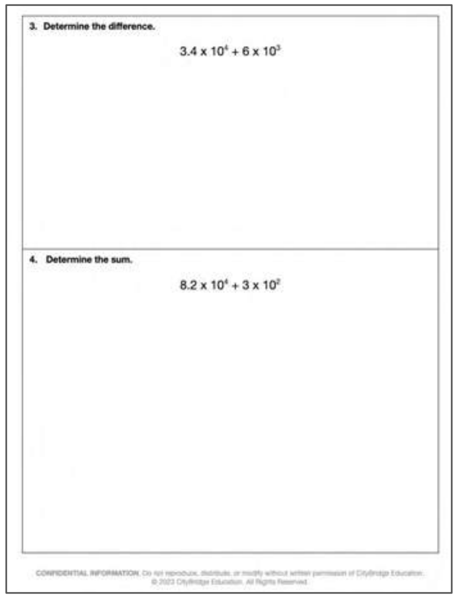
____

2. Consider the expression below.

$$7 \times 10^3 + 1.2 \times 10^4$$

a. Rewrite the expression so each addend has the same power of 10.

b. What is the sum?



3. Determine the difference.

$$3.4 \times 10^4 + 6 \times 10^3$$

4. Determine the sum.

$$8.2 \times 10^4 + 3 \times 10^2$$

Consider the expression $4 \times 10^2 + 7 \times 10^2$.

1. Write 4×10^2 in standard form.

2. Write 7×10^2 in standard form.

3. Find the sum in standard form.

4. Rewrite the sum in scientific notation.

5. Explain why the answer you got in scientific notation makes sense based on the original problem.

____

____

____

Let’s explore what happens if the exponents are different.

Consider the expression $4 \times 10^2 + 7 \times 10^3$.

6. Write 4×10^2 in standard form.

7. Write 7×10^3 in standard form.

8. Write the sum in scientific notation.

9. Why could we not add the leading factors in these expressions?

10. Rewrite 7×10^3 as an expression using 10^2 so both values are written with the same power of 10. Then find the sum. Write the answer in scientific notation.

Try a couple more similar problems.

11. Find the sum in scientific notation.

$$6.2 \times 10^5 + 8.2 \times 10^3$$

12. Find the difference in scientific notation.

$$7.8 \times 10^6 + 5 \times 10^5$$

1. Write each number in standard form, and then find the sum.

$$3 \times 10^4 + 9 \times 10^5$$

Write the sum in scientific notation.

____

2. Consider the expression below.

$$7 \times 10^3 + 1.2 \times 10^4$$

a. Rewrite the expression so each addend has the same power of 10.

b. What is the sum?

3. **Determine the difference.**

$$3.4 \times 10^4 - 6 \times 10^3$$

4. **Determine the sum.**

$$8.2 \times 10^4 + 3 \times 10^2$$

Name: KEY

Consider the expression $4 \times 10^2 + 7 \times 10^2$.

1. Write 4×10^2 in standard form.

   400

2. Write 7×10^2 in standard form.

   700

3. Find the sum in standard form.

   $400 + 700 = 1{,}100$

4. Rewrite the sum in scientific notation.

   1.1×10^3

5. Explain why the answer you got in scientific notation makes sense based on the original problem.

   If we added the original leading factors, we'd have 11×10^2 which is equivalent to 1.1×10^3.

Let's explore what happens if the exponents are different.

Consider the expression $4 \times 10^2 + 7 \times 10^3$.

6. Write 4×10^2 in standard form.

   400

7. Write 7×10^3 in standard form.

   7000

8. Write the sum in scientific notation.

   $7{,}400 \rightarrow 7.4 \times 10^3$

9. Why could we not add the leading factors in these expressions?

They didn't have the same power of 10. It would be like adding values of different units.

10. Rewrite 7×10^3 as an expression using 10^2 so both values are written with the same power of 10. Then find the sum. Write the answer in scientific notation.

$(70 \times 10^2) + (4 \times 10^2)$

$74 \times 10^2 = 7.4 \times 10^3$

Try a couple more similar problems.

11. Find the sum in scientific notation.

$$6.2 \times 10^5 + 8.2 \times 10^3$$

620×10^3 8.2×10^3

628.2×10^3

6.282×10^5

12. Find the difference in scientific notation.

$$7.8 \times 10^6 + 5 \times 10^5$$

$(7.8 \times 10^6) + (0.5 \times 10^6)$

8.3×10^6

Name: KEY

1. Write each number in standard form, and then find the sum.

$$3 \times 10^4 + 9 \times 10^5$$

30,000 + 900,000

930,000

Write the sum in scientific notation.

9.3×10^5

2. Consider the expression below.

$$7 \times 10^3 + 1.2 \times 10^4$$

a. Rewrite the expression so each addend has the same power of 10.

$0.7 \times 10^4 + 1.2 \times 10^4$

b. What is the sum?

1.9×10^4

3. **Determine the difference.**

$$3.4 \times 10^4 - 6 \times 10^3$$

$3.4 \times 10^4 - 0.6 \times 10^4$

2.8×10^4

4. **Determine the sum.**

$$8.2 \times 10^4 + 3 \times 10^2$$

$8.2 \times 10^4 + 0.03 \times 10^4$

8.23×10^4

Eighth Grade Math Lesson Materials

CITYTUTOR DC

G8 Unit 6:

Pythagorean Theorem and Irrational Numbers

G8 U6 Lesson 1
Find the area of a tilted square

G8 U6 Lesson 1 - Students will find the area of a tilted square

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we're learning a new way to calculate the area of a square. But here's the twist – the square will be tilted! This might sound tricky, but we'll use a decomposition strategy to make it easy. By the end of this lesson, you'll be able to find the area of any tilted square on a grid.

Let's Review (Slide 3): Let's recall what you remember about area. Possible Student Answers, Key Points:

- Area is the amount of space inside of a shape.
- We can use tiles to find the total area of rectangles.
- We can multiply the length by the width to find the area of rectangles and squares.
- We can cut shapes into smaller shapes, like rectangles, to find the area.
- The area should be measured in square units.

These are all true about the area. Today, we are going to focus on squares. How might we find the area of the square on the screen? Possible Student Answers, Key Points:

- We can add the number of smaller units inside of the square.
- We can count how many units the height and width are and multiply them.
- Since squares have equal side lengths, the height and width can be multiplied.
- The area of a square is s^2 or s x s.
- The area is 16 units squared.

Awesome! You all remember a lot about area and how to find it. Since we are working with squares today, your understanding that squares have the same side lengths and that the area of a square is s^2 will be a great foundation for our lesson.

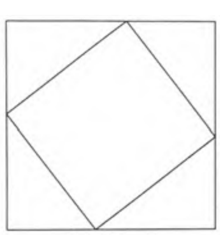
Let's Talk (Slide 4): Now, let's look at another square *(pass out Tilted Square handout)*. What do you notice about the first square? Possible Student Answers, Key Points:

- The square is on a grid.
- The square is tilted.

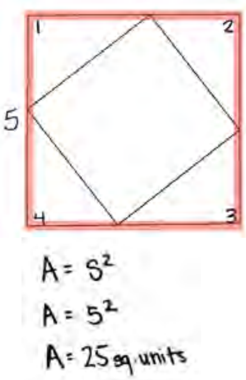
Earlier, you said that to find the area of the square, you must know a side length. If you know the side length, you can find the area of the square by multiplying "s" times "s." How might we find the area of this square that is tilted? Possible Student Answers, Key Points:

- The sides cut across the unit boxes.
- Count the units inside of the square.
- The area is 8 units squared.
- Count all the times and subtract the purple ones.

Let's Think (Slide 5): On the last slide, we saw a tilted square, and it was easy to count the units and measure the side lengths. However, sometimes, when a square is tilted, it doesn't fit neatly into the grid squares. However, we can still find its area by breaking it down into smaller, more manageable shapes.

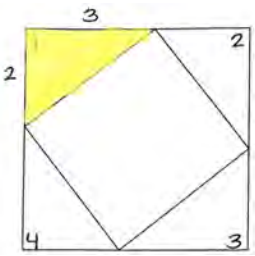


We are going to find the area of squares that are tilted on grids today (d*raw a tilted square with an enclosing square and tell students to look at Grid #2 on Tilted Square Handout)* First, let's identify some shapes that we see on the grid. Squares! Triangles!

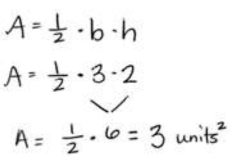


Right! There is a larger square that is not tilted but outlines the tilted square (*trace with fingers*); we call that the enclosing square. We also see four right triangles created by the two squares. We are going to decompose, which means to break down these shapes to help us solve for the area of the red square.

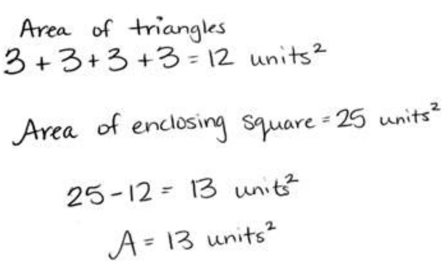
Let's work through this together. First, we'll find the area of the enclosing square, the bigger square, and afterward, we will find the areas of the triangles outside the square. This is called decomposition. Now let's label the triangle numbers 1 - 4. The enclosing square has a side length of five units (*label the side, 5. Write* $A = s^2$). That means that its area is five squared or five times five, 25 units squared.



Now, we want to find the area of the triangles made by the squares. To find the area of a triangle, we use the formula ½ x base x height of the triangle *(write formula)*. As we see in Triangle 1, the base is 3, and the height is 2 *(label)*. If we multiply 3 x 2 and divide by 2, our area would equal 3 square units.



For triangle 2, our base is two, and our height is three. We need to use the same formula as before ½ x base x height of the triangle, 2 times 3 divided by 2. The area is three units squared. For triangle 3, our base is 3, and our height is 2. This triangle has the same area as triangle 1. We multiply 3 by 2 and divide by 2. The area is 3 square units. For the last triangle, number 4, we will find the area by multiplying the base by 2, the height by 3, and then dividing by 2. The area is 3 square units, just like the other triangles.



Remember, because we decomposed, we can take the enclosed square's area–the big square (trace)–and subtract the triangles' areas outside the square.

Let's add the area of the 4 triangles, each one was 3. So, 3 + 3 + 3 + 3 = 12 units squared, and the area of the enclosing square is 25 units squared. We can subtract to find the area of the red square: 25 - 12 = 13 units squared. The area of the tilted square is 13 units squared.

Let's Try it (Slides 6): Now, let's practice finding the area of some other tilted squares using the decomposition method. Remember, we find the area of the enclosing square and then we take away the area of all four of the triangles.

WARM WELCOME

Today we will find the area of a tilted square

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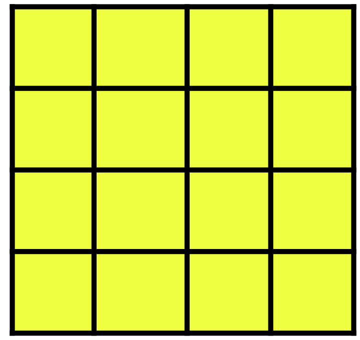
Let's Review:

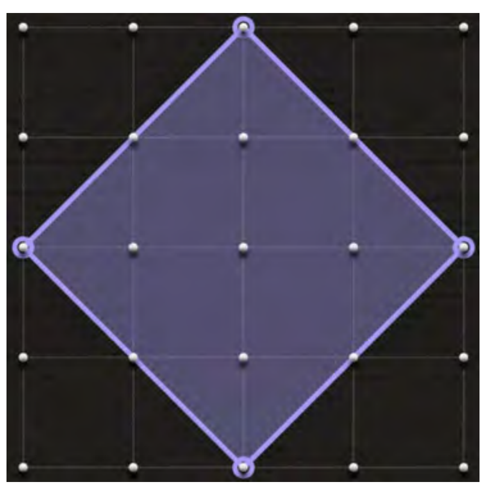
What do you remember about area?



How might we find the area of the given square?

Let's Talk:

How might we find the area of this square?



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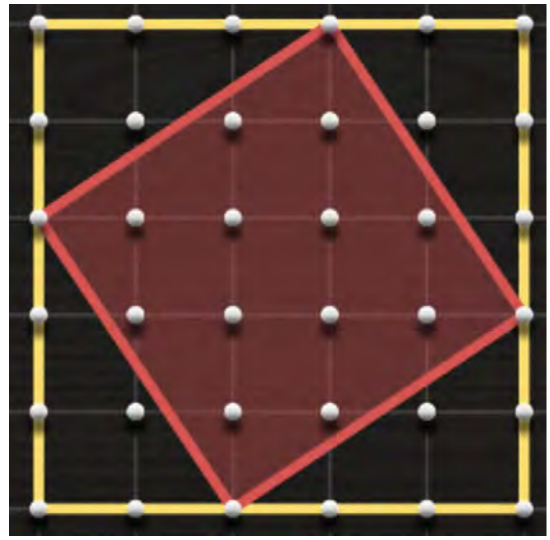
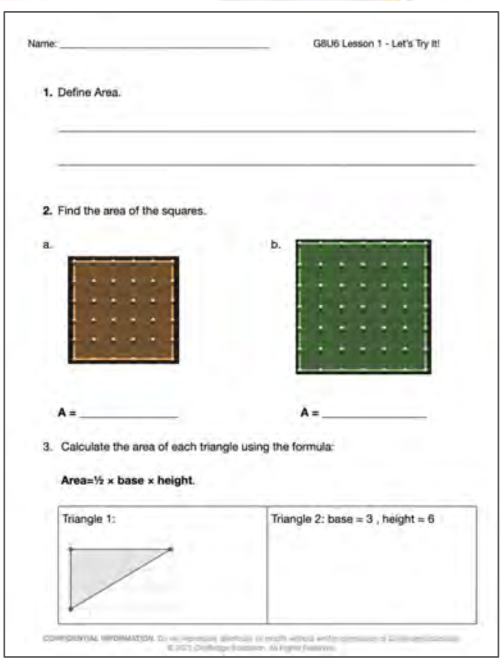
Let's Think:

Let's find the area of the red square.



Let's Try It:

Let's explore finding the area of tilted squares together.



Name: ______________________ G8U6 Lesson 1 - Let's Try It!

1. Define Area.

____

____

2. Find the area of the squares.

a.

A = __________

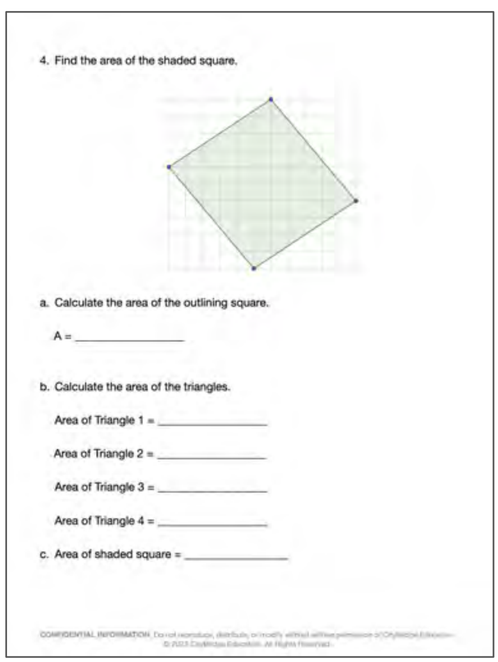
b.

A = __________

3. Calculate the area of each triangle using the formula:

Area=½ × base × height

Triangle 1:	Triangle 2: base = 3 , height = 6



4. Find the area of the shaded square.

a. Calculate the area of the outlining square.

A = __________

b. Calculate the area of the triangles.

Area of Triangle 1 = __________

Area of Triangle 2 = __________

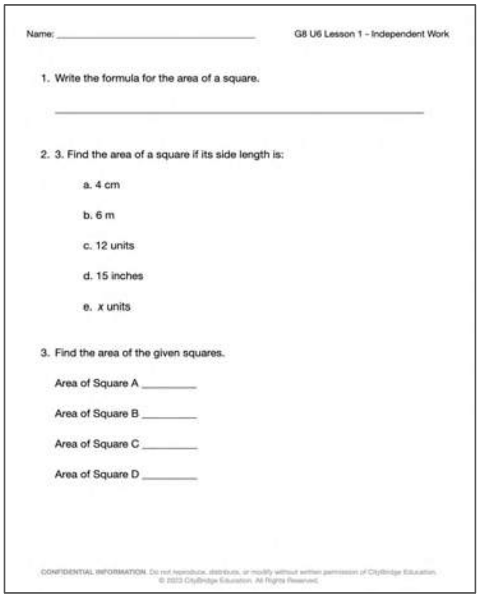
Area of Triangle 3 = __________

Area of Triangle 4 = __________

c. Area of shaded square = __________

On your Own:

Now it's time to try finding the area of tilted squares on your own.



Name: ______________________ G8 U6 Lesson 1 - Independent Work

1. Write the formula for the area of a square.

____

2. 3. Find the area of a square if its side length is:

a. 4 cm

b. 6 m

c. 12 units

d. 15 inches

e. x units

3. Find the area of the given squares.

Area of Square A ________

Area of Square B ________

Area of Square C ________

Area of Square D ________

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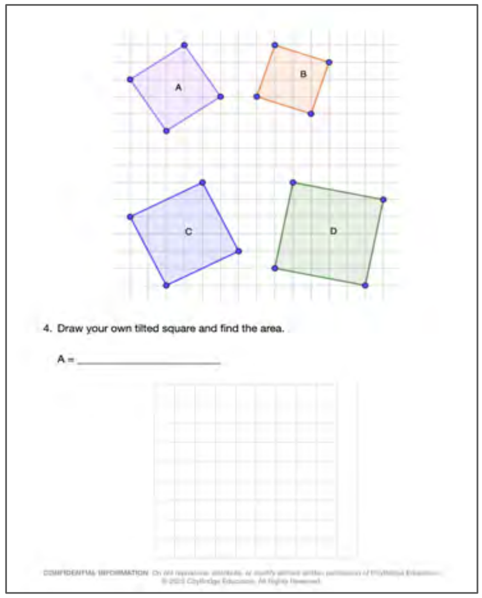

4. Draw your own tilted square and find the area.

A = ________________

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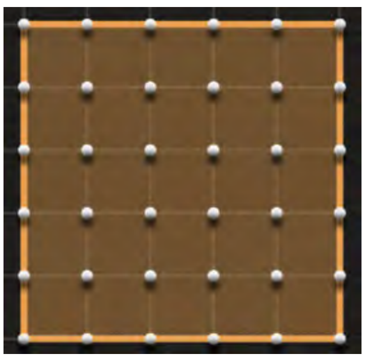
1. Define Area.

____

____

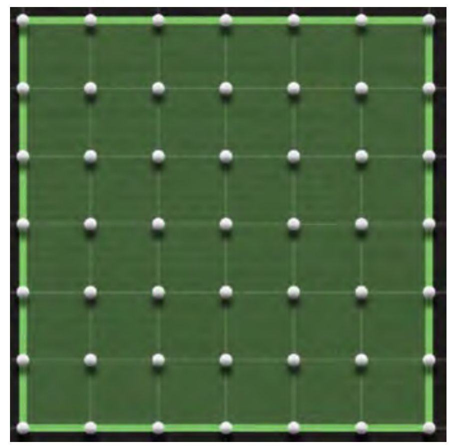
2. Find the area of the squares.

a.



A = ________________

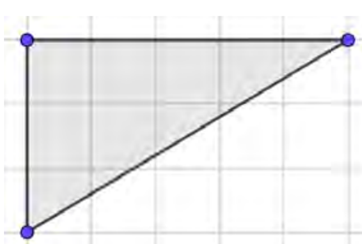
b.



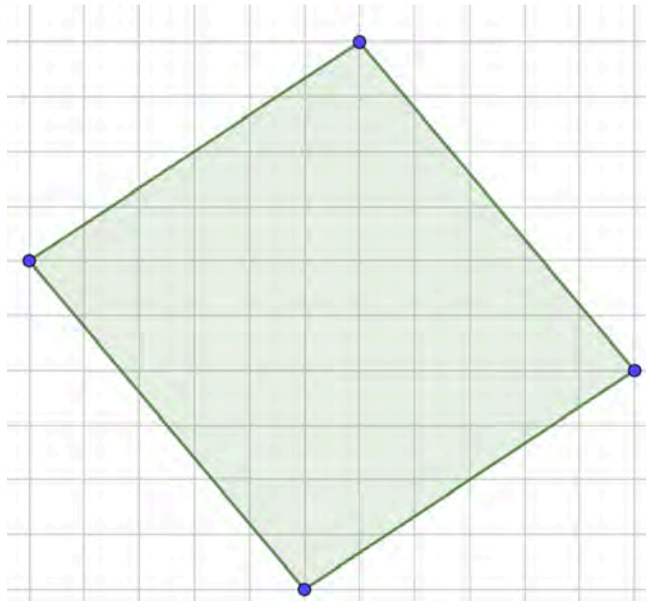
A = ________________

3. Calculate the area of each triangle using the formula:

Area=½ × base × height.

Triangle 1:	Triangle 2: base = 3 , height = 6
	

4. Find the area of the shaded square.



a. Draw your enclosing square. Calculate the area of the enclosing square.

A = ___________________

b. Calculate the area of the triangles.

Area of Triangle 1 = ___________________

Area of Triangle 2 = ___________________

Area of Triangle 3 = ___________________

Area of Triangle 4 = ___________________

c. Area of shaded square = __________________

1. Write the formula for the area of a square.

____

2. 3. Find the area of a square if its side length is:

a. 4 cm ; A = _________________

b. 6 m ; A = _________________

c. 12 units ; A = _________________

d. 15 inches ; A = _________________

e. x units ; A = _________________

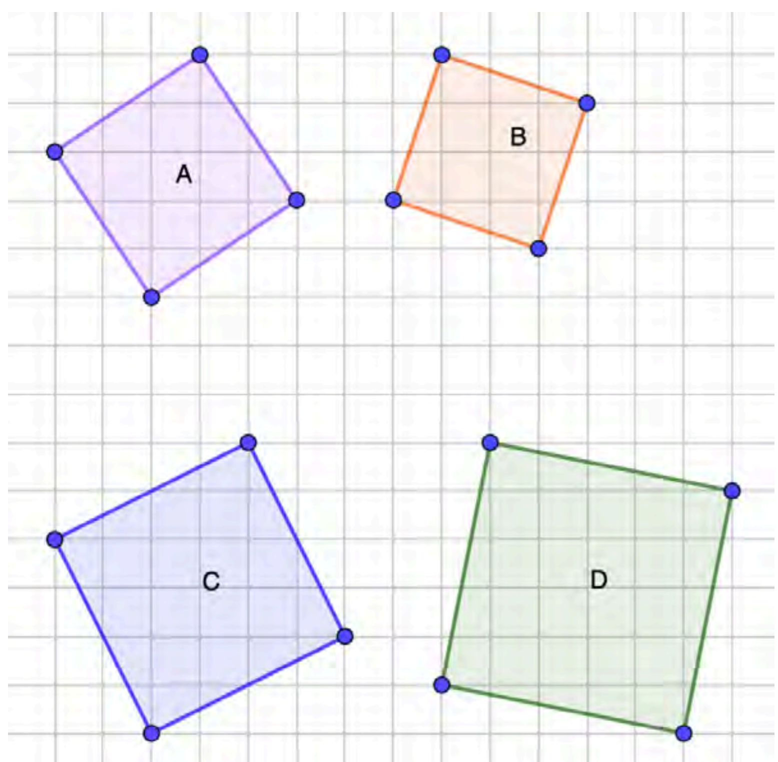
3. Find the area of the given squares.

Area of Square A = _______________

Area of Square B = _______________

Area of Square C = _______________

Area of Square D = _______________

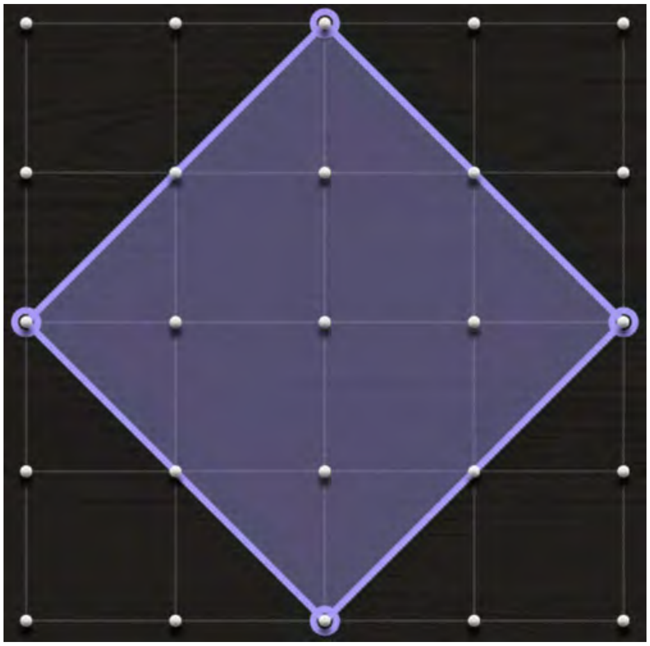


4. Draw your own tilted square and find the area.

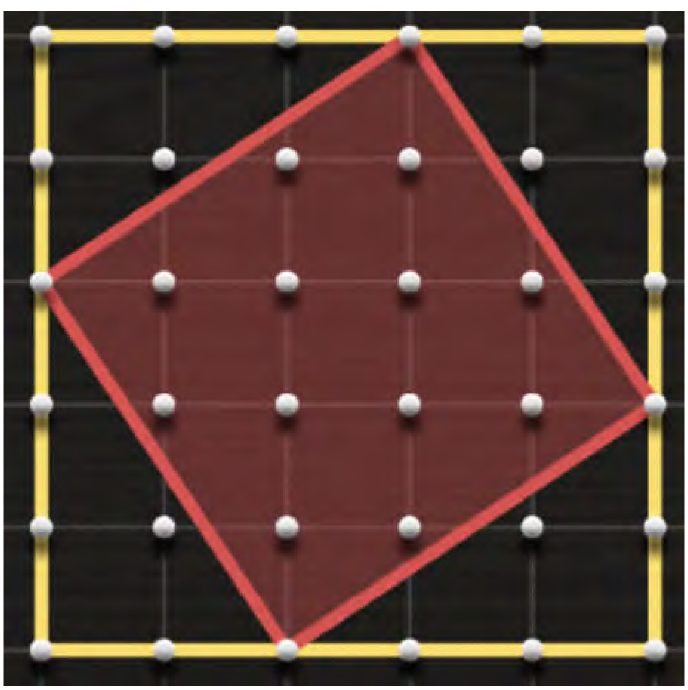
A = ________________________

Name: ______________________________________

1.



2.



Name: Answer Key

1. Define Area.

Area is the space enclosed by a shape.

2. Find the area of the squares.

a.

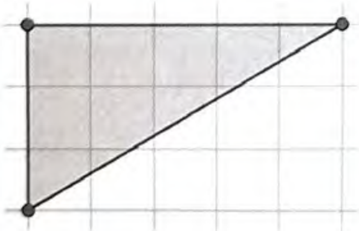
A = $25 u^2$

b.

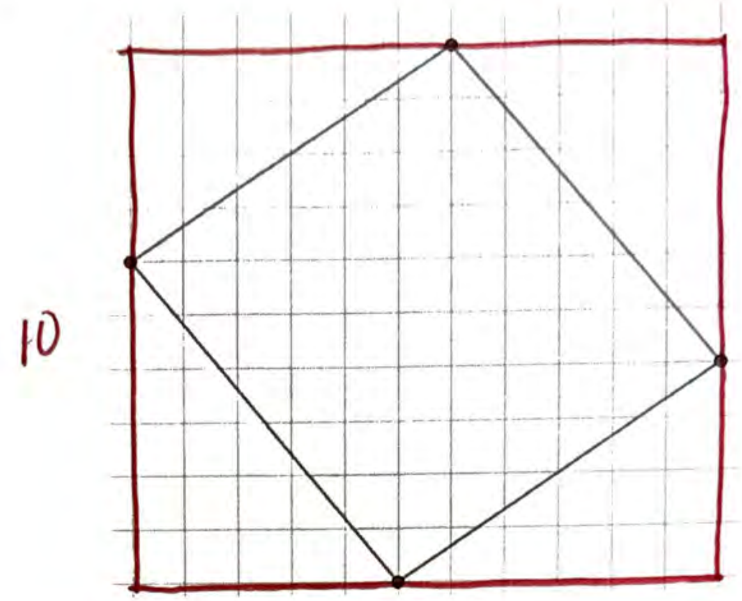
A = $36 u^2$

3. Calculate the area of each triangle using the formula:

Area=½ × base × height.

Triangle 1:	Triangle 2: base = 3 , height = 6
 $A=\frac{1}{2}\cdot b\cdot h$ $A=\frac{1}{2}\cdot 3\cdot 5$ $A=\frac{15}{2}$ or $7.5 u^2$	$A=\frac{1}{2}\cdot b\cdot h$ $A=\frac{1}{2}\cdot 3\cdot 6$ $A=9 u^2$

4. Find the area of the shaded square.



a. Draw your enclosing square. Calculate the area of the enclosing square.

A = $100u^2$

b. Calculate the area of the triangles.

Area of Triangle 1 = $12u^2$

Area of Triangle 2 = $12u^2$

Area of Triangle 3 = $12u^2$

Area of Triangle 4 = $12u^2$

c. Area of shaded square = $100 - 48 = 52u^2$

Name: ______________________________

1. Write the formula for the area of a square.

$A = s^2$

2. 3. Find the area of a square if its side length is:

a. 4 cm ; A = $16\ cm^2$

b. 6 m ; A = $36\ m^2$

c. 12 units ; A = $144\ u^2$

d. 15 inches ; A = $225\ in^2$

e. x units ; A = $x^2\ u^2$

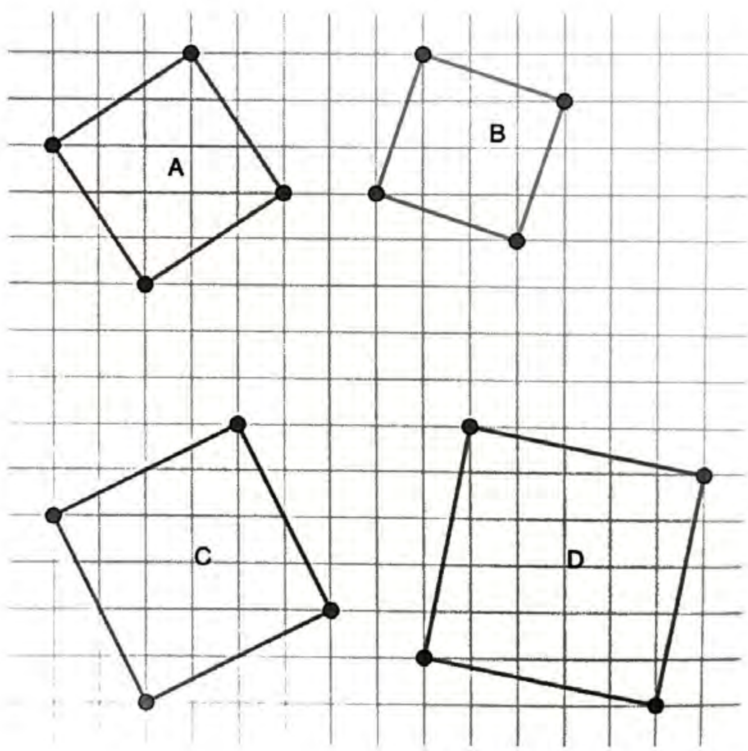
3. Find the area of the given squares.

