



# Seventh Grade Math Lesson Materials

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Identification of the copyrighted work claimed to have been infringed, or, if multiple copyrighted works allegedly have been infringed, then a representative list of such copyrighted works;

Identification of the material that is claimed to be infringing and that is to be removed or access to which is to be disabled, and information reasonably sufficient to permit us to locate the allegedly infringing material, e.g., the specific web page address on the Platform;

Information reasonably sufficient to permit us to contact the party alleging infringement, including an email address;

A statement that the party alleging infringement has a good-faith belief that use of the copyrighted work in the manner complained of is not authorized by the copyright owner or its agent, or is not otherwise permitted under the law; and

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# G7 Unit 1:

Scale Drawings

# **G7 U1 Lesson 1**

Differentiate between scaled and non-scaled copies of a figure

## G7 U1 Lesson 1 - Students will differentiate between scaled and non-scaled copies of a figure

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today is our first lesson in our unit that's all about scale drawings. A scale drawing is a two-dimensional representation of an actual object or place. Scale drawings and scale models are all around us. We see them when we look at maps or at blueprints of buildings. Model-builders, video game designers, and artists often use scale drawings to help recreate figures in different sizes. The applications for scale drawings are seemingly limitless, so I'm excited to start exploring this concept with you. Let's jump in!

**Let's Talk (Slide 3):** Take a look at the original image of the kitten. Now, look at the other images. What do you notice about the images? **Possible Student Answers, Key Points:**

- I notice the first one is a smaller version of the original and the second one is a larger version of the original.
- I notice the third one is kind of slanted and skinny. I notice the fourth one is kind of flattened.

Each image is a different version of the original, but only two are what we would call *scaled copies*. A scaled copy is when an image is changed in a way where each part of the images is a certain number of times bigger or smaller than the original. A scaled copy is never stretched or distorted. Based on what I shared, which of these images could we consider scaled copies of the original? How do you know? **Possible Student Answers, Key Points:**

- I think the first two could be considered scaled copies. They look like the same kitten in the original, just a little bigger or a little smaller.
- The other two images distort the kitten so it doesn't look similar. It's stretched or pulled or squished in a way that doesn't match the original.

The first two images are scaled copies of the original. A scaled copy can be larger or smaller than an original, but never stretched out or distorted like the other two images appear.

**Let's Think (Slide 4):** Take a second to look at the original rectangle here. What do you notice about the original rectangle? **Possible Student Answers, Key Points:**

- I notice it's small and narrow.
- I notice it has a length of 3 units and a width of 1 unit. I notice the area is 3 square units.

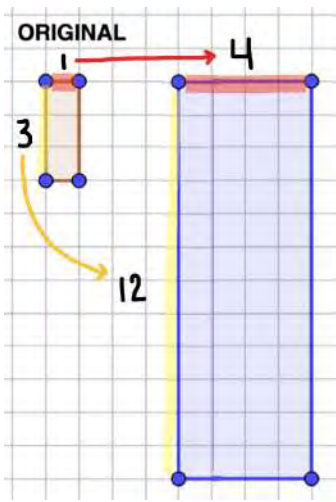
One of the two other rectangles can be considered a scaled copy of the original rectangle. We're going to try to see if we can determine which rectangle is a scaled copy just by looking at the original. I'll also show you a way to mathematically prove whether something is a scaled copy of an original.

Let's just think visually for a second. You already told me some attributes you noticed for the original rectangle. Based on those attributes do you think it's more likely that the purple rectangle or the pink rectangle is a scaled copy of the original? **Possible Student Answers, Key Points:**

- I think the first/purple rectangle is a scaled copy. It's still narrow and tall like the original.
- I don't think the second/pink rectangle is a scaled copy. It's not narrow like the original. It's more square-shaped.

The purple rectangle is a scaled copy of the original. We can tell this visually because, even though the rectangle is bigger than the original, it still maintains the general shape of the original. The pink rectangle is a square, which isn't the same shape as the original.

If we want to take a more precise, mathematical approach to determining whether something is a scaled copy, we can draw our attention to the side lengths. The side lengths in a scaled copy are related to the corresponding side lengths in the original in a consistent way.



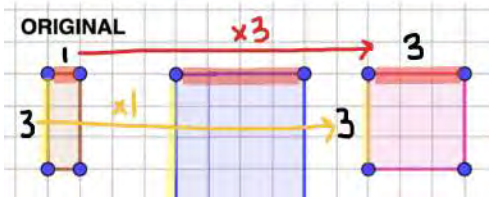
Let's start by looking at each shorter side length. What is the short side length of the original rectangle? (1 unit) What is the short side length of the scaled copy? (4 units) (highlight the corresponding lengths and label them with the measurements, drawing an arrow from the original to the scaled copy) The original side length is 1 and the corresponding side length in the scaled copy is 4.

Let's look at the longer side length. What is the long side length of the original rectangle? (3 units) What is the long side length of the scaled copy? (12 units) (highlight the corresponding lengths and label them with the measurements, drawing an arrow from the original to the scaled copy) The original side length is 3 and the corresponding side length in the scaled copy is 12.

What do you notice about the measurements of each pair of corresponding side lengths? Possible Student Answers, Key Points:

- I know  $1 \times 4 = 4$  and I know  $3 \times 4 = 12$ . Each one can be multiplied by 4 to find the scaled copy.
- I know each side length of the scaled copy is 4 times as long as the corresponding side of the original.

The side lengths of the original and the scaled copy are related in a consistent way. Each side length in the scaled copy is 4 times as long as the corresponding side length in the original.



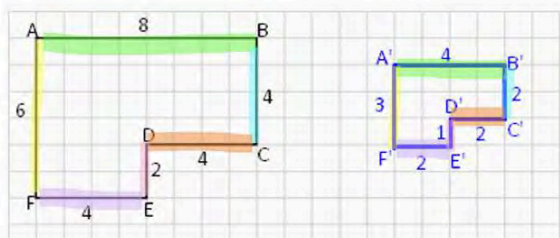
We already said the pink rectangle did not look like a scaled copy, because the shape was different. We can prove that by looking at the corresponding side lengths. (label and highlight corresponding side lengths as shown) The top side length of the scaled copy is three times as long as the corresponding side length of the original figure. The left side length of the scaled copy is one times as long as the corresponding side length of the original. Since the side lengths are not related in a consistent way, we know for certain that the pink rectangle cannot be a scaled copy of the original.

We can visually check to see if an image is a scaled copy of an original figure, and we also saw how we can look at how corresponding sides are related to determine if an image is a scaled copy of the original figure. Both strategies can work, but I'll caution you that just going off of visuals can sometimes be misleading. Let's look at one more.

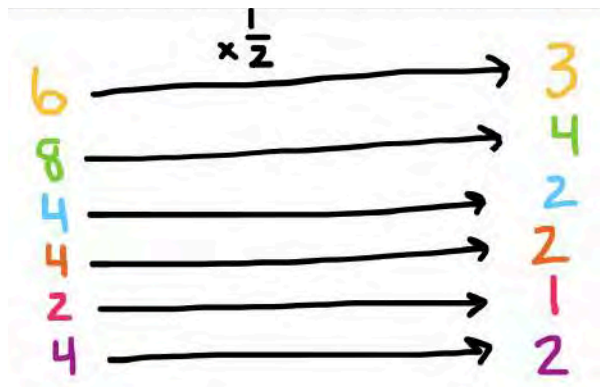
**Let's Think (Slide 5):** This problem wants us to determine whether the second polygon is a scaled copy of the original polygon. Visually, does the second figure appear to be a scaled copy? Possible Student Answers, Key Points:

- It looks like it's the same shape, but just a little smaller. I think it could be a scaled copy.
- It's hard to tell for sure. This polygon has more sides than the last example we saw, so I'm not sure.





The second image appears to be similar in shape to the original image, but it can be hard to tell just by looking. Let's look at corresponding sides to see. I'll highlight each corresponding side. (*highlight each corresponding side systematically*)



Let's look at the length of each corresponding side in the original compared to the potential scaled copy. (*list sides as shown, color-coding as possible to match the visual, then draw an arrow between corresponding sides*) Are the side lengths related in a consistent way? What do you notice? **Possible Student Answers, Key Points:**

**Possible Student Answers, Key Points:**

- I think they are related in a consistent way. Each measurement on the scaled copy is half of the original measurement.
- I can divide the original side length by 2 each time, and the result will be the length of the other polygon.

(*write  $\times \frac{1}{2}$  between the corresponding sides*) Excellent! By looking at how the corresponding sides are related, we can see that the second polygon is a scaled copy of the first polygon. Each side length of the scaled copy is half the length of the original. We can multiply each original length by  $\frac{1}{2}$ , and the result will be the corresponding length in the other polygon.

**Let's Try it (Slides 6 - 7):** Now let's work on a few more examples together before you get a chance to practice on your own. Sometimes we can visually tell if an image is a scaled copy, because the copy is the same shape; the copy isn't distorted or stretched or squished in any way. We also saw how we can look more precisely at corresponding sides. If all corresponding sides are related in a similar way, then we can be certain that an image is a scaled copy. Let's keep these ideas in mind as we look at our next few examples. Let's go for it!

# WARM WELCOME



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**Today we will differentiate between scaled and non-scaled copies of a figure.**

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Let's Talk:

What do you notice about the images shown here?

Original



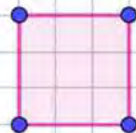
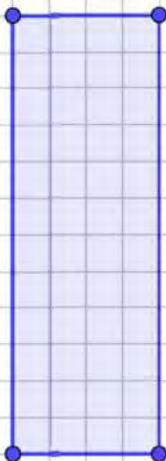
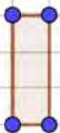
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Let's Think:

Which rectangle is a scaled copy of the original?

Explain how you know visually.

ORIGINAL



Explain how you know using numbers.

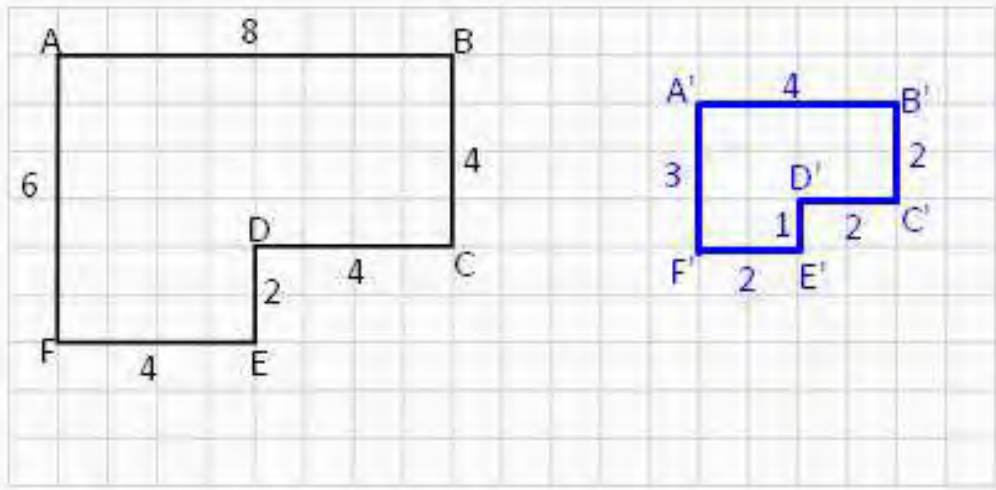
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# Let's Think:

## Is the blue polygon a scaled image of the original?

### ORIGINAL



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# Let's Try It:

## Let's explore differentiating between scaled and non-scaled copies of a figure together.

Name: \_\_\_\_\_ Q7 U1 Lesson 1 - Let's Try It

Look at the original image of the hamster.

1. Circle the image that looks most like the original.

2. When an image gets larger or smaller, and nothing else about the image changes, we call it a \_\_\_\_\_.

Look at the original image of the rectangle.

3. Circle the rectangle that is a scaled copy of the original.

4. Explain how you know [just by looking].

5. Label the length of each short side on the original and the scaled copy. How are they related?

6. Label the length of each long side on the original and the scaled copy. How are they related?

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Look at the original rectangle.

7. Label the side lengths of the original rectangle.

8. Label the side lengths of Rectangle A and Rectangle B.

9. Which rectangle is a scaled copy of the original?  
a. Rectangle A  
b. Rectangle B

10. How do you know?

Look at the original rectangles below. The top rectangle is the original.

11. Circle the rectangle that is a scaled copy of the original.

12. Why did you *not* circle the other rectangles?

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# On your Own:

Now it's time to differentiate between scaled and non-scaled copies of a figure on your own.

Name: \_\_\_\_\_ Q7 U1 Lesson 1 - Independent Work

1. Select **all** the rectangles that are scaled copies of the original rectangle.

a.

b.

c.

d.

e.

2. Circle each turtle that is a scaled copy of the original turtle.

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3. Maria and Zeke tried to draw scaled copies of the original rectangle below.

Maria's rectangle had a length of 10 cm and a width of 4 cm.  
Zeke's rectangle had a length of 40 cm and a width of 16 cm.

Which statement below is true?

- Both students are incorrect.
- Maria is correct. Zeke is incorrect.
- Zeke is correct. Maria is incorrect.
- Both students are correct.

4. Daniel said that both of the figures below are scaled copies of the original figure. He said they are both L-shaped and enlarged. Do you agree or disagree? Explain.

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**Look at the original image of the hamster.**

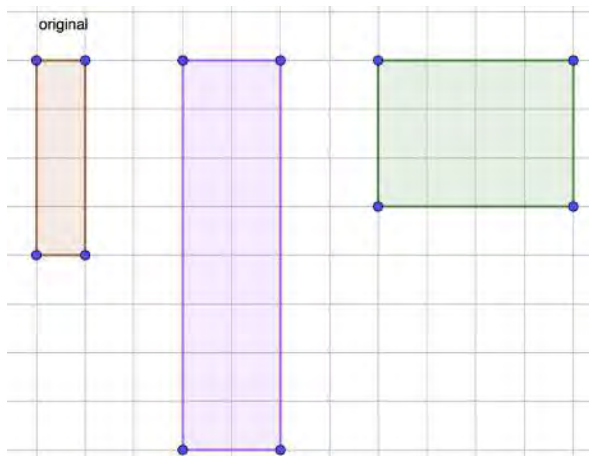
**ORIGINAL**

1. Circle the image that looks most like the original.



2. When an image gets larger or smaller, and nothing else about the image changes, we call it a \_\_\_\_\_.

**Look at the original image of the rectangle.**



3. Circle the rectangle that is a scaled copy of the original.

4. Explain how you know just by looking.

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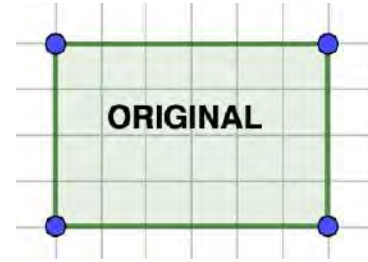
5. Label the length of each short side on the original and the scaled copy. How are they related?

6. Label the length of each long side on the original and the scaled copy. How are they related?

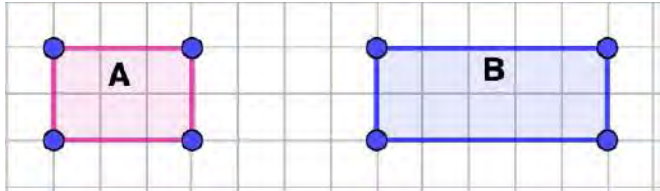


Look at the original rectangle.

7. Label the side lengths of the original rectangle.



8. Label the side lengths of Rectangle A and Rectangle B.



9. Which rectangle is a scaled copy of the original?

- a. Rectangle A
- b. Rectangle B

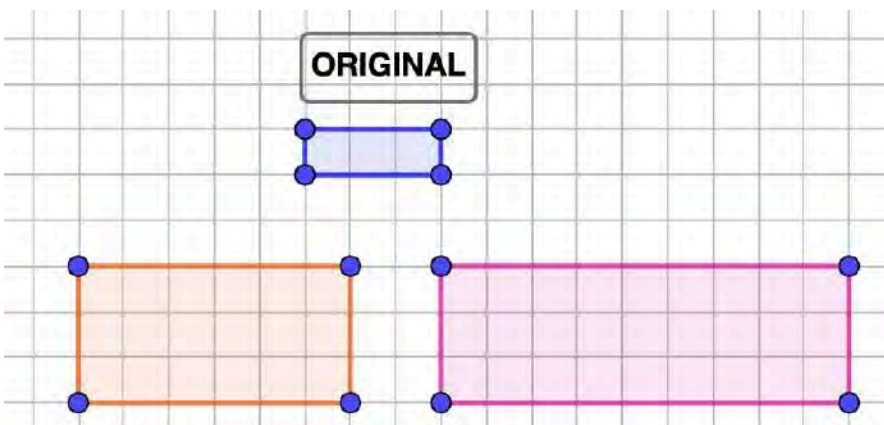
10. How do you know?

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Look at the original rectangles below. The top rectangle is the original.



11. Circle the rectangle that is a scaled copy of the original.

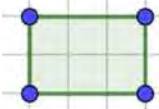
12. Why did you *not* circle the other rectangle?

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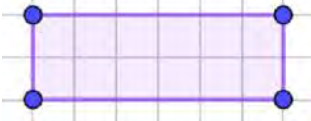
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1. Select all the rectangles that are scaled copies of the original rectangle.



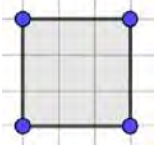
a.



b.



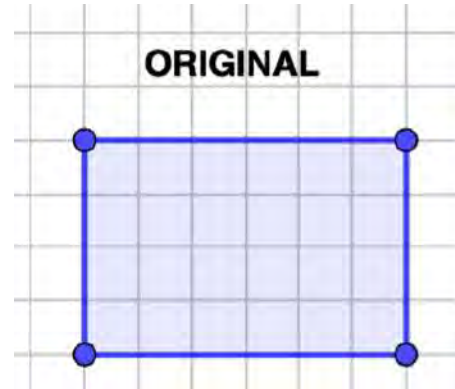
c.



d.



e.



ORIGINAL

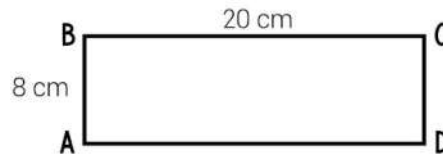
2. Circle each turtle that is a scaled copy of the original turtle.

Original





3. Maria and Zeke tried to draw scaled copies of the original rectangle below.



Maria's rectangle had a length of 10 cm and a width of 4 cm.  
Zeke's rectangle had a length of 40 cm and a width of 16 cm.

Which statement below is true?

- a. Both students are incorrect.
- b. Maria is correct. Zeke is incorrect.
- c. Zeke is correct. Maria is incorrect.
- d. Both students are correct.

4. Daniel said that both of the figures below are scaled copies of the original figure. He said they are both L-shaped and enlarged. Do you agree or disagree? Explain.

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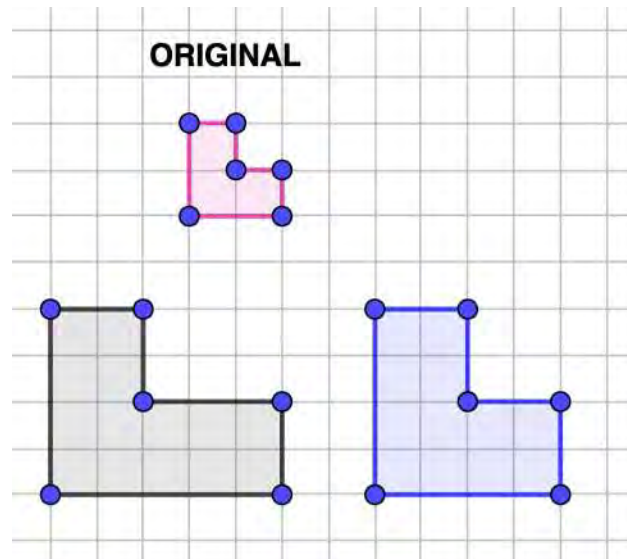
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Look at the original image of the hamster.

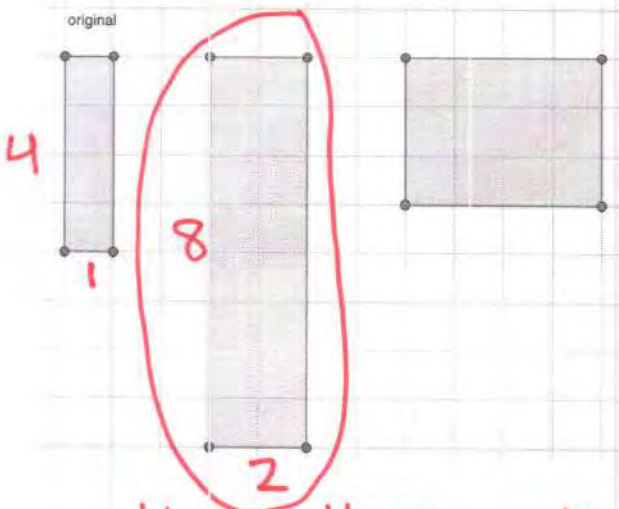
ORIGINAL

1. Circle the image that looks most like the original.



2. When an image gets larger or smaller, and nothing else about the image changes, we call it a scaled copy.

Look at the original image of the rectangle.



3. Circle the rectangle that is a scaled copy of the original.

4. Explain how you know just by looking.

It is most similar in terms of shape. It's narrow and tall, while the other option is shorter and wider.

5. Label the length of each short side on the original and the scaled copy. How are they related?

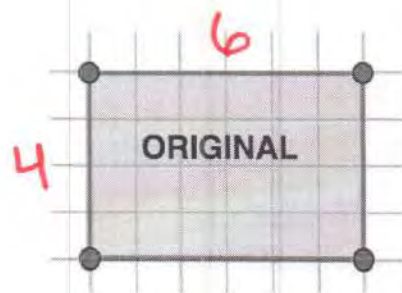
The copy is twice as long.

6. Label the length of each long side on the original and the scaled copy. How are they related?

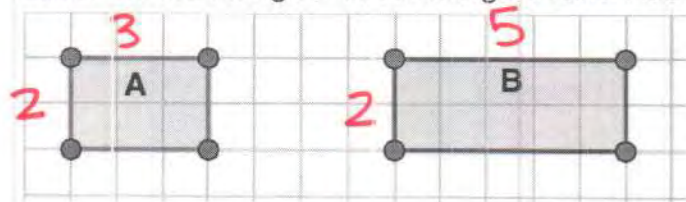
The copy is twice as long.

Look at the original rectangle.

7. Label the side lengths of the original rectangle.



8. Label the side lengths of Rectangle A and Rectangle B.



9. Which rectangle is a scaled copy of the original?

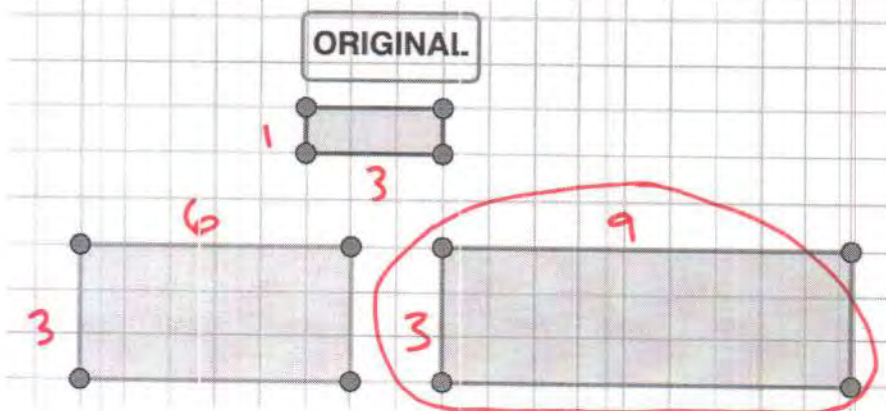
a. Rectangle A

b. Rectangle B

10. How do you know?

The sides of "A" are related in a consistent way to the original. Each side of "A" is half as long as the original side.

Look at the original rectangles below. The top rectangle is the original.



11. Circle the rectangle that is a scaled copy of the original.

12. Why did you *not* circle the other rectangle?

The width is 3 times longer while the length is 2

times longer. The relationship between sides is inconsistent.



1. Select all the rectangles that are scaled copies of the original rectangle.

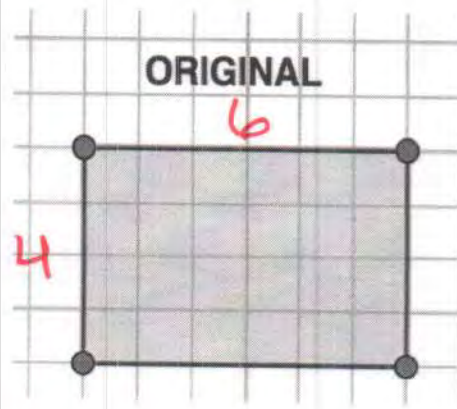
a.  $6 \times \frac{1}{2} = 3$   
 $4 \times \frac{1}{2} = 2$

~~b.~~  $6 \times 2 = 12$   
 $4 \times 2 = 8$

c.  $6 \times 2 = 12$   
 $4 \times 2 = 8$

~~d.~~  $3 \times 3 = 9$

~~e.~~  $12 \times 4 = 48$



2. Circle each turtle that is a scaled copy of the original turtle.

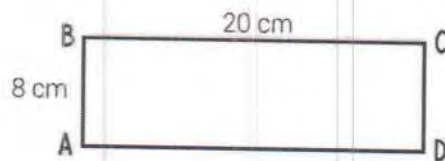
Original

same shape

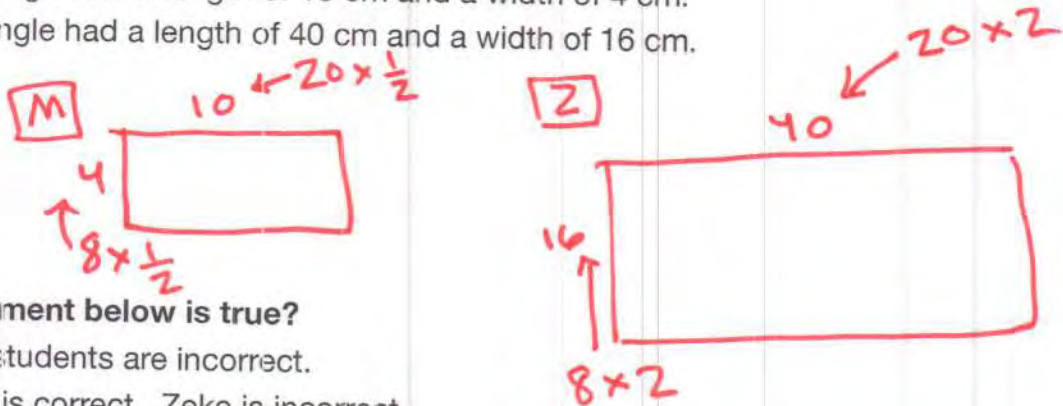
stretched out

tilted

3. Maria and Zeke tried to draw scaled copies of the original rectangle below.



Maria's rectangle had a length of 10 cm and a width of 4 cm.  
 Zeke's rectangle had a length of 40 cm and a width of 16 cm.

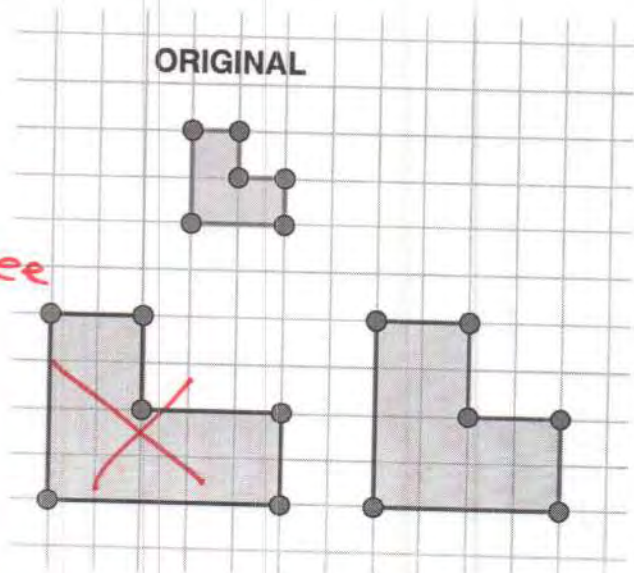


Which statement below is true?

- a. Both students are incorrect.
- b. Maria is correct. Zeke is incorrect.
- c. Zeke is correct. Maria is incorrect.
- d. Both students are correct.

4. Daniel said that both of the figures below are scaled copies of the original figure. He said they are both L-shaped and enlarged. Do you agree or disagree? Explain.

I agree that they  
are both L-shaped and  
enlarged, but I disagree  
that they are both  
scaled copies. The  
first shape is stretched  
out so the bottom part is longer than it  
should be. The shape is distorted.



# **G7 U1 Lesson 2**

Identify corresponding parts and determine the scale factor between two figures



**G7 U1 Lesson 2 - Students will identify corresponding parts and determine the scale factor between two figures**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we started to think about scaled copies. We saw images that were scaled copies of an original and some that were not. What makes an image of scaled copy of an original? What makes an image NOT a scaled copy of an original? **Possible Student Answers, Key Points:**

- An image is a scaled copy if it's the same shape, just bigger or smaller.
- An image is not a scaled copy if it is distorted in any way, like if it's widened or stretched out.

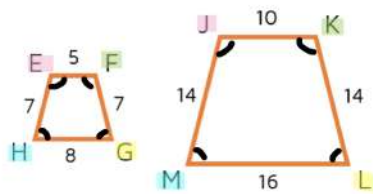
When we saw sets of images, we were able to tell if a figure was a scaled copy just by looking. In other cases, it helped to look at corresponding sides. Today, we'll continue thinking about scaled figures and how we can use the mathematical relationship between corresponding sides to help us think precisely about scaled copies.

**Let's Talk (Slide 3):** Take a look at the figures shown here. What do you notice about the figures? What do you wonder? **Possible Student Answers, Key Points:**

- I notice they are trapezoids. I notice the corresponding sides are color-coded. I notice there are different letters at each vertex. I notice the second one is smaller and rotated a little bit.
- I wonder what the side lengths are. I wonder if the second one is actually a scaled copy or not.

The second trapezoid appears to be a scaled copy of the original trapezoid, just a bit smaller and turned at an angle. It appears as though all corresponding sides are related to each other in a consistent way. It would be helpful to have numbers labeling the side lengths so we can be sure that each side of the original can be multiplied by a consistent factor that results in the corresponding side length. This consistent factor relating corresponding sides is called the scale factor. Let's look at a few problems dealing with scale factor together.

**Let's Think (Slide 4):** The two trapezoids shown here are related. Trapezoid EFGH is the original. Trapezoid JKLM is the scaled copy. The problem wants us to list corresponding sides and angles, and then determine the scale factor.



Let's start by looking at corresponding angles. I see Angle E and Angle J are corresponding, because they both are at the same position within their corresponding figure. *(highlight each angle and mark the angles with an arc se you name them)* Angle K and F are also corresponding. What other corresponding angles do you see? **(Angles H and M as well as angles G and L)** I notice that each pair of corresponding angles are the same size. For example, Angle E is an obtuse angle that appears to be a little more than 90 degrees. It's corresponding angle, Angle J, is also an obtuse angle that appears to be a little more than 90 degrees. Angles in scaled copies should have the same measurement as their corresponding angles.

EFGH	JKLM
5	10
7	14
8	16
7	14

Now, let's look at corresponding sides. *(make a t-chart with corresponding sides, highlighting the sides on the figure if that is helpful)* Which sides are corresponding, and how do you know? **Possible Student Answers, Key Points:**

- 5 and 10 are corresponding measurements, since they're both the tops of the trapezoids. 8 and 16 are corresponding, since they're both the bottoms of the figures. Each 7 corresponds with a side length of 14 in the scaled copy.

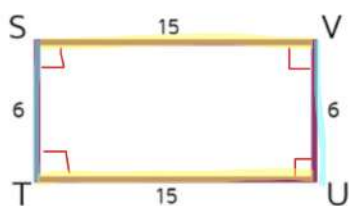
What do you notice about each pair of corresponding sides? **Possible Student Answers, Key Points:**



- Each side of JKLM is two times the length of the corresponding side in EFGH.
- I notice a pattern.  $5 \times 2 = 10$ ,  $7 \times 2 = 14$ ,  $8 \times 2 = 16$ , and  $7 \times 2 = 14$ .

(draw arrows between each corresponding side and write  $\times 2$  next to the table) We can multiply each side length on the original trapezoid to get the corresponding side length on the scaled copy. Since we can multiply each side by 2, we can say that the scale factor used to make the scaled copy is 2. The scale factor is 2. I think you're ready for another example.

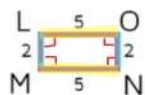
**Let's Think (Slide 5):** This problem shows two rectangles. It says the rectangle STUV is the original, which means that LMNO is a scaled copy. Let's work to determine the scale factor used to create rectangle LMNO.



Let's start by thinking about the angles for a moment. We know the corresponding angles in scaled copies should be equivalent to the angles in the original figure. Do we see that in these figures? How do you know?

**Possible Student Answers, Key Points:**

- Yes, all of the angles are 90 degrees.
- Yes, since both shapes are rectangles, I know every angle is a right angle. All the corresponding angles are equivalent.



(mark each angle with a box signifying a right angle) Since each angle in a rectangle is 90 degrees, we know all the corresponding angles are congruent.

Let's move on to thinking about corresponding sides. Which sides are corresponding? (highlight and color-code as student shares) (SV and TU correspond with LO and MN while ST and VU correspond with LM and ON)

STUV	LMNO	scale factor
15	5	$\frac{1}{3}$
15	5	$\frac{1}{3}$
6	2	$\frac{1}{3}$
6	2	$\frac{1}{3}$

I'll make a t-chart so we can think about how the corresponding sides are related. I'll add a column to the right, so we can think about the scale factor that this question asks about. (sketch a chart as shown and fill in the corresponding sides) What do you notice about the corresponding sides in our chart? **Possible Student Answers, Key Points:**

- There are two pairs of corresponding sides that measure 15 and 5, and two pairs of corresponding sides that measure 6 and 2.
- I notice the sides in LMNO are shorter than their corresponding side in STUV.
- I notice that each length in STUV is three times as long as the corresponding length in LMNO.

Each side of STUV is 3 times larger than its corresponding side in LMNO. The factor we can multiply lengths from STUV by to get their corresponding lengths in LMNO is  $\frac{1}{3}$ . (fill in  $\frac{1}{3}$  in the last column as you narrate) 15 times  $\frac{1}{3}$  is 5. 15 times  $\frac{1}{3}$  is 5. 6 times  $\frac{1}{3}$  is 2. 6 times  $\frac{1}{3}$  is 2. The scale factor used to create LMNO was  $\frac{1}{3}$ .

**Let's Try it (Slides 6 - 7):** We'll try a few more together before you practice independently. We saw today that corresponding angles in scale copies are identical or congruent. We also noticed that we can find the scale factor by identifying corresponding sides, often using a table, and thinking about how they are mathematically related. The number we can multiply a given length in an original figure to produce the



corresponding length in the scaled copy is called the scale factor. Let's work together to identify corresponding parts and determine the scale factor between two figures.

# WARM WELCOME

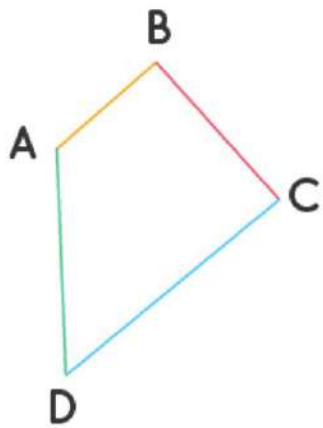


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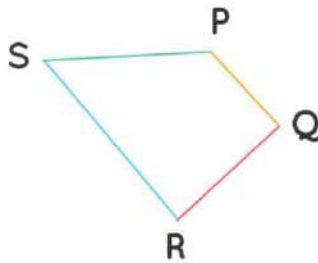
**Today we will identify corresponding parts and determine the scale factor between two figures.**

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## Let's Talk:



ORIGINAL



SCALED COPY

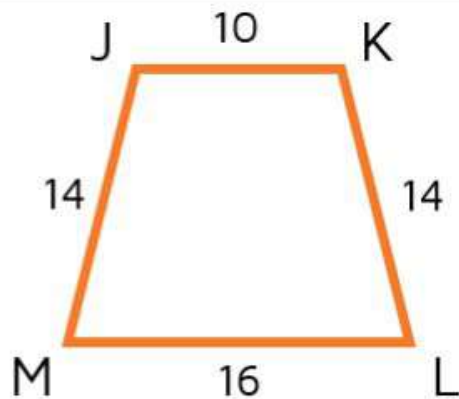
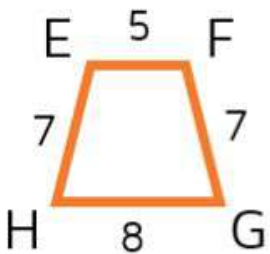
**What do you notice?**

**What do you wonder?**

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## Let's Think:

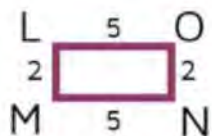
**Trapezoid EFGH is the original. List the corresponding sides and angles. What is the scale factor?**



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# Let's Think:

Rectangle STUV is the original. What is the scale factor?



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# Let's Try It:

Let's explore identifying corresponding parts and determining the scale factor between two figures together.

Name: \_\_\_\_\_ G7 U1 Lesson 2 - Let's Try It

The triangle below is the original triangle.

1. How do you know the second triangle is a scaled copy of the first triangle?

2. Complete the table to show the corresponding side lengths.

SIDE LENGTHS OF ORIGINAL	SIDE LENGTHS OF SCALED COPY
5	10

3. Complete the table to show the corresponding angles.

ANGLES IN ORIGINAL	ANGLES IN SCALED COPY
angle ABC	
angle BCA	

4. Each corresponding side of the scaled copy is \_\_\_\_\_ times as long as the corresponding side of the original figure.

5. What is the scale factor? \_\_\_\_\_

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Rectangle A is the original rectangle. Rectangle B is a scaled copy.

6. Complete the table with the corresponding side lengths of the scaled copy.

ORIGINAL RECTANGLE	SCALED COPY
2	
2	
6	
6	

7. The scale factor is...  
 a. greater than 1,  
 b. less than 1.

8. What is the scale factor? How do you know?  
 \_\_\_\_\_  
 \_\_\_\_\_

Polygon P is the original polygon. Polygon Q is the scaled copy.

9. The scale factor is...  
 a. greater than 1,  
 b. less than 1.

10. What is the scale factor? How do you know?  
 \_\_\_\_\_  
 \_\_\_\_\_

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# On your Own:

Now it's time to identify corresponding parts and determine the scale factor between two figures on your own.

Name: \_\_\_\_\_ 67 U1 LESSON 2 - Independent Work

1. Complete each table based on the original triangle and the scaled copy.

ORIGINAL TRIANGLE	SCALED COPY	ANGLES IN THE ORIGINAL TRIANGLE	CORRESPONDING ANGLES IN THE SCALED COPY
2	6	$\angle XYZ$	
3	9	$\angle YZX$	
3	9	$\angle ZXY$	

2. Complete the table with the corresponding side lengths. Then fill in the column to show the scale factor based on the corresponding side lengths.

EDGE LENGTH IN ORIGINAL POLYGON	CORRESPONDING EDGE LENGTH IN SCALED COPY	SCALE FACTOR
2	6	
3	9	
4	12	
5	15	
6	18	

3. Complete the table. What is the scale factor from the original trapezoid to the scaled copy?

EDGE LENGTH IN ORIGINAL POLYGON	CORRESPONDING EDGE LENGTH IN SCALED COPY	SCALE FACTOR
10	5	
25	12.5	

4. Jebbrel says the scale factor from the original polygon to the scaled copy is 3. Curtis says the scale factor from the original polygon to the scaled copy is  $\frac{1}{3}$ . Who is correct? Explain how you know.

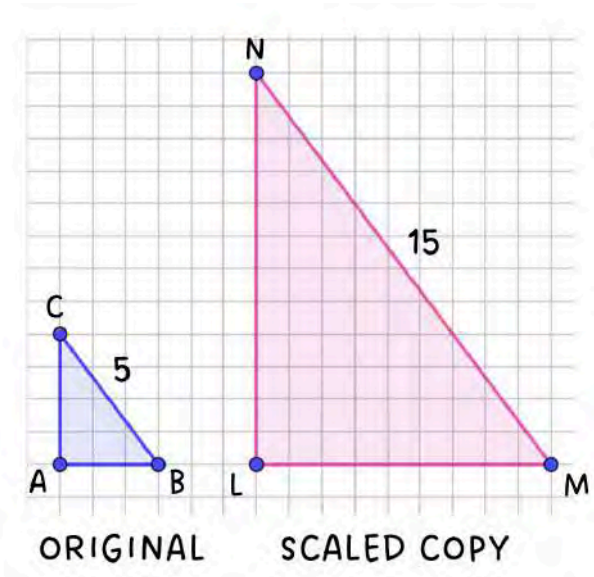
\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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The first triangle below is the original triangle.



1. How do you know the second triangle is a scaled copy of the first triangle?

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2. Complete the table to show the corresponding side lengths.

SIDE LENGTHS OF ORIGINAL	SIDE LENGTHS OF SCALED COPY
5	15

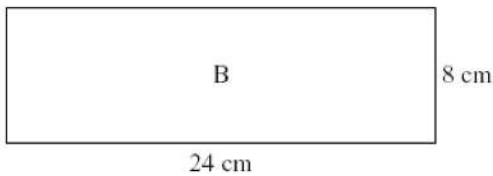
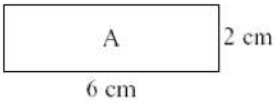
3. Complete the table to show the corresponding angles.

ANGLES IN ORIGINAL	ANGLES IN SCALED COPY
angle ABC	
angle BCA	

4. Each corresponding side of the scaled copy is \_\_\_\_\_ times as long as the corresponding side of the original figure.

5. What is the scale factor? \_\_\_\_\_

Rectangle A is the original rectangle. Rectangle B is a scaled copy.



6. Complete the table with the corresponding side lengths of the scaled copy.

ORIGINAL RECTANGLE	SCALED COPY
2	
2	
6	
6	

7. The scale factor is...
- greater than 1.
  - less than 1.

8. What is the scale factor? How do you know?

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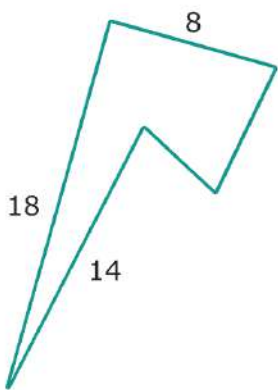


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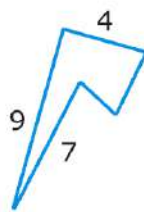


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Polygon P is the original polygon. Polygon Q is the scaled copy.



Polygon P



Polygon Q

9. The scale factor is...
- greater than 1.
  - less than 1.

10. What is the scale factor? How do you know?

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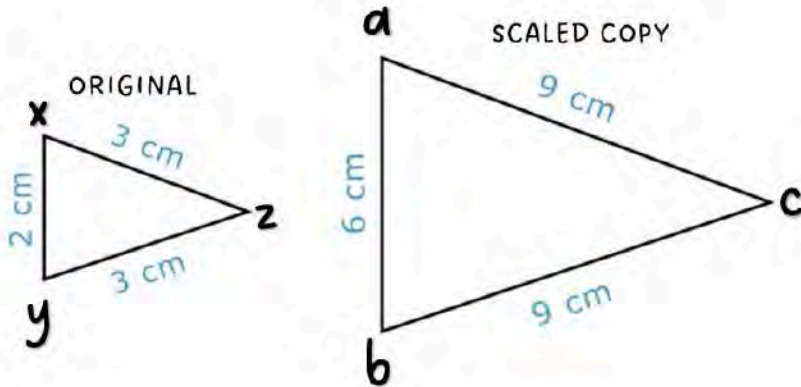


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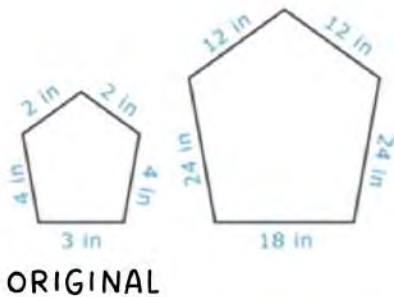
1. Complete each table based on the original triangle and the scaled copy.



ORIGINAL TRIANGLE	SCALED COPY
2	
3	
3	

ANGLES IN THE ORIGINAL TRIANGLE	CORRESPONDING ANGLES IN THE SCALED COPY
$\angle XYZ$	
$\angle YZX$	
$\angle ZXY$	

2. Complete the table with the corresponding side lengths. Then fill in the column to show the scale factor based on the corresponding side lengths.



SIDE LENGTH IN ORIGINAL POLYGON	CORRESPONDING SIDE LENGTH IN SCALED COPY	SCALE FACTOR
3		
4		
2		
2		
4		
3		



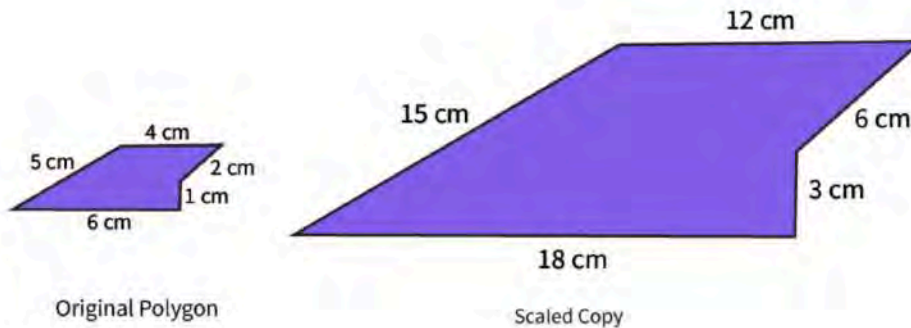
3. Complete the table. What is the scale factor from the original trapezoid to the scaled copy?



ORIGINAL

SIDE LENGTH IN ORIGINAL POLYGON	CORRESPONDING SIDE LENGTH IN SCALED COPY	SCALE FACTOR
15		
10		
25		

4. Jebbrel says the scale factor from the original polygon to the scaled copy is 3. Curtis says the scale factor from the original polygon to the scaled copy is  $\frac{1}{3}$ . Who is correct? Explain how you know.




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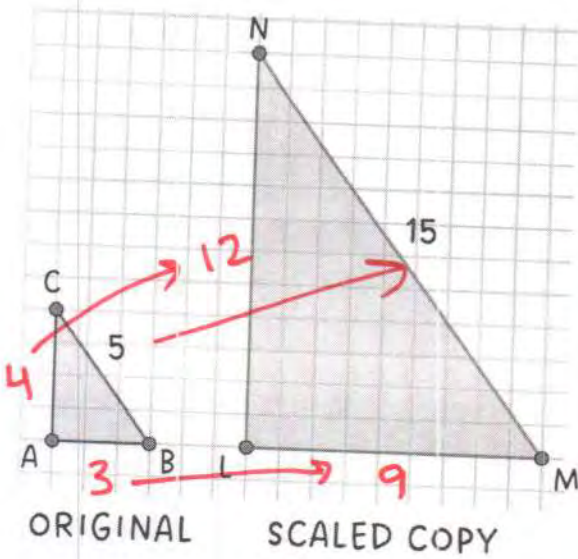


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The first triangle below is the original triangle.



1. How do you know the second triangle is a scaled copy of the first triangle?

Each corresponding side is 3 times as long as the original. It's the same shape, just a different size.

2. Complete the table to show the corresponding side lengths.

SIDE LENGTHS OF ORIGINAL	SIDE LENGTHS OF SCALED COPY
5	15
4	12
3	9

3. Complete the table to show the corresponding angles.

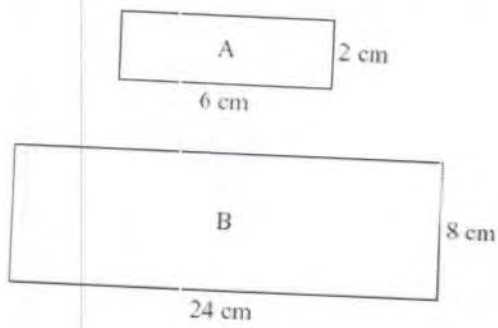
ANGLES IN ORIGINAL	ANGLES IN SCALED COPY
angle ABC	$\angle LMN$
angle BCA	$\angle MNL$
angle CAB	$\angle NLM$

4. Each corresponding side of the scaled copy is 3 times as long as the corresponding side of the original figure.

5. What is the scale factor? 3



Rectangle A is the original rectangle. Rectangle B is a scaled copy.



6. Complete the table with the corresponding side lengths of the scaled copy.

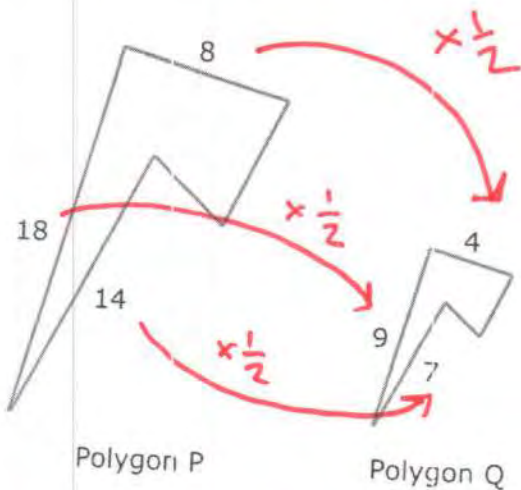
ORIGINAL RECTANGLE		SCALED COPY
2	$\xrightarrow{\times 4}$	8
2	$\longrightarrow$	8
6	$\longrightarrow$	24
6	$\longrightarrow$	24

7. The scale factor is...
- greater than 1.
  - less than 1.

8. What is the scale factor? How do you know?

The scale factor is 4. Each side of the original can be multiplied by 4 to get the corresponding side.

Polygon P is the original polygon. Polygon Q is the scaled copy.

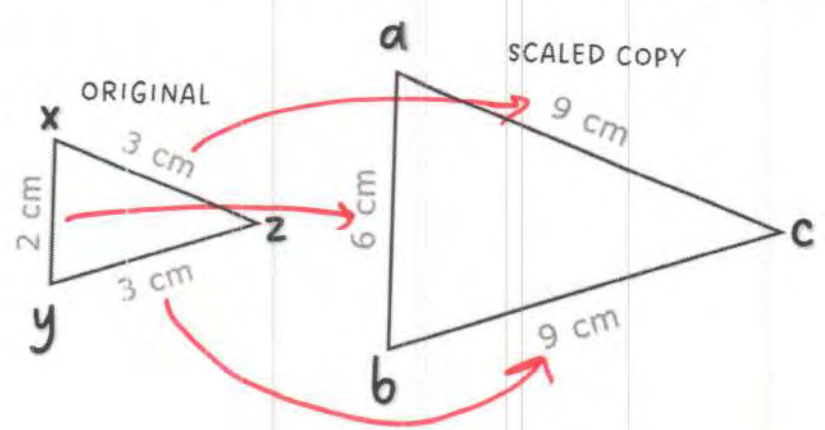


9. The scale factor is...
- greater than 1.
  - less than 1.

10. What is the scale factor? How do you know?

The scale factor is  $\frac{1}{2}$ , because each side can be multiplied by a factor of  $\frac{1}{2}$  to make the corresponding length in the scaled copy.

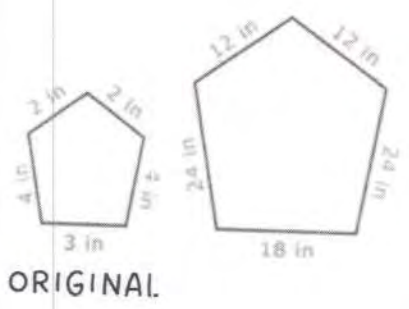
1. Complete each table based on the original triangle and the scaled copy.



ORIGINAL TRIANGLE	SCALED COPY
2	6
3	9
3	9

ANGLES IN THE ORIGINAL TRIANGLE	CORRESPONDING ANGLES IN THE SCALED COPY
$\angle XYZ$	$\angle abc$
$\angle YZX$	$\angle bca$
$\angle ZXY$	$\angle cab$

2. Complete the table with the corresponding side lengths. Then fill in the column to show the scale factor based on the corresponding side lengths.

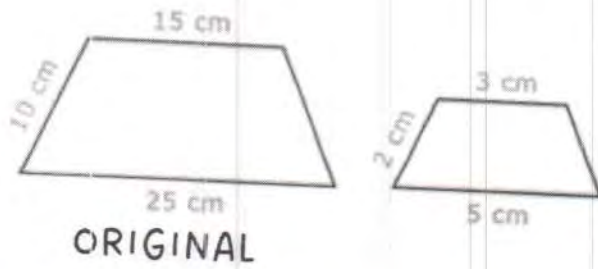


SIDE LENGTH IN ORIGINAL POLYGON	CORRESPONDING SIDE LENGTH IN SCALED COPY	SCALE FACTOR
3	18	6
4	24	6
2	12	6
2	12	6
4	24	6
3	18	6

$\times 6$



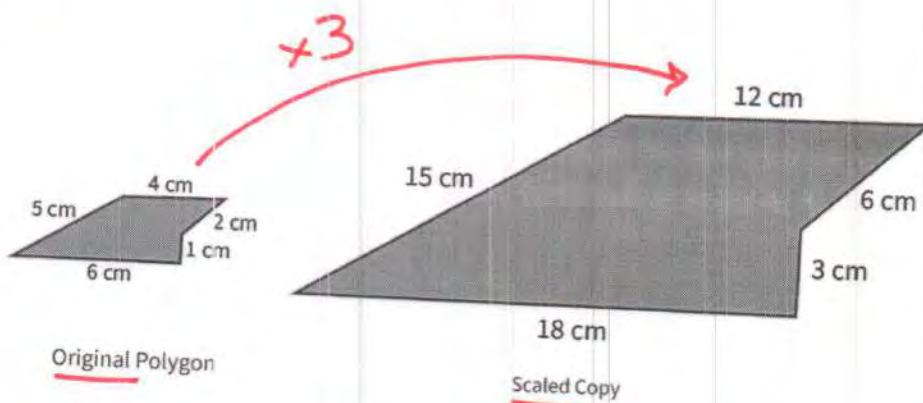
3. Complete the table. What is the scale factor from the original trapezoid to the scaled copy?



SIDE LENGTH IN ORIGINAL POLYGON	CORRESPONDING SIDE LENGTH IN SCALED COPY	SCALE FACTOR
15	→ 3	$\frac{1}{5}$
10	→ 2	$\frac{1}{5}$
25	→ 5	$\frac{1}{5}$
10	→ 2	$\frac{1}{5}$

$\times \frac{1}{5}$

4. Jebbrel says the scale factor from the original polygon to the scaled copy is 3. Curtis says the scale factor from the original polygon to the scaled copy is  $\frac{1}{3}$ . Who is correct? Explain how you know.



Jebbrel is correct. Each side of the scaled copy is 3 times the length of the corresponding side in the original.

# **G7 U1 Lesson 3**

Draw a scaled copy of a given figure using  
a given scale factor

## G7 U1 Lesson 3 - Students will draw a scaled copy of a given figure using a given scale factor

**Warm Welcome (Slide 1):** Tutor choice

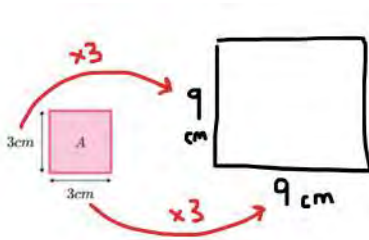
**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we'll continue working with scale drawings. Our goal today will be to identify or draw scaled copies of a figure based on a given scale factor. As we saw previously, a scale factor is a number that we can multiply the dimensions of an original figure by that results in a scaled copy of the original figure. Let's dive in!

**Let's Talk (Slide 3):** Here we have a square. The side lengths of the square are all 3 units, or 3 centimeters in this case. What do you notice or wonder about the thought bubbles? **Possible Student Answers, Key Points:**

- I notice one is thinking about a scale factor of 3 which is a whole number. I notice the other is thinking of a scale factor of  $\frac{1}{3}$  which is a fraction. I notice 3 and  $\frac{1}{3}$  are multiplicative inverses.
- I wonder what the scaled copies will look like.

What would happen if we took this pink square and made a scale copy using a scale factor of 3? **Possible Student Answers, Key Points:**

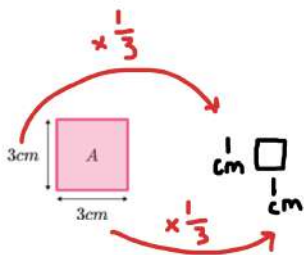
- The scale copy of the square would be bigger than the original.
- The side lengths would be 9cm.



If we used a scale factor of 3, each side length of the original would be multiplied by a factor of 3. The resulting scaled copy would be a square that measures 9 centimeters on each side. *(sketch and label square representing the scaled copy)* The length would be 9, since 3 times 3 equals 9. The width would also be 9, since 3 times 3 equals 9. *(draw arrows and "x 3" to connect each corresponding side)*

What about a scale factor of  $\frac{1}{3}$ ? What would be the same? What would be different? **Possible Student Answers, Key Points:**

- We would still end up with a scaled copy of the square, it would just be smaller.
- The side lengths would be 1 centimeter.

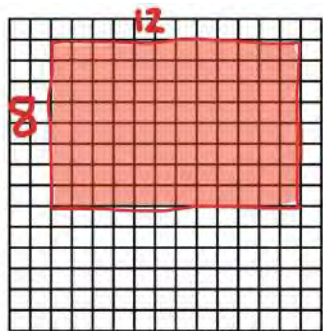


If we used a scale factor of  $\frac{1}{3}$  to make a scale copy, we would need to multiply each side by  $\frac{1}{3}$ . I know 3 times  $\frac{1}{3}$  is equal to 1, so each side length of the new square would be 1 centimeter long. *(sketch new square, including arrows showing the scale factor)*

The size of the resulting scaled copy depends on the scale factor being used. We can see here that multiplying the dimensions of the square by a factor of 3 resulted in a larger scaled copy. Multiplying the dimensions by  $\frac{1}{3}$  resulted in a smaller scaled copy. We'll explore this more in future lessons, but keep it in the back of your mind as we work through today's examples.

**Let's Think (Slide 4):** This problem wants us to create two different scaled drawings of the same original rectangle. What are the two scale factors the problem wants us to consider? How do you think each scale factor will impact the scaled copies we draw? **Possible Student Answers, Key Points:**

- They want us to think about a scale factor of 2 and  $\frac{1}{2}$ .
- The scale factor of 2 will require us to make a bigger rectangle, multiplying each side of the original by 2. I think the scale of  $\frac{1}{2}$  will result in a smaller scaled copy, since we'll be multiplying by a fraction.



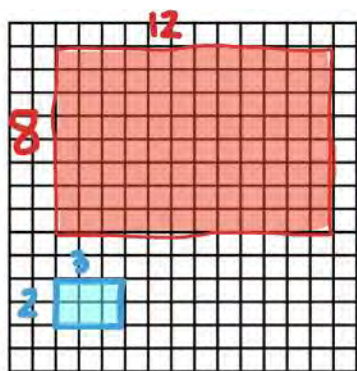
$$4 \times 2 = 8$$

$$6 \times 2 = 12$$

Let's see if what you described is true. Let's begin by thinking about a scale factor of 2. A scale factor of 2 means we must multiply each dimension of the original rectangle by 2. What will the dimensions of the scaled copy be? (8 units and 12 units) (write  $4 \times 2 = 8$  and  $6 \times 2 = 12$  as student shares)

When we multiply the length and width each by 2, we end up with a scaled copy with a length of 12 and a width of 8. Let's sketch that on the grid here. (sketch and label a rectangle measuring 12 units by 8 units leaving enough room along the bottom of the grid to draw the next scaled copy)

We just drew a scaled copy of the original rectangle by multiplying each dimension by the scale factor, then using the grid to draw the new figure. Let's do the same work with the second scale factor of  $\frac{1}{2}$ .



$$4 \times \frac{1}{2} = 2$$

$$6 \times \frac{1}{2} = 3$$

How can I find the dimensions of a scaled copy using a scale factor of  $\frac{1}{2}$ ? Possible Student Answers, Key Points:

- You can multiply each dimension by the scale factor of  $\frac{1}{2}$ .
- I know 4 times  $\frac{1}{2}$  is  $\frac{4}{2}$  or 2. I know 6 times  $\frac{1}{2}$  is  $\frac{6}{2}$  or 3.

(write  $4 \times \frac{1}{2} = 2$  and  $6 \times \frac{1}{2} = 3$  as student shares) I notice these dimensions are smaller than the original rectangle's dimensions. That makes sense, since we multiplied each dimension by  $\frac{1}{2}$ . This scaled copy will have a length of 3 and a width of 2. Let's draw that on the grid. (sketch a 2 x 3 rectangle underneath

the previous scaled copy) We have two different scaled copies of the same rectangle. As we can see, the scale factor being used impacts the size of the scaled copy. The shape, of course, stays the same.

Let's consider one more example that asks us to think about scale factor in a slightly different way.

**Let's Think (Slide 5):** Instead of having us draw a new scaled copy, this problem wants us to determine which of the three triangles shown here could be scaled copies of the original triangle at the top. We know a scaled copy is a figure whose corresponding sides have a consistent relationship with the original figure. Let's see if we can find a consistent scale factor that could result in each scaled copy. If we can find a consistent scale factor, the figure is a scaled copy. If we can't find a consistent scale factor, then the figure is not a scaled copy.

Look at the first potential scaled copy, then look at the original triangle. Which sides are corresponding sides? (12 and 6, 5 and  $2\frac{1}{2}$ , and 13 and  $6\frac{1}{2}$ ) I want to think about what factor each original side can be multiplied by to result in the dimensions of the scaled copy.

$$12 \times \frac{1}{2} = 6 \checkmark$$

$$5 \times \frac{1}{2} = 2\frac{1}{2} \checkmark$$

$$13 \times \frac{1}{2} = 6\frac{1}{2} \checkmark$$

(write  $12 \times \underline{\quad} = 6$ ,  $5 \times \underline{\quad} = 2\frac{1}{2}$ , and  $13 \times \underline{\quad} = 6\frac{1}{2}$ ) I can think about what number I multiply 12 by to end up with 6. I know  $12 \times \frac{1}{2} = 6$ , because half of 12 is 6. (write  $\frac{1}{2}$  in blank) 12 times a scale factor of  $\frac{1}{2}$  results in a scaled side equal to 6 units.

I know 5 times  $\frac{1}{2}$  results in  $2\frac{1}{2}$ . 5 times  $\frac{1}{2}$  is  $\frac{5}{2}$ , which is equal to  $2\frac{1}{2}$ . (fill in  $\frac{1}{2}$ ) What about 13? 13 times what results in  $6\frac{1}{2}$ ? ( $\frac{1}{2}$ ) (fill in  $\frac{1}{2}$ ) Since each side length can be multiplied by a scale factor of  $\frac{1}{2}$  and result in the side length of the scaled copy, I know the first triangle IS a scaled copy of the original.



$$12 \times 2 = 24 \quad \checkmark$$

$$5 \times 2 = 10 \quad \checkmark$$

$$13 \times 2 = 26 \quad \checkmark$$

Let's use similar thinking to consider the second triangle. (*point to sides as you name them*) I know 12 corresponds with 24. I know 5 corresponds with 10. I know 13 corresponds with 26. Is this a scaled copy of the original? How do you know? **Possible Student Answers, Key Points:**

- It is a scaled copy of the original. The scale factor is 2.
- $12 \times 2 = 24$ ,  $5 \times 2 = 10$ , and  $13 \times 2 = 26$ . I can multiply each side of the original figure by 2.

(*write equations as shown*) We can multiply each side of the original triangle by 2 and we will end up with the side lengths of this second triangle. This second triangle must be a scaled copy. The scale factor used to make it is 2.

$$12 \times 2 = 24 \quad \times$$

$$5 \times 3 = 15 \quad \checkmark$$

$$13 \times 3 = 39 \quad \checkmark$$

Let's use the same thinking to see if this last triangle is a scaled copy. (*write each equation and point to the named sides as you narrate*)

I know  $12 \times 2 = 24$ . I know  $5 \times 3 = 15$ . I know  $13 \times 3 = 39$ . Is this triangle a scaled copy of the original? How do you know? **Possible Student Answers, Key Points:**

- It is not a scaled copy.
- We can use multiplication to find the corresponding side lengths, but one side needs to be multiplied by 2 and the other sides need to be multiplied by 3. It's not a consistent relationship.

The relationship between the sides of the original triangle and the sides of the copy are not consistent. There is not one consistent scale factor we can use to find the resulting side lengths. Therefore, this third triangle cannot be a scaled copy of the original. The scale factor being used must be the same throughout the entire figure.

**Let's Try it (Slides 6 - 7):** Let's try out a few more problems before you show what you know independently. Today we'll use a scale factor to multiply dimensions of an original figure and consider how to draw or identify the resulting scaled copy. Depending on the number we use as the scale factor, we can end up with different scaled copies. We also saw today that is important to multiply each dimension or side by the same scale factor, otherwise the resulting figure cannot be considered a true scaled copy. We'll keep all this in mind as we try the next set of problems.

# WARM WELCOME

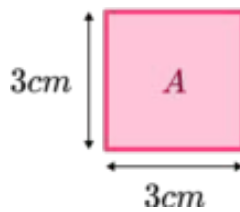


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**Today we will draw a scaled copy of a given figure using a given scale factor.**

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## Let's Talk:



Scale factor  
of 3...?

Scale factor  
of  $\frac{1}{3}$ ...?

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## Let's Think:

**A rectangle is shown below.**

Sketch a scaled copy using a scale factor of 2.

Sketch another scaled copy using a scale factor of  $\frac{1}{2}$ .

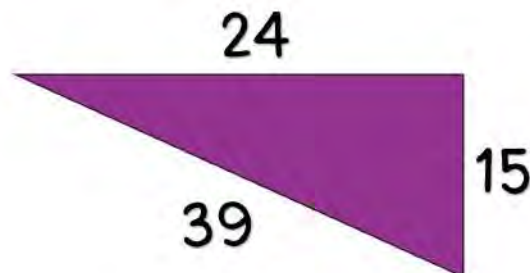
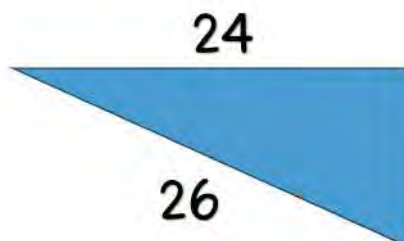
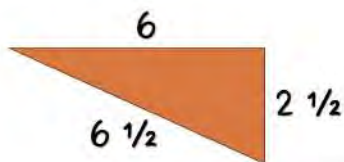
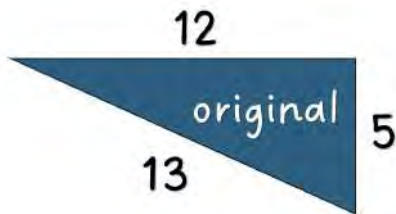


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# Let's Think:

Which of the triangles below could be scaled copies of the original triangle?



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# Let's Try It:

Let's explore drawing a scaled copy of a given figure using a given scale factor together.

Name: \_\_\_\_\_ G7 U1 Lesson 3 - Let's Try It

Fill in the blank.

1. A figure that has been enlarged or reduced from the original but nothing else changes is called a \_\_\_\_\_.

An original rectangle and a scaled copy are shown here.

2. What is the scale factor?

3. Use the scale factor to find the missing length on the scaled copy.

Use the rectangles shown here to answer the questions that follow.

4. What is the scale factor?

5. Use the scale factor to find the missing length on the scaled copy.

6. What would be the dimensions of a scaled copy if the scale factor was 3?

Consider drawing a scaled copy of the rectangle shown here using a scale factor of  $\frac{1}{3}$ .

7. Multiply 12 by the scale factor.

8. Multiply 6 by the scale factor.

9. Use the grid to draw the scaled copy.

10. What would be different if the scale factor was 3 instead of  $\frac{1}{3}$ ?

Consider triangle ABC.

11. Use corresponding sides to determine whether each figure could be a scaled copy of triangle ABC.

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# On your Own:

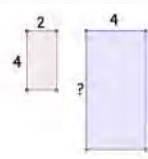
Now it's time to draw a scaled copy of a given figure using a given scale factor on your own.

Name: \_\_\_\_\_ G7 U1 Lesson 3 - Independent Work

1. The smaller rectangle is the original.

a. What is the scale factor?

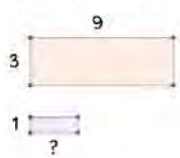
b. Use the scale factor to find the missing side.



2. The larger rectangle is the original.

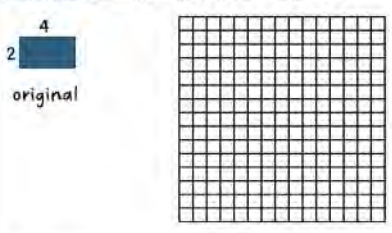
a. What is the scale factor?

b. Use the scale factor to find the missing side.



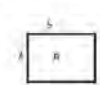
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3. Use the grid to create a scaled copy of the original figure with a scale factor of 3. On the same grid, create a scaled copy of the original figure with a scale factor of  $\frac{1}{2}$ .



4. Which dimensions could represent a rectangle that is a scaled copy of Rectangle A? Select all that apply.

a. LENGTH: 10, WIDTH: 4  
 b. LENGTH: 20, WIDTH: 8  
 c. LENGTH: 2.5, WIDTH: 1  
 d. LENGTH: 6, WIDTH: 3  
 e. LENGTH: 10, WIDTH: 2



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**Fill in the blank.**

1. A figure that has been enlarged or reduced from the original but nothing else changes is called a \_\_\_\_\_.

**An original rectangle and a scaled copy are shown here.**

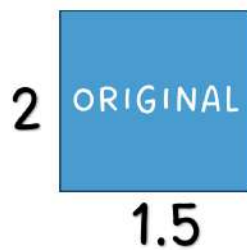
2. What is the scale factor?



3. Use the scale factor to find the missing length on the scaled copy.

**Use the rectangles shown here to answer the questions that follow.**

4. What is the scale factor?

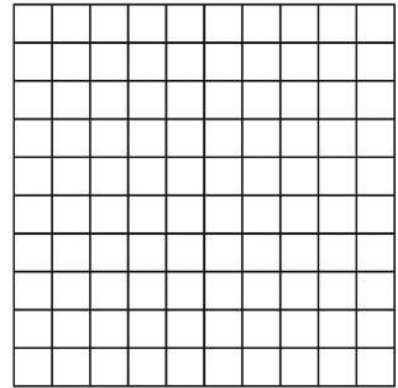


5. Use the scale factor to find the missing length on the scaled copy.

6. What would be the dimensions of a scaled copy if the scale factor was 3?

Consider drawing a scaled copy of the rectangle shown here using a scale factor of  $\frac{1}{3}$ .

7. Multiply 12 by the scale factor.



8. Multiply 6 by the scale factor.

9. Use the grid to draw the scaled copy.

10. What would be different if the scale factor was 3 instead of  $\frac{1}{3}$ ?

---



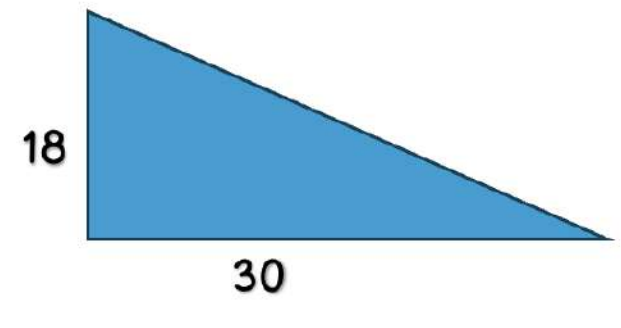
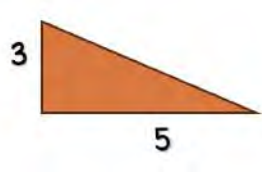
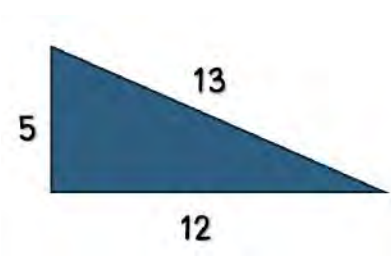
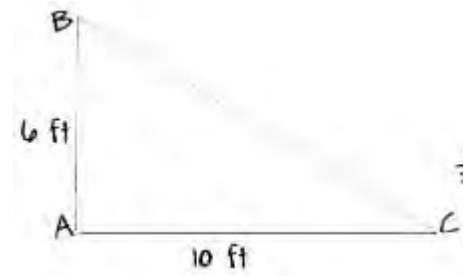
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Consider triangle ABC.

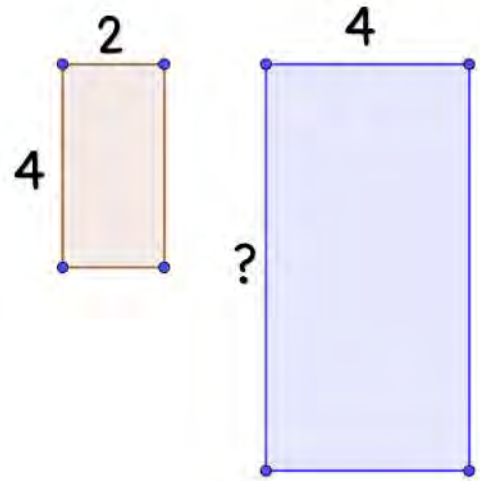
11. Use corresponding sides to determine whether each figure could be a scaled copy of triangle ABC.



**1. The smaller rectangle is the original.**

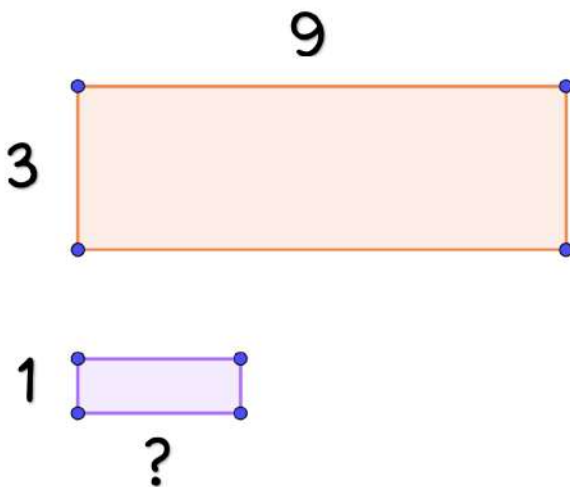
a. What is the scale factor?

b. Use the scale factor to find the missing side.

**2. The larger rectangle is the original.**

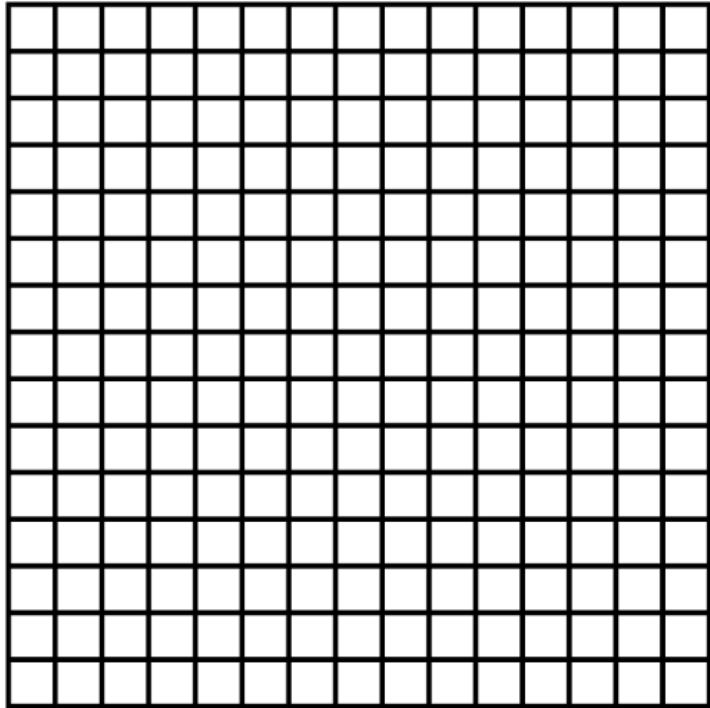
a. What is the scale factor?

b. Use the scale factor to find the missing side.



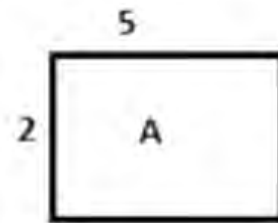


3. Use the grid to create a scaled copy of the original figure with a scale factor of 3. On the same grid, create a scaled copy of the original figure with a scale factor of  $\frac{1}{2}$ .



4. Which dimensions could represent a rectangle that is a scaled copy of Rectangle A?  
Select all that apply.

- a. LENGTH: 10, WIDTH: 4
- b. LENGTH: 20, WIDTH: 8
- c. LENGTH: 2.5, WIDTH: 1
- d. LENGTH: 6, WIDTH: 3
- e. LENGTH: 10, WIDTH: 2



Name: KEY

Fill in the blank.

1. A figure that has been enlarged or reduced from the original but nothing else changes is called a scaled copy.

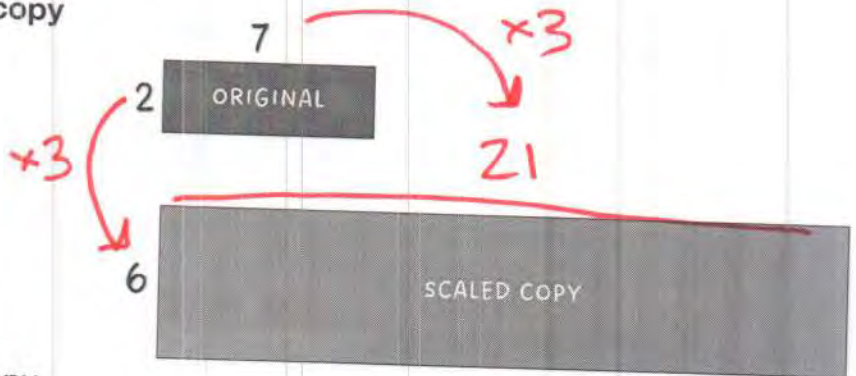
An original rectangle and a scaled copy are shown here.

2. What is the scale factor?

3

3. Use the scale factor to find the missing length on the scaled copy.

$$7 \times 3 = \underline{21}$$



Use the rectangles shown here to answer the questions that follow.

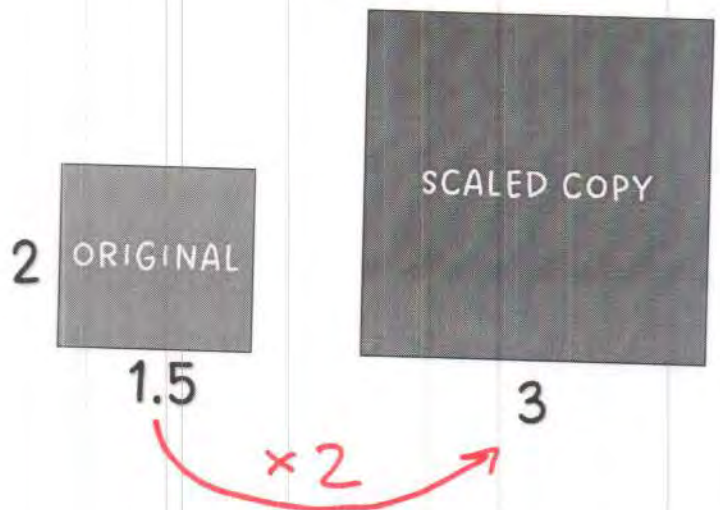
4. What is the scale factor?

$$\underline{1.5} \times 2 = 3 \quad \underline{2}$$

5. Use the scale factor to find the missing length on the scaled copy.

$$2 \times 2 = 4$$

4



6. What would be the dimensions of a scaled copy if the scale factor was 3?

$$2 \times 3 = 6$$
$$1.5 \times 3 = 4.5$$

6 and 4.5

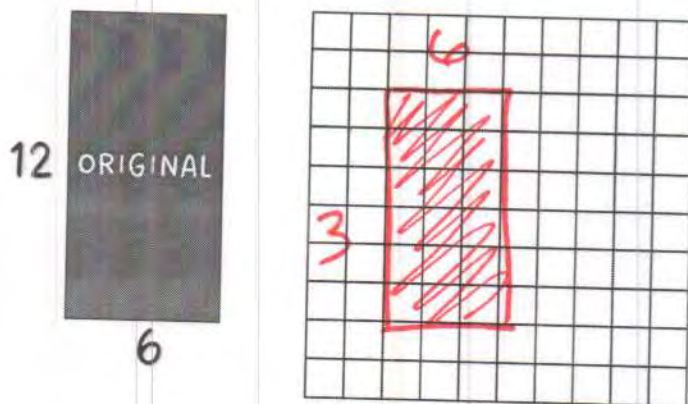
Consider drawing a scaled copy of the rectangle shown here using a scale factor of  $\frac{1}{3}$ .

7. Multiply 12 by the scale factor.

$$12 \times \frac{1}{3} = 4$$

8. Multiply 6 by the scale factor.

$$6 \times \frac{1}{3} = 2$$



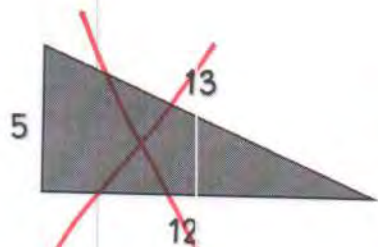
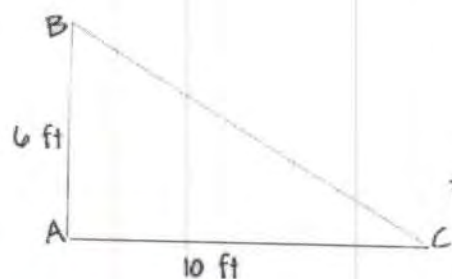
9. Use the grid to draw the scaled copy.

10. What would be different if the scale factor was 3 instead of  $\frac{1}{3}$ ?

The rectangle would be 3 times bigger than the original. I would multiply each side by 3.

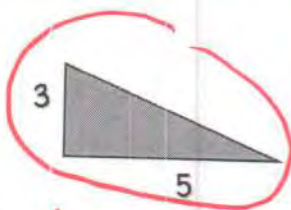
Consider triangle ABC.

11. Use corresponding sides to determine whether each figure could be a scaled copy of triangle ABC.



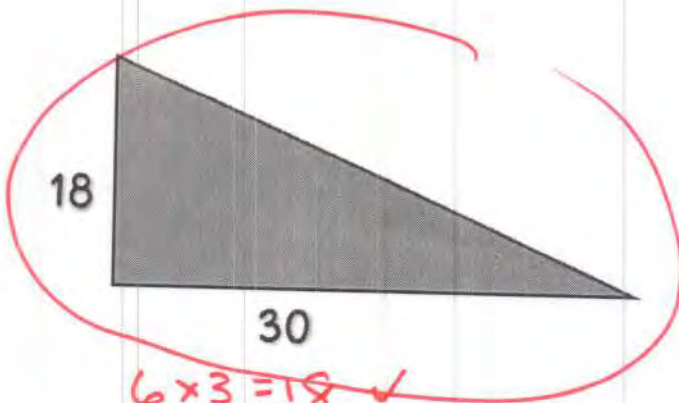
$$6 \times ? = 5$$

$$10 \times ? = 12$$



$$6 \times \frac{1}{2} = 3 \checkmark$$

$$10 \times \frac{1}{2} = 5 \checkmark$$



$$6 \times 3 = 18 \checkmark$$

$$10 \times 3 = 30 \checkmark$$



1. The smaller rectangle is the original.

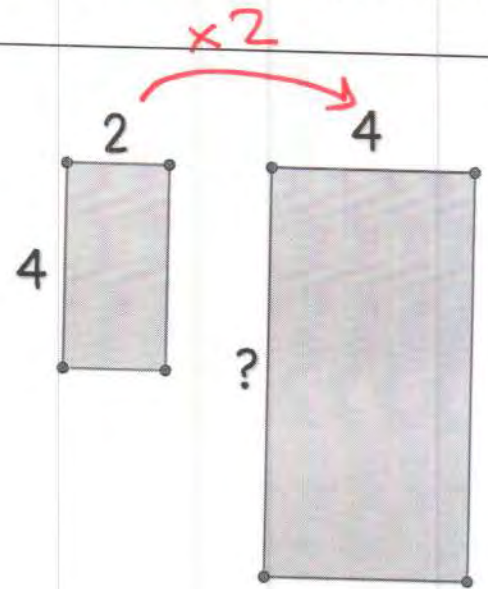
a. What is the scale factor?

(2)

b. Use the scale factor to find the missing side.

$$4 \times 2 = ?$$

(8)



2. The larger rectangle is the original.

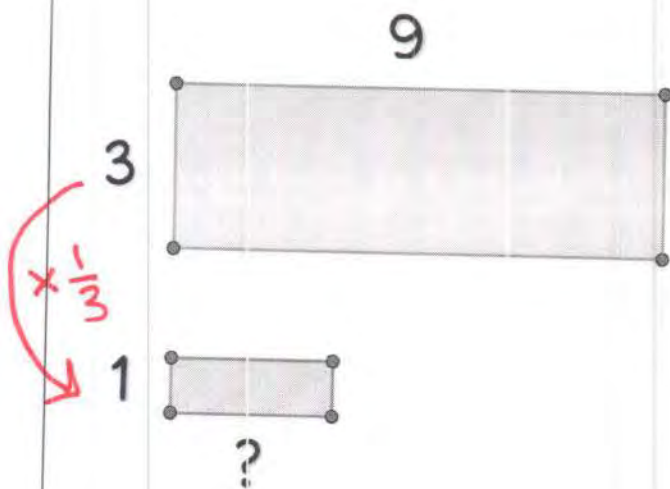
a. What is the scale factor?

( $\frac{1}{3}$ )

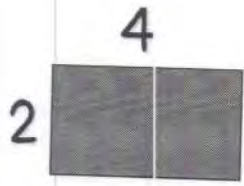
b. Use the scale factor to find the missing side.

$$9 \times \frac{1}{3} = \frac{9}{3} = 3$$

(3)



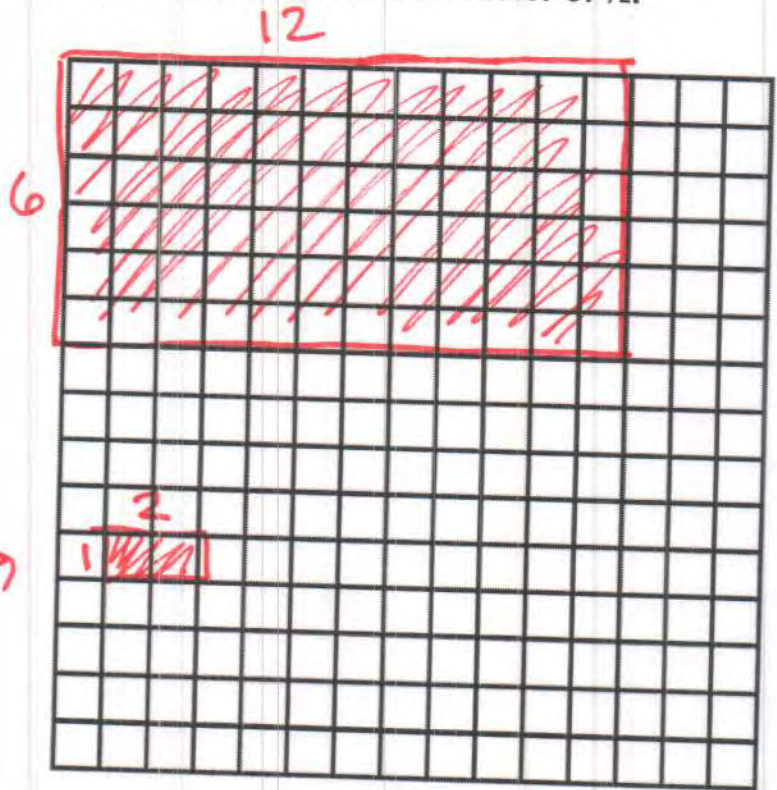
3. Use the grid to create a scaled copy of the original figure with a scale factor of 3. On the same grid, create a scaled copy of the original figure with a scale factor of  $\frac{1}{2}$ .



original

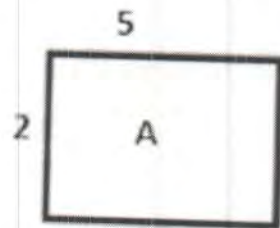
$$\left. \begin{array}{l} 4 \times 3 = 12 \\ 2 \times 3 = 6 \end{array} \right\}$$

$$\left. \begin{array}{l} 4 \times \frac{1}{2} = 2 \\ 2 \times \frac{1}{2} = 1 \end{array} \right\}$$



4. Which dimensions could represent a rectangle that is a scaled copy of Rectangle A? Select all that apply.

- a. LENGTH: 10, WIDTH: 4
  - b. LENGTH: 20, WIDTH: 8
  - c. LENGTH: 2.5, WIDTH: 1
  - d. ~~LENGTH: 6, WIDTH: 3~~
  - e. ~~LENGTH: 10, WIDTH: 2~~
- Handwritten calculations and arrows:
- For (a):  $5 \times 2 = 10$ ,  $2 \times 2 = 4$
  - For (b):  $5 \times 4 = 20$ ,  $2 \times 4 = 8$
  - For (c):  $5 \times \frac{1}{2} = 2.5$ ,  $2 \times \frac{1}{2} = 1$



## **G7 U1 Lesson 4**

Use corresponding sides, corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another

## G7 U1 Lesson 4 - Students will use corresponding sides, corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** For the past several lessons, we've been thinking about scaled copies. Today, we're going to act like detectives. We're going to look closely at corresponding sides and corresponding angles to determine whether one figure is a scaled copy of another or not. Let's jump in!

**Let's Talk (Slide 3):** Look at the two rectangles shown here, Figure A and Figure B. What do you notice about the figures? What do you wonder? **Possible Student Answers, Key Points:**

- They are both rectangles. Each figure has four 90-degree angles. Figure A is bigger than Figure B.
- I notice Figure A's width of 9 is multiplied by  $\frac{1}{3}$  to make Figure B's width of 3. I notice Figure A's length of 16 is multiplied by  $\frac{1}{2}$  to make Figure B's length of 8. They can't be scaled copies, since the relationship is not consistent.
- I wonder if whoever did this was trying to make a scaled copy.

These figures might look like scaled copies at first glance, but you'll notice they are not actually scaled copies. In order to be a scaled copy, the figure's corresponding angles must be the same measure and the corresponding sides must be related in a consistent way. In this example, the corresponding angles are the same size, but we can see that the scale factor used to create Figure B are inconsistent. Figure B is not a scaled copy of Figure A.

**Let's Think (Slide 4):** Let's look at our first problem. Here we have two quadrilaterals. The question wants us to figure out whether Figure Y is a scaled copy of Figure X? Before we look closer, does Figure Y visually look like it could be a scaled copy? **Possible Student Answers, Key Points:**

- Visually it looks like it could be a scaled copy. It looks a little smaller, but the shape is similar, and it doesn't look stretched or compressed.

They definitely look similar, so there's a possibility Figure Y is a scaled copy of Figure X. Let's make sure it's a scaled copy by checking for two things. First, we need to make sure corresponding angles are the same. Second, we need to make sure that corresponding sides are related using a consistent scale factor. If both those components are true, then we can be certain that Figure Y is a scaled copy of Figure X.



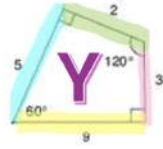
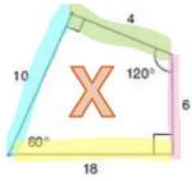
Look at the angles. Let's highlight angles that are corresponding. I know this top left angle on Figure X corresponds with the top left angle on Figure Y. *(have student point to the remaining corresponding sides and highlight each set in a different color if possible)* Corresponding angles must be congruent or the same in a scaled copy.

By marking the corresponding angles, I can see that each pair has the same measure. *(point as you narrate)* We have a pair of corresponding angles that measure 90 degrees, a pair that measure 120 degrees, another pair that measure 90 degrees, and a pair that measure 60 degrees. The first component of a scaled copy, identical corresponding angles, checks out.

Let's look at the sides now. What do you notice about the sides of Figure X and Figure Y? **Possible Student Answers, Key Points:**

- Figure X has longer sides than Figure Y.
- Figure X has sides that are two times longer than the corresponding side of Figure Y.
- I think the scale factor is  $\frac{1}{2}$ .





(highlight corresponding sides, color-coding if possible) I'm going to mark my corresponding sides to help me keep track of my thinking. I see 4 corresponds with 2, 6 corresponds with 3, 18 corresponds with 9, and 10 corresponds with 5. Is there a consistent scale factor that can be used to get from Figure X to Figure Y? **Possible Student Answers, Key Points:**

- The scale factor is  $\frac{1}{2}$ .
- It is consistent, because I can multiply each side of Figure X by the same factor and end up with the corresponding side length in Figure Y.

$$4 \times \frac{1}{2} = 2$$

$$6 \times \frac{1}{2} = 3$$

$$18 \times \frac{1}{2} = 9$$

$$10 \times \frac{1}{2} = 5$$

(write equations as you narrate) A scale factor of  $\frac{1}{2}$  can take us from Figure X to Figure Y. 4 times a scale factor of  $\frac{1}{2}$  gets us to 2. 6 times a scale factor of  $\frac{1}{2}$  gets us to 3. 18 times a scale factor of  $\frac{1}{2}$  gets us to 9. 10 times a scale factor of  $\frac{1}{2}$  gets us to 5.

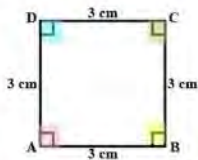
Since the angles in Figure Y were identical to the corresponding angles in Figure X, and since we were able to use a consistent scale factor for each pair of corresponding sides, we can say that Figure Y is a scale copy of Figure X.

We made a prediction that Figure Y looked like it could be a scale copy of Figure X, and now we were able to prove it mathematically by looking at the corresponding sides and angles. Let's look at one more using the same thought process.

**Let's Think (Slide 5):** Take a look at Quadrilateral PQRS and Quadrilateral ABCD. The question wants us to determine if ABCD is a scaled copy of PQRS. Let's make a prediction just based on how the figure look. Do you think ABCD will be a scaled copy of PQRS? Why or why not? **Possible Student Answers, Key Points:**

- I don't think it will be a scaled copy, because PQRS is a rhombus and ABCD is a square.
- I don't think it will be a scaled copy, because PQRS looks slanted compared to ABCD.
- It might be a scaled copy, because it looks like the person used a scale of 2. I'm not sure.

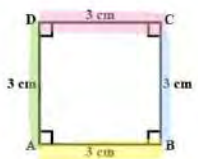
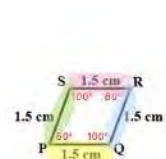
Let's look closely at the corresponding angles and side lengths to officially determine whether ABCD can be considered a scaled copy of figure PQRS. Remember, the corresponding angles have to be the same size and corresponding sides must be related in a consistent way using a single scale factor.



(highlight or point to corresponding angles) Are the corresponding angles in these figures identical? **Possible Student Answers, Key Points:**

- They are not the same.
- Angle S is 100 degrees and Angle D is 90 degrees. Angle R is 80 degrees and Angle C is 90 degrees. None of the other corresponding angle pairs match.

All corresponding angles must be the same in a figure for it to be a scaled copy of another figure. In this case, none of the corresponding angles are the same. Based on this, we already know quadrilateral ABCD cannot be a scaled copy of quadrilateral PQRS.



Even though we know ABCD is not a scaled copy, let's take a minute to still look at the corresponding side lengths. (highlight corresponding sides using different colors if possible) All the side lengths in the original figure are 1.5 centimeters. All the side lengths in figure ABCD measure 3 centimeters. Is there a scale factor that could be used to create ABCD from PQRS?

**Possible Student Answers, Key Points:**



- The scale factor is 2.
- I know  $1.5 \times 2$  is equal to 3. You can multiply each side by a scale factor of 2 to end up with a new side length of 3 centimeters.

$$1.5 \times 2 = 3$$

$$1.5 \times 2 = 3$$

$$1.5 \times 2 = 3$$

$$1.5 \times 2 = 3$$

(write  $1.5 \times 2 = 3$  for each corresponding side pair as shown) If you multiply each side of the original figure by 2, we can get the side lengths in the second figure. Even though it seems like corresponding sides are related in a consistent way, we still can't call figure ABCD a scaled copy of figure PQRS because the corresponding angles are not identical.

Both things must be true in order for a figure to be a scaled copy. One, the corresponding angles must be equivalent. Two, the corresponding side lengths must be related in a consistent way using a consistent scale factor. If only one thing is true, or neither are true, then the figure cannot be considered a scaled copy.

**Let's Try it (Slides 6 - 7):** Now it's time to practice a few more together. Once we finish these next few, as usual, you'll get a chance to show what you know on your own. As you work through these next problems with me, what two components will we look for to ensure a given figure is a scaled copy of an original?

Possible Student Answers, Key Points:

- The corresponding angles must be identical.
- The corresponding sides must be related by a common scale factor.

We'll look closely at corresponding angles and corresponding sides to determine whether a figure is a scaled copy of another figure. We can always make a visual prediction, but we want to carefully use angles and sides to make sure our original prediction is correct.

# WARM WELCOME



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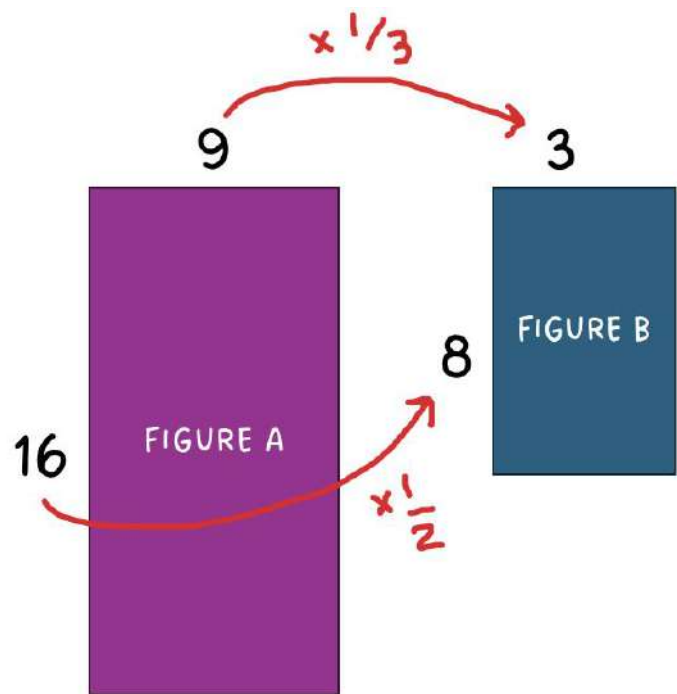
**Today we will use corresponding sides,  
corresponding distances and  
corresponding angles to tell whether  
one figure is a scaled copy of another.**

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Let's Talk:

What do you notice?

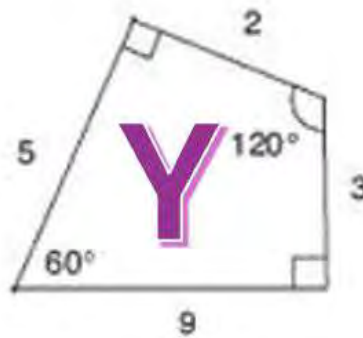
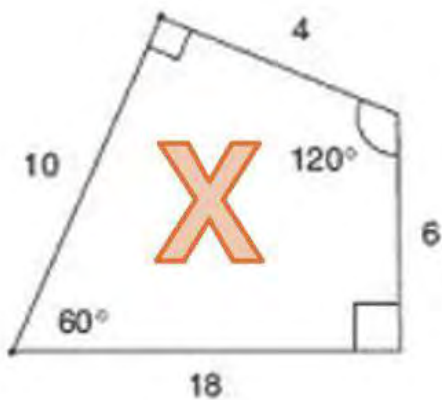
What do you wonder?



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Let's Think:

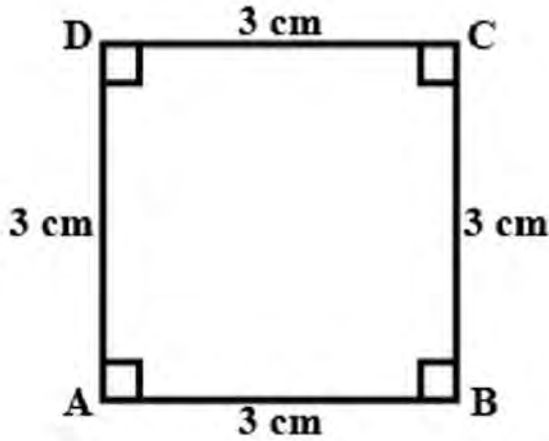
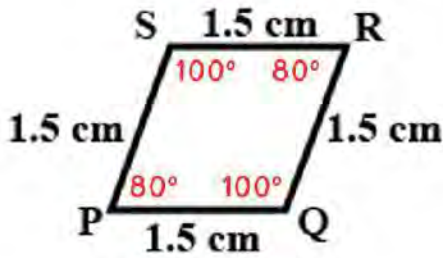
Is Figure Y a scaled copy of Figure X?



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# Let's Think:

Is Quadrilateral ABCD a scaled copy of Quadrilateral PQRS?



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# Let's Try It:

Let's explore using corresponding sides, corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another together.

Name: \_\_\_\_\_ (37 U1 Lesson 6 - Let's Try It)

Consider trapezoid EFGH and trapezoid WXYZ.

- Complete the table with each corresponding side length.
- Find the scale factor for each pair of corresponding sides.
- Are the side lengths scaled by the same factor? \_\_\_\_\_
- Now look at the corresponding angles. Do the corresponding angles have the same measures? \_\_\_\_\_
- If a figure uses the same scale factor for each corresponding side AND if the corresponding angle measures are equal, then it is a scaled copy. Is trapezoid WXYZ a scaled copy of trapezoid EFGH?

TRAPEZOID EFGH (Trapezoid EFGH)	CORRESPONDING SIDE LENGTH (Trapezoid WXYZ)	SCALE FACTOR
5	15	
11	33	
4	12	
4	12	

Figure A and Figure B are both rectangles.

- Complete the table to find the corresponding side lengths and each scale factor.

TRAPAZOID FIGURE A	CORRESPONDING SIDE LENGTH (FIGURE B)	SCALE FACTOR
6	2	
14	7	

- Are the side lengths scaled by the same factor? \_\_\_\_\_
- Do the corresponding angles have the same measure? \_\_\_\_\_
- If a figure uses the same scale factor for each corresponding side AND if the corresponding angle measures are equal, then it is a scaled copy. Is Figure B a scaled copy of Figure A?

Consider Polygon G and Polygon H.

- Are the side lengths scaled by the same factor? \_\_\_\_\_
- Do the corresponding angles have the same measure? \_\_\_\_\_
- Is Polygon H a scaled copy of Polygon G? \_\_\_\_\_

Consider Polygon ABCD and Polygon WXYZ.

- Are the side lengths scaled by the same factor? \_\_\_\_\_
- Do the corresponding angles have the same measure? \_\_\_\_\_
- Is Polygon WXYZ a scaled copy of Polygon ABCD? \_\_\_\_\_
- How do you know? \_\_\_\_\_

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# On your Own:

Now it's time to use corresponding sides, corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another on your own.

Name: \_\_\_\_\_ G7 U1 Lesson 4 – Independent Work

1. Complete the table below to show the corresponding side lengths and the scale factor.

SIDE LENGTH (X)	CORRESPONDING SIDE LENGTH (Y)	SCALE FACTOR
2	4	
9	18	
4	8	
9	18	

Select **all** true statements below.

- The side lengths are the same in both figures.
- The corresponding angle measures are the same in both figures.
- The side lengths are scaled by the same factor.
- Figure B is a scaled copy of Figure A.

2. Complete the table below to show the corresponding side lengths and the scale factor.

SIDE LENGTH (A)	CORRESPONDING SIDE LENGTH (B)	SCALE FACTOR
6	2	
6	2	
6	2	
6	2	

Select **all** true statements below.

- The side lengths are the same in both figures.
- The corresponding angle measures are the same in both figures.
- The side lengths are scaled by the same factor.
- Figure B is a scaled copy of Figure A.

3. Is Figure Y a scaled copy of Figure X? Explain how you know. Include the terms **scale factor**, **corresponding sides**, and **corresponding angles** in your response.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

4. Is ADEF a scaled copy of  $\triangle ABC$ ? If so, include the scale factor that was used to create the scaled copy.

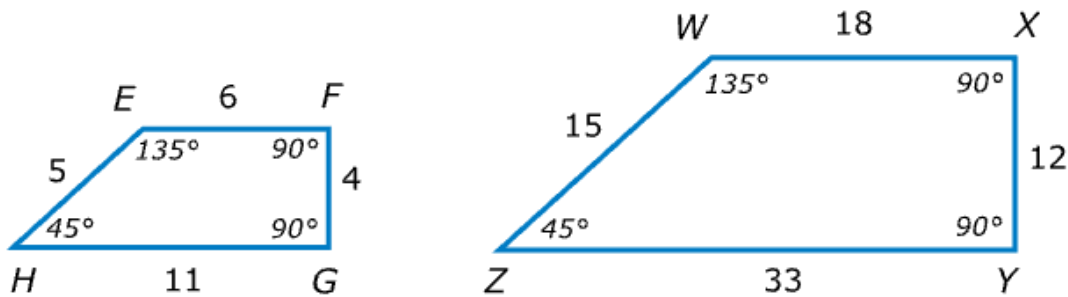
\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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Consider trapezoid EFGH and trapezoid WXYZ.



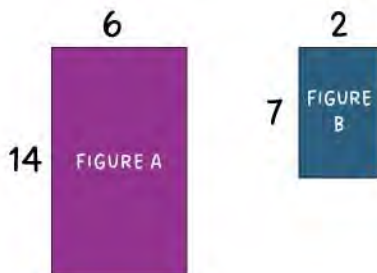
- Complete the table with each corresponding side length.
- Find the scale factor for each pair of corresponding sides.

SIDE LENGTH (Trapezoid EFGH)	CORRESPONDING SIDE LENGTH (Trapezoid WXYZ)	SCALE FACTOR
6		
4		
5		
11		

- Are the side lengths scaled by the same factor? \_\_\_\_\_
- Now look at the corresponding angles. Do the corresponding angles have the same measures? \_\_\_\_\_
- If a figure uses the same scale factor for each corresponding side AND if the corresponding angle measures are equal, then it is a scaled copy.** Is trapezoid WXYZ a scaled copy of trapezoid EFGH?

Figure A and Figure B are both rectangles.

- Complete the table to find the corresponding side lengths and each scale factor.



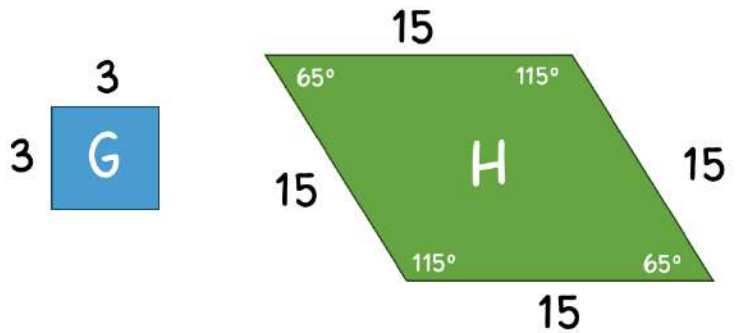
SIDE LENGTH (Figure A)	CORRESPONDING SIDE LENGTH (Figure B)	SCALE FACTOR
6		
14		



7. Are the side lengths scaled by the same factor? \_\_\_\_\_
8. Do the corresponding angles have the same measure? \_\_\_\_\_
9. **If a figure uses the same scale factor for each corresponding side AND if the corresponding angle measures are equal, then it is a scaled copy.** Is Figure B a scaled copy of Figure A?

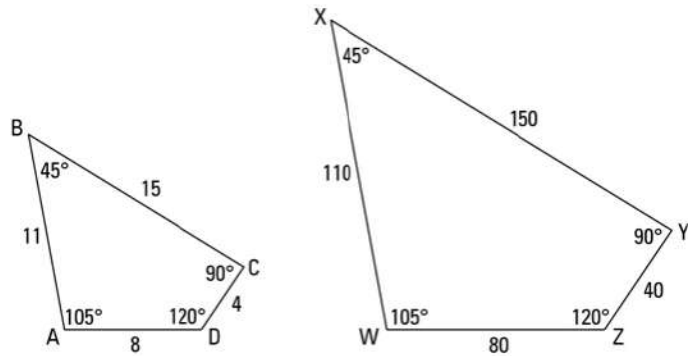
**Consider Polygon G and Polygon H.**

10. Are the side lengths scaled by the same factor? \_\_\_\_\_
11. Do the corresponding angles have the same measure? \_\_\_\_\_
12. Is Polygon H a scaled copy of Polygon G? \_\_\_\_\_



**Consider Polygon ABCD and Polygon WXYZ.**

13. Are the side lengths scaled by the same factor? \_\_\_\_\_
14. Do the corresponding angles have the same measure? \_\_\_\_\_
15. Is Polygon WXYZ a scaled copy of Polygon ABCD? \_\_\_\_\_
16. How do you know?




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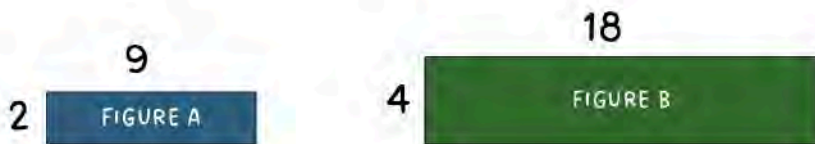


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1. Complete the table below to show the corresponding side lengths and the scale factor.

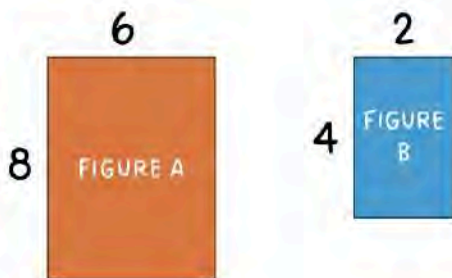


SIDE LENGTH (A)	CORRESPONDING SIDE LENGTH (B)	SCALE FACTOR
2		
9		

Select all true statements below.

- The side lengths are the same in both figures.
- The corresponding angle measures are the same in both figures.
- The side lengths are scaled by the same factor.
- Figure B is a scaled copy of Figure A.

2. Complete the table below to show the corresponding side lengths and the scale factor.

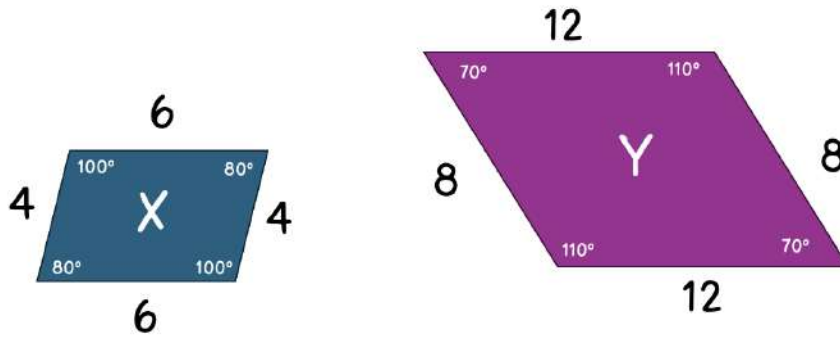


SIDE LENGTH (A)	CORRESPONDING SIDE LENGTH (B)	SCALE FACTOR
6		
8		

Select all true statements below.

- The side lengths are the same in both figures.
- The corresponding angle measures are the same in both figures.
- The side lengths are scaled by the same factor.
- Figure B is a scaled copy of Figure A.

3. Is Figure Y a scaled copy of Figure X? Explain how you know. Include the terms *scale factor*, *corresponding sides*, and *corresponding angles* in your response.




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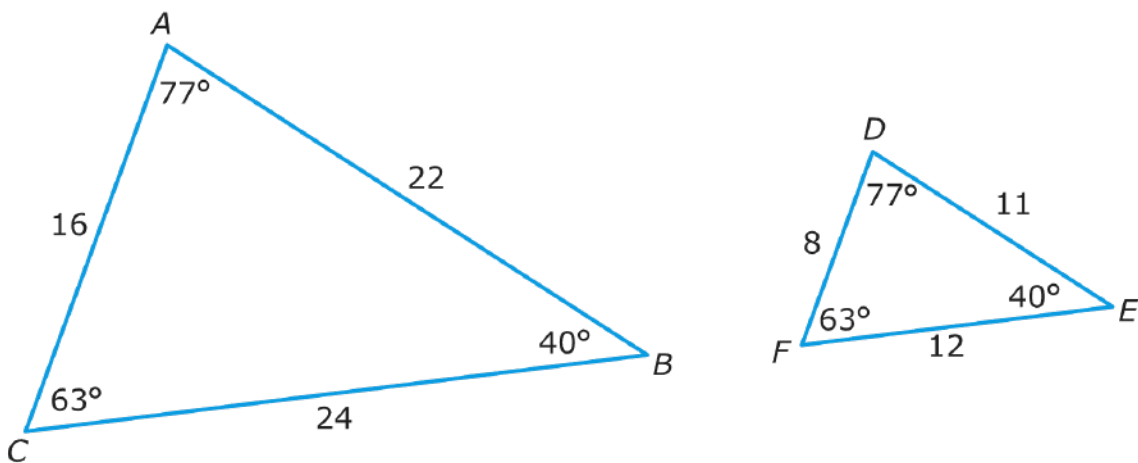


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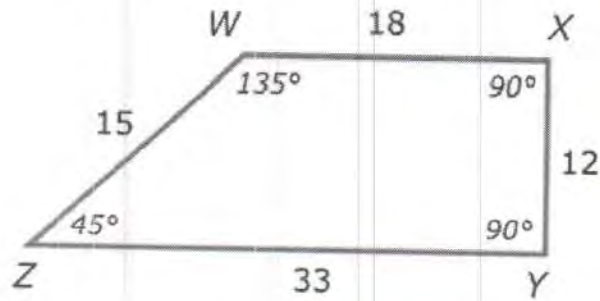
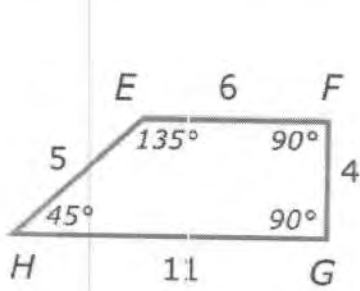


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4. Is  $\triangle DEF$  a scaled copy of  $\triangle ABC$ ? If so, include the scale factor that was used to create the scaled copy.



Consider trapezoid EFGH and trapezoid WXYZ.



1. Complete the table with each corresponding side length.

SIDE LENGTH (Trapezoid EFGH)	CORRESPONDING SIDE LENGTH (Trapezoid WXYZ)	SCALE FACTOR
6	$\xrightarrow{\times 3} 18$	3
4	$\longrightarrow 12$	3
5	$\longrightarrow 15$	3
11	$\longrightarrow 33$	3

2. Find the scale factor for each pair of corresponding sides.

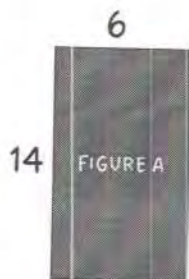
3. Are the side lengths scaled by the same factor? YES

4. Now look at the corresponding angles. Do the corresponding angles have the same measures? YES

5. If a figure uses the same scale factor for each corresponding side AND if the corresponding angle measures are equal, then it is a scaled copy. Is trapezoid WXYZ a scaled copy of trapezoid EFGH? YES

Figure A and Figure B are both rectangles.

6. Complete the table to find the corresponding side lengths and each scale factor.



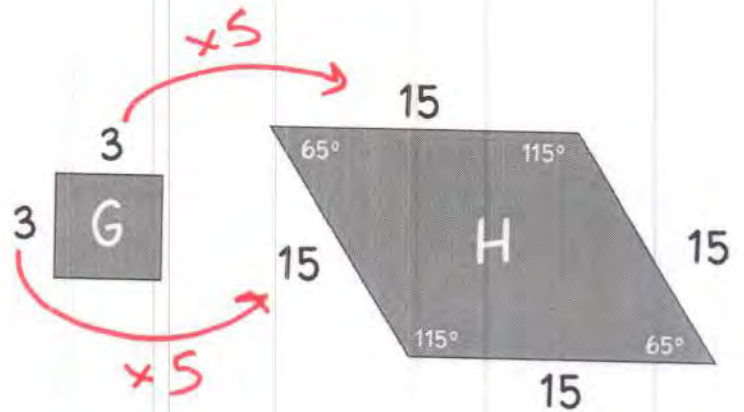
SIDE LENGTH (Figure A)	CORRESPONDING SIDE LENGTH (Figure B)	SCALE FACTOR
6	$\xrightarrow{\times 1/3} 2$	$1/3$
14	$\xrightarrow{\times 1/2} 7$	$1/2$
6	$\xrightarrow{\times 1/3} 2$	$1/3$
14	$\xrightarrow{\times 1/2} 7$	$1/2$



7. Are the side lengths scaled by the same factor? NO
8. Do the corresponding angles have the same measure? YES
9. If a figure uses the same scale factor for each corresponding side AND if the corresponding angle measures are equal, then it is a scaled copy. Is Figure B a scaled copy of Figure A? NO

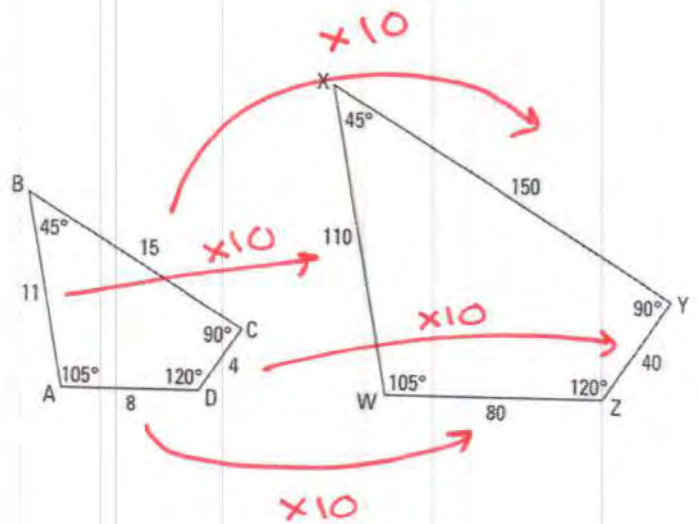
Consider Polygon G and Polygon H.

10. Are the side lengths scaled by the same factor? YES
11. Do the corresponding angles have the same measure? NO
12. Is Polygon H a scaled copy of Polygon G? NO



Consider Polygon ABCD and Polygon WXYZ.

13. Are the side lengths scaled by the same factor? YES
14. Do the corresponding angles have the same measure? YES
15. Is Polygon WXYZ a scaled copy of Polygon ABCD? YES
16. How do you know?



The angle measures are identical and the scale factor is consistent for each pair of corresponding sides.

1. Complete the table below to show the corresponding side lengths and the scale factor.



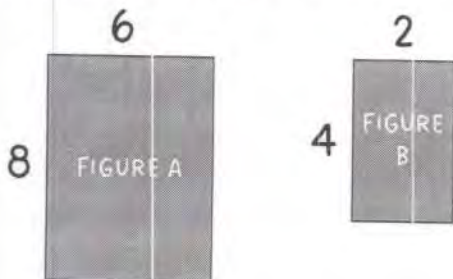
SIDE LENGTH (A)	CORRESPONDING SIDE LENGTH (B)	SCALE FACTOR
2	4	2
9	18	2

*Handwritten notes: Red arrows connect 2 to 4 and 9 to 18. A red 'x2' is written below the arrows.*

Select all true statements below.

- a. The side lengths are the same in both figures.
- b. The corresponding angle measures are the same in both figures.
- c. The side lengths are scaled by the same factor.
- d. Figure B is a scaled copy of Figure A.

2. Complete the table below to show the corresponding side lengths and the scale factor.



SIDE LENGTH (A)	CORRESPONDING SIDE LENGTH (B)	SCALE FACTOR
6	2	1/3
8	4	1/2

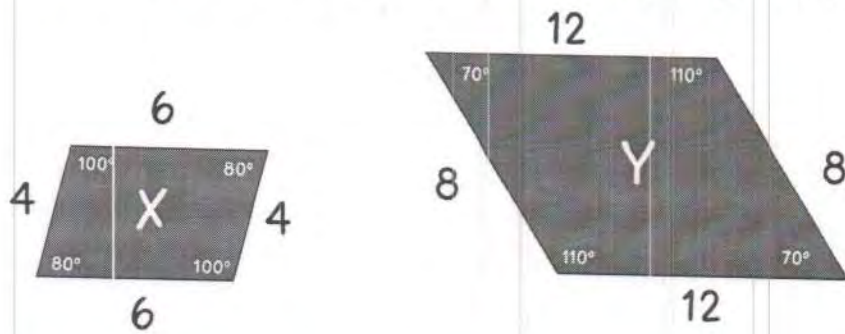
*Handwritten notes: Red arrows connect 6 to 2 and 8 to 4. A red 'x1/3' is written between 6 and 2, and a red 'x1/2' is written between 8 and 4.*

Select all true statements below.

- a. The side lengths are the same in both figures.
- b. The corresponding angle measures are the same in both figures.
- c. The side lengths are scaled by the same factor.
- d. Figure B is a scaled copy of Figure A.

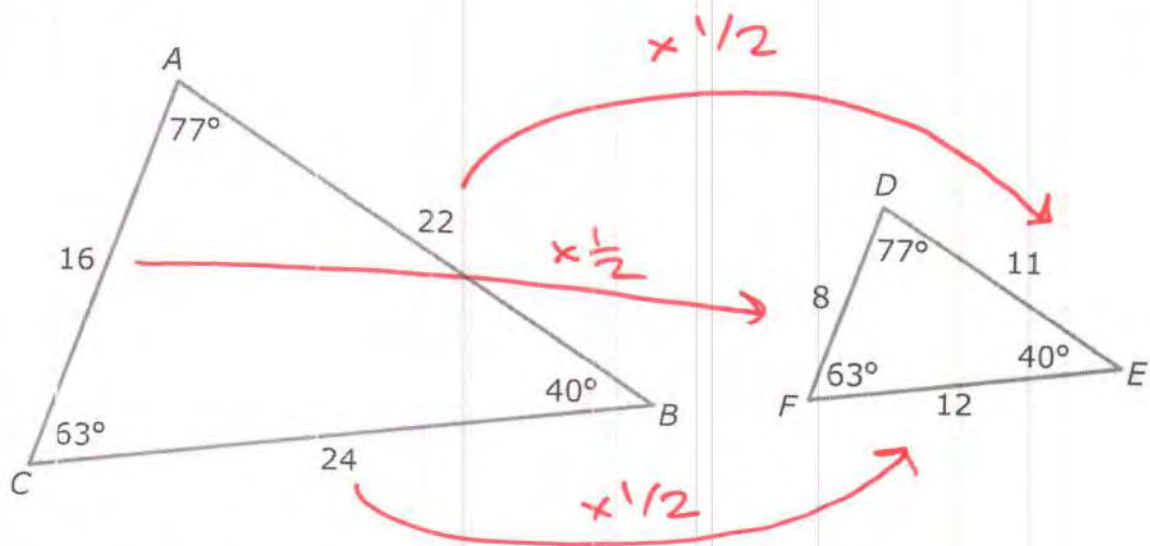


3. Is Figure Y a scaled copy of Figure X? Explain how you know. Include the terms *scale factor*, *corresponding sides*, and *corresponding angles* in your response.



No, Figure Y is not a scaled copy. The side lengths appear to be related by a scale factor of 2, but the corresponding angles are not the same measures.

4. Is  $\triangle DEF$  a scaled copy of  $\triangle ABC$ ? If so, include the scale factor that was used to create the scaled copy.



Yes,  $\triangle DEF$  is a scaled copy. The scale factor is  $\frac{1}{2}$ .

## **G7 U1 Lesson 5**

Describe how scale factors of 1, less than 1, and greater than 1 affect the size of a scaled copy, and explain how scaling can be reversed using reciprocal scale factors

**G7 U1 Lesson 5 - Students will describe how scale factors of 1, less than 1, and greater than 1 affect the size of a scaled copy, and explain how scaling can be reversed using reciprocal scale factors**

**Warm Welcome (Slide 1):** Tutor choice

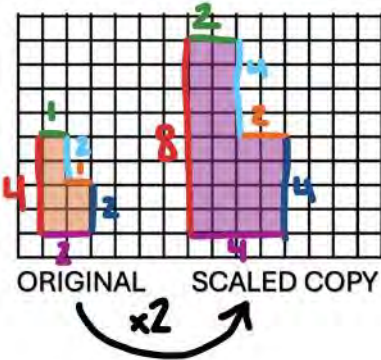
**Frame the Learning/Connect to Prior Learning (Slide 2):** We've been working hard to build our understanding of scale copies and scale factor. We've learned how to identify a scaled copy, we've learned the importance of looking at corresponding sides and angles, and we've even drawn our own scaled copies of figures. Today, we're going to look closely at how different types of scale factors can impact a scaled copy of an image. Let's jump right in!

**Let's Talk (Slide 3):** Take a moment to look at the two sets of images here. What do you notice is the same? What's different? **Possible Student Answers, Key Points:**

- I notice both grids have the same two figures on them. I notice they both show an original and a scaled copy. I notice all the shapes are hexagons.
- I notice that in the first set, the scaled copy is larger than the original. In the second set, the scaled copy is smaller than the original.

These two pairs of images shows the same figures, but in a different order. In the first set, the scaled copy is a bigger version of the original. In the second set, the scaled copy is a smaller version of the original. Let's explore the difference in how these scaled copies were created. I want you to keep an eye out for anything you notice about the scale factors used in each set of figures.

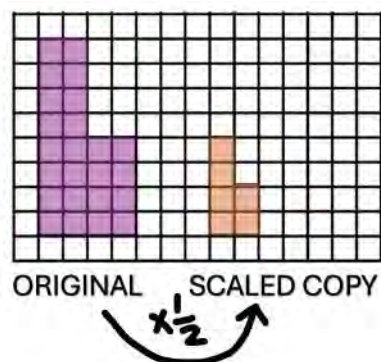
**Let's Think (Slide 4):** The first problem we'll tackle wants us to find the scale factor for the two pairs of images we just looked at.



Let's look at the first set. Which sides are corresponding? What scale factor was used? (*highlight and label corresponding sides as the student shares, supporting as needed*) **Possible Student Answers, Key Points:**

- 4 corresponds with 8. 1 corresponds with 2. 2 corresponds with 4. 1 corresponds with 2. 2 corresponds with 4. 2 corresponds with 4.
- The scale factor is 2, since we can multiply each side length in the original by 2 to end up with the side lengths in the scaled copy.

The scale factor here is 2. (*write  $\times 2$  with an arrow connecting both images as shown*) 4 times 2 is 8, 1 times 2 is 2, 2 times 2 is 4, 1 times 2 is 2, 2 times 2 is 4, and 2 times 2 is 4. We can multiply each side length in the original image by 2 to end up with the corresponding side lengths in the scaled copy. Each side of the scaled copy is two times larger than the sides in the original figure.



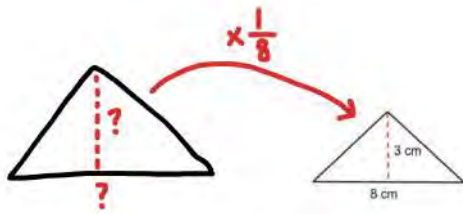
Now we'll look at the other set of figures. We already know the corresponding side lengths, since the images are the same; they're just in a different order. The bigger figure is the original, and the smaller figure is the scaled copy. Thinking about the long side on the left of the figure, we knew that  $4 \times 2 = 8$  before. Now, I need to think about 8 times what number would equal 4? The scale factor is  $\frac{1}{2}$ , because  $8 \times \frac{1}{2} = 4$ . All of the other sides in the original can be multiplied by  $\frac{1}{2}$  to create the smaller scaled figure.

How does this scale factor compare to the previous scale factor? [Possible Student Answers, Key Points:](#)

- The first scale factor was a whole number. This scale factor is a fraction.
- They are inverses.  $\frac{1}{8}$  is the inverse of 8. They are reciprocals of each other.

A few things are important to note here. First, we've seen this before, but multiplying an original by a scale factor greater than 1 will result in a larger scaled copy. Multiplying an original by a scale factor less than 1 will result in a smaller scaled copy. The other thing that's important to note is that these scales are reciprocals or multiplicative inverses. If you want to reverse or undo the effects of a scaling an image, you can use the reciprocal or the multiplicative inverse to get the figure back to its original form.

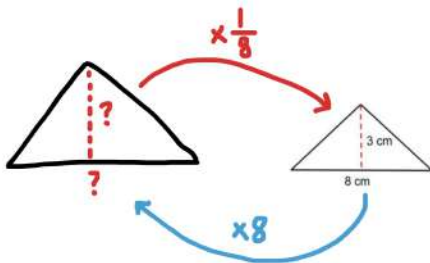
**Let's Think (Slide 5):** Let's try one more problem together.



This one shows us a triangle, and the directions say this triangle is a scaled copy of an original that we don't see shown. The scaled copy was made using a scale factor of  $\frac{1}{8}$ . If I want to visualize this, I know the original triangle must be bigger, since this scaled copy we see is  $\frac{1}{8}$  the size of the original. *(sketch a larger triangle as shown, and draw an arrow showing a scale of  $\frac{1}{8}$  was used to go from the larger triangle to the smaller scaled copy)*

The first question asks us what scale factor we can use to return this scaled copy back to its original size. We can think about the last problem we solved to help us. If they originally used a scale of  $\frac{1}{8}$ , how can we reverse that or undo that using a different scale factor? [Possible Student Answers, Key Points:](#)

- We can find the reciprocal or the multiplicative inverse of  $\frac{1}{8}$ .
- I know the reciprocal or multiplicative inverse of  $\frac{1}{8}$  is  $\frac{8}{1}$  or 8.



$$3 \times 8 = 24 \text{ cm}$$

$$8 \times 8 = 64 \text{ cm}$$

To “undo” or reverse the effects of the scaling, we can use the multiplicative inverse of the scale factor. The multiplicative inverse of  $\frac{1}{8}$  is  $\frac{8}{1}$  or 8. The scale factor we can use to scale the triangle to its original form is 8. *(draw an arrow from the scaled copy to the sketch of the original triangle and label it with  $\times 8$ )* If we want to find the original height and base, we just need to multiply the dimensions of the scaled copy by 8.

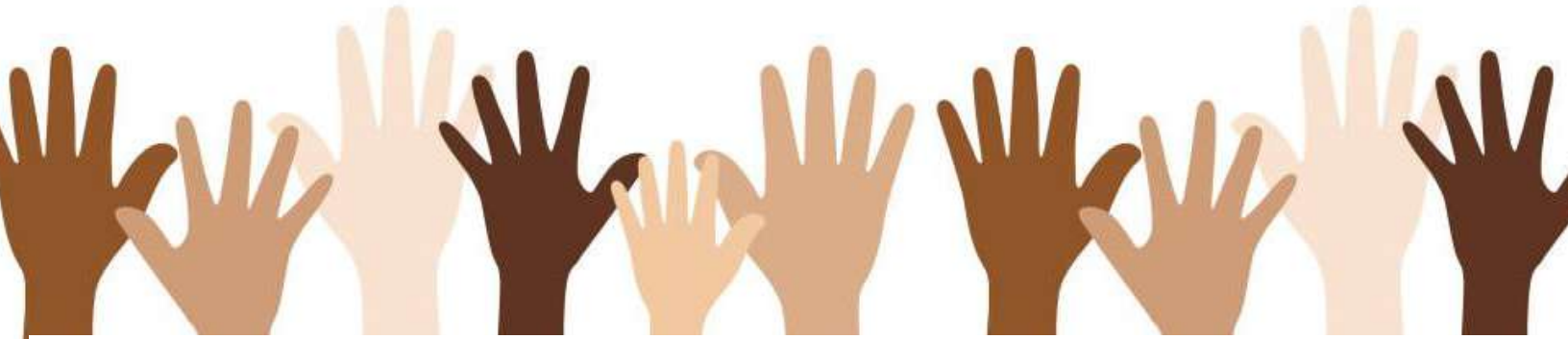
What is the original height? What is the original base? How do you know? [Possible Student Answers, Key Points:](#)

- The original height is 24 centimeters, because  $3 \times 8 = 24$ .
- The original base is 64 centimeters, because  $8 \times 8 = 64$ .