Area of Square A = $13\ u^2$

Area of Square B = $10\ u^2$

Area of Square C = $20\ u^2$

Area of Square D = $26\ u^2$



Name: Answer Key

1. Write the formula for the area of a square.

$A = s^2$

2. 3. Find the area of a square if its side length is:

a. 4 cm ; A = $16\ cm^2$

b. 6 m ; A = $36\ m^2$

c. 12 units ; A = $144\ u^2$

d. 15 inches ; A = $225\ in^2$

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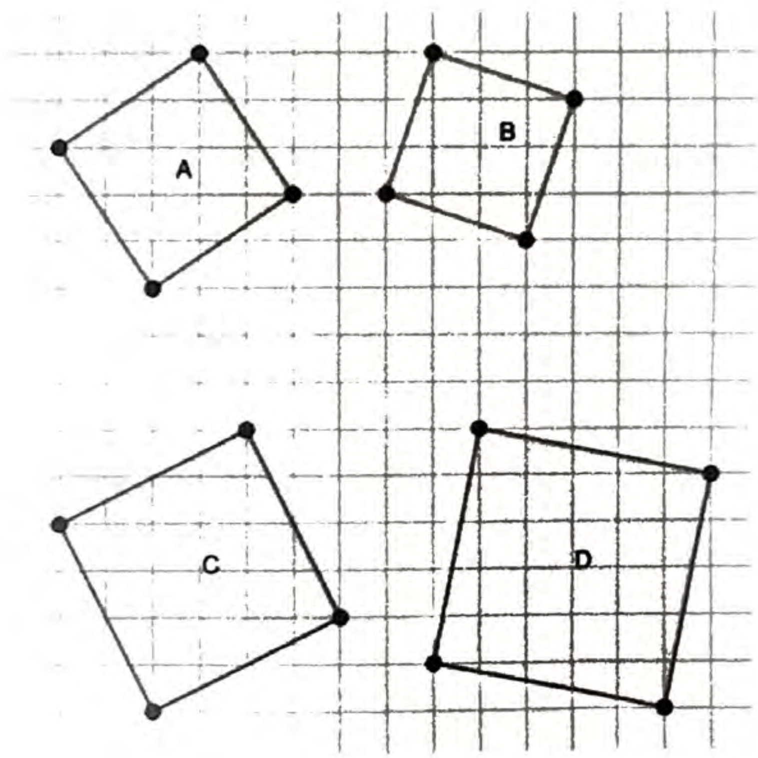
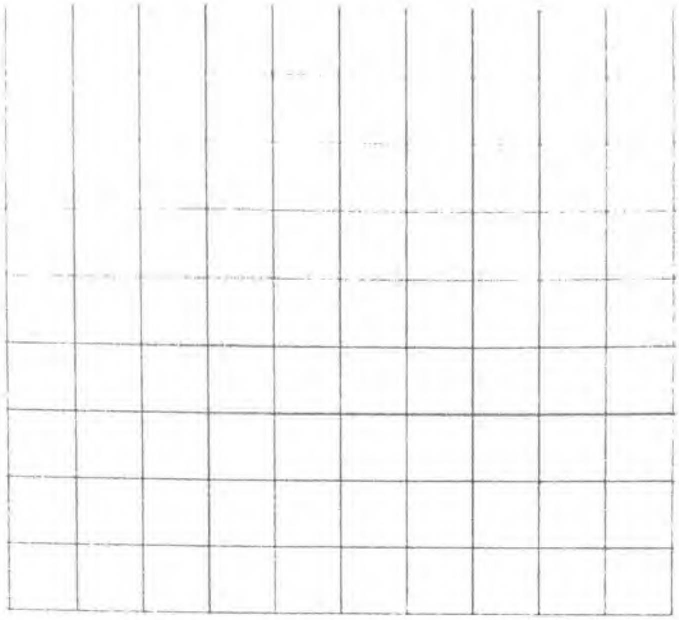


4. Draw your own tilted square and find the area.

A = ____________________ Answers may vary.



G8 U6 Lesson 2
Comprehend “square root” and use square root notation to express the side length of a square, given its area.

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G8 U6 Lesson 2 - Students will use the Square Root Notation to express the side length of a square, given its area.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we will learn about square roots. By the end of this lesson, you'll understand what a square root is and how to use square root notation to find the side length of a square when you're given its area.

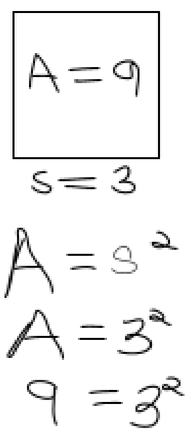
Let's Review (Slide 3): Let's brainstorm! **What does it mean to square a number?** Possible Student Answers, Key Points:

- To multiply the number by itself

When we square a number, we multiply that number by itself. For example, the square of 3 is 3 times 3, which equals 9. Who can give me another example? (*Allow 1- 2 students to share.)*

Let's Talk (Slide 4): We reviewed how to square a number. Can **someone tell me the relationship between the side length of a square and its area?** Possible Student Answers, Key Points:

- The area is the side length squared.
- $A = s^2$

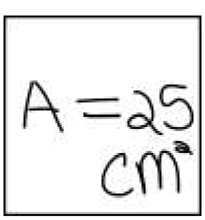


(Draw square on the Whiteboard.) The area of a square is the side length times itself, or $A = s^2$. If you know the side length, you can find the area. In this case, we see that the side length is 3 units. If we use the formula $A = s^2$, then $3^2 = 9$.

So, the relationship between the side length and the area of a square is that the side length squared equals the area of the square.

Let's Think (Slide 5): Sometimes, we may not be given the side length but will be given the Area, to find the side length. **How might we find the side length of a square if we only know the area?** Possible Student Answers, Key Points:

- Take the square root
- Do the opposite of squaring a number



(Draw a square on the board with an area of 25 cm^2.)

To find the side length, we must take the square root of the area.

$\sqrt{25} = 5$

$5 \cdot 5 = 25$

Let's start with the term 'square root.' The square root of a number is a value that, when multiplied by itself, gives the original number. For example, the square root of 25 is 5 because 5 times 5 equals 25." *(Teacher writes √25 = 5 on the board.)*

The symbol we use for square root is called a radical, and it looks like this *(point to radical symbol)*: So *√25 = 5* means that 5 is the square root of 25.

Now, let's relate this to squares. If you have a square with an area of 25 square units, the length of each side of the square is the square root of 25.

(Teacher writes on the board:)

Area = side length x side length

25 cm = side length x side length

side length = √25 = 5

So, the side length of a square with an area of 25 cm^2 is 5 cm.

Let's Try it (Slides 6): Now it's your turn. Remember that the square root of a number is the value that, when multiplied by itself, gives the original number. We use the radical symbol √ for square roots. We can also use square roots to find the side lengths of squares when we know their areas.

WARM WELCOME

Today we will comprehend "square root" and use square root notation to express the side length of a square, given its area.

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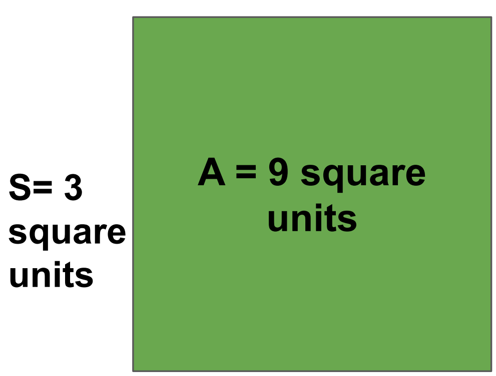
Let's Review:

What does it to mean to square a number? Give an example.

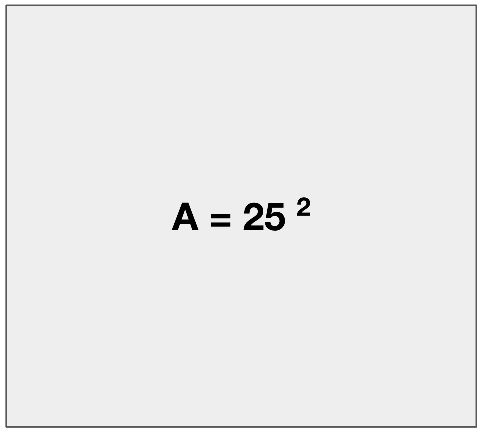
Let's Talk:

What is the relationship between the side length and the area of a square?



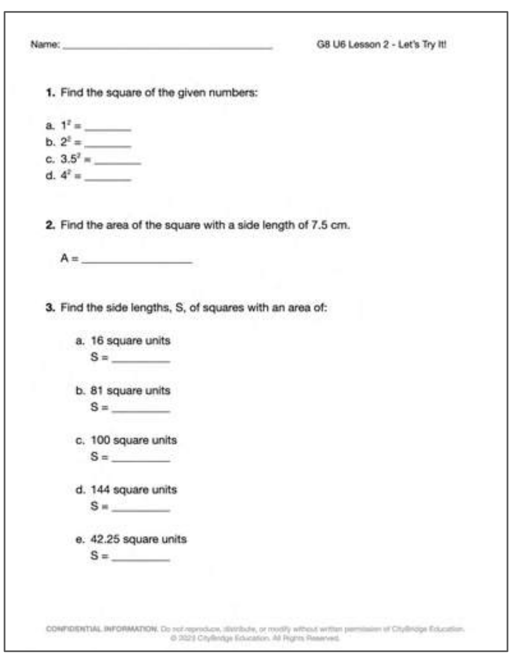
Let's Think:

How might we find the side length of a square if we only know the area?



Let's Try It:

Let's explore using the Square Root Notation to express the side length of a square, given its area together.



Name: ______________________ G8 U6 Lesson 2 - Let's Try It!

1. Find the square of the given numbers:

a. 1^2 = _______
b. 2^2 = _______
c. 3.5^2 = _______
d. 4^2 = _______

2. Find the area of the square with a side length of 7.5 cm.

A = ______________

3. Find the side lengths, S, of squares with an area of:

a. 16 square units
S = ________

b. 81 square units
S = ________

c. 100 square units
S = ________

d. 144 square units
S = ________

e. 42.25 square units
S = ________

4. Find the side lengths of the square.

S = ________

A = 196 cm^2

On your Own:

Now it's time to use the Square Root Notation to express the side length of a square, given its area on your own.

Name: ______________________ G8 U6 Lesson 2 - Independent Work

1. Find the square of the given numbers:

a. 9^2 = __________

b. 5.5^2 = __________

c. 15^2 = __________

2. Find the area of the square.

15 m

A = ______

3. What does the "square root" of a number mean?

____

____

4. Find the square root:

a. $\sqrt{289}$ = ______

b. $\sqrt{121}$ = ______

c. $\sqrt{225}$ = ______

5. Complete the table with the missing side lengths and area.

side length, s	1.5		5.5		12.5
area, a		4		400	

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Name: ____ G8 U6 Lesson 2 - Let's Try It

1. Find the square of the given numbers:

a. 1^2 = ________
b. 2^2 = ________
c. 3.5^2 = ________
d. 4^2 = ________

2. Find the area of the square with a side length of 7.5 cm.

A = ___________________

3. Find the side lengths, *S*, of squares with area, *A*.

a. *A = 16 square units*
S = __________

b. *A = 81 square units*
S = __________

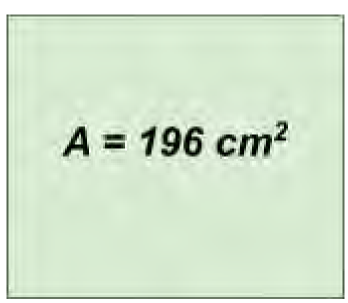
c. *A = 100 square units*
S = __________

d. *A = 144 square units*
S = __________

e. *A = 42.25 square units*
S = __________

4. Find the side length of the square.

S = __________



Name: ____

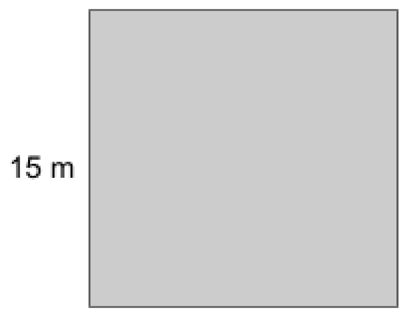
1. Find the square of the given numbers:

a. 9^2 = _____________

b. 5.5^2 = ____________

c. 15^2 = _____________

2. Find the area of the square.



A = _______

3. What does the “square root” of a number mean?

____

____

4. Find the square root:

a. $\sqrt{289}$ = _______

b. $\sqrt{121}$ = _______

c. $\sqrt{225}$ = _______

5. Complete the table with the missing side lengths and area.

side length, *s*	1.5		5.5		12.5
area, *a*		4		400	

Name: Answer Key

1. Find the square of the given numbers:

a. $1^2 =$ 1
b. $2^2 =$ 4
c. $3.5^2 =$ 12.25
d. $4^2 =$ 16

2. Find the area of the square with a side length of 7.5 cm.

$A =$ 56.25 cm^2

3. Find the side lengths, *S*, of squares with area, *A*.

a. *A = 16 square units*
S = 4 units

b. *A = 81 square units*
S = 9 units

c. *A = 100 square units*
S = 10 units

d. *A = 144 square units*
S = 12 units

e. *A = 42.25 square units*
S = 6.5 units

4. Find the side length of the square.

S = 14 cm

A = 196 cm²

Name: Answer Key

1. Find the square of the given numbers:

a. 9^2 = 81

b. 5.5^2 = 30.25

c. 15^2 = 225

2. Find the area of the square.

15 m

A = 225 m^2

3. What does the "square root" of a number mean?

The square root of a number is a number multiplied by itself gives the original number.

4. Find the square root:

a. √289 = 17

b. √121 = 11

c. √225 = 15

5. Complete the table with the missing side lengths and area.

side length, *s*	1.5	2	5.5	20	12.5
area, *a*	2.25	4	30.25	400	156.25

G8 U6 Lesson 3
Classify rational and irrational numbers

G8 U6 Lesson 3 - Students will classify rational and irrational numbers.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we will learn about rational and irrational numbers. By the end of this lesson, you will be able to explain what each term means and identify examples of each.

Can anyone tell me what a fraction is? What about a decimal? Great! Today, we will see how these concepts fit into our new terms: rational and irrational numbers.

Let's Talk (Slide 3): Let's start by looking at a number line. **What do you notice and wonder about this number line?** Possible Student Answers, Key Points:

- There are fractions, decimals, and percents.
- Some numbers are positive and negative. (positive & negative integers)
- There are whole numbers.

These are great observations and wonderings. This number line shows that numbers can be represented by fractions, decimals, and whole numbers. Numbers on a number line can be classified as rational and irrational numbers.

Let's Think (Slide 4): So, let's look at examples of rational numbers. **What do you notice and wonder about the examples of rational numbers?** Possible Student Answers, Key Points:

- There are fractions and decimals
- Repeating decimals
- Whole Numbers
- It can be negative or positive.
- Square Roots (perfect square)

Thank you for your noticings and wonderings. Yes, a rational number is any number that can be expressed as a fraction, where both the numerator and the denominator are integers, and the denominator is not zero.

(Write on the board: ***Rational Number = a/b and a and b are both integers, b ≠ 0.****)*
(Reference each example in the table as a rational number.)

Let's Think (Slide 5): Now let's look at irrational numbers; what **do you notice and wonder about these numbers?** Possible Student Answers, Key Points:

- There are fractions and decimals.
- Math Symbols
- Square roots (non-perfect square)
- Decimals that do not repeat.

Good noticings and wonderings. An irrational number is a number that cannot be expressed as a fraction. These numbers have non-repeating, non-terminating decimal parts.

(Write on the board: ***Irrational Number = a number cannot be written as a/b and a and b are both integers, b ≠ 0.****)*

(Reference each example in the table as an irrational number.)

Now, let's practice identifying rational and irrational numbers together. I'll show you a number, and you tell me if it's rational or irrational.

(Give examples: ⅘, 1.25, .333…, π, and √7 .)

- ⅘ - rational
- 1.25 - rational
- .333…- irrational
- π - irrational
- √7 - irrational

Let's Try it (Slides 6): Let's continue practicing classifying numbers as either rational or irrational.

WARM WELCOME

Today we will classify rational and irrational numbers.

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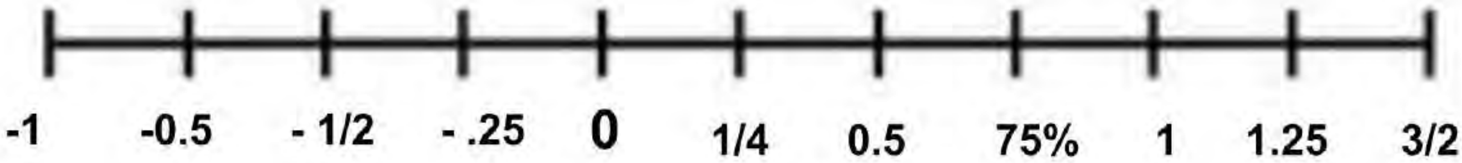
Let's Talk:

What do you notice and wonder about the number line?



Let's Think:

What do you notice and wonder about rational numbers?

Examples:

1	**9.45454545…**	**-¾**
0.75 (since 0.75 = ¾)	**-2** (since $-2 = -\frac{2}{1}$)	**$\sqrt{49}$**

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Let's Think:

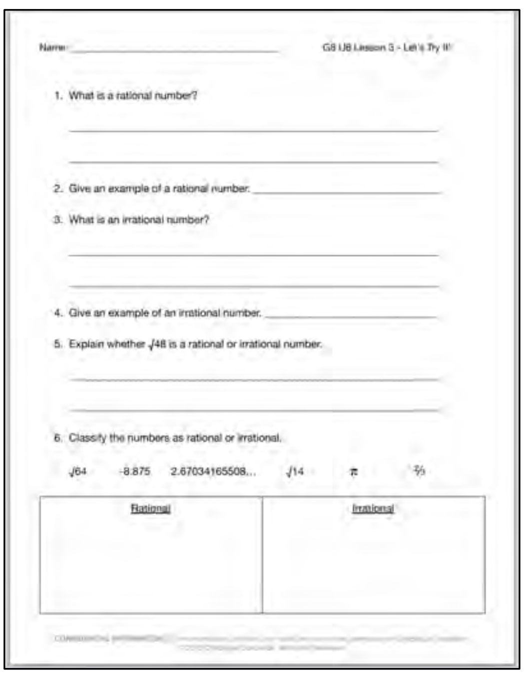
What do you notice and wonder about irrational numbers?

Examples:

π (pi)	**$\sqrt{3}$**	**$\sqrt{4}$**
1.243487…	**e** (Euler's Number)	**$\sqrt{5}$**

Let's Try It:

Let's explore classifying rational and irrational numbers together.



Name: ______________________ G8 U8 Lesson 3 - Let's Try It!

1. What is a rational number?

2. Give an example of a rational number. ______________

3. What is an irrational number?

4. Give an example of an irrational number. ______________

5. Explain whether $\sqrt{48}$ is a rational or irrational number.

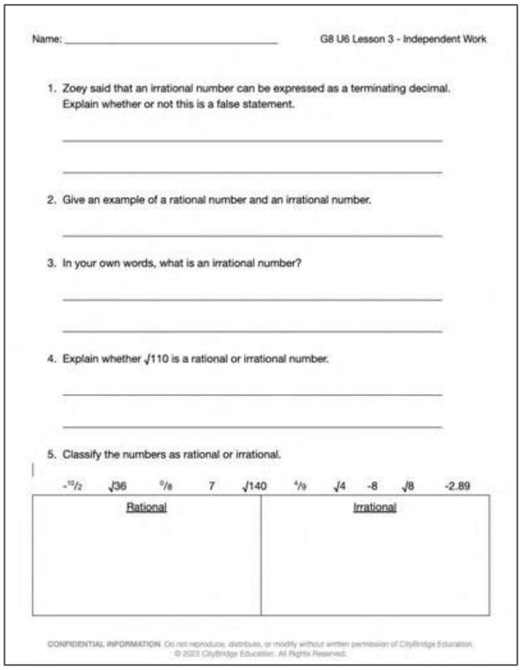
6. Classify the numbers as rational or irrational.

$\sqrt{64}$ -8.875   2.67034165508… $\sqrt{14}$ π $\frac{2}{3}$

Rational	Irrational

On your Own:

Now it's time to classify rational and irrational numbers on your own.



Name: ______________________ G8 U6 Lesson 3 - Independent Work

1. Zoey said that an irrational number can be expressed as a terminating decimal. Explain whether or not this is a false statement.

2. Give an example of a rational number and an irrational number.

3. In your own words, what is an irrational number?

4. Explain whether $\sqrt{110}$ is a rational or irrational number.

5. Classify the numbers as rational or irrational.

$-^{10}/_{2}$ $\sqrt{36}$ $^{0}/_{8}$ 7 $\sqrt{140}$ $^{4}/_{9}$ $\sqrt{4}$ -8 $\sqrt{8}$ -2.89

Rational	Irrational

Name: ____ G8 U6 Lesson 3 - Let's Try It

1. What is a rational number?

____

____

2. Give an example of a rational number. ______________________________

3. What is an irrational number?

____

____

4. Give an example of an irrational number. ____________________________

5. Explain whether $\sqrt{48}$ is a rational or irrational number.

____

____

6. Classify the numbers as rational or irrational.

$\sqrt{64}$ -8.875 2.67034165508… $\sqrt{14}$ π $\frac{2}{3}$

Rational	Irrational

1. Zoey said that an irrational number can be expressed as a terminating decimal. Explain whether or not this is a false statement.

   ____

   ____

2. Give an example of a rational number and an irrational number.

   ____

3. In your own words, what is an irrational number?

   ____

   ____

4. Explain whether $\sqrt{110}$ is a rational or irrational number. **(Use a calculator if needed)**

   ____

   ____

5. Classify the numbers as rational or irrational.

$-{}^{10}/_{2}$ $\sqrt{36}$ ${}^{0}/_{8}$ 7 $\sqrt{140}$ ${}^{4}/_{9}$ $\sqrt{4}$ -8 $\sqrt{8}$ -2.89

Rational	Irrational

Name: Answer Key

G8 U6 Lesson 3 - Let's Try It!

1. What is a rational number?

Any number that can be written as a fraction, the numerator ÷ denominator are integers, denominator ≠ 0

2. Give an example of a rational number. answers may vary

3. What is an irrational number?

a real number that cannot be expressed as a ratio of integers.

4. Give an example of an irrational number. answers may vary

5. Explain whether √48 is a rational or irrational number.

√48 is an irrational number, it cannot express it as a fraction.

6. Classify the numbers as rational or irrational.

√64 -8.875 2.67034165508... √14 π ⅔

Rational	Irrational
√64 ⅔ -8.875	π √14 2.67034165508...

Name: Answer Key

1. Zoey said that an irrational number can be expressed as a terminating decimal. Explain whether or not this is a false statement.

Zoey is incorrect. Irrational numbers are non-terminating decimals

2. Give an example of a rational number and an irrational number.

Answers may vary

3. In your own words, what is an irrational number?

A real number that cannot be expressed as a ratio of integers

4. Explain whether $\sqrt{110}$ is a rational or irrational number. **(Use a calculator if needed)**

$\sqrt{110}$ is an irrational number because it cannot be written as a fraction or ratio.

5. Classify the numbers as rational or irrational.

$-^{10}/_{2}$ $\sqrt{36}$ $^{0}/_{8}$ 7 $\sqrt{140}$ $^{4}/_{9}$ $\sqrt{4}$ -8 $\sqrt{8}$ -2.89

Rational	Irrational
$\sqrt{36}$ $-10/2$ 7 $\sqrt{4}$ -8 0/8 4/9 -2.89	$\sqrt{8}$ $\sqrt{140}$

G8 U6 Lesson 4
Find decimal approximation for square roots

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G8 U6 Lesson 4 - Students will find a decimal approximation for square roots and locate its approximation on the number line.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we're going to learn how to find a decimal approximation for square roots and how to locate these approximations on the number line. First, let's review what a square root is.

Let's Review (Slide 3): We have worked with square roots over the last two lessons. **What does it mean to find the square root?** Possible Student Answers, Key Points:

- Find the number multiplied by itself that gives the original number

Nice! So, let's share some examples of perfect square roots. *(**Think-Pair-Share:** Give students 30 seconds to think about an example. Give students 30 seconds to share with an elbow partner, then share out loud with the group)*

Great! But not all numbers are perfect squares. Numbers like 2, 3, and 5 are not perfect squares and have square roots that are not whole numbers. Today, we'll learn how to approximate these square roots.

Let's Talk (Slide 4): Since we know that all square roots are not perfect**, how might we find the approximation of** $\sqrt{2}$? Possible Student Answers, Key Points:

- Use a calculator
- Use perfect squares that we know to help us approximate.

$\sqrt{2}$

$\sqrt{1}=1$ $\sqrt{4}=2$

(Write $\sqrt{2}$ on the board) First, we need to identify the two closest perfect squares. What are they? 1 and 4

1.2
×1.2
24
120
1.44

Correct! Since, $\sqrt{1} = 1$ and $\sqrt{4} = 2$, we know $\sqrt{2}$ is between 1 and 2. To get a better estimate and more accurate, we can use a method of guess and check. Since 2 is closer to 1 than 4, we can guess a decimal closer to 1, let's choose 1.2. If we multiply 1.2 by itself *(write on the board and calculate)*, we get 1.44.

1.4
×1.4
56
+140
1.96

Let's try one more decimal, 1.4, and multiply it by itself. *(write on the board and calculate)* We get 1.96, which is pretty close to 2. That is a more accurate approximation of $\sqrt{2}$. (*Write $\sqrt{2}$ is approximately ≈ 1.4.*) So $\sqrt{2} \approx 1.4$

We will only use calculators to check our approximations.

$\sqrt{2} \approx 1.4$

Let's Think (Slide 5):

$\sqrt{10}$

$\sqrt{9} = 3$ $\sqrt{16} = 4$

Now let's try one more: what is the approximation of $\sqrt{10}$; This time we are going to plot it on a number line *(Write $\sqrt{10}$ on the board)* What are the two perfect squares that are closest to $\sqrt{10}$? 9 and 16 Since $\sqrt{9} = 3$ and $\sqrt{16} = 4$, $\sqrt{10}$ it is located between 3 and 4. Would it be closer to 3 or 4? 3. That's right $\sqrt{10}$ is closer to 3 because 10 is closer to 9 and not 16.

$$\begin{array}{r} 3.2 \\ \times 3.2 \\ \hline 64 \\ +960 \\ \hline 10.24 \end{array}$$

$\sqrt{10} \approx 3.2$

So, let's guess a decimal to begin with. Let's try 3.2. We can multiply 3.2 by itself *(write on the board and calculate);* when we do, we get 10.24. This is very close to 10, but a little over. So we could try another decimal that is a little smaller, but we'll stop here and say that $\sqrt{10} \approx 3.2$. *(Write on the board)*

Now let's look at the number line, we have found an approximation and we can use that to plot $\sqrt{10}$ on the number line. We know that it is closer to 3, but a little over. *(Draw a number line on the board 1 - 10; make a dot closer to 3)*

Let's Try it (Slides 6): Now it's turn to practice. Remember we can find decimal approximations for square roots by identifying the two closest perfect squares and refining our estimates. We can also plot these approximations on a number line.

WARM WELCOME

Today we will learn how to find a decimal approximation for square roots and locate these approximations on the number line.

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Let's Review:

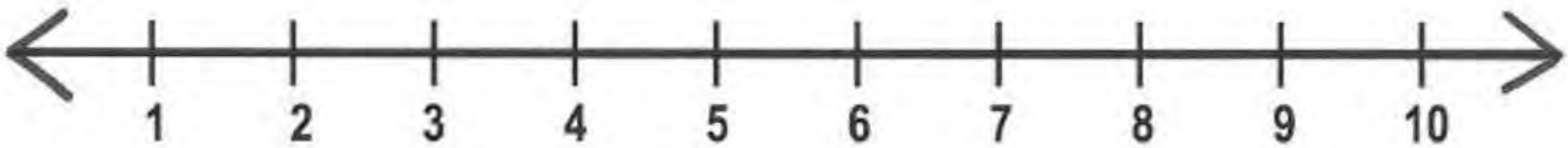
What does it mean to find the square root? Give an example of perfect squares.

Let's Talk:

How might we approximate the value of $\sqrt{2}$?

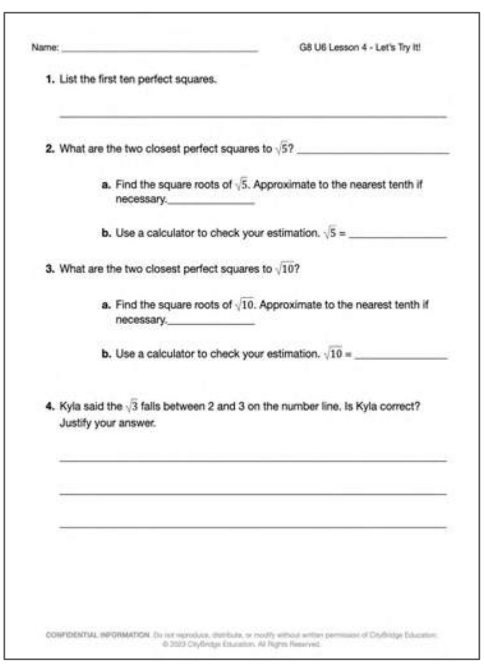
Let's Think:

Where might $\sqrt{10}$ be placed on the number line?



Let's Try It:

Let's find a decimal approximation for square roots and locate its approximation on the number line together.



Name: ______________________ G8 U6 Lesson 4 - Let's Try It!

1. List the first ten perfect squares.

____

2. What are the two closest perfect squares to $\sqrt{5}$? ______________

   a. Find the square roots of $\sqrt{5}$. Approximate to the nearest tenth if necessary.__________

   b. Use a calculator to check your estimation. $\sqrt{5}$ = __________

3. What are the two closest perfect squares to $\sqrt{10}$?

   a. Find the square roots of $\sqrt{10}$. Approximate to the nearest tenth if necessary.__________

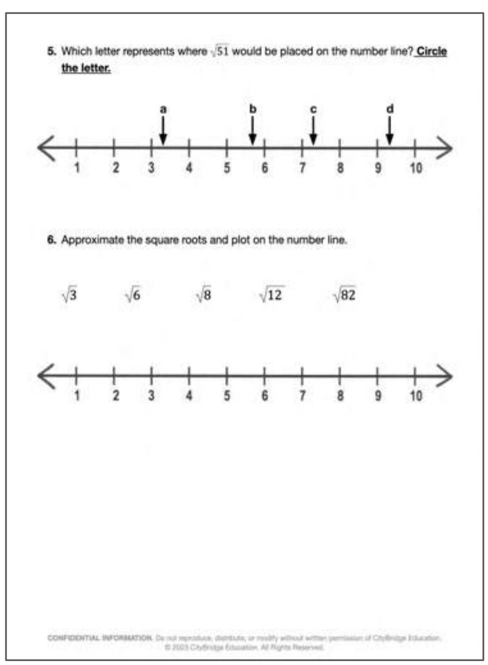
   b. Use a calculator to check your estimation. $\sqrt{10}$ = __________

4. Kyla said the $\sqrt{3}$ falls between 2 and 3 on the number line. Is Kyla correct? Justify your answer.

____

____

____



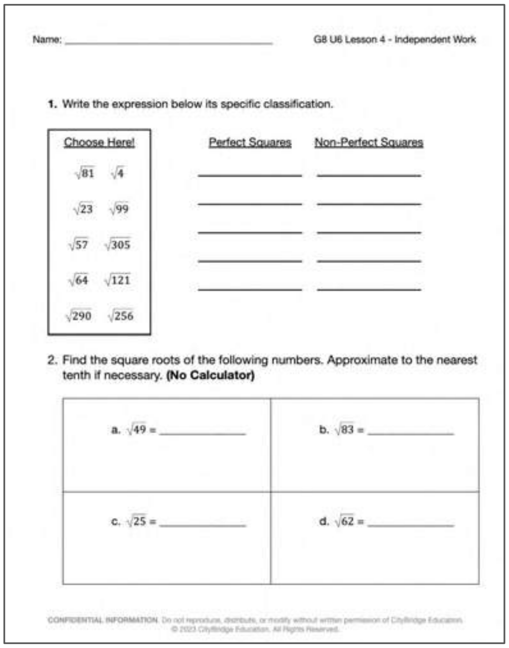
5. Which letter represents where $\sqrt{51}$ would be placed on the number line? **<u>Circle the letter.</u>**

6. Approximate the square roots and plot on the number line.

$\sqrt{3}$ $\sqrt{6}$ $\sqrt{8}$ $\sqrt{12}$ $\sqrt{82}$

On your Own:

Now it's your time to find a decimal approximation for square roots and locate it on the number line on your own.



Name: ______________________ G8 U6 Lesson 4 - Independent Work

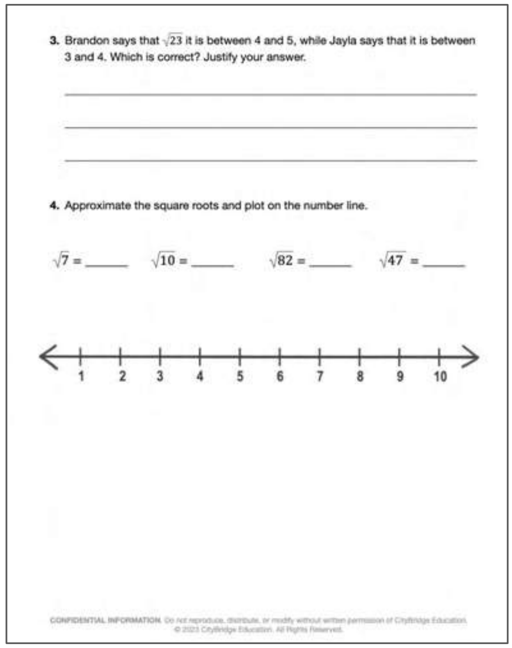
1. Write the expression below its specific classification.

Choose Here!	
$\sqrt{81}$	$\sqrt{4}$
$\sqrt{23}$	$\sqrt{99}$
$\sqrt{57}$	$\sqrt{305}$
$\sqrt{64}$	$\sqrt{121}$
$\sqrt{290}$	$\sqrt{256}$

Perfect Squares	Non-Perfect Squares
________	________
________	________
________	________
________	________
________	________

2. Find the square roots of the following numbers. Approximate to the nearest tenth if necessary. **(No Calculator)**

a. $\sqrt{49}$ = ________	b. $\sqrt{83}$ = ________
c. $\sqrt{25}$ = ________	d. $\sqrt{62}$ = ________



3. Brandon says that $\sqrt{23}$ it is between 4 and 5, while Jayla says that it is between 3 and 4. Which is correct? Justify your answer.

____

____

____

4. Approximate the square roots and plot on the number line.

$\sqrt{7}$ = _____ $\sqrt{10}$ = _____ $\sqrt{82}$ = _____ $\sqrt{47}$ = _____

Name: ____ G8 U6 Lesson 4 - Let's Try It

1. List the first ten perfect squares.

____

2. What are the two closest perfect squares to $\sqrt{5}$? _________________________________

   a. Find the square roots of $\sqrt{5}$. Approximate to the nearest tenth if necessary.________________

   b. Use a calculator to check your estimation. $\sqrt{5}$ = ________________________

3. What are the two closest perfect squares to $\sqrt{10}$? _________________________________

   a. Find the square roots of $\sqrt{10}$. Approximate to the nearest tenth if necessary.________________

   b. Use a calculator to check your estimation. $\sqrt{10}$ = ________________________

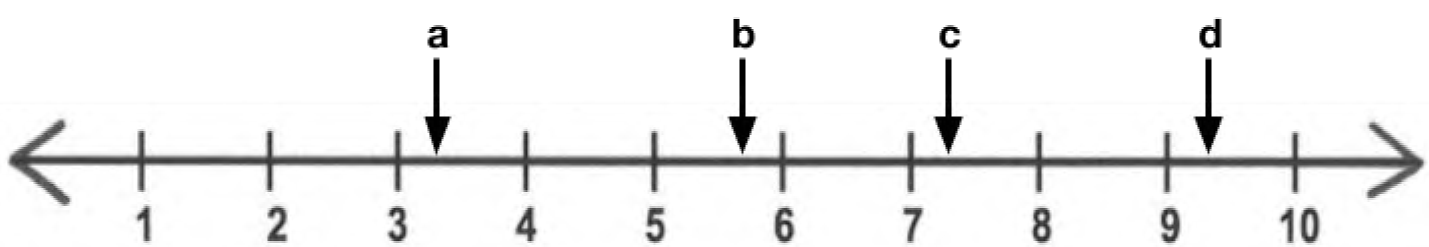
4. Kyla said the $\sqrt{3}$ falls between 2 and 3 on the number line. Is Kyla correct? Justify your answer.

____

____

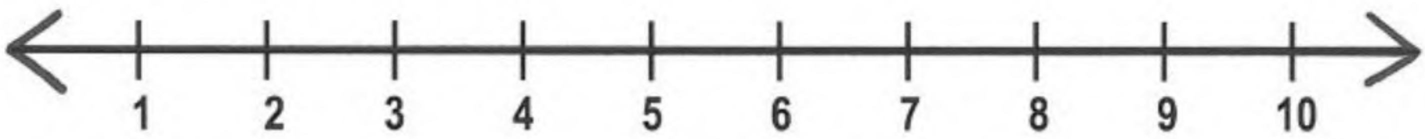
____

5. Which letter represents where $\sqrt{51}$ would be placed on the number line? **Circle the letter.**



6. Approximate the square roots and plot on the number line.

$\sqrt{3}$ $\sqrt{6}$ $\sqrt{8}$ $\sqrt{12}$ $\sqrt{82}$



1. Write the expression below its specific classification.

Choose Here!	
$\sqrt{81}$	$\sqrt{4}$
$\sqrt{23}$	$\sqrt{99}$
$\sqrt{57}$	$\sqrt{305}$
$\sqrt{64}$	$\sqrt{121}$
$\sqrt{290}$	$\sqrt{256}$

Perfect Squares	Non-Perfect Squares
____________	____________
____________	____________
____________	____________
____________	____________
____________	____________

2. Find the square roots of the following numbers. Approximate to the nearest tenth if necessary. **(No Calculator)**

a. $\sqrt{49}$ = ____________	b. $\sqrt{83}$ = ____________
c. $\sqrt{25}$ = ____________	d. $\sqrt{62}$ = ____________

3. Brandon says that $\sqrt{23}$ it is between 4 and 5, while Jayla says that it is between 3 and 4. Which is correct? Justify your answer.

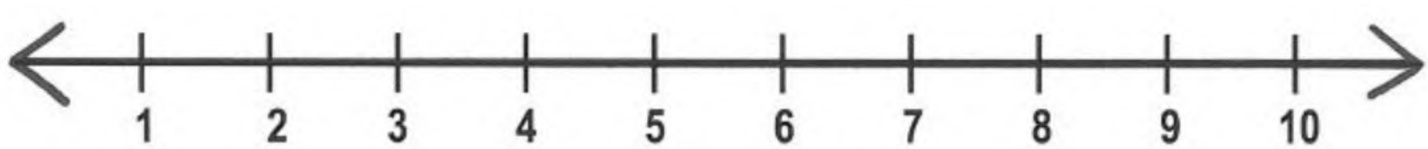
____

____

____

4. Approximate the square roots and plot on the number line.

$\sqrt{7}$ = ________ $\sqrt{10}$ = ________ $\sqrt{82}$ = ________ $\sqrt{47}$ = ________



Name: Answer Key

1. List the first ten perfect squares.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

2. What are the two closest perfect squares to $\sqrt{5}$? 4 and 9

   a. Find the square roots of $\sqrt{5}$. Approximate to the nearest tenth if necessary. 2.2

   b. Use a calculator to check your estimation. $\sqrt{5}$ = 2.23

3. What are the two closest perfect squares to $\sqrt{10}$? 9 and 16

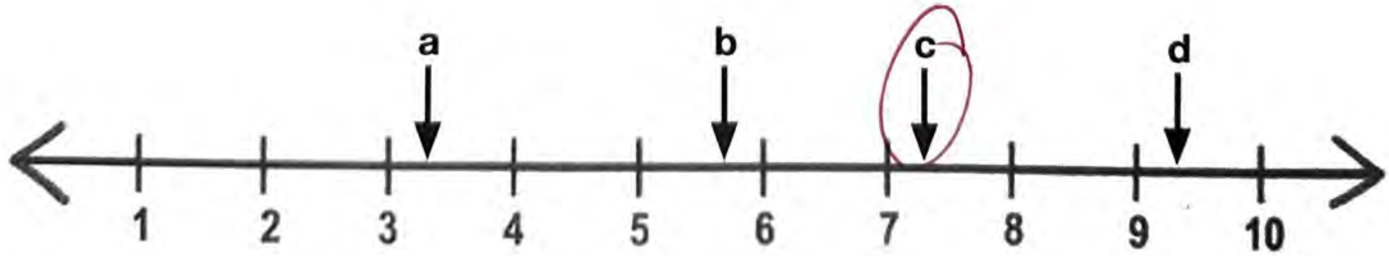
   a. Find the square roots of $\sqrt{10}$. Approximate to the nearest tenth if necessary. 3.1

   b. Use a calculator to check your estimation. $\sqrt{10}$ = 3.16

4. Kyla said the $\sqrt{3}$ falls between 2 and 3 on the number line. Is Kyla correct? Justify your answer.

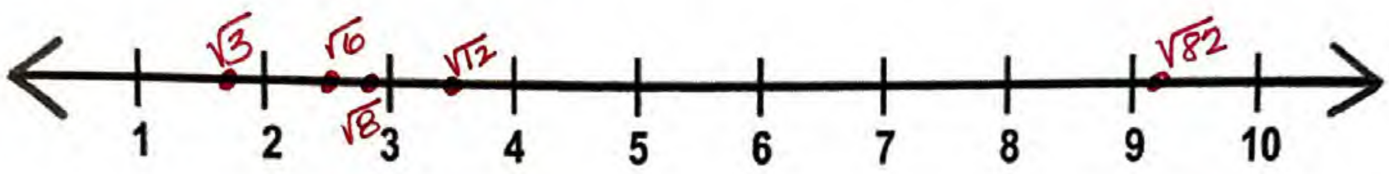
Kyla is incorrect. $\sqrt{3}$ falls between 1 and 4. $\sqrt{1}=1$ and $\sqrt{4}=2$, which means it would be between 1 and 2.

5. Which letter represents where $\sqrt{51}$ would be placed on the number line? **Circle the letter.**



6. Approximate the square roots and plot on the number line.

$\sqrt{3}$ $\sqrt{6}$ $\sqrt{8}$ $\sqrt{12}$ $\sqrt{82}$



Name: Answer Key

1. Write the expression below its specific classification.

Choose Here!	
$\sqrt{81}$	$\sqrt{4}$
$\sqrt{23}$	$\sqrt{99}$
$\sqrt{57}$	$\sqrt{305}$
$\sqrt{64}$	$\sqrt{121}$
$\sqrt{290}$	$\sqrt{256}$

Perfect Squares	Non-Perfect Squares
$\sqrt{81}$	$\sqrt{23}$
$\sqrt{64}$	$\sqrt{57}$
$\sqrt{4}$	$\sqrt{99}$
$\sqrt{64}$	$\sqrt{290}$
$\sqrt{256}$	$\sqrt{305}$

2. Find the square roots of the following numbers. Approximate to the nearest tenth if necessary. **(No Calculator)**

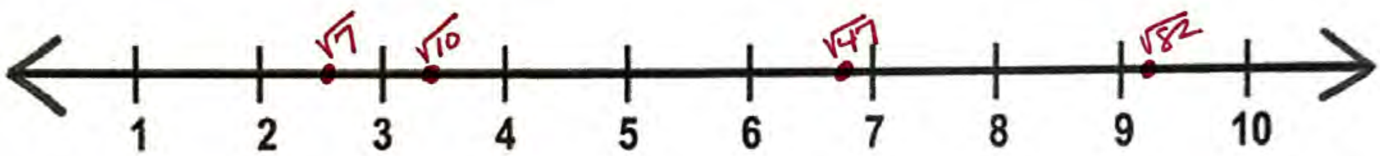
a. $\sqrt{49}$ = 7	b. $\sqrt{83}$ = 9.1
c. $\sqrt{25}$ = 5	d. $\sqrt{62}$ = 7.9

3. Brandon says that $\sqrt{23}$ it is between 4 and 5, while Jayla says that it is between 3 and 4. Which is correct? Justify your answer.

Brandon is correct $\sqrt{23}$ falls betwen 4 and 5.
$\sqrt{23}$ is betwen $\sqrt{16}$ and $\sqrt{25}$. $\sqrt{16}=4$ and
$\sqrt{25}=5$

4. Approximate the square roots and plot on the number line.

$\sqrt{7}$ = 2.6 $\sqrt{10}$ = 3.2 $\sqrt{82}$ = 9.1 $\sqrt{47}$ = 6.9



G8 U6 Lesson 5
Find a decimal approximation for square roots and locate its approximation on the number line

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G8 U6 Lesson 5 - Students will identify the two whole number values that a square root is between.

Materials: **Index Cards**

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we will learn how to identify the two whole numbers that a square root is between. Remember from our previous lesson that the square roots of perfect squares are whole numbers, but the square roots of other numbers fall between two whole numbers.

Let's Review (Slide 3): *(Pass out index cards)* Let's take 2 minutes to list the first 10 square numbers. Use your index card to jot them down. Possible Student Answers, Key Points:

- 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 *(Have students call out and write them on the board)*

These numbers are important because they help us find the square roots of numbers that aren't perfect squares.

Let's Talk (Slide 4): **Can anyone explain how we know that the $\sqrt{31}$ is between 5 and 6 on a number line?** Possible Student Answers, Key Points:

- Find the approximation of the square root.
- Find the two perfect squares that $\sqrt{31}$ is between.

$$\sqrt{31}$$

(Write $\sqrt{31}$ on the board) Good explanations. Let's look at the number 31. Is 31 a perfect square? No, it's not. But we can find the two square numbers it's between. What are the square numbers closest to 31? *(Refer to the list on the board)* 25 and 36

$$\sqrt{31}$$
$$\sqrt{25} \quad \sqrt{36}$$
$$5 < \sqrt{31} < 6$$

Exactly! The square root of 31 is somewhere between the square roots of 25 and 36. So, we know that $\sqrt{25} = 5$ and $\sqrt{36} = 6$. Therefore, $5 < \sqrt{31} < 6$.

Remember that square roots of perfect squares can help us find two whole numbers that non-perfect square roots fall between.

Let's Think (Slide 5): Now, think about this another way. We can tell which number the non-perfect square root is closest to. **Explain how you know that $\sqrt{35}$ is a little less than 6.** Possible Student Answers, Key Points:

- Find which squared numbers $\sqrt{35}$ is closest to, 25 and 36.
- $\sqrt{36} = 6$.
- $\sqrt{35}$ is closest to $\sqrt{36}$, but less.

$$\sqrt{35}$$
$$\sqrt{25} \quad \sqrt{36}$$
$$\sqrt{25} = 5 \quad \sqrt{36} = 6$$

(Write $\sqrt{35}$ on the board) Those are all good ideas! Just as before we can find the two closest square roots to $\sqrt{35}$. Those two would be 25 and 36. Remember that $\sqrt{25} = 5$ and $\sqrt{36} = 6$. Therefore, $\sqrt{35}$ would be a little less than 6 since, 35 is closest to 36.

Remember as you work today, you can use a calculator as tool to check your work.

Let's Try it (Slides 6): Now, I want you to try a few on your own. I'll be here to help if you need it. Remember, find the closest square numbers first and then identify the two whole numbers the square root is between.

WARM WELCOME

Today we will learn how to find the two whole numbers that a square root is between.

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Let's Review:

List the first ten square numbers.

Let's Talk:

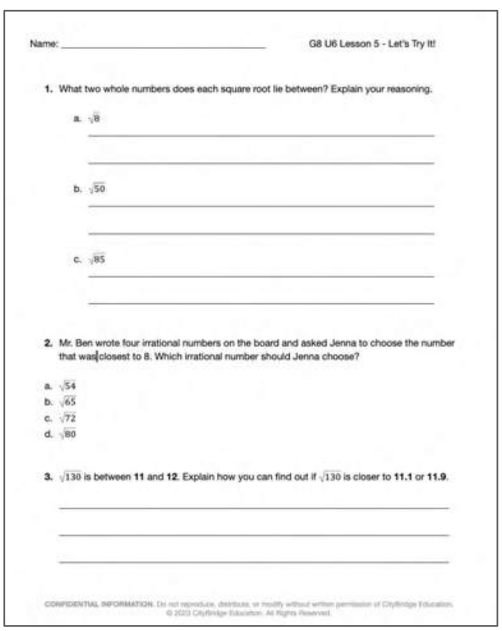
Explain how you know that $\sqrt{31}$ lies between 5 and 6 on a number line.

Let's Think:

Explain how you know that $\sqrt{35}$ is a little less than 6.

Let's Try It:

Let's explore how identify the two whole number values that a square root is between together.

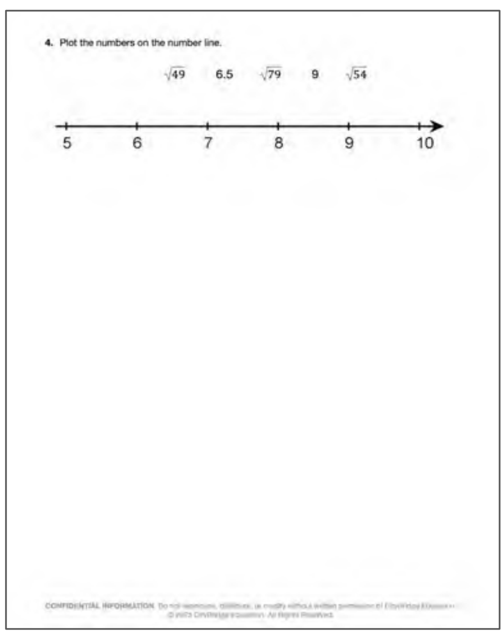

Name: ______________________ G8 U6 Lesson 5 - Let's Try It!

1. What two whole numbers does each square root lie between? Explain your reasoning.
   a. $\sqrt{8}$
   b. $\sqrt{50}$
   c. $\sqrt{85}$

2. Mr. Ben wrote four irrational numbers on the board and asked Jenna to choose the number that was closest to 8. Which irrational number should Jenna choose?

a. $\sqrt{54}$
b. $\sqrt{65}$
c. $\sqrt{72}$
d. $\sqrt{80}$

3. $\sqrt{130}$ is between **11** and **12**. Explain how you can find out if $\sqrt{130}$ is closer to **11.1** or **11.9**.


4. Plot the numbers on the number line.

$\sqrt{49}$ 6.5 $\sqrt{79}$ 9 $\sqrt{54}$

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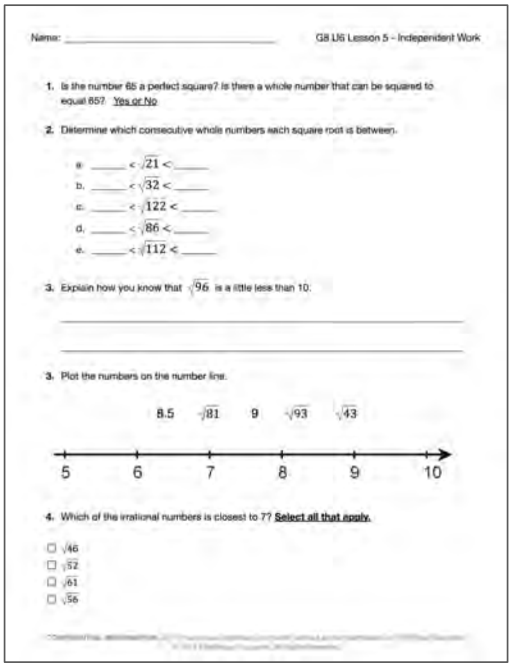
On your Own:

Now it's your time identify the two whole number values that a square root is between on your own.



Name: ______________________ G8 U6 Lesson 5 - Independent Work

1. Is the number 65 a perfect square? Is there a whole number that can be squared to equal 65? Yes or No

2. Determine which consecutive whole numbers each square root is between.

   a. ____ $< \sqrt{21} <$ ____
   b. ____ $< \sqrt{32} <$ ____
   c. ____ $< \sqrt{122} <$ ____
   d. ____ $< \sqrt{86} <$ ____
   e. ____ $< \sqrt{112} <$ ____

3. Explain how you know that $\sqrt{96}$ is a little less than 10.

____

____

3. Plot the numbers on the number line.

8.5 $\sqrt{81}$ 9 $\sqrt{93}$ $\sqrt{43}$

5 6 7 8 9 10

4. Which of the irrational numbers is closest to 7? **Select all that apply.**

☐ $\sqrt{46}$
☐ $\sqrt{52}$
☐ $\sqrt{61}$
☐ $\sqrt{56}$

1. What two whole numbers does each square root lie between? Explain your reasoning.

a. $\sqrt{8}$

____

____

b. $\sqrt{50}$

____

____

c. $\sqrt{85}$

____

____

2. Mr. Ben wrote four irrational numbers on the board and asked Jenna to choose the number closest to 8. Which irrational number should Jenna choose?

a. $\sqrt{54}$
b. $\sqrt{65}$
c. $\sqrt{72}$
d. $\sqrt{80}$

3. $\sqrt{130}$ is between **11** and **12.** Explain how you can find out if $\sqrt{130}$ is closer to **11.1** or **11.9.**

____

____

____

4. Plot the numbers on the number line.

$\sqrt{49}$ 6.5 $\sqrt{79}$ 9 $\sqrt{54}$

1. Is the number 65 a perfect square? Is there a whole number that can be squared to equal 65? Yes or No

2. Determine which consecutive whole numbers each square root is between.

a. _______ $< \sqrt{21} <$ _______

b. _______ $< \sqrt{32} <$ _______

c. _______ $< \sqrt{122} <$ _______

d. _______ $< \sqrt{86} <$ _______

e. _______ $< \sqrt{112} <$ _______

3. Explain how you know that $\sqrt{96}$ is a little less than 10.

____

____

3. Plot the numbers on the number line.

8.5 $\sqrt{81}$ 9 $\sqrt{93}$ $\sqrt{43}$

4. Which of the irrational numbers is closest to 7? **Select all that apply.**

☐ $\sqrt{46}$
☐ $\sqrt{52}$
☐ $\sqrt{61}$
☐ $\sqrt{56}$

Name: Answer Key

1. What two whole numbers does each square root lie between? Explain your reasoning.

   a. $\sqrt{8}$

   $\sqrt{4} < \sqrt{8} < \sqrt{9}$, $2 < \sqrt{8} < 3$

   b. $\sqrt{50}$

   $\sqrt{49} < \sqrt{50} < \sqrt{64}$

   $7 < \sqrt{50} < 8$

   c. $\sqrt{85}$

   $\sqrt{81} < \sqrt{85} < \sqrt{100}$

   $9 < \sqrt{85} < 10$

2. Mr. Ben wrote four irrational numbers on the board and asked Jenna to choose the number closest to 8. Which irrational number should Jenna choose?

a. $\sqrt{54}$
b. $\sqrt{65}$ (circled)
c. $\sqrt{72}$
d. $\sqrt{80}$

3. $\sqrt{130}$ is between **11** and **12**. Explain how you can find out if $\sqrt{130}$ is closer to **11.1** or **11.9**.

You can multiply 11.1 by itself and 11.9 by itself to see which is closer to 130.

4. Plot the numbers on the number line.

$\sqrt{49}$ 6.5 $\sqrt{79}$ 9 $\sqrt{54}$

Name: Answer Key

1. Is the number 65 a perfect square? Is there a whole number that can be squared to equal 65? Yes or (No)

2. Determine which consecutive whole numbers each square root is between.

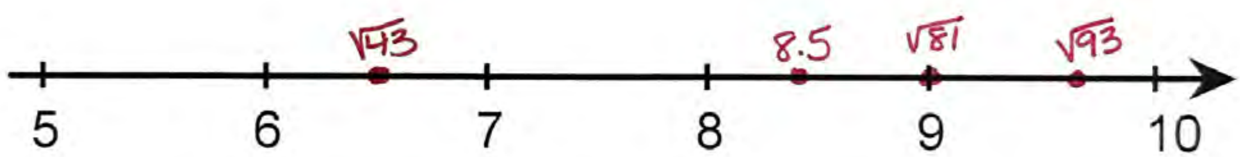
   a. 4 $< \sqrt{21} <$ 5
   b. 5 $< \sqrt{32} <$ 6
   c. 11 $< \sqrt{122} <$ 12
   d. 9 $< \sqrt{86} <$ 10
   e. 10 $< \sqrt{112} <$ 11

3. Explain how you know that $\sqrt{96}$ is a little less than 10.

$\sqrt{100} = 10$, since 96 is less than 100, then $\sqrt{96}$ would be less than $\sqrt{100}$

3. Plot the numbers on the number line.

8.5 $\sqrt{81}$ 9 $\sqrt{93}$ $\sqrt{43}$



4. Which of the irrational numbers is closest to 7? **Select all that apply.**

- [x] $\sqrt{46}$
- [x] $\sqrt{52}$
- [ ] $\sqrt{61}$
- [ ] $\sqrt{56}$

G8 U6 Lesson 6
Comprehend the term "Pythagorean Theorem" as the equation a2 + b2 = c2.

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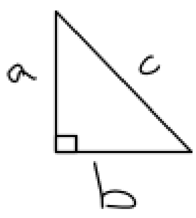
G8 U6 Lesson 6 - Students will comprehend the term "Pythagorean Theorem" as the equation $a^2 + b^2 = c^2$.

Warm Welcome (Slide 1): Tutor choice

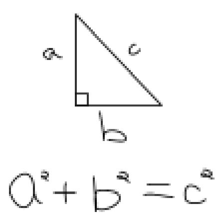
Frame the Learning/Connect to Prior Learning (Slide 2): Today, we're going to learn about a very important theorem in geometry, the Pythagorean Theorem. A theorem is a statement that has been proven or can be proven. We've learned about squaring numbers and finding the square root, which will be helpful today.

Let's Talk (Slide 3): Before we dive in, can anyone tell me what a right triangle is? Possible Student Answers, Key Points:

- Has a right angle or 90-degree angle
- Has three sides
- has two legs and a hypotenuse

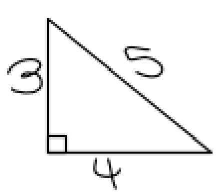


Exactly! A right triangle has one angle that is 90 degrees. Now, let's look at this right triangle." *(Draw a right triangle on the board and label the sides a (leg), b (leg), and c, with c being the hypotenuse.)* "a" and "b" are legs of the triangle and "c" is the longest side of the right triangle, which is also always across from the right angle, we call it the hypotenuse. Now let's say it together: HYPOTENUSE!



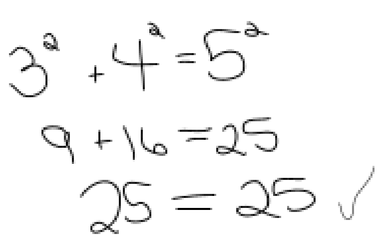
The Pythagorean Theorem states that in a right triangle, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the other two sides. We can write this as $a^2 + b^2 = c^2$.

Let's Think (Slide 4): Remember I said that the Pythagorean Theorem can be proved, and it can help us prove things about right triangles. Using the Pythagorean Theorem, $a^2 + b^2 = c^2$, determine if this triangle is indeed a right triangle. The Pythagorean Theorem can only be true for right triangles.



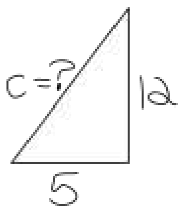
(Draw the triangle on the board.) So which sides could be labeled "a" and "b"? 3 or 4. Great. Which of the sides is the hypotenuse? The side that is 5.

So if we square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides, then we know that this is a right triangle.

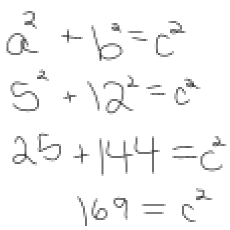


We first set 3 squared plus 4 squared equal to the hypotenuse squared, which is 5 squared. 3 squared is 9, and 4 squared is 16. 5 squared is 25. 9 + 16 equals 25, so that means that 25 = 25. This is a true statement. Therefore, these side lengths make a right triangle because the hypotenuse squared equals the sum of the squares of the other two sides.

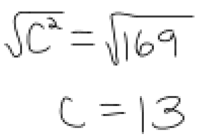
Let's Think (Slide 5): Now let's look at how we can find the length of the hypotenuse if we know the side lengths of the other two sides. Remember that $a^2 + b^2 = c^2$.