*(write  $3 \times 8 = 24$  and  $8 \times 8 = 64$  as student shares)* The original height is 24 centimeters, because 3 times a scale factor of 8 is 24. The original base is 64 centimeters, because 8 times a scale factor of 8 is 64. Nice work!

**Let's Try it (Slides 6 - 7):** We've seen today that when we multiply an original by scale factor that is a fraction less than 1, the scaled copy will shrink. If we multiply by a scale copy by a scale factor greater than 1, the scaled copy will get bigger. We also explored how we can use the multiplicative inverse to undo or reverse the effects of scaling an image. Let's keep these important ideas in mind as we tackle a few more problems together. After you're feeling even more confident, you'll get the opportunity to try some out on your own. Let's get started!

# WARM WELCOME



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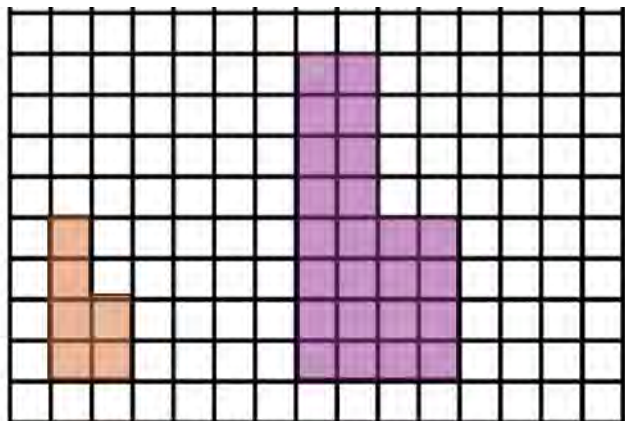
**Today we will describe how scale factors of 1, less than 1, and greater than 1 affect the size of a scaled copy, and explain how scaling can be reversed using reciprocal scale factors.**

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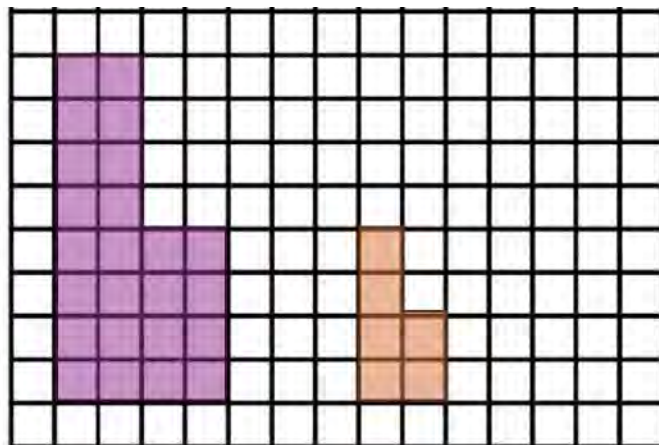


## Let's Talk:

**What's the same?  
What's different?**



ORIGINAL      SCALED COPY

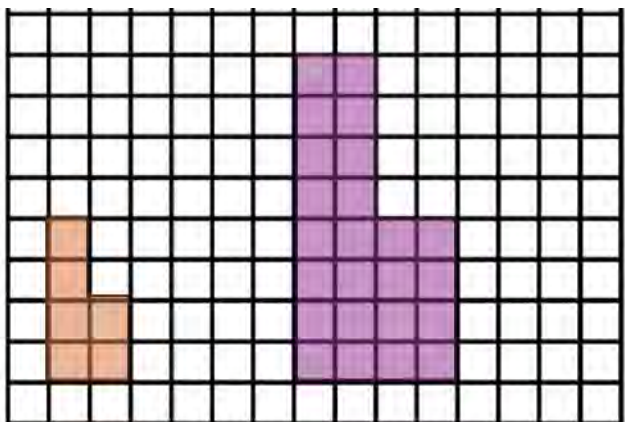


ORIGINAL      SCALED COPY

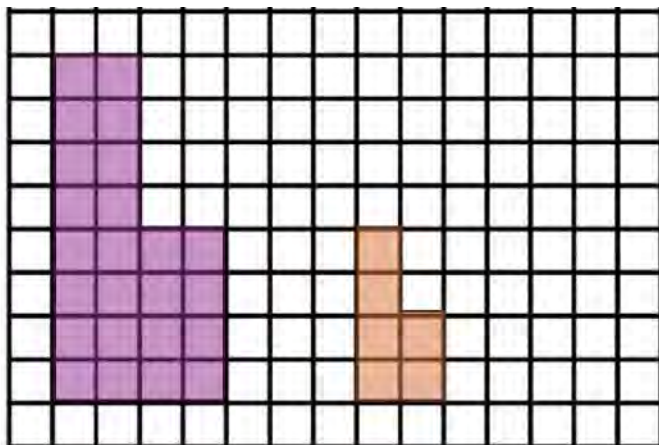
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## Let's Think:

**Determine the scale factor for each set of polygons.**



ORIGINAL      SCALED COPY



ORIGINAL      SCALED COPY

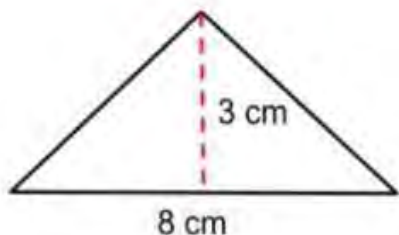
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# Let's Think:

The triangle here is a scaled copy. It was scaled by a factor of  $\frac{1}{8}$ .

a. What scale factor can you use to scale the triangle to its original size?



b. What is the triangle's original height? Base?

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# Let's Try It:

Let's explore describing how scale factors of 1, less than 1, and greater than 1 affect the size of a scaled copy, and explaining how scaling can be reversed using reciprocal scale factors together.

Name: \_\_\_\_\_ G7 U1 Lesson 5 - Let's Try It

Consider the two rectangles shown here.

- Find the side lengths of both rectangles.
- What is the scale factor?

Consider the two rectangles shown here.

- Find the side lengths of both rectangles.
- What is the scale factor?

5. What do you notice about the scale factor for the first set of rectangles compared to the second set of rectangles?

Consider the triangles below.

6. Find the scale factor given each pair of triangles.

- When the scale factor is greater than 1, the scaled copy is \_\_\_\_\_ than the original.
- When the scale factor is less than 1, the scaled copy is \_\_\_\_\_ than the original.

This rectangle is a scaled copy. It was scaled by factor of 4.

- What scale factor can you use to return the figure to its original size? (HINT: What is the reciprocal of the scale factor?)
- What are the dimensions of the original rectangle?
 

LENGTH = \_\_\_\_\_ m

WIDTH = \_\_\_\_\_ m

This rectangle is a scaled copy. It was scaled by a factor of  $\frac{1}{5}$ .

- What scale factor can you use to return the figure to its original size?
- What are the dimensions of the original rectangle?
 

LENGTH = \_\_\_\_\_ m

WIDTH = \_\_\_\_\_ m

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# On your Own:

Now it's time to describe how scale factors of 1, less than 1, and greater than 1 affect the size of a scaled copy, and explain how scaling can be reversed using reciprocal scale factors on your own.

Name: \_\_\_\_\_ Q7 U1 Lesson 5 - Independent Work

1. Complete the table based on the original rectangle and its scaled copy. What is the scale factor?

SIDE LENGTH (ORIGINAL)	CORRESPONDING SIDE LENGTH (SCALED COPY)

2. Complete the table based on the original rectangle and its scaled copy. What is the scale factor?

SIDE LENGTH (ORIGINAL)	CORRESPONDING SIDE LENGTH (SCALED COPY)

3. Explain what happens to a scaled copy when the scale factor is greater than 1. Explain what happens to a scaled copy when a scale factor is less than 1. Use your work from Question #1 and Question #2 to support your thinking, if that's helpful.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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4. The trapezoid shown here is a scaled copy. It was scaled by a factor of  $\frac{1}{3}$ .

a. What scale factor could be used to scale it back to its original size?

b. What are the side lengths of the original trapezoid?

5. The triangle shown here is a scaled copy. It was scaled by a factor of 3.

a. What scale factor could be used to scale it back to its original size?

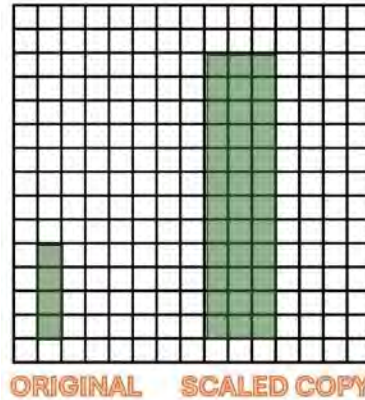
b. What is the length of the original base? Height?

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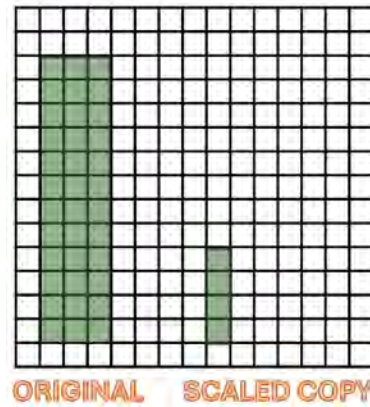
Consider the two rectangles shown here.

1. Find the side lengths of both rectangles.
2. What is the scale factor?



Consider the two rectangles shown here.

3. Find the side lengths of both rectangles.
4. What is the scale factor?



5. What do you notice about the scale factor for the first set of rectangles compared to the second set of rectangles?

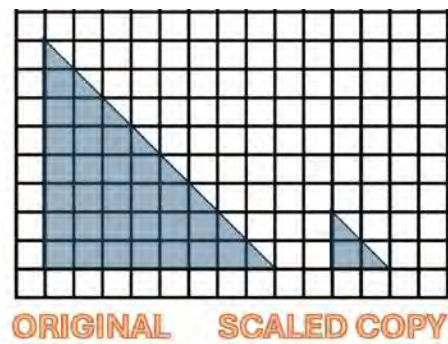
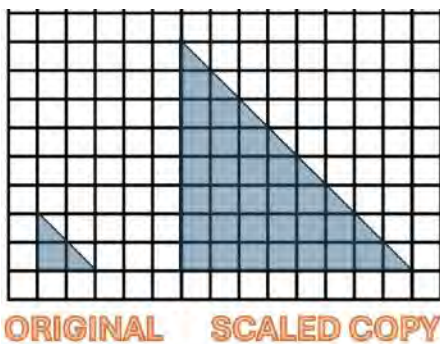
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Consider the triangles below.

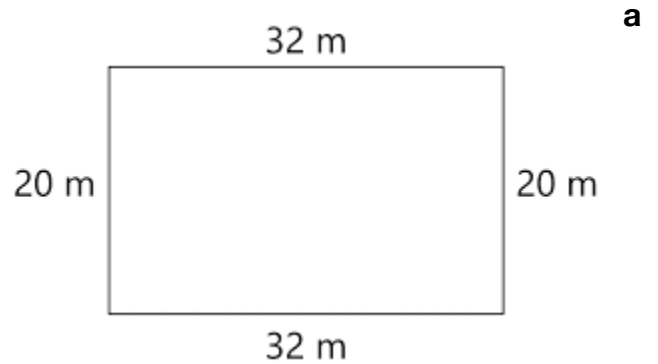
6. Find the scale factor given each pair of triangles.



7. When the scale factor is greater than 1, the scaled copy is \_\_\_\_\_ than the original.
8. When the scale factor is less than 1, the scaled copy is \_\_\_\_\_ than the original.

This rectangle is a scaled copy. It was scaled by factor of 4.

9. What scale factor can you use to return the figure to its original size? (HINT: What is the reciprocal of the scale factor?)



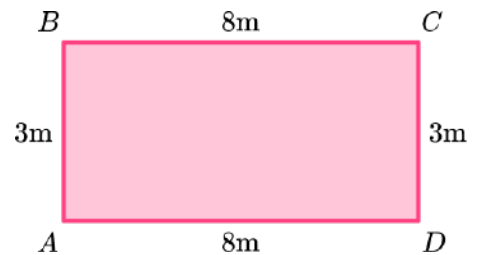
10. What are the dimensions of the original rectangle?

LENGTH = \_\_\_\_\_ m

WIDTH = \_\_\_\_\_ m

This rectangle is a scaled copy. It was scaled by a factor of  $\frac{1}{5}$ .

11. What scale factor can you use to return the figure to its original size?

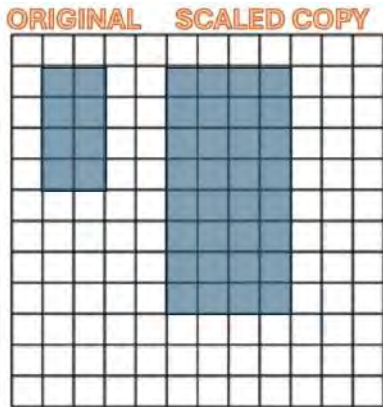


12. What are the dimensions of the original rectangle?

LENGTH = \_\_\_\_\_ m

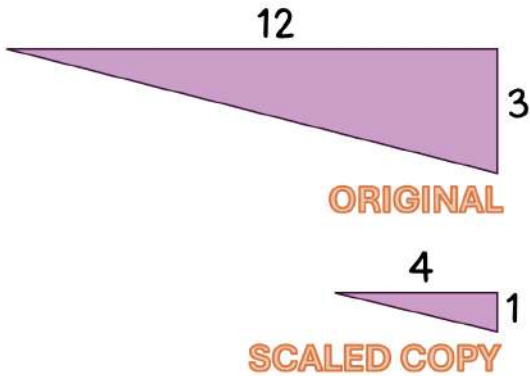
WIDTH = \_\_\_\_\_ m

1. Complete the table based on the original rectangle and its scaled copy. What is the scale factor?



SIDE LENGTH (ORIGINAL)	CORRESPONDING SIDE LENGTH (SCALED COPY)

2. Complete the table based on the original rectangle and its scaled copy. What is the scale factor?



SIDE LENGTH (ORIGINAL)	CORRESPONDING SIDE LENGTH (SCALED COPY)

3. Explain what happens to a scaled copy when the scale factor is greater than 1. Explain what happens to a scaled copy when a scale factor is less than 1. Use your work from Question #1 and Question #2 to support your thinking, if that's helpful.

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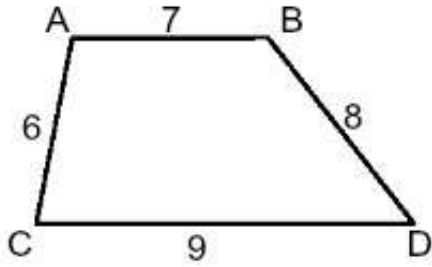
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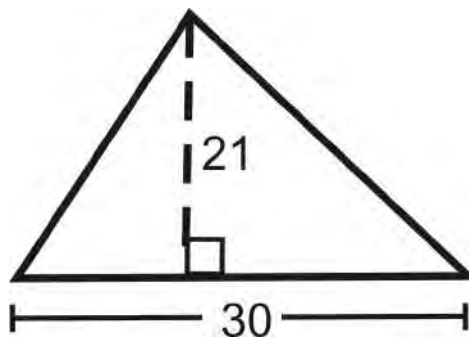


4. The trapezoid shown here is a scaled copy. It was scaled by a factor of  $\frac{1}{2}$ .



- What scale factor could be used to scale it back to its original size?
- What are the side lengths of the original trapezoid?

5. The triangle shown here is a scaled copy. It was scaled by a factor of 3.



- What scale factor could be used to scale it back to its original size?
- What is the length of the original base? Height?

Name: KEY

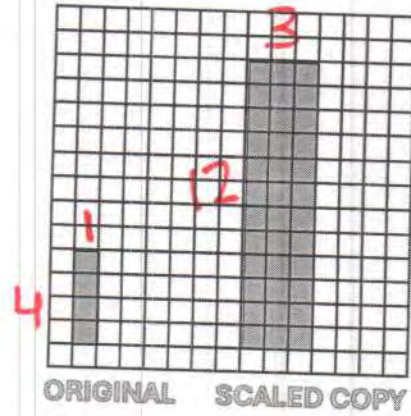
Consider the two rectangles shown here.

- Find the side lengths of both rectangles.

$$\times 3 \begin{pmatrix} 1, 4 \\ \downarrow \quad \downarrow \\ 3, 12 \end{pmatrix} \times 3$$

- What is the scale factor?

$\textcircled{3}$



Consider the two rectangles shown here.

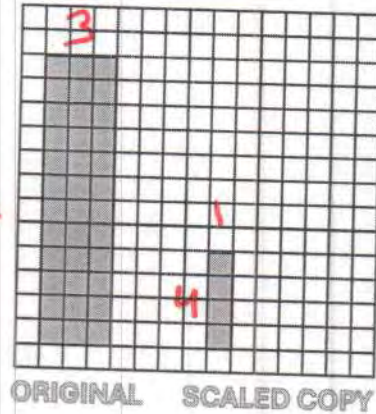
- Find the side lengths of both rectangles.

(same as above)  $\rightarrow$  12

- What is the scale factor?

$$\textcircled{\frac{1}{3}} \quad 3 \times \frac{1}{3} = 1$$

$$12 \times \frac{1}{3} = 4$$

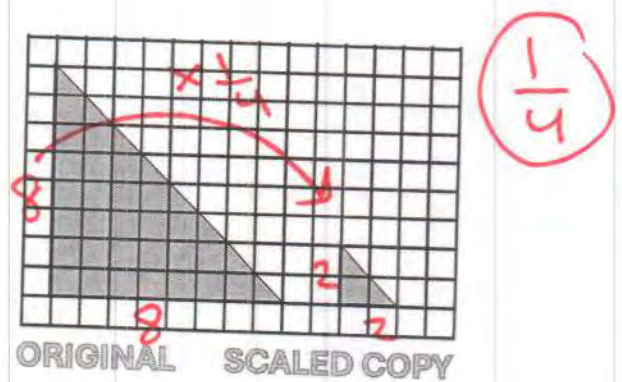
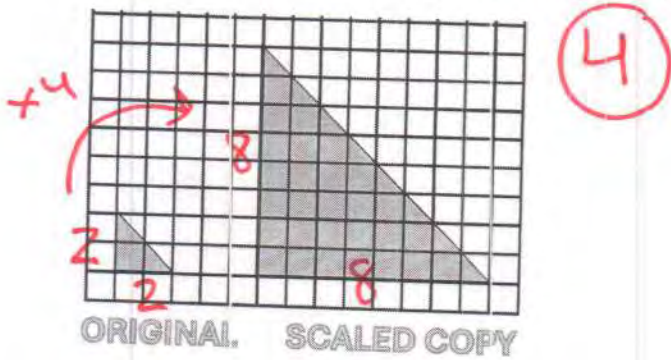


- What do you notice about the scale factor for the first set of rectangles compared to the second set of rectangles?

The first is 3, and the second is  $\frac{1}{3}$ .  
They are reciprocals or multiplicative inverses.

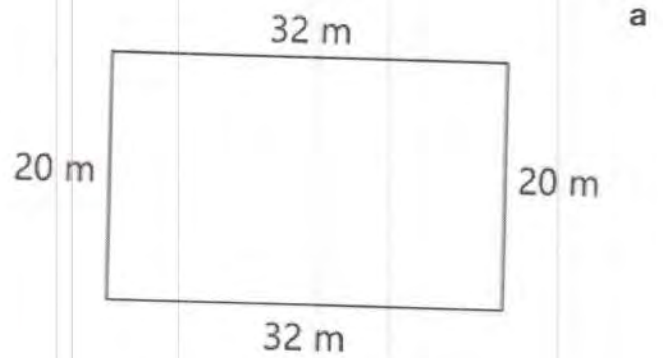
Consider the triangles below.

- Find the scale factor given each pair of triangles.



7. When the scale factor is greater than 1, the scaled copy is bigger than the original.
8. When the scale factor is less than 1, the scaled copy is smaller than the original.

This rectangle is a scaled copy. It was scaled by factor of 4.



9. What scale factor can you use to return the figure to its original size? (HINT: What is the reciprocal of the scale factor?)

$$\left(\frac{1}{4}\right)$$

10. What are the dimensions of the original rectangle?

LENGTH = 8 m  
 WIDTH = 5 m

$$32 \times \frac{1}{4} = 8$$

$$20 \times \frac{1}{4} = 5$$

This rectangle is a scaled copy. It was scaled by a factor of  $\frac{1}{5}$ .

11. What scale factor can you use to return the figure to its original size?

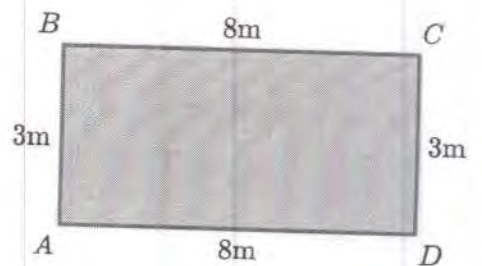
$$(5)$$

12. What are the dimensions of the original rectangle?

LENGTH = 40 m  
 WIDTH = 15 m

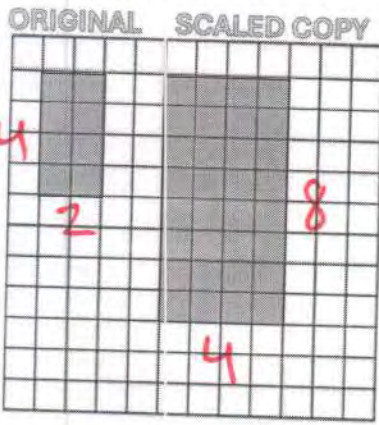
$$8 \times 5 = 40$$

$$3 \times 5 = 15$$





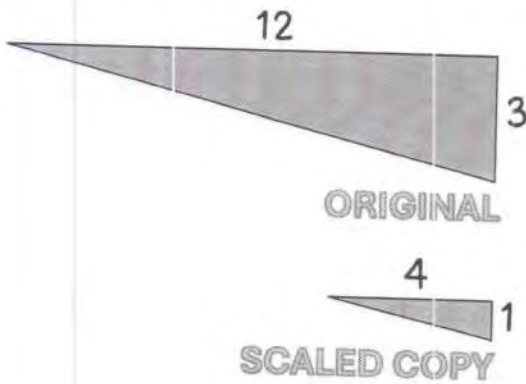
1. Complete the table based on the original rectangle and its scaled copy. What is the scale factor?



SIDE LENGTH (ORIGINAL)	CORRESPONDING SIDE LENGTH (SCALED COPY)
4	8
2	4

Scale factor = 2

2. Complete the table based on the original rectangle and its scaled copy. What is the scale factor?



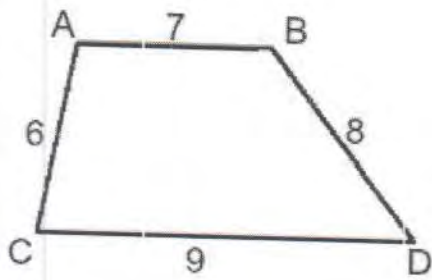
SIDE LENGTH (ORIGINAL)	CORRESPONDING SIDE LENGTH (SCALED COPY)
12	4
3	1

Scale factor =  $\frac{1}{3}$

3. Explain what happens to a scaled copy when the scale factor is greater than 1. Explain what happens to a scaled copy when a scale factor is less than 1. Use your work from Question #1 and Question #2 to support your thinking, if that's helpful.

When the scale factor is greater than 1, the scaled copy is bigger than the original. When the scale factor is less than 1, the scaled copy is smaller than the original.

4. The trapezoid shown here is a scaled copy. It was scaled by a factor of  $\frac{1}{2}$ .



a. What scale factor could be used to scale it back to its original size?

(2)

b. What are the side lengths of the original trapezoid?

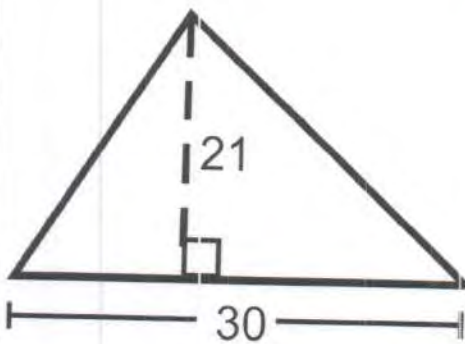
$$7 \times 2 = (14)$$

$$8 \times 2 = (16)$$

$$6 \times 2 = (12)$$

$$9 \times 2 = (18)$$

5. The triangle shown here is a scaled copy. It was scaled by a factor of 3.



a. What scale factor could be used to scale it back to its original size?

( $\frac{1}{3}$ )

b. What is the length of the original base? Height?

$$30 \times \frac{1}{3} = (10) \text{ base}$$

$$21 \times \frac{1}{3} = (7) \text{ height}$$



# **G7 U1 Lesson 6**

Use a scale drawing and its scale to calculate actual distances

## G7 U1 Lesson 6 - Students will use a scale drawing and its scale to calculate actual distances

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We've been working with scale factor and scaled copies for the past several lessons. Today, we'll keep thinking about those things, but we'll look specifically at scale drawings. A scale drawing is a drawing where all lengths correspond to the original object by the same scale. Scale drawings are used in a number of fields of study. A blueprint for a house is a type of scale drawing. An engineer might make a scale drawing or a scale model of a bridge they are designing. Artists sometimes use scale drawings to precisely make a work of art or create a computer rendering of something.

Why do you think it's useful for designers, engineers, scientists, or artists to make bigger or smaller scale drawings of figures? **Possible Student Answers, Key Points:**

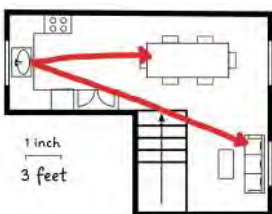
- If an object is too big to work with, it might be helpful to make a smaller scale drawing to more easily work with whatever the object is.
- If something is really small, like a cell or a small insect, a scale drawing might help them more clearly see and think about what they're working with.

**Let's Talk (Slide 3):** Here is one example of a scale drawing. This is a scale drawing of Manny's apartment. What do you notice about the scale drawing? What do you wonder? **Possible Student Answers, Key Points:**

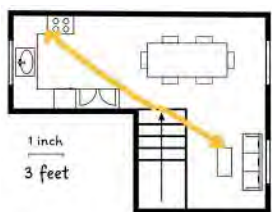
- I notice there is a scale that says 1 inch is equivalent to 3 feet. I notice his apartment has stairs, a sink, a stove, a dining table, a sofa, and a coffee table.
- I wonder what some of the symbols mean. I wonder how big his apartment is in real life. I wonder why he made this scale drawing.

That's interesting! I noticed a lot of the objects in his apartment are represented by symbols. I also saw the scale that he used where 1 inch is equivalent to 3 feet in his actual apartment. Let's look at a problem that involves a scale drawing.

**Let's Think (Slide 4):** This problem wants us to answer a few questions about Manny's scale drawing. Since it's a scale drawing, it's safe to assume that the dimensions of every object in the drawing correspond with the dimensions of every object in real life using the same scale.



The first question wants us to determine whether Manny's sink is closer to the table or the couch. We can't actually see Manny's apartment in real life, but since the drawing is done using a consistent scale, we can use the drawing to determine which object the sink is closer to. *(draw two lines pointing from the sink to the each of the named objects)* Without seeing the apartment in real life, we can still know that Manny's actual sink is closer to the dining room table than the couch.



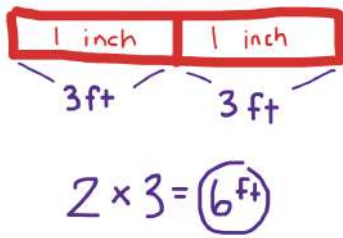
The next question wants us to determine whether the stairs are closer to the stove or the coffee table. How can the scale drawing help us? Which object is the stairs closer to? **Possible Student Answers, Key Points:**

- The lengths in real life would be longer than the lengths in the drawing, but we can use the drawing since it was made using a consistent scale.
- The stairs are closer to the coffee table than the stove.

Without having to visit Manny's apartment in person, the scale drawing helped us to easily think about the proximity of an object to other objects around it.

The third question tells us that the dining room table in the scale drawing is 2 inches long, and it wants us to figure out how long the table would be in real life. What information does this problem give us that will be helpful in determining the actual length of the table? **Possible Student Answers, Key Points:**

- The table is 2 inches in the drawing.
- The model is drawn using a scale where 1 inch is equivalent to 3 feet.



The drawing includes the scale that was used to make it. I see here that it says 1 inch on the drawing is equal to 3 feet in real life. *(point to scale in image)* The table is 2 inches. *(draw a tape diagram partitioned into two boxes that are each labeled as 1 inch)* I know each inch corresponds with 3 feet in real life. *(label 3 feet underneath each box of the tape diagram)* So, if the drawing shows a table that is 2 inches, I can see that the actual table would be 3 feet. I can also show that by multiplying  $2 \times 3$ , if I didn't want to draw a tape diagram.

The last question about Manny's scale drawing asks us to determine the actual length of the coffee table. What information does this problem give us that will be helpful in determining the actual length of the coffee table? **Possible Student Answers, Key Points:**

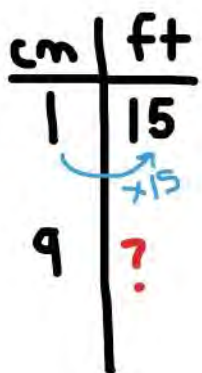
- The coffee table is  $1 \frac{1}{3}$  inches in the drawing.
- The model is drawn using a scale where 1 inch is equivalent to 3 feet.

$$1 \frac{1}{3} \times 3 = 4 \text{ ft}$$

I know each inch on the drawing represents 3 feet in real life. I can multiply the number of inches on the drawing by 3 to find the actual length of any object in the scale drawing. Take a moment to find the value of  $1 \frac{1}{3}$  times 3. *(write  $1 \frac{1}{3} \times 3 =$ ) (4 or  $3 \frac{3}{3}$ )* I know  $3 \times 1$  is 3, and  $3 \times \frac{1}{3}$  is 1. So the total product of  $1 \frac{1}{3}$  times 3 is 4. The actual length of the coffee table is 4 feet. We used the scale to help us multiply to find the actual length of the coffee table.

**Let's Think (Slide 5):** In this problem, we see a scale drawing of part of a neighborhood. Take a moment to look over the visual. What do you notice? What stands out to you as being potentially important? **Possible Student Answers, Key Points:**

- I notice there are 3 houses.
- I notice House X and House Y are 9 centimeters apart in the drawing.
- I notice there is a scale that states that 1 centimeter on the drawing is equivalent to 15 feet in real life.



Part A asks us how far House X is from House Y in actuality. I can see from the picture that they measure 9 centimeters apart in the scale drawing. *(draw t-chart showing centimeters on the left and feet on the right)* The scale that is provided shows that 1 centimeter is equal to 15 feet. *(fill 1 and 15 into respective columns)* This means that if I know the number of centimeters, I know the number of feet in real life will be 15 times that amount. *(draw arrow from 1 to 15 labeled as "x 15")*

$$9 \times 15 = 135 \text{ ft}$$

The distance in the scale drawing is 9 centimeters. *(fill 9 in the centimeters column of the t-chart)* Now, all we need to do is find the product of  $9 \times 15$ . Take a moment. What is the product of 9 and 15? **(135)** *(write equation)* 9 times 15 is 135, so I know 9 centimeters in the drawing is equivalent to 135 feet in real life. House X is 135 feet away from House Y.

The last part of this question wants us to estimate, so our answer might not be exact. Specifically, it asks us to estimate the actual distance between House Y and House Z.

Look at the image. Just by looking, what do you notice about the distance between House Y and Z?

Possible Student Answers, Key Points:

- It's not labeled.
- House Z and House Y are closer together than House X and House Y.
- It looks like maybe 3 or 4 centimeters, based on the distance that is labeled.

These houses are closer together than the other pair we looked at, but it's not labeled. If the other distance in the scale drawing is 9 centimeters, about how long do you think this unlabeled distance will be? (maybe 2 or 3 centimeters) I think 2 or 3 centimeters looks about right.

If we say the distance between House Y and House Z in the drawing is about 3 centimeters. How can we use the given scale to find the actual distance between the houses? Possible Student Answers, Key Points:

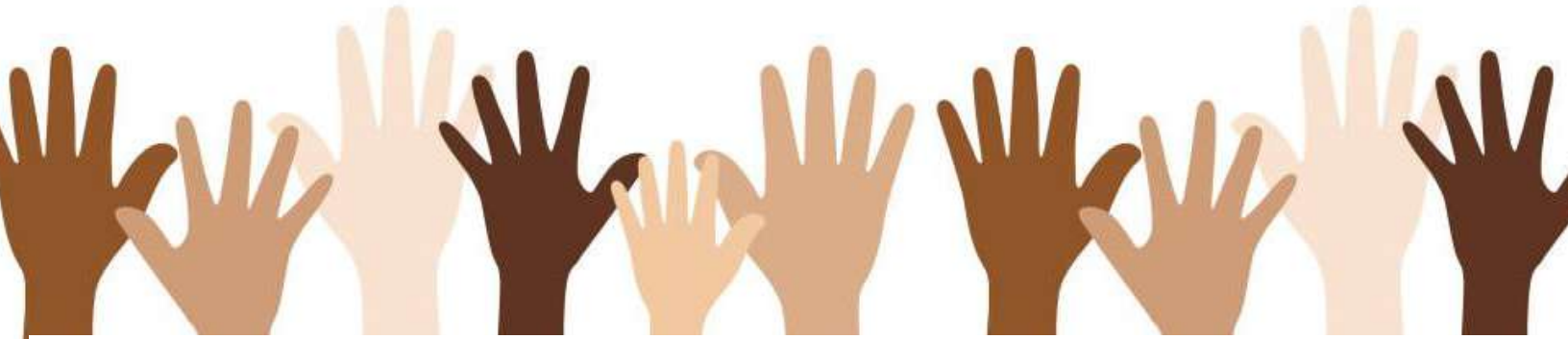
- I can multiply 3 times 15.
- I know 1 centimeter is 15 feet in real life, so I can add 15 ft + 15 ft + 15 ft to find three centimeters.

$$3 \times 15 = 45 \text{ ft}$$

The number of feet in real life is fifteen times the number of centimeters in the scale drawing. I know 3 times 15 is equal to 45. (write equation) The actual distance between House Y and House Z is 45 feet.

**Let's Try it (Slides 6 - 7):** We just looked at two different problems involving scale drawings. One scale drawing was a floorplan of an apartment. One scale drawing was a map of a neighborhood. Scale drawings are drawings where the lengths in the drawing correspond to lengths in real life in a consistent way using a consistent scale. As we work through other examples today, we'll want to carefully notice any scale that is provided. We'll also want to pay attention to our units, as scale drawings often involve scales with varied units. Let's work a bit more together, and then you'll get a chance to show what you know independently.

# WARM WELCOME



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**Today we will use a scale drawing and its scale to calculate actual distances.**

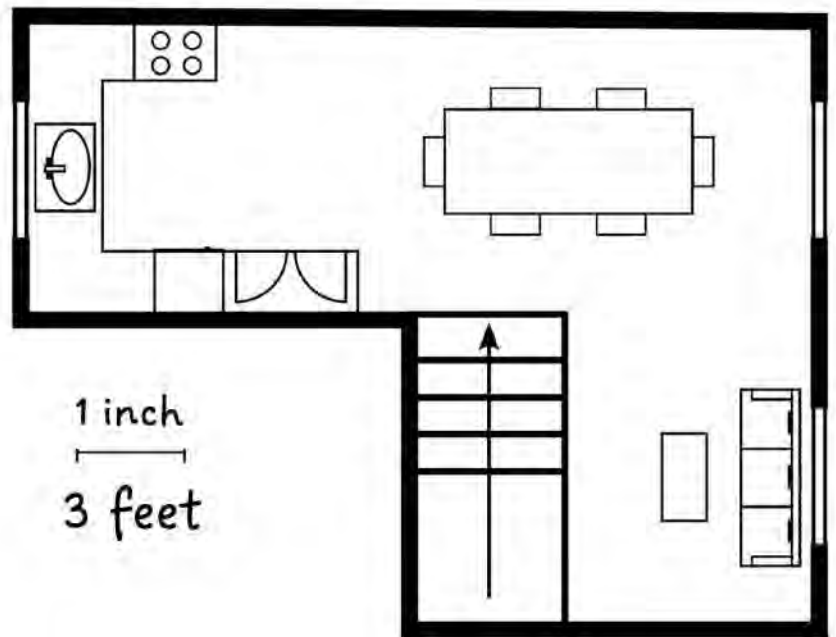
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## Let's Talk:

**This is a scale drawing of Manny's apartment.**

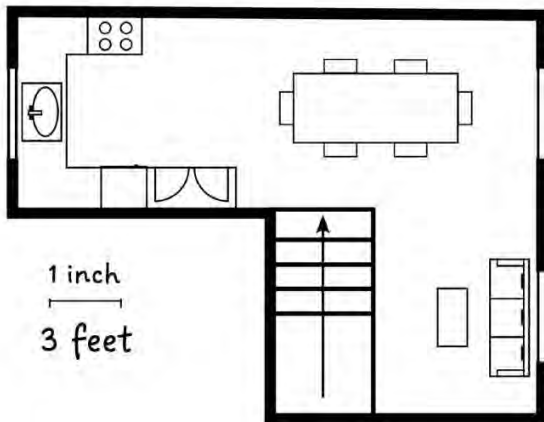
What do you notice?  
What do you wonder?



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## Let's Think:

**Use the scale drawing to answer the following questions.**



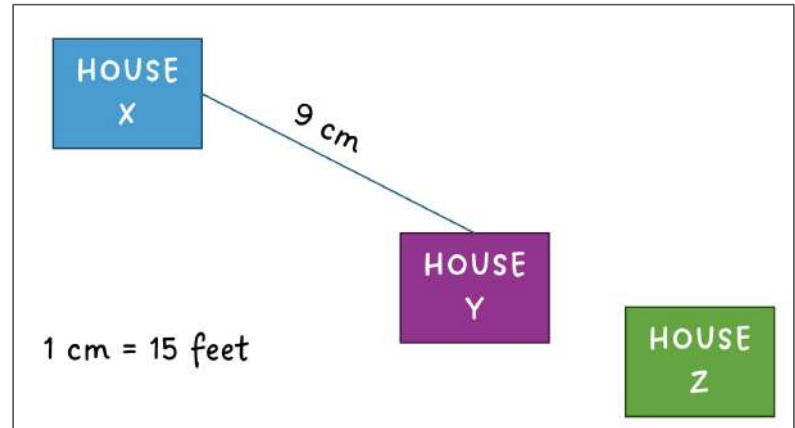
1. Is the sink closer to the dining table or the couch?
2. Are the stairs closer to the stove or the coffee table?
3. In the drawing, the dining room table is 2 inches long. How long is it in real life?
4. In the drawing, the coffee table is  $1\frac{1}{3}$  inches long. What is the actual length?

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# Let's Think:

Use the scale drawing to answer the questions.

- How far is House Y from House X in real life?
- About how apart are House Y and House Z in real life?



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# Let's Try It:

Let's explore using a scale drawing and its scale to calculate actual distances together.

Name: \_\_\_\_\_ G7 U1 Lesson 6 - Let's Try It

Two students attempted to draw the desk fan shown here.

- Circle the drawing that best represents a scaled representation.
- Explain why you did not circle the other image?

In a scale drawing, all lengths correspond to the lengths in the original image by the same scale.

Look at the scaled drawing of the bedroom and bathroom below to estimate distances and lengths.

- Compare the length of the bed to the length of the bathtub.
  - The bed is longer.
  - The bathtub is longer.
- The sinks are closer to the
  - bathub
  - toilet
- Based on the scale, we know 1 centimeter on the drawing is equal to \_\_\_\_\_ feet in real life.

6. Janya measured that the length of the bed in the drawing is 3 centimeters. How long is the actual bed?

- Michael measures that the width of the bedroom in the scaled drawing is 10 centimeters. How wide is the actual bedroom?
- Leo knows that the length of the bathtub in the scaled image is 2 centimeters. How long is the bathtub in real life?
- Morah notices the length of a sink is  $\frac{1}{2}$  centimeter. What is the actual length of the sink?

Look at the scale drawings of Bridge A, Bridge B, and Bridge C.

10. About how long is Bridge B? Explain how you know.

11. About how long is Bridge C? Explain how you know.

12. About how much longer is Bridge C than Bridge A? Explain how you know.

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# On your Own:

Now it's time to use a scale drawing and its scale to calculate actual distances on your own.

Name: \_\_\_\_\_ G7 U1 Lesson 6 - Indoor Work

1. Use the scaled image to answer the questions.



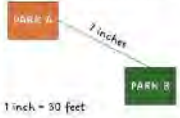
a. Is the dining room table closer to the sofa or the TV stand?

b. Is the coffee table closer to the rug or the chair?

c. The sofa measures 4 centimeters in the drawing. What is the actual length of the sofa?

d. The dining room table measures 1 3/4 cm wide in the drawing. How wide is the dining room table in real life?

2. On a scaled map, Park A is 7 inches from Park B. Use the information in the image below to determine the actual distance between Park A and Park B.



1 inch = 30 feet

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3. Patrick drew a scaled drawing of his local playground. He used a scale where 1 inch is equivalent to 3.5 meters. What is the actual length of Patrick's local playground?



4. Malaysia was trying to find the actual height of a house based on the scale drawing shown here. The drawing uses a scale where 1 inch = 4 feet. Malaysia said the house would actually be 12.5 feet, because  $8.5 \div 4 = 12.5$ .



8.5 in

Explain her error, and include the correct height in your response.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

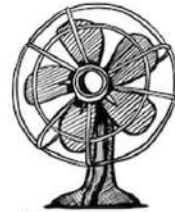
\_\_\_\_\_

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Two students attempted to draw the desk fan shown here.

1. Circle the drawing that best represents a scaled representation.
2. Explain why you did not circle the other image?



Betty



Zion

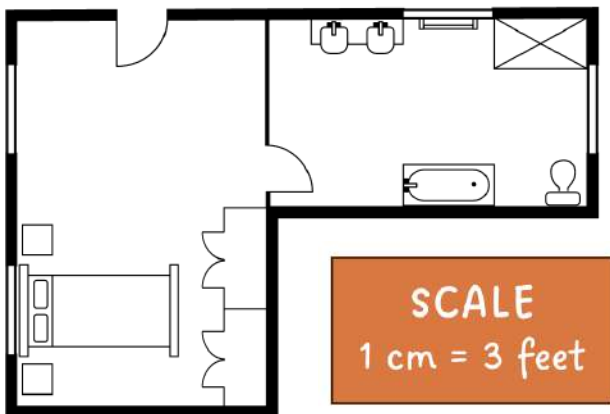
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In a scale drawing, all lengths correspond to the lengths in the original image by the same scale.

Look at the scale drawing of the bedroom and bathroom below to estimate distances and lengths.



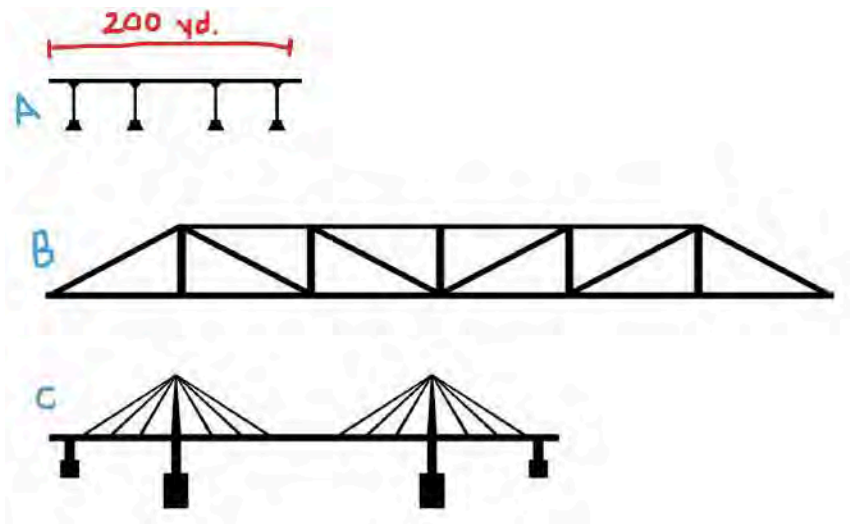
3. Compare the length of the bed to the length of the bathtub.
  - a. The bed is longer.
  - b. The bathtub is longer.
4. The sinks are closer to the \_\_\_\_\_.
  - a. bathtub
  - b. toilet
5. Based on the scale, we know 1 centimeter on the drawing is equal to \_\_\_\_\_ feet in real life.

6. Janiya measured that the length of the bed in the drawing is 3 centimeters. How long is the actual bed?

7. Michael measures that the width of the bedroom in the scaled drawing is 10 centimeters. How wide is the actual bedroom?
  
8. Leo knows that the length of the bathtub in the scaled image is 2 centimeters. How long is the bathtub in real life?
  
9. Moriah notices the length of a sink is  $\frac{1}{2}$  centimeter. What is the actual length of the sink?

**Look at the scale drawings of Bridge A, Bridge B, and Bridge C.**

10. About how long is Bridge B?  
Explain how you know.




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11. About how long is Bridge C? Explain how you know.

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12. About how much longer is Bridge C than Bridge A? Explain how you know.

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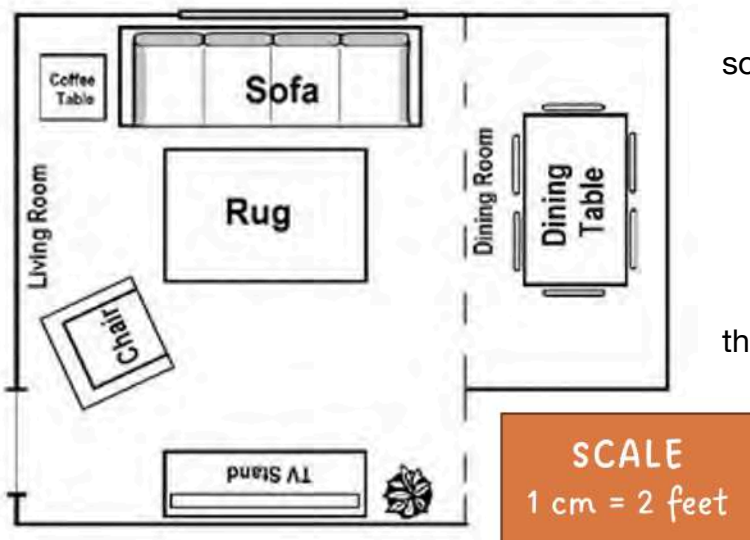
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1. Use the scaled image to answer the questions.



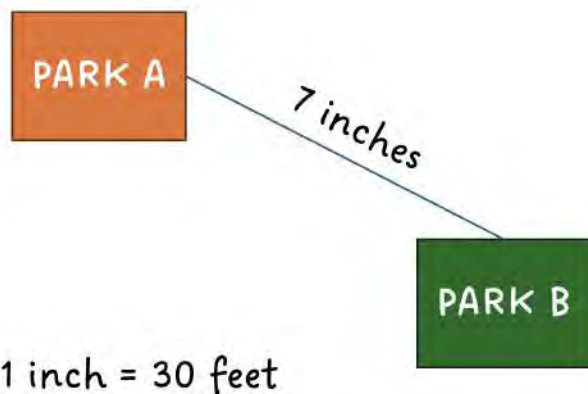
a. Is the dining room table closer to the sofa or the TV stand?

b. Is the coffee table closer to the rug or the chair?

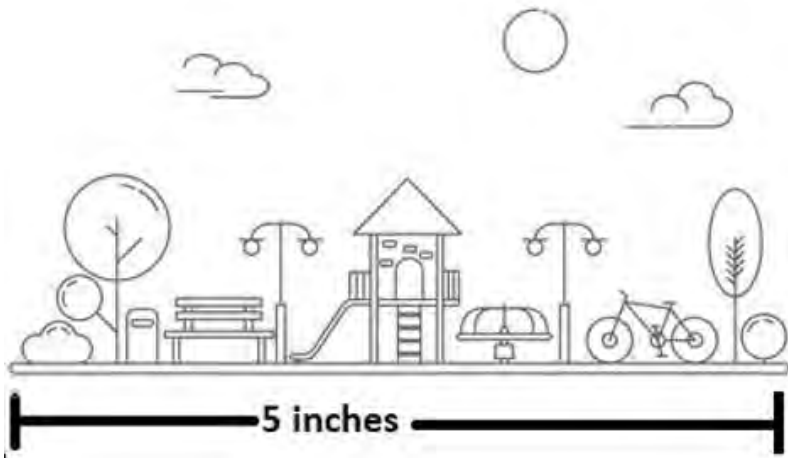
c. The sofa measures 4 centimeters in the drawing. What is the actual length of the sofa?

d. The dining room table measures  $1\frac{1}{2}$  cm wide in the drawing. How wide is the dining room table in real life?

2. On a scaled map, Park A is 7 inches from Park B. Use the information in the image below to determine the actual distance between Park A and Park B.



3. Patrick drew a scaled drawing of his local playground. He used a scale where 1 inch is equivalent to 3.5 meters. What is the actual length of Patrick's local playground?



4. Malaysia was trying to find the actual height of a house based on the scale drawing shown here. The drawing uses a scale where 1 inch = 4 feet. Malaysia said the house would actually be 12.5 feet, because  $8.5 + 4 = 12.5$ .



Explain her error, and include the correct height in your response.

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Two students attempted to draw the desk fan shown here.

1. Circle the drawing that best represents a scaled representation.
2. Explain why you did not circle the other image?

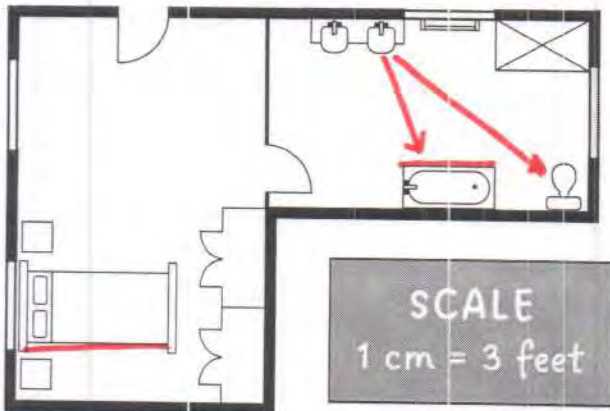


Zion's

image is distorted. It's squished and looks flattened.

In a scale drawing, all lengths correspond to the lengths in the original image by the same scale.

Look at the scale drawing of the bedroom and bathroom below to estimate distances and lengths.



3. Compare the length of the bed to the length of the bathtub.
  - a. The bed is longer.
  - b. The bathtub is longer.
4. The sinks are closer to the \_\_\_\_\_.
  - a. bathtub
  - b. toilet
5. Based on the scale, we know 1 centimeter on the drawing is equal to 3 feet in real life.

6. Janiya measured that the length of the bed in the drawing is 3 centimeters. How long is the actual bed?

cm	ft
1	3
3	9
x3	

(9 ft)

$3 \times 3 = 9$



7. Michael measures that the width of the bedroom in the scaled drawing is 10 centimeters. How wide is the actual bedroom?

$$10 \times 3 = 30 \quad (30 \text{ ft})$$

8. Leo knows that the length of the bathtub in the scaled image is 2 centimeters. How long is the bathtub in real life?

$$2 \times 3 = 6 \quad (6 \text{ ft})$$

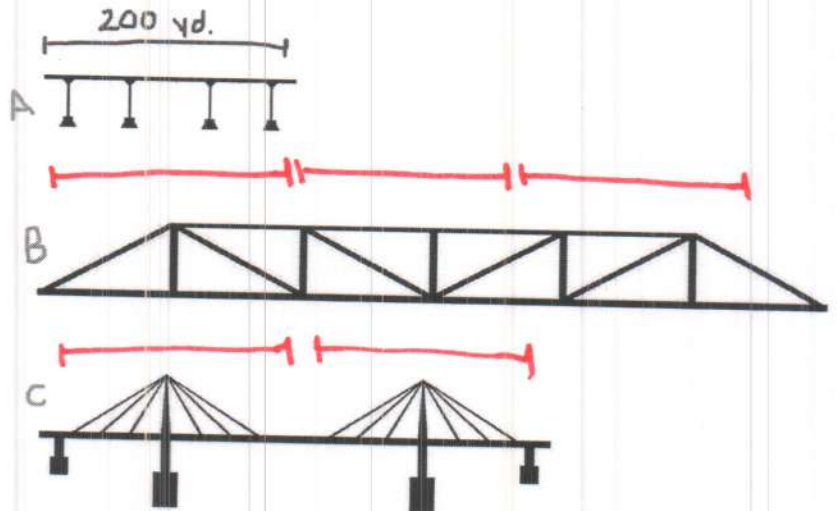
9. Moriah notices the length of a sink is  $\frac{1}{2}$  centimeter. What is the actual length of the sink?

$$\frac{1}{2} \times 3 = \frac{3}{2} \quad (1\frac{1}{2} \text{ ft})$$

Look at the scale drawings of Bridge A, Bridge B, and Bridge C.

10. About how long is Bridge B?

Explain how you know.



Bridge B looks  
about 3 times as  
long as A, so  
(200 x 3) it is  
about 600 yards.

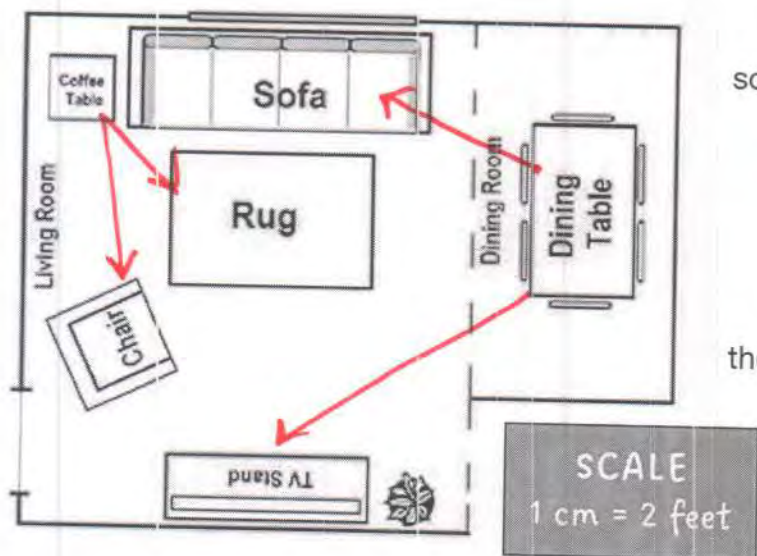
11. About how long is Bridge C? Explain how you know.

Bridge C looks twice as long as  
Bridge A. It is about 400 yards  
long. (200 x 2)

12. About how much longer is Bridge C than Bridge A? Explain how you know.

Bridge C is about 400 yards and Bridge A  
is 200 yards. Bridge C is about 200  
yards longer than Bridge A. (400 - 200)

1. Use the scaled image to answer the questions.



a. Is the dining room table closer to the sofa or the TV stand?

It is closer to the sofa.

b. Is the coffee table closer to the rug or the chair?

It is closer to the rug.

c. The sofa measures 4 centimeters in the drawing. What is the actual length of the sofa?

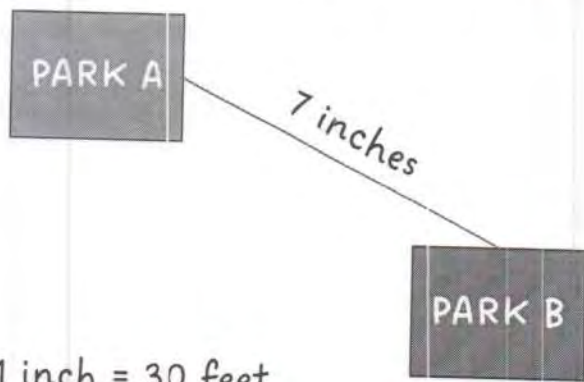
cm	ft
1	2
4	?

$$4 \times 2 = 8 \text{ ft}$$

d. The dining room table measures  $1\frac{1}{2}$  cm wide in the drawing. How wide is the dining room table in real life?

$$1\frac{1}{2} \times 2 = ? \quad \frac{3}{2} \times 2 = \frac{6}{2} = 3 \text{ ft}$$

2. On a scaled map, Park A is 7 inches from Park B. Use the information in the image below to determine the actual distance between Park A and Park B.



1 inch = 30 feet

in	ft
1	30
7	210

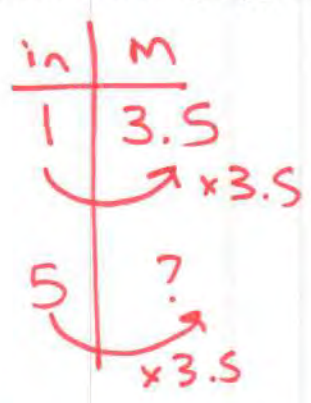
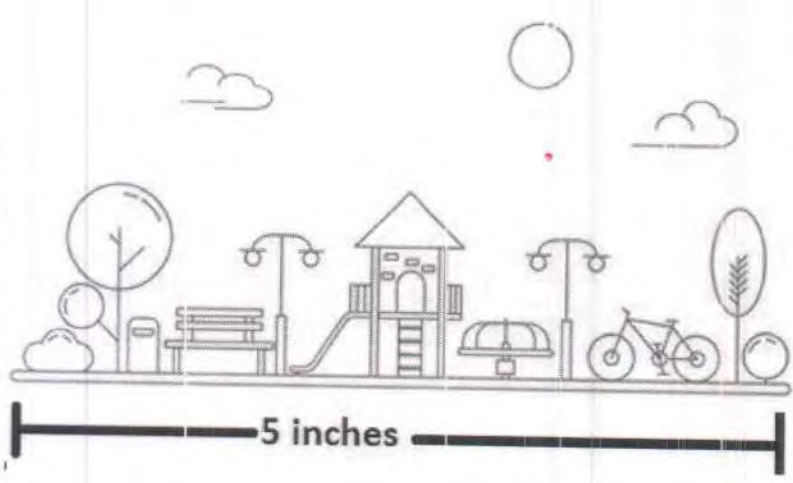
$\times 30$  (under 1)

$\times 30$  (under 7)

210 ft



3. Patrick drew a scaled drawing of his local playground. He used a scale where 1 inch is equivalent to 3.5 meters. What is the actual length of Patrick's local playground?



$$\begin{array}{r} 2 \\ 3.5 \\ \times 5 \\ \hline 17.5 \end{array}$$

17.5 m

4. Malaysia was trying to find the actual height of a house based on the scale drawing shown here. The drawing uses a scale where 1 inch = 4 feet. Malaysia said the house would actually be 12.5 feet, because  $8.5 + 4 = 12.5$ .



$$8.5 \times 4 = ?$$

$$(8.5 \times 2) + (8.5 \times 2)$$

$$17 + 17 = 34 \text{ ft.}$$

Explain her error, and include the correct height in your response.

If the scale is 1 inch = 4 feet, she should multiply 8.5 by 4. She should not have added 4. The correct height is 34 feet.

# **G7 U1 Lesson 7**

Determine the scale and the dimensions of a scale drawing when given the actual dimensions of an object

**G7 U1 Lesson 7 - Students will determine the scale and the dimensions of a scale drawing when given the actual dimensions of an object**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we calculated real-life, actual distances or measurements based on a scale drawing. Today, we'll consider how to use the actual distances or measurements and a scale to determine dimensions of a scale drawing. It's almost like we're doing the opposite of what we did in the previous lesson.

**Let's Talk (Slide 3):** Take a look at the images here. What do you notice? What do you wonder? **Possible Student Answers, Key Points:**

- I notice one picture shows a computer image of a statue, and one image shows the actual statue. I notice the height of the statue on the screen is 5 centimeters, but the height of the statue in real life is 20 feet.
- I wonder if the image on the computer is a scale model of the actual statue. I wonder what the scale factor is.

The picture on the computer screen represents a scale model of the actual statue. What scale do you think was used to make the image? **Possible Student Answers, Key Points:**

- I know 5 inches is the same as 20 feet in real life. So,  $5 \text{ in} = 20 \text{ ft}$  could be the scale.
- I know every inch represents 4 feet of the statue. So, the scale is  $1 \text{ in} = 4 \text{ ft}$ .

$$5 \text{ in} = 20 \text{ ft}$$

$$1 \text{ in} = 4 \text{ ft}$$

(write  $5 \text{ in} = 20 \text{ ft}$  and  $1 \text{ in} = 4 \text{ ft}$ ) In our last lesson, we would have thought through how to move from the scaled image to the actual statue. We would have multiplied each inch by 4 to find the height of the actual statue in feet. Today, we're going to be thinking about how we could go from the statue to the scaled image. It's like we're moving in the opposite direction.

If we could multiply the inches by 4 to find the number of feet, what do you think we could do to go from the number of actual feet to the number of inches in the scale model? **Possible Student Answers, Key Points:**

- We could divide the number of feet by 4, since division is the inverse of multiplication.
- We could multiply the number of feet by  $\frac{1}{4}$ , since  $\frac{1}{4}$  is the multiplicative inverse of 4.

Interesting! Let's try out a couple problems together to see if what you're considering is true or not!

**Let's Think (Slide 4):** This problem gives us the dimensions of an actual swimming pool.



They want us to consider the dimensions of two different scale drawings. Since there is no visual of the pool, I'm going to sketch a rectangle so I can picture what the actual pool looks like. (sketch a rectangle labeled with 50 m and 20 m)

in	m
1	2
?	50
?	20

*(A red arrow points from the '1' in the first row to the '2' in the first row, with 'x2' written next to it.)*

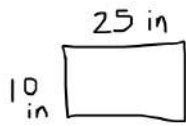
Part A wants us to find the dimensions of a scale drawing if the scale used is 1 inch = 2 meters. I'll use a t-chart to think about that relationship. (draw t-chart labeled with inches and meters, and include 1 inch and 2 meters as the first row of values) I know 1 inch in the scale drawing is equal to 2 meters in the actual pool. That means the number of meters in real life is 2 times the number of inches in the drawing. (draw arrow from 1 in to 2 m labeled with "x 2") We need to use that relationship to think about how many inches long and how many inches wide the scale drawing will be. (fill in 50 and 20 in the meter column of the chart and two question marks in the inch column as shown)

We know we can multiply the number of inches by 2 to find the number of meters. With that information in mind, what math can we do to find the number of inches instead? **Possible Student Answers, Key Points:**

- We could divide the number of meters by 2, since division is the inverse of multiplication.
- We could multiply the number of meters by  $\frac{1}{2}$ , since  $\frac{1}{2}$  is the multiplicative inverse of 2.

$$50 \times \frac{1}{2} = 25$$

$$20 \times \frac{1}{2} = 10$$



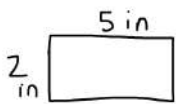
I know 50 times  $\frac{1}{2}$  is 25. I know 20 times  $\frac{1}{2}$  is 10. (write equations) The dimensions of the scale drawing will be 25 inches and 10 inches. (sketch and label a rectangle with the dimensions of the scale drawing)

in	m
$\frac{1}{2}$	5
1	10
?	50
?	20

Part B gives us a different scale. This scale says that  $\frac{1}{2}$  inch on the drawing is equal to 5 meters in real life. Let's set up a table to show that. (sketch a table and write  $\frac{1}{2}$  and 5 in the inch and meter columns respectively) It's a little tricky for me to think about  $\frac{1}{2}$  and 5 are related. I could figure out how many times I need to multiply  $\frac{1}{2}$  to make 5, but it might be a bit easier if I scale this relationship up. If I know  $\frac{1}{2}$  an inch represents 5 meters, then if I double that, I know 1 inch would represent 10 meters. That's a simpler scale to think about. (add 1 inch and 10 meters in their columns and label with an arrow showing "x 10") The number of meters is ten times the number of inches in the scale drawing. (write 50 and 20 in the meters column)

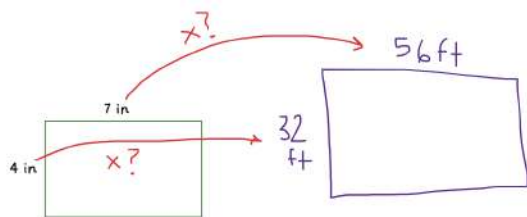
How can I use this scale now to find the dimensions of the scale drawing? **Possible Student Answers, Key Points:**

- I know  $50 \times \frac{1}{10}$  is 5. I know  $20 \times \frac{1}{10}$  is 2.
- We can divide. I know 50 divided by 10 is 5, and 20 divided by 10 is 2.



Great! Using the scale of  $\frac{1}{2}$  inch = 5 meters, we figured out that the dimensions of the scale drawing will be 5 inches by 2 inches. (sketch a and label a rectangle accordingly) We just used two different scales to think about the same swimming pool.

**Let's Think (Slide 5):** Our final problem before we jump into some practice gives us information about a rectangular playground. We also see a scale drawing of the playground. Instead of finding the dimensions of the scale drawing like in our previous problem, we are asked to find the scale that was used to create the drawing.



Since the problem does not provide us a visual of the actual playground, I'll sketch and label a rectangle to represent the actual playground. This will help me compare corresponding sides. (sketch a rectangle and label with the dimensions of the playground) Which sides are corresponding?

What relationship do you notice between corresponding sides? **Possible Student Answers, Key Points:**

- The longer sides correspond with each other, and the shorter sides correspond with each other. I know 7 inches corresponds with 56 feet. I know 4 inches corresponds with 32 feet.
- I notice you can multiply each side length in the scale drawing by 8 to end up with the number of feet on the actual playground.

$$1 \text{ in} = 8 \text{ ft}$$

The drawing helps me see that the corresponding sides are related by a factor of 8. Every 1 inch in the scale drawing corresponds with 8 feet in the actual playground. (write 1 inch = 8 feet)

We just determined the scale when given the dimensions of an actual object and its scale drawing. Well done!

**Let's Try it (Slides 6 - 7):** Now let's practice! We'll do a few more problems together before you get a chance to work independently. As we saw today, tables can help us keep track of our units and think carefully about the relationship between corresponding sides. We also saw that if we know the relationship between units in one direction, like inches to meters, we can use the inverse or opposite operation to move between units in the other direction, like meters back to inches. Let's keep these things in mind as we try out a few more similar problems.



# WARM WELCOME



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**Today we will determine the scale and the dimensions of a scale drawing when given the actual dimensions of an object.**

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## Let's Talk:

**What do you notice?**

**What do you wonder?**



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## Let's Think:

**A swimming pool measures 50 meters long and 20 meters wide. Find the dimensions of a scale drawing using the given scales.**

a.  $1 \text{ inch} = 2 \text{ meters}$

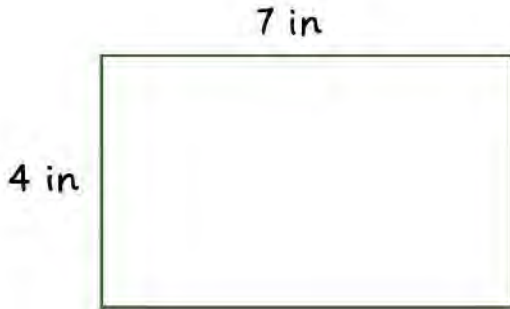
b.  $\frac{1}{2} \text{ inch} = 5 \text{ meters}$

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# Let's Think:

The actual dimensions of a playground are 32 feet by 56 feet. A scale drawing of the playground is shown here. What is the scale?



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# Let's Try It:

Let's explore determining the scale and the dimensions of a scale drawing when given the actual dimensions of an object together.

Name: \_\_\_\_\_ G7/U1 Lesson 7 - Let's Try It

Mr. Roth makes a scale drawing of the volleyball court shown here. Notice the dimensions.

- Mr. Roth makes a scale of 1 inch = 2 meters. What does that scale mean?  
\_\_\_\_\_
- Label the dimensions on Mr. Roth's scaled drawing.  
\_\_\_\_\_
- Find the area of the volleyball court in Mr. Roth's scaled drawing.  
\_\_\_\_\_

Ms. Kiernan makes a different scaled drawing of the same volleyball court. She uses a scale of  $\frac{1}{4}$  inch = 2 meters.

- What does Ms. Kiernan's scale mean?  
\_\_\_\_\_
- What are the dimensions of Ms. Kiernan's scaled drawing?  
\_\_\_\_\_
- Find the perimeter of the volleyball court in Ms. Kiernan's drawing.  
\_\_\_\_\_

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A museum brochure has an image of a piece of artwork. The image is 3 inches wide and 5 inches tall. The actual piece of artwork is 25 inches tall.

- What dimension in the brochure corresponds with the actual height of the artwork, 75 inches?  
\_\_\_\_\_
- What scale is being used for the image in the brochure?  
\_\_\_\_\_
- Use the scale to determine the actual width of the piece of artwork.  
\_\_\_\_\_

A rectangular conference table measures 24 feet long and 12 feet wide. Consider the two different scale drawings of the conference table.

- What scale was used to create Scale Drawing A?  
\_\_\_\_\_
- What scale was used to create Scale Drawing B?  
\_\_\_\_\_
- Draw and label another scale drawing of the conference table using a different scale.  
\_\_\_\_\_

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# On your Own:

Now it's time to determine the scale and the dimensions of a scale drawing when given the actual dimensions of an object on your own.


Name: \_\_\_\_\_ G7 U1 Lesson 7 - Independent Work

1. Zscai made a scale drawing of the soccer field at his school. He used the scale 1 cm = 10 feet.

What are the dimensions of Zscai's scale drawing?



What is the area of the scaled drawing?




2. Adrienne is opening up a restaurant with a large rectangular patio that measures 48 meters long and 56 meters wide. The blueprint for the patio is a scaled drawing that uses a scale of 1 inch = 4 meters.

What is the area of the patio on the blueprint? Include a drawing as part of your response.

3. A farmer plants cabbage in a rectangular field that has a length of 30 yards and a width of 24 yards. The farmer makes a scale drawing using a scale of  $\frac{1}{3}$  inch = 3 yards.

Label the dimensions on the farmer's scale drawing, then find the perimeter of the field in the scale drawing.



4. Allen drew a scaled drawing of his classroom. His drawing is 9 inches wide and 6 inches long. The actual length of the classroom is 30 feet.

**PART A:** What scale is being used for Allen's drawing? Explain how you know.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**PART B:** What is the actual width of Allen's classroom?

\_\_\_\_\_

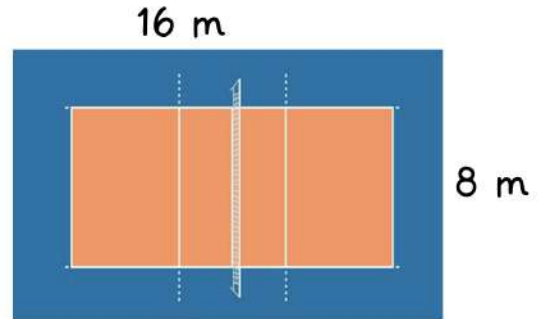
\_\_\_\_\_

\_\_\_\_\_

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Name: \_\_\_\_\_

**Mr. Roth makes a scale drawing of the volleyball court shown here. Notice the dimensions.**



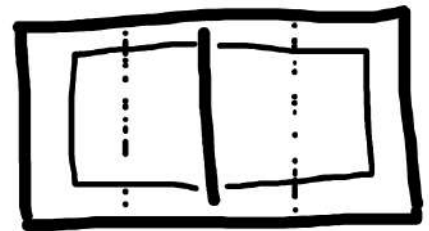
1. Mr. Roth makes a scale of 1 inch = 2 meters. What does that scale mean?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

2. Label the dimensions on Mr. Roth's scaled drawing.
3. Find the area of the volleyball court in Mr. Roth's scaled drawing.



**Ms. Kiernan makes a different scaled drawing of the same volleyball court. She uses a scale of  $\frac{1}{2}$  inch = 2 meters.**

4. What does Ms. Kiernan's scale mean?

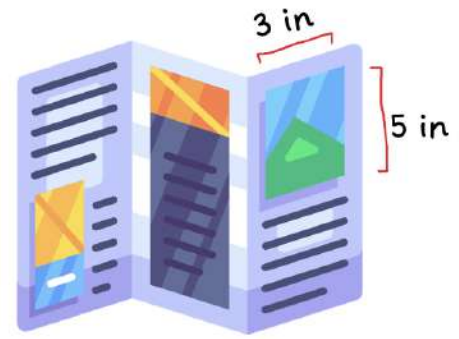
\_\_\_\_\_

\_\_\_\_\_

5. What are the dimensions of Ms. Kiernan's scaled drawing?
6. Find the perimeter of the volleyball court in Ms. Kiernan's drawing.



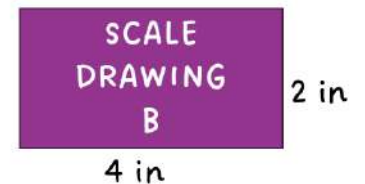
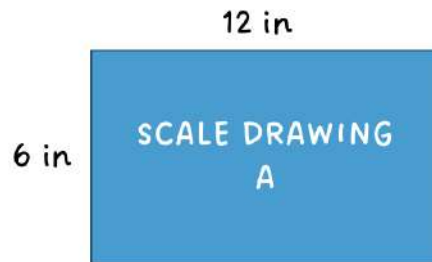
A museum brochure has an image of a piece of artwork. The image is 3 inches wide and 5 inches tall. The actual piece of artwork is 75 inches tall.



7. What dimension in the brochure corresponds with the actual height of the artwork, 75 inches?
8. What scale is being used for the image in the brochure?
9. Use the scale to determine the actual width of the piece of artwork.

A rectangular conference table measures 24 feet long and 12 feet wide. Consider the two different scale drawings of the conference table.

10. What scale was used to create Scale Drawing A?

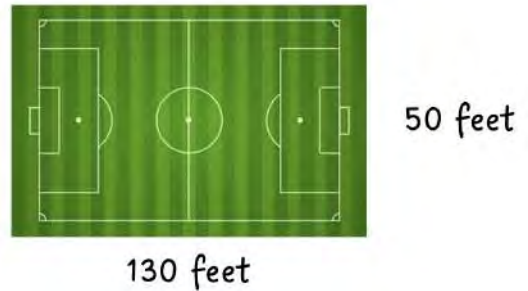


11. What scale was used to create Scale Drawing B?

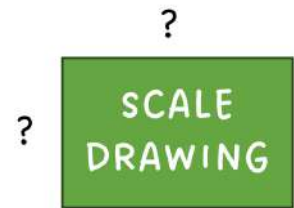
12. Draw and label another scale drawing of the conference table using a different scale.

1. Zacai made a scale drawing of the soccer field at his school. He used the scale  $1 \text{ cm} = 10 \text{ feet}$ .

What are the dimensions of Zacai's scale drawing?



What is the area of the scaled drawing?

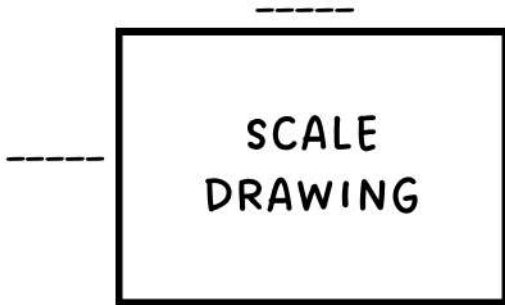


2. Adrienne is opening up a restaurant with a large rectangular patio that measures 48 meters long and 56 meters wide. The blueprint for the patio is a scaled drawing that uses a scale of  $1 \text{ inch} = 4 \text{ meters}$ .

What is the area of the patio on the blueprint? Include a drawing as part of your response.

3. A farmer plants cabbage in a rectangular field that has a length of 30 yards and a width of 24 yards. The farmer makes a scale drawing using a scale of  $\frac{1}{2}$  inch = 3 yards.

Label the dimensions on the farmer's scale drawing, then find the perimeter of the field in the scale drawing.



4. Allen drew a scaled drawing of his classroom. His drawing is 9 inches wide and 6 inches long. The actual length of the classroom is 30 feet.

**PART A:** What scale is being used for Allen's drawing? Explain how you know.

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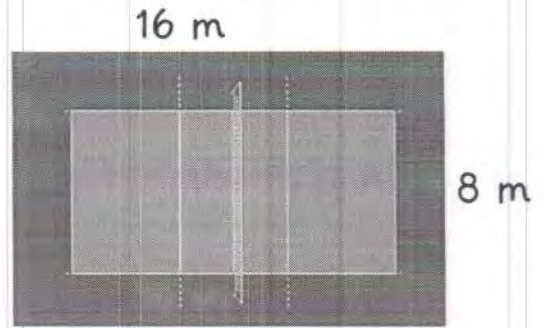
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**PART B:** What is the actual width of Allen's classroom?

Name: KEY

Mr. Roth makes a scale drawing of the volleyball court shown here. Notice the dimensions.



1. Mr. Roth makes a scale of 1 inch = 2 meters. What does that scale mean?

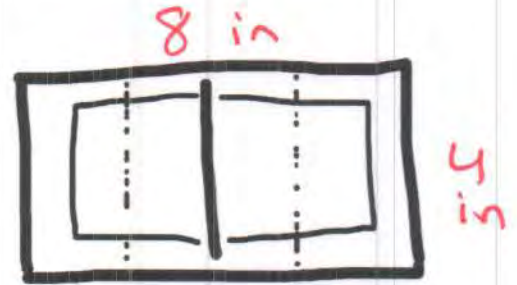
Every 1 inch of the scale drawing represents 2 meters on the actual court.

2. Label the dimensions on Mr. Roth's scaled drawing.

$$16 \div 2 = 8 \quad 8 \div 2 = 4$$

3. Find the area of the volleyball court in Mr. Roth's scaled drawing.

$$8 \times 4 = 32 \text{ in}^2$$



Ms. Kiernan makes a different scaled drawing of the same volleyball court. She uses a scale of  $\frac{1}{2}$  inch = 2 meters.

4. What does Ms. Kiernan's scale mean?

Every half inch in the drawing represents 2 meters in real life. Also, 1 inch = 4 m.

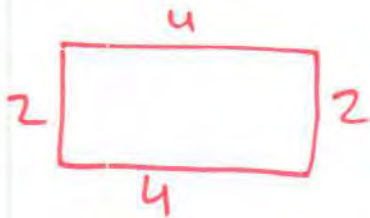
5. What are the dimensions of Ms. Kiernan's scaled drawing?

$$16 \div 4 = 4$$

$$8 \div 4 = 2$$

4 in by 2 in

6. Find the perimeter of the volleyball court in Ms. Kiernan's drawing.



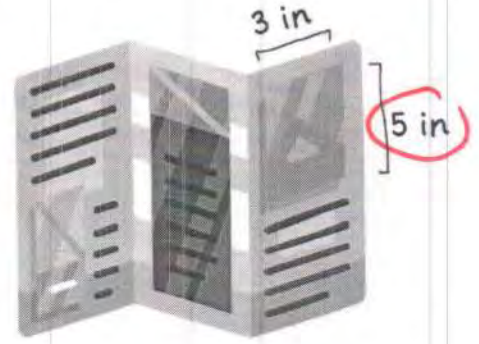
$$4 + 4 + 2 + 2$$

$$\begin{array}{c} \vee \quad \vee \\ 8 \quad 4 \\ \vee \\ 12 \end{array}$$

12 in



A museum brochure has an image of a piece of artwork. The image is 3 inches wide and 5 inches tall. The actual piece of artwork is ~~25~~ 75 inches tall.



7. What dimension in the brochure corresponds with the actual height of the artwork, 75 inches?

5 in

8. What scale is being used for the image in the brochure?

$$5 \times ? = 75$$

OR

$$75 \div 5 = ?$$

$$? = 15$$

$$1 \text{ in} = 15 \text{ in}$$

9. Use the scale to determine the actual width of the piece of artwork.

$$3 \times 15 = 45 \text{ in}$$

A rectangular conference table measures 24 feet long and 12 feet wide. Consider the two different scale drawings of the conference table.

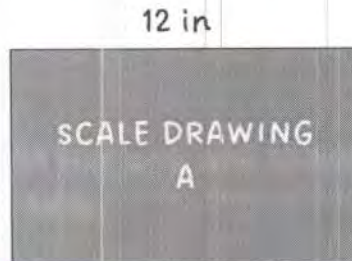
10. What scale was used to create Scale Drawing A?

$$24 \times \frac{1}{2} = 12$$

$$12 \times \frac{1}{2} = 6$$

$$1 \text{ in} = 2 \text{ ft}$$

6 in



SCALE DRAWING B

2 in

4 in

11. What scale was used to create Scale Drawing B?

$$1 \text{ in} = 6 \text{ ft}$$

$$4 \times 6 = 24$$

$$2 \times 6 = 12$$

12. Draw and label another scale drawing of the conference table using a different scale.

$$1 \text{ in} = 3 \text{ ft}$$





1. Zacai made a scale drawing of the soccer field at his school. He used the scale 1 cm = 10 feet.

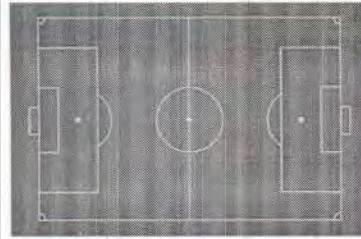
What are the dimensions of Zacai's scale drawing?

$$? \times 10 = 50$$

$$5 \text{ cm}$$

$$? \times 10 = 130$$

$$13 \text{ cm}$$



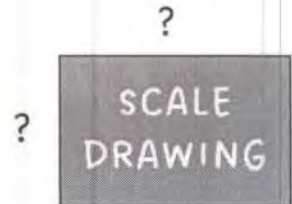
50 feet

130 feet

What is the area of the scaled drawing?

$$5 \times 13 = ?$$

$$65 \text{ cm}^2$$



2. Adrienne is opening up a restaurant with a large rectangular patio that measures 48 meters long and 56 meters wide. The blueprint for the patio is a scaled drawing that uses a scale of 1 inch = 4 meters.

What is the area of the patio on the blueprint? Include a drawing as part of your response.



$$56 \div 4 = 14$$

$$48 \div 4 = 12$$



$$14 \times 12 = ?$$

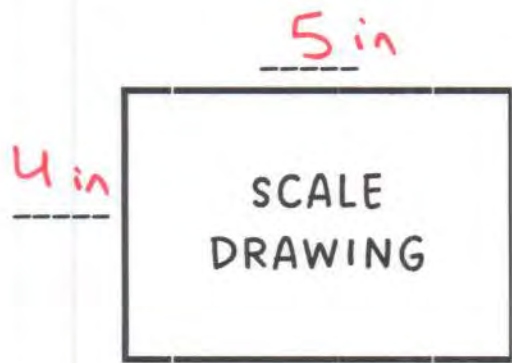
$$(14 \times 10) + (14 \times 2)$$

$$140 + 28$$

$$168 \text{ in}^2$$

3. A farmer plants cabbage in a rectangular field that has a length of 30 yards and a width of 24 yards. The farmer makes a scale drawing using a scale of  $\frac{1}{2}$  inch = 3 yards.

Label the dimensions on the farmer's scale drawing, then find the perimeter of the field in the scale drawing.



$1 \text{ inch} = 6 \text{ yards}$

$30 \div 6 = 5$

$24 \div 6 = 4$

$5 + 5 + 4 + 4 = 18 \text{ in}$

4. Allen drew a scaled drawing of his classroom. His drawing is 9 inches wide and 6 inches long. The actual length of the classroom is 30 feet.

**PART A:** What scale is being used for Allen's drawing? Explain how you know.

He used a scale of 1 inch = 5 feet.

I know because 30 divided by 6 is 5. So an inch on the drawing represents 5 feet in real life.

**PART B:** What is the actual width of Allen's classroom?

$9 \times 5 = 45$

$45 \text{ ft}$

## **G7 U1 Lesson 8**

Reproduce a scale drawing at a different scale and determine how much actual area is represented by one square unit in a scale drawing

## G7 U1 Lesson 8 - Students will reproduce a scale drawing at a different scale and determine how much actual area is represented by one square unit in a scale drawing

**Warm Welcome (Slide 1):** Tutor choice

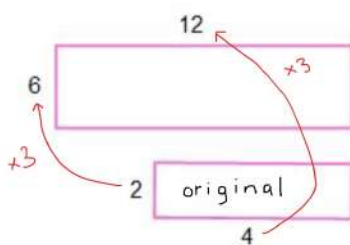
**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we're going to keep up our work with scale drawings. When you think about working with scale factor and scale drawings, what comes to mind for you? What have you learned so far that feels important or memorable? **Possible Student Answers, Key Points:**

- A scaled copy's dimensions are related to a figure's dimensions in a consistent way.
- When we scale something, we make it bigger or smaller, but we don't distort it in any other way.
- Sometimes scale drawings can involve different units, like 1 inch can represent 2 kilometers.
- A scale factor is a factor we can multiply by to make a scale drawing or model. The scale factor must be the same for all dimensions.
- Maps, blueprints, and floorplans are all examples of scale drawings.

We've learned a lot! We'll keep thinking about all those ideas today, and we'll draw special attention to the area of our scale drawings compared to the area of the actual object.

**Let's Talk (Slide 3):** Here we see two rectangles. What do you notice about how the sides are related? **Possible Student Answers, Key Points:**

- The scale factor is 3.
- The sides of the scale copy are 3 times the length of the sides in the original.



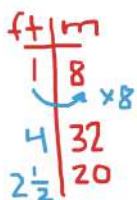
The sides in the original can be multiplied by a scale factor of 3, and the resulting figure will be the scale copy we see here. *(draw arrows labeled with "x 3" from each side of the original to its corresponding side)*

Now think about the area. What is the area of each rectangle? How are the area's related? **Possible Student Answers, Key Points:**

- The area of the original rectangle is 8 square units, because  $2 \times 4 = 8$ . The area of the scale copy is 72 square units, because  $6 \times 12 = 72$ .
- The area of the scale copy is significantly greater than the area of the original rectangle. It's 9 times bigger.

The area of the original rectangle is 8. The area of the scale copy is 72. We noticed the sides of the scale copy were 3 times bigger than the sides of the original. Is the area 3 times bigger than the original? **(No.)** No, the area of the scale copy is 9 times bigger than the area of the original. Since both the length and the width are multiplied by 3, the area is multiplied by 3 squared, or 9. 8 times 3 squared, or 8 times 9, equals 72. The area of the scale copy is 9 times the area of the original. Let's try out a few problems involving area of scale drawings.

**Let's Think (Slide 4):** Here we have an image that shows the actual dimensions of a parking lot. We're trying to figure out the dimensions of a scale model if we use a scale where 1 foot represents 8 meters. *(sketch a t-chart with columns representing feet and meters, and put 1 and 8 in their respective columns)*



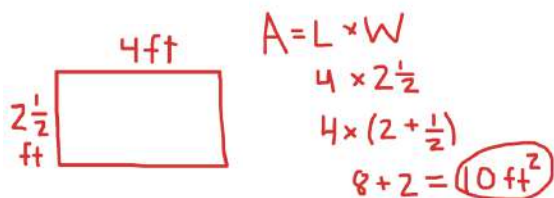
We can multiply the number of feet by 8 to find the number of meters. How can we use that relationship to find how many feet represent 32 meters and 20 meters? *(write 32 and 20 in the meters column)* **Possible Student Answers, Key Points:**

- We can multiply 32 and 20 by  $\frac{1}{8}$ , since  $\frac{1}{8}$  is the multiplicative inverse of 8.
- We can divide 32 and 20 by 8, since division is the opposite of multiplication.



$$20 \div 8 = \frac{20}{8} = 2\frac{4}{8}$$

32 times  $\frac{1}{8}$ , or divided by 8, is 4. (write 4 in the t-chart) 20 times  $\frac{1}{8}$ , or divided by 8, isn't quite as intuitive since 8 is not a whole number factor of 20. We can still figure it out. (write equation as you narrate) 20 divided by 8 is  $\frac{20}{8}$ . We can rewrite that as a mixed number of  $2\frac{4}{8}$  or  $2\frac{1}{2}$ .

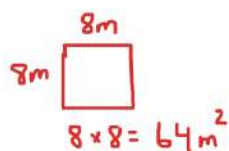


The dimensions of the scale model will be 4 feet long by  $2\frac{1}{2}$  feet wide. (sketch and label a rectangle showing the dimensions of the scale model) Now that we know the side lengths, we can find the area of the scale model. How can we find the area of the model? (Write the multiplication equation as the student shares, even if the steps are different than the example shown here)

#### Possible Student Answers, Key Points:

- We can multiply length times width.
- I know  $4 \times 2\frac{1}{2}$  is like  $4 \times 2$  and  $4 \times \frac{1}{2}$ .  $4 \times 2$  is 8 and  $4 \times \frac{1}{2}$  is 2.  $8 + 2$ , means the area is 10 square units.
- I can multiply 4 times  $\frac{5}{2}$ . 4 times  $\frac{5}{2}$  is  $\frac{20}{2}$ , which is equivalent to 10.

The area is of the model 10 square feet. Now, we can think about the second prompt. The question wants to know how many square meters are represented by 1 square foot. We can think of this in two different ways.



One way is to picture 1 square foot. (sketch a square) This square is 1 square foot. The scale tells me that each foot represents 8 meters, so instead of labeling this square foot with side lengths of 1 foot, I'll just label the side lengths 8 meters. (label 8 meters on each side of the square) What is the area of this square foot when we think about it in terms of meters? How do you know? Possible Student Answers, Key Points:

- The area is 64 square meters. I can multiply the length times the width.
- I know the area, because 8 meters times 8 meters is 64 square meters.

So, 1 square foot in the model represents 64 square meters in the actual parking lot.

The other way we can think about this is by comparing the areas of the scale model to the actual parking lot. We found the area of the scale model to be 10 square feet. Look at the image of the actual parking lot's dimensions. What would be the area of the actual parking lot? Possible Student Answers, Key Points:

- It would be 640 square meters. I can multiply the length times the width.
- 32 times 20 equals 640.

$$10 \times \underline{\quad} = 640$$

If the scale model's area is 10 square inches, and the actual parking lot's area is 640 square meters. I can think: 10 times what number gets me to 640? (write  $10 \times \underline{\quad} = 640$ ) I know the answer is 64. So, each square foot of the model represents 64 square meters of the actual parking lot.

Either strategy can help us think about how much 1 square unit of a model represents in real life. We can visualize 1 square unit and actually calculate the area, or we can think about the relationship between the area of the scale drawing and the actual area.

**Let's Think (Slide 5):** Let's look at one more problem involving scale drawings and area. In this problem, Jayla is using a table to keep track of the dimensions of a national park and her scale drawing of the national park. Take a moment to look closely at the information in the table. (pause) What information do you see in the table? Possible Student Answers, Key Points:



- I notice there are widths and lengths. I notice Jayla calculated the area. I notice the scale is missing in the second row.

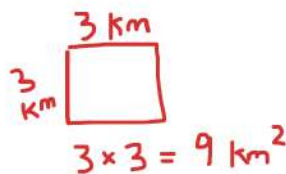
Let's use the table to determine the scale Jayla used for her drawing. How are the dimensions of the actual national park related to the dimensions of the drawing? **Possible Student Answers, Key Points:**

- The number of centimeters is  $\frac{1}{3}$  the number of kilometers.
- The number of kilometers is 3 times the number of centimeters.

	WIDTH	LENGTH
cm	15 km	12 km
	5 cm	4 cm

$$1 \text{ cm} = 3 \text{ km}$$

(draw arrows between corresponding measurements and label with  $\times \frac{1}{3}$ ) The table shows us that the number of centimeters in the drawing is  $\frac{1}{3}$  the number of kilometers. Or we can think of it as the number of kilometers is three times the number of centimeters. The scale Jayla uses is 1 centimeter = 3 kilometers. (write  $1 \text{ cm} = 3 \text{ km}$ )



Now we'll answer part B. It wants us to find how many square kilometers are represented by 1 square centimeter. Let's do this both ways we've learned. We'll start by visualizing 1 square centimeter. (sketch a square) Instead of labeling each side as being 1 cm, I'll label each side as being 3 km, since the scale tells me I can represent 1 cm as 3 km. (label square) What's the area of this square? (9 square kilometers) One square centimeter on the scale drawing represents 9 square kilometers in real life.

	WIDTH	LENGTH	AREA
	15 km	12 km	180 sq. km
	5 cm	4 cm	20 sq. cm

We can also think about this by comparing the areas of the scale drawing and the actual park. The area of the drawing is 20 square centimeters. The area of the actual park is 180 square kilometers. I can think 20 times what number is equal to 180? (draw arrow from 20 to 180) 20 times 9 is equal to 180, so I know each square centimeter represents 9 square kilometers.

**Let's Try it (Slides 6 - 7):** We just solved a number of problems involving scale drawings with a particular focus on area of scale figures. We saw that the area isn't related in the same way as the side lengths. For example, if I scale length by a factor of 2 and width by a factor of 2, the area will scale by a factor of 4. We also explored two different ways to consider how much area is represented by 1 square unit on a scale drawing. We can visualize 1 square unit using the given scale, or we can compare the area of the scale image to the area of the actual image. Let's try a few more problems to practice some of this thinking. When we wrap up, you'll get a chance to try some out independently.

# WARM WELCOME



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**Today we will reproduce a scale drawing at a different scale and determine how much actual area is represented by one square unit within a scale drawing.**

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## Let's Talk:

How do the side lengths compare?

How do the areas compare?

12

6



2

original



4

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## Let's Think:

The dimensions of a parking lot are shown.

What is the area of a scale model of the parking lot using a scale of 1 foot = 8 meters?

32 m

20 m



20 m

32 m

How many square meters are represented by 1 square foot?

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## Let's Think:

Jayla made a scale drawing of a national park that is actually 15 km wide and 12 km long. She recorded dimensions in the table below.

SCALE	WIDTH	LENGTH	AREA
1 km = 1 km	15 km	12 km	180 sq. km
	5 cm	4 cm	20 sq. cm

- What scale did Jayla use?
- How many square kilometers are represented by 1 square centimeter?

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## Let's Try It:

Let's explore reproducing a scale drawing at a different scale and determining how much actual area is represented by one square unit in a scale drawing together.

Name: \_\_\_\_\_ G7 U1 Lesson 8 - Let's Try It

A city park measures 54 meters long and 18 meters wide. A scale drawing is made using a scale of 1 in = 3 meters.

- Sketch a rectangle to represent the scaled drawing.
- Use the scale to find and label the length on your scaled drawing.
- Use the scale to find and label the width on your scaled drawing.
- Use a scale of 1 in = 6 meters to sketch and label a different scaled drawing of the city park.
- The smaller the number of meters represented by 1 inch, the \_\_\_\_\_ the scale drawing is.
  - smaller
  - larger
- Use your two scaled drawings to complete the columns for scale, width, and length. The information for the actual city park is already provided.
 

SCALE	WIDTH	LENGTH	AREA
1 m = 1 m	18 m	54 m	972 sq. m
- Find the area of each scaled drawing. Record the information in the table.

- Using the first scaled drawing, how many square meters are represented by 1 square inch? Use the equation below to help arrive at your answer.
 
$$108 \times \underline{\hspace{1cm}} = 972$$
 1 square inch = \_\_\_\_\_ square meters.
- Using the second scaled drawing, how many square meters are represented by 1 square inch? Use the equation below to help arrive at your answer.
 
$$27 \times \underline{\hspace{1cm}} = 972$$
 1 square inch = \_\_\_\_\_ square meters.

Jeremiah made a scale drawing of his backyard. Look at the information provided in the table.

SCALE	WIDTH	LENGTH	AREA
1 ft = 1 ft	20 ft	8 ft	160 sq. ft
	4 in.	1 in.	4 sq. in.

- Find the scale used for Jeremiah's scale drawing.
- How many square feet are represented by 1 square inch?
- Describe how the scale impacts the area of a scaled drawing.

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# On your Own:

Now it's time to reproduce a scale drawing at a different scale and determine how much actual area is represented by one square unit in a scale drawing on your own.

Name: \_\_\_\_\_ G7 U1 Lesson 6 – Independent Work

1. Chelsea made a scale drawing of a poster that has a scale of 1 centimeter = 4 inches.

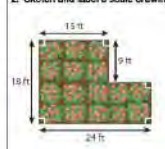


24 in  
36 in

a. Label the dimensions of Chelsea's scale drawing.

b. What is the area of Chelsea's scale drawing?

2. Sketch and label a scale drawing of the rose garden using a scale of 1 inch = 3 feet.



15 ft  
10 ft  
24 ft  
9 ft

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3. A banquet hall measures 60 feet by 30 feet.

a. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 10 feet?

b. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 2 feet?

c. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 15 feet?

d. Which of the three scales produces the scaled drawing with the greatest area?

4. Michelle made a scale drawing of her garden.

SCALE	WIDTH	LENGTH	AREA
1 ft = 1 ft	24 ft	18 ft	432 sq. ft
?	6 cm	6 cm	48 sq. cm

Find the scale she used for the scale drawing of her garden based on the information provided.

How many square feet are represented by 1 square centimeter?

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8. Using the first scaled drawing, how many square meters are represented by 1 square inch? Use the equation below to help arrive at your answer.

$$108 \times \underline{\hspace{2cm}} = 972$$

$$1 \text{ square inch} = \underline{\hspace{2cm}} \text{ square meters}$$

9. Using the second scaled drawing, how many square meters are represented by 1 square inch? Use the equation below to help arrive at your answer.

$$27 \times \underline{\hspace{2cm}} = 972$$

$$1 \text{ square inch} = \underline{\hspace{2cm}} \text{ square meters}$$

**Jeremiah made a scale drawing of his backyard. Look at the information provided in the table.**

SCALE	WIDTH	LENGTH	AREA
1 ft = 1 ft	20 ft	5 ft	100 sq. ft
	4 in	1 in	4 sq. in

10. Find the scale used for Jeremiah's scale drawing.
11. How many square feet are represented by 1 square inch?
12. Describe how the scale impacts the area of a scaled drawing.

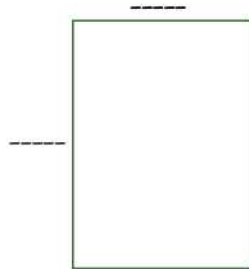
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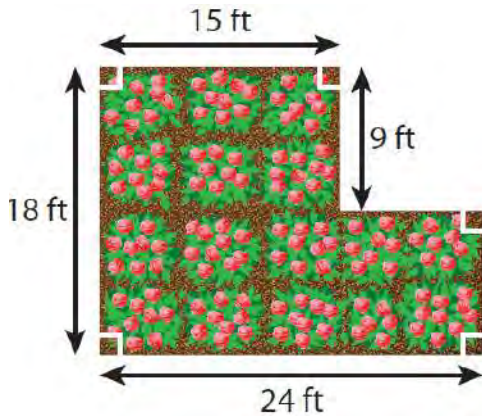
1. Chelsea made a scale drawing of a poster that has a scale of 1 centimeter = 4 inches.



a. Label the dimensions of Chelsea's scale drawing.

b. What is the area of Chelsea's scale drawing?

2. Sketch and label a scale drawing of the rose garden using a scale of 1 inch = 3 feet.



**3. A banquet hall measures 60 feet by 30 feet.**

- a. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 10 feet?
  
- b. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 2 feet?
  
- c. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 15 feet?
  
- d. Which of the three scales produces the scaled drawing with the greatest area?

**4. Michelle made a scale drawing of her garden.**

SCALE	WIDTH	LENGTH	AREA
1 ft = 1 ft	24 ft	18 ft	432 sq. ft
?	8 cm	6 cm	48 sq. cm

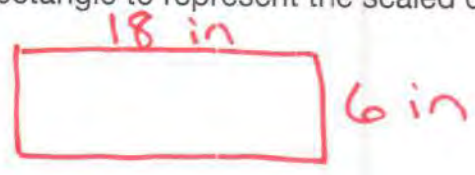
Find the scale she used for the scale drawing of her garden based on the information provided.

How many square feet are represented by 1 square centimeter?

Name: KEY

A city park measures 54 meters long and 18 meters wide. A scale drawing is made using a scale of 1 in = 3 meters.

1. Sketch a rectangle to represent the scaled drawing.



2. Use the scale to find and label the length on your scaled drawing.

$$54 \div 3 = 18$$

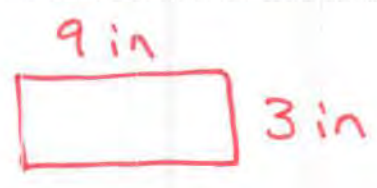
$$18 \div 3 = 6$$

3. Use the scale to find and label the width on your scaled drawing.

4. Use a scale of 1 in = 6 meters to sketch and label a different scaled drawing of the city park.

$$54 \div 6 = 9$$

$$18 \div 6 = 3$$



5. The smaller the number of meters represented by 1 inch, the \_\_\_\_\_ the scale drawing is.

- a. smaller
- b. larger

6. Use your two scaled drawings to complete the columns for scale, width, and length. The information for the actual city park is already provided.

SCALE	WIDTH	LENGTH	AREA
1 m = 1 m	18 m	54 m	972 sq. m
1 in = 3 m	6 in	18 in	108 in <sup>2</sup>
1 in = 6 m	3 in	9 in	27 in <sup>2</sup>

7. Find the area of each scaled drawing. Record the information in the table.

$$6 \times 18 = 108$$

$$3 \times 9 = 27$$



8. Using the first scaled drawing, how many square meters are represented by 1 square inch? Use the equation below to help arrive at your answer.

$$108 \times \underline{9} = 972$$

$$1 \text{ square inch} = \underline{9} \text{ square meters}$$

9. Using the second scaled drawing, how many square meters are represented by 1 square inch? Use the equation below to help arrive at your answer.

$$27 \times \underline{36} = 972$$

$$1 \text{ square inch} = \underline{36} \text{ square meters}$$

Jeremiah made a scale drawing of his backyard. Look at the information provided in the table.

SCALE	WIDTH	LENGTH	AREA
1 ft = 1 ft	20 ft	5 ft	100 sq. ft
1 in = 5 ft	4 in	1 in	4 sq. in

10. Find the scale used for Jeremiah's scale drawing.

$$1 \text{ in} = 5 \text{ ft}$$

11. How many square feet are represented by 1 square inch?

$$\underline{25 \text{ ft}^2}$$

$$4 \times \underline{25} = 100$$

12. Describe how the scale impacts the area of a scaled drawing.

In this case, ~~everywhere~~ the length is multiplied by 5 and the width is multiplied by 5. This means the area is scaled by  $5^2$  or 25.

1. Chelsea made a scale drawing of a poster that has a scale of 1 centimeter = 4 inches.



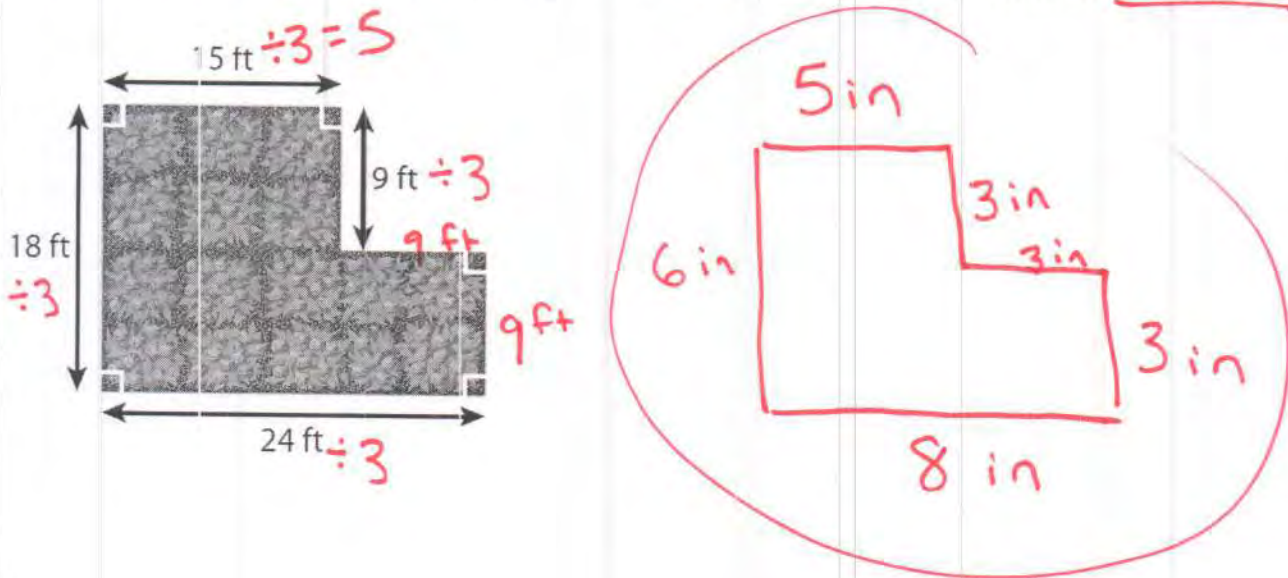
a. Label the dimensions of Chelsea's scale drawing.

b. What is the area of Chelsea's scale drawing?

$$6 \times 9 = 54$$

$$(54 \text{ cm}^2)$$

2. Sketch and label a scale drawing of the rose garden using a scale of 1 inch = 3 feet.





3. A banquet hall measures 60 feet by 30 feet.

a. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 10 feet?

$$60 \div 10 = 6$$

$$30 \div 10 = 3$$

$$6 \text{ in} \times 3 \text{ in}$$

b. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 2 feet?

$$60 \div 2 = 30$$

$$30 \div 2 = 15$$

$$30 \text{ in} \times 15 \text{ in}$$

c. What are the dimensions of a scale drawing of the banquet hall if the scale used is 1 inch = 15 feet?

$$60 \div 15 = 4$$

$$30 \div 15 = 2$$

$$4 \text{ in} \times 2 \text{ in}$$

d. Which of the three scales produces the scaled drawing with the greatest area?

The drawing using a scale of 1 inch = 2 feet!

$$6 \times 3 = 18$$

$$30 \times 15 = 450 \checkmark$$

$$4 \times 2 = 8$$

4. Michelle made a scale drawing of her garden.

SCALE	WIDTH	LENGTH	AREA
1 ft = 1 ft	24 ft	18 ft	432 sq. ft
?	8 cm	6 cm	48 sq. cm

Find the scale she used for the scale drawing of her garden based on the information provided.

$$1 \text{ cm} = 3 \text{ ft}$$

How many square feet are represented by 1 square centimeter?

$$48 \times ? = 432$$

$$9 \text{ ft}^2$$

# **G7 U1 Lesson 9**

Explain how to use scales without units to determine scaled or actual distances.

## G7 U1 Lesson 9 - Students will explain how to use scales without units to determine scaled or actual distances

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We've almost reached the end of our unit that's all about scale. Our work today will have us think about how we can use scales with units and scales without units to think about scaled or actual distances. Let's jump right in

**Let's Talk (Slide 3):** Here we see an image of a giraffe. Take a second to look over the provided information. What do you notice? What do you wonder? **Possible Student Answers, Key Points:**

- I notice the giraffe is 18 units tall. I notice the scale is  $1 = 10$ .
- I wonder how tall the giraffe actually is. I wonder what units these numbers represent.

This scale does not have units. Not all scales are expressed using units. Since the scale is  $1 = 24$  and does not include units, we can assume that the actual giraffe is 24 times as tall as the scale image we're provided. Let's look at some examples of problems that include scales without units.

**Let's Think (Slide 4):** Here we see a scale image of parade float. The height of the house on the float's model is labeled as 5 inches. Does the provided scale include units? What do you think the scale mean in this context? **Possible Student Answers, Key Points:**

- No, this scale does not include units.
- The scale is  $1 = 24$ , so the parade float is 24 times bigger than the image we're provided.

$$5 \times 24 = ?$$
$$120 \text{ in.}$$

We can use the scale to find the actual height of the house. Since the scale is not expressed in units, we can assume that the height we find will also be in inches. We can multiply 5 by 24, since we know the height of the actual house will be 24 times the size of the house in the drawing. (*write  $5 \times 24 = ?$* ) Take a second and calculate. What is  $5 \times 24$ ? (120) The height of the house on the parade float will be 120 inches. The unit stays constant since the scale did not tell us to use a different unit.

Now that we have the height of the house in inches, we can find the height of the house in feet? What math can I do to convert 120 inches into feet? **Possible Student Answers, Key Points:**

- I know 1 foot is 12 inches. I can think about 120 divided by 12.
- I can use multiplication and think about how many groups of 12 inches will get me up to 120 inches.

$$120 \div 12 = ?$$
$$12 \times ? = 120$$
$$\boxed{10 \text{ ft}}$$

There are 12 inches in 1 foot. I can divide 120 by 12 to find the answer in feet, or I can think about 12 times *what* equals 120. (*write equations as shown as you narrate*) No matter how we think about it, the height of the house on the actual float will be 10 feet. (*write 10 feet*)

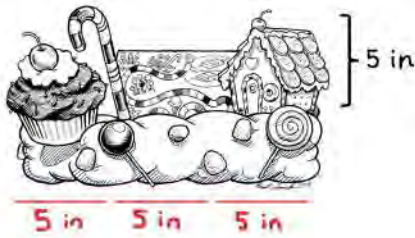
Let's move on to Part B. They want us to estimate the entire length of the float in inches. Is the length of the float labeled? (No.) If the height of the house portion of the float is 5 inches on the scale image, how can I use that to estimate the length of the scale image? **Possible Student Answers, Key Points:**

- I can use my fingers to show the length of the 5-inch bracket, and then see how many of that distance goes across the length of the float.
- It looks like the float is maybe 3 or 4 times as long as the house is tall based on the size of the bracket.

We can iterate the length of the 5-inch bracket to get a general sense of how long the image of the float is. Once we figure that out, we can use the scale to determine how long the float will be in real life. Let's do that now.



(sketch lines that are about as long as the 5-inch bracket along the bottom of the image, labeling each as 5 inches)



I can see that the float is about 3 brackets long, or about 3 groups of 5 inches. Based on that, how long can we estimate the image to be? (15 inches) If the length of the scale image of the float is about 15 inches, how can I calculate the actual length of the float? Possible Student Answers, Key Points:

- We can multiply 15 times 24, since each inch on the image is equal to 24 inches on the actual float.
- I know  $15 \times 24 = 360$ .

$$15 \times 24 = ?$$

$$360 \text{ in.}$$

Each inch in the image is equal to 24 inches. 15 times 24 is 360, so I know the actual length of the float is 360 inches. (write  $15 \times 24 = 360$ )

**Let's Think (Slide 5):** Let's try one more problem. (read through the problem once) Both Frank and Kierra are correct, but they took different approaches. What do you think is different about the scales each person used?

Possible Student Answers, Key Points:

- I see that Frank used a scale of 1 inch to 2 feet. His scale includes units.
- I notice Kierra's scale is  $1 = 24$ , which does not include units.

(F)

in.	ft.
1	2
?	20

10 in. ✓

The question asks us to figure out how they can both end up with a correct length of 10 inches, even though they used seemingly different scales. Let's start by thinking about Frank's approach. I'll put Frank's scale into a t-chart. (sketch a t-chart labeled with inches and feet, and put 1 and 2 in their respective columns) Based on Frank's scale, I know that at the number of feet will be two times the number of inches. (draw arrow from 1 to 2 showing "x 2") How do you think Frank used that information to arrive at a scale drawing length of 10 inches? Possible Student Answers, Key Points:

- He could take 20 and divide it by 2. Or he could take 20 and multiply it by  $\frac{1}{2}$ .
- He could think about what number times 10 would equal 20.

If Frank knows that the number of feet is twice the number of inches, he knows that the number of inches that are needed to represent 20 feet would be 10 inches.

in.	ft.
1	24
?	20

Let's think about what would happen if Kierra took the same approach. (draw a t-chart labeled with inches and feet, putting 1 in the inches column and 24 in the feet column) This wouldn't work. The relationship here would mean that the number of feet is 24 times the number of inches in the scale drawing. Is Kierra going to get an answer of 10? (No.) 10 times 24 would not be 20. (cross out the t-chart) Kierra must have thought about the problem in a different way.

The fact that Kierra's scale does not include units, clues us into the fact that she likely thought about the scale drawing and the actual room in terms of the same unit. I know 2 feet is the same as 24 inches, so this leads me to believe that she converted feet into inches.

If the room is 20 feet long, how long would that be in inches? How do you know? [Possible Student Answers, Key Points:](#)

- I know 1 foot is 12 inches, so I can use multiplication to find how many inches 20 feet would be.
- I know  $20 \times 12 = 240$ , so 20 feet is equivalent to 240 inches.

$20 \times 12 = 240 \text{ in}$  (write  $20 \times 12 = 240 \text{ inches}$ ) We can multiply 20 by 12 inches to see that 20 feet is the same as 240 inches.

in.	in.
1	24
?	240

10 in. ✓

Now, Kierra can use the scale of 1 to 24 to determine the length of the scale drawing. (sketch t-chart showing 1 inch to 24 inches and ? to 240 inches as shown) We can multiply the number of inches in the scale drawing by 24 to find the length of the actual room, so I know we can do the opposite to find the length of the scale drawing. What is 240 divided by 24, or 240 times  $1/24$ ? (10) Kierra also ended up with an answer of 10 inches.

We just walked through what both students likely did to arrive at 10 inches. In your own words, how were they able to arrive at the same answer with different scales? [Possible Student Answers, Key Points:](#)

- Frank thought about how many feet each inch represents. His scale compared inches to feet.
- Kierra converted the length of the room into inches, so it was the same unit as the scale drawing. This meant her scale looked a little different, but did not require units since the units she was comparing were the same.

Both showed a scale of 1 to 24, but in different ways. Frank thought of it as 1 inch to 2 feet, and Kierra thought of it as 1 inch to 24 inches. 2 feet and 24 inches are equivalent, so they were able to arrive at the same scaled length.

**Let's Try it (Slides 6 - 7):** We're becoming scale experts. We've spent several days becoming experts with scales that include units, and today we were able to brush up on our skills with scales that don't include units. When we are comparing objects with similar units, we know that we don't need to use units in our scale. When our objects are measured in different units, it's important that we name the units as part of our scale. Let's try a few more together, and then you'll get a chance to practice on your own.

# WARM WELCOME



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**Today we will explain how to use scales without units to determine scaled or actual distances.**

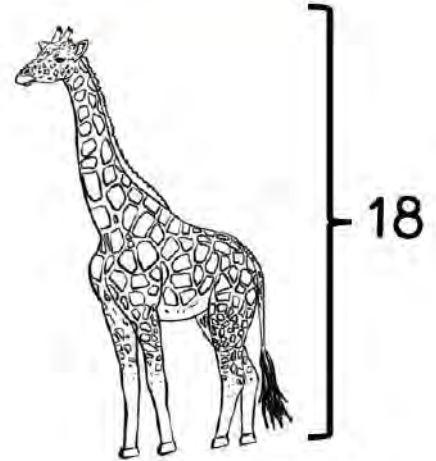
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## Let's Talk:

SCALE  
1 = 10

A scale image of a giraffe is shown.

What do you notice?  
What do you wonder?



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## Let's Think:

SCALE  
1 = 24

The image below shows a scale image of a parade float.

- What's the height of the actual house in inches? Feet?
- Estimate the length of the entire float in inches.



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## Let's Think:

A room measures 20 feet long. Frank and Kierra make scale drawings of the field.

- Frank uses a scale of 1 inch to 2 feet.
- Kierra uses a scale of 1 to 24.

They both end up with a scale drawing that is 10 inches long. How is that possible?


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## Let's Try It:

Let's explore explaining how to use scales without units to determine scaled or actual distances together.

Name: \_\_\_\_\_ G7 UT Lesson 9 - Let's Try It

Consider the scale drawing of a living room. The scale used to make the drawing is 1 = 12.



- Does the scale have units?
  - yes
  - no
- Use the scale to find the length of the couch in inches.
- How long is the couch in feet? Remember, 1 foot is equivalent to 12 inches.
- Based on the image, estimate the width of the fireplace in the scaled drawing.
- Estimate the length of the actual fireplace in inches.
- About how long is the fireplace in feet?

A bus measures 30 feet long.

- Coco uses a scale of 1 inch = 2 feet to make a scale model of the bus. How many inches long is Coco's scale model?
- Wallace uses a scale of 1 inch = 3 feet to make a scale model of the bus. How many inches long is Wallace's model?

Isabel wants to make a scale model of the same bus, but she wants to use a scale factor without units. She starts by converting the length of the bus into inches.

- How many inches are equivalent to 1 foot?
- The bus is 30 feet long. How many inches are equivalent to 30 feet?
- Isabel uses a scale factor of 1 = 15 to make her scale model. How long is her scale model?

Look at each scale below.

1 = 3	1 = 12	1/3 = 1	3 = 1	1/12 = 1
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- How many feet are equivalent to 1 yard? \_\_\_\_\_
- Which scales are equivalent to 1 foot = 1 yard? How do you know?

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## On your Own:

Now it's time to explain how to use scales without units to determine scaled or actual distances on your own.

Name: \_\_\_\_\_ G7 U1 Lesson 9 - Independent Work

1. Which scales are equivalent to 1 centimeter to 1 meter? Select ALL that apply.

- a. 1 to 100
- b.  $\frac{1}{100}$  to 1
- c.  $\frac{1}{10}$  to 1
- d. 10 to 1
- e. 100 to 1

2. A scale model of a playground slide is built at a scale of 1 to 48. If the model slide is 5 inches tall, how tall is the actual slide in inches? Feet?

3. A rectangular parking lot is 40 meters long and 25 meters wide.

- Blake made a scale drawing of the parking lot using a scale of 1 cm to 5 meters.
- Lydia made a scale drawing of the parking lot using a scale of 1 to 500.

They both ended up with a scale drawing that measured 8 centimeters by 5 centimeters. Explain how that is possible.

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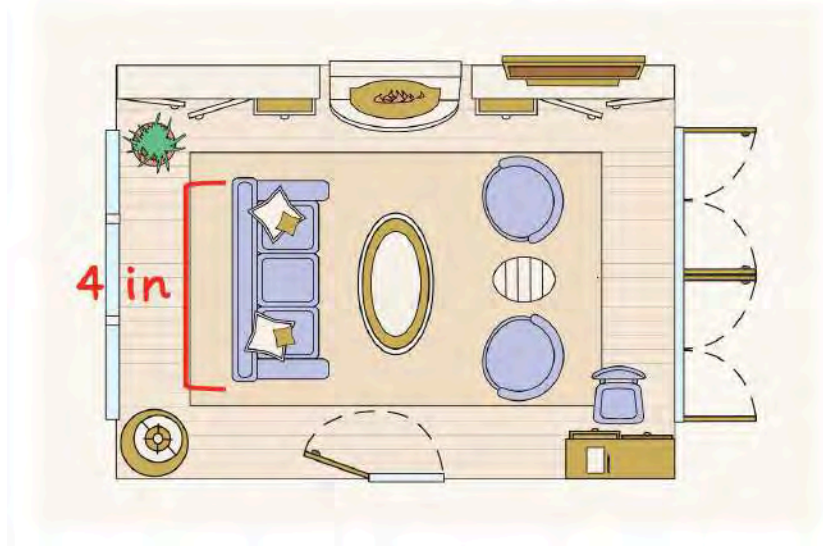
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4. Darryl drew a plan of a swimming pool using a scale of 1 to 80. The length of the swimming pool in his drawing is 1.75 inches. What is the actual measurement of the length of the swimming pool?

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Consider the scale drawing of a living room. The scale used to make the drawing is  $1 = 12$ .

1. Does the scale have units?
  - a. yes
  - b. no
2. Use the scale to find the length of the couch in inches.



3. How long is the couch in feet? Remember, 1 foot is equivalent to 12 inches.
4. Based on the image, estimate the width of the fireplace in the scaled drawing.
5. Estimate the length of the actual fireplace in inches.
6. About how long is the fireplace in feet?

**A bus measures 30 feet long.**

7. Coco uses a scale of 1 inch = 2 feet to make a scale model of the bus. How many inches long is Coco's scale model?
8. Wallace uses a scale of 1 inch = 3 feet to make a scale model of the bus. How many inches long is Wallace's model?

**Isabel wants to make a scale model of the same bus, but she wants to use a scale factor without units. She starts by converting the length of the bus into inches.**

9. How many inches are equivalent to 1 foot?
10. The bus is 30 feet long. How many inches are equivalent to 30 feet?
11. Isabel uses a scale factor of 1 = 15 to make her scale model. How long is her scale model?

**Look at each scale below.**

$$1 = 3$$

$$1 = 12$$

$$1/3 = 1$$

$$3 = 1$$

$$1/12 = 1$$

12. How many feet are equivalent to 1 yard? \_\_\_\_\_
13. Which scales are equivalent to 1 foot = 1 yard? How do you know?

Name: \_\_\_\_\_

**1. Which scales are equivalent to 1 centimeter to 1 meter? Select ALL that apply.**

- a. 1 to 100
- b.  $1/100$  to 1
- c. 1 to 10
- d. 10 to 1
- e. 100 to 1

**2. A scale model of a playground slide is built at a scale of 1 to 48. If the model slide is 5 inches tall, how tall is the actual slide in inches? Feet?**

**3. A rectangular parking lot is 40 meters long and 25 meters wide.**

- Blake made a scale drawing of the parking lot using a scale of 1 cm to 5 meters.
- Layla made a scale drawing of the parking lot using a scale of 1 to 500.

**They both ended up with a scale drawing that measured 8 centimeters by 5 centimeters. Explain how that is possible.**

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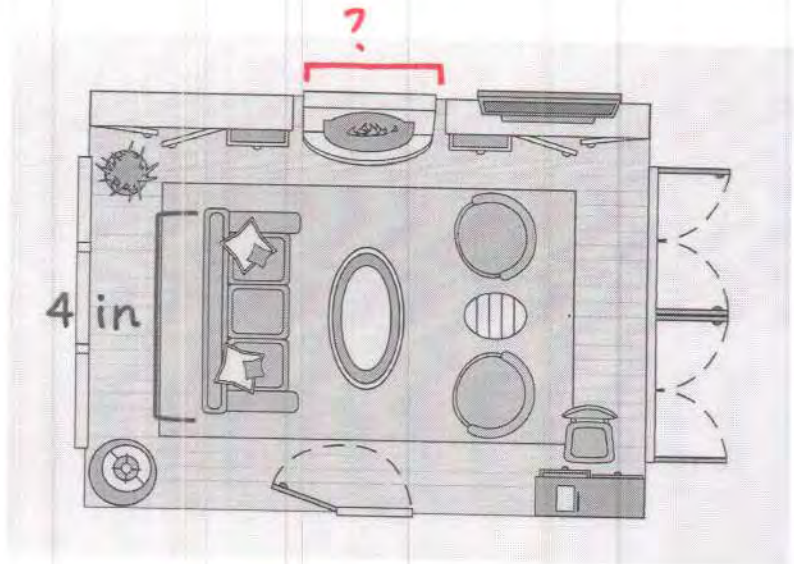
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**4. Darryl drew a plan of a swimming pool using a scale of 1 to 60. The length of the swimming pool in his drawing is 1.75 inches. What is the actual measurement of the length of the swimming pool?**



Name: KEY

Consider the scale drawing of a living room. The scale used to make the drawing is 1 = 12.



1. Does the scale have units?

a. yes

b. no

2. Use the scale to find the length of the couch in inches.

$$\begin{array}{r} 12 \\ \downarrow \\ 4 \end{array} \begin{array}{l} \times 12 \\ ? \\ \times 12 \end{array}$$

48 in

3. How long is the couch in feet? Remember, 1 foot is equivalent to 12 inches.

$$48 \div 12 = 4 \text{ feet}$$

4. Based on the image, estimate the width of the fireplace in the scaled drawing.

about  
2 inches

5. Estimate the length of the actual fireplace in inches.

$$2 \times 12 = 24 \text{ inches}$$

6. About how long is the fireplace in feet?

$$24 \div 12 = 2 \text{ feet}$$

A bus measures 30 feet long.

7. Coco uses a scale of 1 inch = 2 feet to make a scale model of the bus. How many inches long is Coco's scale model?

$$30 \div 2 = 15$$

15 inches

8. Wallace uses a scale of 1 inch = 3 feet to make a scale model of the bus. How many inches long is Wallace's model?

$$30 \div 3 = 10$$

10 inches

Isabel wants to make a scale model of the same bus, but she wants to use a scale factor without units. She starts by converting the length of the bus into inches.

9. How many inches are equivalent to 1 foot?

12 inches

10. The bus is 30 feet long. How many inches are equivalent to 30 feet?

$$30 \times 12 = 360$$

360 inches

11. Isabel uses a scale factor of 1 = 15 to make her scale model. How long is her scale model?

$$360 \div 15 = ?$$

$$15 \times ? = 360$$

24 inches

Look at each scale below.

1 = 3

1 = 12

$\frac{1}{3} = 1$

3 = 1

$\frac{1}{12} = 1$

12. How many feet are equivalent to 1 yard? 3 ft

13. Which scales are equivalent to 1 foot = 1 yard? How do you know?

1 foot is  $\frac{1}{3}$  of a yard.

$1 = 3$   
foot feet

$\frac{1}{3} = 1$   
yard yard



1. Which scales are equivalent to 1 centimeter to 1 meter? Select ALL that apply.

a. 1 to 100

b. 1/100 to 1

c. ~~1 to 10~~

d. ~~10 to 1~~

e. ~~100 to 1~~

$$1 \text{ cm} = \frac{1}{100} \text{ m}$$

$$100 \text{ cm} = 1 \text{ m}$$

2. A scale model of a playground slide is built at a scale of 1 to 48. If the model slide is 5 inches tall, how tall is the actual slide in inches? Feet?

$$5 \times 48 = ?$$

$$\begin{array}{r} 48 \\ \times 5 \\ \hline 240 \end{array}$$

240  
inches

$$240 \div 12 = ?$$

20  
feet

3. A rectangular parking lot is 40 meters long and 25 meters wide.

- Blake made a scale drawing of the parking lot using a scale of 1 cm to 5 meters.
- Layla made a scale drawing of the parking lot using a scale of 1 to 500.

They both ended up with a scale drawing that measured 8 centimeters by 5 centimeters. Explain how that is possible.

They seemingly used different scales, but Blake's uses units and Layla's does not. Blake can divide each dimension by 5 to end up with 8 cm x 5 cm. Layla's scale means "1 cm to 500 cm" which is the same as "1 cm to 5 m" since  $500 \text{ cm} = 5 \text{ m}$ . She would need to convert 40 m x 25 m to 4000 cm x 2500 cm first.

4. Darryl drew a plan of a swimming pool using a scale of 1 to 60. The length of the swimming pool in his drawing is 1.75 inches. What is the actual measurement of the length of the swimming pool?

$$\begin{array}{r} 1.75 \\ \times 60 \\ \hline 105.00 \end{array}$$

105 in

# **G7 U1 Lesson 10**

Use different scales, with or without units, to describe the same drawings.



## G7 U1 Lesson 10 - Students will use different scales, with or without units, to describe the same drawings

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today is our final day learning about scale and scale factor for now. Like our previous lesson, today we'll see some scales with units and some scales without. We were able to navigate this before, so we can think of today as being mostly an opportunity to enhance our skills.

**Let's Talk (Slide 3):** As we've seen before, it helps to be familiar with unit conversions when dealing with scale drawings. Before we jump into our main problems today, let's make sure we're clear around some important unit conversions. Look at the first, red box. These are customary units of length. About how big is each of these units? **Possible Student Answers, Key Points:**

- A foot is the length of a ruler or about the length of a notebook.
- An inch is about the length of the top part of my thumb or the size of an inch-tile I've used in math.
- A yard is about the width of a door or three rulers.

1 foot = 12 inches  
1 yard = 3 feet

*(if student shares conversions, fill them in the blanks as they share)* We know that 1 foot is equivalent to 12 inches. We also know that 1 yard is equivalent to 3 feet. We'll use these customary units throughout our work today.

Now look at the second, blue box. These are metric units of length. About how big is each of these units?

**Possible Student Answers, Key Points:**

- A meter is about the size of yard. That's about the width of the door.
- A centimeter is smaller than an inch. It's like the width of my fingernail.

1 meter = 100 centimeters

*(if student shares conversions, fill them in the blanks as they share)* We know that 1 meter is equivalent to 100 centimeters. We'll use this information in some problems today. Now that we've refreshed on our customary and metric length units, let's try out a couple problems.

**Let's Think (Slide 4):** Our first problem wants us to find the actual height of this building in meters. What unit is the scale image measured in? **(Centimeters.)** Does the provided scale include units? **(No.)** Since the scale does not include units, I know if I multiply 6 using the scale, the resulting height will be in centimeters. I'll want to keep that in mind, since the question wants us to find the height of the actual building in meters.

$$6 \times 700 = 4200$$

4200 cm

Let's use the scale to find the height of the building in centimeters. The scale tells us that the actual height of the building is 700 times the height of the drawing. What is  $6 \times 700$ ? **(4,200)** *(write  $6 \times 700 = 4,200$ )* The height of the building is 4,200 centimeters.

If we want to convert this height to now be in meters, how can I use what we just reviewed about meters and centimeters to help us? **Possible Student Answers, Key Points:**

- I know 100 centimeters is equal to 1 meter.
- I can divide 4,200 centimeters by 100 to find how many meters.
- I can think about a number I can multiply 100 by to get to 4,200.

Let's set up a table to help us keep track of our units. I'll draw a t-chart and label one column with centimeters and one column with meters. I'll put the conversion we know in the top row of the table.

cm	m
100	1
4200	?

*(Handwritten: An arrow points from 100 cm to 1 m with the label  $\times \frac{1}{100}$ )*

(sketch t-chart as described, putting 100 centimeters and 1 meter in the top row) We know 100 centimeters is equivalent to 1 meter. So to convert from centimeters to meters, I can divide by 100 or multiply by  $\frac{1}{100}$ . (draw arrow from 100 cm to 1 m showing “ $\times \frac{1}{100}$ ”) Take a minute to figure out what 4,200 centimeters is in terms of meters. How do you know? **Possible Student Answers, Key Points:**

- I know 4,200 divided by 100 is 42.
- I know 4,200 times  $\frac{1}{100}$  is  $4,200/100$  or 42.

$$4200 \times \frac{1}{100} = \frac{4200}{100}$$

(write equation and answer as you narrate) We can multiply 4,200 by  $\frac{1}{100}$  to get  $4,200/100$ . That’s equivalent to 42. 4,200 centimeters is equivalent to 42 meters.

**42 m**

Using the scale without units meant that the actual height we calculated was in centimeters, the same unit we started with. The only thing we had to do to answer the question in terms of meters was to convert the centimeters to meters. The table was a helpful way to keep our thinking organized.

**Let’s Think (Slide 5):** Let’s try one more. What do you notice is the same and different about this problem compared to the last problem we did? **Possible Student Answers, Key Points:**

- It’s the same in that we’re finding the actual height of a building. It’s the same in that the scale does not include units.
- It’s different because the building looks different. It’s different because it uses customary units.

This problem is very similar to the last one, just with customary units. Rather than solve it the exact same way, let me show you another way we can tackle problems like this. In our last problem, we used the scale and then converted our answer into the designated unit. For this one, let’s try converting *first* so our answer comes out in the correct unit automatically.

$$\begin{array}{l} 1 = 24 \\ \text{in} \quad \text{in} \\ \downarrow \\ 1 = 2 \\ \text{in} \quad \text{ft.} \end{array}$$

The scale is 1 in = 24 in. (write 1 in = 24 in) Instead of thinking of this as 1 inch representing 24 inches, I can think of this same scale as 1 inch representing 2 feet. 24 inches is the same thing as 2 feet. (write 1 in = 2 ft directly underneath 1 in = 24 in) We didn’t change the meaning of the scale, we just made the scale work for the units in the problem. Now if we use this scale, the answer we get will be in feet.

in.	ft.
1	2
20	?

*(Handwritten: An arrow points from 1 in to 2 ft with the label  $\times 2$ )*

I’m going to sketch a t-chart to help us find the actual height of the building given the scale image of 20 inches. (draw t-chart with a column for inches and a column for feet, and put 1 inch and 2 feet in the top row) The scale of 1 inch to 2 feet is in my table. I know to move from inches to feet, we can multiply by 2. How can I use that to find the actual height of the building? **Possible Student Answers, Key Points:**

- We can multiply the number of inches by 2 to find the number of feet.
- I know  $20 \times 2 = 40$ , so the actual height of the building is 40 feet.