(Draw triangle on whiteboard.) Suppose we have a right triangle where one leg is 5 units long and the other leg is 12 units long. We want to find the length of the hypotenuse. According to the Pythagorean Theorem, we have a = 5, b = 12, and we need to find c

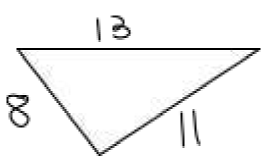


We can substitute the values we know using the formula $a^2 + b^2 = c^2$. 5 squared plus 12 squared equals "c" squared." We know that 5 squared equals 25 and 12 squared equals 144. The sum of the squares equals 169.

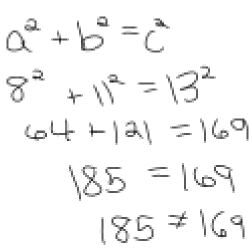


Now, we take the square root of both sides to find c. Remember, when we square root a squared number or variable, they cancel each other out. The square root of 169 equals 13. The hypotenuse is 13 units long.

Let's Think (Slide 6): Now let's look at one more example: **Is this a right triangle? Explain why or why not.**



(Draw non-right triangle on the whiteboard.) What can we use to help us determine whether this is a right triangle or not? The Pythagorean Theorem. Yes, we can test out this triangle using the Pythagorean Theorem. Since we know that the longest side is the hypotenuse, and we label that as "c," we will substitute 13 for c, and 8 and 11 will be our "a" and "b."



8 squared is 64, and 11 squared is 121. Remember, we set the sum of the squares equal to our longest side, in this case, 13 squared. 64 plus 121 is equal to 185, and 13 squared is equal to 169. Since 185 does not equal 185, we can say that this triangle is not a right triangle. The sides' lengths do not make for a right triangle, and the Pythagorean Theorem allowed us to test it.

Let's Try it (Slides 7): Great job today! Remember, the Pythagorean Theorem is a powerful tool for working with right triangles. We can use $a^2 + b^2 = c^2$ to us determine if a triangle is a right triangle or not and we can find missing side lengths by using the theorem.

WARM WELCOME

Today we will comprehend the term "Pythagorean Theorem" as the equation $a^2 + b^2 = c^2$..

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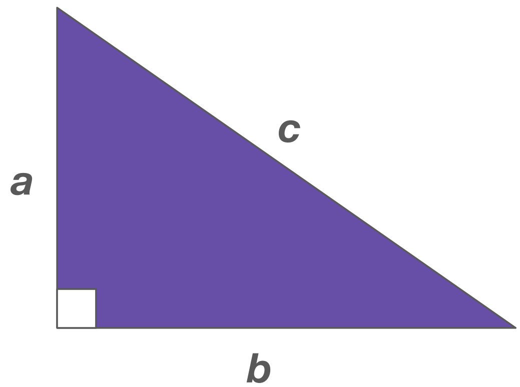
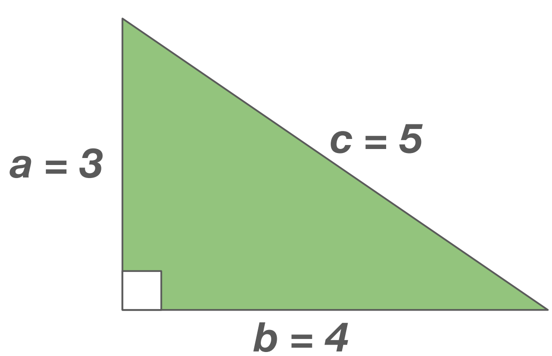
Let's Talk:

What is a right triangle?



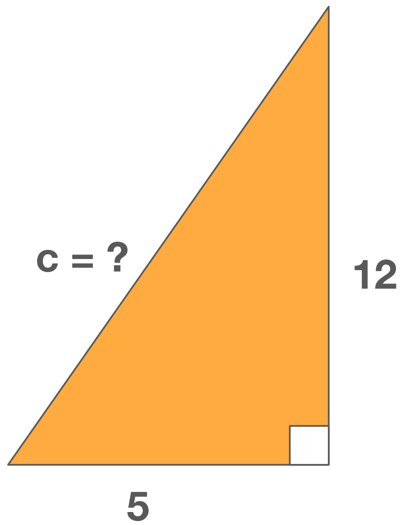
Let's Think:

Using the Pythagorean Theorem, $a^2 + b^2 = c^2$, determine if this a right triangle?



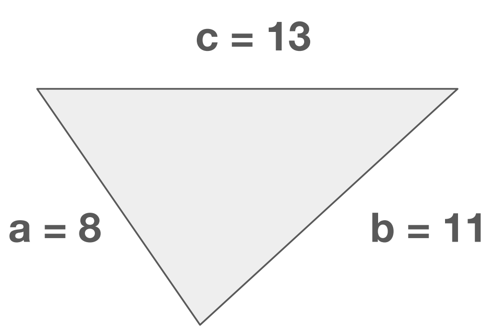
Let's Think:

How can we use the Pythagorean Theorem to solve for a the hypotenuse?



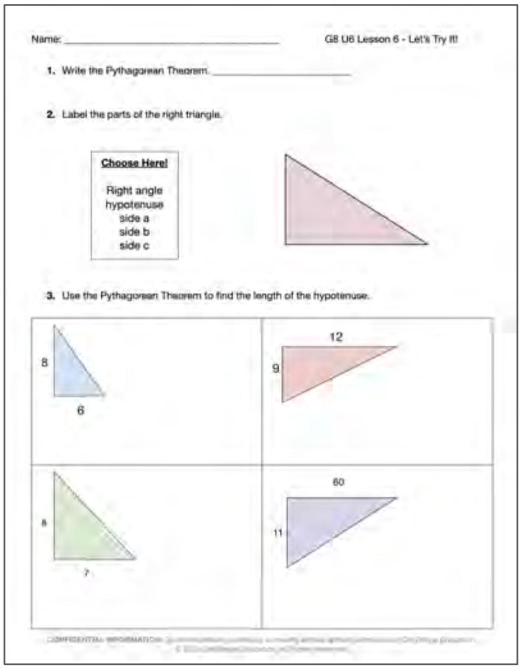
Let's Think:

Is this a right triangle? Explain why or why not.



Let's Try It:

Let's explore using the Pythagorean Theorem together.



Name: ______________________ G8 U6 Lesson 6 - Let's Try It!

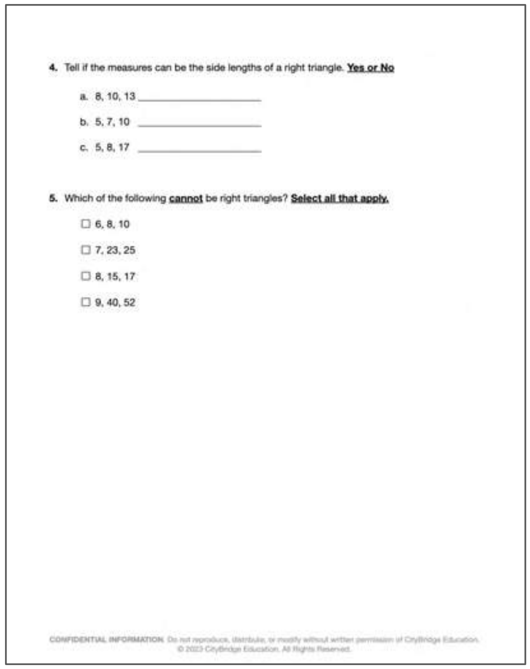
1. Write the Pythagorean Theorem. ______________________

2. Label the parts of the right triangle.

Choose Here!

Right angle
hypotenuse
side a
side b
side c

3. Use the Pythagorean Theorem to find the length of the hypotenuse.



4. Tell if the measures can be the side lengths of a right triangle. **Yes or No**

a. 8, 10, 13 ______________

b. 5, 7, 10 ______________

c. 5, 8, 17 ______________

5. Which of the following **cannot** be right triangles? **Select all that apply.**

☐ 6, 8, 10

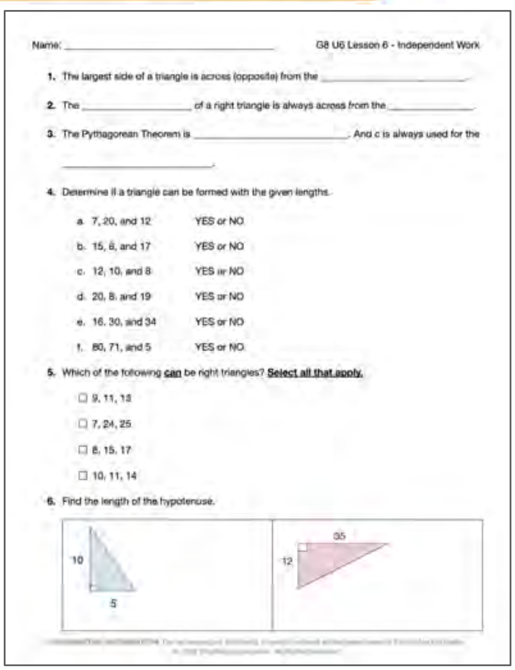
☐ 7, 23, 25

☐ 8, 15, 17

☐ 9, 40, 52

On your Own:

Now it's time use the Pythagorean Theorem on your own.



Name: ______________________ G8 U6 Lesson 6 - Independent Work

1. The largest side of a triangle is across (opposite) from the ______________.

2. The ______________ of a right triangle is always across from the ______________.

3. The Pythagorean Theorem is ______________________. And c is always used for the ______________________.

4. Determine if a triangle can be formed with the given lengths.

a. 7, 20, and 12 YES or NO

b. 15, 8, and 17 YES or NO

c. 12, 10, and 8 YES or NO

d. 20, 8, and 19 YES or NO

e. 16, 30, and 34 YES or NO

f. 80, 71, and 5 YES or NO

5. Which of the following **can** be right triangles? **Select all that apply.**

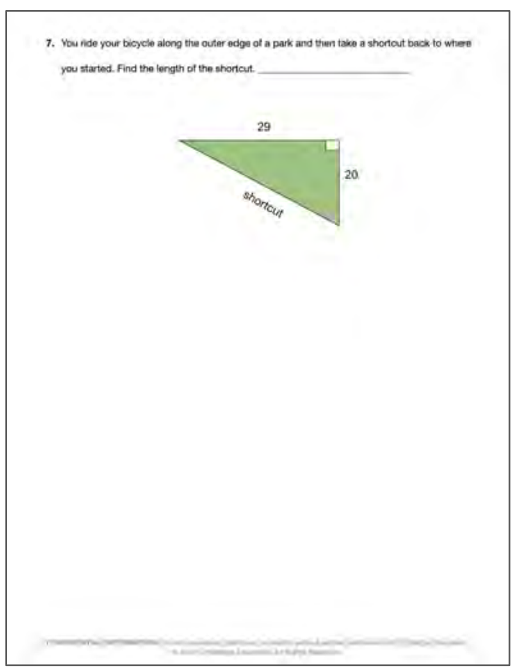
☐ 9, 11, 13

☐ 7, 24, 25

☐ 8, 15, 17

☐ 10, 11, 14

6. Find the length of the hypotenuse.



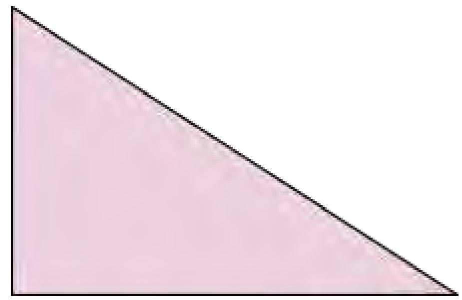
7. You ride your bicycle along the outer edge of a park and then take a shortcut back to where you started. Find the length of the shortcut. ______________

Name: ____ G8 U6 Lesson 6 - Let’s Try It

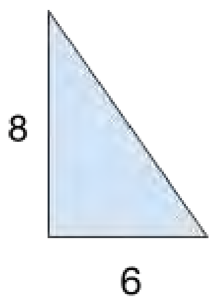
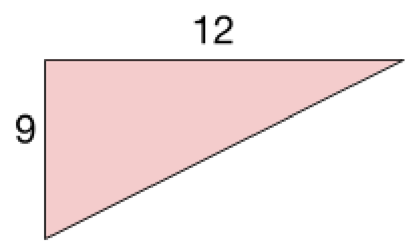
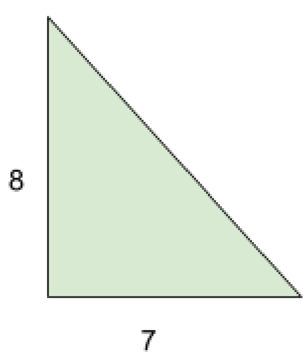
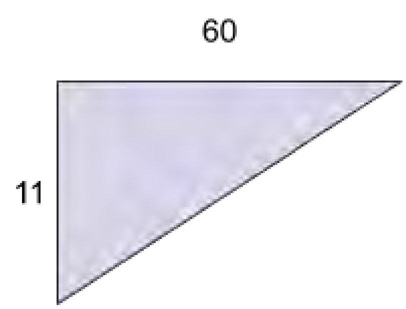
1. Write the Pythagorean Theorem. __________________________

2. Label the parts of the right triangle.

Choose Here!
Right angle
hypotenuse
side a
side b
side c



3. Use the Pythagorean Theorem to find the length of the hypotenuse.

4. Tell if the measures can be the side lengths of a right triangle. **Yes or No**

a. 8, 10, 13 ________________________

b. 5, 7, 10 ________________________

c. 5, 8, 17 ________________________

5. Which of the following **cannot** be right triangles? **Select all that apply.**

- ☐ 6, 8, 10
- ☐ 7, 23, 25
- ☐ 8, 15, 17
- ☐ 9, 40, 52

Name: ____

1. The largest side of a triangle is across (opposite) from the ____________________________.

2. The _____________________ of a right triangle is always across from the ________________.

3. The Pythagorean Theorem is ______________________________. And c is always used for the

_____________________________.

4. Determine if a triangle can be formed with the given lengths.

a. 7, 20, and 12 YES or NO

b. 15, 8, and 17 YES or NO

c. 12, 10, and 8 YES or NO

d. 20, 8, and 19 YES or NO

e. 16, 30, and 34 YES or NO

f. 80, 71, and 5 YES or NO

5. Which of the following **can** be right triangles? **Select all that apply.**

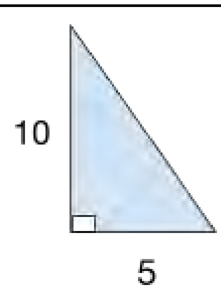
☐ 9, 11, 13

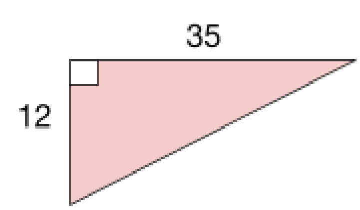
☐ 7, 24, 25

☐ 8, 15, 17

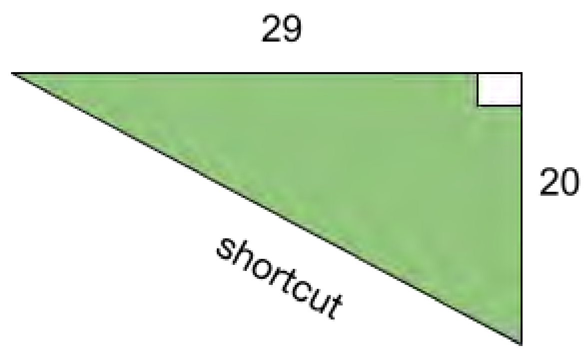
☐ 10, 11, 14

6. Find the length of the hypotenuse.





7. You ride your bicycle along the outer edge of a park and then take a shortcut back to where you started. Find the length of the shortcut. ___________________________



Name: Answer Key

1. The largest side of a triangle is across (opposite) from the right angle.

2. The hypotenuse of a right triangle is always across from the right angle.

3. The Pythagorean Theorem is $a^2 + b^2 = c^2$. And c is always used for the hypotenuse.

4. Determine if a triangle can be formed with the given lengths.

   a. 7, 20, and 12 — YES or **NO** (circled)

   b. 15, 8, and 17 — **YES** (circled) or NO

   c. 12, 10, and 8 — YES or **NO** (circled)

   d. 20, 8, and 19 — YES or **NO** (circled)

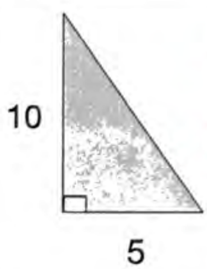
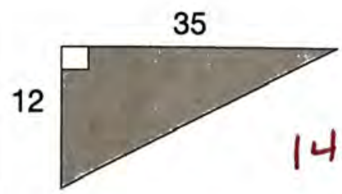
   e. 16, 30, and 34 — **YES** (circled) or NO

   f. 80, 71, and 5 — YES or **NO** (circled)

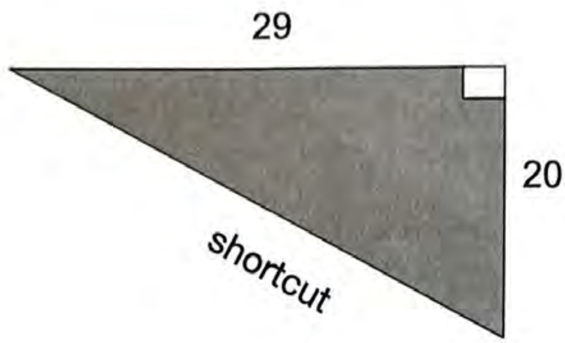
5. Which of the following **can** be right triangles? **Select all that apply.**

   - [ ] 9, 11, 13
   - [x] 7, 24, 25
   - [x] 8, 15, 17
   - [ ] 10, 11, 14

6. Find the length of the hypotenuse.

 $5^2 + 10^2 = c^2$ $25 + 100 = c^2$ $125 = c^2$ $c = \sqrt{125}$	 $12^2 + 35^2 = c^2$ $144 + 1225 = c^2$ $1369 = c^2$ $\sqrt{1369} = c$ $c = 37$

7. You ride your bicycle along the outer edge of a park and then take a shortcut back to where you started. Find the length of the shortcut. 35.2 units



$20^2 + 29^2 = c^2$

$400 + 841 = c^2$

$1241 = c^2$

$c = \sqrt{1241} \approx 35.2$

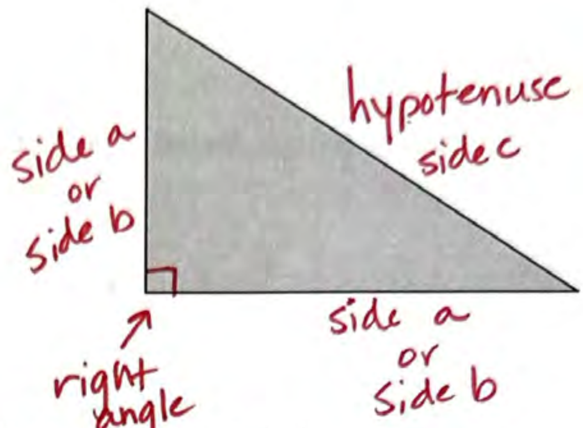
Name: Answer Key

G8 U6 Lesson 6 - Let's Try It!

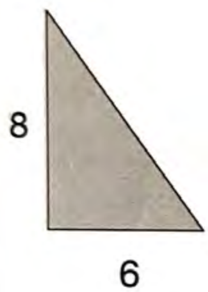
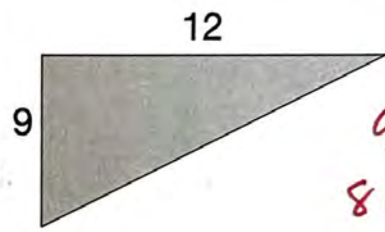
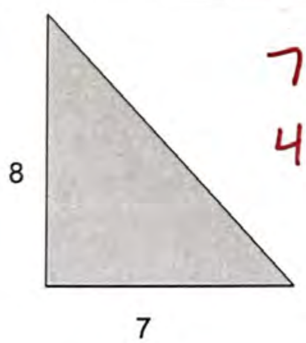
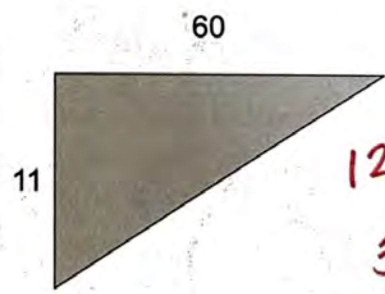
1. Write the Pythagorean Theorem. $a^2 + b^2 = c^2$

2. Label the parts of the right triangle.

Choose Here!
Right angle hypotenuse side a side b side c



3. Use the Pythagorean Theorem to find the length of the hypotenuse.

 $6^2 + 8^2 = c^2$ $36 + 64 = c^2$ $100 = c^2$ $\sqrt{100} = c$ $c = 10$	 $9^2 + 12^2 = c^2$ $81 + 144 = c^2$ $225 = c^2$ $\sqrt{225} = c$ $c = 15$
 $7^2 + 8^2 = c^2$ $49 + 64 = c^2$ $113 = c^2$ $\sqrt{113} = c$ $c \approx 10.6$	 $11^2 + 60^2 = c^2$ $121 + 3600 = c^2$ $3721 = c^2$ $\sqrt{3721} = c$ $c = 61$

4. Tell if the measures can be the side lengths of a right triangle. **Yes or No**

a. 8, 10, 13 No

b. 5, 7, 10 No

c. 5, 8, 17 No

5. Which of the following **cannot** be right triangles? **Select all that apply.**

- ☐ 6, 8, 10
- ☑ 7, 23, 25
- ☐ 8, 15, 17
- ☑ 9, 40, 52

G8 U6 Lesson 7
Explain the Pythagorean Theorem proof and calculate unknown sides

CITYTUTOR DC

G8 U6 Lesson 7 - Students will explain the Pythagorean proof and calculate unknown sides.

Warm Welcome (Slide 1): Tutor choice

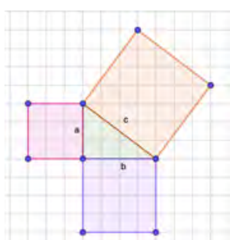
Frame the Learning/Connect to Prior Learning (Slide 2): Today, we're going to continue to explore one of the most famous theorems in mathematics: the Pythagorean Theorem. By the end of this lesson, you will understand a proof of the theorem and be able to calculate an unknown side length of a right triangle using the Pythagorean Theorem.

Let's Talk (Slide 3): In our last lesson, we learned about the Pythagorean Theorem. **When thinking about that, how could we determine if this is a right triangle or not?** Possible Student Answers, Key Points:

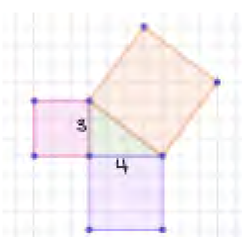
- Pythagorean Theorem
- Add the sum of the squares of sides "a" and "b", and they should equal the square of side "c."

That's right, we could use the Pythagorean Theorem. The Pythagorean Theorem states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides. This can be written as $a^2 + b^2 = c^2$.

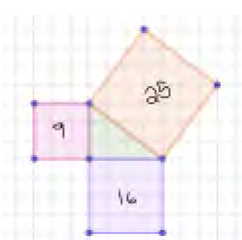
But why is the Pythagorean Theorem true? Let's see why this is true with a visual proof.



(Draw a large right triangle on the whiteboard, then draw squares on each triangle's sides. Show that the area of the square on the hypotenuse equals the sum of the areas of the squares on the other two sides.)



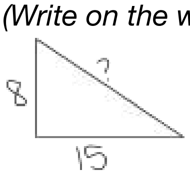
Here, we have a right triangle with legs of 3 and 4 units. Let's calculate the area of their squares. $A = 3^2 = 9$ and $A = 4^2 = 16$



According to the Pythagorean Theorem, the hypotenuse should be 5 units. The area of the square on the hypotenuse is 25, which is the same as the sum of the areas of the squares on the other two sides, 9 and 16.

So we see in this proof that the sum of the squares of the legs equals the square of the hypotenuse. This should always work for the Pythagorean Theorem.

Let's Think (Slide 4): We can use the Pythagorean Theorem to find the hypotenuse. If we know two side lengths, then we can find the hypotenuse. Can someone help me get started? *(Allow Students to walk you through the steps)* What are sides "a" and "b?" 8 and 15. What is the Pythagorean Theorem? $a^2 + b^2 = c^2$ *(Write on the whiteboard and Draw a right triangle)*



$$a^2 + b^2 = c^2$$
$$8^2 + 15^2 = c^2$$
$$64 + 225 = c^2$$
$$289 = c^2$$

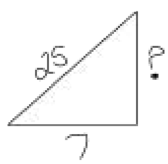
Let's substitute our values for "a" and "b." We get $8^2 + 15^2 = c^2$. $8^2 = 64$ and $15^2 = 225$. When we find the sum of the squares, we get 289. We have to find the value of "c," but now we have "c^2." We have to take the "Square root" of both sides of the equal sign to find "c." The square root of "c^2" cancels out the squared part, leaving you with "c." The square root of 289 is 17. That means the side "c", the hypotenuse, equals 17.

$$289 = c^2$$
$$\sqrt{289} = \sqrt{c^2}$$
$$c = 17$$

Let's Think (Slide 5): Now let's imagine that we know one leg (or side) and the hypotenuse of the right triangle; how could we use the Pythagorean Theorem to find the other side? Possible Student Answers, Key Points:

- Substitute into the Pythagorean
- Subtract the square of the hypotenuse and the square of the given side.

Those are great ideas! We can still use the Pythagorean Theorem, instead of adding the sum of squares when we know two sides, this time we are going to subtract the square of the hypotenuse and the square of the side that we know.



We can substitute what we know. We know that "a" is equal to "7" and that "c" is equal to "25." First, let's write the Pythagorean Theorem, $a^2 + b^2 = c^2$.

- $7^2 + b^2 = 25^2$
  - $b^2 = 25^2 - 7^2$
  - $b^2 = 625 - 49$
  - $b^2 = 576$
  - $\sqrt{b^2} = \sqrt{576}$
  - $b = 24$

$$a^2 + b^2 = c^2$$
$$7^2 + b^2 = 25^2$$
$$b^2 = 25^2 - 7^2$$
$$b^2 = 625 - 49$$
$$b^2 = 576$$
$$\sqrt{b^2} = \sqrt{576}$$
$$b = 24$$

So, the other leg is 24 units.

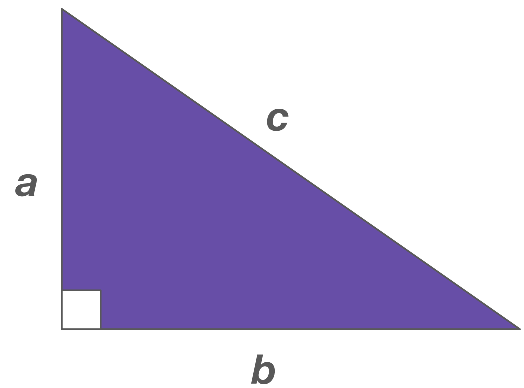
Let's Try it (Slides 6): Now it's your turn to try. Remember that the Pythagorean Theorem can help us find the length of the hypotenuse if we know two given side lengths, and we can find a side length if we know the hypotenuse and the other side. This is always true for right triangles.

WARM WELCOME

Today you will Explain the Pythagorean Theorem proof and calculate unknown sides.

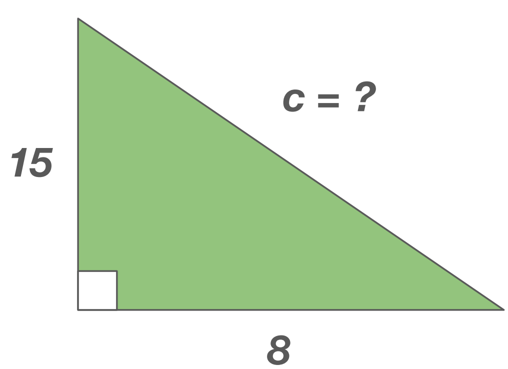
Let's Talk:

How could we determine if this is a right triangle or not?



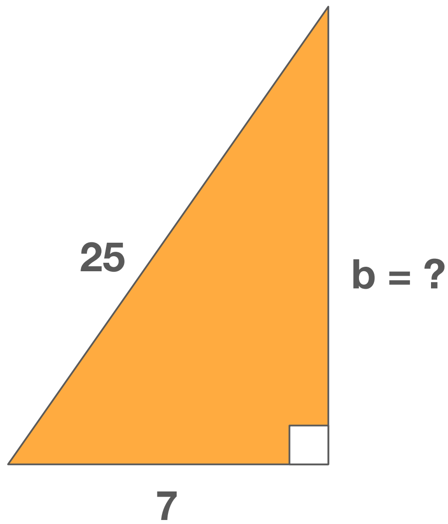
Let's Think:

How can we use the Pythagorean Theorem to solve for the hypotenuse?



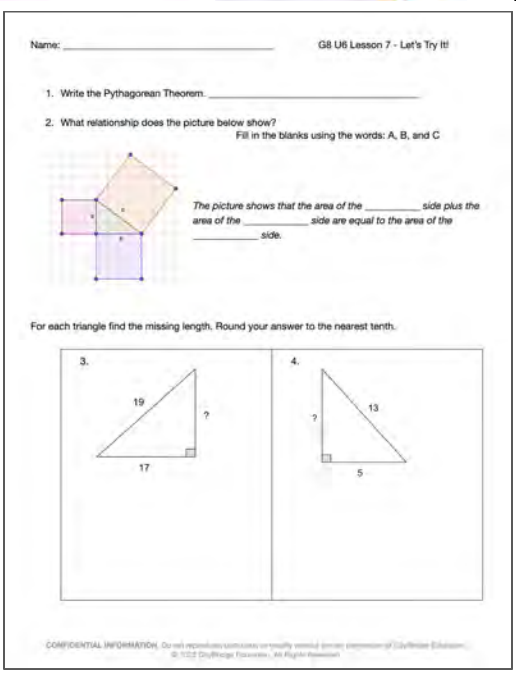
Let's Think:

How can we use the Pythagorean Theorem to solve for a missing leg?



Let's Try It:

Let's explore how to explain the Pythagorean proof and calculate unknown sides together.



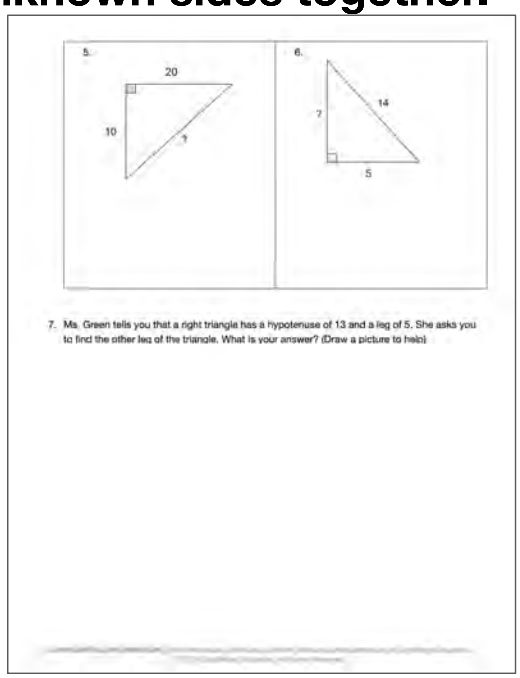
Name: ______________________ G8 U6 Lesson 7 - Let's Try It!