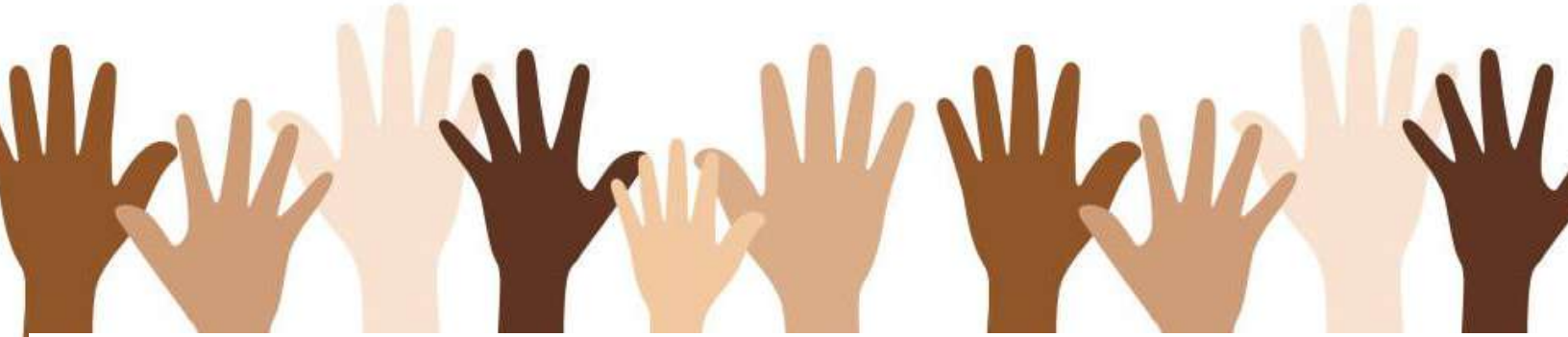
$$20 \times 2 = 40 \text{ ft}$$

The drawing is 20 inches. If we multiply 20 by 2, we end up with 40. The actual height of the building is 40 feet. (write  $20 \times 2 = 40$ )

Because we already converted inches to feet when thinking about our scale, we don’t need to convert now. Whether you prefer solving problems like this the original way we did with the first example or this way, you’ll end up with the same answer.

**Let's Try it (Slides 6 - 7):** Now that we've seen two different ways to tackle similar problems, let's practice a little more. When we're finished, you'll get to try out a few independently. We'll want to look closely at the scales we are given. If they don't have units, we'll want to pay close attention to the units in our actual objects and scale drawings. As we saw with our two examples, we can scale and then convert our answer *or* we can reason about units for the scale and avoid having to convert at the end. Pick which option works best for you, and know that it might vary from problem to problem. Let's give it a try!

# WARM WELCOME



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**Today we will use different scales, with or without units, to describe the same drawings.**

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## Let's Talk:

1 foot = \_\_\_\_ inches

1 yard = \_\_\_\_ feet

1 meter = \_\_\_\_ centimeters



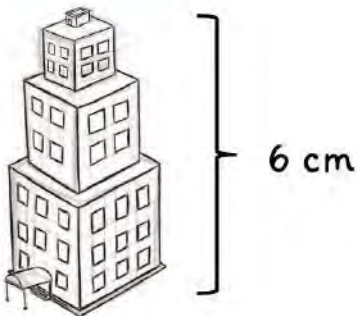
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## Let's Think:

Find the actual height of the building in meters by finding the height in centimeters first.

SCALE

1 = 700



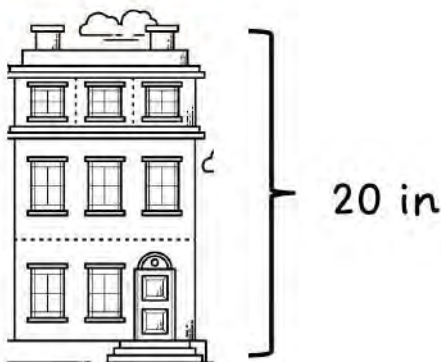
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# Let's Think:

Find the actual height of the building in feet by converting the scale factor into feet.

**SCALE**  
1 = 24



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# Let's Try It:

Let's explore using different scales, with and without units, to describe the same drawings together.

A scale drawing of a train engine is shown here.

1. Does the scale have units?  
a. no  
b. yes

2. When a scale does not have units, it means the units in the actual object are \_\_\_\_\_ the units in the drawing.  
a. the same as  
b. different from

3. Find the length of the actual train engine in inches.

4. One foot is \_\_\_\_\_ than one inch.  
a. larger  
b. smaller

5. It will take \_\_\_\_\_ feet to measure the train engine than inches.  
a. more  
b. fewer

6. How many inches are in 1 foot?

7. Find the length of the actual train engine in feet by converting the length of the actual train in inches into feet.

8. Find the length of the actual train engine in yards.

A scale drawing of a house is shown here.

9. Does the scale have units?  
a. no  
b. yes

10. Find the height of the actual house in centimeters.

11. Find the height of the actual house in meters by converting centimeters into meters.

Let's find the height of the actual house in meters another way.

12. Rewrite the scale using meters instead of centimeters.  
1 cm = 100 cm    1 cm = \_\_\_\_\_ m

13. Use the rewritten scale to find the height of the house in meters.

14. Which method of finding the house in meters do you prefer? Why?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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# On your Own:

Now it's time to use different scales, with or without units, to describe the same drawings on your own.


Name: \_\_\_\_\_ 67 U1 Lesson 10 - Independent Work

1. This is a scale image of a basketball hoop. Use the scale to find the height of the actual basketball hoop.



16 cm  
SCALE  
1:50

2. This is a scale drawing of a kangaroo. Use the scale to find the length of the actual kangaroo.




SCALE  
1:35  
3 1/2 in.

3. Rewrite each scale using feet.

1 inch = 12 inches	1 inch =
1 inch = 24 inches	1 inch =
1 inch = 60 inches	1 inch =


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4. The length of a scale drawing of a park bench is 50 centimeters. Find the length of the actual bench in meters.



SCALE  
1:6  
50 cm      7 meters

5. Jason was trying to determine how many feet the actual penguin is based on the scale drawing shown here. He said the answer is 36 feet. Explain why Jason is incorrect. Include the correct answer in your response.



SCALE  
1:9  
4 in      7 feet

\_\_\_\_\_

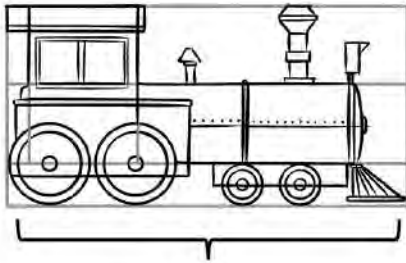
\_\_\_\_\_

\_\_\_\_\_

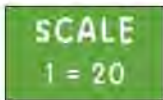
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A scale drawing of a train engine is shown here.



12 in



1. Does the scale have units?

- a. no
- b. yes

2. When a scale does not have units, it means the units in the actual object are \_\_\_\_\_ the units in the drawing.

- a. the same as
- b. different from

3. Find the length of the actual train engine in inches.

4. One foot is \_\_\_\_\_ than one inch.

- a. larger
- b. smaller

5. It will take \_\_\_\_\_ feet to measure the train engine than inches.

- a. more
- b. fewer

6. How many inches are in 1 foot?

7. Find the length of the actual train engine in feet by converting the length of the actual train in inches into feet.

8. Find the length of the actual train engine in yards.

A scale drawing of a house is shown here.



9. Does the scale have units?

- a. no
- b. yes

10. Find the height of the actual house in centimeters.

11. Find the height of the actual house in meters by converting centimeters into meters.

**Let's find the height of the actual house in meters another way.**

12. Rewrite the scale using meters instead of centimeters.

$$1 \text{ cm} = 100 \text{ cm} \quad 1 \text{ cm} = \underline{\hspace{2cm}} \text{ m}$$

13. Use the rewritten scale to find the height of the house in meters.

14. Which method of finding the house in meters do you prefer? Why?

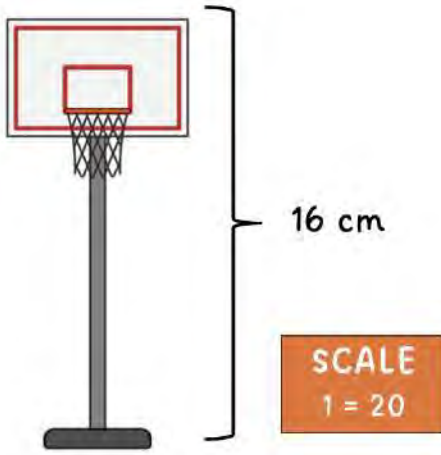
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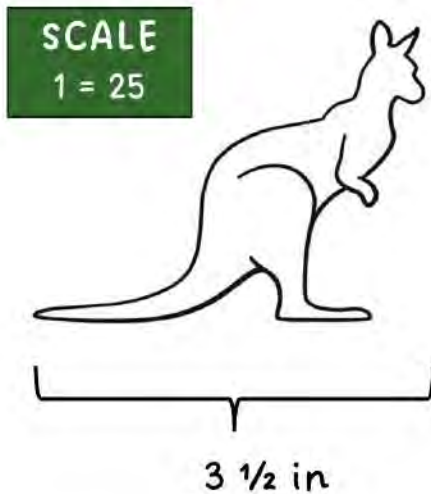
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1. This is a scale image of a basketball hoop. Use the scale to find the height of the actual basketball hoop.



2. This is a scale drawing of a kangaroo. Use the scale to find the length of the actual kangaroo.



3. Rewrite each scale using feet.

$$1 \text{ inch} = 12 \text{ inches}$$

$$1 \text{ inch} =$$

$$1 \text{ inch} = 24 \text{ inches}$$

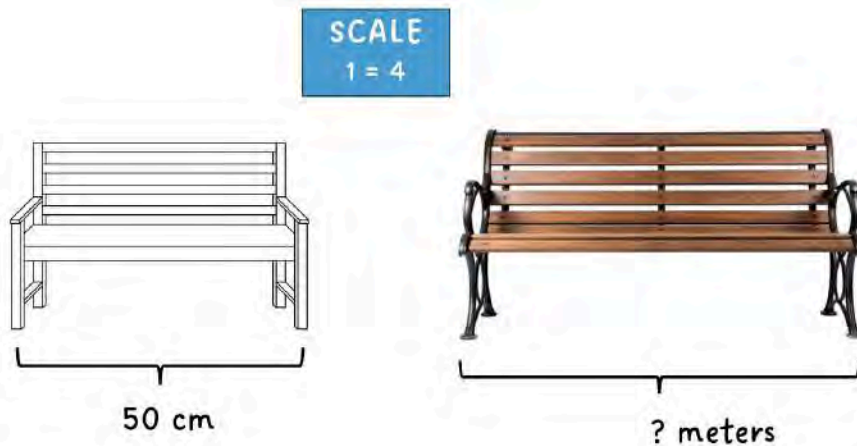
$$1 \text{ inch} =$$

$$1 \text{ inch} = 60 \text{ inches}$$

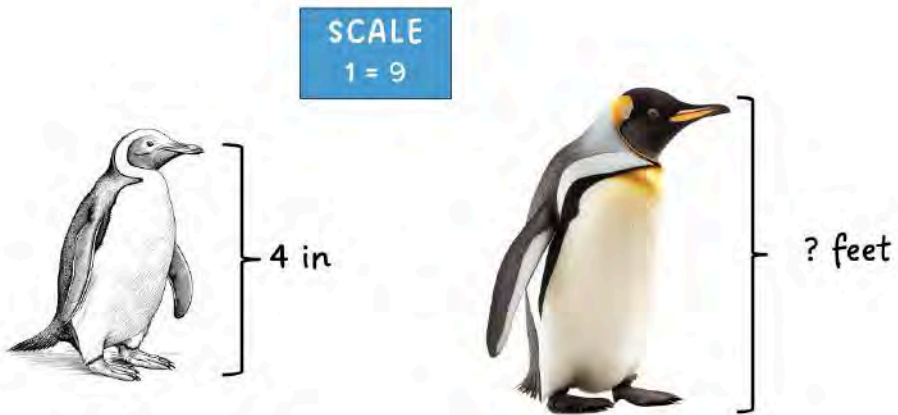
$$1 \text{ inch} =$$



4. The length of a scale drawing of a park bench is 50 centimeters. Find the length of the actual bench in meters.



5. Jason was trying to determine how many feet the actual penguin is based on the scale drawing shown here. He said the answer is 36 feet. Explain why Jason is incorrect. Include the correct answer in your response.



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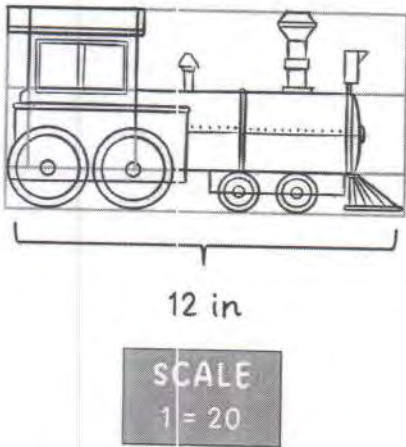
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Name: KEY

A scale drawing of a train engine is shown here.



1. Does the scale have units?

- a. no
- b. yes

2. When a scale does not have units, it means the units in the actual object are \_\_\_\_\_ the units in the drawing.

- a. the same as
- b. different from

3. Find the length of the actual train engine in inches.

$$12 \times 20 = 240$$

240 inches

4. One foot is \_\_\_\_\_ than one inch.

- a. larger
- b. smaller

5. It will take \_\_\_\_\_ feet to measure the train engine than inches.

- a. more
- b. fewer

6. How many inches are in 1 foot?

12 inches

7. Find the length of the actual train engine in feet by converting the length of the actual train in inches into feet.

$$240 \div 12 = 20 \text{ ft}$$

8. Find the length of the actual train engine in yards.

$$20 \div 3 = \frac{20}{3} = 6 \frac{2}{3} \text{ yd}$$

A scale drawing of a house is shown here.



9. Does the scale have units?

- a. no
- b. yes

10. Find the height of the actual house in centimeters.

$$7 \times 100 = 700 \text{ cm}$$

11. Find the height of the actual house in meters by converting centimeters into meters.

$$700 \div 100 = 7 \text{ m}$$

Let's find the height of the actual house in meters another way.

12. Rewrite the scale using meters instead of centimeters.

$$1 \text{ cm} = 100 \text{ cm} \quad 1 \text{ cm} = \underline{1} \text{ m}$$

13. Use the rewritten scale to find the height of the house in meters.

$$\begin{array}{r|l} \text{cm} & \text{m} \\ \hline 1 & 1 \times 1 \\ 7 & 7 \end{array}$$

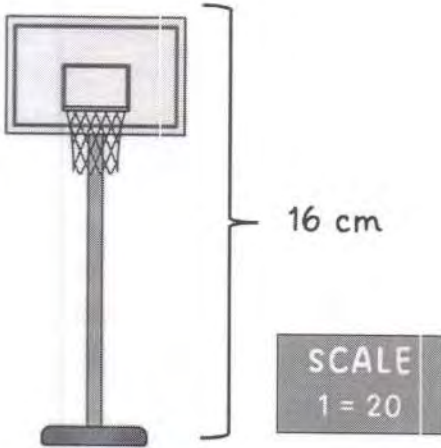
$$7 \times 1 = 7 \text{ m}$$

14. Which method of finding the house in meters do you prefer? Why? (answers can vary!)

It depends on the problem. In this example I prefer using the given scale and converting units last.



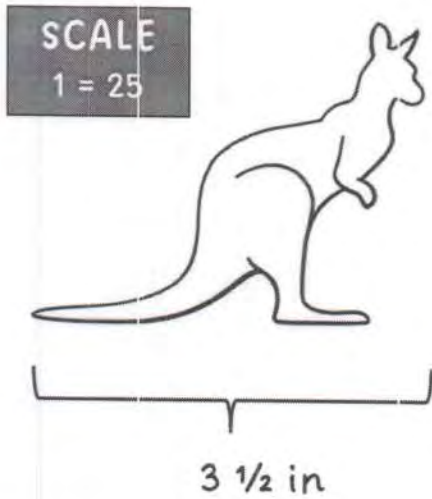
1. This is a scale image of a basketball hoop. Use the scale to find the height of the actual basketball hoop.



$$16 \times 20 = ?$$

$$\boxed{320 \text{ cm}}$$

2. This is a scale drawing of a kangaroo. Use the scale to find the length of the actual kangaroo.



$$3\frac{1}{2} \times 25$$

$$(25 \times 3) + (25 \times \frac{1}{2})$$

$$75 + 12\frac{1}{2}$$

$$\boxed{87\frac{1}{2} \text{ in}}$$

3. Rewrite each scale using feet.

1 inch = 12 inches

1 inch = 1 foot

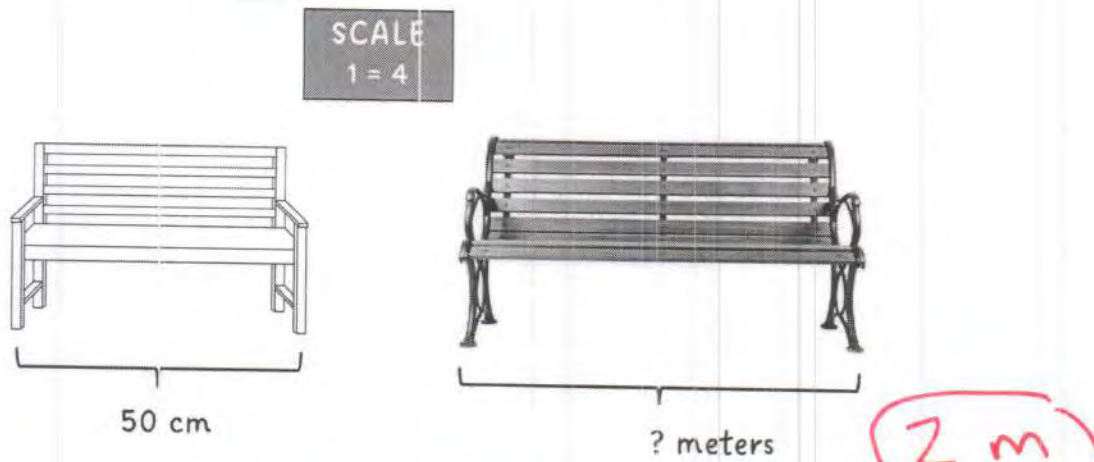
1 inch = 24 inches

1 inch = 2 feet

1 inch = 60 inches

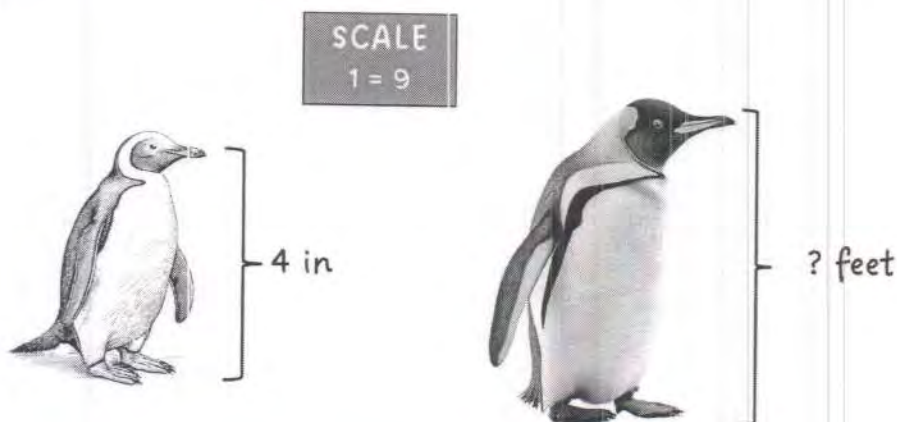
1 inch = 5 feet

4. The length of a scale drawing of a park bench is 50 centimeters. Find the length of the actual bench in meters.



$$50 \times 4 = 200 \text{ cm}$$
$$200 \div \frac{100}{1} = \cancel{2000} 2$$

5. Jason was trying to determine how many feet the actual penguin is based on the scale drawing shown here. He said the answer is 36 feet. Explain why Jason is incorrect. Include the correct answer in your response.



Jason did not pay attention to his units. A 36-foot penguin is unrealistic. He can use the 1 = 9 scale to find the penguin's height in inches ( $4 \times 9 = 36 \text{ in}$ ). Then he can convert inches to feet ( $36 \div 12 = 3$ ). It is 3 feet tall.





# G7 Unit 2:

Introducing Proportional Relationships

# **G7 U2 Lesson 1**

Compare and create representations to compare ratios in the context of recipes or scaled copies.

## G7 U2 Lesson 1 - Today we will use the unit rate to decide if ratios are proportional.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** The next 15 lessons are going to be about proportions! Proportions are everywhere, all around us. I bet you've already been using proportions and you didn't even realize it. There will be some new words to describe the proportion idea. But you are going to understand these ideas as long as you take your time to think.

**Let's Review (Slide 3):** Let's review a word from last year, which is "ratio." The question here says, "What is the ratio of teddies to kids?" Does anyone know? **Possible Student Answers, Key Points:**

- 6 to 3
- 2 per kid
- There are 6 teddies and 3 kids.



6:3 6 to 3

We write a ratio with a colon between the numbers like this. The ratio of teddies to kids is "six colon three." We say it is "6 TO 3." Sometimes people write the numbers as a fraction like this: "six over three." Sometimes people simplify the numbers too. But we'll stick with what we see.

Is that ratio the same as the ratio of kids to teddies? NO! The words are in a different order so we would have to write our numbers in a different order too. We would write, "three colon six" or "three to six" or "three over six." So there are two things to remember since the last time you worked with ratios. First, we are thinking about two amounts because there are two different things that have a relationship. In this case, teddies and kids. Second, the order of the words we use really matters. Teddies to kids is not the same as kids to teddies. It can get confusing so we are really going to have to label our numbers and be careful about the order.

**Let's Talk (Slide 4):** Now we will use the ratio of teddies to kids to decide which of the daycares here would be more fun. I see Miss. Joya's Fun Place of Sweetness and Mr. Grump's Serious Building for Kids. I bet you already have an opinion about which one would be nice for the little guys who have to go here. But let's use math to be really exact. One way to compare is to share the kids to compare. What operation do we use when we are sharing or splitting something? **Possible Student Answers, Key Points:**

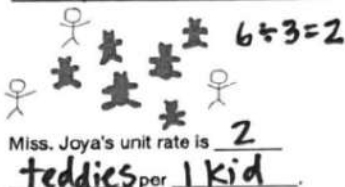
- Division
- Dealing
- Repeated Subtraction

We can share the teddies with kids to compare! Sharing is the same as dividing.

We use division! Sharing is the same as dividing. And when we divide teddies by kids then we will find how many teddies for each kid or how many teddies for just one kid.

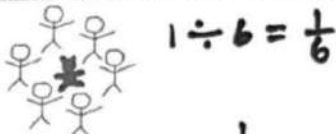
*Read the sentence slowly from the slide twice because this is a key point.* "In ratios when we know the amount of something for JUST ONE of the other thing, it is called the UNIT RATE." So when we divide and find how many teddies for each kid, we will be finding the unit rate. Let's do it.

Miss. Joya's Fun Place of Sweetness



There are 6 teddies and 3 kids so I am going to do 6 divided by 3. That's 2. Miss. Joya's unit rate is 2 teddies per 1 kid. Usually we don't write 1 after the word per because we know it's there. But I'll write it for today.

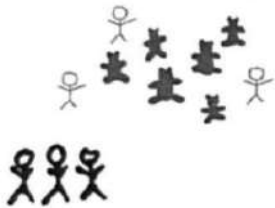
Mr. Grump's Serious Building for Kids



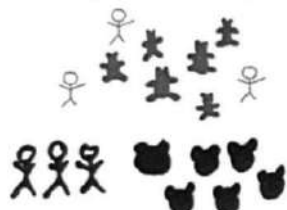
Mr. Grump's unit rate is  $\frac{1}{6}$   
teddy per 1 kid.

Now let's look at Mr. Grump's Serious Building for Kids. We can find the unit rate here too. The unit rate is still the amount of teddies for one kid and we still find it using division. The kids still have to share the teddies, right? Now there is 1 teddy. And there are 1 - 2 - 3 - 4 - 5 - 6 kids! Oh no! If I do 1 divided by 6, I don't even get a whole number! I get 1 sixth. Mr. Grump's unit rate is 1 sixth of a teddy per 1 kid. That's just a piece of a teddy per kid.

Now, this problem was obvious and silly. But the big idea is we can divide to find the unit rate. And the unit rate will help us compare ratios.



**Let's Think (Slide 5):** Ratios are called PROPORTIONAL when their unit rate is the same. Here's an example. Let's imagine that Miss. Joya decides to double the size of her daycare! I am going to double the kids. I am going to draw three more kids.



Now, Miss. Joya has a nice daycare. She cares about her kids. If she gets double the number of kids, she's not going to keep the same amount of teddies. If she gets double the number of kids, what do you think she is going to do with the number of teddies? [Possible Student Answers, Key Points:](#)

- She is going to get 6 more teddies.
- She is going to double the number of teddies.

If Miss. Joya gets double the amount of kids, she is going to have double the amount of teddies. There were 6 before so she needs 6 more.

Teddies	Kids
6	3

We can fill in the new amounts on the table and we will notice some super important things. First, remember that we said the order of the words is super important. Notice that this column is teddies. *Point to the teddies column.* And this column is kids. *Point to the kids column.* Let's fill in the numbers for the first picture. There were 6 teddies and 3 kids.

Teddies	Kids
6	3
12	6

Then Miss. Joys doubled the size of her daycare. Let's count. *Count all the teddies.* She needed 12 teddies for 6 kids. What do you notice about our table? [Possible Student Answers, Key Points:](#)

- The teddies are times two.
- The kids are times two.
- If you go down, it goes plus 6 and plus 3.
- If you go across, it is divided by 2.

There are going to be lots of ways to describe ratios when they have this special doubling relationship. Or a tripling relationship. That's why we need a whole fifteen lessons for this unit! But for today, let's focus on the unit rate. This says, "what is the new unit rate?" How do we find the unit rate again?

[Possible Student Answers, Key Points:](#)

- Divide.
- Divide 12 by 6.

What is the new UNIT RATE? 2 teddies per 1 kid

We divide! In this case, 12 divided by 6 is 2. And that's 2 teddies per 1 kid.

Wow! This is really super important! I see that my new unit rate is the same as my old unit rate! Before it was 2 teddies per kid and it's still 2 teddies per kid! That is important! The unit rate stayed the same because when we doubled the kids, we doubled the teddies too. And the relationship between teddies and kids stayed the same. When the relationship between two amounts is the same, it has a special name. It is called a PROPORTION. We can write, the first ratio is PROPORTIONAL to the second ratio.

The the first ratio is proportional to the second ratio.

*Reread the heading of the slide.* This is the main idea for today: "Ratios are proportional when their unit rate is the same."

**Let's Try It (Slide 6):** Let's practice writing division and fractions together from stories. I will walk you through step by step and we will make sure we figure out which number is the dividend so it can go before the division sign.



# WARM WELCOME



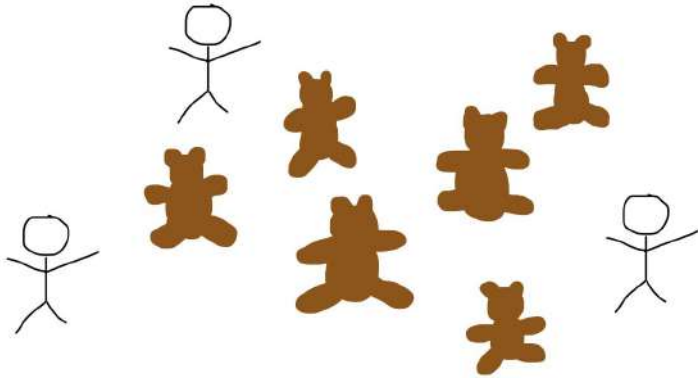
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**Today we will use the unit rate to decide  
if ratios are proportional.**

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## Let's Review:

What is the ratio of teddies to kids?  
Is that the same as the ratio of kids  
to teddies?



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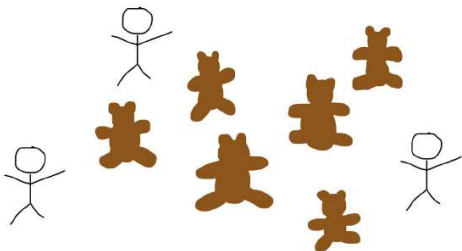
## Let's Talk:

Use the ratio of teddies to kids to decide  
which daycare would be more fun.

We can share the teddies with kids to compare! Sharing is the same as \_\_\_\_\_.

In ratios when we know the amount of something for JUST ONE of the other thing,  
it is called the **UNIT RATE**.

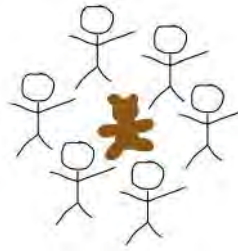
Miss. Joya's Fun Place of Sweetness



Miss. Joya's unit rate is \_\_\_\_\_

\_\_\_\_\_ per \_\_\_\_\_.

Mr. Grump's Serious Building for Kids



Mr. Grump's unit rate is \_\_\_\_\_

\_\_\_\_\_ per \_\_\_\_\_.

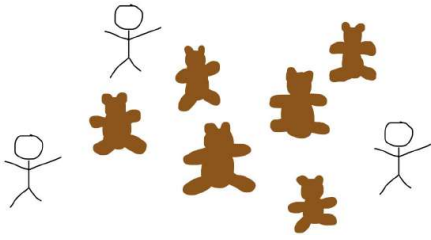
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## Let's Think:

Ratios are **PROPORTIONAL** when their unit rate is the same.

Let's imagine that Miss. Joya decides to double the size of her daycare!



Teddies	Kids

What is the new UNIT RATE? \_\_\_\_\_ per \_\_\_\_\_

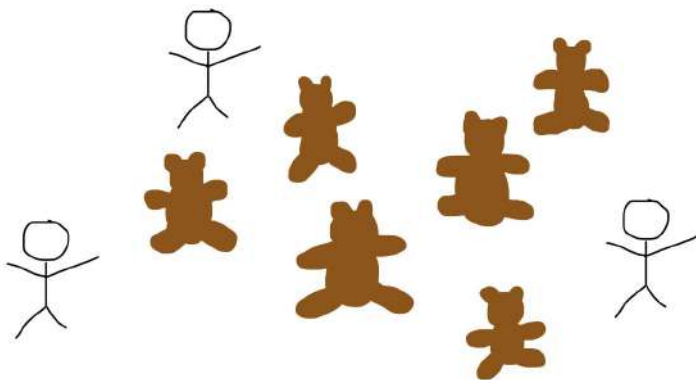
The the first ratio is \_\_\_\_\_ to the second ratio.

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## Let's Review:

What is the ratio of teddies to kids?  
Is that the same as the ratio of kids to teddies?



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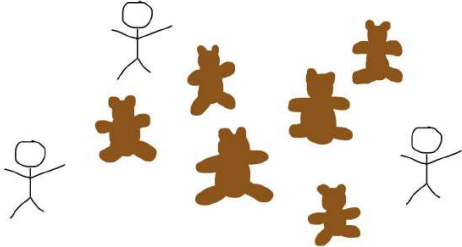
## Let's Talk:

Use the ratio of teddies to kids to decide which daycare would be more fun.

We can share the teddies with kids to compare! Sharing is the same as \_\_\_\_\_.

In ratios when we know the amount of something for JUST ONE of the other thing, it is called the **UNIT RATE**.

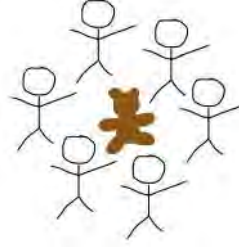
### Miss. Joya's Fun Place of Sweetness



Miss. Joya's unit rate is \_\_\_\_\_

\_\_\_\_\_ per \_\_\_\_\_.

### Mr. Grump's Serious Building for Kids



Mr. Grump's unit rate is \_\_\_\_\_

\_\_\_\_\_ per \_\_\_\_\_.

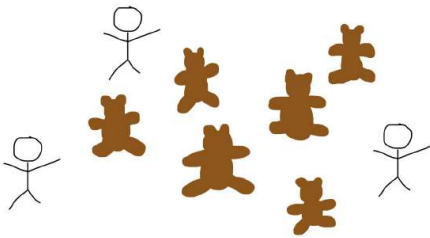
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## Let's Think:

Ratios are **PROPORTIONAL** when their unit rate is the same.

Let's imagine that Miss. Joya decides to double the size of her daycare!



Teddies	Kids

What is the new UNIT RATE? \_\_\_\_\_ per \_\_\_\_\_

The first ratio is \_\_\_\_\_ to the second ratio.

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## Let's Try It:

Let's practice finding unit rates and deciding if the ratios are proportional.

Name: \_\_\_\_\_

G7 U2 Lesson 1 - Let's Try It

1. Draw a picture of the story below.

**Ratio A:** Jerry has 15 flowers for 3 vases.

2. Divide to find the unit rate.

3. The unit rate of Ratio A is \_\_\_\_\_ per \_\_\_\_\_

4. Draw a picture of the story below.

**Ratio B:** Sara has 20 flowers for 5 vases.

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_

G7 U2 Lesson 1 - Independent Work

Remember: Ratios are PROPORTIONAL when their unit rate is the same.

Find the unit rate for each ratio. Then circle the words that best complete the sentence.

<p>1.</p> <p><b>Ratio A:</b> Lisa mixed 9 cups of water with 3 tablespoons of lemonade mix.</p> <p>The unit rate of Ratio A is _____</p> <p>_____ per _____</p> <p><b>Ratio B:</b> Sam mixed 4 cups of water with 2 tablespoons of lemonade mix.</p> <p>The unit rate of Ratio B is _____</p> <p>_____ per _____</p> <p>Circle one:</p> <p>Ratio A is proportional to Ratio B.</p> <p>OR</p> <p>Ratio A is NOT proportional to Ratio B.</p>	<p>2.</p> <p><b>Ratio A:</b> There are 6 preschoolers and 3 kindergartners at the playground.</p> <p>The unit rate of Ratio A is _____</p> <p>_____ per _____</p> <p><b>Ratio B:</b> There are 12 preschoolers and 6 kindergartners on the field.</p> <p>The unit rate of Ratio B is _____</p> <p>_____ per _____</p> <p>Circle one:</p> <p>Ratio A is proportional to Ratio B.</p> <p>OR</p> <p>Ratio A is NOT proportional to Ratio B.</p>
<p>3.</p> <p><b>Ratio A:</b> Ms. Allen's basket of treats came with _____</p>	<p>4.</p> <p><b>Ratio A:</b> Rachel's tree has 24 red ornaments</p>

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Name: \_\_\_\_\_

1. Draw a picture of the story below.

**Ratio A:** Jerry has 15 flowers for 3 vases.

2. Divide to find the unit rate.

3. The unit rate of Ratio A is \_\_\_\_\_ per \_\_\_\_\_

4. Draw a picture of the story below.

**Ratio B:** Sara has 20 flowers for 5 vases.

5. Divide to find the unit rate.

6. The unit rate of Ratio B is \_\_\_\_\_ per \_\_\_\_\_

7. Circle one:

The unit rates are the same.

OR

The unit rates are different.

8. Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

9. Draw a picture of the story below.

**Ratio A:** A 20 gallon tank requires 5 drops of special fish solution to purify the water.

10. Divide to find the unit rate.

11. The unit rate of Ratio A is \_\_\_\_\_ per \_\_\_\_\_

12. Draw a picture of the story below.

**Ratio B:** A 12 gallon fish tank requires 3 drops of special fish solution to purify the water.

13. Divide to find the unit rate.

14. The unit rate of Ratio B is \_\_\_\_\_ per \_\_\_\_\_

15. Circle one:

The unit rates are the same.

OR

The unit rates are different.

16. Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

Remember: Ratios are PROPORTIONAL when their unit rate is the same.

Find the unit rate for each ratio. Then circle the words that best complete the sentence.

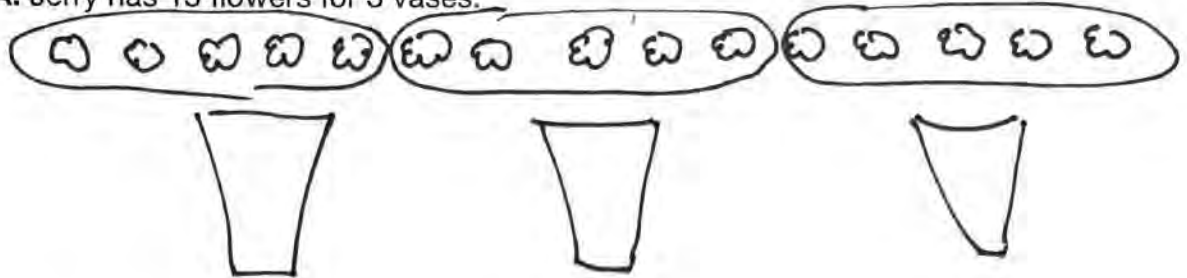
<p>1. <b>Ratio A:</b> Lisa mixed 9 cups of water with 3 tablespoons of lemonade mix.</p> <p>The unit rate of Ratio A is _____ _____ per _____</p> <p><b>Ratio B:</b> Sam mixed 4 cups of water with 2 tablespoons of lemonade mix.</p> <p>The unit rate of Ratio B is _____ _____ per _____</p> <p>Circle one: Ratio A is proportional to Ratio B. OR Ratio A is NOT proportional to Ratio B.</p>	<p>2. <b>Ratio A:</b> There are 6 preschoolers and 3 kindergarteners at the playground.</p> <p>The unit rate of Ratio A is _____ _____ per _____</p> <p><b>Ratio B:</b> There are 12 preschoolers and 6 kindergarteners on the field.</p> <p>The unit rate of Ratio B is _____ _____ per _____</p> <p>Circle one: Ratio A is proportional to Ratio B. OR Ratio A is NOT proportional to Ratio B.</p>
<p>3. <b>Ratio A:</b> Ms. Allen's basket of treats came with 15 cookies and 3 brownies.</p> <p>The unit rate of Ratio A is _____ _____ per _____</p> <p><b>Ratio B:</b> Mr Buford's basket of treats came with 20 cookies and 4 brownies.</p> <p>The unit rate of Ratio B is _____ _____ per _____</p> <p>Circle one: Ratio A is proportional to Ratio B. OR Ratio A is NOT proportional to Ratio B.</p>	<p>4. <b>Ratio A:</b> Rachel's tree has 24 red ornaments and 6 gold ornaments.</p> <p>The unit rate of Ratio A is _____ _____ per _____</p> <p><b>Ratio B:</b> Peter's tree has 5 gold ornaments and 20 red ornaments.</p> <p>The unit rate of Ratio B is _____ _____ per _____</p> <p>Circle one: Ratio A is proportional to Ratio B. OR Ratio A is NOT proportional to Ratio B.</p>

Find the unit rate for each ratio. Then circle the words that best complete the sentence.

<p>1.</p> <p><b>Ratio A:</b> At the class party, there are 20 juice boxes for 4 kids.</p> <p>The unit rate of Ratio A is _____</p> <p>_____ per _____</p> <p><b>Ratio B:</b> In the lunch room, there are 10 juice boxes for 10 kids.</p> <p>The unit rate of Ratio B is _____</p> <p>_____ per _____</p> <p>Circle one:</p> <p style="text-align: center;">Ratio A is proportional to Ratio B.</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Ratio A is NOT proportional to Ratio B.</p>	<p>2.</p> <p><b>Ratio A:</b> Susannah got paid \$12 for babysitting 3 hours.</p> <p>The unit rate of Ratio A is _____</p> <p>_____ per _____</p> <p><b>Ratio B:</b> Susannah got paid \$10 for mowing lawns for 2 hours.</p> <p>The unit rate of Ratio B is _____</p> <p>_____ per _____</p> <p>Circle one:</p> <p style="text-align: center;">Ratio A is proportional to Ratio B.</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Ratio A is NOT proportional to Ratio B.</p>
<p>3.</p> <p><b>Ratio A:</b> Rose's bowl of fruit salad has 3 strawberries and 6 blueberries.</p> <p>The unit rate of Ratio A is _____</p> <p>_____ per _____</p> <p><b>Ratio B:</b> Nathaniel's bowl of fruit salad has 4 strawberries and 8 blueberries.</p> <p>The unit rate of Ratio B is _____</p> <p>_____ per _____</p> <p>Circle one:</p> <p style="text-align: center;">Ratio A is proportional to Ratio B.</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Ratio A is NOT proportional to Ratio B.</p>	<p>4.</p> <p><b>Ratio A:</b> Dennis got 6 mg of Vitamin C by eating 2 pieces of fruit.</p> <p>The unit rate of Ratio A is _____</p> <p>_____ per _____</p> <p><b>Ratio B:</b> Lila got 2 mg of Vitamin C by eating 9 pieces of fruit.</p> <p>The unit rate of Ratio B is _____</p> <p>_____ per _____</p> <p>Circle one:</p> <p style="text-align: center;">Ratio A is proportional to Ratio B.</p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Ratio A is NOT proportional to Ratio B.</p>

1. Draw a picture of the story below.

**Ratio A:** Jerry has 15 flowers for 3 vases.



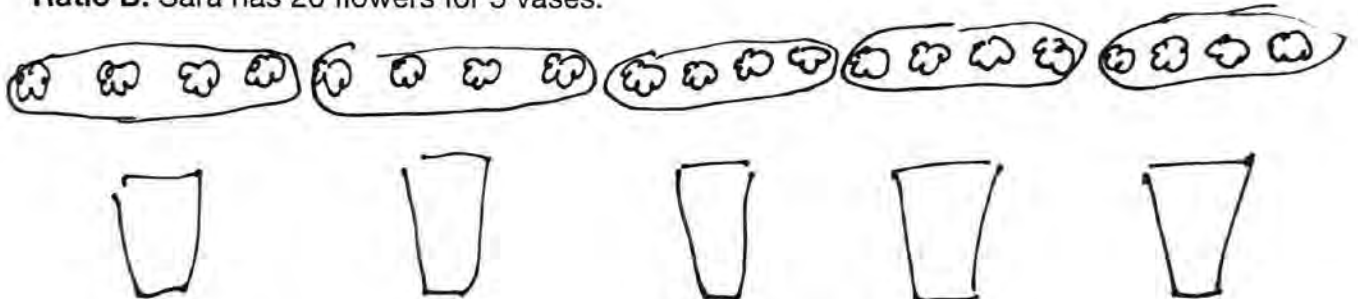
2. Divide to find the unit rate.

$$15 \div 3 = 5$$

3. The unit rate of Ratio A is 5 flowers per 1 vase

4. Draw a picture of the story below.

**Ratio B:** Sara has 20 flowers for 5 vases.



5. Divide to find the unit rate.

$$20 \div 5 = 4$$

6. The unit rate of Ratio B is 4 flowers per 1 vase

7. Circle one:

The unit rates are the same.

OR

The unit rates are different.

8. Circle one:

Ratio A is proportional to Ratio B.

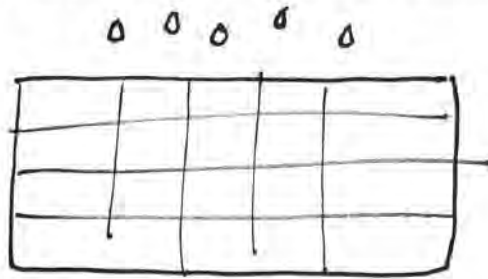
OR

Ratio A is NOT proportional to Ratio B.



9. Draw a picture of the story below.

**Ratio A:** A 20 gallon tank requires 5 drops of special fish solution to purify the water.



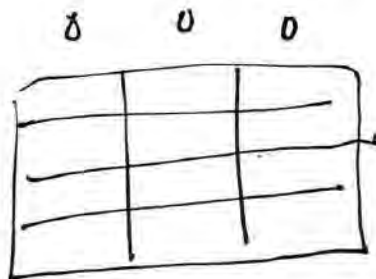
10. Divide to find the unit rate.

$$20 \div 5 = 4 \text{ gallons per drop}$$

11. The unit rate of Ratio A is 4 gallons per 1 drop

12. Draw a picture of the story below.

**Ratio B:** A 12 gallon fish tank requires 3 drops of special fish solution to purify the water.



13. Divide to find the unit rate.

$$12 \div 3 = 4 \text{ gallons per drop}$$

14. The unit rate of Ratio B is 4 gallons per 1 drop

15. Circle one:

The unit rates are the same.

OR

The unit rates are different.

16. Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

Name: \_\_\_\_\_

# ANSWER KEY

G7 U2 Lesson 1 - Independent Work

Remember: Ratios are PROPORTIONAL when their unit rate is the same.

Find the unit rate for each ratio. Then circle the words that best complete the sentence.

1.  
**Ratio A:** Lisa mixed 9 cups of water with 3 tablespoons of lemonade mix.

The unit rate of Ratio A is 3  
cups per tablespoon

**Ratio B:** Sam mixed 4 cups of water with 2 tablespoons of lemonade mix.

The unit rate of Ratio B is 2  
cups per tablespoon

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

2.  
**Ratio A:** There are 6 preschoolers and 3 kindergarteners at the playground.

The unit rate of Ratio A is 2  
preschoolers per kindergartener

**Ratio B:** There are 12 preschoolers and 6 kindergarteners on the field.

The unit rate of Ratio B is 2  
preschoolers per kindergarten

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

3.  
**Ratio A:** Ms. Allen's basket of treats came with 15 cookies and 3 brownies.

The unit rate of Ratio A is 5  
cookies per brownie

**Ratio B:** Mr Buford's basket of treats came with 20 cookies and 4 brownies.

The unit rate of Ratio B is 5  
cookies per brownie

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

4.  
**Ratio A:** Rachel's tree has 24 red ornaments and 6 gold ornaments.

The unit rate of Ratio A is 4  
red ornaments per gold ornament

**Ratio B:** Peter's tree has 5 gold ornaments and 20 red ornaments.

The unit rate of Ratio B is  $\frac{1}{4}$   
red ornaments per gold ornament

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

Find the unit rate for each ratio. Then circle the words that best complete the sentence.

1.

**Ratio A:** At the class party, there are 20 juice boxes for 4 kids.

The unit rate of Ratio A is 5

juice boxes per kid

**Ratio B:** In the lunch room, there are 10 juice boxes for 10 kids.

The unit rate of Ratio B is 1

juice box per kid

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

2.

**Ratio A:** Susannah got paid \$12 for babysitting 3 hours.

The unit rate of Ratio A is 4

dollars per hour

**Ratio B:** Susannah got paid \$10 for mowing lawns for 2 hours.

The unit rate of Ratio B is 5

dollars per hour

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

3.

**Ratio A:** Rose's bowl of fruit salad has 3 strawberries and 6 blueberries.

The unit rate of Ratio A is  $\frac{1}{2}$  or  $\frac{3}{6}$

strawberries per blueberry

**Ratio B:** Nathaniel's bowl of fruit salad has 4 strawberries and 8 blueberries.

The unit rate of Ratio B is  $\frac{1}{2}$  or  $\frac{4}{8}$

strawberries per blueberry

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

4.

**Ratio A:** Dennis got 6 mg of Vitamin C by eating 2 pieces of fruit.

The unit rate of Ratio A is 3

mg per piece of fruit

**Ratio B:** Lila got 2 mg of Vitamin C by eating 6 pieces of fruit.

The unit rate of Ratio B is  $\frac{2}{6}$  or  $\frac{1}{3}$

mg per piece of fruit

Circle one:

Ratio A is proportional to Ratio B.

OR

Ratio A is NOT proportional to Ratio B.

## **G7 U2 Lesson 2**

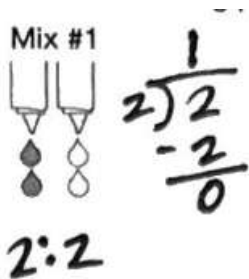
Use a table to describe a proportional relationship, calculate the constant of proportionality, and find missing values.

## G7 U2 Lesson 2 - Today we will generate proportions to find the constant of proportionality.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we're going to generate proportions to find the constant of proportionality. You're going to see that we're just using the same ratios that we've been learning about. Let's go!

**Let's Review (Slide 3):** We learned in our last class that we decide if two ratios are proportional using their unit rate. That means the ratio when the second amount is just one. Let's use that here. *Read the text and then point to each mix as you discuss it.* I see that for Mix #1, Rose used two drops of red and two drops of yellow. Red and yellow make orange. What is the ratio of red drops to yellow drops?



Possible Student Answers, Key Points:

- 2 to 2
- They are the same.

The ratio is 2 to 2. There are the same amount of each. Let's find the unit rate. How do I do that? Possible Student Answers, Key Points:

- Divide!
- Red divided by yellow
- 2 divided by 2

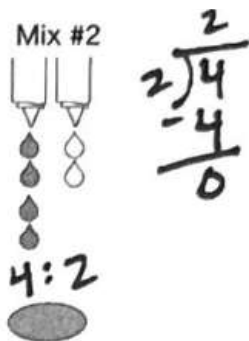
I divide! 2 divided by 2 is 1. That's 1 red drop per 1 yellow drop.

Let's look at the next mix. Look at that! It's NOT the same color orange! Why isn't it the same color orange? Possible Student Answers, Key Points:

- She put more red.

What is the ratio of red drops to yellow drops? Possible Student Answers, Key Points:

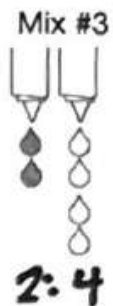
- 4 to 2
- There are double.



The ratio is 4 to 2. Let's find the unit rate. How do I do that? Possible Student Answers, Key Points:

- Divide!
- Red divided by yellow
- 4 divided by 2

I divide! 4 divided by 2 is 2. That's 2 red drops per 1 yellow drop. This is important! Notice that the unit rate is NOT the same. That makes sense though, right? The color is NOT the same. Rose switched up her formula for Mix #2, right? She got a much darker orange because there are 2 drops of red for every 1 drop of yellow.



Let's look at Mix #3. What is the ratio of red drops to yellow drops? Possible Student Answers, Key Points:

- 2 to 4
- There are half as many.



$$4 \overline{) 2} \begin{array}{r} 0.5 \\ -0 \\ \hline 2 \end{array} = \frac{1}{2}$$

The ratio is 2 to 4. Now this is going to be a little trickier but does anyone think they know the unit rate? **Possible Student Answers, Key Points:**

- We divide 2 by 4.
- It is half.

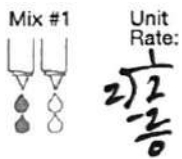
We have to keep the same order that we used for the other mixes, which was red drops divided by yellow drops. So this time it's not 4 divided by 2. It's 2 divided by 4. I can't get a whole number if I do 2 divided by 4 because 2 is smaller than 4 so I get a fraction, 2 over 4. You can simplify that. I happen to know 2 is half of 4. Notice

AGAIN that the unit rate is NOT the same. And that makes sense because the color is NOT the same. She got a much lighter orange this time because there was only half the red as yellow.

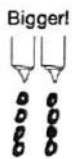
The ratios are **NOT proportional**

Let's fill in this blank. We saw that the unit rates are NOT the same. So we say that the ratios are NOT proportional. And we can see that because the paint mixes are different colors.

**Let's Talk (Slide 4):** This is our big idea for the day. *Point to the top of the slide and read the main idea in bold.* "When unit rates are the same, the number is the **CONSTANT OF PROPORTIONALITY.**" We're going to make some mixes where the unit rates are the same and find the constant of proportionality. Let's read. *Read the story about Rose.*



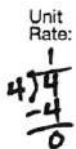
Let's start with Mix #1. We already know the unit rate for this one because we did it on the last slide. 2 divided by 2 is 1. That's 1 red drop per 1 yellow drop.



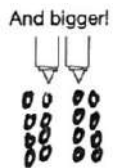
Now Rose wants to make bigger amounts of the SAME color. She doesn't want a darker orange or a lighter orange this time. So let's imagine she doubles the amount of red. Now she has four drops of red. What do you think Rose has to do with the yellow? **Possible Student Answers, Key Points:**

- She has to double the amount of yellow.
- She needs four drops of yellow.

If she is going to put in more red then she needs to put in more yellow the exact same way so the color stays the same. If she doubles the red, she has to double the yellow.



Let's see what happens to our unit rate! I do 4 divided by 4. That's 1! 1 red drop per 1 yellow drop. This time, it's the SAME unit rate. We kept the relationship between red and yellow. These ratios are proportional!



Let's do even more! I'm going to double the red again! Now I have 8 drops of red. What do you think Rose has to do with the yellow? **Possible Student Answers, Key Points:**

- She has to double the amount of yellow.
- She needs eight drops of yellow.

If she is going to put in more red then she needs to put in more yellow the exact same way so the color stays the same. If she doubles the red, she has to double the yellow.

Unit Rate:

$$\begin{array}{r} 1 \\ 8 \overline{)8} \\ \underline{8} \\ 0 \end{array}$$

Let's see what happens to our unit rate! I do 8 divided by 8. That's still 1! 1 red drop per 1 yellow drop. Once again, it's the SAME unit rate. We kept the relationship between red and yellow so we kept the unit rate. These ratios are still proportional!

The constant of proportionality is 1.

Now, we said that when unit rates are the same, the number is called the constant of proportionality. So there isn't extra math to do here. The unit rate was 1 and then it was 1 and then it was 1. So the constant of proportionality is 1.



Unit Rate:

$$\begin{array}{r} 2 \\ 4 \overline{)8} \\ \underline{4} \\ 0 \end{array}$$

**Let's Talk (Slide 5):** We can find the constant of proportionality using any set of proportional ratios. This says, "Let's make bigger amounts of Mix #2. Draw a picture. Find the unit rate." Now we're working with the darker orange. We already found the unit rate. It was 4 divided by 2. Now we have 2 drops of red per 1 drop of yellow.

Let's try to make more paint. We want it to be the same color. So let's imagine we double the amount of red. Now there are 8 drops of red. What do we have to do with the yellow? [Possible Student Answers, Key Points:](#)



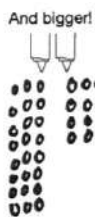
- We have to double the amount of yellow.
- We need four drops of yellow.

If we are going to put in more red then we need to put in more yellow. We increase it the exact same way we increased the red so the color stays the same. If she doubles the red, she has to double the yellow. That means 4 drops of yellow.

Unit Rate:

$$\begin{array}{r} 2 \\ 4 \overline{)8} \\ \underline{8} \\ 0 \end{array}$$

Let's see what happens to our unit rate! I do 8 divided by 4. That's 2! 2 red drops per 1 yellow drop. Look! It's the SAME unit rate as before. We kept the relationship between red and yellow. These ratios are proportional!



Okay, now we're going to go really crazy. Let's TRIPLE the red! That would be  $8 \times 3$ . That would be 24 drops of red. That's tough to even draw. What do you think Rose has to do with the yellow? [Possible Student Answers, Key Points:](#)

- She has to triple the amount of yellow.
- She needs 12 drops of yellow.

Once again, if she is going to put in more red then she needs to put in more yellow the exact same way so the color stays the same. If she triples the red, she has to triple the yellow.

Unit Rate:

$$\begin{array}{r} 2 \\ 12 \overline{)24} \\ \underline{24} \\ 00 \end{array}$$

Let's see what happens to our unit rate! I do 24 divided by 12. That's 2! We tripled our mix but it's still 2 red drops per 1 yellow drop. It's still the SAME unit rate. We kept the relationship between red and yellow, and these ratios are proportional!

The constant of proportionality is 2.

Now, just like before, we said that when unit rates are the same, the number is called the constant of proportionality. So there isn't extra math to do here. The unit rate was 2 and then it was 2 and then it was 2. So the constant of proportionality is 2.

yellow drops	red drops

**Let's Think (Slide 6):** We are going to be spending a lot more time with graphs but let's see our work lined up on a table so you can see why the constant of proportionality is so important. We're going to follow these steps. *Read the first step.* I did red drops divided by yellow drops. So I am going to put red on the right and yellow on the left. You'll see why in a minute.

yellow drops	red drops
2	4

Now I have to put in the numbers from my mixes. On the last slide, there were 4 drops of red and 2 drops of yellow.

yellow drops	red drops
2	4
4	8

Then we doubled it so there were 8 drops of red and 4 drops of yellow.

yellow drops	red drops
2	4
4	8
12	24

And then we tripled that so there were 24 drops of red and 12 drops of yellow.

yellow drops	red drops
2 x 2 4	
4 x 2 8	
12 x 2 24	

Next step! *Read the second step.* I told you, you'd see why we write the number we divided on the right hand side. Because now we can use it for a related fact and we can multiply across. Look! I use that constant of proportionality. 2 times 2 makes 4. 4 times 2 makes 8. 12 times 2 makes 24. This will work no matter how big our table gets.

yellow drops	red drops
2 x 2 4	
4 x 2 8	
12 x 2 24	

Next step! *Read the third step.* You might notice that we can also see patterns going up and down. That's because we doubled and tripled. So I can show that as 2 plus itself and 4 plus itself. You might have done some of this repeated addition on both sides in 6th grade.

yellow drops	red drops
2 x 2 4	
4 x 2 8	
12 x 2 24	

For the next line, we could think of it as 4 plus itself plus itself and 8 plus itself plus itself. You don't need to worry about all of this now. But these relationships are going to help us make sure that the numbers in our table really are proportional. *Read the fourth step.* For now, let's just focus on the constant of proportionality.

**Let's Try It (Slide 7):** Let's practice finding the constant of proportionality together. We just have to remember that it's the same as the unit rate so division each time. I am going to take you through step by step.

**Today we will generate proportions to find the constant of proportionality.**

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**WARM WELCOME**

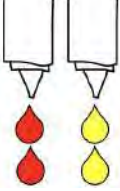







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## Let's Review:

We decide if two ratios are proportional using their unit rate.

Rose was mixing paints. What is the ratio of red to yellow drops for each mixture?

Mix #1	Mix #2	Mix #3
		
		

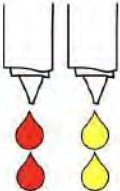
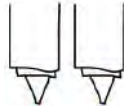
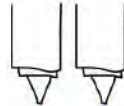



The ratios are \_\_\_\_\_.

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## Let's Talk:

When unit rates are the same, the number is the **CONSTANT OF PROPORTIONALITY**.

Rose wants to make bigger amounts of Mix #1. Draw a picture of what she could do. Find the unit rate of each mixture.

Mix #1	Unit Rate:	Bigger!	Unit Rate:	And bigger!	Unit Rate:
					
					

The constant of proportionality is \_\_\_\_\_.

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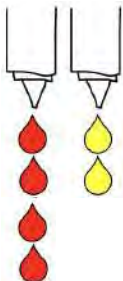


## Let's Talk:

We can find the constant of proportionality using any set of proportional ratios.

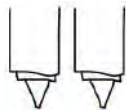
Let's make bigger amounts of Mix #2. Draw a picture. Find the unit rate.

Mix #2



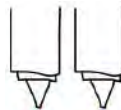
Unit Rate:

Bigger!



Unit Rate:

And bigger!



Unit Rate:



The constant of proportionality is \_\_\_\_\_.

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## Let's Think:

When we record proportional ratios in a table, we will see many relationships.

1. We usually put the number we divided on the right side of the table.
2. Then we can use our constant of proportionality to multiply HORIZONTALLY.
3. We will also see that the numbers are adding repeated VERTICALLY on each side.
4. In our next lesson, we can use this to find out other values that would work on our chart.

_____	_____
drops	drops

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# Let's Try It:

## Let's practice finding the constant of proportionality.

Name: \_\_\_\_\_

G7 U2 Lesson 2 - Let's Try It

1. Everyone knows that there are 4 tires on 1 car. What is the unit rate of tires per car?



_____	_____

2. Put the labels on the top of the table.

3. Use the information from problem #1 to fill in the first row.

4. Let's double the number of cars! Add on to the picture above.

5. Now what is the unit rate? \_\_\_\_\_ per \_\_\_\_\_

6. Use the information for problem #4 to fill in the next row of the table.

7. Imagine that there were 6 cars. Draw the picture below.

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# On your Own:





## Now it's time for you to do it on your own.

Name: \_\_\_\_\_

G7 U2 Lesson 2 - Independent Work

Remember: When the unit rates are the same, that number is the constant of proportionality.

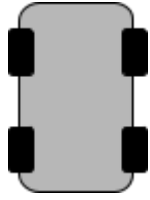
Double the picture then double it again. Use your pictures to complete the table then find the constant of proportionality.

<p>1.</p>   <table border="1" style="width: 100%;"> <tr><th style="width: 50%;">circles</th><th style="width: 50%;">triangles</th></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </table> <p>What is the constant of proportionality?</p>	circles	triangles							<p>2.</p>   <table border="1" style="width: 100%;"> <tr><th style="width: 50%;">circles</th><th style="width: 50%;">triangles</th></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </table> <p>What is the constant of proportionality?</p>	circles	triangles						
circles	triangles																
circles	triangles																
<p>3.</p> 	<p>4.</p> 																

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Name: \_\_\_\_\_

1. Everyone knows that there are 4 tires on 1 car. What is the unit rate of tires per car?



_____	_____

2. Put the labels on the top of the table.

3. Use the information from problem #1 to fill in the first row.

4. Let's double the number of cars! Add on to the picture above.

5. Now what is the unit rate? \_\_\_\_\_ per \_\_\_\_\_

6. Use the information for problem #4 to fill in the next row of the table.

7. Imagine that there were 3 cars. Draw the picture below.

8. Now what is the unit rate? \_\_\_\_\_ per \_\_\_\_\_

9. Use the information for problem #7 to fill in the next row of the table.

10. What do you notice about the table going from left to right?

---

11. What do you notice about the table going from top to bottom?

---

12. What is the constant of proportionality for this table? \_\_\_\_\_

13. Imagine that it costs \$6 to buy 2 cupcakes.



_____	_____

14. What is the unit rate?

\_\_\_\_\_ per \_\_\_\_\_

15. Put the labels on the top of the table.

16. Use the information from problem #13 to fill in the first row.

17. Let's draw another box in the picture above.

18. Now what is the unit rate? \_\_\_\_\_ per \_\_\_\_\_

19. Use the information for problem #16 to fill in the next row of the table.

20. Let's draw ANOTHER box in the picture above.

21. Now what is the unit rate? \_\_\_\_\_ per \_\_\_\_\_

22. Use the information for problem #19 to fill in the next row of the table.

23. What do you notice about the table going from left to right?

---

24. What do you notice about the table going from top to bottom?

---

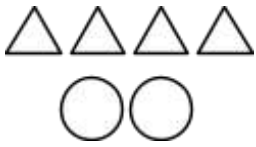
25. What is the constant of proportionality for this table? \_\_\_\_\_

Name: \_\_\_\_\_

Remember: When the unit rates are the same, that number is the constant of proportionality.

Double the picture then double it again. Use your pictures to complete the table then find the constant of proportionality.

1.



circles	triangles

What is the constant of proportionality?

2.



circles	triangles

What is the constant of proportionality?

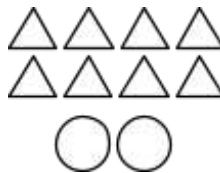
3.



circles	triangles

What is the constant of proportionality?

4.



circles	triangles

What is the constant of proportionality?



Double the picture then double it again. Use your pictures to complete the table then find the constant of proportionality.

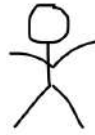
5.



eyes	ears

What is the constant of proportionality?

6.



arms	legs

What is the constant of proportionality?

7.



eyes	arms

What is the constant of proportionality?

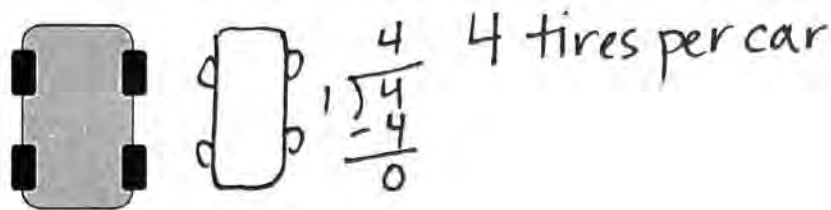
8.



fangs	nose

What is the constant of proportionality?

1. Everyone knows that there are 4 tires on 1 car. What is the unit rate of tires per car?



<u>car</u>	<u>tires</u>
1	4
2	8
3	12

2. Put the labels on the top of the table.

3. Use the information from problem #1 to fill in the first row.

4. Let's double the number of cars! Add on to the picture above.

5. Now what is the unit rate? 4 tires per car

$$\begin{array}{r} 4 \\ 2 \overline{)8} \\ \underline{-8} \\ 0 \end{array}$$

6. Use the information for problem #4 to fill in the next row of the table.

7. Imagine that there were **3** cars. Draw the picture below.



8. Now what is the unit rate? 4 tires per car

$$\begin{array}{r} 04 \\ 3 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

9. Use the information for problem #7 to fill in the next row of the table.

10. What do you notice about the table going from left to right?

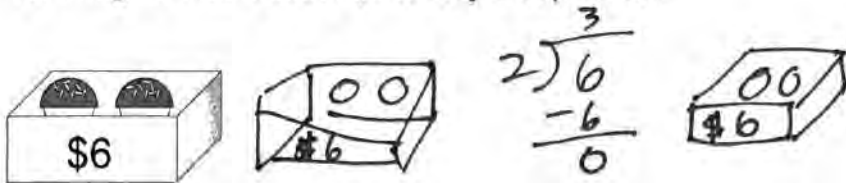
It is cars time 4 to get tires.

11. What do you notice about the table going from top to bottom?

It kept adding the number over and over on each side.

12. What is the constant of proportionality for this table? 4

13. Imagine that it costs \$6 to buy 2 cupcakes.



cupcake	dollars
2	\$6
4	\$12

14. What is the unit rate?

~~3~~ 3 dollars per cupcake

15. Put the labels on the top of the table.

16. Use the information from problem #13 to fill in the first row.

17. Let's draw another box in the picture above.

18. Now what is the unit rate? 3 dollars per cupcake

$$\begin{array}{r} 03 \\ 4 \overline{)12} \\ \underline{-12} \\ 0 \end{array}$$

19. Use the information for problem #16 to fill in the next row of the table.

20. Let's draw ANOTHER box in the picture above.

21. Now what is the unit rate? 3 dollars per cupcake

$$\begin{array}{r} 03 \\ 6 \overline{)18} \\ \underline{-18} \\ 00 \end{array}$$

22. Use the information for problem #19 to fill in the next row of the table.

23. What do you notice about the table going from left to right?

It is cupcakes times 3 to get dollars.

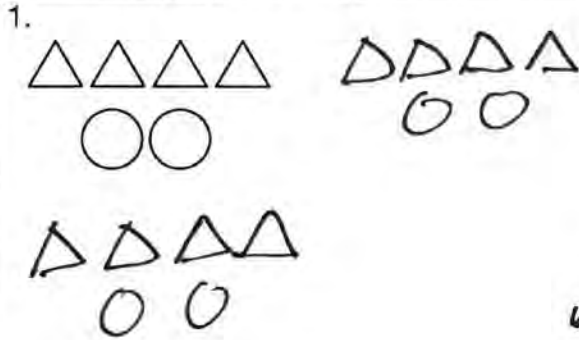
24. What do you notice about the table going from top to bottom?

It adds the 1st number over and over on each side.

25. What is the constant of proportionality for this table? 3

Remember: When the unit rates are the same, that number is the constant of proportionality.

Double the picture then double it again. Use your pictures to complete the table then find the constant of proportionality.

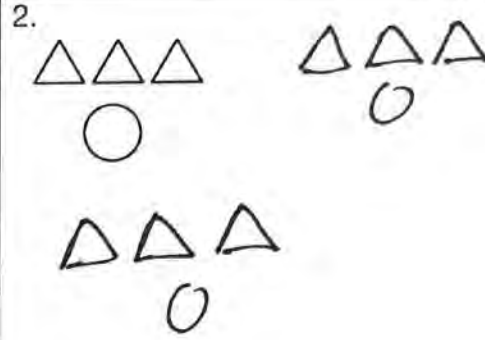


$$\begin{array}{r} 02 \\ 6 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

$$\begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 4 \overline{)8} \\ \underline{-8} \\ 0 \end{array}$$

circles	triangles
2	4
4	8
6	12

What is the constant of proportionality? 2

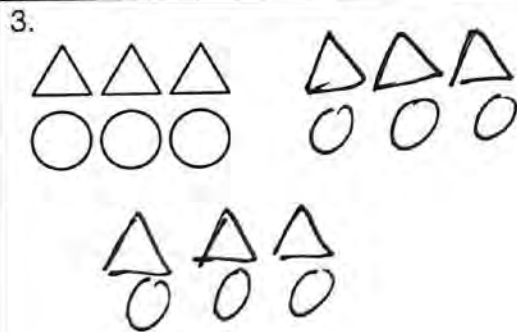


circles	triangles
1	3
2	6
3	9

$$\begin{array}{r} 3 \\ 1 \overline{)3} \\ \underline{-3} \\ 0 \end{array} \quad \begin{array}{r} 3 \\ 2 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

$$\begin{array}{r} 3 \\ 3 \overline{)9} \\ \underline{-9} \\ 0 \end{array}$$

What is the constant of proportionality? 3

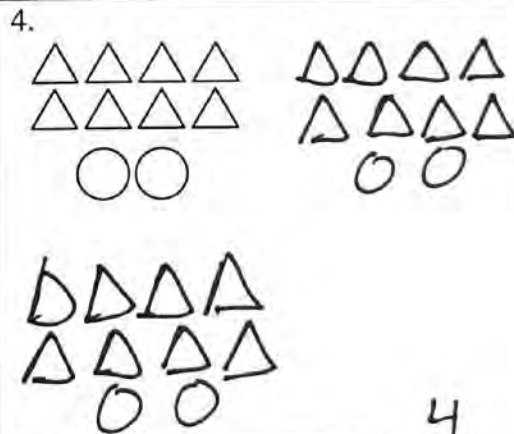


circles	triangles
3	3
6	6
9	9

$$\begin{array}{r} 1 \\ 3 \overline{)3} \\ \underline{-3} \\ 0 \end{array} \quad \begin{array}{r} 1 \\ 6 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 9 \overline{)9} \\ \underline{-9} \\ 0 \end{array}$$

What is the constant of proportionality? 1



circles	triangles
2	8
4	16
6	24

$$\begin{array}{r} 4 \\ 2 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 4 \\ 4 \overline{)16} \\ \underline{-16} \\ 00 \end{array}$$

$$\begin{array}{r} 06 \\ 4 \overline{)24} \\ \underline{-24} \\ 00 \end{array}$$

What is the constant of proportionality? 4

Double the picture then double it again. Use your pictures to complete the table then find the constant of proportionality.

5.



eyes	ears
1	2
2	4
3	6

$$\begin{array}{r} 2 \\ 1 \overline{)2} \\ \underline{-2} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 3 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

What is the constant of proportionality?  $2$

6.



arms	legs
2	2
4	4
6	6

$$\begin{array}{r} 1 \\ 2 \overline{)2} \\ \underline{-2} \\ 0 \end{array} \quad \begin{array}{r} 1 \\ 4 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 1 \\ 6 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

What is the constant of proportionality?  $1$

7.



eyes	arms
2	5
4	10
6	15

$$\begin{array}{r} 2\frac{1}{2} \\ 2 \overline{)5} \\ \underline{-4} \\ 1 \end{array} \quad \begin{array}{r} 2\frac{2}{4} \\ 4 \overline{)10} \\ \underline{-8} \\ 2 \end{array}$$

$$\begin{array}{r} 02\frac{3}{6} \\ 6 \overline{)15} \\ \underline{-12} \\ 3 \end{array}$$

What is the constant of proportionality?  $2\frac{1}{2}$

8.



fangs	nose
2	1
4	2
6	<del>2</del> 3

$$\begin{array}{r} 0\frac{1}{2} \\ 2 \overline{)1} \\ \underline{-0} \\ 1 \end{array} \quad \begin{array}{r} 0\frac{2}{4} \\ 4 \overline{)2} \\ \underline{-0} \\ 2 \end{array}$$

$$\begin{array}{r} 0\frac{3}{6} \\ 6 \overline{)3} \\ \underline{-0} \\ 3 \end{array}$$

What is the constant of proportionality?  $\frac{1}{2}$



Double the picture then double it again. Use your pictures to complete the table then find the constant of proportionality.

5.



eyes	ears
1	2
2	4
3	6

$$\begin{array}{r} 2 \\ 1 \overline{)2} \\ \underline{-2} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 3 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

What is the constant of proportionality?  $2$

6.



arms	legs
2	2
4	4
6	6

$$\begin{array}{r} 1 \\ 2 \overline{)2} \\ \underline{-2} \\ 0 \end{array} \quad \begin{array}{r} 1 \\ 4 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 1 \\ 6 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

What is the constant of proportionality?  $1$

7.



eyes	arms
2	5
4	10
6	15

$$\begin{array}{r} 2\frac{1}{2} \\ 2 \overline{)5} \\ \underline{-4} \\ 1 \end{array} \quad \begin{array}{r} 2\frac{2}{4} \\ 4 \overline{)10} \\ \underline{-8} \\ 2 \end{array} \quad \begin{array}{r} 2\frac{3}{6} \\ 6 \overline{)15} \\ \underline{-12} \\ 3 \end{array}$$

What is the constant of proportionality?  $2\frac{1}{2}$

8.



fangs	nose
2	1
4	2
6	<del>3</del>

$$\begin{array}{r} 0\frac{1}{2} \\ 2 \overline{)1} \\ \underline{-0} \\ 1 \end{array} \quad \begin{array}{r} 0\frac{2}{4} \\ 4 \overline{)2} \\ \underline{-0} \\ 2 \end{array} \quad \begin{array}{r} 0\frac{3}{6} \\ 6 \overline{)3} \\ \underline{-0} \\ 3 \end{array}$$

What is the constant of proportionality?  $\frac{1}{2}$

## **G7 U2 Lesson 3**

Find the constant of proportionality from information given on a table and use the constant of proportionality to fill information on a table.

**G7 U2 Lesson 3 - Today we will use the constant of proportionality to fill information on a table.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we're going to keep working with the constant of proportionality. You have already found it so today we'll explore how it can be used to fill in a ratio table.

**Let's Review (Slide 3):** We already learned that we can collect ratios on a table and check if they are proportional. Let's do an example with these pictures. It says, "Use the pictures to fill in the first 3 rows of the table." I look at my table and there are triangles. *Point to where it says "triangles" on top of the table.* And there are sides. *Point to where it says "sides" on top of the table.* I just need to count to fill this in. *Point to the triangles as you count.* Here, I see 1, 2. So I put a 2 in the triangles column. Now let's trace the sides. *Use your fingertip to trace each side as you count.* I see 1, 2, 3, 4. So I put a 4 in the side column.

triangles	sides
2	4

triangles	sides
2	4
4	8

triangles	sides
2	4
4	8
6	12

Let's do the next one. *Model counting the triangles out loud while you point. Then count the side out loud while you trace them with your fingers.* There are 4 triangles and 8 sides. I am going to write that in my table.

Let's do the next one. *Model counting the triangles out loud while you point. Then count the side out loud while you trace them with your fingers.* There are 6 triangles and 12 sides. I am going to write that in my table.

Use the pictures to fill in the first 3 rows of the table.

Do you think the ratios are proportional?

If so, what is the constant of proportionality?

triangles	sides
2	4
4	8
6	12

Now it says, "Do you think the ratios are proportional?"

What do you think? [Possible Student Answers, Key Points:](#)

**Points:**

- They are proportional because it is just the same picture over and over.
- They are proportional because it is always 2 sides per triangle.
- They are proportional because they have the same unit rate.
- They are proportional because they have the same constant of proportionality.
- They are proportional because you can keep adding 2 to the triangles and 4 to the sides columns.

In our last lesson we learned that if the unit rate is the same for a set of ratios then they are proportional. So let's find the unit rate. For the first row, I am going to do 4 divided by 2 is 2. That's 2 sides per triangle. Let's do the next one. It's 8 divided by 4 is 2. That's 2 sides per triangle. Let's do the next one. It's 12 divided by 6 is 2. That's 2 sides per triangle. So, are they proportional? YES! Because the unit rates are the same.

Use the pictures to fill in the first 3 rows of the table.

Do you think the ratios are proportional?

If so, what is the constant of proportionality?

triangles	sides
2	4
4	8
6	12

So then, what is the constant of proportionality? That's easy. It's just 2. It's just the unit rate we already found.

triangles	sides
2	4
4	8
6	12

**Let's Talk (Slide 4):** Now here's the cool thing. *Read the top of the slide.* "When there is a constant of proportionality, we can use it to find missing values." The first thing we need to think about is here: "Notice that the constant of proportionality multiplies each row from left to right. Write it in the circles." So I am going to write in the circle, "times 2" and "times 2" and "times 2." And I don't even have any numbers in this bottom row but I know whatever numbers we put will have to have that same relationship so "times 2."

triangles	sides
2	4
4	8
6	12
12	24

*Read from the slide.* "Now we can use this to find out how many sides there would be for a picture with 12 triangles. Let's use the table then draw to check." If I use the table, I am going to put a 12 in the triangle column. I have to be careful because there could be a different problem that asks about 12 sides. So it is not always going to be the first column. It is all based on the words. The problem said "12 triangles" so 12 goes in the triangles column. Now I can just do the math I have in the circle. 12 times 2 is 24 so there must be 24 sides.



Let's draw a picture to check. I drew 12 triangles. Now let's count the sides. We were right!

**Let's Think (Slide 5):** There's one more idea that we need to put together here. This is the same table we just saw. The numbers were just brought over from the last slide to this slide. Now we can keep exploring. *Read from the slide.* "If we can multiply by the constant of proportionality then we can divide by it." That's because multiplication and division are **OPPOSITES!**

...because multiplication and division are opposites

...because multiplication and division are opposites

So if we can multiply by the constant of proportionality from left to right then we can divide by it from right to left.

Instead of asking "how many sides would on a picture with 12 triangles" we can ask, "how many triangles would be picture with 20 sides?"

*Keep reading the slide as you fill in the words.* So if we can multiply by the constant of proportionality from left to right then we can also **DIVIDE** by it from **RIGHT** to **LEFT**. Let's look at it row by row. *Point from the left column across to the right column.* We can do 2 times 2 makes 4. *Point from the right column across to the left column.* But we can also think of it as 4 divided by 2 makes 2. Let's keep going. 4 times 2 is 8 or 8 divided by 2 is 4. Next row, 6 times 2 is 12 and 12 divided by 2 is 6.

triangles	sides
2	4
4	8
6	12
	20

*Read the rest of the slide.* Now, instead of asking "how many sides would be on a picture with 12 triangles" we can ask, "how many triangles would be on a picture with 20 sides? This is a totally **DIFFERENT** problem than the last one because it is asking about 20 sides now - not 20 triangles. So now I am going to put the 20 in the sides column.

triangles	sides
2	4
4	8
6	12
10	20

We can still think of it as multiplication but now instead of multiplying 20, we are asking, "what times 2 makes 20?" Or we can do the opposite! Division! It would be 20 divided by 2 makes 10.



Let's draw a picture to check. I am going to draw until I have 20 sides. Now let's count the triangles. There are 10. So, from now on, we are going to find the constant of proportionality by dividing like we did before. But now

when we put it on our table. We can multiply from left to right or divided from right to left.

**Let's Try It (Slide 6):** Let's practice! I am going to take you through step by step.



# WARM WELCOME



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**Today we will use the constant of proportionality to fill information on a table.**

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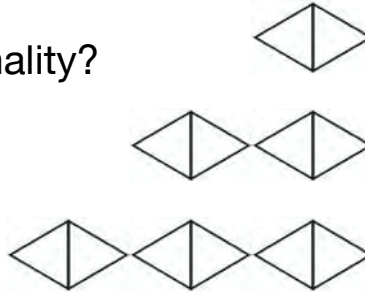
## Let's Review:

We can collect ratios on a table and check if they are proportional.

Use the pictures to fill in the first 3 rows of the table.

Do you think the ratios are proportional?

If so, what is the constant of proportionality?



triangles	sides

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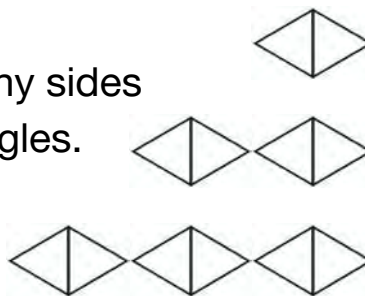
## Let's Talk:

When there is a constant of proportionality, we can use it to find missing values.

Notice that the constant of proportionality multiplies each row from left to right. Write it in the circles.

Now we can use this to find out how many sides there would be for a picture with 12 triangles.

Let's use the table then draw to check.



triangles	sides
2	○ 4
4	○ 8
6	○ 12
	○

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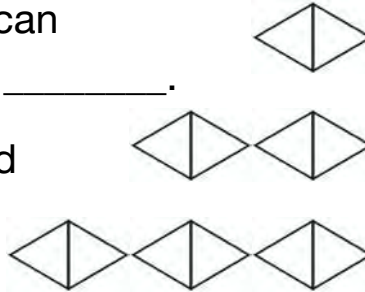
# Let's Think:

If we can multiply by the constant of proportionality then we can divide by it.

...because multiplication and division are \_\_\_\_\_!

So if we can multiply by the constant of proportionality from left to right then we can \_\_\_\_\_ by it from \_\_\_\_\_ to \_\_\_\_\_.

Instead of asking "how many sides would on a picture with 12 triangles" we can ask, "how many triangles would be picture with 20 sides?"



triangles		sides
2	(x2)	4
4	(x2)	8
6	(x2)	12
	(x2)	

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# Let's Try It:

Let's use the constant of proportionality to fill in tables together.

Name: \_\_\_\_\_

G7 U2 Lesson 3 - Let's Try It

1. In our last lesson, we created a table based on 4 tires per 1 car. Use the pictures to fill in the top 3 rows of the table.

	cars	tires
		( )
		( )
		( )
		( )
		( )

2. What is the constant of proportionality? \_\_\_\_\_

3. Use the constant of proportionality to write the correct multiplication in the circles on the table.

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# On your Own:

## Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 3 - Independent Work

Remember: We can multiply or divide by the constant of proportionality.

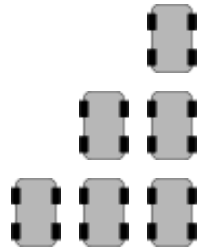
Find the constant of proportionality and complete the sentence. Put your answers in the table.

<p>1. Jeff gets paid \$10 for 2 hours of work.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>hours</th> <th>dollars</th> </tr> </thead> <tbody> <tr><td>2</td><td>10</td></tr> <tr><td>4</td><td>20</td></tr> <tr><td>6</td><td>30</td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table> <p>What is the constant of proportionality? _____</p> <p>9 hours of work earns \$ ____.</p> <p>_____ hours of work earns \$25.</p>	hours	dollars	2	10	4	20	6	30					<p>2. We need 4 tbsp of sugar for 2 cups of flour.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>cups</th> <th>tbsp</th> </tr> </thead> <tbody> <tr><td>2</td><td>4</td></tr> <tr><td>4</td><td>8</td></tr> <tr><td>6</td><td>12</td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table> <p>What is the constant of proportionality? _____</p> <p>12 cups of flour needs _____ tbsp of sugar.</p> <p>_____ cups of flour need 20 tbsp of sugar.</p>	cups	tbsp	2	4	4	8	6	12				
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Name: \_\_\_\_\_

1. In our last lesson, we created a table based on 4 tires per 1 car. Use the pictures to fill in the top 3 rows of the table.



cars	tires
	○
	○
	○
	○
	○

2. What is the constant of proportionality? \_\_\_\_\_

3. Use the constant of proportionality to write the correct multiplication in the circles on the table.

**Use the table to find how many tires would be on 7 cars.**

4. Write 7 cars in the correct place on the table.

5. Write the constant of proportionality in the correct place on the table.

6. Find the number of tires that would complete the table and complete the sentence below.

7 cars would have \_\_\_\_\_ tires.

**Use the table to find how many cars would have 40 tires.**

7. Write 40 tires in the correct place on the table.

8. Write the constant of proportionality in the correct place on the table.

9. Find the number of cars that would complete the table and complete the sentence below.

\_\_\_\_\_ cars would have 40 tires.



10. In our last lesson, we also used the cupcakes shown below. Use the pictures to fill in the top 3 rows of the table.

cupcakes	dollars
	○
	○
	○
	○
	○

11. What is the constant of proportionality? \_\_\_\_\_

12. Use the constant of proportionality to write the correct multiplication in the circles on the table.

**Use the table to find the cost of 9 cupcakes.**

13. Write 9 cupcakes in the correct place on the table.

14. Write the constant of proportionality in the correct place on the table.

15. Find the number of dollars that would complete the expression and complete the sentence.

9 cupcakes would cost \_\_\_\_\_ dollars.

**Use the table to find how many cupcakes can be bought with \$30.**

16. Write 30 dollars in the correct place on the table.

17. Write the constant of proportionality in the correct place on the table.

18. Find the number of cupcakes that would complete the expression and complete the sentence.

\_\_\_\_\_ cupcakes would cost \$30.

Name: \_\_\_\_\_

Remember: We can multiply or divide by the constant of proportionality.

Find the constant of proportionality and complete the sentence. Put your answers in the table.

1. Jeff gets paid \$10 for 2 hours of work.

hours	dollars
2	10
4	20
6	30

What is the constant of proportionality? \_\_\_\_\_

9 hours of work earns \$\_\_\_\_\_.

\_\_\_\_\_ hours of work earns \$25.

2. We need 4 tbsp of sugar for 2 cups of flour.

cups	tbsp
2	4
4	8
6	12

What is the constant of proportionality? \_\_\_\_\_

12 cups of flour needs \_\_\_\_\_ tbsp of sugar.

\_\_\_\_\_ cups of flour need 20 tbsp of sugar.

3. It takes 20 minutes to bike 5 miles.

miles	minutes
5	20
10	40
15	60

What is the constant of proportionality? \_\_\_\_\_

2 miles will take \_\_\_\_\_ minutes.

\_\_\_\_\_ miles will take 12 minutes.

4. Lisa gets a \$3 discount off every \$9 spent.

\$ discount	\$ spent
3	9
6	18
9	27

What is the constant of proportionality? \_\_\_\_\_

Lisa will get a \_\_\_\_\_ discount on \$12 spent.

Lisa will get a \$5 discount on \$\_\_\_\_\_ spent.

Find the constant of proportionality and complete the sentence. Put your answers in the table.

5. Jeff gets paid \$10 for 2 hours of work.

hours	dollars
2	10
4	20
6	30

What is the constant of proportionality? \_\_\_\_\_

9 hours of work earns \$\_\_\_\_\_.

\_\_\_\_\_ hours of work earns \$25.

6. We need 4 tbsp of sugar for 2 cups of flour.

cups	tbsp
2	4
4	8
6	12

What is the constant of proportionality? \_\_\_\_\_

12 cups of flour needs \_\_\_\_\_ tbsp of sugar.

\_\_\_\_\_ cups of flour need 20 tbsp of sugar.

7. It takes 20 minutes to bike 5 miles.

miles	minutes
5	20
10	40
15	60

What is the constant of proportionality? \_\_\_\_\_

2 miles will take \_\_\_\_\_ minutes.

\_\_\_\_\_ miles will take 12 minutes.

8. Lisa gets a \$3 discount off every \$9 spent.

\$ discount	\$ spent
3	9
6	18
9	27

What is the constant of proportionality? \_\_\_\_\_

Lisa will get a \_\_\_\_\_ discount on \$12 spent.

Lisa will get a \$5 discount on \$\_\_\_\_\_ spent.

1. In our last lesson, we created a table based on 4 tires per 1 car. Use the pictures to fill in the top 3 rows of the table.



cars		tires
1	(x4)	4
2	(x4)	8
3	(x4)	12
7	(x4)	28
10	(x4)	40

2. What is the constant of proportionality? 4

$$\begin{array}{r} 4 \\ 1 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 4 \\ 2 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 04 \\ 3 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

3. Use the constant of proportionality to write the correct multiplication in the circles on the table.

**Use the table to find how many tires would be on 7 cars.**

- Write 7 cars in the correct place on the table.
- Write the constant of proportionality in the correct place on the table.
- Find the number of tires that would complete the table and complete the sentence below.

$$7 \times 4 = ?$$

$$7 \times 4 = 28$$

7 cars would have 28 tires.

**Use the table to find how many cars would have 40 tires.**

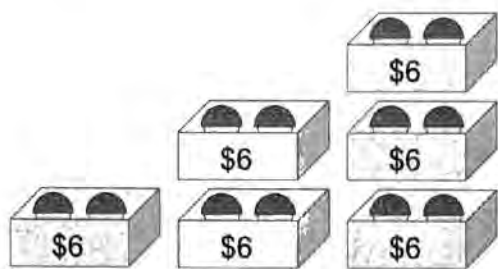
- Write 40 tires in the correct place on the table.
- Write the constant of proportionality in the correct place on the table.
- Find the number of cars that would complete the table and complete the sentence below.

$$? \times 4 = 40$$

$$10 \times 4 = 40$$

10 cars would have 40 tires.

10. In our last lesson, we also used the cupcakes shown below. Use the pictures to fill in the top 3 rows of the table.



cupcakes		dollars
2	$\times 3$	6
4	$\times 3$	12
6	$\times 3$	18
9	$\times 3$	27
10	$\times 3$	30

11. What is the constant of proportionality? 3

$$\begin{array}{r} 3 \\ 2 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

$$\begin{array}{r} 03 \\ 4 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

$$\begin{array}{r} 03 \\ 6 \overline{)18} \\ \underline{-18} \\ 00 \end{array}$$

12. Use the constant of proportionality to write the correct multiplication in the circles on the table.

**Use the table to find the cost of 9 cupcakes.**

13. Write 9 cupcakes in the correct place on the table.

14. Write the constant of proportionality in the correct place on the table.

15. Find the number of dollars that would complete the expression and complete the sentence.

$$\begin{array}{l} 9 \times 3 = ? \\ 9 \times 3 = 27 \end{array}$$

9 cupcakes would cost 27 dollars.

**Use the table to find how many cupcakes can be bought with \$30.**

16. Write 30 dollars in the correct place on the table.

17. Write the constant of proportionality in the correct place on the table.

18. Find the number of cupcakes that would complete the expression and complete the sentence.

$$\begin{array}{l} ? \times 3 = 30 \\ 10 \times 3 = 30 \end{array}$$

10 cupcakes would cost \$30.



Remember: We can multiply or divide by the constant of proportionality.

Find the constant of proportionality and complete the sentence. Put your answers in the table.

1. Jeff gets paid \$10 for 2 hours of work.

hours		dollars
2	$\times 5$	10
4	$\times 5$	20
6	$\times 5$	30
9	$\times 5$	45
5	$\times 5$	25

$$\begin{array}{r} 05 \\ 2 \overline{)10} \\ \underline{-10} \\ 00 \end{array} \quad \begin{array}{r} 05 \\ 4 \overline{)20} \\ \underline{-20} \\ 00 \end{array}$$

$$\begin{array}{r} 05 \\ 6 \overline{)30} \\ \underline{-30} \\ 00 \end{array}$$

What is the constant of proportionality? 5

9 hours of work earns \$ 45.

5 hours of work earns \$25.

2. We need 4 tbsp of sugar for 2 cups of flour.

cups		tbsp
2	$\times 2$	4
4	$\times 2$	8
6	$\times 2$	12
12	$\times 2$	24
10	$\times 2$	20

$$\begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 4 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 02 \\ 6 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

What is the constant of proportionality? 2

12 cups of flour needs 24 tbsp of sugar.

10 cups of flour need 20 tbsp of sugar.

3. It takes 20 minutes to bike 5 miles.

miles		minutes
5	$\times 4$	20
10	$\times 4$	40
15	$\times 4$	60
2	$\times 4$	8
3	$\times 4$	12

$$\begin{array}{r} 04 \\ 5 \overline{)20} \\ \underline{-20} \\ 00 \end{array} \quad \begin{array}{r} 04 \\ 10 \overline{)40} \\ \underline{-40} \\ 00 \end{array} \quad \begin{array}{r} 04 \\ 15 \overline{)60} \\ \underline{-60} \\ 00 \end{array}$$

What is the constant of proportionality? 4

2 miles will take 8 minutes.

3 miles will take 12 minutes.

4. Lisa gets a \$3 discount off every \$9 spent.

\$ discount		\$ spent
3	$\times 3$	9
6	$\times 3$	18
9	$\times 3$	27
4	$\times 3$	12
5	$\times 3$	15

$$\begin{array}{r} 3 \\ 3 \overline{)9} \\ \underline{-9} \\ 0 \end{array} \quad \begin{array}{r} 03 \\ 6 \overline{)18} \\ \underline{-18} \\ 00 \end{array} \quad \begin{array}{r} 03 \\ 9 \overline{)27} \\ \underline{-27} \\ 00 \end{array}$$

What is the constant of proportionality? 3

Lisa will get a \$4 discount on \$12 spent.

Lisa will get a \$5 discount on \$ 12 spent.

Find the constant of proportionality and complete the sentence. Put your answers in the table.

5. Jeff gets paid \$10 for 2 hours of work.

hours		dollars
2	$\times 5$	10
4	$\times 5$	20
6	$\times 5$	30
9	$\times 5$	45
5	$\times 5$	25

$$\begin{array}{r} 05 \\ 2 \overline{)10} \\ \underline{-10} \\ 00 \end{array} \quad \begin{array}{r} 05 \\ 4 \overline{)20} \\ \underline{-20} \\ 00 \end{array} \quad \begin{array}{r} 05 \\ 6 \overline{)30} \\ \underline{-30} \\ 00 \end{array}$$

What is the constant of proportionality? 5

9 hours of work earns \$ 45.

5 hours of work earns \$25.

6. We need 4 tbsp of sugar for 2 cups of flour.

cups		tbsp
2	$\times 2$	4
4	$\times 2$	8
6	$\times 2$	12
12	$\times 2$	24
10	$\times 2$	20

$$\begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 2 \\ 4 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 02 \\ 6 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

What is the constant of proportionality? 2

12 cups of flour needs 24 tbsp of sugar.

10 cups of flour need 20 tbsp of sugar.

7. It takes 20 minutes to bike 5 miles.

miles		minutes
5	$\times 4$	20
10	$\times 4$	40
15	$\times 4$	60
2	$\times 4$	8
3	$\times 4$	12

$$\begin{array}{r} 04 \\ 5 \overline{)20} \\ \underline{-20} \\ 00 \end{array} \quad \begin{array}{r} 04 \\ 10 \overline{)40} \\ \underline{-40} \\ 00 \end{array} \quad \begin{array}{r} 04 \\ 15 \overline{)60} \\ \underline{-60} \\ 00 \end{array}$$

What is the constant of proportionality? 4

2 miles will take 8 minutes.

3 miles will take 12 minutes.

8. Lisa gets a \$3 discount off every \$9 spent.

\$ discount		\$ spent
3	$\times 3$	9
6	$\times 3$	18
9	$\times 3$	27
4	$\times 3$	12
5	$\times 3$	15

$$\begin{array}{r} 3 \\ 3 \overline{)9} \\ \underline{-9} \\ 0 \end{array} \quad \begin{array}{r} 03 \\ 6 \overline{)18} \\ \underline{-18} \\ 00 \end{array} \quad \begin{array}{r} 03 \\ 9 \overline{)27} \\ \underline{-27} \\ 00 \end{array}$$

What is the constant of proportionality? 3

Lisa will get a 4 discount on \$12 spent.

Lisa will get a \$5 discount on \$ 15 spent.

## **G7 U2 Lesson 4**

Write equations to represent a proportional relationship described in a table.

**G7 U2 Lesson 4 - Today we will write two equations to represent a proportional relationship in a table.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will be writing two equations to represent a proportional relationship in a table. We will see how this is not different than what we've been doing. It's just another way to represent the same idea.

stacks	blocks
1	3
12	
	12

**Let's Review (Slide 3):** We know we can multiply or divide by the constant of proportionality. Let's apply it to a problem. *Read the story.* "Jacob is using stacks of blocks to build a wall. Use the picture to fill in the first 3 rows of the table." I see in this first picture that I have 1 stack so I will write 1 under the word stacks. I count the blocks and there are 1 - 2 - 3. So I will write 3 under the word blocks.

stacks	blocks
1	3
2	6
12	
	12

You tell me what to write for the next picture. **Possible Student Answers, Key Points:**

- There are 2 stacks and 6 blocks.
- Put 2 in the stacks column and 6 in the blocks column.

stacks	blocks
1	3
2	6
3	9
12	
	12

You tell me what to write for the next picture. **Possible Student Answers, Key Points:**

- There are 3 stacks and 9 blocks.
- Put 3 in the stacks column and 9 in the blocks column.

stacks	blocks
1	3
2	6
3	9
12	
	12

Now, how do I find the constant of proportionality? **Possible Student Answers, Key Points:**

- You can divide each ratio.
- You can see that it is always "times 3" going across.

I can kind of see that it is "times 3" going across. But if I couldn't see it then I can just divide each ratio. I am going to divide 3 by 1 and get 3. I am going to divide 6 by 2 and get 3. I am going to divide 9 by 3 and get 3.

stacks	blocks
1	3
2	6
3	9
12	
	12

The constant of proportionality is 3. And I can write that in circles that show me what math to do from left to write.

stacks	blocks
1	3
2	6
3	9
12	36
4	12

Now, the final question is, "how can we use the constant of proportionality to find the missing values?" This is what we learned in our last lesson. It is easy to see the 12 x 3. That's makes 36. What about this 12 in the blocks column. It is NOT 12 times 3. It is something times 3 to make 12. This is where I have to work backwards. I have to do the opposite. I have to divide. 12 divided by 3 makes 4. And that makes sense right, there are always fewer stacks than blocks.

**Let's Talk (Slide 4):** That is some really special thinking that we just did. And in mathematics, we like to find a way to represent the special thinking especially when it will work over and over for any number. *Read the heading.* "An equation with variables can represent the operations we see on the table." Variables are letters that represent a place where you can plug in any number. So if I have x in an equation. Then I am saying you can put 1 in the place of x or 2 in the place of x, etc. In the case of our example with Jacob, it says, "Let x stand for the number of stacks and y stand for the number of

x stacks	y blocks
1	3
2	6
3	9

blocks.” That’s important because remember on the last slide, when there were 12 stacks, we multiplied. But when there were 12 blocks, we divided. I am going to write an x over the stacks column and a y over the blocks column so that I remember which side is which.

Let's list the equations we wrote in numbers then we'll substitute words then we'll substitute variables:

$$\begin{aligned} 1 \times 3 &= 3 \\ 2 \times 3 &= 6 \\ 3 \times 3 &= 9 \end{aligned}$$

Now, let’s list the equations we wrote in numbers. We have 1 x 3 equals 3, 2 x 3 equals 6 and 3 x 3 equals 9.

Let's list the equations we wrote in numbers then we'll substitute words then we'll substitute variables:

$$\begin{aligned} 1 \times 3 &= 3 \\ 2 \times 3 &= 6 \\ 3 \times 3 &= 9 \\ \text{stacks} \times 3 &= \text{blocks} \end{aligned}$$

If I substitute words for these equations, every time I multiplied, it was stacks x 3 equals blocks.

Let's list the equations we wrote in numbers then we'll substitute words then we'll substitute variables:

$$\begin{aligned} 1 \times 3 &= 3 \\ 2 \times 3 &= 6 \\ 3 \times 3 &= 9 \\ \text{stacks} \times 3 &= \text{blocks} \\ x \cdot 3 &= y \quad y = 3x \end{aligned}$$

And now for the final step, if I put in my variables instead of the words, we can write x times 3 is y. Sometimes we write it like this:  $3x = y$  or  $y = 3x$ . They all mean the same thing: multiply the numbers on the x side of the table which means multiply the stacks.

Let's list the equations we wrote in numbers then we'll substitute words then we'll substitute variables:

$$\begin{aligned} 1 \times 3 &= 3 \\ 2 \times 3 &= 6 \\ 3 \times 3 &= 9 \\ \text{stacks} \times 3 &= \text{blocks} \\ x \cdot 3 &= y \quad y = 3x \quad \frac{y}{3} = x \end{aligned}$$

But we know that we can always do the opposite operation too, right? So let’s do our division list. It is 3 divided by 1 equals 3 and 6 divided by 2 equals 3 and 9 divided by 3 equals 3. If I substitute words for these equations, every time I divided, it was blocks divided by 3 equals stacks. And below, for the final step, if I put in my variables instead of the words, we can write y divided by 3 is x. Sometimes we write it like this:  $y \text{ over } 3$

equals x or  $x = y \text{ divided by } 3$ . They all mean the same thing: divide the numbers on the y side of the table which means divide the blocks.

stacks	blocks
1	3
2	6
3	9
15	

**Let's Think (Slide 5):** Let’s put this into practice for a real life example. You’re not going to need to do all this today but I want you to see how we use our equations. This says, “we can use any of our equivalent equations to solve for a missing value.” Let’s read. “Imagine Jacob made 15 stacks. Put the number on the table and solve. Then use an equation to solve.” 15 stacks mean I want to put 15 in the stacks column. 15 is x.

Imagine Jacob made 15 stacks. Put the number on the table and solve. Then use an equation to solve.

$$\begin{aligned} y &= 3x \\ y &= 3 \cdot 15 \\ y &= 45 \end{aligned}$$

So I can use the equation x times 3 = y. When I substitute 15 for x, I get 15 times 3 equals y or 15 times 3 equals 45.

stacks	blocks
1	3
2	6
3	9
15	45

Now I know 45 goes on the y side of the equation.

stacks	blocks
1	3
2	6
3	9
15	45
	15

Let’s read the next one, “Imagine Jacob used 15 blocks. Put the number on the table and solve. Then use an equation to solve.” In this case, it is not 15 stacks, it is 15 blocks. I would write 15 on this side of the equation. And so I wouldn’t multiply by 3, I would do the opposite, I would divide. 15 divided by 3 is 5.



Imagine Jacob used 15 blocks. Put the number on the table and solve. Then use an equation to solve.

$$\begin{array}{l} 3x = 4 \\ 3x = 15 \\ \hline 3 \quad 3 \\ x = 5 \end{array}$$

Now let's think about using an equation. If I use the equation, I just used, it works but I have to put the 15 in a different place. The equation was  $x$  times 3 equals  $y$ . Now  $y$  is 15 so I substitute the  $y$ . I get  $x$  times 3 equals 15. Do you see how I don't multiply the 15? I need something times 3 to make 15. I can work backwards using algebra to solve. I divide by 3 on both sides. I get  $x = 5$ . Notice that I ended up dividing after all.

Imagine Jacob used 15 blocks. Put the number on the table and solve. Then use an equation to solve.

$$\begin{array}{l} 3x = 4 \\ 3x = 15 \\ \hline 3 \quad 3 \\ x = 5 \end{array} \quad \begin{array}{l} y \div 3 = x \\ 15 \div 3 = x \\ \hline 5 = x \end{array}$$

Another way that I could do this is to use the other related equation. Instead of  $x$  times 3 equals  $y$ , I could use  $y$  divided by 3 equals  $x$ . Then I substitute 15 for  $y$  just like before. But the equation says 15 divided by 3 equals  $x$ . Which becomes  $5 = x$ . We get the same answer.

stacks	blocks
1	3
2	6
3	9
15	45
5	15

I am going to put the 5 on my table. Here's the main idea: if you can think of two opposite equations for your table then you can solve for the missing values in either column. I'm not going to ask you to find missing values today but you are going to have to write two opposite equations.

**Let's Try It (Slide 6):** Let's practice! I am going to take you through step by step.

# WARM WELCOME



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**Today we will write two equations to represent a proportional relationship in a table.**

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## Let's Review:

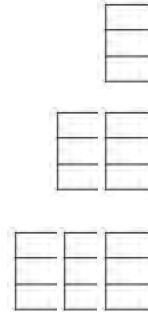
**We can multiply or divide by the constant of proportionality.**

Jacob is using stacks of blocks to build a wall.

Use the pictures to fill in the first 3 rows of the table.

What is the constant of proportionality?

How can we use the constant of proportionality to find the missing values?



stacks	blocks
12	
	12

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## Let's Talk:

**An equation with variables can represent the operations we see on the table.**

Let  $x$  stand for the number of stacks and  $y$  stand for the number of blocks.

Let's list the equations we wrote in numbers then we'll substitute words then we'll substitute variables:

stacks	blocks
1	3
2	6
3	9

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## Let's Think:

We can use any equivalent equation to solve for a missing value.

Imagine Jacob made 15 stacks. Put the number on the table and solve. Then use an equation to solve.

Imagine Jacob used 15 blocks. Put the number on the table and solve. Then use an equation to solve.

stacks	blocks
1	3
2	6
3	9

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## Let's Try It:

Let's find two related equations for each table together.

Name: \_\_\_\_\_ G7 U2 Lesson 4 - Let's Try It

**Find the constant of proportionality. Then write TWO equations that describe the table.**

1. The spider below has 6 legs and 2 eyes. Fill in the top three rows of the table based on the spiders you see.



x	y
eyes	legs
	○
	○
	○
	○
	○

2. What is the constant of proportionality? \_\_\_\_ Put it in the circles.

3. Write a multiplication equation using the variables,  $x$  and  $y$ . \_\_\_\_\_

4. Write a division equation using the variables,  $x$  and  $y$ . \_\_\_\_\_

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# On your Own:

## Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 4 - Independent Work

Remember: We can multiply or divide by the constant of proportionality.

Find the constant of proportionality. Then write TWO equations that describe the table. Use the equation to fill in the rest of the rows.

1. Ben gets 2 hours of HW for 8 hours of class.

$x$	$y$
hours of HW	hours of class
2	8
4	16
6	24

Equations:

---



---

2. There are 3 teachers for every 15 kids.

$x$	$y$
teachers	kids
3	15
6	30
9	45

Equations:

---



---

3. It takes 20 minutes to solve 10 math facts.

$x$	$y$
-----	-----

4. At his bake shop, Ren sells 8 cookies for \$2.

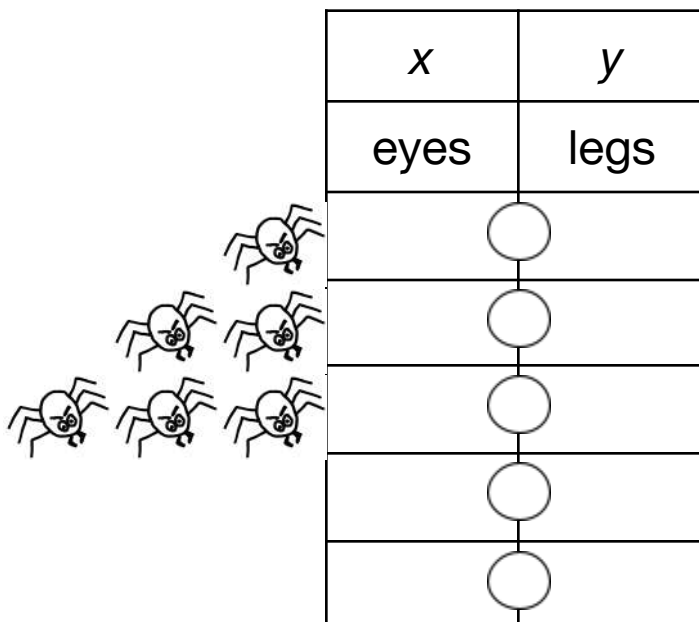
$x$	$y$
-----	-----

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**Find the constant of proportionality. Then write TWO equations that describe the table.**

1. The spider below has 6 legs and 2 eyes. Fill in the top three rows of the table based on the spiders you see.



$x$	$y$
eyes	legs
	○
	○
	○
	○
	○
	○

2. What is the constant of proportionality? \_\_\_\_\_ Put it in the circles.

3. Write a multiplication equation using the variables,  $x$  and  $y$ . \_\_\_\_\_

4. Write a division equation using the variables,  $x$  and  $y$ . \_\_\_\_\_

**Use the equation to fill in the rest of the rows.**

5. Let's imagine  $x$  is \_\_\_\_\_. Plug  $x$  into the equation you wrote for #3 and solve.

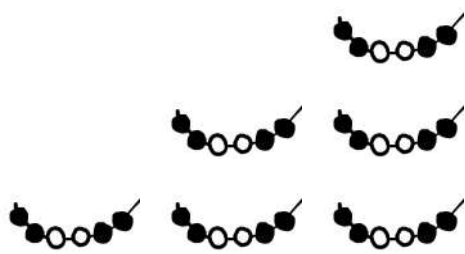
6. Put your numbers in a row of the table. Draw a picture to check your work.

7. Let's imagine  $y$  is \_\_\_\_\_. Plug  $y$  into the equation you wrote for #6 and solve.

8. Put your numbers in a row of the table. Draw a picture to check your work.

**Find the constant of proportionality. Then write TWO equations that describe the table.**

1. Jade made a necklace with 4 black beads and 2 white beads. Fill in the top three rows of the table based on the spiders you see.



$x$	$y$
white	black
	○
	○
	○
	○
	○

2. What is the constant of proportionality? \_\_\_\_\_ Put it in the circles.

3. Write a multiplication equation using the variables,  $x$  and  $y$ . \_\_\_\_\_

4. Write a division equation using the variables,  $x$  and  $y$ . \_\_\_\_\_

**Use the equation to fill in the rest of the rows.**

5. Let's imagine  $x$  is \_\_\_\_\_. Plug  $x$  into the equation you wrote for #3 and solve.

6. Put your numbers in a row of the table. Draw a picture to check your work.

7. Let's imagine  $y$  is \_\_\_\_\_. Plug  $y$  into the equation you wrote for #6 and solve.

8. Put your numbers in a row of the table. Draw a picture to check your work.

Name: \_\_\_\_\_

Remember: We can multiply or divide by the constant of proportionality.

Find the constant of proportionality. Then write TWO equations that describe the table. Use the equation to fill in the rest of the rows.

1. Ben gets 2 hours of HW for 8 hours of class.

$x$	$y$
hours of HW	hours of class
2	8
4	16
6	24

Equations:

---

---

2. There are 3 teachers for every 15 kids.

$x$	$y$
teachers	kids
3	15
6	30
9	45

Equations:

---

---

3. It takes 20 minutes to solve 10 math facts.

$x$	$y$
math facts	minutes
10	20
20	40
30	60

Equations:

---

---

4. At his bake shop, Ren sells 8 cookies for \$2.

$x$	$y$
\$	cookies
2	8
4	16
6	24

Equations:

---

---

Find the constant of proportionality. Then write TWO equations that describe the table. Use the equation to fill in the rest of the rows.

5. We need 3 scoops of cocoa in 3 cups of milk.

$x$	$y$
scoops	cups
3	3
6	6
9	9

Equations:

---



---

6. Taylor Swift rehearses 100 hours for a 20 minute concert.

$x$	$y$
minutes	hours
20	100
40	200
60	300

Equations:

---



---

7. The trail mix has 2 scoops of nuts for every 6 scoops of pretzels.

$x$	$y$
scoops of nuts	scoops of pretzels
2	6
4	12
6	18

Equations:

---



---

8. You can drive 60 miles on 2 gallons of gas.

$x$	$y$
gallons	miles
2	60
4	120
6	180

Equations:

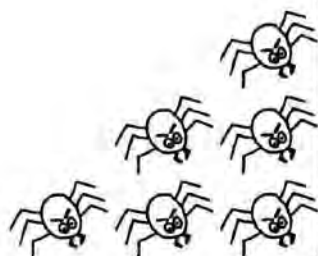
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Find the constant of proportionality. Then write TWO equations that describe the table.

1. The spider below has 6 legs and 2 eyes. Fill in the top three rows of the table based on the spiders you see.



x		y
eyes		legs
2	(x3)	6
4	(x3)	12
6	(x3)	18
10	(x3)	30
20	(x3)	60

$$\begin{array}{r} 3 \\ 2 \overline{)6} \\ \underline{-6} \\ 0 \end{array} \quad \begin{array}{r} 63 \\ 4 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

$$\begin{array}{r} 03 \\ 6 \overline{)18} \\ \underline{-18} \\ 00 \end{array}$$

2. What is the constant of proportionality? 3 Put it in the circles.

3. Write a multiplication equation using the variables, x and y.  $x \cdot 3 = y$  or  $3x = y$

4. Write a division equation using the variables, x and y.  $y \div 3 = x$  or  $\frac{y}{3} = x$

Use the equation to fill in the rest of the rows.

5. Let's imagine x is 10. Plug x into the equation you wrote for #3 and solve.

$$x \cdot 3 = y$$

$$10 \cdot 3 = y$$

$$30 = y$$

6. Put your numbers in a row of the table. Draw a picture to check your work.

7. Let's imagine y is 60. Plug y into the equation you wrote for #6 and solve.

$$y \div 3 = x$$

$$60 \div 3 = x$$

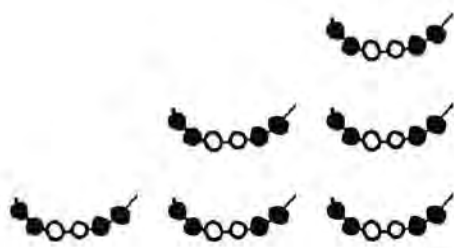
$$20 = x$$

8. Put your numbers in a row of the table. Draw a picture to check your work.



Find the constant of proportionality. Then write TWO equations that describe the table.

1. Jade made a necklace with 4 black beads and 2 white beads. Fill in the top three rows of the table based on the spiders you see.



x		y
white		black
2	(x2)	4
4	(x2)	8
6	(x2)	12
10	(x2)	20
5	(x2)	10

$$\begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 4 \overline{)8} \\ \underline{-8} \\ 0 \end{array}$$

$$\begin{array}{r} 02 \\ 6 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

2. What is the constant of proportionality? 2 Put it in the circles.

3. Write a multiplication equation using the variables, x and y.  $x \cdot 2 = y$  or  $2x = y$

4. Write a division equation using the variables, x and y.  $y \div 2 = x$  or  $\frac{y}{2} = x$

Use the equation to fill in the rest of the rows.

5. Let's imagine x is 10. Plug x into the equation you wrote for #3 and solve.

$$x \cdot 2 = y$$

$$10 \cdot 2 = y$$

$$\boxed{20 = y}$$

6. Put your numbers in a row of the table. Draw a picture to check your work.

7. Let's imagine y is 10. Plug y into the equation you wrote for #4 and solve.

$$y \div 2 = x$$

$$10 \div 2 = x$$

$$\boxed{5 = x}$$

8. Put your numbers in a row of the table. Draw a picture to check your work.

# Name: ANSWER KEY

Remember: We can multiply or divide by the constant of proportionality.

Find the constant of proportionality. Then write TWO equations that describe the table. Use the equation to fill in the rest of the rows.

1. Ben gets 2 hours of HW for 8 hours of class.

x		y
hours of HW		hours of class
2	$\times 4$	8
4	$\times 4$	16
6	$\times 4$	24
5	$\times 4$	20
8	$\times 4$	32

$$\begin{array}{r} 4 \\ 2 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 4 \\ 4 \overline{)16} \\ \underline{-16} \\ 00 \end{array}$$

$$\begin{array}{r} 4 \\ 6 \overline{)24} \\ \underline{-24} \\ 00 \end{array}$$

many right answers

Equations:

$$x \cdot 4 = y \text{ or } 4x = y$$

$$y \div 4 = x \text{ or } \frac{y}{4} = x$$

2. There are 3 teachers for every 15 kids.

x		y
teachers		kids
3	$\times 5$	15
6	$\times 5$	30
9	$\times 5$	45
many right answers		

$$\begin{array}{r} 5 \\ 3 \overline{)15} \\ \underline{-15} \\ 00 \end{array} \quad \begin{array}{r} 5 \\ 6 \overline{)30} \\ \underline{-30} \\ 00 \end{array}$$

$$\begin{array}{r} 5 \\ 9 \overline{)45} \\ \underline{-45} \\ 00 \end{array}$$

Equations:

$$x \cdot 5 = y \text{ or } 5x = y$$

$$y \div 5 = x \text{ or } \frac{y}{5} = x$$

3. It takes 20 minutes to solve 10 math facts.

x		y
math facts		minutes
10	$\times 2$	20
20	$\times 2$	40
30	$\times 2$	60
many right answers		

$$\begin{array}{r} 2 \\ 10 \overline{)20} \\ \underline{-20} \\ 00 \end{array} \quad \begin{array}{r} 2 \\ 20 \overline{)40} \\ \underline{-40} \\ 00 \end{array}$$

$$\begin{array}{r} 2 \\ 30 \overline{)60} \\ \underline{-60} \\ 00 \end{array}$$

Equations:

$$x \cdot 2 = y \text{ or } 2x = y$$

$$y \div 2 = x \text{ or } \frac{y}{2} = x$$

4. At his bake shop, Ren sells 8 cookies for \$2.

x		y
\$		cookies
2	$\times 4$	8
4	$\times 4$	16
6	$\times 4$	24
many right answers		

$$\begin{array}{r} 4 \\ 2 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 4 \\ 4 \overline{)16} \\ \underline{-16} \\ 00 \end{array}$$

$$\begin{array}{r} 4 \\ 6 \overline{)24} \\ \underline{-24} \\ 00 \end{array}$$

Equations:

$$x \cdot 4 = y \text{ or } 4x = y$$

$$y \div 4 = x \text{ or } \frac{y}{4} = x$$

Find the constant of proportionality. Then write TWO equations that describe the table. Use the equation to fill in the rest of the rows.

5. We need 3 scoops of cocoa in 3 cups of milk.

x		y
scoops		cups
3	x 1	3
6	x 1	6
9	x 1	9
many right answers		

$$\begin{array}{r} 1 \\ 3 \overline{)3} \\ \underline{-3} \\ 0 \end{array} \quad \begin{array}{r} 1 \\ 6 \overline{)6} \\ \underline{-6} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 9 \overline{)9} \\ \underline{-9} \\ 0 \end{array}$$

Equations:

$$1x = y \text{ or } x = y$$

$$y \div 1 = x \text{ or } \frac{y}{1} = x \text{ or } y = x$$

6. Taylor Swift rehearses 100 hours for a 20 minute concert.

x		y
minutes		hours
20	x 5	100
40	x 5	200
60	x 5	300
many right answers		

$$\begin{array}{r} 005 \\ 20 \overline{)100} \\ \underline{-100} \\ 000 \end{array} \quad \begin{array}{r} 005 \\ 40 \overline{)200} \\ \underline{-200} \\ 000 \end{array}$$

$$\begin{array}{r} 005 \\ 60 \overline{)300} \\ \underline{-300} \\ 000 \end{array}$$

Equations:

$$x \cdot 5 = y \text{ or } 5x = y$$

$$y \div 5 = x \text{ or } \frac{y}{5} = x$$

7. The trail mix has 2 scoops of nuts for every 6 scoops of pretzels.

x		y
scoops of nuts		scoops of pretzels
2	x 3	6
4	x 3	12
6	x 3	18
many right answers		

$$\begin{array}{r} 3 \\ 2 \overline{)6} \\ \underline{-6} \\ 0 \end{array} \quad \begin{array}{r} 03 \\ 4 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

$$\begin{array}{r} 03 \\ 6 \overline{)18} \\ \underline{-18} \\ 00 \end{array}$$

Equations:

$$x \cdot 3 = y \text{ or } 3x = y$$

$$y \div 3 = x \text{ or } \frac{y}{3} = x$$

8. You can drive 60 miles on 2 gallons of gas.

x		y
gallons		miles
2	x 30	60
4	x 30	120
6	x 30	180
many right answers		

$$\begin{array}{r} 30 \\ 2 \overline{)60} \\ \underline{-60} \\ 0 \end{array} \quad \begin{array}{r} 030 \\ 4 \overline{)120} \\ \underline{-120} \\ 000 \end{array}$$

$$\begin{array}{r} 030 \\ 6 \overline{)180} \\ \underline{-180} \\ 000 \end{array}$$

Equations:

$$x \cdot 30 = y \text{ or } 30x = y$$

$$y \div 30 = x \text{ or } \frac{y}{30} = x$$

## **G7 U2 Lesson 5**

Write two equations that represent the same proportional relationship.

**G7 U2 Lesson 5 - Today we will use a table and equation to solve problems that involve proportional relationships.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will use a table and equation to solve problems that involve proportional relationships. We are not really learning anything new. We are just practicing using what we've already learned. You're going to be great at it.

x Liters	y balls

**Let's Review (Slide 3):** The big idea of our last lesson was that there will always be at least two related equations for a proportion table. Let's find them for this story. Read along with me silently while I read out loud. "Ethan can fit 10 ping pong balls in a 2 Liter bottle. Let x equal the number of liters. Let y equal the number of balls. What two equations represent the table?" The first thing I am going to do is write in x and y on my table. X is the liters and y is the balls.

x Liters	y balls
2	10
4	20
6	30

Next I can use this picture to fill in the table. Here we have a 2 liter bottle with 10 balls so I put 2 under liters and 10 under balls. Let's count the next picture. There are 2 - 4 liters. I put 4 under liters. Now we count the balls. *Touch each one as you count out loud to 20.* I put 20 under balls. Let's keep going. There are 2 - 4 - 6 liters. I put 6 under liters. Now we count the balls. *Touch each one as you count out loud to 30.* I put 30 under balls.

x Liters	y balls
2 x5	10
4 x5	20
6 x5	30

The correct operation might already be jumping out at you! If not, you can divide the right side by the left side. I don't think you need to do that though. What is the operation that is happening? **Possible Student Answers, Key Points:**

- It is times 5.
- We multiply the left side by 5 to get the left side.

I can write x5 is little circles in the middle of my table.

Now let's write our equations. *Point from left to right as you name the equations.* I see 2 times 5 is 10. I see 4 times 5 is 20. I see 6 times 5 is 30. If I wanted to explain what is happening in words, I would say liters times 5 equals balls. And if I substitute in x and y, it is x times 5 equals y.  $x \cdot 5 = y$   $5x = y$  y. A lot of times we rewrite that is  $5x = y$ .

Now let's think about it going the opposite way. *Point from right to left as you name the equations.* I see 10 divided by 5 is 2. I see 20 divided by 5 is 4. I see 30 divided by 5 is 6. If I wanted to explain what is happening in words, I would say balls divided by 5 is liters. And if I substitute in x and y, it is y divided by 5 equal x. A lot of times we rewrite that is y over 5 equal x. These two equations both represent the same proportional relationship just in opposite ways.  $y \div 5 = x$   $\frac{y}{5} = x$

Liters	balls
2	10
4	20
6	30
10	

**Let's Talk (Slide 4):** Once we have equations, we can plug in one variable and find the other. Read silently with me while I read out loud. "Imagine Ethan has 10 Liters of space. How many ping pong balls can he hold?" I am going to put 10 under liters on my table. I can already guess what this side is going to be but let's use the equation.

The equation is x times 5 equals y or y equals 5 x. Now we have one very important question to ask ourselves. That is, "should 10 go in the place of x or the place of y?" Remember we

can substitute either variable so we have to decide based on what the variables represent. X is liters and Y is balls. So should 10 go in the place of the x or the place of the y? [Possible Student Answers](#), [Key Points](#):

- 10 should go in place of the x because it is 10 Liters and x is Liters.
- 10 is on the x side of the table.
  - X equals 10.

Imagine Ethan has 10 Liters of space. How many ping pong balls can he hold?

$$y = 5x$$

$$y = 5 \cdot 10$$

$$y = 50$$

back your work:

The 10 should go in place of the x because it is 10 Liters and x is Liters. Also, we wrote the 10 on the x side of the table because it was Liters. So I put 10 in place of x. Then I can solve for y. 5 times 10 is 50.

Liters	balls
2	10
4	20
6	30
10	50

So I can put 50 on the table.

Imagine Ethan has 10 Liters of space. How many ping pong balls can he hold?

$$y = 5x$$

$$y = 5 \cdot 10$$

$$y = 50$$

Draw a picture to check your work:

Let's draw a picture to check our work. I will draw 2 liters then 4 liters then 6 liters then 8 liters then 10 liters. Now let's fill them with balls: 10 - 20 - 30 - 40 - 50! Our pictures matches the table and the equation. Who can put our final answer in a complete sentence using the words from the story? I'll help you start. We would say, "With 10 Liters of space, Ethan..." [Possible Student Answers](#), [Key Points](#):

- With 10 Liters of space, Ethan can hold 50 ping pong balls.

Liters	balls
2	10
4	20
6	30
10	50
	100

**Let's Talk (Slide 5):** Now let's try a different problem with the same table. Read silently with me while I read out loud. "Imagine Ethan wants to hold 100 ping pong balls. How many Liters does he need?" I am going to put 100 under balls on my table this time. It's not Liters. And I can already guess what this other side is going to be but let's use the equation.

Now, I can use a related division equation but let's stick with the same equation, x times 5 equals y, just to see what happens. We are going to ask ourselves the same very important question. That is, "should 100 go in the place of x or the place of y?" Remember we can substitute either variable so we have to decide based on what the variables represent. X is liters and Y is balls. So should 100 go in the place of the x or the place of the y? [Possible Student Answers](#), [Key Points](#):

- 100 should go in place of the y because it is 100 balls and y is balls.
- 100 is on the y side of the table.
- Y equals 100.

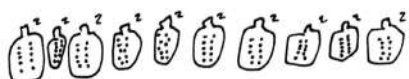
The 100 should go in place of the y because it is 100 ping pong balls and y is balls. Also, we wrote the 100 on the y side of the table because it was balls. So I put 100 in place of y. Now I need to solve. I can't solve by multiplying by 5 this time. Instead, this equation is asking, what times five makes 100. I can solve by using the algebra you learned in 6th grade, which mean doing the opposite operations. I am going to divide by 5 on this side and on this side. Then I get  $y = 20$ . So I can put 20 on the Liters side of the table.

$$y = 5x$$

$$\frac{100 = 5x}{5 \quad 5}$$

$$20 = x$$

Liters	balls
2	10
4	20
6	30
10	50
20	100



Let's draw a picture to check our work. I will dots so I can do this a bit faster. There would be 10 balls then 20 - 30 - 40 - 50 - 60 - 70 - 80 - 90 - 100. Let's label the Liters. This is 2 Liters, 4 Liters, 6, 8, 10.



Our pictures matches the table and the equation. Who can put our final answer in a complete sentence using the words from the story? I'll help you start. We would say, "To hold 100 ping pong balls, Ethan..." **Possible Student Answers, Key Points:**

- To hold 100 ping pong balls, Ethan needs 10 Liters of space.

x	y
1	2
2	4
3	6
14	

**Let's Think (Slide 6):** Even if we only have one equation, we can use algebra to solve. Read this example problem along with me silently while I read out loud. "Susie used the equation  $2x = y$  to fill in the table. What is the value of y when x is 14." 14 goes in the x side of the table.

Susie used the equation  $2x = y$  to fill in the table. What is the value of y when x is 14?

$$\begin{aligned} 2x &= y \\ 2 \cdot 14 &= y \\ 28 &= y \end{aligned}$$

You can probably see the operation on the table but let's practice with the equation. First, I write the equation as it is given,  $2x = y$ . Now I put in 14 for x. I get 2 times 14 equals y. That's 28 equals y.

x	y
1	2
2	4
3	6
14	28

I can put 28 in the y column.

x	y
1	2
2	4
3	6
14	28
	14

Now, this might seem like the exact same question but it's not. It says, "what is the value of x when y is 14?" So now y is 14 not x. We put 14 in the y column.

What is the value of x when y is 14?

$$\begin{aligned} 2x &= y \\ 2x &= 14 \\ \frac{2x}{2} &= \frac{14}{2} \\ x &= 7 \end{aligned}$$

Let's use the same equation but replace y this time. I write  $2x = y$ . Then I rewrite it with y as 14 so  $2x = 14$ . To solve this, I have to work backwards and divide by 2 on each side. The 2 divided by 2 is cancelled out so we get x on this side. 14 divided by 2 is 7. So our answer is  $x = 7$ .

x	y
1	2
2	4
3	6
14	28
7	14

I will put 7 on the table, and it looks right, doesn't it. So I can see that I thought of all this correctly.

**Let's Try It (Slide 7):** Great thinking! Now, let's practice! I am going to take you through step by step.

# WARM WELCOME



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**Today we will use a table and equation to solve problems that involve proportional relationships.**

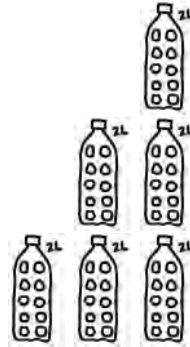
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## Let's Review:

There will always be at least two related equations for a proportion table.

Ethan can fit 10 ping pong balls in a 2 Liter bottle. Let  $x$  equal the number of liters. Let  $y$  equal the number of balls.

What two equations represent the table?



Liters	balls

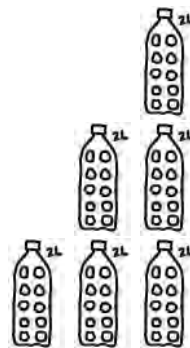
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## Let's Talk:

Once we have equations, we can plug in one variable and find the other.

Imagine Ethan has 10 Liters of space. How many ping pong balls can he hold?

Draw a picture to check your work:



Liters	balls
2	10
4	20
6	30

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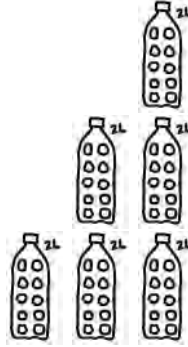


## Let's Talk:

Once we have equations, we can plug in one variable and find the other.

Imagine Ethan wants to hold 100 ping pong balls. How many Liters does he need?

Draw a picture to check your work:



Liters	balls
2	10
4	20
6	30
10	50

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## Let's Think:

Even if we only have one equation, we can use algebra to solve.

Susie used the equation  $2x = y$  to fill in the table. What is the value of  $y$  when  $x$  is 14?

What is the value of  $x$  when  $y$  is 14?

$x$	$y$
1	2
2	4
3	6

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# Let's Try It:




## Let's do some practice together!

Name: \_\_\_\_\_

G7 U2 Lesson 5 - Let's Try It

Use the constant of proportionality to write two equations.

1. In our last lesson, we saw the spider below has 6 legs and 2 eyes. Fill in the top three rows of the table based on the spiders you see.

x	y
eyes	legs
	○
	○
	○
	○
	○

2. Write a multiplication equation using the variables,  $x$  and  $y$ .

3. Write a division equation using the variables,  $x$  and  $y$ .

How many legs would you expect to see if you saw 24 eyes?

4. Put 24 in the correct place on the table. Is 24, the  $x$  or the  $y$  in your equation? \_\_\_\_\_

5. Plug 24 into the correct equation and solve.

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# On your Own:

## Now it's time for you to do it on your own.

Name: \_\_\_\_\_

G7 U2 Lesson 5 - Independent Work

Remember: We can multiply or divide by the constant of proportionality.

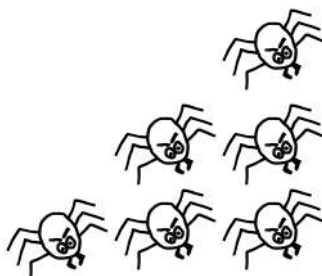
Find the constant of proportionality to answer the questions and record answers in the table.

1. Ben gets 2 hours of HW for 8 hours of class.		2. There are 3 teachers for every 15 kids.	
hours of HW	hours of class	teachers	kids
2	8	3	15
4	16	6	30
6	24	9	45
How many hours of HW will Ben get after 12 hours of class?		How many teachers are needed for 20 kids?	
How many hours of class did Ben have to receive 5 hours of HW?		How many kids must there be if there are 10 teachers?	
3. It takes 20 minutes to solve 10 math facts.		4. At his bake shop, Ren sells 8 cookies for \$2.	
math facts	minutes	\$	cookies
10	20	2	8

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**Use the constant of proportionality to write two equations.**

1. In our last lesson, we saw the spider below has 6 legs and 2 eyes. Fill in the top three rows of the table based on the spiders you see.



$x$	$y$
eyes	legs
	○
	○
	○
	○
	○

2. Write a multiplication equation using the variables,  $x$  and  $y$ .

\_\_\_\_\_

3. Write a division equation using the variables,  $x$  and  $y$ .

\_\_\_\_\_

**How many legs would you expect to see if you saw 24 eyes?**

4. Put 24 in the correct place on the table. Is 24, the  $x$  or the  $y$  in your equation? \_\_\_\_\_

5. Plug 24 into the correct equation and solve.

6. Draw a picture to check your answer.

7. Write a complete answer sentence using the correct words from the problem:

\_\_\_\_\_

**How many eyes would you expect to see if you saw 24 legs?**

8. Put 24 in the correct place on the table. Is 24, the  $x$  or the  $y$  in your equation? \_\_\_\_\_

9. Plug 24 into the correct equation and solve.

10. Draw a picture to check your answer.

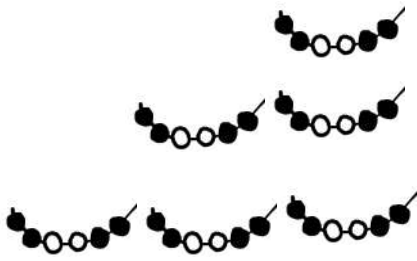
11. Write a complete answer sentence using the correct words from the problem:

\_\_\_\_\_



Use the constant of proportionality. Then write **TWO** equations that describe the table.

12. In our last lesson, Jade made a necklace with 4 black beads and 2 white beads. Fill in the top three rows of the table based on the spiders you see.



$x$	$y$
white	black
	○
	○
	○
	○
	○

13. Write a multiplication equation using the variables,  $x$  and  $y$ .

\_\_\_\_\_

14. Write a division equation using the variables,  $x$  and  $y$ .

\_\_\_\_\_

**How many white beads would you expect to see if you saw 40 black beads?**

15. Put 40 in the correct place on the table. Is 40, the  $x$  or the  $y$  in your equation? \_\_\_\_\_

16. Plug 40 into the correct equation and solve.

17. Draw a picture to check your answer.

18. Write a complete answer sentence using the correct words from the problem:

\_\_\_\_\_

**How many black beads would you expect to see if you saw 40 white beads?**

19. Put 40 in the correct place on the table. Is 40, the  $x$  or the  $y$  in your equation? \_\_\_\_\_

20. Plug 40 into the correct equation and solve.

21. Draw a picture to check your answer.

22. Write a complete answer sentence using the correct words from the problem:

\_\_\_\_\_

Name: \_\_\_\_\_

Remember: We can multiply or divide by the constant of proportionality.