1. Write the Pythagorean Theorem. ______________________
2. What relationship does the picture below show?
   Fill in the blanks using the words: A, B, and C

   The picture shows that the area of the __________ side plus the area of the __________ side are equal to the area of the __________ side.

For each triangle find the missing length. Round your answer to the nearest tenth.

3. 19, ?, 17
4. ?, 13, 5

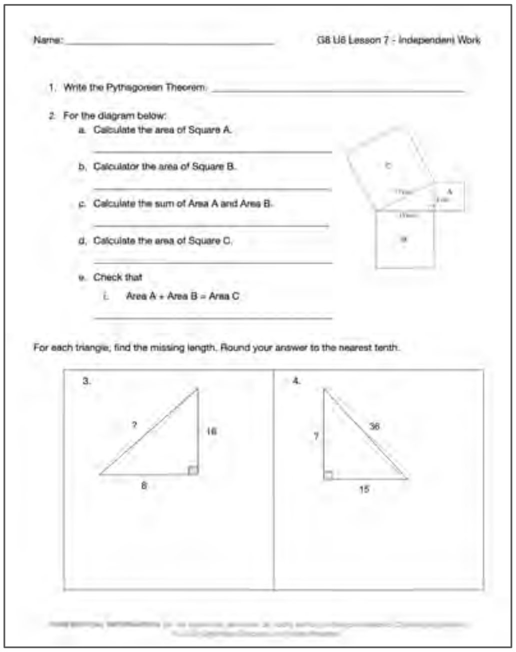


5. 20, 10, ?
6. ?, 14, 5

7. Ms. Green tells you that a right triangle has a hypotenuse of 13 and a leg of 5. She asks you to find the other leg of the triangle. What is your answer? (Draw a picture to help)

On your Own:

Now it's time to try explain the Pythagorean proof and calculate unknown sides on your own.



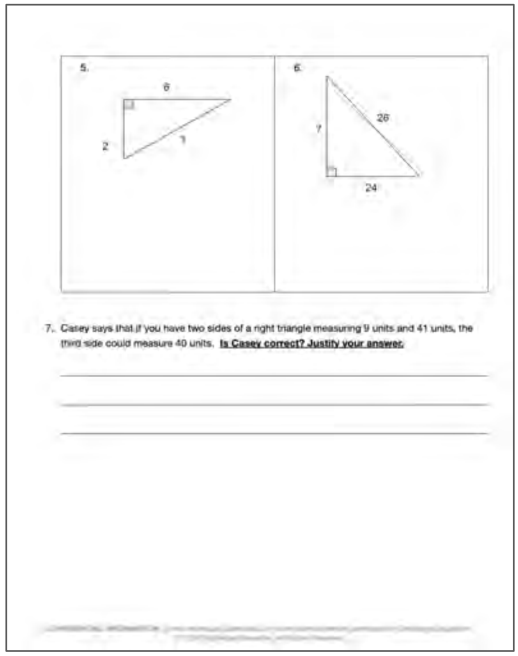
Name:____________________ G8 U8 Lesson 7 - Independent Work

1. Write the Pythagorean Theorem. ____________________
2. For the diagram below:
   a. Calculate the area of Square A.
   b. Calculator the area of Square B.
   c. Calculate the sum of Area A and Area B.
   d. Calculate the area of Square C.
   e. Check that
      i. Area A + Area B = Area C

For each triangle, find the missing length. Round your answer to the nearest tenth.

3. ? 16 8

4. ? 36 15



5. 6 2 ?

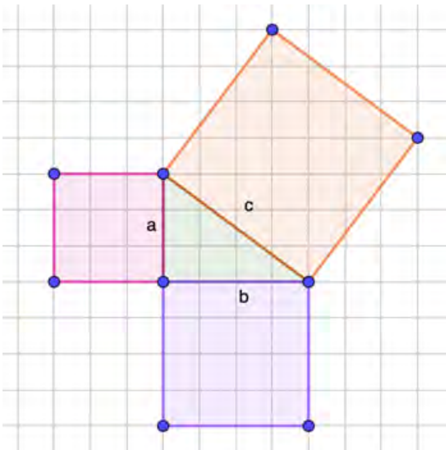
6. ? 26 24

7. Casey says that if you have two sides of a right triangle measuring 9 units and 41 units, the third side could measure 40 units. **Is Casey correct? Justify your answer.**

1. Write the Pythagorean Theorem. ____

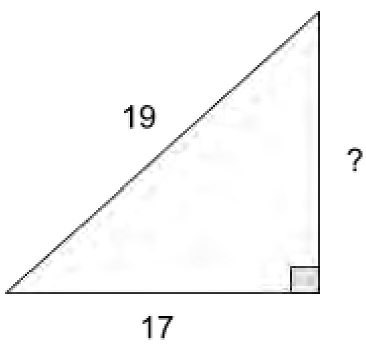
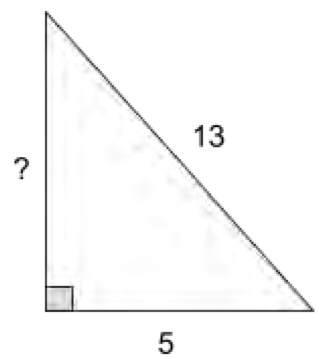
2. What relationship does the picture below show?

Fill in the blanks using the words: A, B, and C



The picture shows that the area of the ___________ side plus the area of the _____________ side are equal to the area of the _____________ side.

For each triangle find the missing length. Round your answer to the nearest tenth.

3.	4.
	

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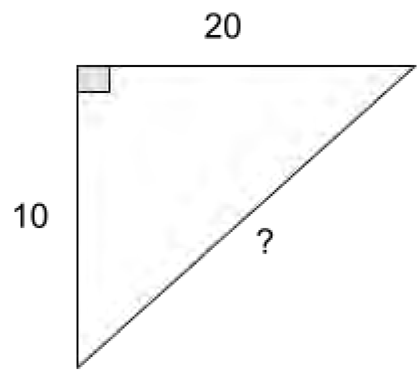

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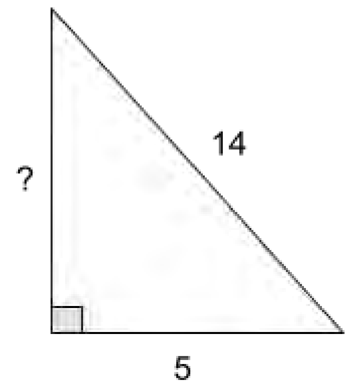
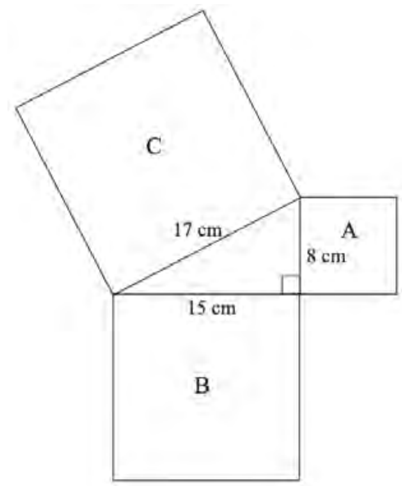


7. Ms. Green tells you that a right triangle has a hypotenuse of 101 and a leg of 20. She asks you to find the other leg of the triangle. What is your answer? (Draw a picture to help)

Name: ___________________________________ G8 U6 Lesson 7 - Independent Work

1. Write the Pythagorean Theorem. ____

2. For the diagram below:



   a. Calculate the area of Square A.

   ____

   b. Calculator the area of Square B.

   ____

   c. Calculate the sum of Area A and Area B.

   ____

   d. Calculate the area of Square C.

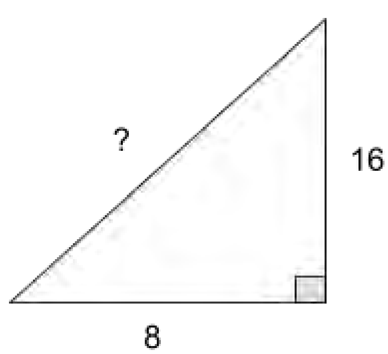
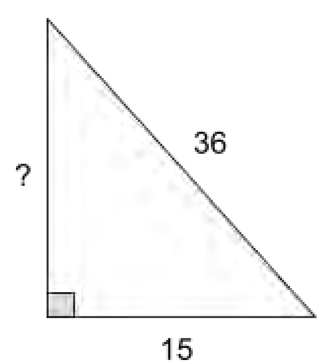
   ____

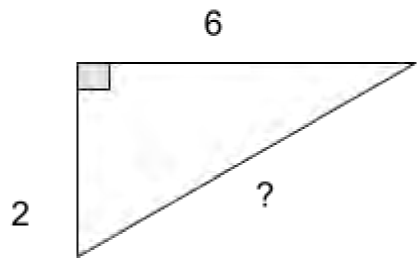
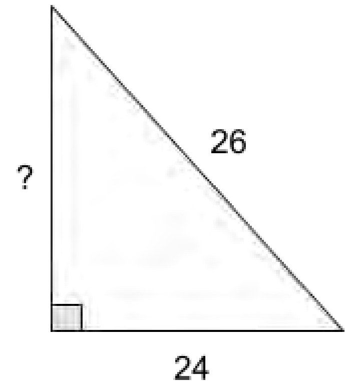
   e. Check that

      i. Area A + Area B = Area C

   ____

For each triangle, find the missing length. Round your answer to the nearest tenth.

3. 	4. 

5.	6.
	

7. Casey says that if you have two sides of a right triangle measuring 9 units and 41 units, the third side could measure 40 units. **<u>Is Casey correct? Justify your answer.</u>**

____

____

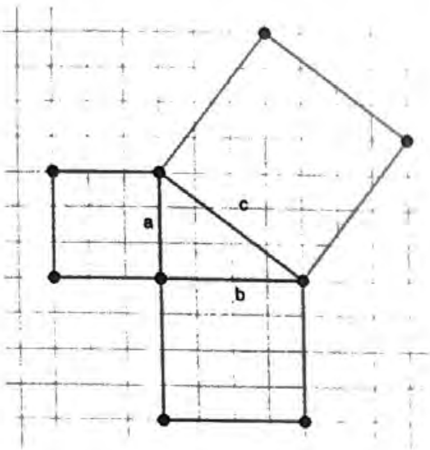
____

Name: Answer Key

1. Write the Pythagorean Theorem. $a^2 + b^2 = c^2$

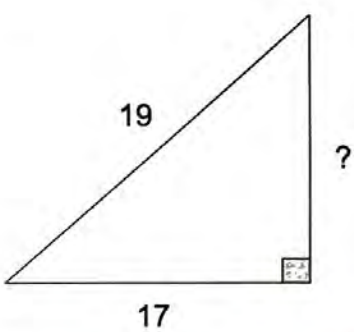
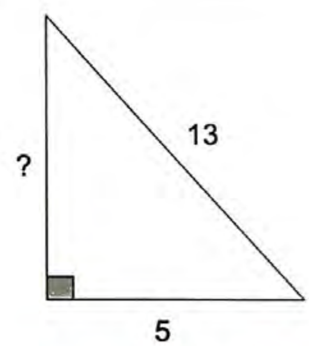
2. What relationship does the picture below show?

Fill in the blanks using the words: A, B, and C

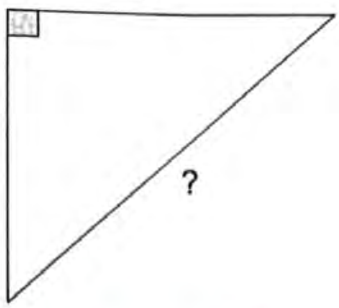


The picture shows that the area of the A *side plus the area of the* B *side are equal to the area of the* C *side.*

For each triangle find the missing length. Round your answer to the nearest tenth.

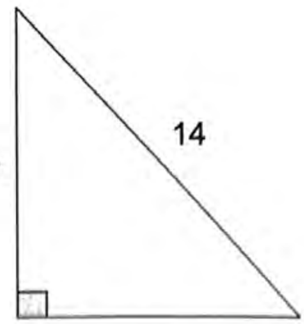
3.	4.
	
$17^2 + b^2 = 19^2$ $b^2 = 19^2 - 17^2$ $b^2 = 361 - 289$ $b^2 = 72$ $b = 8.5$	$a^2 + 5^2 = 13^2$ $a^2 = 13^2 - 5^2$ $a^2 = 169 - 25$ $a^2 = 144$ $a = 12$

5.



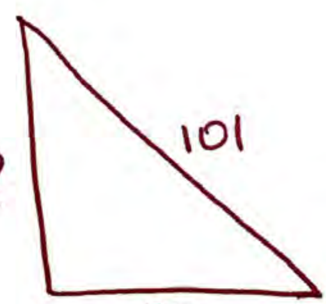
$10^2 + 20^2 = c^2$
$100 + 400 = c^2$
$500 = c^2$
$c = 22.4$

6.



$5^2 + b^2 = 14^2$
$b^2 = 14^2 - 5^2$
$b = 196 - 25$
$b^2 = 171$
$b = 13.1$

7. Ms. Green tells you that a right triangle has a hypotenuse of 101 and a leg of 20. She asks you to find the other leg of the triangle. What is your answer? (Draw a picture to help)

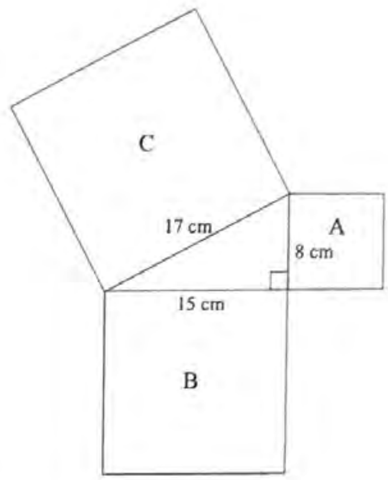


$20^2 + b^2 = 101^2$
$b^2 = 101^2 - 20^2$
$b^2 = 10201 - 400$
$b^2 = 9801$
$b = 99$

Name: Answer Key

1. Write the Pythagorean Theorem. $a^2 + b^2 = c^2$

2. For the diagram below:



   a. Calculate the area of Square A.
   $A = 8^2 = 64$

   b. Calculator the area of Square B.
   $A = 15^2 = 225$

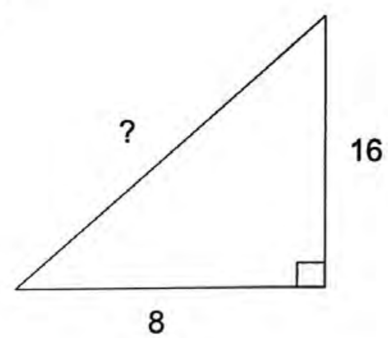
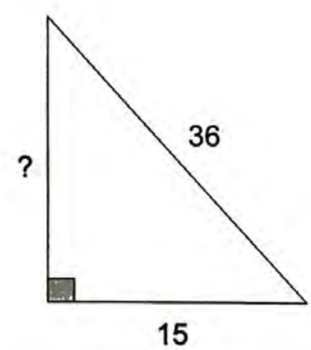
   c. Calculate the sum of Area A and Area B.
   $225 + 64 = 289$

   d. Calculate the area of Square C.
   $A = 17^2 = 289$

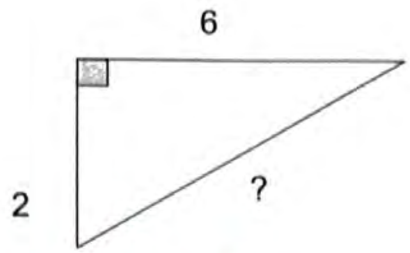
   e. Check that

      i. Area A + Area B = Area C
      $64 + 225 = 289$ ✓

For each triangle, find the missing length. Round your answer to the nearest tenth.

3.	4.
	
$8^2 + 16^2 = c^2$ $64 + 256 = c^2$ $320 = c^2$ $c = 17.9$	$a^2 + 15^2 = 36^2$ $a^2 = 36^2 - 15^2$ $a^2 = 1296 - 225$ $a^2 = 1071$ $a = 32.7$

5.



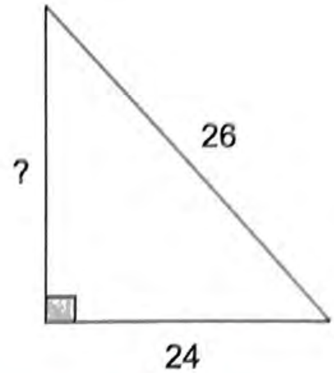
$2^2 + 6^2 = c^2$

$4 + 36 = c^2$

$40 = c^2$

$c = 6.3$

6.



$a^2 + 24^2 = 26^2$

$a^2 = 26^2 - 24^2$

$a^2 = 676 - 576$

$a^2 = 100$

$a = 10$

7. Casey says that if you have two sides of a right triangle measuring 9 units and 41 units, the third side could measure 40 units. **<u>Is Casey correct? Justify your answer.</u>**

Casey is correct. 9 and 40 could be side lengths and 41 is the hypotenuse. $9^2 + 40^2 = 41^2$

G8 U6 Lesson 8
Calculate unknown side lengths using the Pythagorean Theorem

CITYTUTOR DC

G8 U6 Lesson 8 - Students will calculate unknown side lengths of a right triangle by using the Pythagorean Theorem.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we are still practicing using the Pythagorean Theorem to find the unknown side lengths of a right triangle. Remember, we can find the hypotenuse side or the leg side of a right triangle using the Pythagorean Theorem.

Let's Talk (Slide 3): If we look at these statements, **which do you feel doesn't belong? Justify your answer.** Possible Student Answers, Key Points:

- B.
- 10 is the hypotenuse length, and 6 is the leg length; you must subtract their squares to find the other leg length.

That's a valid observation. When we look at all the answer choices, we can tell that sides "a" and "b" are "8" and "6" and that the hypotenuse is "10" because it's the largest number. Let's look at each choice.

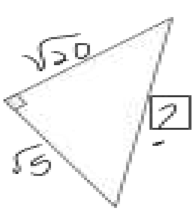
Choice A: What are we missing? Side "b." Yes, this is how we would initially set up the Pythagorean Theorem.
Choice B: We have the hypotenuse and a leg length. We would need to subtract the squares to find the other leg length.
Choice C: We are missing the other side but have the Pythagorean Theorem and one leg length. In this case, we would subtract their squares to find the missing leg.
Choice D: This is how the Pythagorean Theorem would look with all the side lengths substituted in. We could test to make sure that this triangle is a right triangle.

It's important to remember that when we have leg lengths, we add their squares to find the hypotenuse length. But if we have the hypotenuse and a leg length, we subtract their squares to find the other leg length.

Let's Think (Slide 4): Let's look at this example. **Can we use the Pythagorean Theorem to find the missing side? Justify your answer.** Possible Student Answers, Key Points:

- Yes, we can use the Pythagorean Theorem
- We know the lengths of both legs, but we are missing the hypotenuse.

We know the length of the two legs, but we are missing the hypotenuse. What can the Pythagorean Theorem help us find? Either the hypotenuse or a leg length. Exactly, we are going to use it in this case to find the hypotenuse.



(Draw a triangle on the board.) Let's solve for the hypotenuse by substituting what we know into the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$(\sqrt{5})^2 + (\sqrt{20})^2 = c^2$$
$$5 + 20 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = \sqrt{c^2}$$
$$c = 5$$

What is the Pythagorean Theorem? $a^2 + b^2 = c^2$
What are the values of "a" and "b"? $\sqrt{5}$ and $\sqrt{20}$
Let's substitute into "a" and "b". Remember that when we square square roots, they cancel out each other. We are left with 5 and 20, and the sum of the squares is "25." Now we take the square root of both sides of the equal sign, and we find that the hypotenuse is 25 units long.

Let's Try it (Slides 5): Today, we will work on finding any unknown side of a right triangle. Always pay attention to what you are given, when we have leg lengths, we add their squares to find the hypotenuse length. But if we have the hypotenuse and a leg length, we subtract their squares to find the other leg length.

WARM WELCOME

Today we will calculate unknown side lengths using the Pythagorean Theorem.

Let's Talk:

Which one does not belong? Justify your answer.

A. $8^2 + b^2 = 10^2$

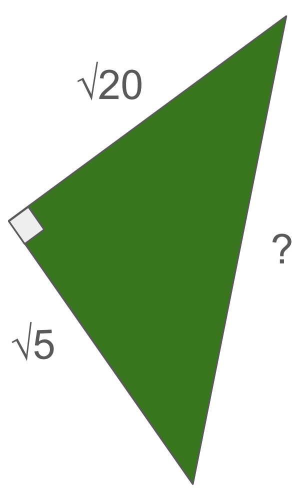
B. $10^2 + 6^2 = b^2$

C. $a^2 = 10^2 - 6^2$

D. $8^2 + 6^2 = 10^2$

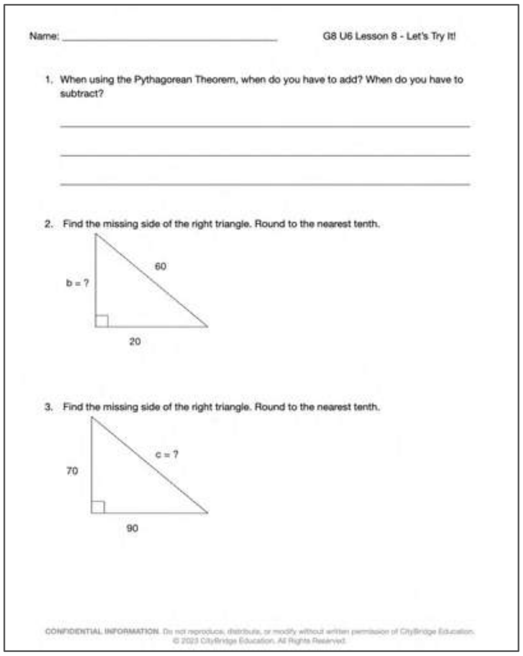
Let's Think:

Can we use the Pythagorean Theorem to find the missing side? Justify your answer.



Let's Try It:

Let's try calculating unknown side lengths of a right triangle by using the Pythagorean Theorem together.

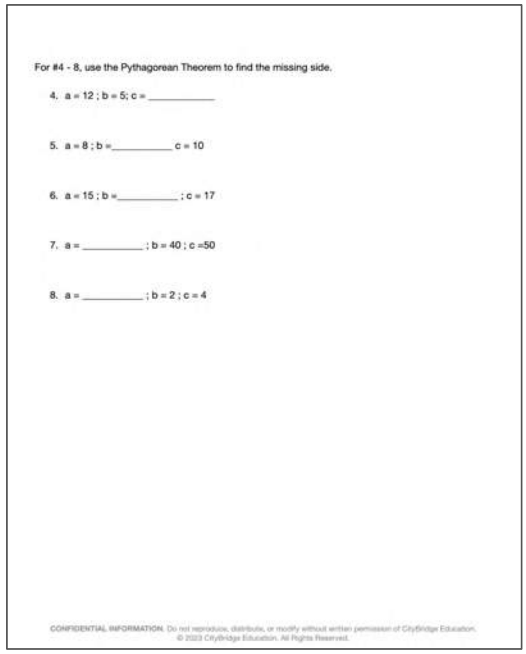


Name: ______________________ G8 U6 Lesson 8 - Let's Try It!

1. When using the Pythagorean Theorem, when do you have to add? When do you have to subtract?

2. Find the missing side of the right triangle. Round to the nearest tenth.

3. Find the missing side of the right triangle. Round to the nearest tenth.

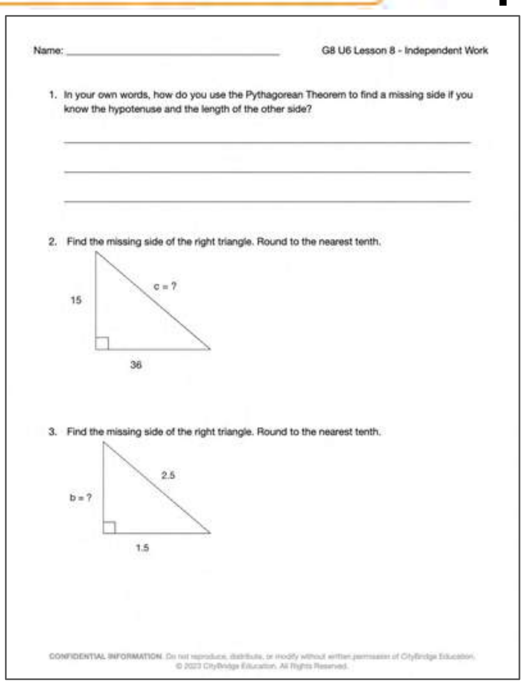


For #4 - 8, use the Pythagorean Theorem to find the missing side.

4. a = 12 ; b = 5; c = ________
5. a = 8 ; b = ________ c = 10
6. a = 15 ; b = ________ ; c = 17
7. a = ________ ; b = 40 ; c =50
8. a = ________ ; b = 2 ; c = 4

On your Own:

Now it's time to calculate unknown side lengths of a right triangle by using the Pythagorean Theorem on your own.

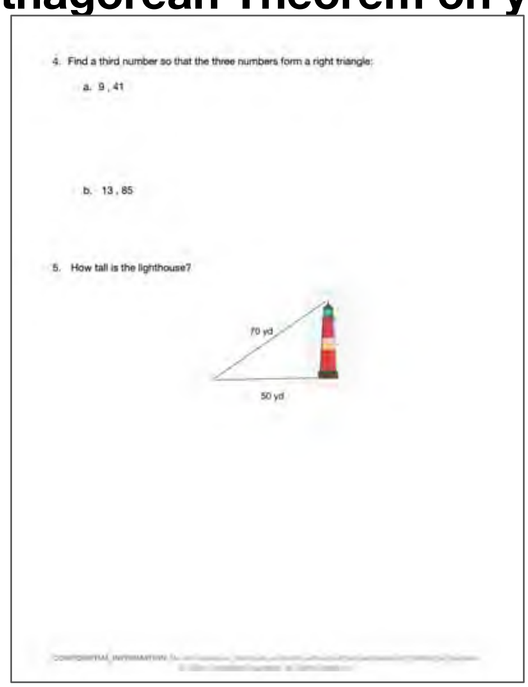


Name: ______________________ G8 U6 Lesson 8 - Independent Work

1. In your own words, how do you use the Pythagorean Theorem to find a missing side if you know the hypotenuse and the length of the other side?

2. Find the missing side of the right triangle. Round to the nearest tenth.

3. Find the missing side of the right triangle. Round to the nearest tenth.



4. Find a third number so that the three numbers form a right triangle:
   a. 9 , 41
   b. 13 , 85
5. How tall is the lighthouse?

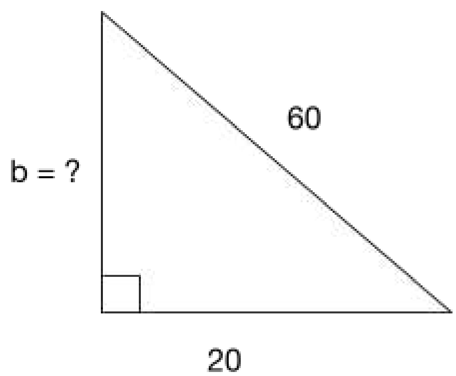
1. When using the Pythagorean Theorem, when do you have to add? When do you have to subtract?

____

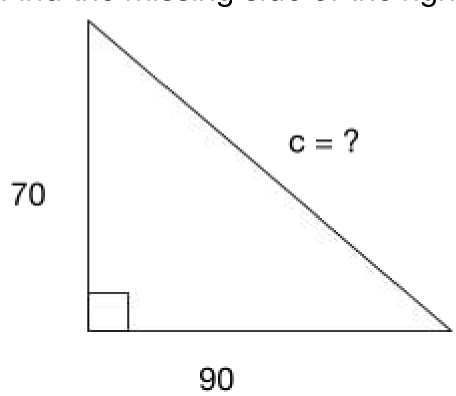
____

____

2. Find the missing side of the right triangle. Round to the nearest tenth.



3. Find the missing side of the right triangle. Round to the nearest tenth.



For #4 - 8, use the Pythagorean Theorem to find the missing side.

4. a = 12 ; b = 5; c = ____________

5. a = 8 ; b =___________ c = 10

6. a = 15 ; b =___________ ; c = 17

7. a = ___________ ; b = 40 ; c =50

8. a = ___________ ; b = 2 ; c = 4

Name: ______________________________________ G8 U6 Lesson 8 - Independent Work

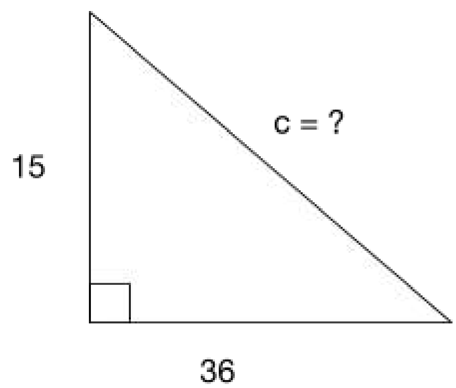
1. In your own words, how do you use the Pythagorean Theorem to find a missing side if you know the hypotenuse and the length of the other side?

____

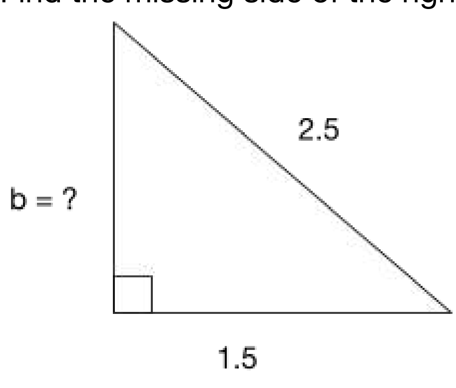
____

____

2. Find the missing side of the right triangle. Round to the nearest tenth.



3. Find the missing side of the right triangle. Round to the nearest tenth.

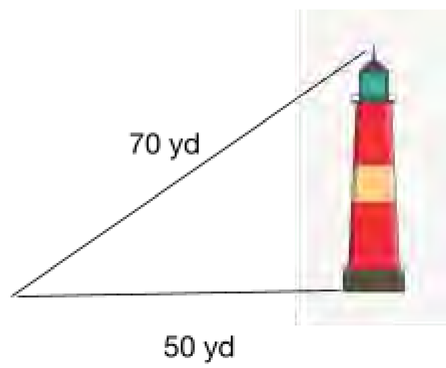


4. Find a third number so that the three numbers form a right triangle:

   a. 9 , 41

   b. 13 , 85

5. How tall is the lighthouse?

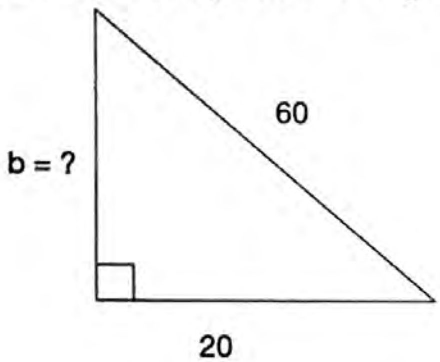


Name: Answer Key

1. When using the Pythagorean Theorem, when do you have to add? When do you have to subtract?

When you know both leg lengths, you add their squares. When you know the hypotenuse and a leg, you subtract their squares.