Find the constant of proportionality to answer the questions and record answers in the table.

1. Ben gets 2 hours of HW for 8 hours of class.

hours of HW	hours of class
2	8
4	16
6	24

How many hours of HW will Ben get after 12 hours of class?

How many hours of class did Ben have to receive 5 hours of HW?

2. There are 3 teachers for every 15 kids.

teachers	kids
3	15
6	30
9	45

How many teachers are needed for 20 kids?

How many kids must there be if there are 10 teachers?

3. It takes 20 minutes to solve 10 math facts.

math facts	minutes
10	20
20	40
30	60

How long would it take to solve 34 math facts?

How many math facts will be solved in 100 min?

4. At his bake shop, Ren sells 8 cookies for \$2.

\$	cookies
2	8
4	16
6	24

How many cookies can be bought with \$5?

How much would it cost for 20 cookies?

Find the constant of proportionality to answer the questions and record answers in the table.

5. We need 3 scoops of cocoa in 3 cups of milk.

scoops	cups
3	3
6	6
9	9

How many cups of milk use 5 scoops of cocoa?

How many scoops of cocoa do we need for 30 cups of milk?

6. Taylor Swift rehearses 100 hours for a 20 minute concert.

minutes	hours
20	100
40	200
60	300

How long does Taylor Swift rehearse for a 120 minute concert?

How long must the concert be if Taylor Swift rehearses for 500 hours?

7. DC charges \$5 tax on a \$50 purchase.

\$ tax	\$ purchase
5	50
10	100
15	150

How much was the purchase if there was \$30 in tax?

How much was the tax if there was a \$30 purchase?

8. The 3 person swim team needs 2 quarts of gatorade after each game.

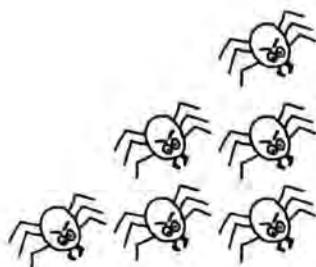
quarts	people
2	3
4	6
6	9

How many quarts of gatorade are needed for 30 people?

How many people would served with 30 quarts of gatorade?

Use the constant of proportionality to write two equations.

1. In our last lesson, we saw the spider below has 6 legs and 2 eyes. Fill in the top three rows of the table based on the spiders you see.



x		y
eyes		legs
2	(x3)	6
4	(x3)	12
6	(x3)	18
24	(x3)	72
8	(x3)	24

2. Write a multiplication equation using the variables, x and y.

$$\underline{x \cdot 3 = y \text{ or } 3x = y}$$

3. Write a division equation using the variables, x and y.

$$\underline{y \div 3 = x \text{ or } \frac{y}{3} = x}$$

How many legs would you expect to see if you saw 24 eyes?

4. Put 24 in the correct place on the table. Is 24, the x or the y in your equation? x

5. Plug 24 into the correct equation and solve.

$$\begin{aligned} x \cdot 3 &= y \\ 24 \cdot 3 &= y \\ 72 &= y \end{aligned} \qquad \begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array}$$

6. Draw a picture to check your answer.



7. Write a complete answer sentence using the correct words from the problem:

Spiders with 24 eyes would have 72 legs.

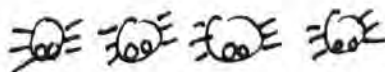
How many eyes would you expect to see if you saw 24 legs?

8. Put 24 in the correct place on the table. Is 24, the x or the y in your equation? y

9. Plug 24 into the correct equation and solve.

$$\begin{aligned} y \div 3 &= x \\ 24 \div 3 &= x \\ 8 &= x \end{aligned}$$

10. Draw a picture to check your answer.

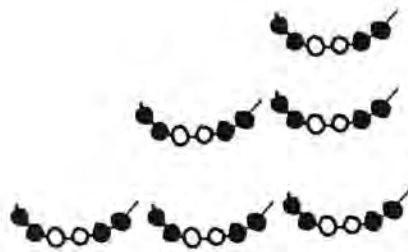


11. Write a complete answer sentence using the correct words from the problem:

Spiders with 24 legs would have 8 eyes.

Use the constant of proportionality. Then write TWO equations that describe the table.

12. In our last lesson, Jade made a necklace with 4 black beads and 2 white beads. Fill in the top three rows of the table based on the spiders you see.



x		y
white		black
2	(x2)	4
4	(x2)	8
6	(x2)	12
20	(x2)	40
40	(x2)	80

13. Write a multiplication equation using the variables, x and y.

$$x \cdot 2 = y \text{ or } 2x = y$$

14. Write a division equation using the variables, x and y.

$$y \div 2 = x \text{ or } \frac{y}{2} = x$$

How many white beads would you expect to see if you saw 40 black beads?

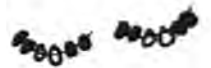
15. Put 40 in the correct place on the table. Is 40, the x or the y in your equation? y

16. Plug 40 into the correct equation and solve.

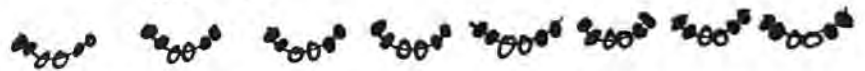
$$y \div 2 = x$$

$$40 \div 2 = x$$

$$20 = x$$



17. Draw a picture to check your answer.



18. Write a complete answer sentence using the correct words from the problem:

Necklaces with 40 black beads would have 20 white beads.

How many black beads would you expect to see if you saw 40 white beads?

19. Put 40 in the correct place on the table. Is 40, the x or the y in your equation? x

20. Plug 40 into the correct equation and solve.

$$x \cdot 2 = y$$

$$40 \cdot 2 = y$$

$$80 = y$$



21. Draw a picture to check your answer.



22. Write a complete answer sentence using the correct words from the problem:

Necklaces with 40 white beads would have 80 black beads.

Remember: We can multiply or divide by the constant of proportionality.

Find the constant of proportionality to answer the questions and record answers in the table.

1. Ben gets 2 hours of HW for 8 hours of class.

hours of HW		hours of class
2	$\times 4$	8
4	$\times 4$	16
6	$\times 4$	24
3	$\times 4$	12
5	$\times 4$	20

$$x \cdot 4 = y$$

$$y \div 4 = x$$

How many hours of HW will Ben get after 12 hours of class?

$$y \div 4 = x$$

$$12 \div 4 = x$$

$$\boxed{3 = x}$$

How many hours of class did Ben have to receive 5 hours of HW?

$$x \cdot 4 = y$$

$$5 \cdot 4 = y$$

$$\boxed{20 = y}$$

2. There are 3 teachers for every 15 kids.

teachers		kids
3	$\times 5$	15
6	$\times 5$	30
9	$\times 5$	45
4	$\times 5$	20
10	$\times 5$	50

$$x \cdot 5 = y$$

$$y \div 5 = x$$

How many teachers are needed for 20 kids?

$$y \div 5 = x$$

$$20 \div 5 = x$$

$$\boxed{4 = x}$$

How many kids must there be if there are 10 teachers?

$$x \cdot 5 = y$$

$$10 \cdot 5 = y$$

$$\boxed{50 = y}$$

3. It takes 20 minutes to solve 10 math facts.

math facts		minutes
10	$\times 2$	20
20	$\times 2$	40
30	$\times 2$	60
34	$\times 2$	68
50	$\times 2$	100

$$x \cdot 2 = y$$

$$y \div 2 = x$$

How long would it take to solve 34 math facts?

$$x \cdot 2 = y$$

$$34 \cdot 2 = y$$

$$\boxed{68 = y}$$

How many math facts will be solved in 100 min?

$$y \div 2 = x$$

$$100 \div 2 = x$$

$$\boxed{50 = x}$$

4. At his bake shop, Ren sells 8 cookies for \$2.

\$		cookies
2	$\times 4$	8
4	$\times 4$	16
6	$\times 4$	24
5	$\times 4$	20
5	$\times 4$	20

$$x \cdot 4 = y$$

$$y \div 4 = x$$

How many cookies can be bought with \$5?

$$x \cdot 4 = y$$

$$5 \cdot 4 = y$$

$$\boxed{20 = y}$$

How much would it cost for 20 cookies?

$$y \div 4 = x$$

$$20 \div 4 = x$$

$$\boxed{5 = x}$$



Find the constant of proportionality to answer the questions and record answers in the table.

5. We need 3 scoops of cocoa in 3 cups of milk.

scoops		cups
3	$\times 1$	3
6	$\times 1$	6
9	$\times 1$	9
5	$\times 1$	5
30	$\times 1$	30

$$1x = y$$

$$\frac{y}{1} = x$$

$$x = y$$

How many cups of milk use 5 scoops of cocoa?

$$\begin{aligned} x \cdot 1 &= y \\ 5 \cdot 1 &= y \\ \boxed{5} &= y \end{aligned}$$

How many scoops of cocoa do we need for 30 cups of milk?

$$\begin{aligned} y \div 1 &= x \\ 30 \div 1 &= x \\ \boxed{30} &= x \end{aligned}$$

6. Taylor Swift rehearses 100 hours for a 20 minute concert.

minutes		hours
20	$\times 5$	100
40	$\times 5$	200
60	$\times 5$	300
120	$\times 5$	600
	$\times 5$	

$$x \cdot 5 = y$$

$$y \div 5 = x$$

How long does Taylor Swift rehearse for a 120 minute concert?

$$\begin{aligned} x \cdot 5 &= y \\ 120 \cdot 5 &= y \\ \boxed{600} &= y \end{aligned}$$

$$\begin{array}{r} \times 120 \\ 5 \\ \hline 600 \end{array}$$

How long must the concert be if Taylor Swift rehearses for 500 hours?

$$\begin{aligned} y \div 5 &= x \\ 500 \div 5 &= x \\ \boxed{100} &= x \end{aligned}$$

7. DC charges \$5 tax on a \$50 purchase.

\$ tax		\$ purchase
5	$\times 10$	50
10	$\times 10$	100
15	$\times 10$	150
30	$\times 10$	300
3	$\times 10$	30

$$x \cdot 10 = y$$

$$y \div 10 = x$$

How much was the purchase if there was \$30 in tax?

$$\begin{aligned} x \cdot 10 &= y \\ 30 \cdot 10 &= y \\ \boxed{300} &= y \end{aligned}$$

How much was the tax if there was a \$30 purchase?

$$\begin{aligned} y \div 10 &= x \\ 30 \div 10 &= x \\ \boxed{3} &= x \end{aligned}$$

8. The 3 person swim team needs 2 quarts of gatorade after each game.

quarts		people
2	$\times 1\frac{1}{2}$	3
4	$\times 1\frac{1}{2}$	6
6	$\times 1\frac{1}{2}$	9
20	$\times 1\frac{1}{2}$	30
30	$\times 1\frac{1}{2}$	

$$\begin{array}{r} 1\frac{1}{2} \\ 2 \overline{)3} \\ \underline{-2} \\ 1 \end{array}$$

$$x \cdot 1\frac{1}{2} = y$$

$$y \div 1\frac{1}{2} = x$$

How many quarts of gatorade are needed for 30 people?

$$\begin{aligned} y \div 1\frac{1}{2} &= x & 30 \div 1\frac{1}{2} \\ 30 \div 1\frac{1}{2} &= x & 30 \times \frac{2}{3} = 20 \\ \boxed{20} &= x & \end{aligned}$$

How many people would served with 30 quarts of gatorade?

$$\begin{aligned} x \cdot 1\frac{1}{2} &= y & 30 \times \frac{3}{2} = 45 \\ 30 \cdot 1\frac{1}{2} &= y & \\ \boxed{45} &= y & \end{aligned}$$

# **G7 U2 Lesson 6**

Use tables and equations to solve problems involving proportional relationships.



**Let's Talk (Slide 4):** Now let's do the same steps but with a story that's a little different because there are other relationships with more than one step that we can explore. Read the story silently in your mind while I read it out loud. "Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Fill in the table with the total cost for 2 tickets then 4 tickets then 6 tickets." Let's draw pictures again. There's a \$2 entry fee. I'll draw one of those bracelets you sometimes get when you go inside a place. Then we'll start with two tickets. It costs \$2 per ticket. That is 2 groups of 2, which is \$4.

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Fill in the table with the total cost for 2 tickets then 4 tickets then 6 tickets.  
Find the unit rate for each row.

tickets	dollars
2	6

But we have this entry fee so we have to add that \$2 in too. That means the cost will be \$6. So far, it's the same as the last problem. Let's see if it stays the same.

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Fill in the table with the total cost for 2 tickets then 4 tickets then 6 tickets.  
Find the unit rate for each row.

tickets	dollars
2	6
4	10
6	

Now we need 4 tickets so I'll draw more. They still cost \$2 each. That is 4 groups of 2, which is \$8. But we still have entry fee so we have to add that \$2 in too. That means the cost will be \$10. That's not the same as the last problem. Because the tickets are cheaper but there's this entrance fee, right?

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Fill in the table with the total cost for 2 tickets then 4 tickets then 6 tickets.  
Find the unit rate for each row.

tickets	dollars
2	6
4	10
6	14

Let's do 6 tickets now. I'll draw some more. They still cost \$2 each. That is 6 groups of 2, which is \$12. But we still have entry fee so we have to add that \$2 in too. That means the cost will be \$14.

Now, the next step says to find the unit rate for each row. This might seem like something we've already done. But stick with me. Something interesting is about to happen. How do I find the unit rate? [Possible Student Answers, Key Points:](#)

- You divide one quantity by the other.
- You divide 6 by 2 and 10 by 4 and 14 by 6.

We always divide one quantity by the other. So, let's do each row. I will do 6 divided by 2. That's 3. Next, I will do 10 divided by 4. That's 2 and then we subtract 8 so we have 2 leftover. I can write my final answer as 2 and 2 fourths. One more. We do 14 divided by 6. 6 goes into 14 twice. I subtract 12 and 2 left so the final answer is 2 and 2 sixths. Interesting!

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Fill in the table with the total cost for 2 tickets then 4 tickets then 6 tickets.  
Find the unit rate for each row.

tickets	dollars
2	6
4	10
6	14

Find the unit rate for each row.

$$\frac{3}{2} \quad \frac{2\frac{1}{2}}{4} \quad \frac{2\frac{1}{3}}{6}$$

$$\begin{array}{r} 3 \\ 2 \overline{)6} \\ \underline{6} \\ 0 \end{array} \quad \begin{array}{r} 2\frac{1}{4} \\ 4 \overline{)10} \\ \underline{8} \\ 2 \end{array} \quad \begin{array}{r} 2\frac{1}{3} \\ 6 \overline{)14} \\ \underline{12} \\ 2 \end{array}$$

There is something very important to notice here! What did you notice? [Possible Student Answers, Key Points:](#)

- The unit rates are different.
- There isn't a constant of proportionality.
- We didn't get the same answer to our division.

We didn't get the same answer to our division. In other words, the unit rates are different and so there is no constant of proportionality. Is this relationship proportional? No! And of course, that makes sense because we always had to add this entrance fee, right? It didn't change as we got more tickets the way that the cost in the last table was totally only based on tickets.

Is this relationship proportional? no

**Let's Think (Slide 5):** We can write equations for many non-proportional tables too. I want to do that for the problem we just did. But I'm not going to ask you to do that on your independent practice. I just

want you to see what we write and start thinking about it. Read along with your eyes as I read aloud. It starts out as the same problem we just did. "Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Write an equation to represent the table. Let x represent the number of tickets and y represent the total cost in dollars." Remember the last time we started writing equations with variables, we started by listing out equations for what we did with numbers. We did 2 times 3 plus 2 equals 6. We did 4 times 2 plus 2 equals 10. We did 6 times 2 plus 2 equals 14.

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Write an equation to represent the table. Let x represent the number of tickets and y represent the total cost in dollars.

$$2 \times 2 + 2 = 6$$

$$4 \times 2 + 2 = 10$$

$$6 \times 2 + 2 = 14$$

x	y
tickets	dollars
2	6
4	10
6	14

represent the table. Let x represent the number of tickets and y represent the total cost in dollars." Remember the last time we started writing equations with variables, we started by listing out equations for what we did with numbers. We did 2 times 3 plus 2 equals 6. We did 4 times 2 plus 2 equals 10. We did 6 times 2 plus 2 equals 14.

Notice what is the same here. I see times 2 plus 2 and times 2 plus 2 and times 2 plus 2. This part that is the same isn't a variable because variables can change. Now remember that in our last lesson, we also wrote an equation with words. This first number would be tickets. So that's tickets times 2 plus 2 equals dollars. And finally, we can put in x and y. It says to let x represent the number of tickets so I will put x in place of tickets. It says let y represent the total cost in dollars so I will put y in place of dollars. Our equation is x times 2 plus 2 equals y. Sometimes we write it as 2x plus 2 equals y. What do you notice about this equation compared to the equations we've been working with? [Possible Student Answers, Key Points:](#)

- It has addition.
- It is not just multiplication.
- It has two operations.
- It is a longer equation.

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Write an equation to represent the table. Let x represent the number of tickets and y represent the total cost in dollars.

$$2 \times 2 + 2 = 6$$

$$4 \times 2 + 2 = 10$$

$$6 \times 2 + 2 = 14$$

$$\text{tickets} \times 2 + 2 = \text{dollars}$$

$$x \cdot 2 + 2 = y$$

x	y
tickets	dollars
2	6
4	10
6	14

The equation doesn't just have multiplication. It has addition too! It is a two step equation. We'll talk more about that next week. For now, it's enough to notice that an equation that looks like this is NOT proportional.

**Let's Try It (Slide 6):** Great listening today! Now, let's practice! I am going to take you through step by step.

# WARM WELCOME



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**Today we will use a table and equation to solve problems that are not always proportional.**

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## Let's Review:

**We know a relationship is proportional when the unit rates are the same.**

Ralph went to the fair! Now he needs to decide how many tickets to buy at the price of \$3 per ticket. Fill in the table with the total cost for 2 tickets then 4 tickets then 6 tickets.

Find the unit rate for each row.

tickets	dollars
2	
4	
6	

Is this relationship proportional? \_\_\_\_\_

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## Let's Talk:

**There are other relationships with more than one step that we can explore.**

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Fill in the table with the total cost for 2 tickets then 4 tickets then 6 tickets.

Find the unit rate for each row.

tickets	dollars
2	
4	
6	

Is this relationship proportional? \_\_\_\_\_

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## Let's Think:

We can write equations for many non-proportional tables too.

Pete went to a different fair with a \$2 entrance fee. Now he needs to decide how many tickets to buy at the price of \$2 per ticket. Write an equation to represent the table. Let  $x$  represent the number of tickets and  $y$  represent the total cost in dollars.

tickets	dollars
2	6
4	10
6	14

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## Let's Try It:

Let's use equations to solve problems together.

Name: \_\_\_\_\_ G7\_U2 Lesson 6 - Let's Try It

**Let's represent the stories and determine if they are proportional.**

When Joey completes a math test, she spends 2 minutes per question. Use the table to show how long she spends on 2 questions then 4 questions then 6 questions.	When Rachel completes a math test, she spends 1 minute per question. Then she takes 5 minutes at the end to check it over. Use the table to show how long she spends on 2 questions then 4 questions then 6 questions.	When Nathaniel completes a math test, he spends $\frac{1}{2}$ minute per question. Use the table to show how long he spends on 2 questions then 4 questions then 6 questions.																														
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# On your Own:

## Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 6 - Independent Work

Remember: Ratios must have a constant of proportionality in order to be proportional.  
Use the story to complete the table. Then determine if it is proportional.

1. Lisa always buys one bag of chips and apples at the grocery store. A bag of chips costs \$2. Each apple costs \$1. Complete the chart with the total grocery bill in dollars for the different amounts of apples.

apples	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

2. Jan stops by 7-11 for soda every weekend. It costs \$3 for a bottle of soda. Complete the chart with the total cost in dollars for the different amounts of soda.

soda	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

3. Sam's Club charges a \$20 membership fee. Then you can buy a roasted chicken for \$10. Complete the chart with the total grocery bill in dollars for the different amounts of chicken.

4. Kris doesn't shop anywhere with a membership fee. He just buys his chicken at Safeway for \$15. Complete the chart with the total grocery bill in dollars for the different

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**Let's represent the stories and determine if they are proportional.**

When Joey completes a math test, she spends 2 minutes per question. Let  $x$  represent the number of questions and let  $y$  represent the number of minutes. Use the table to show how long she spends on 2 questions then 4 questions then 6 questions.

When Rachel completes a math test, she spends 1 minute per question. Then she takes 5 minutes at the end to check it over. Let  $x$  represent the number of questions and let  $y$  represent the number of minutes. Use the table to show how long she spends on 2 questions then 4 questions then 6 questions.

When Nathaniel completes a math test, he spends  $\frac{1}{2}$  minute per question. Let  $x$  represent the number of questions and let  $y$  represent the number of minutes. Use the table to show how long he spends on 2 questions then 4 questions then 6 questions.

x	y
2	
4	
6	

x	y
2	
4	
6	

x	y
2	
4	
6	

1. Is there a constant of proportionality? If so, what?

\_\_\_\_\_

2. Is there a constant of proportionality? If so, what?

\_\_\_\_\_

3. Is there a constant of proportionality? If so, what?

\_\_\_\_\_

4. Is the relationship proportional?

5. Is the relationship proportional?

6. Is the relationship proportional?

6. What do you notice about the types of stories that had a constant of proportionality?

Remember: Ratios must have a constant of proportionality in order to be proportional.

Use the story to complete the table. Then determine if it is proportional.

1. Lisa always buys one bag of chips and apples at the grocery store. A bag of chips costs \$2. Each apple costs \$1. Complete the chart with the total grocery bill in dollars for the different amounts of apples.

apples	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

3. Sam's Club charges a \$20 membership fee. Then you can buy a roasted chicken for \$10. Complete the chart with the total grocery bill in dollars for the different amounts of chicken.

Roasted chicken	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

2. Jan stops by 7-11 for soda every weekend. It costs \$3 for a bottle of soda. Complete the chart with the total cost in dollars for the different amounts of soda.

soda	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

4. Kris doesn't shop anywhere with a membership fee. He just buys his chicken at Safeway for \$15. Complete the chart with the total grocery bill in dollars for the different amounts of chicken.

Roasted chicken	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

Use the story to complete the table. Then determine if it is proportional.

5. Willie gets paid \$10 per hour and then he always gets a \$5 tip. Complete the chart with the total amount that Willie can earn for different hours.

hours	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

6. Stacie gets paid \$12 per hour. She doesn't get any tips! Complete the chart with the total amount that Stacie can earn for different hours.

hours	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

7. Alex rented a truck for \$8 per hour. Complete the chart with the total amount that Alex will have to pay for different amounts of time.

hours	dollars
1	
2	
3	
7	

Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

8. Adam rented a truck for \$10 per hour but she had a coupon for \$6 off. Complete the chart with the total amount that Adam will have to pay for different amounts of time.

hours	dollars
1	
2	
3	
7	

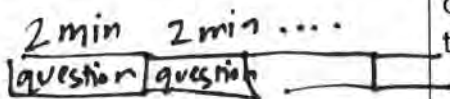
Is the relationship proportional? \_\_\_\_

If so, what's the constant of proportionality? \_\_\_\_

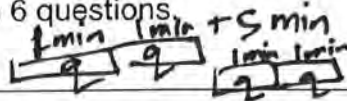


Let's represent the stories and determine if they are proportional.

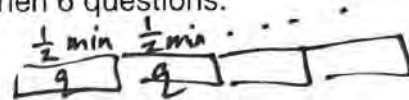
When Joey completes a math test, she spends 2 minutes per question. Let  $x$  represent the number of questions and let  $y$  represent the number of minutes. Use the table to show how long she spends on 2 questions then 4 questions then 6 questions.



When Rachel completes a math test, she spends 1 minute per question. Then she takes 5 minutes at the end to check it over. Let  $x$  represent the number of questions and let  $y$  represent the number of minutes. Use the table to show how long she spends on 2 questions then 4 questions then 6 questions.



When Nathaniel completes a math test, he spends  $\frac{1}{2}$  minute per question. Let  $x$  represent the number of questions and let  $y$  represent the number of minutes. Use the table to show how long he spends on 2 questions then 4 questions then 6 questions.



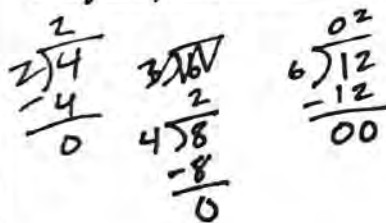
x		y
questions		minutes
2	$\times 2$	4
4	$\times 2$	8
6	$\times 2$	12

x		y
questions		minutes
2	$\times 1 + 5$	7
4	$\times 1 + 5$	9
6	$\times 1 + 5$	11

x		y
questions		minutes
2	$\div 2$	1
4	$\div 2$	2
6	$\div 2$	3

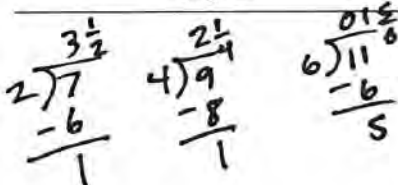
1. Is there a constant of proportionality? If so, what?

Yes, it is 2.



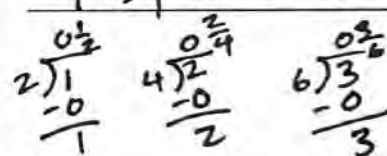
2. Is there a constant of proportionality? If so, what?

No



3. Is there a constant of proportionality? If so, what?

Yes, it is  $\frac{1}{2}$



4. Is the relationship proportional?

Yes

5. Is the relationship proportional?

No

6. Is the relationship proportional?

yes

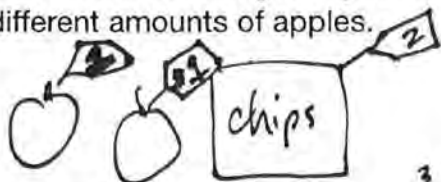
6. What do you notice about the types of stories that had a constant of proportionality?

The stories with a constant of proportionality have a single number for each question and that is all. It is one step just like the  $y = kx$  equation.

Remember: Ratios must have a constant of proportionality in order to be proportional.

Use the story to complete the table. Then determine if it is proportional.

1. Lisa always buys one bag of chips and apples at the grocery store. A bag of chips costs \$2. Each apple costs \$1. Complete the chart with the total grocery bill in dollars for the different amounts of apples.



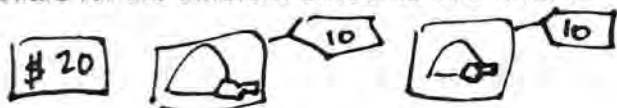
apples	dollars
1	$1 \times 1 + 2 = 3$
2	$2 \times 1 + 2 = 4$
3	$3 \times 1 + 2 = 5$
7	$7 \times 1 + 2 = 9$

$$\begin{array}{r} 1 \overline{)3} \\ -3 \\ \hline 0 \end{array} \quad \begin{array}{r} 2 \overline{)4} \\ -4 \\ \hline 0 \end{array}$$

Is the relationship proportional? NO

If so, what's the constant of proportionality? none

3. Sam's Club charges a \$20 membership fee. Then you can buy a roasted chicken for \$10. Complete the chart with the total grocery bill in dollars for the different amounts of chicken.



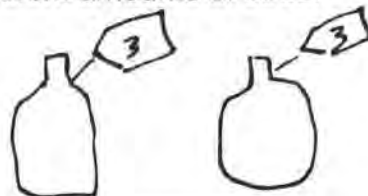
Roasted chicken	dollars
1	$1 \times 10 + 20 = 30$
2	$2 \times 10 + 20 = 40$
3	$3 \times 10 + 20 = 50$
7	$7 \times 10 + 20 = 90$

$$\begin{array}{r} 30 \\ 1 \overline{)30} \\ -30 \\ \hline 0 \end{array} \quad \begin{array}{r} 20 \\ 2 \overline{)40} \\ -40 \\ \hline 0 \end{array}$$

Is the relationship proportional? NO

If so, what's the constant of proportionality? none

2. Jan stops by 7-11 for soda every weekend. It costs \$3 for a bottle of soda. Complete the chart with the total cost in dollars for the different amounts of soda.



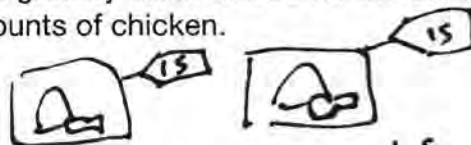
soda	dollars
1	$1 \times 3 = 3$
2	$2 \times 3 = 6$
3	$3 \times 3 = 9$
7	$7 \times 3 = 21$

$$\begin{array}{r} 3 \\ 1 \overline{)3} \\ -3 \\ \hline 0 \end{array} \quad \begin{array}{r} 3 \\ 2 \overline{)6} \\ -6 \\ \hline 0 \end{array}$$

Is the relationship proportional? YES

If so, what's the constant of proportionality? 3

4. Kris doesn't shop anywhere with a membership fee. He just buys his chicken at Safeway for \$15. Complete the chart with the total grocery bill in dollars for the different amounts of chicken.



Roasted chicken	dollars
1	$1 \times 15 = 15$
2	$2 \times 15 = 30$
3	$3 \times 15 = 45$
7	$7 \times 15 = 105$

$$\begin{array}{r} 15 \\ 1 \overline{)15} \\ -15 \\ \hline 0 \end{array} \quad \begin{array}{r} 15 \\ 2 \overline{)30} \\ -30 \\ \hline 0 \end{array}$$

Is the relationship proportional? Yes

If so, what's the constant of proportionality? 15

Use the story to complete the table. Then determine if it is proportional.

5. Willie gets paid \$10 per hour and then he always gets a \$5 tip. Complete the chart with the total amount that Willie can earn for different hours.

\$5/hr  
\$10/hr \$10/hr

hours	dollars
1	$\times 10 + 5$ 15
2	$\times 10 + 5$ 25
3	$\times 10 + 5$ 35
7	$\times 10 + 5$ 75

$$\begin{array}{r} 15 \\ 1 \overline{)15} \\ \underline{-15} \\ 0 \end{array} \quad \begin{array}{r} 12\frac{1}{2} \\ 2 \overline{)25} \\ \underline{-24} \\ 05 \\ \underline{-4} \\ 1 \end{array}$$

Is the relationship proportional? No

If so, what's the constant of proportionality? none

6. Stacie gets paid \$12 per hour. She doesn't get any tips! Complete the chart with the total amount that Stacie can earn for different hours.

\$12/hr \$12/hr

hours	dollars
1	$\times 12$ 12
2	$\times 12$ 24
3	$\times 12$ 36
7	$\times 12$ 84

$$\begin{array}{r} 12 \\ 1 \overline{)12} \\ \underline{-12} \\ 0 \end{array} \quad \begin{array}{r} 12 \\ 2 \overline{)24} \\ \underline{-24} \\ 0 \end{array}$$

Is the relationship proportional? yes

If so, what's the constant of proportionality? 12

7. Alex rented a truck for \$8 per hour. Complete the chart with the total amount that Alex will have to pay for different amounts of time.

\$8/hr \$8/hr \$8/hr

hours	dollars
1	$\times 8$ 8
2	$\times 8$ 16
3	$\times 8$ 24
7	$\times 8$ 56

$$\begin{array}{r} 8 \\ 1 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 08 \\ 2 \overline{)16} \\ \underline{-16} \\ 00 \end{array}$$

Is the relationship proportional? yes

If so, what's the constant of proportionality? 8

8. Adam rented a truck for \$10 per hour but she had a coupon for \$6 off. Complete the chart with the total amount that Adam will have to pay for different amounts of time.

\$10/hr \$10/hr \$6 off

hours	dollars
1	$\times 10 - 6$ 4
2	$\times 10 - 6$ 14
3	$\times 10 - 6$ 24
7	$\times 10 - 6$ 64

$$\begin{array}{r} 4 \\ 1 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 07 \\ 2 \overline{)14} \\ \underline{-14} \\ 00 \end{array}$$

Is the relationship proportional? No

If so, what's the constant of proportionality? none

# **G7 U2 Lesson 7**

Use a table of values to determine if a relationship is proportional.

**2G7 U2 Lesson 7 - Today we will recognize that proportional relationships are characterized by equations in the form  $y = kx$ .**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will recognize that proportional relationships are characterized by equations in the form  $y = kx$ . You have already done a lot of work with tables and equations so you are going to do great!

**Let's Review (Slide 3):** If we see a pattern, we can write an equation from the table. Let's do an example. Read along silently with me while I read out loud. "Mary fills the fruit baskets that she sells at her shop with apples and oranges. The table shows how much fruit she buys for different amounts of baskets. Let  $x$  represent the number of oranges and  $y$  represent the number of apples. Write an equation to represent the amount of each fruit that Maria buys." Remember the last time we started writing equations with variables, we started by listing out equations for what we did with numbers. What operation do you see on this table? It has to be the same operation for each row?

$x$	$y$
oranges	apples
2	4
4	8
6	12

$2 \times 2 = 4$   
 $4 \times 2 = 8$   
 $6 \times 2 = 12$

**Possible Student Answers, Key Points:**

- It is multiplication.
- It is times 2.

It is  $2 \times 2$  equals 4 and 4 times 2 equals 8 and 6 times 2 equals 12. Let me write those down.

We see that the "times 2" is the same every time. The variable is needed for the other parts that can change. Now remember that in our last lesson, we also wrote an equation with words. This first

number would be oranges. So that's oranges times 2 equals apples. And finally, we can put in  $x$  and  $y$ . It says to let  $x$  represent the number of oranges so I will put  $x$  in place of oranges. It says let  $y$  represent the number of apples so I will put  $y$  in place of apples. Our equation is  $x$  times 2 equals  $y$ . Sometimes we write it as  $2x$  equals  $y$ . This is all very familiar. And most importantly, we can see that this is a proportion because it has a constant of proportionality.

**Let's Review:** If we see a pattern, we can write an equation from the table.

Mary fills the fruit baskets that she sells at her shop with apples and oranges. The table shows how much fruit she buys for different amounts of baskets. Let  $x$  represent the number of oranges and  $y$  represent the number of apples. Write an equation to represent the amount of each fruit that Maria buys.

$x$	$y$
oranges	apples
2	4
4	8
6	12

$2 \times 2 = 4$   
 $4 \times 2 = 8$   
 $6 \times 2 = 12$   
 oranges  $\times 2 =$  apples  
 $x \cdot 2 = y$  or  $y = 2x$

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Which table shows a proportion? What do you notice about the pictures? What do you notice about the equation for the table?

$x$	$y$
towers	squares
1	4
2	8
3	12

**Let's Talk (Slide 4):** Equations from proportion tables always have the same form. Let's look at two more examples. On this first table, I see it is a times 4 relationship. I am going to call the towers  $x$  and the squares  $y$ .

Which table shows a proportion? What do you notice about the pictures? What do you notice about the equation for the table?

$x$	$y$
towers	squares
1	6
2	8
3	10

$x \cdot 4 = y$

Then it would be 1 times 4 equals 4, 2 times 4 equals 8, 3 times 4 equals 12. In other words, towers times 4 equals squares, which means  $x$  times 4 =  $y$  which we usually write as  $4x = y$ . That is because the number next to a letter means multiply. Three  $x$  is a quicker way to write the equation. I can see in this picture how there's a relationship and that relationship just keeps increasing and increasing the same way.

Now let's look at the next table. Can you look and see an obvious operation? Not really. It's not times 4 because 1 times 6 equals 6 but 2 times 6 doesn't equal 6. It would have to be 2 times 4. And I can't



think of something to turn 3 into 10. This is a signal to me that something special must be happening. I'm already thinking that there might not be a constant of proportionality. Maybe this is like the non-proportional stories we explored in our last lesson! Let's use the picture to help us figure out an equation. I see that there are towers of 2. 1 group of 2 then 2 groups of 2 then 3 groups of 2. That's

Which table shows a proportion? What do you notice about the pictures? What do you notice about the equation for the table?

x	y
towers	squares
1	4
2	8
3	12

$x \cdot 4 = y$

x	y
towers	squares
1	6
2	8
3	10

$1 \times 2 + 4$   
 $2 \times 2 + 4$   
 $3 \times 2 + 4$

like times 2 over and over. But each of these pictures also has an extra tower at the end with 4 squares. That's like a plus 4. Let's see if times 2 plus 4 works for our table. *Point from left to right in each row as you do the math.* 1 times 2 is 2 plus 4 is 4. That works! Next row, 2 times 2 is 4 plus 4 is 8. That works! Next row, 2 times 3 is 6 plus 4 is 10. That works! Yay!

Which table shows a proportion? What do you notice about the pictures? What do you notice about the equation for the table?

x	y
towers	squares
1	4
2	8
3	12

$x \cdot 4 = y$  or  $y = 4x$

x	y
towers	squares
1	6
2	8
3	10

$1 \times 2 + 4$   
 $2 \times 2 + 4$   
 $3 \times 2 + 4$   
 $x \cdot 2 + 4 = y$   
 or  $y = 2x + 4$

So we did times 2 plus 2 every time. That's towers times 2 plus 2 equals squares. So, let's label towers as x and squares as y. That means our equation is x times 2 plus 2 equal y. Or if we want to rewrite the equation, it can be written as  $2x + 2 = y$ . Let's go back to these questions on the slide. First, which table shows a proportion? It is this first one that had a constant of proportionality.

Next, what do you notice about the pictures? The one on the left, the proportional one, is repetitive groups. The one of the right had an extra bit added on. It was NOT proportional. Now, most importantly, what do you notice about the equations? **Possible Student Answers, Key Points:**

- The first equation only has multiplication.
- The second equation has addition.
- The second equation has two operations.

The first equation only has multiplication. The second multiplication has multiplication and addition. The proportional equation has multiplication. The NON proportional equation has two operations. That is really helpful for us to notice because now we can just look at equations and we will already know if they are proportions. In fact, it is so important that mathematicians gave that kind of equation a special name. They call it  $y = kx$ . K stands for a number of some kind. So this just means y equals some number times x. And that's what we have for  $y = 3x$  on the left. But it is NOT  $y = kx$  on the right. It is  $y = 2x + 2$ . It is NOT proportional.

**Let's Think (Slide 5):** So, this is our big idea for today. *Read the top line of the slide.* "An equation in the form  $y = kx$  will always be proportional." Let's look at an equation we've been given here. It says, "Use the equation,  $y = 3x + 1$  to complete the table. Find the unit rates for each row." I'm already noticing that this equation isn't just multiplication. It is a two part equation. But, let's get some numbers. All we have to do to complete the table is plug in the number in the x column into the where the letter x is. So, I will write the equation as it is. Then on the next line I will substitute x. Then I do 3 times 1, which is 3. And I am going to recopy everything else to make a full next line. Now I see 3 plus 1 so  $y = 4$ . Let's do the next one. I will write the equation as it is. Then on the next line I will substitute x. Then I do 3 times 2, which is 6. And I am going to recopy everything else to make a full next line.

$$y = 3x + 1$$

$$y = 3 \cdot 1 + 1$$

$$y = 3 + 1$$

$$y = 4$$
  

$$y = 3x + 1$$

$$y = 3 \cdot 2 + 1$$

$$y = 6 + 1$$

$$y = 7$$
  

$$y = 3x + 1$$

$$y = 3 \cdot 3 + 1$$

$$y = 9 + 1$$

$$y = 10$$

x	y
1	4
2	7
3	10

Now I see 6 plus 1 so  $y = 7$ . Let's do the next one. I will write the equation as it is. Then on the next line I will substitute x. Then I do 3 times 3, which is 9. And I am going to recopy everything else to make a full next line. Now I see 9 plus 1 so  $y = 10$ .



$$\begin{array}{r} 4 \\ 1 \overline{)4} \\ \underline{-4} \\ 0 \end{array}$$

$$\begin{array}{r} 3\frac{1}{2} \\ 2 \overline{)7} \\ \underline{-6} \\ 1 \end{array}$$

$$\begin{array}{r} 3\frac{1}{3} \\ 3 \overline{)10} \\ \underline{-9} \\ 1 \end{array}$$

Now we can find the unit rates. In the first row, we do 4 divided by 1, which is 4. For the next row, we do 7 divided by 2. 2 goes into seven 3 times. I subtract 6 and have a remainder of 1 so my answer is two and one half. For the next row, we do 10 divided by 3. 3 goes into ten 3 times. I subtract 9 and have a remainder of 1 so my answer is two and one third.

Are the equation and table here proportional? How do you know? [Possible Student Answers, Key Points:](#)

- No because the equation has an extra plus 1 in it.
- No because the unit rates are not the same.
- No because there isn't a constant of proportionality.
- No because it doesn't keep increasing the same way.

No, they are not proportional! First of all, we could predict that because we see the equation  $3x + 1$  has this extra plus 1. It is not in the form  $y = kx$ . But also, when we used the equation to fill in the table and found the unit rates, we saw that all the unit rates were different. So there isn't a constant of proportionality.

**Let's Think (Slide 6):** Now we can look at equations and just know if they are proportional even without doing the math. We just have to know that an equation in the form  $y = kx$  will always be proportional. And if they can't be written in that form, they aren't proportional. This says, "Cross out all the equation that you think are NOT proportional. In other words, cross out the equations that are not

(a)  $y = 7x$  ✓

in the form,  $y = kx$ ." I am going to let you show me with a SILENT thumbs up or thumbs down. Let's start with  $y = 7x$ . Is that in the form  $y = kx$ ? Is that proportional? YES! So I am not going to cross it out.

(b)  ~~$y = x + 4$~~

Let's look at the next one. Show me with a SILENT thumbs up or thumbs down. Is that in the form  $y = kx$ ? Is that proportional? NO! It has addition! I am going to cross it out.

Now this one might seem tricky but don't get tricked. Equations can be written in equivalent ways where they still mean the same thing. So, show me with a SILENT thumbs up or thumbs down. Is  $4x =$

(c)  $4x = y$  ✓

$y$  in the form  $y = kx$ ? Is it proportional? It is! The  $y$  and the  $4x$  are on opposite sides of the equal sign but it still means the same idea. So it is still in the form  $y = kx$ . I'm not going to cross it out.

(d)  ~~$5x - 2 = y$~~

Let's look at the next one. It says  $5x - 2 = y$ . Show me with a SILENT thumbs up or thumbs down. Is  $5x - 2 = y$  in the form  $y = kx$ ? Is it proportional? NO! It has subtraction. That is not  $kx$ .  $Kx$  is multiplication. I am going to cross it out.

(e)  $y = \frac{1}{2}x$  ✓

Let's look at the next one. It is  $y$  equals one half  $x$ . Show me with a SILENT thumbs up or thumbs down. Is it in the form  $y = kx$ ? Is it proportional? YES! It has a fraction but it is still just multiplying  $x$ . It does not have addition or subtraction. It is proportional.

(f)  $y = x + 2$  ✓

Okay, now for the last one. This one is the trickiest and it is going to teach us something new. I'll give you one big hint - division is just the opposite of multiplication. So what do you think? Show me with a SILENT thumbs up or thumbs down. Is it in the form  $y = kx$ ? It is! Dividing by 2 is the same as

multiplying by one half. So really this division equation is related to a multiplication equation. It is proportional.

**Let's Try It (Slide 7):** Great thinking today! Now, let's practice! I am going to take you through step by step.

# WARM WELCOME



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**Today we will recognize that proportional relationships are characterized by equations in the form  $y = kx$ .**

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## Let's Review:

If we see a pattern, we can write an equation from the table.

Mary fills the fruit baskets that she sells at her shop with apples and oranges. The table shows how much fruit she buys for different amounts of baskets. Let  $x$  represent the number of oranges and  $y$  represent the number of apples. Write an equation to represent the amount of each fruit that Maria buys.

oranges	apples
2	4
4	8
6	12


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## Let's Talk:

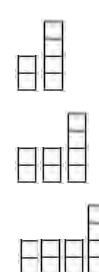
Equations from proportion tables always have the same form.

Which table shows a proportion? What do you notice about the pictures? What do you notice about the equation for the table?

towers	squares
1	4
2	8
3	12



towers	squares
1	6
2	8
3	10



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Let's Think:

**An equation in the form  $y = kx$  will always be proportional.**

Use the equation,  $y = 3x + 1$ , to complete the table.

x	y
1	
2	
3	

Find the unit rates for each row.

Are the equation and table proportional? \_\_\_\_\_ How do you know.

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Let's Think:

**An equation in the form  $y = kx$  will always be proportional.**

Cross out all the equation that you think are NOT proportional. In other words, cross out the equations that are not in the form,  $y = kx$ .

- (a)  $y = 7x$
- (b)  $y = x + 4$
- (c)  $4x = y$
- (d)  $5x - 2 = y$
- (e)  $y = \frac{1}{2}x$
- (f)  $y = x \div 2$

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## Let's Try It:

Let's determine if equations are proportional together.

Name: \_\_\_\_\_

G7 U2 Lesson 7 - Let's Try It

Let's look for patterns in the equations for proportional relationships.

Maizy, Lea and Connor each recorded the amount they got paid for mowing a lawn based on the number of hours they worked.

x	y	x	y	x	y
hours	dollars	hours	dollars	hours	dollars
1	10	1	13	4	12
2	20	2	14	8	24
3	30	3	15	12	36
4	40	4	16	16	48

1. Is there a constant of proportionality? If so, what? _____	2. Is there a constant of proportionality? If so, what? _____	3. Is there a constant of proportionality? If so, what? _____
--	--	--

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_

G7 U2 Lesson 7 - Independent Work

Remember: Ratios must have a constant of proportionality in order to be proportional.

Use the equation to fill in the table. Then determine if it is proportional.

1. Is $y = 6x$ proportional? _____ <table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td>2</td><td></td></tr> <tr><td>4</td><td></td></tr> <tr><td>6</td><td></td></tr> <tr><td>11</td><td></td></tr> <tr><td>20</td><td></td></tr> </tbody> </table> <p>How do you know?</p> <p>If so, what's the constant of proportionality? _____</p>	x	y	2		4		6		11		20		2. Is $y = x + 2$ proportional? _____ <table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td>2</td><td></td></tr> <tr><td>4</td><td></td></tr> <tr><td>6</td><td></td></tr> <tr><td>11</td><td></td></tr> <tr><td>20</td><td></td></tr> </tbody> </table> <p>How do you know?</p> <p>If so, what's the constant of proportionality? _____</p>	x	y	2		4		6		11		20	
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2																									
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20																									
x	y																								
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6																									
11																									
20																									
3. Is $y = x - 1$ proportional? _____ <table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td></td><td></td></tr> </tbody> </table>	x	y			4. Is $y = 4x$ proportional? _____ <table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td></td><td></td></tr> </tbody> </table>	x	y																		
x	y																								
x	y																								

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**Let's look for patterns in the equations for proportional relationships.**

Maizy, Lea and Connor each recorded the amount they got paid for mowing a lawn based on the number of hours they worked.

x	y	x	y	x	y
hours	dollars	hours	dollars	hours	dollars
1	10	1	13	4	12
2	20	2	14	8	24
3	30	3	15	12	36
4	40	4	16	16	48

1. Is there a constant of proportionality? If so, what?

\_\_\_\_\_

2. Is there a constant of proportionality? If so, what?

\_\_\_\_\_

3. Is there a constant of proportionality? If so, what?

\_\_\_\_\_

4. Which equation can be used to represent the table?

- (a)  $y = 10x$
- (b)  $y = x + 10$
- (c)  $y = 3x$
- (d)  $y = x + 12$

5. Which equation can be used to represent the table?

- (a)  $y = 10x$
- (b)  $y = x + 10$
- (c)  $y = 3x$
- (d)  $y = x + 12$

6. Which equation can be used to represent the table?

- (a)  $y = 10x$
- (b)  $y = x + 10$
- (c)  $y = 3x$
- (d)  $y = x + 12$

6. What do you notice about the types of equations that had a constant of proportionality?

Use the equation to fill in the table:  $y = 2x + 1$ .

6. Notice what operations are happening to  $x$  in the equation. Put that in each circle of the table.

7. Use the operation in the circle on  $x$  to find  $y$ .

8. Check for the constant of proportionality.

$x$	$y$
3	○
6	○
9	○
30	○
24	○

7. Are the equation and table above proportional? \_\_\_\_\_ How do you know? \_\_\_\_\_

---

---

---

8. If they are proportional, what is the constant of proportionality? \_\_\_\_\_

Use the equation to fill in the table:  $y = x \div 3$ .

9. Notice what operations are happening to  $x$  in the equation. Put that in each circle of the table.

10. Use the operation in the circle on  $x$  to find  $y$ .

11. Check for the constant of proportionality.

$x$	$y$
3	○
6	○
9	○
30	○
24	○

12. Are the equation and table above proportional? \_\_\_\_\_ How do you know? \_\_\_\_\_

---

---

---

13. If they are proportional, what is the constant of proportionality? \_\_\_\_\_

Name: \_\_\_\_\_

Remember: Ratios must have a constant of proportionality in order to be proportional.

Use the equation to fill in the table. Then determine if it is proportional.

1. Is  $y = 6x$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_

2. Is  $y = x + 2$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_

3. Is  $y = x - 1$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_

4. Is  $y = 4x$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_

Use the equation to fill in the table. Then determine if it is proportional.

5. Is  $y = x \div 2$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_\_

6. Is  $y = 2x + 1$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_\_

7. Is  $y = \frac{1}{2}x$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_\_

8. Is  $y = 3x - 2$  proportional? \_\_\_\_\_

$x$	$y$
2	
4	
6	
11	
20	

How do you know?

If so, what's the constant of proportionality? \_\_\_\_\_

Let's look for patterns in the equations for proportional relationships.

Maizy, Lea and Connor each recorded the amount they got paid for mowing a lawn based on the number of hours they worked.

x		y
hours		dollars
1	$\times 10$	10
2	$\times 10$	20
3	$\times 10$	30
4	$\times 10$	40

x		y
hours		dollars
1	$+12$	13
2	$+12$	14
3	$+12$	15
4	$+12$	16

x		y
hours		dollars
4	$\times 3$	12
8	$\times 3$	24
12	$\times 3$	36
16	$\times 3$	48

1. Is there a constant of proportionality? If so, what?

10

$$\begin{array}{r} 10 \\ 1 \overline{)10} \\ \underline{-10} \\ 00 \end{array} \quad \begin{array}{r} 10 \\ 2 \overline{)20} \\ \underline{-20} \\ 00 \end{array} \quad \begin{array}{r} 10 \\ 3 \overline{)30} \\ \underline{-30} \\ 00 \end{array}$$

2. Is there a constant of proportionality? If so, what?

NO

$$\begin{array}{r} 13 \\ 1 \overline{)13} \\ \underline{-13} \\ 00 \end{array} \quad \begin{array}{r} 07 \\ 2 \overline{)14} \\ \underline{-14} \\ 00 \end{array} \quad \begin{array}{r} 05 \\ 3 \overline{)15} \\ \underline{-15} \\ 00 \end{array}$$

3. Is there a constant of proportionality? If so, what?

3

$$\begin{array}{r} 03 \\ 4 \overline{)12} \\ \underline{-12} \\ 00 \end{array} \quad \begin{array}{r} 03 \\ 8 \overline{)24} \\ \underline{-24} \\ 00 \end{array} \quad \begin{array}{r} 03 \\ 12 \overline{)36} \\ \underline{-36} \\ 00 \end{array}$$

4. Which equation can be used to represent the table?

- (a)  $y = 10x$
- (b)  $y = x + 10$
- (c)  $y = 3x$
- (d)  $y = x + 12$

5. Which equation can be used to represent the table?

- (a)  $y = 10x$
- (b)  $y = x + 10$
- (c)  $y = 3x$
- (d)  $y = x + 12$

6. Which equation can be used to represent the table?

- (a)  $y = 10x$
- (b)  $y = x + 10$
- (c)  $y = 3x$
- (d)  $y = x + 12$

6. What do you notice about the types of equations that had a constant of proportionality?

All the equations had some number times  $x$ .  
They are in the form  $y = kx$ .

Use the equation to fill in the table:  $y = 2x + 1$ .

6. Notice what operations are happening to  $x$  in the equation. Put that in each circle of the table.

7. Use the operation in the circle on  $x$  to find  $y$ .

8. Check for the constant of proportionality.

$$\begin{array}{r} 2\bar{3} \\ 3 \overline{)7} \\ \underline{-6} \\ 1 \end{array} \quad \begin{array}{r} 02\bar{1} \\ 6 \overline{)13} \\ \underline{-12} \\ 1 \end{array}$$

$x$		$y$
3	$\odot \times 2 + 1$	7
6	$\odot \times 2 + 1$	13
9	$\odot \times 2 + 1$	19
30	$\odot \times 2 + 1$	61
24	$\odot \times 2 + 1$	49

7. Are the equation and table above proportional? NO How do you know? \_\_\_\_\_

There is no constant of proportionality.

The equation is not in the form  $y = Kx$ .

8. If they are proportional, what is the constant of proportionality? none

Use the equation to fill in the table:  $y = x \div 3$ .

9. Notice what operations are happening to  $x$  in the equation. Put that in each circle of the table.

10. Use the operation in the circle on  $x$  to find  $y$ .

11. Check for the constant of proportionality.

$$\begin{array}{r} 0\bar{3} \\ 3 \overline{)1} \\ \underline{-0} \\ 1 \end{array} \quad \begin{array}{r} 02\bar{6} \\ 6 \overline{)2} \\ \underline{-0} \\ 2 \end{array} \quad \begin{array}{r} 02\bar{3} \\ 9 \overline{)3} \\ \underline{-0} \\ 3 \end{array}$$

$x$		$y$
3	$\odot \div 3$	1
6	$\odot \div 3$	2
9	$\odot \div 3$	3
30	$\odot \div 3$	10
24	$\odot \div 3$	8

12. Are the equation and table above proportional? Yes How do you know? \_\_\_\_\_

There is a constant of proportionality.

Dividing by 3 is like multiplying by  $\frac{1}{3}$  so the equation is  $y = \frac{1}{3}x$  which is in the form  $y = Kx$ .

13. If they are proportional, what is the constant of proportionality?  $\frac{1}{3}$



# Name: ANSWER KEY

Remember: Ratios must have a constant of proportionality in order to be proportional.

Use the equation to fill in the table. Then determine if it is proportional.

1. Is  $y = 6x$  proportional? yes

x	y
2	12
4	24
6	36
11	66
20	120

$$\begin{array}{r} 2 \overline{)12} \\ \underline{-12} \\ 00 \end{array} \quad \begin{array}{r} 4 \overline{)24} \\ \underline{-24} \\ 00 \end{array}$$

How do you know?

They have a constant of proportionality.  
and/or

The equation is in the form  $y=kx$ . It is not in  $y=kx$  form.

If so, what's the constant of proportionality? 6

2. Is  $y = x + 2$  proportional? no

x	y
2	4
4	6
6	8
11	13
20	22

$$\begin{array}{r} 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 4 \overline{)6} \\ \underline{-4} \\ 2 \end{array}$$

How do you know?

There is not a constant of proportionality.  
and/or

The equation is in the form  $y=kx$ . It is not in  $y=kx$  form.

If so, what's the constant of proportionality? none

3. Is  $y = x - 1$  proportional? no

x	y
2	1
4	3
6	5
11	10
20	19

$$\begin{array}{r} 0 \frac{1}{2} \\ 2 \overline{)1} \\ \underline{-0} \\ 1 \end{array} \quad \begin{array}{r} 0 \frac{3}{4} \\ 4 \overline{)3} \\ \underline{-0} \\ 3 \end{array}$$

How do you know?

There is not a constant of proportionality.  
and/or

It is not in  $y=kx$  form.

If so, what's the constant of proportionality? none

4. Is  $y = 4x$  proportional? yes

x	y
2	8
4	16
6	24
11	44
20	80

$$\begin{array}{r} 4 \\ 2 \overline{)8} \\ \underline{-8} \\ 0 \end{array} \quad \begin{array}{r} 4 \\ 4 \overline{)16} \\ \underline{-16} \\ 0 \end{array}$$

How do you know?

They have a constant of proportionality.  
and/or

The equation is in  $y=kx$  form.

If so, what's the constant of proportionality? 4

Use the equation to fill in the table. Then determine if it is proportional.

5.

Is  $y = x \div 2$  proportional? yes

x	y
2	1
4	2
6	3
11	$5\frac{1}{2}$
20	10

$$\begin{array}{r} 0\frac{1}{2} \\ 2 \overline{)1} \\ \underline{-0} \\ 1 \end{array} \quad \begin{array}{r} 0\frac{3}{4} \\ 4 \overline{)2} \\ \underline{-0} \\ 2 \end{array}$$

How do you know?

Dividing by 2 is the same as  $\times \frac{1}{2}$  so the equation is in the form  $y = kx$ .

and/or

There is a constant of proportionality.

If so, what's the constant of proportionality?  $\frac{1}{2}$

6.

Is  $y = 2x + 1$  proportional? no

x	y
2	5
4	9
6	13
11	23
20	41

How do you know?

It is not in  $y = kx$  form.

and/or

There is no constant of proportionality.

If so, what's the constant of proportionality?    

7.

Is  $y = \frac{1}{2}x$  proportional? yes

x	y
2	1
4	2
6	3
11	$5\frac{1}{2}$
20	10

$$\begin{array}{r} 0\frac{1}{2} \\ 2 \overline{)1} \\ \underline{-0} \\ 1 \end{array} \quad \begin{array}{r} 0\frac{3}{4} \\ 4 \overline{)2} \\ \underline{-0} \\ 2 \end{array}$$

How do you know?

It has the same constant of proportionality.

and/or

It is in the form  $y = kx$

If so, what's the constant of proportionality?    

8.

Is  $y = 3x - 2$  proportional? no

x	y
2	4
4	12
6	16
11	31
20	58

$$\begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 3 \\ 4 \overline{)12} \\ \underline{-12} \\ 00 \end{array}$$

How do you know?

There is no constant of proportionality.

and/or

It is not in the form  $y = kx$ .

If so, what's the constant of proportionality? none

## **G7 U2 Lesson 8**

Recognize that proportional relationships are characterized by equations in the form

$$y = kx.$$

**G7 U2 Lesson 8 - Today we will use an equation to solve problems that involve proportional relationships.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will use an equation to solve problems that involve proportional relationships. We are just putting all the ideas from the previous lessons together now.

**Let's Review (Slide 3):** We are going to review algebra for the work that we are going to do today. There are two things to remember. Read along silently with me while I read aloud: "We must keep equations balanced by doing the same opposite operation on both sides. We must substitute the

Solve for y when x = 12.

$$y = 4x$$

$$y = 4 \cdot 12$$

$$y = 48$$

correct letter." These are ideas from 6th grade but let's remind ourselves what this means. It says, "Use  $y = 4x$ . Solve for y when  $x = 12$ ." First, I am going to write the equation just like it is. Then I am going to rewrite the equation except for the part I want to substitute. It's like subbing a player in soccer or any sport. I am going to take out the x and put 12 in its place. So now I have y = 4 times 12. Now in this case, y is alone. All the math is on this sign and I just need to do it. 4 times 12 is 48 so y equals 48.

Let's do the next one. It says, "solve for x when  $y = 12$ ." I still write the equation I've been given without changing a thing. Then I am going to rewrite the equation except for the part that I want to substitute. Except this time, I want to substitute the y not the x. That's why it

Solve for x when y = 12.

$$y = 4x$$

$$\frac{12}{4} = \frac{4x}{4}$$

$$3 = x$$

said we must substitute the correct letter. I am going to rewrite it as  $12 = 4x$ . This time I can't just multiply by 4. I don't know what to multiply it by. That's what I'm trying to figure out. Instead, I'm going to do just what it said. It said, "we must keep equations balanced by doing the same opposite operation on both sides." The opposite of "times 4" is dividing by 4. I have to do that on both sides. Now I have 12 divided by 4 equals 4x divided by 4. 12 divided by 4 is 3 on this side. 4x divided by 4 is just x because 4 divided by 4 is 1. And we have  $3 = x$ .

x	y
dogs	pounds of food
1	
2	
3	

**Let's Talk (Slide 4):** Just like we had to substitute the correct letter with numbers, we must substitute the correct letter in a word problem as well. Let's try this word problem. Read along silently with me while I read it out loud. "Each week, AJ uses the equation  $y = 4x$  to determine how many pounds of dog food to buy for each of his dog. X represents the number of dogs. Y represents the number of pounds of dog food. Complete the table." Let's start by labeling our table with x and y. It said x represents the number of dogs so I'm going to put x above dogs. It said y represents the number of pounds of dog food so I'm going to put y above pounds of food.

x	y
dogs	pounds of food
1	4
2	8
3	12

Now I can see on the table that these numbers are x's. The first row is  $x = 1$  then  $x = 2$  then  $x = 3$ . We can plug these in.  $y = 4x$  so we rewrite it as y = 4 times 1. That's  $y = 4$ . Then we put it on the table. Next one. We start with  $y = 4x$ . We rewrite it as y = 4 times 2. That's  $y = 8$ . Then we put it on the table. Next one. We start with  $y = 4x$ . We rewrite it as y = 4 times 3. That's  $y = 12$ . Then we put it on the table.

x	y
dogs	pounds of food
1	4
2	8
3	12
10	

Those were easier because we labeled the x. To figure out what to plug in for the question, we're going to have to do the same kind of thinking about, "Is this number an x or y?" The question is, "how many pounds of food would AJ need for 10 dogs?" We can think about what variable this is a few ways. First, we can put 10 in the table under dogs and see that it is x. Also, the story said x equals dogs.

$$y = 4x$$

$$y = 4 \cdot 10$$

$$y = 40$$

Our problem says 10 dogs. So it has to go in the x place. We write  $y = 4x$  then substitute x. We get  $y = 4$  times 10 so y equals 40.

dogs	pounds of food
1	4
2	8
3	12
10	<del>40</del>
	20

**Let's Think (Slide 5):** Let's try another question for the same problem. It still says the same story about AJ. But let's read the question. It says, "How many dogs must AJ have to buy 20 pounds of food?" Remember, we have to substitute the correct letter in the word problem. There are a few ways to think about it. We can put 20 on the table. It has to be 20 in the pounds of food column, right? So we can already see that 20 is y.

$$y = 4x$$

$$\frac{20}{4} = \frac{4x}{4}$$

$$5 = x$$

Or we can notice that the word after 20 is pounds of food and it said y represents pounds of food. Either way, we write the equation,  $y = 4x$ . Now we put 20 in the y spot. It is  $20 = 4x$ . This problem is going to be a work backwards problem. I divide by 4 on each side. We get 20 divided by 4 is 5. On the other side we have  $4x$  divided 4 is just x. So  $5 = x$ .

**Let's Try It (Slide 6):** Great thinking! Now, let's practice! I am going to take you through step by step.

# WARM WELCOME



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**Today we will use an equation to solve problems that involve proportional relationships.**

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## Let's Review:

**We must keep equations balanced by doing the same opposite operation on both sides. We must substitute the correct letter.**

Use  $y = 4x$ .

Solve for  $y$  when  $x = 12$ .

Solve for  $x$  when  $y = 12$ .

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## Let's Talk:

**We must substitute the correct letter in a word problem.**

Each week, AJ uses the equation  $y = 4x$  to determine how many pounds of dog food to buy for each of his dog.  $X$  represents the number of dogs.  $Y$  represents the number of pounds of dog food. Complete the table.

How many pounds of food would AJ need for 10 dogs?

<b>dogs</b>	<b>pounds of food</b>
1	
2	
3	

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## Let's Think:

We must substitute the correct letter in a word problem.

Each week, AJ uses the equation  $y = 4x$  to determine how many pounds of dog food to buy for each of his dog. X represents the number of dogs. Y represents the number of pounds of dog food. Complete the table.

How many dogs must AJ have to buy 20 pounds of food?

dogs	pounds of food
1	4
2	8
3	12
10	40

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## Let's Try It:

Let's use equations to solve problems together.

Name: \_\_\_\_\_ G7 U2 Lesson 8 - Let's Try It

Sammy has a dog sitting business. He uses the equation,  $y = 4x$ , to determine how many bones to buy for his dogs each month. He has  $x$  stand for the number of dogs and  $y$  stand for the number of bones. Use the equation to complete the table and answer questions about Sammy's business.

- What does  $x$  represent in the story? \_\_\_\_\_ Write the word above  $x$ .
- What does  $y$  represent in the story? \_\_\_\_\_ Write the word above  $y$ .
- Notice what is happening to the  $x$  in your equation. Put that operation in each circle below.

$x$	$y$
1	<input type="text"/>
2	<input type="text"/>
3	<input type="text"/>

- What related equation could you also use?  
\_\_\_\_\_
- Plug each  $x$  into the first equation and solve for  $y$ . Fill in the table.  
 $x = 1$                        $x = 2$                        $x = 3$

How many bones Sammy would need for 20 dogs?  
 \_\_\_\_\_

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson B - Independent Work

Remember: You must pay attention to the words after the numbers.

Use the equation to fill in the table. Then answer the questions and fill in the final rows.

1. Whellis get a certain amount of time to play video games based on the number of hours that he studies. It can be shown with the equation, $y = 10x$ , where $x$ equals the number of hours Whellis does HW and $y$ equals the number of minutes he plays video games.		2. How many minutes does Whellis get to play video games after 6 hours of studying?													
<table border="1"><thead><tr><th>x</th><th>y</th></tr></thead><tbody><tr><td>1</td><td></td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></tbody></table>	x	y	1		2		3							3. How many minutes does Whellis get to play video games after 6 hours of studying?	
x	y														
1															
2															
3															
4. Meryl calculates the amount she pays the kid who rakes her leaves with the equation $y = 8x$ , where $x$ equals the number of hours the kid		5. How much would Meryl need to pay the kid for 10 hours of raking?													

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**Sammy has a dog sitting business. He uses the equation,  $y = 4x$ , to determine how many bones to buy for his dogs each month. He has  $x$  stand for the number of dogs and  $y$  stand for the number of bones. Use the equation to complete the table and answer questions about Sammy's business.**

1. What does  $x$  represent in the story? \_\_\_\_\_ Write the word above  $x$ .
2. What does  $y$  represent in the story? \_\_\_\_\_ Write the word above  $y$ .
3. Notice what is happening to the  $x$  in your equation. Put that operation in each circle below.

$x$	$y$
1	○
2	○
3	○

4. What related equation could you also use?

\_\_\_\_\_

5. Plug each  $x$  into the first equation and solve for  $y$ . Fill in the table.

$x = 1$

$x = 2$

$x = 3$

**How many bones Sammy would need for 20 dogs?**

6. Put the number on the table under the correct word.
7. Plug it into the correct place of an equation and solve. Write the value on the table.