2. Find the missing side of the right triangle. Round to the nearest tenth.



$20^2 + b^2 = 60^2$

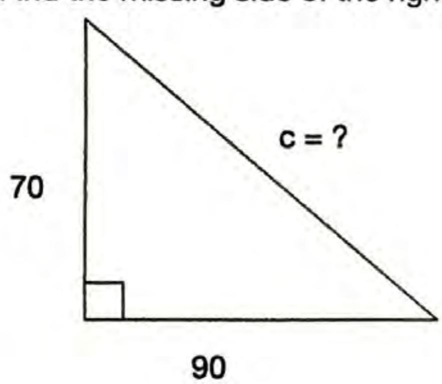
$b^2 = 60^2 - 40^2$

$b^2 = 3600 - 1600$

$b^2 = 2000$

$b = 44.7$

3. Find the missing side of the right triangle. Round to the nearest tenth.



$70^2 + 90^2 = c^2$

$4900 + 8100 = c^2$

$13000 = c^2$

$c = 114$

For #4 - 8, use the Pythagorean Theorem to find the missing side.

4. a = 12 ; b = 5; c = 13

5. a = 8 ; b = 6 c = 10

6. a = 15 ; b = 8 ; c = 17

7. a = 30 ; b = 40 ; c =50

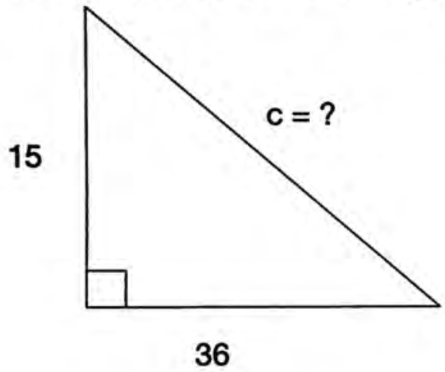
8. a = 3.5 ; b = 2 ; c = 4

Name: Answer Key

1. In your own words, how do you use the Pythagorean Theorem to find a missing side if you know the hypotenuse and the length of the other side?

To find the length of the other side, you can subtract the hypotenuse squared and the square of the leg you know.

2. Find the missing side of the right triangle. Round to the nearest tenth.



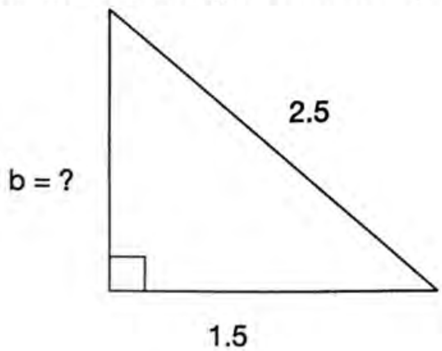
$15^2 + 36^2 = c^2$

$225 + 1296 = c^2$

$1521 = c^2$

$c = 39$

3. Find the missing side of the right triangle. Round to the nearest tenth.



$1.5^2 + b^2 = 2.5^2$

$b^2 = 2.5^2 - 1.5^2$

$b^2 = 6.25 - 2.25$

$b^2 = 4$

$b = 2$

4. Find a third number so that the three numbers form a right triangle:

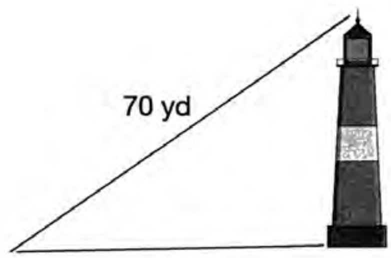
a. 9 , 41

40 or 42

b. 13 , 85

84 or 86

5. How tall is the lighthouse?



$50^2 + b^2 = 70^2$

$b^2 = 70^2 - 50^2$

$b^2 = 4900 - 2500$

$b^2 = 2400$

$b = 49$ yd

G8 U6 Lesson 9
Use the converse of the Pythagorean Theorem to determine right triangles

CITYTUTOR DC

G8 U6 Lesson 9 - Students will use the converse of the Pythagorean Theorem to determine right triangles

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we will use the converse of the Pythagorean Theorem to determine if a triangle is a right triangle. The converse of a statement is when we reverse the order of a statement. For example, if I say, " If the sky is blue, then it is sunny," the converse of that statement would be, "If it is sunny, then the sky is blue." The converse of the Pythagorean theorem will help us determine which lengths make a right triangle.

Let's Talk (Slide 3): **Who can remind me what the Pythagorean Theorem states?** Possible Student Answers, Key Points:

- The area of the square whose side is the hypotenuse is equal to the sum of the areas of the squares on the other two sides.
- $a^2 + b^2 = c^2$
- The sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse.

The Pythagorean Theorem states that, for right triangles, the sum of the squares of the two shorter sides must equal the square of the longest side, the hypotenuse. We could also say that if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

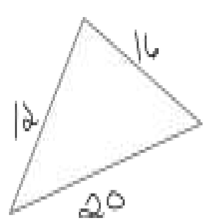
What do you think the converse of that statement might be? *(Allow students to answer.)* Possible Student Answers, Key Points:

- If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.
- If $c^2 = a^2 + c^2$, then the triangle is a right triangle.

Together, the Pythagorean Theorem and its converse provide a one-step test for checking to see if a triangle is a right triangle just using its side lengths. If $a^2 + b^2 = c^2$, it is a right triangle. If $a^2 + b^2 \neq c^2$, it is not a right triangle.

Let's Think (Slide 4): Now that we have discussed the Pythagorean Theorem and the converse of the Pythagorean Theorem. **Terry says that this is a right triangle. Do you agree or disagree? Justify your answer.** Possible Student Answers, Key Points:

- Yes, the square of the length of the longest side, "20", is equal to the sum of the squares of the lengths of the other two sides, "12" and "16", then the triangle is a right triangle.
- If $20^2 = 12^2 + 16^2$, then the triangle is a right triangle.



(Draw a Triangle on the Board)

The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse (the longest side) equals the sum of the squares of the other two sides. But what if we have a triangle and don't know if it's a right triangle? We can use the converse of the Pythagorean Theorem to find out.

$20^2 = 12^2 + 16^2$
$400 = 144 + 256$
$400 = 400$ ✓

We can test it out. If "20 squared" equals the sum of "12 squared and 16 squared, then we have a right triangle. "20 squared" is 400, and the sum of the other two legs equals 400. Therefore, Terry was correct; this is a right triangle.

Let's Try it (Slides 5): Today, you will be testing lengths to verify whether they make a right triangle. Remember, the converse of the Pythagorean Theorem helps us determine whether a triangle is a right triangle. If $c^2 = a^2 + c^2$, then the triangle is a right triangle.

WARM WELCOME

Today we will Use the converse of the Pythagorean Theorem to determine right triangles.

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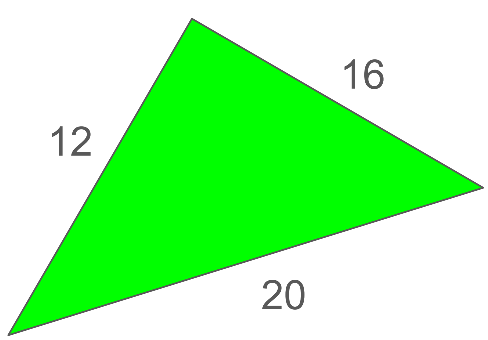
Let's Talk:

What does the Pythagorean Theorem state?

Let's Think:

Terry says that this is a right triangle. Do you agree or disagree? Justify your answer.



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Let's Try It:

Let's use the converse of the Pythagorean Theorem to determine right triangles together.

Name: ______________________ G8 U6 Lesson 9 - Let's Try It!

1. Write the Converse of the Pythagorean Theorem in your own words.

____

____

2.

a	b	c	Is $a^2 + b^2 = c^2$ true?	Is it a right triangle?
20	21	29		
10	15	20		
7	11	12		
12	15	19		
10	24	26		

3. Which set of numbers does not belong? Justify your answers.

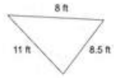
3, 6 ,8   6, 8, 10   5, 12, 13   7, 24, 25

The numbers that do not belong are ________ because ______________

____

____

4. Verify if the triangle is a right triangle.



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On your Own:

Now it's time to determine right triangle using the converse of the Pythagorean Theorem on your own.

Name: ______________________ G8 U6 Lesson 9 - Independent Work

Can you form a right triangle with the three lengths given? Show your work.

1. 20, 99, 101	2. 21, 28, 35
3. 10, 11, 14	4. 7, 10, 11
5. 17, 144, 145	6. $\sqrt{5}$, 5, 5.5

7. What has to be true in order to be sure a triangle is a right triangle?

____

____

____

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Name: ______________________________________ G8 U6 Lesson 9 - Let's Try It

1. Write the Converse of the Pythagorean Theorem in your own words.

____

____

2.

a	b	c	Is $a^2 + b^2 = c^2$ true?	Is it a right triangle?
20	21	29		
10	15	20		
7	11	12		
12	15	19		
10	24	26		

3. Which set of numbers does not belong? Justify your answers.

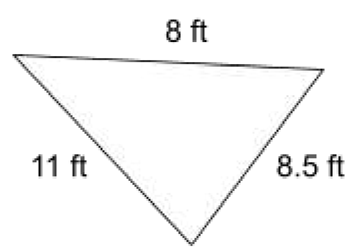
3, 6 ,8   6, 8, 10   5, 12, 13   7, 24, 25

The numbers that do not belong are ____________ because ___________________________

____

____.

4. Verify if the triangle is a right triangle.



Can you form a right triangle with the three lengths given? Show your work.

1. 20, 99, 101	**2.** 21, 28, 35
3. 10, 11, 14	**4.** 7, 10, 11
5. 17, 144, 145	**6.** $\sqrt{5}$, 5, 5.5

7. What has to be true in order to be sure a triangle is a right triangle?

____

____

____

Name: Answer Key

1. Write the Converse of the Pythagorean Theorem in your own words.

If $c^2 = a^2 + b^2$, then the triangle is a right triangle.

2.

a	b	c	Is $a^2 + b^2 = c^2$ true?	Is it a right triangle?
20	21	29	Yes	Yes
10	15	20	No	No
7	11	12	No	No
12	15	19	No	No
10	24	26	Yes	Yes

3. Which set of numbers does not belong? Justify your answers.

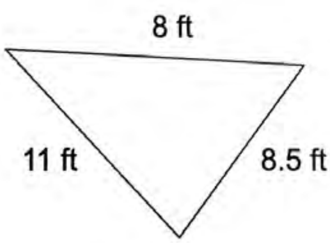
3, 6 ,8 6, 8, 10 5, 12, 13 7, 24, 25

The numbers that do not belong are 3,6,8 because $3^2 + 6^2 \neq 8^2$

The sum of the two shorter sides does not equal the square of the longer side.

4. Verify if the triangle is a right triangle.

It is not a right triangle.



Name: Answer Key

Can you form a right triangle with the three lengths given? Show your work.

1. 20, 99, 101 $101^2 = 20^2 + 99^2$ $10201 = 10201$ ✓ yes	**2.** 21, 28, 35 $35^2 = 21^2 + 28^2$ $1225 = 1225$ ✓ yes
3. 10, 11, 14 $14^2 = 10^2 + 11^2$ $196 \neq 221$ No	**4.** 7, 10, 11 $11^2 = 7^2 + 10^2$ $121 \neq 149$ No
5. 17, 144, 145 $145^2 = 17^2 + 144^2$ $21025 = 21025$ ✓ Yes	**6.** $\sqrt{5}$, 5, 5.5 $5.5^2 = (\sqrt{5})^2 + 5^2$ $30.25 \neq 30$ No

7. What has to be true in order to be sure a triangle is a right triangle?

The sum of the squares of the two shorter legs must be equal to the square of the longest side.

G8 U6 Lesson 10
Use the Pythagorean Theorem to solve problems within a context

CITYTUTOR DC

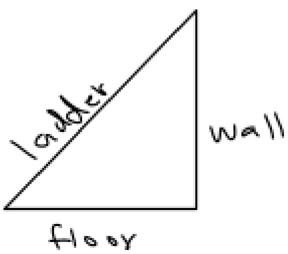
G8 U6 Lesson 10 - Students will use the Pythagorean Theorem to solve problems within a context.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Over the last several lessons, we've used the Pythagorean Theorem to find the lengths of different sides of right triangles. Today, we will learn how to use the Pythagorean Theorem to solve problems that have a more real-world application. This is a chance to see how we may use the Pythagorean Theorem daily.

Let's Talk (Slide 3): How can we determine the length of the ladder if we know the height of the wall and the distance from the wall to the base of the ladder? Possible Student Answers, Key Points:

- The floor and the ladder make a right angle.
- We can use the Pythagorean Theorem.
- $a^2 + b^2 = c^2$



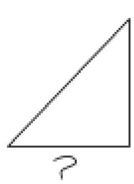
Those are all good answers! We can use the Pythagorean Theorem to determine the length of the ladder. When the ladder is placed against the wall, a right triangle is created. *(Draw a wall, floor, and ladder to demonstrate how the right triangle is made).*

The Pythagorean Theorem helps us find the length of any side of a right triangle if we know the lengths of the other two sides. It's especially useful in solving real-world problems where right triangles are involved.

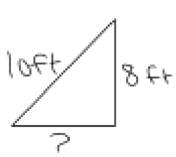
Let's Think (Slide 4): Let's look at an actual example. This is a word problem, and when we read word problems, we need to understand what the problem asks us to do. We will use a strategy used in math called a 3-Read Strategy. This strategy includes reading a math scenario three times with a different goal each time. The first read is to understand what the situation is and to draw a picture. The second read is to understand the mathematics, or what the question asks us to find. The third read is to understand the important information, like the numbers given.

A 10-foot ladder leans against a wall, reaching a height of 8 feet. How far is the base of the ladder from the wall? Let's read this statement three times and pick out all the information.

(Have a student read through once) **What is the situation?** We have a ladder against a wall. Since our last scenario was about a ladder against the wall, let's use the picture that we just drew on the board.



(Have another student read through for the second time.) **What is the question asking us to find?** How far is the base of the ladder from the wall? Let's label our picture with a "?", which is the distance from the ladder's base to the wall.



Now, let's read for the third time. **What is the important information, what was given?** The ladder is 10 feet, reaching 8 feet on the wall. We can add this info to our drawing.

Now, we have a picture that helps us determine how to use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$8^2 + b^2 = 10^2$$
$$b^2 = 10^2 - 8^2$$
$$b^2 = 100 - 64$$
$$b^2 = 36$$
$$\sqrt{b^2} = \sqrt{36}$$
$$b = 6$$

What are the “a”, “b”, and “c”? 8, we don’t know yet, and 10 Let’s write the Pythagorean Theorem, $a^2 + b^2 = c^2$. Then substitute in $8^2 + b^2 = 10^2$. Since we have a shorter leg and they hypotenuse, then we have to subtract the squares that are given and don’t forget to take the square root of both sides of the equal sign. The distance from the ladder's base to the wall is 6 feet.

Let’s Try it (Slides 5): Now, I want you to work in groups to solve these real-world problems. Remember to read the problem 3 times to help you make a picture, identify what you need to find, and what are the most important numbers. Afterward, you will be able to identify the legs and the hypotenuse, set up the Pythagorean Theorem, and solve for the unknown side.

WARM WELCOME

Today we will learn how to use the Pythagorean Theorem to solve problems within a context.

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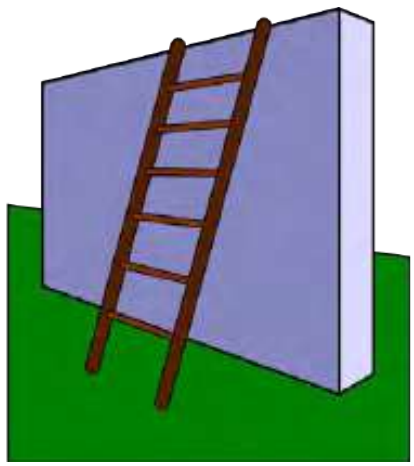
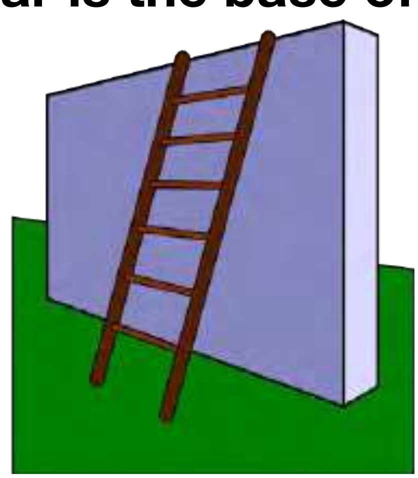
Let's Talk:

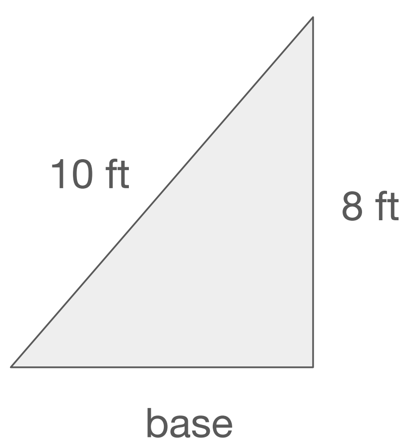
How can we determine the length of the ladder if we know the height of the wall and the distance from the wall to the base of the ladder?



Let's Think:

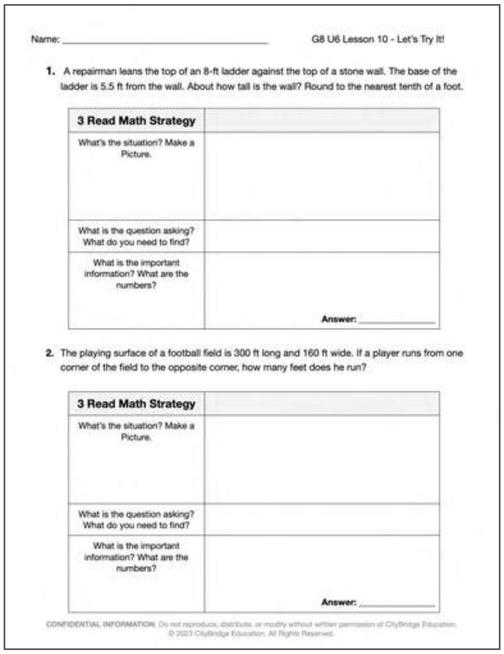
A 10-foot ladder leans against a wall, reaching a height of 8 feet. How far is the base of the ladder from the wall?





Let's Try It:

Let's use the Pythagorean Theorem to solve problems within a context together.



Name: ______________________ G8 U6 Lesson 10 - Let's Try It!

1. A repairman leans the top of an 8-ft ladder against the top of a stone wall. The base of the ladder is 5.5 ft from the wall. About how tall is the wall? Round to the nearest tenth of a foot.

3 Read Math Strategy	
What's the situation? Make a Picture.	
What is the question asking? What do you need to find?	
What is the important information? What are the numbers?	Answer: __________

2. The playing surface of a football field is 300 ft long and 160 ft wide. If a player runs from one corner of the field to the opposite corner, how many feet does he run?

3 Read Math Strategy	
What's the situation? Make a Picture.	
What is the question asking? What do you need to find?	
What is the important information? What are the numbers?	Answer: __________

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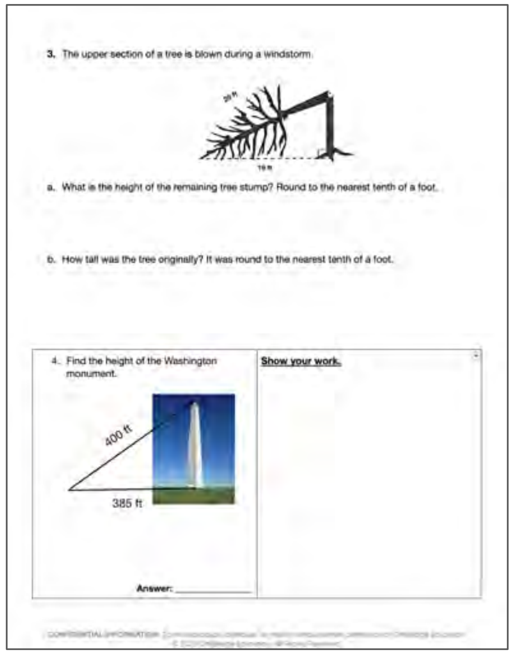


3. The upper section of a tree is blown during a windstorm.

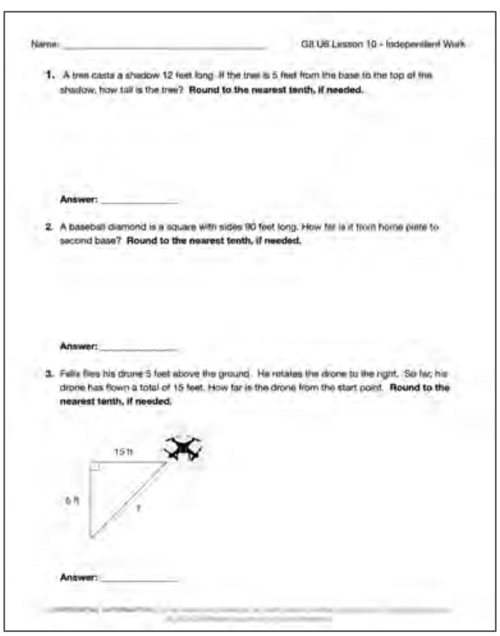
a. What is the height of the remaining tree stump? Round to the nearest tenth of a foot.

b. How tall was the tree originally? It was round to the nearest tenth of a foot.

4. Find the height of the Washington monument.	Show your work.
400 ft 385 ft Answer: __________	

On your Own:

Now it's time to use the Pythagorean Theorem to solve problems within a context on your own.



Name: ______________________ G8 U6 Lesson 10 - Independent Work

1. A tree casts a shadow 12 feet long. If the tree is 5 feet from the base to the top of the shadow, how tall is the tree? **Round to the nearest tenth, if needed.**

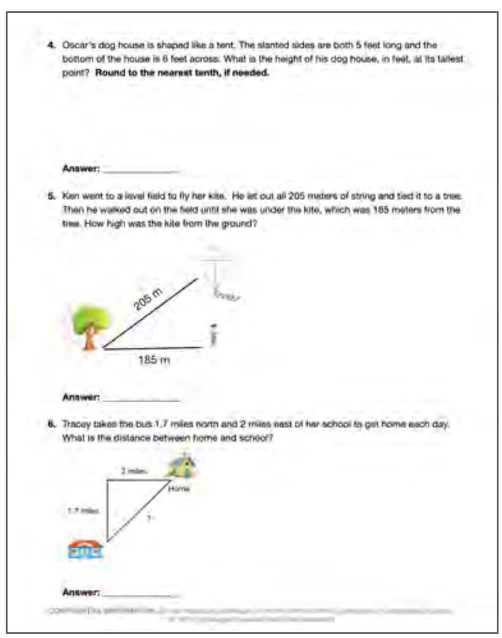
Answer: __________

2. A baseball diamond is a square with sides 90 feet long. How far is it from home plate to second base? **Round to the nearest tenth, if needed.**

Answer: __________

3. Felix flies his drone 5 feet above the ground. He rotates the drone to the right. So far, his drone has flown a total of 15 feet. How far is the drone from the start point. **Round to the nearest tenth, if needed.**

Answer: __________



4. Oscar's dog house is shaped like a tent. The slanted sides are both 5 feet long and the bottom of the house is 6 feet across. What is the height of his dog house, in feet, at its tallest point? **Round to the nearest tenth, if needed.**

Answer: __________

5. Ken went to a level field to fly her kite. He let out all 205 meters of string and tied it to a tree. Then he walked out on the field until she was under the kite, which was 185 meters from the tree. How high was the kite from the ground?

Answer: __________

6. Tracey takes the bus 1.7 miles north and 2 miles east of her school to get home each day. What is the distance between home and school?

Answer: __________

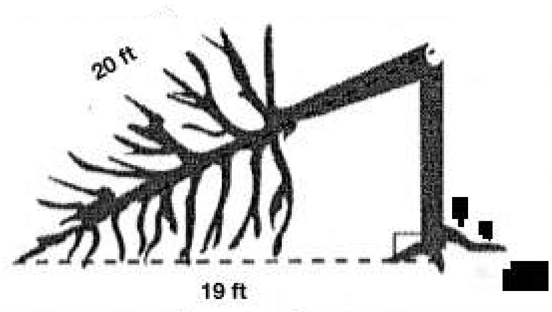
1. A repairman leans the top of an 8-ft ladder against the top of a stone wall. The base of the ladder is 5.5 ft from the wall. About how tall is the wall? Round to the nearest tenth of a foot.

3 Read Math Strategy	
What's the situation? Make a Picture.	
What is the question asking? What do you need to find?	
What is the important information? What are the numbers?	**Answer:** ______________

2. The playing surface of a football field is 300 ft long and 160 ft wide. If a player runs from one corner of the field to the opposite corner, how many feet does he run?

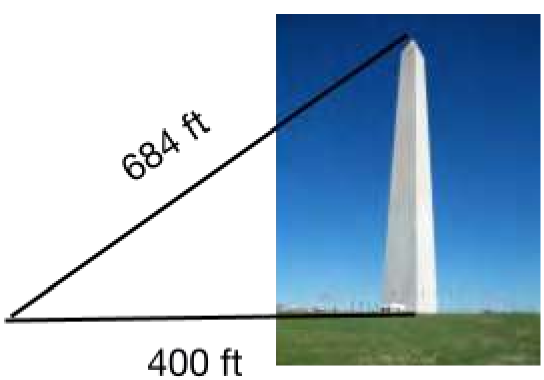
3 Read Math Strategy	
What's the situation? Make a Picture.	
What is the question asking? What do you need to find?	
What is the important information? What are the numbers?	**Answer:** ______________

3. The upper section of a tree is blown during a windstorm.



a. What is the height of the remaining tree stump? Round to the nearest tenth of a foot.

b. How tall was the tree originally? It was round to the nearest tenth of a foot.

4. Find the height of the Washington monument.  **Answer:** ________________	<u>**Show your work.**</u>

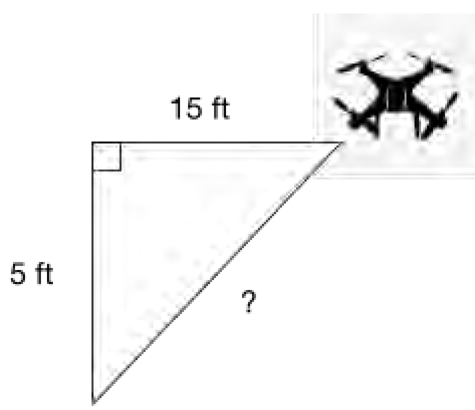
1. A tree casts a shadow 12 feet long. If the tree is 5 feet from the base to the top of the shadow, how tall is the tree? **Round to the nearest tenth, if needed.**

Answer: _______________

2. A baseball diamond is a square with sides 90 feet long. How far is it from home plate to second base? **Round to the nearest tenth, if needed.**

Answer: _______________

3. Felix flies his drone 5 feet above the ground. He rotates the drone to the right. So far, his drone has flown a total of 15 feet. How far is the drone from the start point. **Round to the nearest tenth, if needed.**

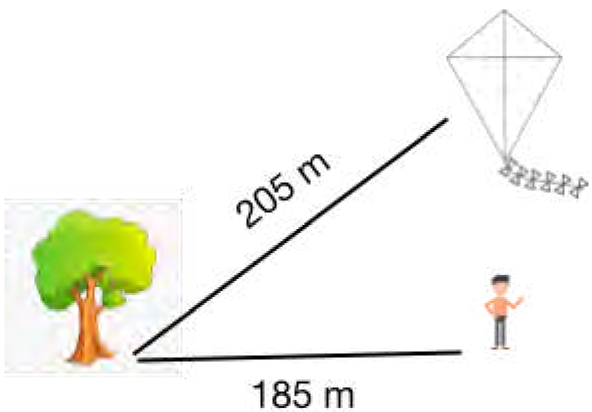


Answer: _______________

4. Oscar's dog house is shaped like a tent. The slanted sides are both 5 feet long and the bottom of the house is 6 feet across. What is the height of his dog house, in feet, at its tallest point? **Round to the nearest tenth, if needed.**

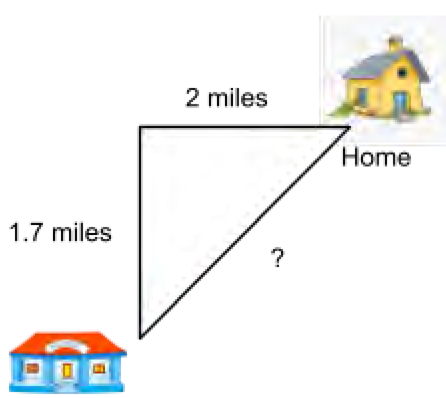
Answer: _______________

5. Ken went to a level field to fly her kite. He let out all 205 meters of string and tied it to a tree. Then he walked out on the field until she was under the kite, which was 185 meters from the tree. How high was the kite from the ground?



Answer: _______________

6. Tracey takes the bus 1.7 miles north and 2 miles east of her school to get home each day. What is the distance between home and school?



Answer: _______________

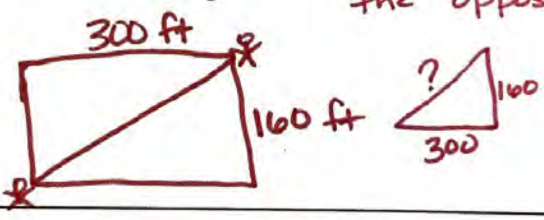
Name: Answer Key

G8 U6 Lesson 10 - Let's Try It!

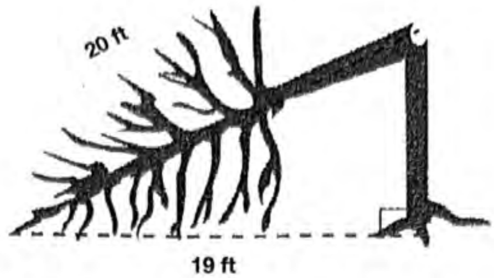
1. A repairman leans the top of an 8-ft ladder against the top of a stone wall. The base of the ladder is 5.5 ft from the wall. About how tall is the wall? Round to the nearest tenth of a foot.

3 Read Math Strategy	
What's the situation? Make a Picture.	Ladder Against a wall $a^2 + 5.5^2 = 8^2$ 
What is the question asking? What do you need to find?	How tall is the wall?
What is the important information? What are the numbers?	The ladder is 8ft Base to wall is 5.5ft **Answer:** 5.8ft

2. The playing surface of a football field is 300 ft long and 160 ft wide. If a player runs from one corner of the field to the opposite corner, how many feet does he run?

3 Read Math Strategy	
What's the situation? Make a Picture.	Football Player runs from one corner to the opposite  $160^2 + 300^2 = c^2$
What is the question asking? What do you need to find?	How many feet does the player run?
What is the important information? What are the numbers?	Foot ball field is 300ft long & 160 ft wide **Answer:** 340 ft

3. The upper section of a tree is blown during a windstorm.

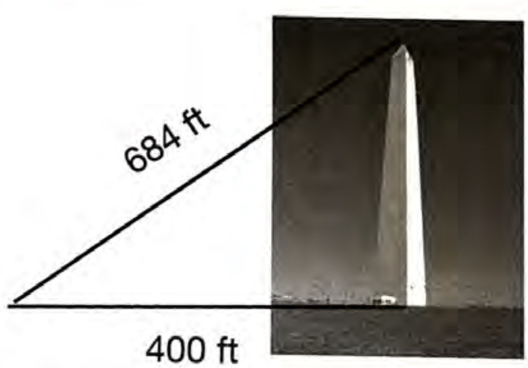


a. What is the height of the remaining tree stump? Round to the nearest tenth of a foot.

6.2 ft

b. How tall was the tree originally? It was round to the nearest tenth of a foot.

26.2 ft

4. Find the height of the Washington monument.	**Show your work.**
 **Answer:** 555 ft	

Name: Answer Key

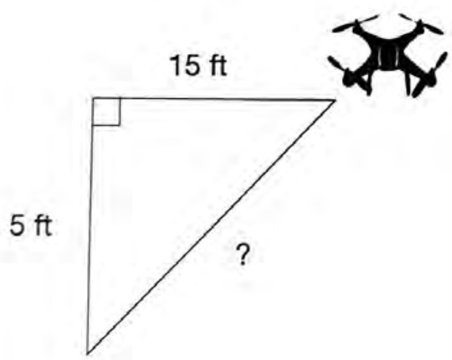
1. A tree casts a shadow 12 feet long. If the tree is 5 feet from the base to the top of the shadow, how tall is the tree? **Round to the nearest tenth, if needed.**

Answer: 13 ft

2. A baseball diamond is a square with sides 90 feet long. How far is it from home plate to second base? **Round to the nearest tenth, if needed.**

Answer: 127.3 ft

3. Felix flies his drone 5 feet above the ground. He rotates the drone to the right. So far, his drone has flown a total of 15 feet. How far is the drone from the start point. **Round to the nearest tenth, if needed.**

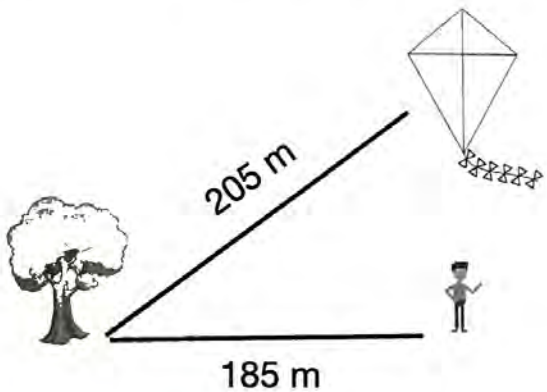


Answer: 15.8 ft

4. Oscar's dog house is shaped like a tent. The slanted sides are both 5 feet long and the bottom of the house is 6 feet across. What is the height of his dog house, in feet, at its tallest point? **Round to the nearest tenth, if needed.**

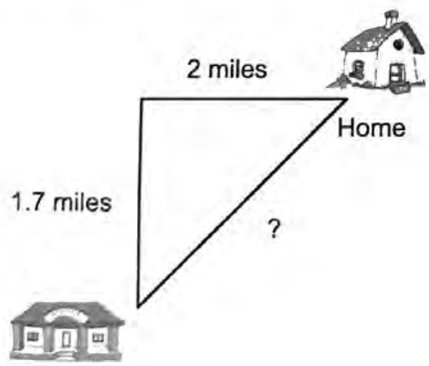
Answer: 4 ft

5. Ken went to a level field to fly her kite. He let out all 205 meters of string and tied it to a tree. Then he walked out on the field until she was under the kite, which was 185 meters from the tree. How high was the kite from the ground?



Answer: 88.3 m

6. Tracey takes the bus 1.7 miles north and 2 miles east of her school to get home each day. What is the distance between home and school?



Answer: 2.6 miles

G8 U6 Lesson 11
Calculate distance in the coordinate plane by using the Pythagorean Theorem

CITYTUTOR DC

G8 U6 Lesson 11 - Students will calculate distance in the coordinate plane by using the Pythagorean Theorem.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Have you ever needed to find the shortest distance between two places on a map, like finding the shortest path from your house to your friend's house or figuring out the quickest way to get to a new restaurant? (Student Responses) Great! When we're looking at maps, we're often looking at a flat surface, just like the coordinate plane we use in math.

Today, we're going to learn a method to find the exact distance between two points on this plane. This method is actually very similar to finding the shortest path on a map.

Let's Review (Slide 3): Our goal today is to learn how to calculate the distance between two points on the coordinate plane using the Pythagorean Theorem. First, we'll review how to plot points. **What are the coordinates of these points?** **A(-3, 4) B(1, 2) C(4, -3) D(-5, -2)**

Remember for every point, there is an "x" and "y" value that makes that one point. "X" values move left and right. "Y" values move up and down.

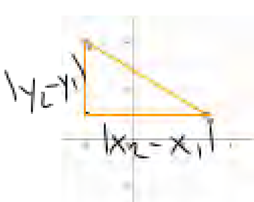
Let's Talk (Slide 4): How can you find the distance between points in the coordinate plane? Possible Student Answers, Key Points:

- Count the units across
- Use the Coordinates and find the horizontal distance

Those are good strategies. These two points make a horizontal line, so it's easier to count the units across. You can also use the coordinates to find the horizontal distance.

Let's Think (Slide 5): *(Hand Out Graph Paper)* Now, on our graph paper, we are going to plot points A and B. Point A is (-2, 4) and Point B (3, 1). **(***Write the coordinates on the board and plot points on the coordinate plane).*

A(-2,4) B(3,1)



Our objective is to find the distance between these two points. First we are going to draw a right triangle by connecting the points with horizontal and vertical lines to form the legs of the triangle, and the hypotenuse will be the line segment from point A to B.

Label the horizontal distance as $|x_2 - x_1|$ and the vertical distance as $|y_2 - y_1|$.

Horizontal Distance $|3-(-2)|=5$

Vertical Distance $|1-4|=3$

To find the horizontal distance, we will use the x-values of our points. The horizontal distance is $|3 - (-2)|$. Distance is always positive, which is why we find the absolute value. The distance would be 5.

The vertical distance uses the y-values of our points. Let's find the vertical distance $|1 - 4|$. The vertical distance is 3.

$$a^2 + b^2 = c^2$$
$$3^2 + 5^2 = c^2$$
$$9 + 25 = c^2$$
$$34 = c^2$$
$$\sqrt{34} = \sqrt{c^2}$$
$$c = 5.8$$

Now that we have our vertical and horizontal distance, we are able to use the Pythagorean Theorem to find the distance between Point A and B. We know the legs have lengths of 3 and 5 units.

The distance between Point A and B is 5.8 units.

Let's Try it (Slides 6): Now it's your turn to find the distance between two points. If they are on the same horizontal or vertical line, we just subtract the coordinates that are different. If they aren't, we can construct a right triangle and use the Pythagorean Theorem.

WARM WELCOME

Today we will Calculate distance in the coordinate plane by using the Pythagorean Theorem.

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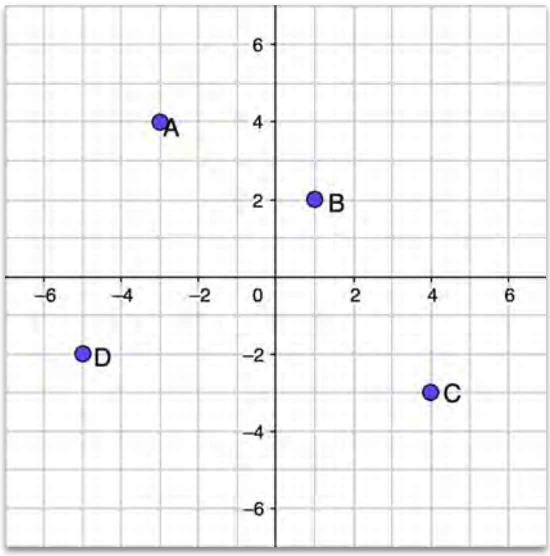
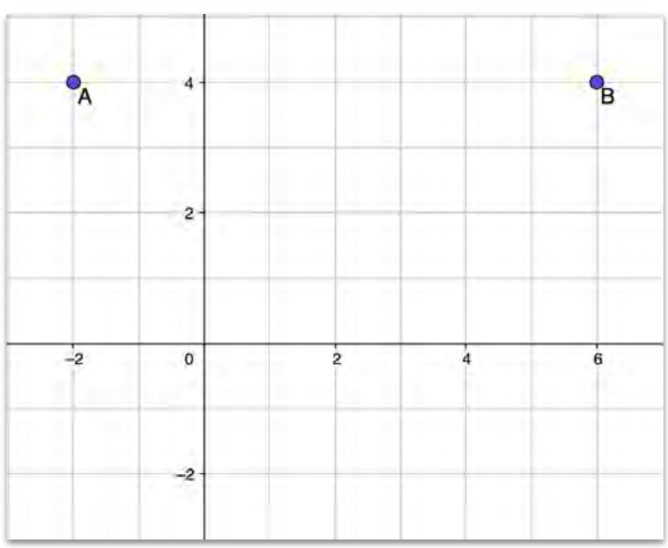
Let's Review:

Find the coordinates of the points.



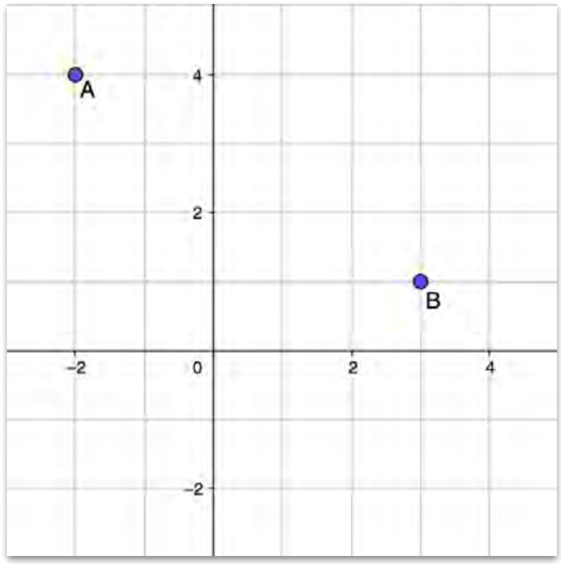
Let's Talk:

How can you find the distance between points in the coordinate plane?



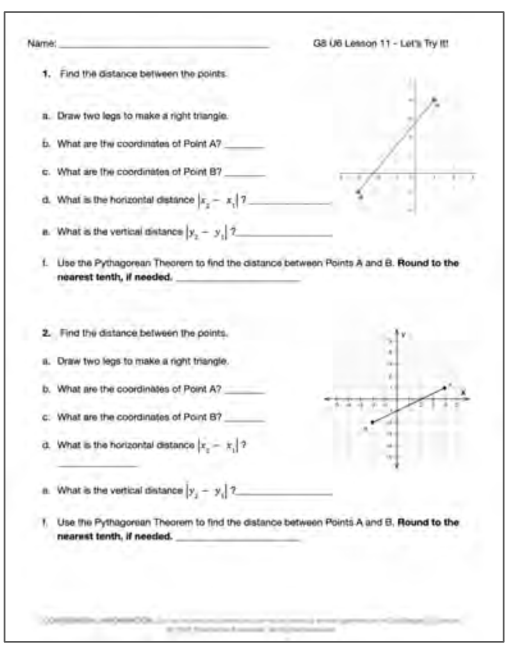
Let's Think:

How can you find the distance between points in the coordinate plane?



Let's Try It:

Let's calculate distance in the coordinate plane using the Pythagorean Theorem together.



Name: ____________________ G8 U6 Lesson 11 - Let's Try It!

1. Find the distance between the points.

a. Draw two legs to make a right triangle.

b. What are the coordinates of Point A? ______

c. What are the coordinates of Point B? ______

d. What is the horizontal distance $|x_2 - x_1|$? ____________

e. What is the vertical distance $|y_2 - y_1|$? ____________

f. Use the Pythagorean Theorem to find the distance between Points A and B. **Round to the nearest tenth, if needed.** ____________

2. Find the distance between the points.

a. Draw two legs to make a right triangle.

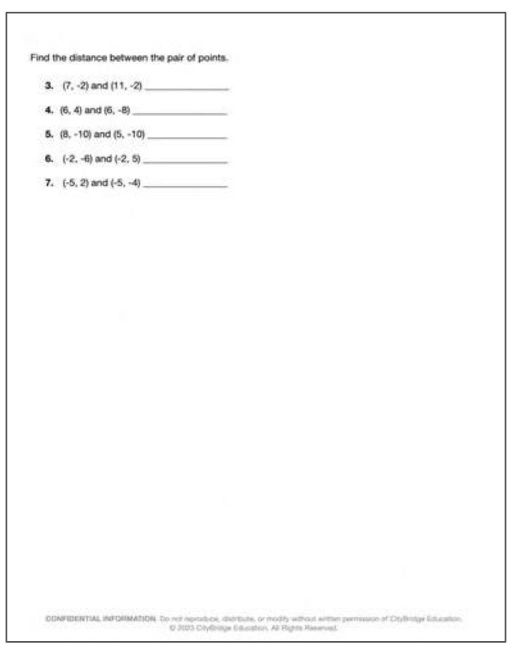
b. What are the coordinates of Point A? ______

c. What are the coordinates of Point B? ______

d. What is the horizontal distance $|x_2 - x_1|$? ____________

e. What is the vertical distance $|y_2 - y_1|$? ____________

f. Use the Pythagorean Theorem to find the distance between Points A and B. **Round to the nearest tenth, if needed.** ____________



Find the distance between the pair of points.

3. (7, -2) and (11, -2) ____________

4. (6, 4) and (6, -8) ____________

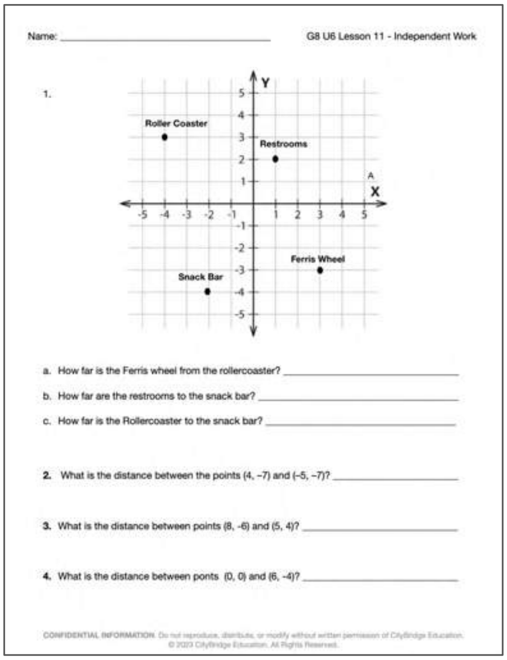
5. (8, -10) and (5, -10) ____________

6. (-2, -6) and (-2, 5) ____________

7. (-5, 2) and (-5, -4) ____________

On your Own:

Now it's time to calculate distance in the coordinate plane using the Pythagorean Theorem on your own.



Name: ______________________ G8 U6 Lesson 11 - Independent Work

1.

a. How far is the Ferris wheel from the rollercoaster? ______________

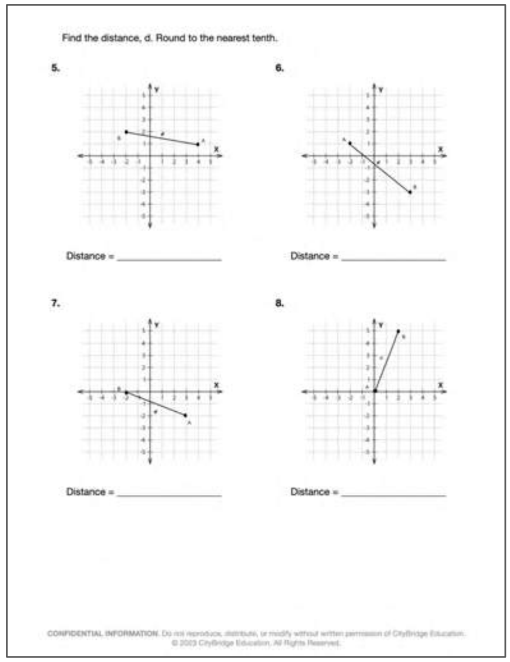
b. How far are the restrooms to the snack bar? ______________

c. How far is the Rollercoaster to the snack bar? ______________

2. What is the distance between the points (4, –7) and (–5, –7)? ______________

3. What is the distance between points (8, -6) and (5, 4)? ______________

4. What is the distance between ponts (0, 0) and (6, -4)? ______________



Find the distance, d. Round to the nearest tenth.

5.

Distance = ______________

6.

Distance = ______________

7.

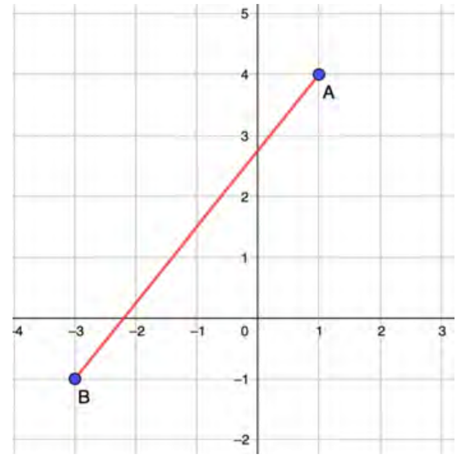
Distance = ______________

8.

Distance = ______________

Name: ______________________________________ G8 U6 Lesson 11 - Let's Try It

1. Find the distance between the points.



a. Draw two legs to make a right triangle.

b. What are the coordinates of Point A? ________

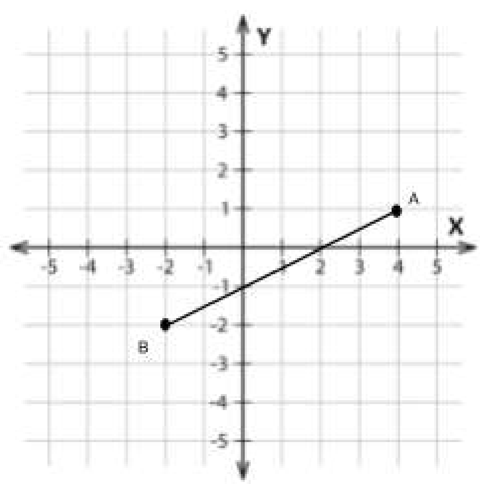
c. What are the coordinates of Point B? ________

d. What is the horizontal distance $|x_2 - x_1|$? ________________

e. What is the vertical distance $|y_2 - y_1|$?___________________

f. Use the Pythagorean Theorem to find the distance between Points A and B. **Round to the nearest tenth, if needed.** _________________________

2. Find the distance between the points.



a. Draw two legs to make a right triangle.

b. What are the coordinates of Point A? ________

c. What are the coordinates of Point B? ________

d. What is the horizontal distance $|x_2 - x_1|$?

____

e. What is the vertical distance $|y_2 - y_1|$?___________________

f. Use the Pythagorean Theorem to find the distance between Points A and B. **Round to the nearest tenth, if needed.** _________________________

Find the distance between the pair of points.

3. (7, -2) and (11, -2) ________________

4. (6, 4) and (6, -8) __________________

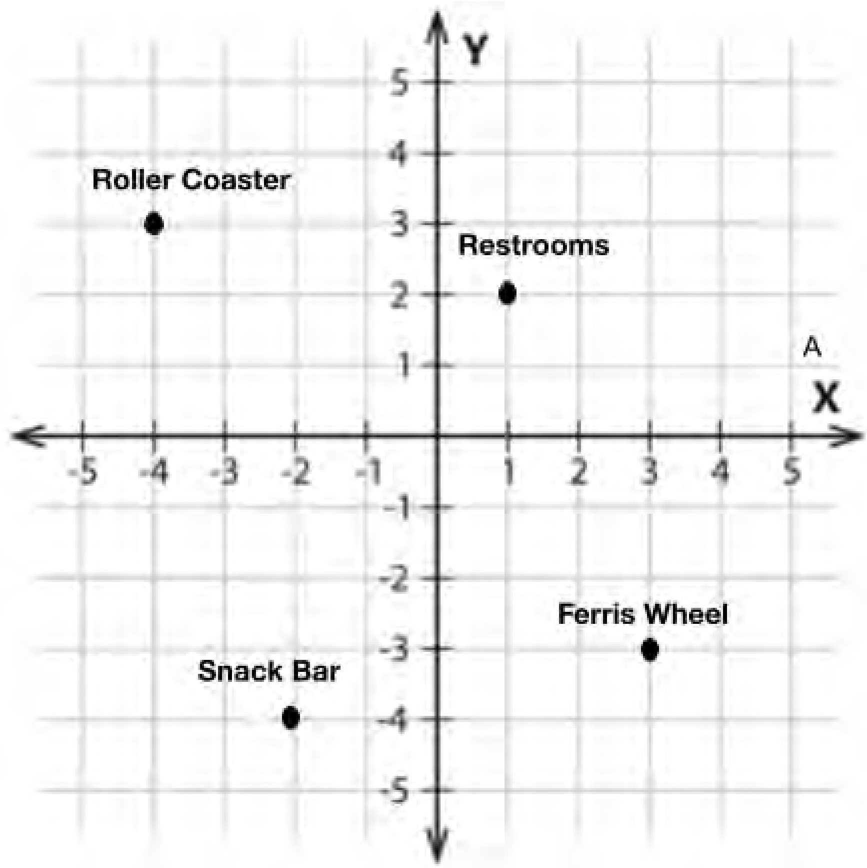
5. (8, -10) and (5, -10) _______________

6. (-2, -6) and (-2, 5) ________________

7. (-5, 2) and (-5, -4) ________________

Name: ____ G8 U6 Lesson 11 - Independent Work

1.



a. How far is the Ferris wheel from the rollercoaster? ________________________________

b. How far are the restrooms to the snack bar? ____

c. How far is the Rollercoaster to the snack bar? ____________________________________

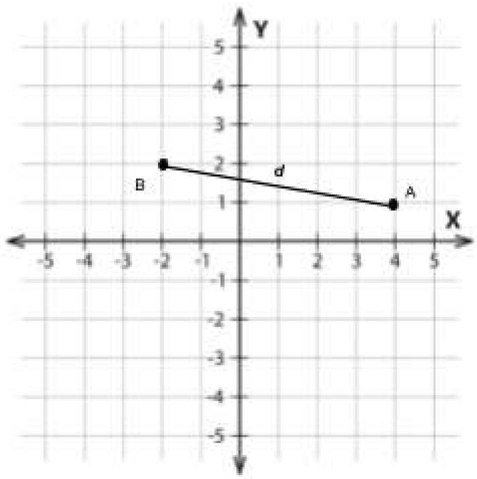
2. What is the distance between the points (4, –7) and (–5, –7)? _______________________

3. What is the distance between points (8, -6) and (5, 4)? _____________________________

4. What is the distance between ponts (0, 0) and (6, -4)? _____________________________

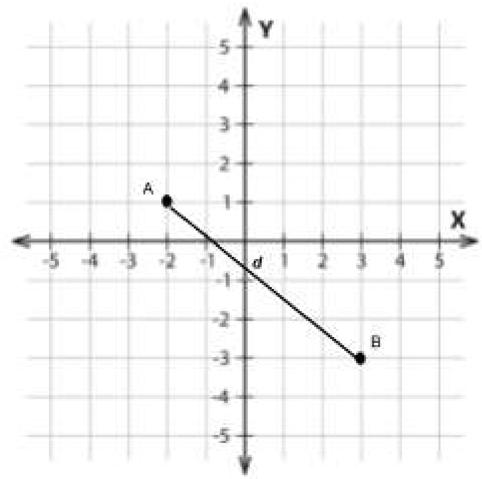
Find the distance, d. Round to the nearest tenth.

5.



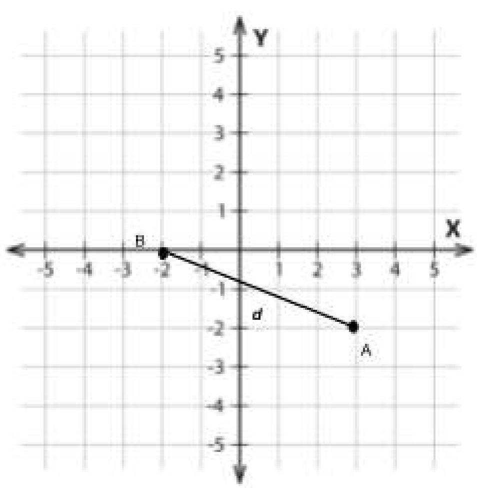
Distance = ____________________

6.



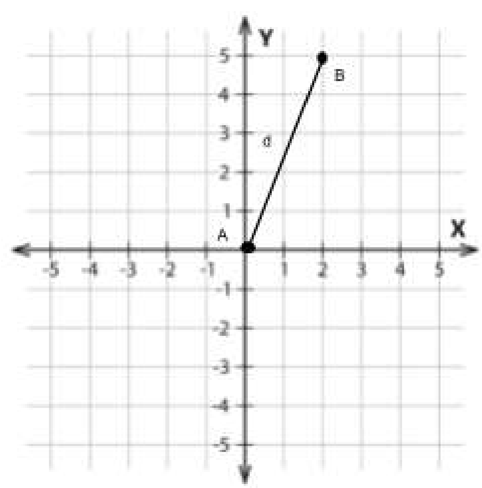
Distance = ____________________

7.

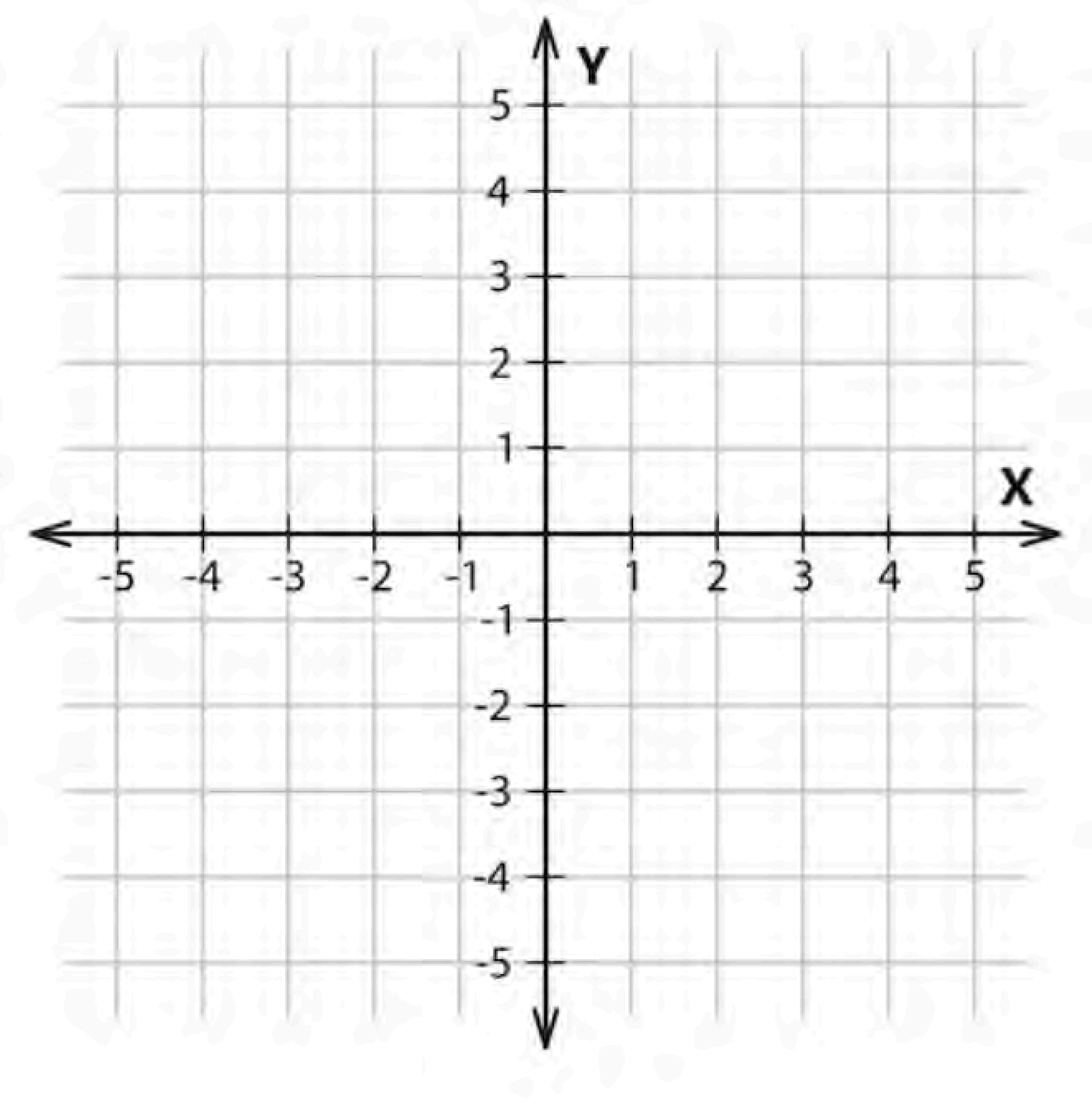


Distance = ____________________

8.

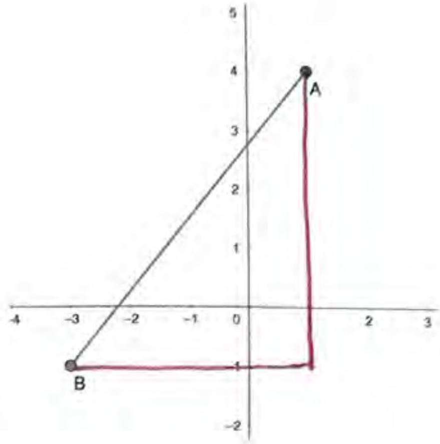


Distance = ____________________


Y
X
5
4
3
2
1
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 1 2 3 4 5

Name: Answer Key

1. Find the distance between the points.



a. Draw two legs to make a right triangle.

b. What are the coordinates of Point A? (1,4)

c. What are the coordinates of Point B? (-3,-1)

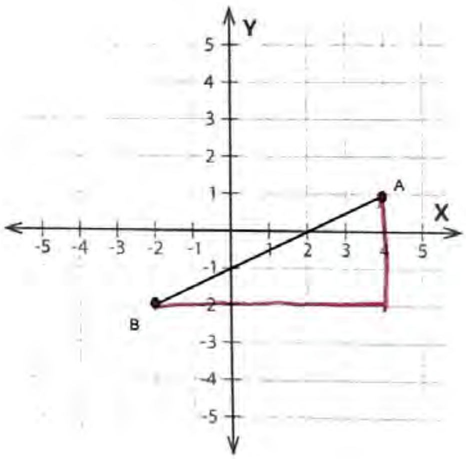
d. What is the horizontal distance $|x_2 - x_1|$? $|-3-1| = 4$

e. What is the vertical distance $|y_2 - y_1|$? $|-1-4| = 5$

f. Use the Pythagorean Theorem to find the distance between Points A and B. **Round to the nearest tenth, if needed.** 6.4 units

$4^2 + 5^2 = c^2$

2. Find the distance between the points.



a. Draw two legs to make a right triangle.

b. What are the coordinates of Point A? (4,1)

c. What are the coordinates of Point B? (-2,-2)

d. What is the horizontal distance $|x_2 - x_1|$?

$|4-(-2)| = 6$

e. What is the vertical distance $|y_2 - y_1|$? $|-2-1| = 3$

f. Use the Pythagorean Theorem to find the distance between Points A and B. **Round to the nearest tenth, if needed.** 6.7 units

$3^2 + 6^2 = c^2$

Find the distance between the pair of points.

3. (7, -2) and (11, -2) 4 units

4. (6, 4) and (6, -8) 12 units

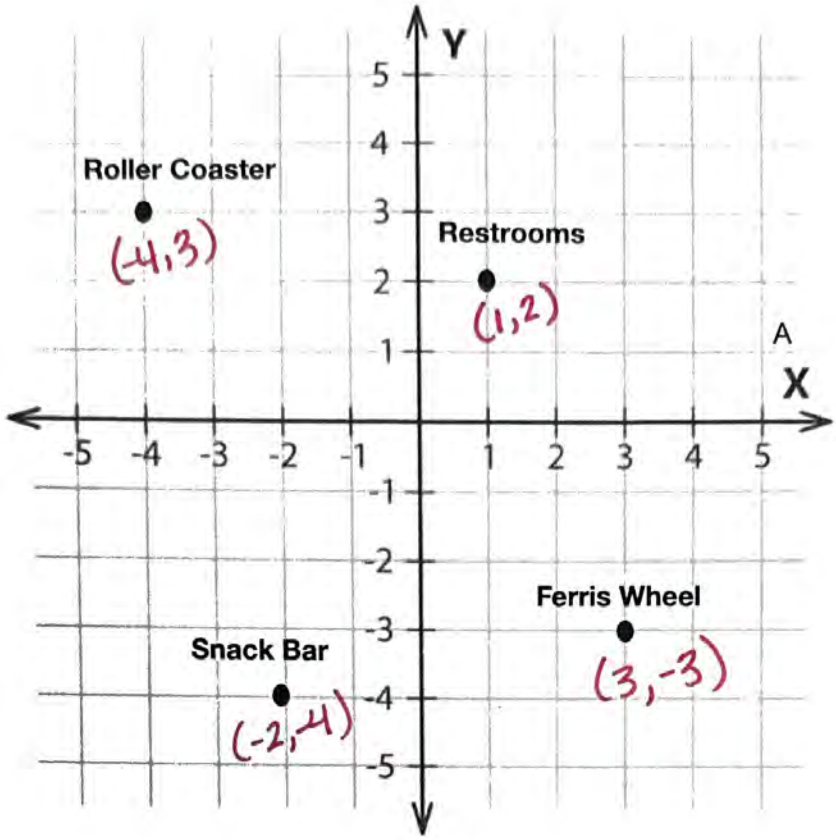
5. (8, -10) and (5, -10) 3 units

6. (-2, -6) and (-2, 5) 11 units

7. (-5, 2) and (-5, -4) 6 units

Name: Answer Key G8 U6 Lesson 11 - Independent Work

1.



a. How far is the Ferris wheel from the rollercoaster? $7^2 + 6^2 = c^2$ $c = 9.2$ units

b. How far are the restrooms to the snack bar? $3^2 + 6^2 = c^2$ $c = 6.7$ units

c. How far is the Rollercoaster to the snack bar? $2^2 + 7^2 = c^2$ $c = 7.3$ units

2. What is the distance between the points (4, –7) and (–5, –7)? $9^2 + 0^2 = c^2$ $c = 9$ units

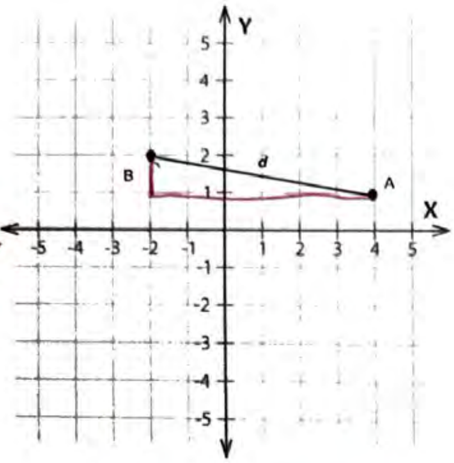
3. What is the distance between points (8, -6) and (5, 4)? $3^2 + 10^2 = c^2$ $c = 10.4$ units

4. What is the distance between ponts (0, 0) and (6, -4)? $6^2 + 4^2 = c^2$ $c = 7.2$ units

Find the distance, d. Round to the nearest tenth.

5.

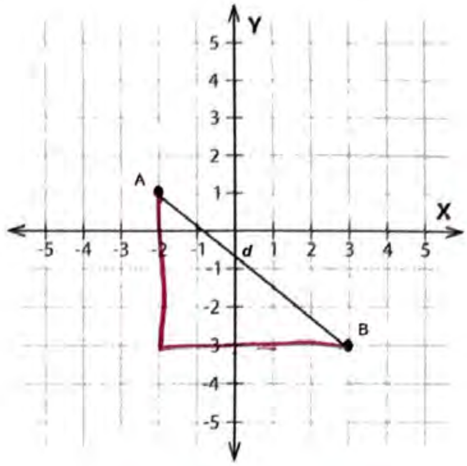
$1^2+6^2=c^2$



Distance = ~6.1 units

6.

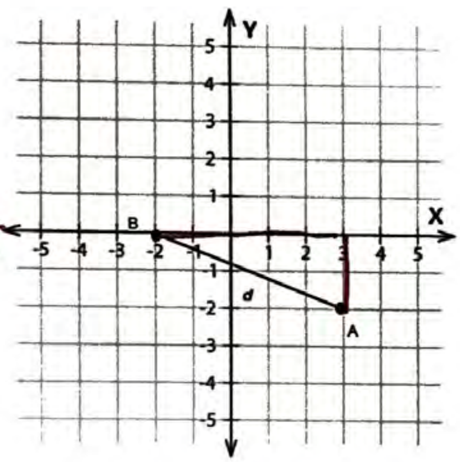
$4^2+5^2=c^2$



Distance = 6.4 units

7.

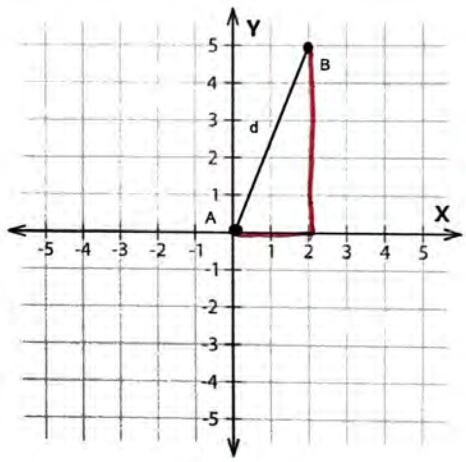
$2^2+5^2=c^2$



Distance = 5.4 units

8.

$2^2+5^2=c^2$



Distance = 5.4 units

G8 U6 Lesson 12
Comprehend the term “cube root of a” and the notation $\sqrt[3]{a}$

CITYTUTOR DC

G8 U6 Lesson 12 - Students will comprehend the term “cube root of a” and the notation $\sqrt[3]{a}$.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we are going to learn about cube roots and the notation used to represent them. By the end of the lesson, you'll understand what the cube root of a number is and how to find it.

Why is understanding cube roots important? Think about packing, construction, and even computing. Cube roots help us solve real-life problems involving volume.

Let’s Talk (Slide 3): Let’s review everything we know about Volume. This box is visual to help you think about what you may remember or have learned about volume. **What do you remember?** Possible Student Answers, Key Points:

- Volume is the total cubic units occupied by it, in a three-dimensional space.
- Volume is measured in units cubed.

Volume is the space occupied within a 3-dimensional space, like cubes, cylinders, and pyramids. What do you know about cubes? Possible Student Answers, Key Points:

- A cube has 6 square faces or sides.
- The length, width, and height of the cube are of equal length.
- The Volume of a cube is s^3.

Cubes have 6 faces that are equal in length, and when we find their volume, we take one side and multiply it by itself 3 times, so the volume is s^3.

Let’s Think (Slide 4): Look at this cube. We know the volume of the cube is 8 cm^3. **How might we figure out the side length of a cube if we know its volume?**

(Write on the board: Cube Root of a Number)
The cube root of a number denoted as $\sqrt[3]{a}$, is the number that, when multiplied by itself three times (cubed), gives the original number, a.
(Write on the board: If $b = \sqrt[3]{a}$ then $a = b^3$)

8
$2 \cdot 2 \cdot 2 = 8$
$\sqrt[3]{8} = 2$

For example, to find the side length of this cube, we can find the cube root of 8. We need to find a number that, when multiplied by itself three times, equals 8.

Let's try 2. 2 × 2 × 2 = 8, so $2 = \sqrt[3]{8}$.

The notation $\sqrt[3]{a}$ helps us quickly identify the cube root operation. For instance, $\sqrt[3]{8}$ means the number which, when cubed, equals 8. That's 2 because 2 x 2 x 2 = 8.

$\sqrt[3]{1000}$
$10 \cdot 10 \cdot 10 = 1000$
$\sqrt[3]{1000} = 10$

Let’s solve one more problem together. What is the $\sqrt[3]{1000}$? *(Write $\sqrt[3]{1000}$ on the board)*

We need to find a number that can be multiplied 3 times and has a product of 1000.
Let's try 10. 10 × 10 × 10 = 1000, so $10 = \sqrt[3]{1000}$.
Let’s Try it (Slides 5): Now it’s your turn to try. Remember that the area of a cube is multiplying a side length three times by itself, and the cube root is finding the number that you multiply three times to get the original number.

WARM WELCOME

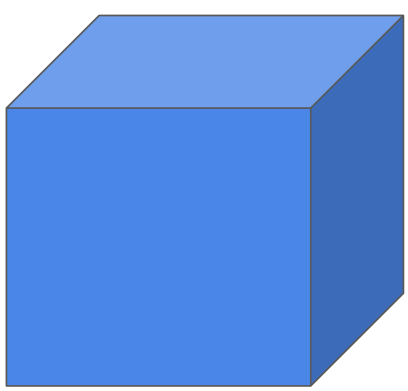
Today, you will comprehend the term "cube root of a" and the notation $\sqrt[3]{a}$.

Let's Talk:

What do you remember about volume?

Let's Think:

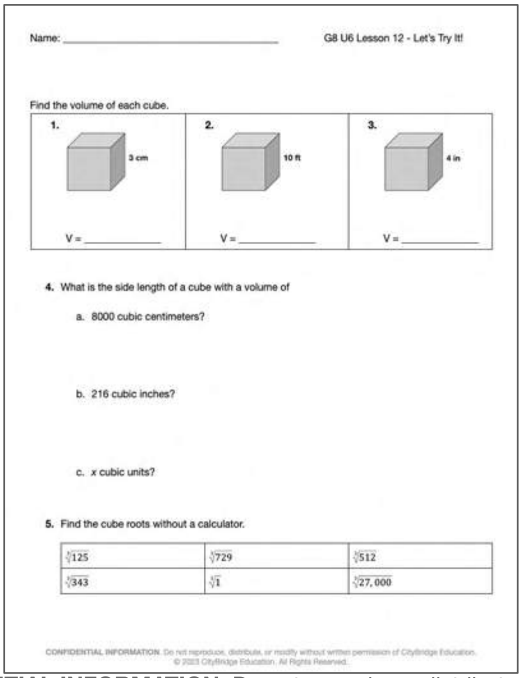
How might we figure out the side length of a cube if we know its volume?



Volume = 8 cm^3

Let's Try It:

Let's explore finding the "cube root of a" and ∛a together.



Name: ______________________ G8 U6 Lesson 12 - Let's Try It!

Find the volume of each cube.

1. 3 cm	2. 10 ft	3. 4 in
V = ________	V = ________	V = ________

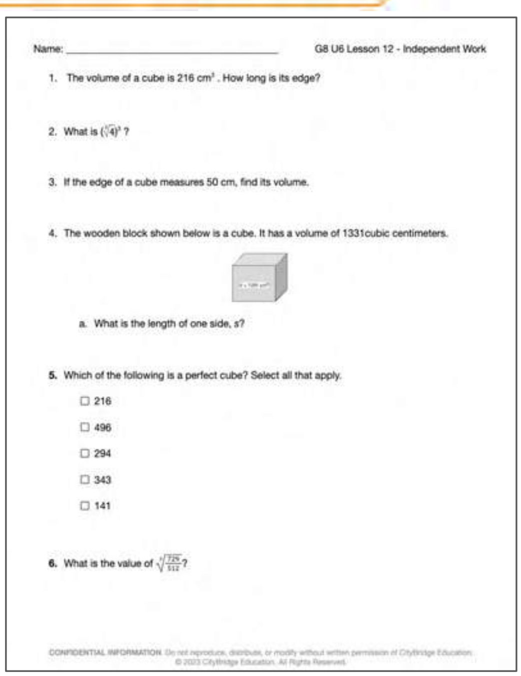
4. What is the side length of a cube with a volume of
   a. 8000 cubic centimeters?
   b. 216 cubic inches?
   c. *x* cubic units?

5. Find the cube roots without a calculator.

$\sqrt[3]{125}$	$\sqrt[3]{729}$	$\sqrt[3]{512}$
$\sqrt[3]{343}$	$\sqrt[3]{1}$	$\sqrt[3]{27,000}$

On your Own:

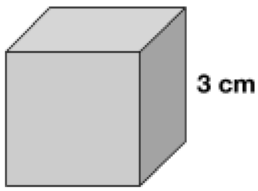
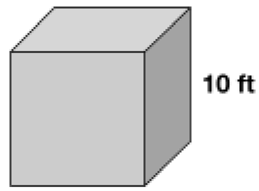
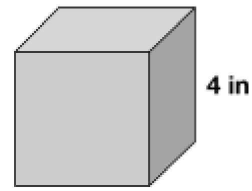
Now it's time to find the "cube root of a" ∛a on your own.



Name: ______________________ G8 U6 Lesson 12 - Independent Work

1. The volume of a cube is 216 cm^3. How long is its edge?
2. What is $(\sqrt[3]{4})^3$?
3. If the edge of a cube measures 50 cm, find its volume.
4. The wooden block shown below is a cube. It has a volume of 1331cubic centimeters.
   a. What is the length of one side, *s*?
5. Which of the following is a perfect cube? Select all that apply.
   - ☐ 216
   - ☐ 496
   - ☐ 294
   - ☐ 343
   - ☐ 141
6. What is the value of $\sqrt[3]{\frac{729}{512}}$?

Find the volume of each cube.

1.  V = ______________	**2.**  V = ______________	**3.**  V = ______________

4. What is the side length of a cube with a volume of

a. 8000 cubic centimeters?

b. 216 cubic inches?

c. *x* cubic units?

5. Find the cube roots without a calculator.

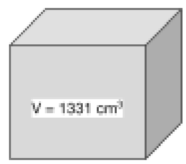
$\sqrt[3]{125}$	$\sqrt[3]{729}$	$\sqrt[3]{512}$
$\sqrt[3]{343}$	$\sqrt[3]{1}$	$\sqrt[3]{27,000}$

1. The volume of a cube is 216 cm^3 . How long is its edge?

2. What is $(\sqrt[3]{4})^3$?

3. If the edge of a cube measures 50 cm, find its volume.

4. The wooden block shown below is a cube. It has a volume of 1331cubic centimeters.



   a. What is the length of one side, *s*?

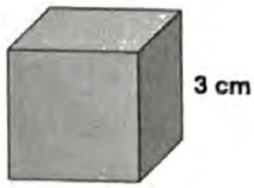
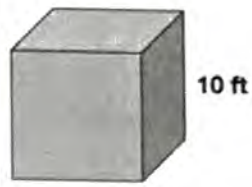
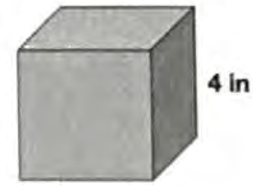
5. Which of the following is a perfect cube? Select all that apply.

- ☐ 216
- ☐ 496
- ☐ 294
- ☐ 343
- ☐ 141

6. What is the value of $\sqrt[3]{\frac{729}{512}}$?

Name: Answer Key

Find the volume of each cube.

1.	2.	3.
		
V = 27 cm^3	V = 1000 ft^3	V = 64 in^3

4. What is the side length of a cube with a volume of

a. 8000 cubic centimeters?

S = 20 cm

b. 216 cubic inches?

S = 6 in

c. *x* cubic units?

S = $\sqrt[3]{x}$ units

5. Find the cube roots without a calculator.

$\sqrt[3]{125}$ = 5	$\sqrt[3]{729}$ = 9	$\sqrt[3]{512}$ = 8
$\sqrt[3]{343}$ = 7	$\sqrt[3]{1}$ = 1	$\sqrt[3]{27,000}$ = 30

Name: Answer Key

G8 U6 Lesson 12 - Independent Work

1. The volume of a cube is 216 cm^3. How long is its edge?

$s = 6$ cm

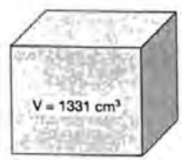
2. What is $(\sqrt[3]{4})^3$?

4

3. If the edge of a cube measures 50 cm, find its volume.

$V = 50^3 = 125{,}000\ cm^3$

4. The wooden block shown below is a cube. It has a volume of 1331cubic centimeters.



a. What is the length of one side, *s*?

$s = 11$ cm

5. Which of the following is a perfect cube? Select all that apply.

- ☑ 216
- ☐ 496
- ☐ 294
- ☑ 343
- ☐ 141

6. What is the value of $\sqrt[3]{\frac{729}{512}}$? $= \frac{9}{8}$

G8 U6 Lesson 13
Determine the whole numbers that a cube root lies between

CITYTUTOR DC

G8 U6 Lesson 13 - Students will determine the whole numbers that a cube root lies between.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today, we're going to learn how to determine the whole numbers between which a cube root lies. This is very similar to when we learned about square roots. Sometimes, the cube roots of numbers are not whole numbers.

Let's Review (Slide 3): List the first ten cubes. This list will help you throughout today's lesson when determining what whole numbers' cube roots lie between.

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

To cube a number is to multiply it by itself three times, so at any time you are unsure about a cube number, you can figure it out.

Let's Talk (Slide 4): Is the $\sqrt[3]{10}$ a perfect cube? If it is, tell me why. If it isn't, tell me why. Possible Student Answers, Key Points:

- No, it is not a perfect cube.
- There is not one number we can multiply by itself three times to equal 10.

As I said before, sometimes, the cube roots of numbers are not whole numbers, and we can find what two whole numbers the cube root is between.

Let's Think (Slide 5): How can we determine what two whole numbers $\sqrt[3]{10}$ lies between? Possible Student Answers, Key Points:

- Use a Calculator.
- Find what two cubed numbers the 10 is between.
- List out the first ten cube roots.

To determine the whole numbers between which the cube root lies, we can use the list we generated earlier with the first ten cubed numbers.

$$\sqrt[3]{10}$$
$$2^3 = 8 \qquad 3^3 = 27$$
$$2 < \sqrt[3]{10} < 3$$

(Write $\sqrt[3]{10}$ and so justification on the board) If we want to find the whole numbers that $\sqrt[3]{10}$ lies between. We know that $2^3 = 8$ and $3^3 = 27$. Since 10 is between 8 and 27, so $\sqrt[3]{10}$ is between 2 and 3.

We can also use a calculator as a tool to check our work.

$$\sqrt[3]{50}$$
$$3^3 = 27 \qquad 4^3 = 64$$
$$3 < \sqrt[3]{50} < 4$$

Let's try another one. (*Write $\sqrt[3]{50}$ and so justification on the board)* Find the whole numbers that $\sqrt[3]{50}$ lies between. We know that $3^3 = 27$ and $4^3 = 64$. Since 50 is between 27 and 64, $\sqrt[3]{50}$ is between 3 and 4.

Let's Try it (Slides 6): Now it's your turn to try! Remember to determine the whole numbers between which a cube root lies, we need to find two perfect cubes that the number lies between.

WARM WELCOME

Today you will determine the whole numbers that a cube root lies between.

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Let's Review:

List the first ten cube roots.

Let's Talk:

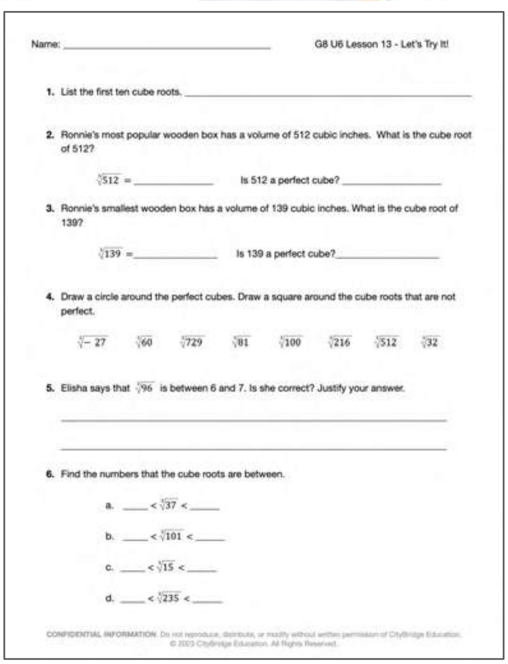
Is the $\sqrt[3]{10}$ a perfect cube? Justify your answer.

Let's Think:

What two whole numbers does the $\sqrt[3]{10}$ fall between?

Let's Try It:

Let's explore finding what whole numbers cube roots are between.



Name: ______________________ G8 U6 Lesson 13 - Let's Try It!

1. List the first ten cube roots. ______________________

2. Ronnie's most popular wooden box has a volume of 512 cubic inches. What is the cube root of 512?

   $\sqrt[3]{512}$ = __________ Is 512 a perfect cube? __________

3. Ronnie's smallest wooden box has a volume of 139 cubic inches. What is the cube root of 139?

   $\sqrt[3]{139}$ = __________ Is 139 a perfect cube? __________

4. Draw a circle around the perfect cubes. Draw a square around the cube roots that are not perfect.

   $\sqrt[3]{-27}$ $\sqrt[3]{60}$ $\sqrt[3]{729}$ $\sqrt[3]{81}$ $\sqrt[3]{100}$ $\sqrt[3]{216}$ $\sqrt[3]{512}$ $\sqrt[3]{32}$

5. Elisha says that $\sqrt[3]{96}$ is between 6 and 7. Is she correct? Justify your answer.

   ____

   ____

6. Find the numbers that the cube roots are between.

   a. ____ $< \sqrt[3]{37} <$ ____

   b. ____ $< \sqrt[3]{101} <$ ____

   c. ____ $< \sqrt[3]{15} <$ ____

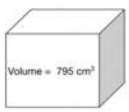
   d. ____ $< \sqrt[3]{235} <$ ____

On your Own:

Now it’s time to try finding what whole numbers cube roots are between on your own.

Name: ____________________ G8 U6 Lesson 13 - Independent Work

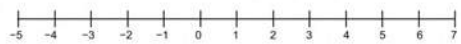
1. List the first ten cube roots. ____________________
2. Find the numbers that the cube roots are between.
   a. ____ $< \sqrt[3]{25} <$ ____
   b. ____ $< \sqrt[3]{2} <$ ____
   c. ____ $< \sqrt[3]{147} <$ ____
   d. ____ $< \sqrt[3]{435} <$ ____
3. Label the following sentences **true** or **false**.
   a. The number 343 is not a perfect square. ____________
   b. The cube root of a perfect cube is an integer. ____________
   c. Taking the cube root and cubing a number are opposite operations. ____________
   d. The symbol for a cube root and a square root is the same. ____________
   e. You can use cube roots to find the side length of a cube given just the volume because all side lengths in a cube are the same length? ____________
4. What two numbers are the side length of the cube between?



Answer: ____________

5. Plot the following numbers approximately on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

$\sqrt[3]{9}$ $\quad -\sqrt[3]{27}$ $\quad \sqrt[3]{729}$ $\quad \sqrt[3]{412}$ $\quad -\sqrt[3]{1}$



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1. List the first ten cubes. ____

2. Ronnie's most popular wooden box has a volume of 512 cubic inches. What is the cube root of 512?

$\sqrt[3]{512} =$ _______________ Is 512 a perfect cube? ___________________

3. Ronnie's smallest wooden box has a volume of 139 cubic inches. What is the cube root of 139?

$\sqrt[3]{139} =$ ________________ Is 139 a perfect cube? ____________________

4. Draw a circle around the perfect cubes. Draw a square around the cube roots that are not perfect.

$\sqrt[3]{-27}$ $\sqrt[3]{60}$ $\sqrt[3]{729}$ $\sqrt[3]{81}$ $\sqrt[3]{100}$ $\sqrt[3]{216}$ $\sqrt[3]{512}$ $\sqrt[3]{32}$

5. Elisha says that $\sqrt[3]{96}$ is between 6 and 7. Is she correct? Justify your answer.

____

____

6. Find the numbers that the cube roots are between.

a. _____ $< \sqrt[3]{37} <$ ______

b. _____ $< \sqrt[3]{101} <$ ______

c. _____ $< \sqrt[3]{15} <$ ______

d. _____ $< \sqrt[3]{235} <$ ______

1. List the first ten cubes. ____

2. Find the numbers that the cube roots are between.

   a. _____ $< \sqrt[3]{25} <$ ______

   b. _____ $< \sqrt[3]{2} <$ ______

   c. _____ $< \sqrt[3]{147} <$ ______

   d. _____ $< \sqrt[3]{435} <$ ______

3. Label the following sentences **true** or **false**.

   a. The number 343 is not a perfect square. __________________

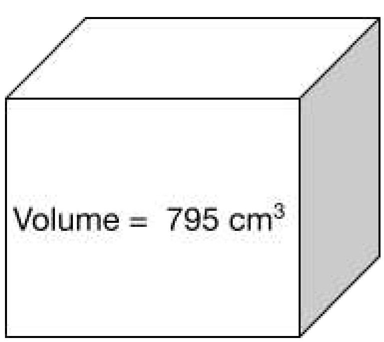
   b. The cube root of a perfect cube is an integer. ________________

   c. Taking the cube root and cubing a number are opposite operations. _______________

   d. The symbol for a cube root and a square root is the same. __________________

   e. You can use cube roots to find the side length of a cube given just the volume because all side lengths in a cube are the same length? _________________

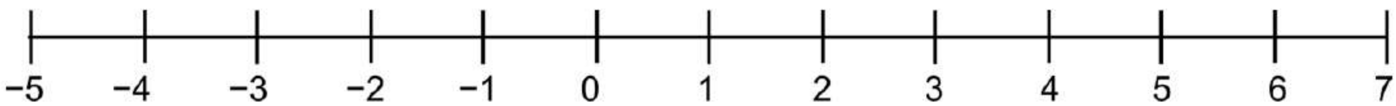
4. What two numbers are the side length of the cube between?



Answer: ___________________

5. Plot the following numbers approximately on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

$\sqrt[3]{9}$ $-\sqrt[3]{27}$ $\sqrt[3]{200}$ $\sqrt[3]{343}$ $-\sqrt[3]{1}$



Name: Answer Key

1. List the first ten cubes. 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

2. Ronnie's most popular wooden box has a volume of 512 cubic inches. What is the cube root of 512?

$\sqrt[3]{512} =$ 8 Is 512 a perfect cube? yes

3. Ronnie's smallest wooden box has a volume of 139 cubic inches. What is the cube root of 139?

$\sqrt[3]{139} =$ 5.18 or 5.2 Is 139 a perfect cube? no

4. Draw a circle around the perfect cubes. Draw a square around the cube roots that are not perfect.

(circled) $\sqrt[3]{-27}$ (squared) $\sqrt[3]{60}$ (circled) $\sqrt[3]{729}$ (squared) $\sqrt[3]{81}$ (squared) $\sqrt[3]{100}$ (circled) $\sqrt[3]{216}$ (circled) $\sqrt[3]{512}$ (squared) $\sqrt[3]{32}$

5. Elisha says that $\sqrt[3]{96}$ is between 6 and 7. Is she correct? Justify your answer.

Elisha is incorrect. Since $4^3 = 64$ and $5^3 = 125$. 96 is between 64 and 125, then $4 < \sqrt[3]{96} < 5$.

6. Find the numbers that the cube roots are between.

a. 3 $< \sqrt[3]{37} <$ 4

b. 4 $< \sqrt[3]{101} <$ 5

c. 2 $< \sqrt[3]{15} <$ 3

d. 6 $< \sqrt[3]{235} <$ 7

Name: Answer Key

1. List the first ten cubes. 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

2. Find the numbers that the cube roots are between.

   a. $2 < \sqrt[3]{25} < 3$

   b. $1 < \sqrt[3]{2} < 2$

   c. $4 < \sqrt[3]{147} < 5$

   d. $7 < \sqrt[3]{435} < 8$

3. Label the following sentences **true** or **false**.

   a. The number 343 is not a perfect square. false

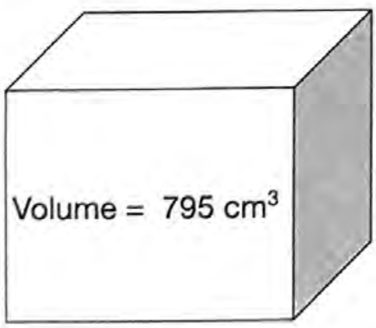
   b. The cube root of a perfect cube is an integer. true

   c. Taking the cube root and cubing a number are opposite operations. true

   d. The symbol for a cube root and a square root is the same. false

   e. You can use cube roots to find the side length of a cube given just the volume because all side lengths in a cube are the same length? true

4. What two numbers are the side length of the cube between?



$9^3 = 729$ $10^2 = 1000$

Answer: $9 < \sqrt[3]{795} < 10$

5. Plot the following numbers approximately on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

$\sqrt[3]{9}$ $-\sqrt[3]{27}$ $\sqrt[3]{200}$ $\sqrt[3]{343}$ $-\sqrt[3]{1}$