**How many dogs must Sammy have if he purchased 20 bones?**

8. Put the number on the table under the correct word.
9. Plug it into the correct place of the equation and solve. Write the value on the table.

Lindsey uses this equation,  $y = 3x$  to plan how to make rice and beans, where  $x$  represents the cups of uncooked beans and  $y$  represents the cups of uncooked rice.

1. What does  $x$  represent in the story? \_\_\_\_\_ Write the word above  $x$ .
2. What does  $y$  represent in the story? \_\_\_\_\_ Write the word above  $y$ .
3. Notice what is happening to the  $x$  in your equation. Put that operation in each circle below.

$x$	$y$
○	6
○	9
○	12

4. What related equation could you also use?

\_\_\_\_\_

5. Plug each  $x$  into the first equation and solve for  $y$ . Fill in the table.

$y = 6$

$y = 9$

$y = 12$

**How many cups of uncooked rice will Lindsey need for 4 cups of beans?**

6. Put the number on the table under the correct word.
7. Plug it into the correct place of an equation and solve. Write the value on the table.

**How many cups of uncooked beans will Lindsey need for 1 cup of rice?**

8. Put the number on the table under the correct word.
9. Plug it into the correct place of the equation and solve. Write the value on the table.

Name: \_\_\_\_\_

Remember: You must pay attention to the words after the numbers.

Use the equation to fill in the table. Then answer the questions and fill in the final rows.

1. Whellis gets a certain amount of time to play video games based on the number of hours that he studies. It can be shown with the equation,  $y = 10x$ , where  $x$  equals the number of hours Whellis does HW and  $y$  equals the number of minutes he plays video games.

x	y
1	
2	
3	

2. How many minutes does Whellis get to play video games after 6 hours of studying?

3. How many minutes does Whellis get to play video games after 8 hours of studying?

4. Meryl calculates the amount she pays the kid who rakes her leaves with the equation  $y = 8x$ , where  $x$  equals the number of hours the kid spends raking and  $y$  equals the number of dollars.

x	y
	16
	32
	64

5. How much would Meryl need to pay the kid for 10 hours of raking?

6. How many hours of raking must have been done if Meryl paid \$80?



Use the equation to fill in the table. Then answer the questions and fill in the final rows.

7. The store stocks short sleeve and long sleeve shirts using the equation,  $y = 2x$ , where  $x$  is the number of long sleeve shirts and  $y$  is the short sleeve shirts.

x	y
1	
2	
3	

8. How many short sleeve shirts will the store have when they have 16 long sleeve shirts?

9. How many long sleeve shirts will the store have when they have 16 short sleeve shirts?

10. The equation for the amount of flea medicine that a kitty needs is  $y = 5x$  where  $x$  is the number of days and  $y$  is the amount of medicine in milligrams.

x	y
	10
	20
	30

11. How many milligrams does a kitty need over 7 days?

12. How many days will 45 mg of medicine last?

13. Jason gave his kitty 9 mg of medicine over 45 days. Was that the correct amount? Explain.

Sammy has a dog sitting business. He uses the equation,  $y = 4x$ , to determine how many bones to buy for his dogs each month. He has  $x$  stand for the number of dogs and  $y$  stand for the number of bones. Use the equation to complete the table and answer questions about Sammy's business.

1. What does  $x$  represent in the story? dogs Write the word above  $x$ .
2. What does  $y$  represent in the story? bones Write the word above  $y$ .
3. Notice what is happening to the  $x$  in your equation. Put that operation in each circle below.

dogs		bones
$x$		$y$
1	( $\times 4$ )	4
2	( $\times 4$ )	8
3	( $\times 4$ )	12
20	( $\times 4$ )	80
5	( $\times 4$ )	20

4. What related equation could you also use?

$y \div 4 = x$

5. Plug each  $x$  into the first equation and solve for  $y$ . Fill in the table.

$x = 1$   
 $y = 4x$   
 $y = 4 \cdot 1$   
 $y = 4$

$x = 2$   
 $y = 4x$   
 $y = 4 \cdot 2$   
 $y = 8$

$x = 3$   
 $y = 4x$   
 $y = 4 \cdot 3$   
 $y = 12$

How many bones Sammy would need for 20 dogs?

6. Put the number on the table under the correct word.
7. Plug it into the correct place of an equation and solve. Write the value on the table.

$y = 4x$   
 $y = 4 \cdot 20$   
 $y = 80$  bones

How many dogs must Sammy have if he purchased 20 bones?

8. Put the number on the table under the correct word.
9. Plug it into the correct place of the equation and solve. Write the value on the table.

$y = 4x$   
 $\frac{20}{4} = \frac{4x}{4}$   
 $5 = x$  dogs

Lindsey uses this equation,  $y = 3x$  to plan how to make rice and beans, where  $x$  represents the cups of uncooked beans and  $y$  represents the cups of uncooked rice.

1. What does  $x$  represent in the story? cups of beans Write the word above  $x$ .

2. What does  $y$  represent in the story? cups of rice Write the word above  $y$ .

3. Notice what is happening to the  $x$  in your equation. Put that operation in each circle below.

cups of beans		cups of rice
$x$		$y$
2	$\times 3$	6
3	$\times 3$	9
4	$\times 3$	12
8	$\times 3$	24
$\frac{1}{3}$	$\times 3$	1

4. What related equation could you also use?

$$\underline{y \div 3 = x}$$

5. Plug each  $x$  into the first equation and solve for  $y$ . Fill in the table.

$$y = 6$$

$$y = 3x$$

$$\frac{6}{3} = \frac{3x}{3}$$

$$\boxed{2 = x}$$

$$y = 9$$

$$y = 3x$$

$$\frac{9}{3} = \frac{3x}{3}$$

$$\boxed{3 = x}$$

$$y = 12$$

$$y = 3x$$

$$\frac{12}{3} = \frac{3x}{3}$$

$$\boxed{4 = x}$$

How many cups of uncooked rice will Lindsey need for 8 cups of beans?

6. Put the number on the table under the correct word.

7. Plug it into the correct place of an equation and solve. Write the value on the table.

$$y = 3x$$

$$y = 3 \cdot 8$$

$$\boxed{y = 24} \text{ cups of rice}$$

How many cups of uncooked beans will Lindsey need for 1 cup of rice?

8. Put the number on the table under the correct word.

9. Plug it into the correct place of the equation and solve. Write the value on the table.

$$\frac{1}{3} = \frac{3x}{3}$$

$$\boxed{\frac{1}{3} = x} \text{ cups of beans}$$

Remember: You must pay attention to the words after the numbers.

Use the equation to fill in the table. Then answer the questions and fill in the final rows.

1. Whellis get a certain amount of time to play video games based on the number of hours that he studies. It can be shown with the equation,  $y = 10x$ , where  $x$  equals the number of hours Whellis does HW and  $y$  equals the number of minutes he plays video games.

hours	minutes
x	y
1	$1 \times 10$ 10
2	$2 \times 10$ 20
3	$3 \times 10$ 30
6	$6 \times 10$ 60
8	$8 \times 10$ 80

2. How many minutes does Whellis get to play video games after 6 hours of studying?

$$y = 10x$$

$$y = 10 \cdot 6$$

$$y = 60$$

3. How many minutes does Whellis get to play video games after 8 hours of studying?

$$y = 10x$$

$$y = 10 \cdot 8$$

$$y = 80$$

4. Meryl calculates the amount she pays the kid who rakes her leaves with the equation  $y = 8x$ , where  $x$  equals the number of hours the kid spends raking and  $y$  equals the number of dollars.

hours	dollars
x	y
2	$2 \times 8$ 16
4	$4 \times 8$ 32
8	$8 \times 8$ 64
10	$10 \times 8$ 80
10	$10 \times 8$ 80

5. How much would Meryl need to pay the kid for 10 hours of raking?

$$y = 8x$$

$$y = 8 \cdot 10$$

$$y = 80$$

6. How many hours of raking must have been done if Meryl paid \$80?

$$y = 8x$$

$$80 = 8x$$

$$\frac{80}{8} = \frac{8x}{8}$$

$$10 = x$$

Use the equation to fill in the table. Then answer the questions and fill in the final rows.

7. The store stocks short sleeve and long sleeve shirts using the equation,  $y = 2x$ , where  $x$  is the number of long sleeve shirts and  $y$  is the short sleeve shirts.

long		short
$x$		$y$
1	$\times 2$	2
2	$\times 2$	4
3	$\times 2$	6
16	$\times 2$	32
8	$\times 2$	16

8. How many short sleeve shirts will the store have when they have 16 long sleeve shirts?

$$y = 2x$$

$$y = 2 \cdot 16$$

$$y = 32$$

9. How many long sleeve shirts will the store have when they have 16 short sleeve shirts?

$$y = 2x$$

$$16 = \frac{2x}{2}$$

$$8 = x$$

10. The equation for the amount of flea medicine that a kitty needs is  $y = 5x$  where  $x$  is the number of days and  $y$  is the amount of medicine in milligrams.

days		mg
$x$		$y$
2	$\times 5$	10
4	$\times 5$	20
6	$\times 5$	30
7	$\times 5$	35
9	$\times 5$	45

11. How many milligrams does a kitty need over 7 days?

$$y = 5x$$

$$y = 5 \cdot 7$$

$$y = 35$$

12. How many days will 45 mg of medicine last?

$$y = 5x$$

$$\frac{45}{5} = \frac{5x}{5}$$

$$9 = x$$

13. Jason gave his kitty 9 mg of medicine over 45 days. Was that the correct amount? Explain.

No, that is not the right amount. Jason switched  $x$  and  $y$ .  $x$  is the number of days and  $y$  is the mg. So  $y = 5x$  means 5 times the number of days. Jason's answer of 9 mg and 45 days would be 5 times the number of milligrams. The right answer is  $45 \times 5$  which is 225 so 45 days needs 225 mg.

# **G7 U2 Lesson 9**

Write an equation to represent a proportional relationship and solve problems about proportional relationships.



**G7 U2 Lesson 9 - Today we will write an equation for story problems and determine if it is a proportion.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will write an equation for story problems and determine if it is a proportion. Most of this will be things you already know. But you are going to have to be super readers to make sure you can understand the story problems. Let's go!

**Let's Review (Slide 3):** To fill in a table, we use the values we are given to find the corresponding values. Let's read the problem and then we'll use the values we are given. "Tamara uses 3 pounds of chicken every time she makes chicken salad. Let x represent the time she makes chicken salad. Let y represent the pounds of chicken she uses. Write an equation to represent the relationship." Someone who read this story problem might think, "It only gives us one number! It's impossible to do any math with just one number!" But we also have numbers to work with on this table. So let's start with the first row. There is a 2 and the 2 is in the salads column so that means I have 2 salads. I am going to draw a picture to think about what's happening in the story. I will draw 2 salads. Then in the story it said that Tamara uses 3 pounds of chicken in each salad. So I will draw 3 in each. I can see that this is 2 groups of 3 which means 2 times 3 which equals 6.

Tamara uses 3 pounds of chicken every time she makes chicken salad. Let x represent the time she makes chicken salad. Let y represent the pounds of chicken she uses. Write an equation to represent the relationship.

③ ③

x	y
salads	pounds
2	6

Tamara uses 3 pounds of chicken every time she makes chicken salad. Let x represent the time she makes chicken salad. Let y represent the pounds of chicken she uses. Write an equation to represent the relationship.

③ ③  
③ ③ ③ ③

x	y
salads	pounds
2	6
4	12

Tamara uses 3 pounds of chicken every time she makes chicken salad. Let x represent the time she makes chicken salad. Let y represent the pounds of chicken she uses. Write an equation to represent the relationship.

③ ③  
③ ③ ③ ③  
④ ④ ④ ④ ④ ④

x	y
salads	pounds
2	6
4	12
6	18

Tamara uses 3 pounds of chicken every time she makes chicken salad. Let x represent the time she makes chicken salad. Let y represent the pounds of chicken she uses. Write an equation to represent the relationship.

③ ③  
③ ③ ③ ③  
④ ④ ④ ④ ④ ④  
 $2 \times 3 = 6$   
 $4 \times 3 = 12$   
 $6 \times 3 = 18$   
salads  $\times$  3 = pounds  
 $x \cdot 3 = y$  or  $y = 3x$

x	y
salads	pounds
2	6
4	12
6	18

I will draw 2 salads. Then in the story it said that Tamara uses 3 pounds of chicken in each salad. So I will draw 3 in each. I can see that this is 2 groups of 3 which means 2 times 3 which equals 6.

If I make this 4 salads now, I will have 3 pounds of chicken in each salad. That would be 4 groups of 3, which is 4 times 3 makes 12.

And then 6 salads with 3 pounds in each is 6 groups of 3 which is 6 times 3 which is 18.

This is something you already know how to do but there is a very important lesson here. You can use any number to draw out a problem and understand what is happening. Then you can go back and write the equation. We did  $2 \times 3 = 6$  and  $4 \times 3 = 12$  and  $6 \times 3 = 18$ . It is always salads times 3 equals pounds. I can put x in for salads and y in for pounds and now we have our final equation: 3 times x equals y. We can also write it as  $y = 3x$  and it means the same thing.

**Let's Talk (Slide 4):** If there aren't any values given, we can make up our own. Let's read this problem and see how that might work. Read along with your eyes while I read out loud. "Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let x represent the pitchers of lemonade. Let y represent the cups of sugar. Write an equation to represent the relationship." It might be tempting to jump to some operation with the 2 and the 5 here. Maybe I add them! Maybe I multiply them! If we just choose an idea that pops into our head without taking time to think, we are likely to get the wrong answer. Instead we're going to do exactly what we did on the last slide. We're going to take a few numbers and draw out the story to understand

x	y
pitchers	cups
1	7
2	9
3	11

what is happening. One of the clues that I should do this is that the problem wants us to use an x and a y. So I need to make a table with x and y. I also want to put the words that the x and y stand for. So, x is pitchers and y is cups. Now, I don't have to use the numbers in the story yet. The variables can be any numbers so let's just start with 1, 2 and 3.

Now this problem is exactly the same as what we just did on the last slide. Remember how we drew a picture and we wrote out the equations and then we put in the variables? It will be the exact same. The important thing we're learning here is that if they don't give us a table of values, we

**Let's Talk:**

If there aren't any values given, we can make up our own.

Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let x represent the pitchers of lemonade. Let y represent the cups of sugar. Write an equation to represent the relationship.

x	y
pitchers	cups
1	7
2	9
3	11

can just draw one ourselves with our own numbers. Let's draw. I have 1 pitcher of lemonade. Underline the sentence in the story as you reread it. Cici buys 2 cups of sugar for every pitcher. So I am going to draw 2 cups in this pitcher. But I'm not done this time. Because it also says she buys an extra 5 cups. So I need to draw an extra 5 cups. That is 1 group of 2, which is 2, plus 5, which is 7. I did  $x \times 2 + 5$ .

**Let's Talk:**

If there aren't any values given, we can make up our own.

Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let x represent the pitchers of lemonade. Let y represent the cups of sugar. Write an equation to represent the relationship.

x	y
pitchers	cups
1	7
2	9
3	11

Let's turn this into 2 pitchers. There are 2 more cups in this pitcher. I don't draw another 5 though. It didn't say 5 for every pitcher. It just said an extra 5 and we already have that extra 5. So this problem is 2 groups of 2 plus 5, which is 4 plus 5. I did  $x \times 2 + 5$  to get 9.

**Let's Talk:**

If there aren't any values given, we can make up our own.

Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let x represent the pitchers of lemonade. Let y represent the cups of sugar. Write an equation to represent the relationship.

x	y
pitchers	cups
1	7
2	9
3	11

Let's turn this into 3 pitchers. There are 2 more cups in this pitcher. I don't draw another 5 though. It didn't say 5 for every pitcher. It just said an extra 5 and we already have that extra 5. So this problem is 3 groups of 2 plus 5, which is 6 plus 5. I did  $x \times 2 + 5$  to get 11.

$1 \times 2 + 5 = 7$   
 $2 \times 2 + 5 = 9$   
 $3 \times 2 + 5 = 11$   
 pitchers  $\times 2 + 5 =$  cups

Our table is complete so now we can list out the equations. I did  $1 \times 2 + 5 = 7$  and  $2 \times 2 + 5 = 9$  and  $3 \times 2 + 5 = 11$ . I can see that it is always "times 2 plus 5" so I am going to put in words. Pitchers  $\times 2 + 5 =$  cups. And now I can put in letters, x is pitchers so x times 2 plus 5 equals y.

**Let's Talk:**

If there aren't any values given, we can make up our own.

Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let x represent the pitchers of lemonade. Let y represent the cups of sugar. Write an equation to represent the relationship.

x	y
pitchers	cups
1	7
2	9
3	11

Remember that the equal sign is just telling us both sides are the same so I can put y on this side and say  $y = 2x + 5$ .

**Let's Think (Slide 5):** This might seem a little new but one thing never changes, "We always evaluate whether a story is proportional with the equation or constant of proportionality" just like we've done for all these lessons. So let's think about the story that we just did. Is it a proportion? [Possible Student Answers, Key Points:](#)

- No, it's not proportional because it's not in the form  $y = kx$ .
- No, it's not proportional because it has a two step equation instead of just multiplication or division.
- No, it's not proportional because you wouldn't get the same unit rate for each row.
- No, it's not proportional because there isn't a constant of proportionality.

Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let  $x$  represent the pitchers of lemonade. Let  $y$  represent the cups of sugar. Write an equation to represent the relationship.

$y = 2x + 5$       **Not proportional!**

$$\begin{array}{r} 7 \\ 1 \overline{)7} \\ \underline{-7} \\ 0 \end{array}$$

$$\begin{array}{r} 4\frac{1}{2} \\ 2 \overline{)9} \\ \underline{-8} \\ 1 \end{array}$$

$$\begin{array}{r} 3\frac{2}{3} \\ 3 \overline{)11} \\ \underline{-9} \\ 2 \end{array}$$

It's NOT proportional! And there are many ways to tell. First, the equation isn't in the form  $y = kx$  where there is just multiplication or the related division. It's a two step equation with some addition in there. Second, when we look at the table that matches the equation, we can find the unit rates for each row and they will not be the same. Let me show you. I do 7 divided by 1 is 7. I do 9 divided by 2. 2 goes into 9 four times with a remainder of 1 so the answer is 4 and one half. I do 11 divided by 3. 3 goes into 11 three times with a remainder of 2 so the answer is 3 and two thirds.

Those unit rates are not the same so there is not a constant of proportionality and this relationship is not proportional. The story isn't proportional. The equation isn't proportional, and the table isn't proportional. They are all different ways of looking at the same thing and none of them are proportional.

**Let's Try It (Slide 6):** Now we will write more equations together. I will take you through step by step.

# WARM WELCOME



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**Today we will write an equation for story problems and determine if it is a proportion.**

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## Let's Review:

To fill in a table, we use the values we are given to find the corresponding values.

Tamara uses 3 pounds of chicken every time she makes chicken salad. Let  $x$  represent the time she makes chicken salad. Let  $y$  represent the pounds of chicken she uses. Write an equation to represent the relationship.

$x$	$y$
salads	pounds
2	
4	
6	

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## Let's Talk:

If there aren't any values given, we can make up our own.

Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let  $x$  represent the pitchers of lemonade. Let  $y$  represent the cups of sugar. Write an equation to represent the relationship.

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## Let's Think:

**We always evaluate whether a story is proportional with the equation or constant of proportionality.**

Cici buys 2 cups of sugar for every pitcher of lemonade she wants to make. And she buys an extra 5 cups just in case she needs it. Let  $x$  represent the pitchers of lemonade. Let  $y$  represent the cups of sugar. Write an equation to represent the relationship.

$$y = 2x + 5$$

<b>x</b>	<b>y</b>
pitchers	cups
1	7
2	9
3	11

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## Let's Try It:

**Now we will write more equations together!**

Name: \_\_\_\_\_ G7 U2 Lesson 9 - Let's Try It

**At Buzz Bakery they always sell bags of cookies with 6 cookies in each bag. Then the owner always add one extra cookie to the cookie order. Let  $x$  represent the number of bags in an order. Let  $y$  represent the number of cookies in an order.**

- Label the top of the table with  $x$  and  $y$ .
- Put the words on the table that correspond with  $x$  and  $y$ .
- Choose any value for  $x$  then draw a picture to represent the story.


- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .
- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .
- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .

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# On your Own:

## Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 9 - Independent Work

Remember: If you need help to understand the story, you can draw a picture.

Make a table to understand the word problem. Then write an equation. Determine if it is proportional.

1. Rudy is a tailor. He gets paid \$10 for every patch he sews. Let  $x$  represent the number of patches. Let  $y$  represent the number of dollars he earns. Write an equation to represent how much Rudy gets paid for sewing patches.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

2. Matt uses 1 gallon of gas for each lawn he mows. He also needs 2 gallons of gas to drive to the area where he mows. Let  $x$  represent the number of lawn he mows. Let  $y$  represent the number of gallons of gas he uses. Write an equation to represent how much gas Matt uses to mow lawns.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

3. Amy gets paid \$30 for every art class she teaches. She has to spend \$20 to buy all the supplies. Let  $x$  represent the number of art classes. Let  $y$  represent the dollars Amy has.

4. Maddie takes 15 mg of Vitamin D per day. Let  $x$  represent the number of days. Let  $y$  represent the number of mg that Maddie takes. Write an equation to represent the mg that Maddie takes

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Name: \_\_\_\_\_

**At Buzz Bakery they always sell bags of cookies with 6 cookies in each bag. Then the owner always add one extra cookie to the cookie order. Let  $x$  represent the number of bags in an order. Let  $y$  represent the number of cookies in an order.**

1. Label the top of the table with  $x$  and  $y$ .
2. Put the words on the table that correspond with  $x$  and  $y$ .
3. Choose any value for  $x$  then draw a picture to represent the story.


4. Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .
5. Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .
6. Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .

7. Make a list of the equations for each row.

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8. Write an equation with  $x$  and  $y$ .

---

9. Find the unit rate for each row.

10. Is the relationship proportional? \_\_\_\_\_

**At Donut Dash, they sell boxes of donuts with 12 donuts per box. Let  $x$  represent the number of boxes in an order. Let  $y$  represent the number of donuts in an order.**

11. Label the top of the table with  $x$  and  $y$ .

12. Put the words on the table that correspond with  $x$  and  $y$ .

13. Choose any value for  $x$  then draw a picture to represent the story.


14. Choose another value for  $x$  then draw a picture to represent the story.

Fill in  $y$ .

15. Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .

16. Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .

17. Make a list of the equations for each row.

---

---

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18. Write an equation with  $x$  and  $y$ .

19. Find the unit rate for each row.

20. Is the relationship proportional? \_\_\_\_\_

Remember: If you need help to understand the story, you can draw a picture.

Make a table to understand the word problem. Then write an equation. Determine if it is proportional.

1. Rudy is a tailor. He gets paid \$10 for every patch he sews. Let  $x$  represent the number of patches. Let  $y$  represent the number of dollars he earns. Write an equation to represent how much Rudy gets paid for sewing patches.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

2. Matt uses 1 gallon of gas for each lawn he mows. He also needs 2 gallons of gas to drive to the area where he mows. Let  $x$  represent the number of lawn he mows. Let  $y$  represent the number of gallons of gas he uses. Write an equation to represent how much gas Matt uses to mow lawns.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

3. Amy gets paid \$30 for every art class she teaches. She has to spend \$20 to buy all the supplies. Let  $x$  represent the number of art classes. Let  $y$  represent the dollars Amy has. Write an equation to show the total amount of money Amy gets from teaching.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

4. Maddie takes 15 mg of Vitamin D per day. Let  $x$  represent the number of days. Let  $y$  represent the number of mg that Maddie takes. Write an equation to represent the mg that Maddie takes over several days.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

Make a table to understand the word problem. Then write an equation. Determine if it is proportional.

5. Rachel's lawn requires 10 gallons of water plus an additional 2 gallons of water for each potted plant. Let  $x$  represent the number of potted plants. Let  $y$  represent the gallons of water. Write an equation for the gallons of water Rachel uses.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

6. It costs \$5 to get into the Spring Fair. Then each ride costs \$2. Let  $x$  represent the number of rides. Let  $y$  represent the total cost. Write an equation for the total cost the Spring Fair based on the number of rides.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

7. Percy gets 10 points for each basket at the Spring Fair game. Let  $x$  represent the number of baskets. Let  $y$  represent the number of points. Write an equation to find the points depending on the number of baskets.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

8. Caryn takes 5 weeks to make a quilt. Let  $x$  represent the number of quilts. Let  $y$  represent the number of weeks she needs to make one. Write an equation for the number of weeks Caryn needs based on the number of quilts.


Equation: \_\_\_\_\_

Is this relationship a proportion? \_\_\_\_\_

# ame: ANSWER KEY

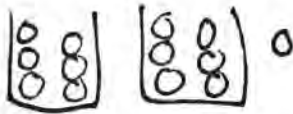
At Buzz Bakery they always sell bags of cookies with 6 cookies in each bag. Then the owner always add one extra cookie to the cookie order. Let  $x$  represent the number of bags in an order. Let  $y$  represent the number of cookies in an order.

$x$	$y$
bags	cookies
1	7
2	13
3	19
4	25

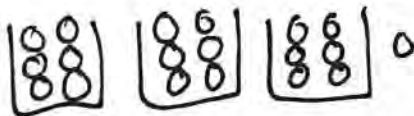
- Label the top of the table with  $x$  and  $y$ .
- Put the words on the table that correspond with  $x$  and  $y$ .
- Choose any value for  $x$  then draw a picture to represent the story.



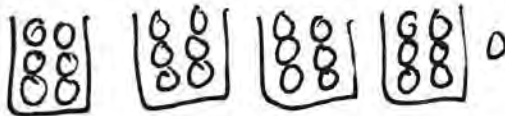
- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .



- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .



- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .



- Make a list of the equations for each row.

$$\begin{aligned} 1 \times 6 + 1 &= 7 \\ 2 \times 6 + 1 &= 13 \\ 3 \times 6 + 1 &= 19 \\ 4 \times 6 + 1 &= 25 \\ x \cdot 6 + 1 &= y \end{aligned}$$

- Write an equation with  $x$  and  $y$ .

- Find the unit rate for each row.

$$\begin{array}{r} 7 \\ 1 \overline{) 7} \\ \underline{-7} \\ 0 \end{array} \quad \begin{array}{r} 6\frac{1}{2} \\ 2 \overline{) 13} \\ \underline{-12} \\ 1 \end{array} \quad \begin{array}{r} 6\frac{1}{3} \\ 3 \overline{) 19} \\ \underline{-18} \\ 1 \end{array} \quad \begin{array}{r} 6\frac{1}{4} \\ 4 \overline{) 25} \\ \underline{-24} \\ 1 \end{array}$$

- Is the relationship proportional? NO



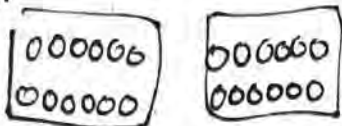
At Donut Dash, they sell boxes of donuts with 12 donuts per box. Let  $x$  represent the number of boxes in an order. Let  $y$  represent the number of donuts in an order.

$x$	$y$
boxes	donuts
1	12
2	24
3	36
4	48

- Label the top of the table with  $x$  and  $y$ .
- Put the words on the table that correspond with  $x$  and  $y$ .
- Choose any value for  $x$  then draw a picture to represent the story.



- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .



- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .



- Choose another value for  $x$  then draw a picture to represent the story. Fill in  $y$ .



- Make a list of the equations for each row.

$$\begin{aligned} & \underline{1 \times 12 = 12} \\ & \underline{2 \times 12 = 24} \\ & \underline{3 \times 12 = 36} \\ & \underline{4 \times 12 = 48} \\ & \underline{x \cdot 12 = y} \end{aligned}$$

- Write an equation with  $x$  and  $y$ .

- Find the unit rate for each row.

$$\begin{array}{r} 12 \\ 1 \overline{)12} \\ \underline{-12} \\ 0 \end{array}$$

$$\begin{array}{r} 12 \\ 2 \overline{)24} \\ \underline{-24} \\ 0 \end{array}$$

$$\begin{array}{r} 12 \\ 3 \overline{)36} \\ \underline{-36} \\ 0 \end{array}$$

$$\begin{array}{r} 12 \\ 4 \overline{)48} \\ \underline{-48} \\ 0 \end{array}$$

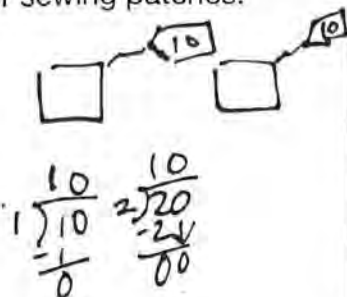
- Is the relationship proportional? Yes

Remember: If you need help to understand the story, you can draw a picture.

Make a table to understand the word problem. Then write an equation. Determine if it is proportional.

1. Rudy is a tailor. He gets paid \$10 for every patch he sews. Let  $x$  represent the number of patches. Let  $y$  represent the number of dollars he earns. Write an equation to represent how much Rudy gets paid for sewing patches.

$x$ patches	$y$ dollars
1	10
2	20
3	30

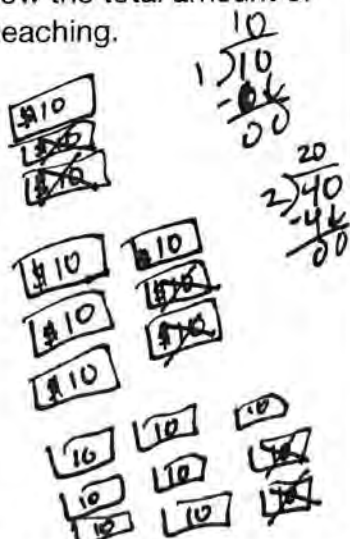


Equation:  $y = 10x$

Is this relationship a proportion? yes

3. Amy gets paid \$30 for every art class she teaches. She has to spend \$20 to buy all the supplies. Let  $x$  represent the number of art classes. Let  $y$  represent the dollars Amy has. Write an equation to show the total amount of money Amy gets from teaching.

$x$ classes	$y$ dollars
1	10
2	40
3	70

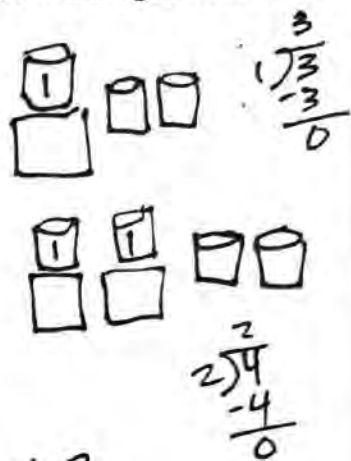


Equation:  $y = 30x - 20$

Is this relationship a proportion? no

2. Matt uses 1 gallon of gas for each lawn he mows. He also needs 2 gallons of gas to drive to the area where he mows. Let  $x$  represent the number of lawn he mows. Let  $y$  represent the number of gallons of gas he uses. Write an equation to represent how much gas Matt uses to mow lawns.

$x$ lawns	$y$ gallons
1	3
2	4
3	5

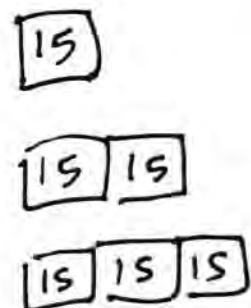


Equation:  $y = 1x + 2$

Is this relationship a proportion? no

4. Maddie takes 15 mg of Vitamin D per day. Let  $x$  represent the number of days. Let  $y$  represent the number of mg that Maddie takes. Write an equation to represent the mg that Maddie takes over several days.

$x$ day	$y$ mg
1	15
2	30
3	45



Equation:  $y = 15x$

Is this relationship a proportion? yes

Make a table to understand the word problem. Then write an equation. Determine if it is proportional.

5. Rachel's lawn requires 10 gallons of water plus an additional 2 gallons of water for each potted plant. Let  $x$  represent the number of potted plants. Let  $y$  represent the gallons of water. Write an equation for the gallons of water Rachel uses.

$x$ plants	$y$ gallons
1	12
2	14
3	16

Handwritten work for problem 5:

- Diagram showing 10 gallons and 2 plants for 12 gallons.
- Diagram showing 10 gallons and 2 plants for 14 gallons.
- Diagram showing 10 gallons and 3 plants for 16 gallons.
- Division problems:  $12 \div 2 = 6$ ,  $14 \div 2 = 7$ ,  $16 \div 2 = 8$ .

Equation:  $y = 2x + 10$

Is this relationship a proportion? no

6. It costs \$5 to get into the Spring Fair. Then each ride costs \$2. Let  $x$  represent the number of rides. Let  $y$  represent the total cost. Write an equation for the total cost the Spring Fair based on the number of rides.

$x$ rides	$y$ dollars
1	7
2	9
3	11

Handwritten work for problem 6:

- Diagram showing \$5 entry fee and 1 ride for \$7.
- Diagram showing \$5 entry fee and 2 rides for \$9.
- Diagram showing \$5 entry fee and 3 rides for \$11.
- Division problems:  $7 \div 2 = 3.5$ ,  $9 \div 2 = 4.5$ ,  $11 \div 2 = 5.5$ .

Equation:  $y = 2x + 5$

Is this relationship a proportion? no

7. Percy gets 10 points for each basket at the Spring Fair game. Let  $x$  represent the number of baskets. Let  $y$  represent the number of points. Write an equation to find the points depending on the number of baskets.

$x$ basket	$y$ point
1	10
2	20
3	30

Handwritten work for problem 7:

- Diagram showing 1 basket for 10 points.
- Diagram showing 2 baskets for 20 points.
- Diagram showing 3 baskets for 30 points.
- Division problems:  $10 \div 1 = 10$ ,  $20 \div 2 = 10$ ,  $30 \div 3 = 10$ .

Equation:  $y = 10x$

Is this relationship a proportion? yes

8. Caryn takes 5 weeks to make a quilt. Let  $x$  represent the number of quilts. Let  $y$  represent the number of weeks she needs to make one. Write an equation for the number of weeks Caryn needs based on the number of quilts.

$x$ quilt	$y$ weeks
1	5
2	10
3	15

Handwritten work for problem 8:

- Diagram showing 1 quilt for 5 weeks.
- Diagram showing 2 quilts for 10 weeks.
- Diagram showing 3 quilts for 15 weeks.
- Division problems:  $5 \div 1 = 5$ ,  $10 \div 2 = 5$ ,  $15 \div 3 = 5$ .

Equation:  $y = 5x$

Is this relationship a proportion? yes

# **G7 U2 Lesson 10**

Generalize that the graph of a proportional relationship lies on a line through the origin.

**G7 U2 Lesson 10 - Today we will represent a proportional relationship with a graph, equation, table and story.**

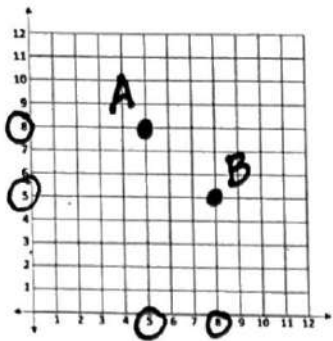
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will represent a proportional relationship with a graph, equation, table and story. It is going to involve applying something you learned in 6th grade to what we've been working on.

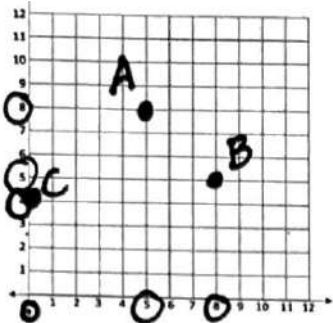
**Let's Review (Slide 3):** We know how to graph coordinate pairs from previous grades. How do I graph (5, 8)? **Possible Student Answers, Key Points:**

- You find the 5 on the horizontal line and 8 on the vertical line and then you see where they meet up.
- You go over 5 and up 8.
- You look for the point above 5 on the x-axis and next to 8 on the y-axis.

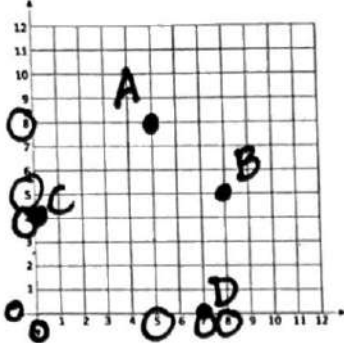
Just to refresh your memory, the first number always is marked on the horizontal line and the second number is marked on the vertical line. I always thinking of it as babies learn to go side to side before they learn to go up and down. So we always do side to side before we go up and down. I circle the 5 on this line. I circle the 8 on this line and then I follow them both until I see where they meet up. This point is (5, 8) and we will label it A. Let's quickly do the next one. Now this one might seem like it's the same because it has the same number just in different order. But it is not the same because the first number is always on the horizontal axis and the second number is always on the vertical axis. We always do side to side before we go up and down. I circle the 8 on this line. I circle the 5 on this line and then I follow them both until I see where they meet up. This is point (8, 5) and we label it B.



Next, let's do (0,4). We always do side to side before we go up and down. This is kind of tricky because of the zero. The zero is right before the one and then I look for the 5 on the vertical line and see where they meet up. I will label it C.



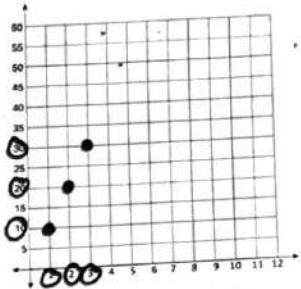
And finally, (7,0). There is another zero but it is second so I am going to find the 7 first on the horizontal line and then I find the 0 on the vertical line which is right under the 1. I follow them both until I see where they meet up. I will label it D. Great job!



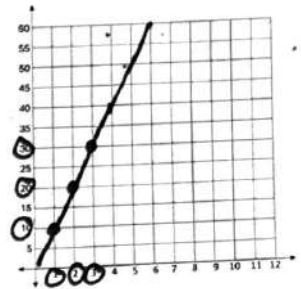


x	y
hours	dollars
1	10
2	20
3	30

**Let's Talk (Slide 4):** We can get coordinates from tables and we'll graph those just like we did in the last slide. Read along with me silently while I read out loud. "The table shows what Ben gets paid for working different numbers of hours. Graph each row as a coordinate pair. This is so easy. You just look at a row, and you can think of the numbers the same way as you thought of the last ones. So, this is really (1,10). This is really (2, 20). This is really (3, 30). The first number is x and the second number is y.

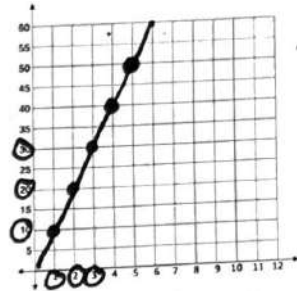


Let's graph the first one. We always do side to side before we go up and down. I circle the 1 on this line. I circle the 10 on this line and then I follow them both until I see where they meet up. Let's graph the next one. We always do side to side before we go up and down. I circle the 2 on this line. I circle the 20 on this line and then I follow them both until I see where they meet up. Let's graph the next one. We always do side to side before we go up and down. I circle the 3 on this line. I circle the 30 on this line and then I follow them both until I see where they meet up.



Now I can take a straight edge and draw a line that goes through all the points. *Be sure to draw the line perfectly straight so that it goes through the points you drew and the additional points you'll be mentioning.* Notice that it goes through more points than just the ones we have. That's useful.

x	y
hours	dollars
1	10
2	20
3	30
4	40
5	50



I see it is here at (0,0) and I can see it is here at (4,40) and (5,50). I can even add those to the table, and you'll notice that the constant of proportionality still works. The first row was "times 10." The next row was "times 10." And so on and so on.

**Let's Think (Slide 5):** We can get coordinates from tables and we'll graph those just like we did the last ones. We just have to do a little extra number crunching. This says, "The equation,  $y = 5x$  represents what Sue gets paid where x is the number of hours and y is the number of dollars." I am going to label the hours x and the dollars y.

x	y
hours	dollars

The equation,  $y = 5x$  represents what Sue gets paid where x is the number of hours and y is the number of dollars.

$$y = 5x$$

$$y = 5 \cdot 1$$

$$y = 5$$

$$y = 5x$$

$$y = 5 \cdot 2$$

$$y = 10$$

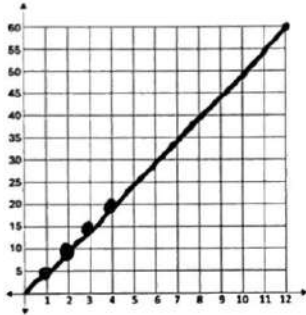
hours	dollars
1	5
2	10

You already know from previous lessons that you can use the equation to get points on the table. Remember we can pick any numbers we want. To keep it simple, I'm just going to pick 1, 2, 3. Now let's plug those in. First, we'll do x equals 1. I write  $y = 5x$  then I put 1 in place of the x so it's y equals 5 times 1. That's 5. I will put that in my table. Next we'll do x equals 2. I write  $y = 5x$  then I put 2 in place of the x so it's y equals 5 times 2. That's 10. I will put that in my table.



hours	dollars
1	5
2	10
3	15
4	20

I can do  $x$  equals 3 so 5 times 3. That's 15. I will put that in my table. I can do  $x$  equals 4 so 5 times 4. That's 20. I will put that in my table.

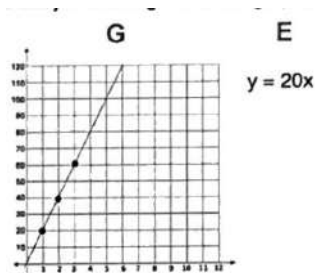


Now we can just look at a row and think of the numbers as regular graphing coordinates. So, the first row is really (1,5) then (2,10) then (3,15) then (4,20). Let's graph the first one. We always do side to side before we go up and down. I look at the 1 on this line. I look at the 5 on this line and then I follow them both until I see where they meet up. Let's graph the next one. We always do side to side before we go up and down. I look at the 2 on this line. I look at the 10 on this line and then I follow them both until I see where they meet up. Let's graph the next one. We always do side to side before we go up and down. I look at the 3 on this line. I look at the 15 on this line and then I follow them both until I see where they

meet up. Let's graph the next one. We always do side to side before we go up and down. I look at the 4 on this line. I look at the 20 on this line and then I follow them both until I see where they meet up. Look! We made a line! Let's use a straight edge to draw it through all the points and beyond.

**Let's Think (Slide 6):** Graphs, equations, tables and stories are all equivalent ways to show a proportion. Just like a story can be told in a book and then they make a TV show or a movie out of it. We can use the word GETS to remember that all four ways can be used. G stands for graph. E stands for equation. T stands for table. S stands for story. G - E - T - S spells GETS. This question asks, "What is the story that can go with our graph, equation and table?" Give the students a whole minute of silent think time. Then collect answers. Be sure that the stories are exactly correct. If not, correct them. Then write down your final right answer. Possible Student Answers, Key Points:

- Jenny gets paid 20 dollars per hour
- It costs 20 dollars for each hour of renting a lawnmower



T

hours	dollars
1	20
2	40
3	60

S  
Joe gets paid \$20 for each hour that he babysits his cousins.

There are millions of possible correct stories. But whatever it is, it is going to involve  $x$  times 10 to make  $y$  or hours times 10 to make  $y$ . Just for today I will write, Joe gets paid \$20 for each hour that he babysits his cousins. We will always be able to have a graph, equation, table and story for any proportion we have, and we can use the acronym, GETS, to remember that.

**Let's Try It (Slide 7):** Now we will graph from tables and equations together. I will take you through step by step.

# WARM WELCOME



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**Today we will represent a proportional relationship with a graph, equation, table and story.**

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## Let's Review:

We know how to graph coordinates from previous grades.

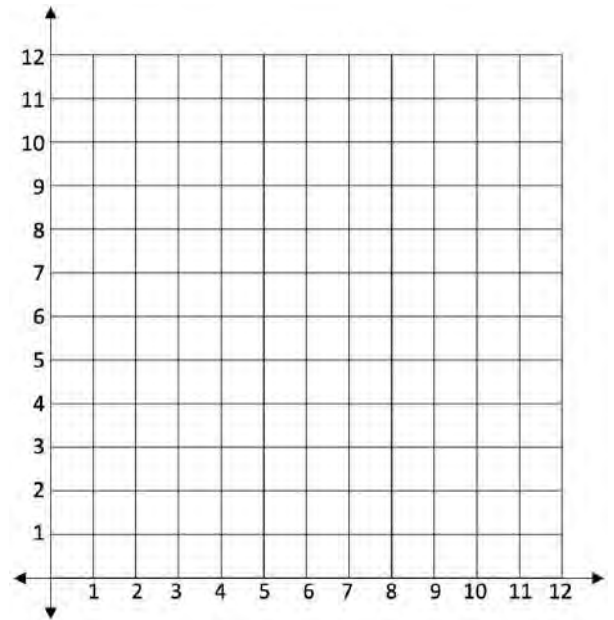
Graph the coordinate pairs.

Point A: (5, 8)

Point B: (8, 5)

Point C: (0, 4)

Point D: (7, 0)



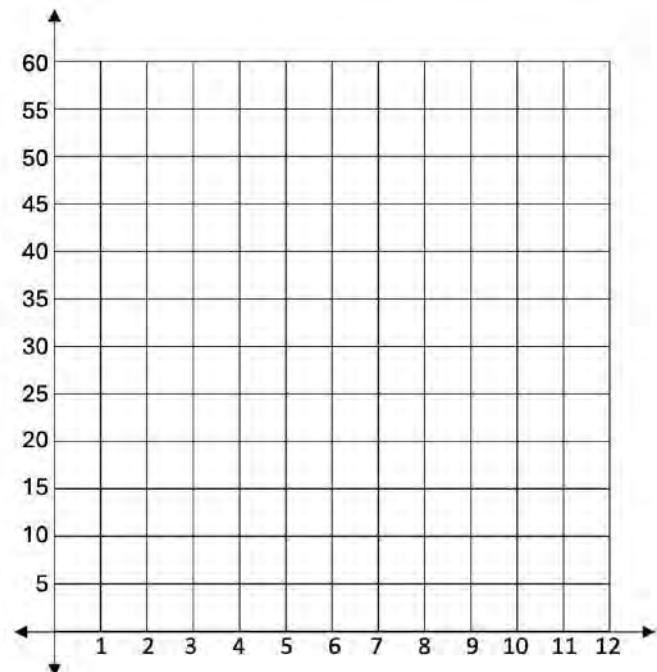
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## Let's Talk:

We can get coordinates from tables.

The table shows what Ben gets paid for working different numbers of hours. Graph each row as a coordinate pair.

hours	dollars
1	10
2	20
3	30

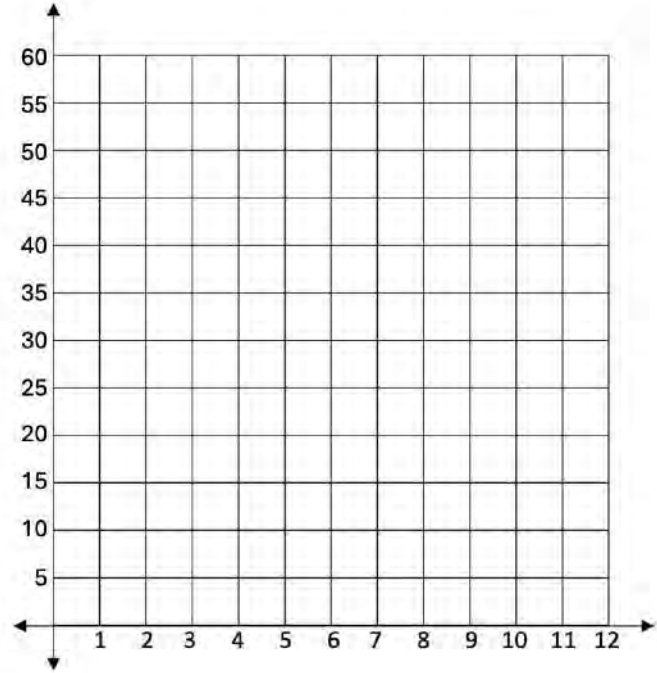


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**Let's Think:** We can get coordinates from equations.

The equation,  $y = 5x$  represents what Sue gets paid where  $x$  is the number of hours and  $y$  is the number of dollars.

hours	dollars

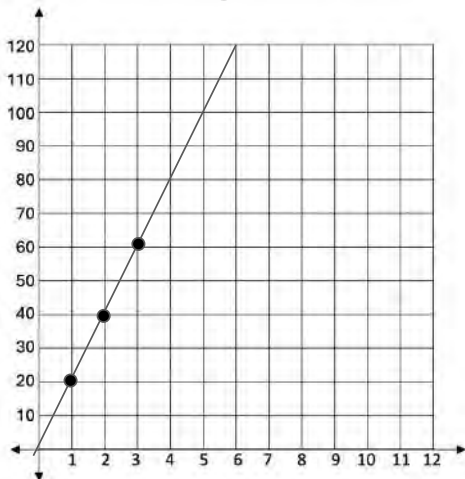


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**Let's Think:** Graphs, equations, tables and stories are all equivalent ways to show a proportion.

We can use the word GETS to remember that all four ways can be used. What is the story that can go with our graph, equation and table?

**G**



**E**

$y = 20x$

**T**

hours	dollars
1	20
2	40
3	60

**S**

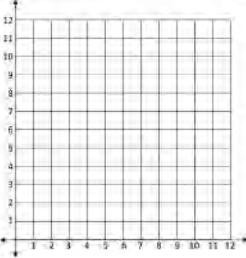
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# Let's Try It: We will do it together step by step.

Name: \_\_\_\_\_ G7 U2 Lesson 10 - Let's Try It

1. Graph the table shown below.

x	y
2	4
4	8
6	12
8	16



2. Write an equation to match the table: \_\_\_\_\_


3. Is the relationship a proportion? \_\_\_\_\_

4. Circle ALL the reasons for your answer:

- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 2.
- (c) The constant of proportionality is 4.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .

5. Complete the table with  $y = 3x$ .

x	y
1	



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# On your Own: Now it's time for you to do it on your own.

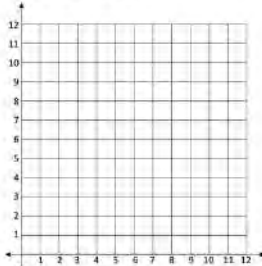
Name: \_\_\_\_\_ G7 U2 Lesson 10 - Independent Work

Remember: Each row of the table is a coordinate pair.

Graph the table or use the equation to make a table and then graph.

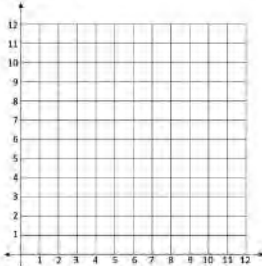
1. Graph the table shown below.

x	y
1	4
2	8
3	12
4	16



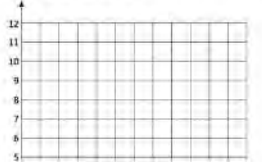
2. Graph the equation:  $y = 3x$

x	y
1	
2	
3	
4	



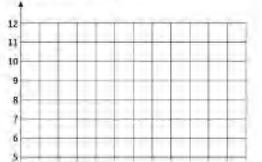
3. Graph the table shown below.

x	y
1	1
2	2
3	3



4. Graph the equation:  $y = x + 3$

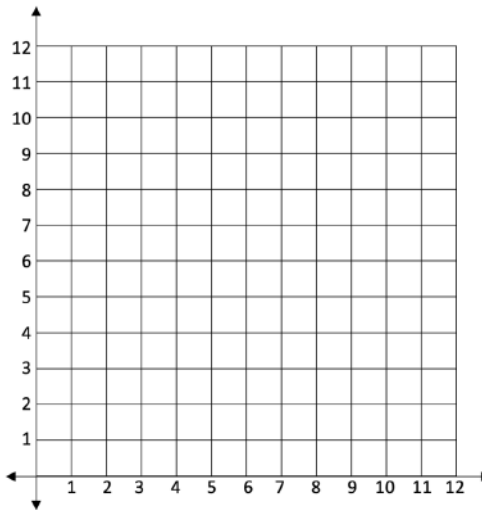
x	y
1	
2	
3	



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1. Graph the table shown below.

x	y
2	4
4	8
6	12
8	16



2. Write an equation to match the table: \_\_\_\_\_

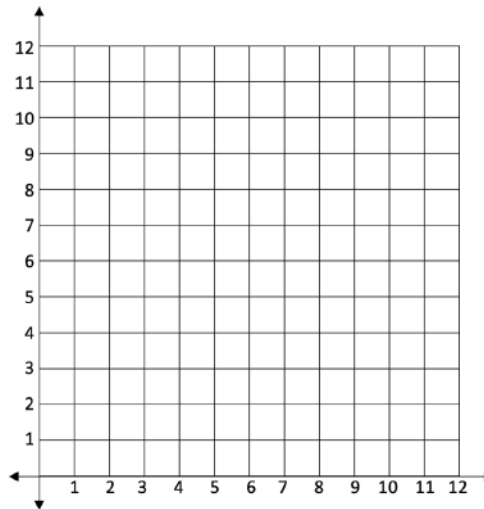
3. Is the relationship a proportion? \_\_\_\_\_

4. Circle ALL the reasons for your answer:

- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 2.
- (c) The constant of proportionality is 4.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .

5. Complete the table with  $y = 3x$ .

x	y
1	
2	
3	
4	



6. Graph the points from the table.

7. Write an equation to match the table: \_\_\_\_\_

8. Is the relationship a proportion? \_\_\_\_\_

9. Circle ALL the reasons for your answer:

- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 1.
- (c) The constant of proportionality is 3.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .



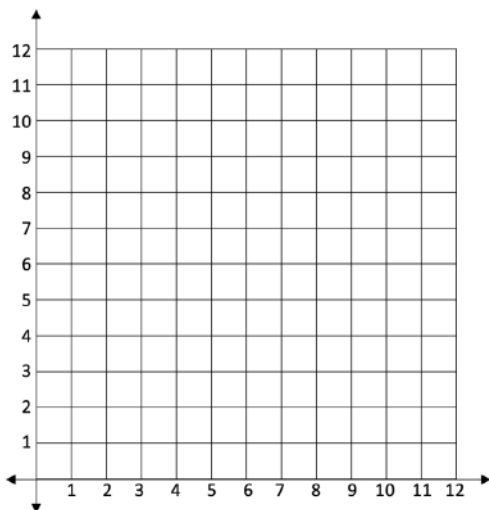
Name: \_\_\_\_\_

Remember: Each row of the table is a coordinate pair.

Graph the table or use the equation to make a table and then graph.

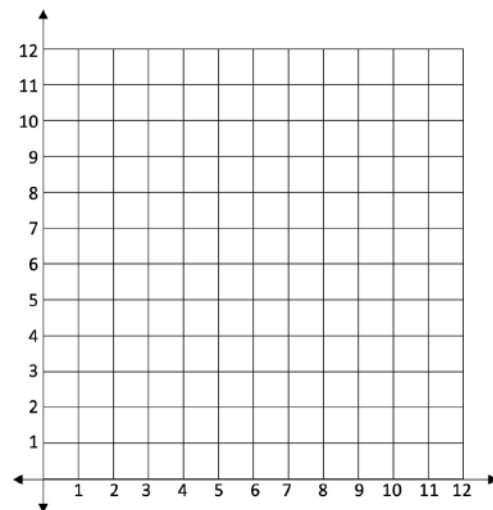
1. Graph the table shown below.

x	y
1	4
2	8
3	12
4	16



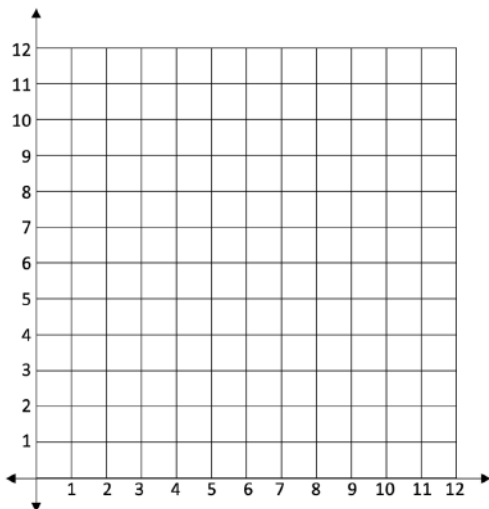
2. Graph the equation:  $y = 3x$

x	y
1	
2	
3	
4	



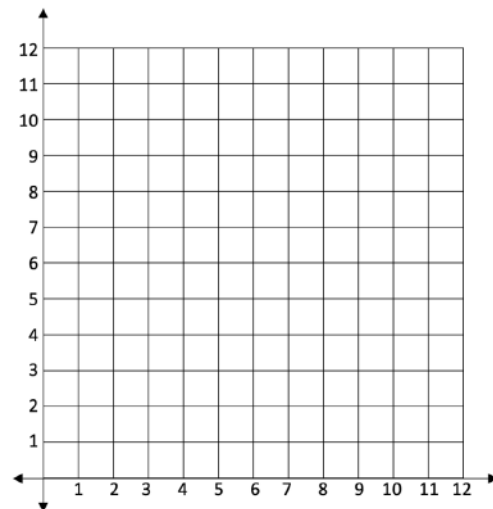
3. Graph the table shown below.

x	y
1	1
2	2
3	3
4	4



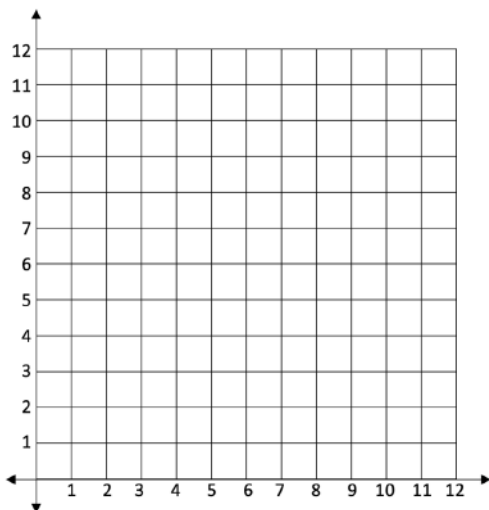
4. Graph the equation:  $y = x + 3$

x	y
1	
2	
3	
4	



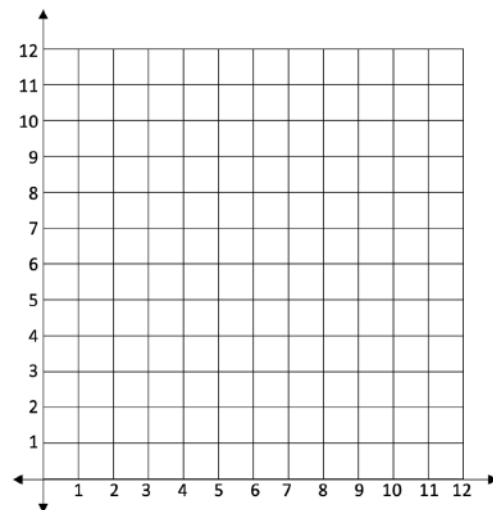
5. Graph the table shown below.

x	y
4	2
8	4
12	6
16	8



6. Graph the equation:  $y = x - 1$

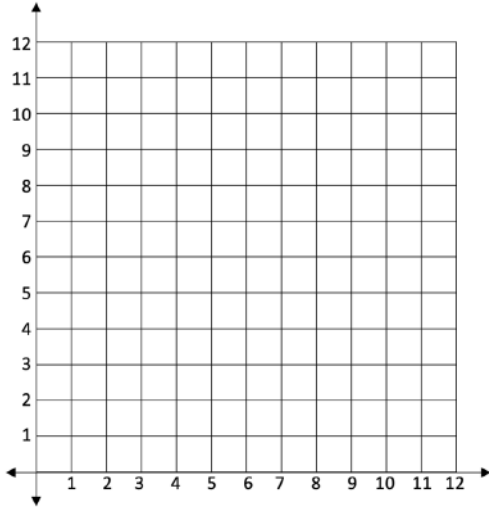
x	y
1	
2	
3	
4	



Graph the table or use the equation to make a table and then graph.

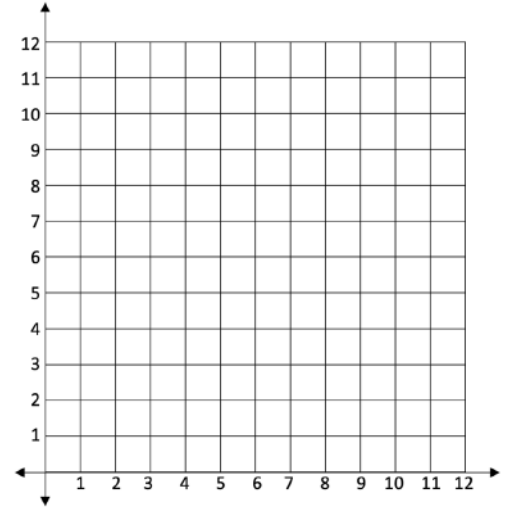
7. Graph the table shown below.

x	y
1	2
2	3
3	4
4	5



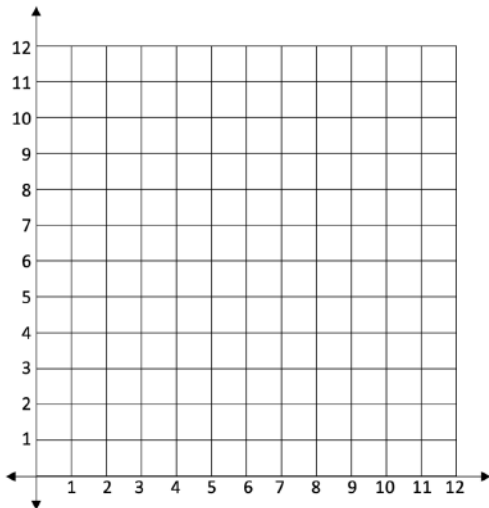
8. Graph the equation:  $y = 2x$

x	y
1	
2	
3	
4	



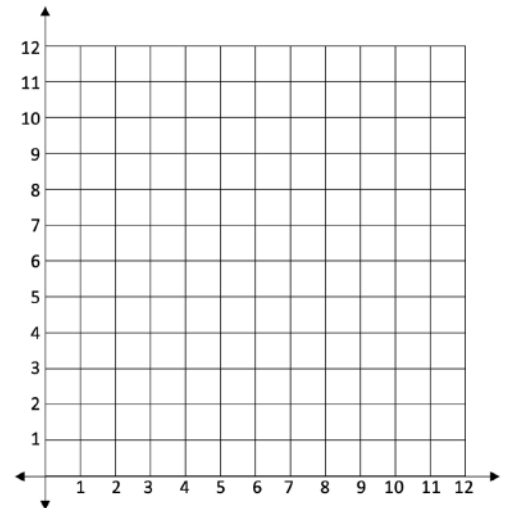
9. Graph the table shown below.

x	y
3	1
6	4
9	7
12	10



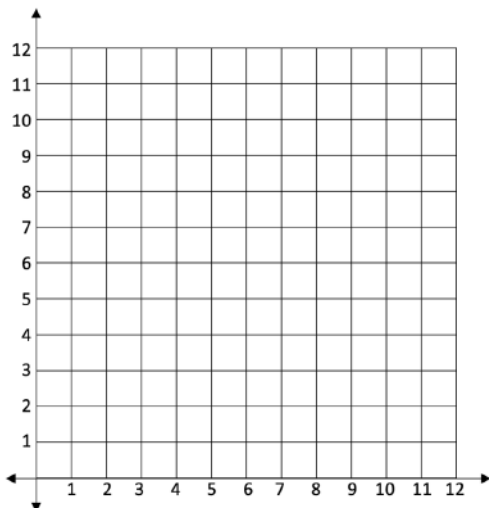
10. Graph the equation:  $y = x \div 2$

x	y
2	
4	
6	
8	



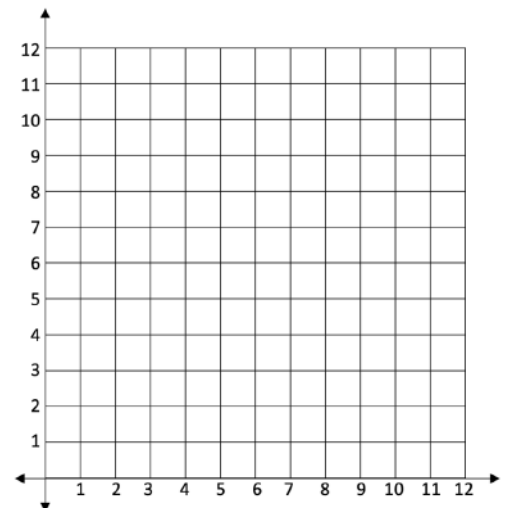
11. Graph the table shown below.

x	y
4	1
8	2
12	3
16	4



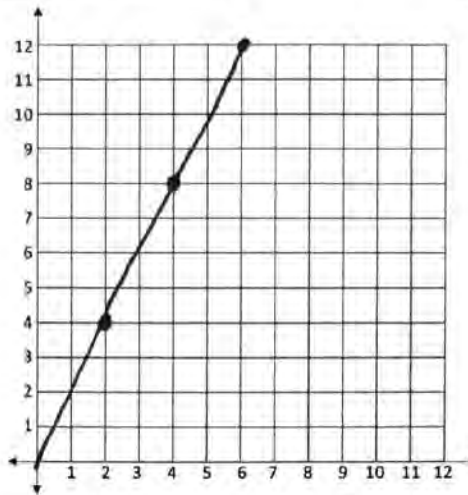
12. Graph the equation:  $y = x$

x	y
1	
2	
3	
4	



1. Graph the table shown below.

x	y
2	4
4	8
6	12
8	16



2. Write an equation to match the table:  $y = 2x$

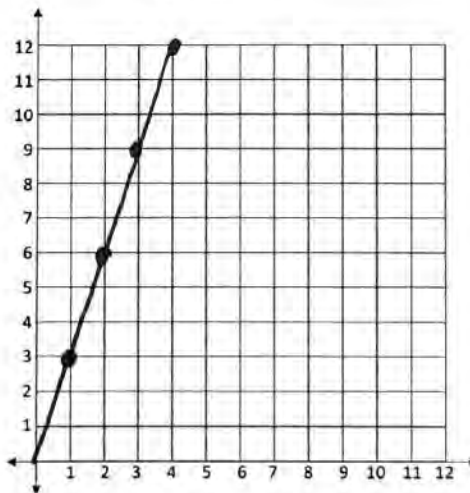
3. Is the relationship a proportion? yes

4. Circle ALL the reasons for your answer:

- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 2.
- (c) The constant of proportionality is 4.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .

5. Complete the table with  $y = 3x$ .

x	y
1	3
2	6
3	9
4	12



6. Graph the points from the table.

7. Write an equation to match the table:  $y = 3x$

8. Is the relationship a proportion? yes

9. Circle ALL the reasons for your answer:

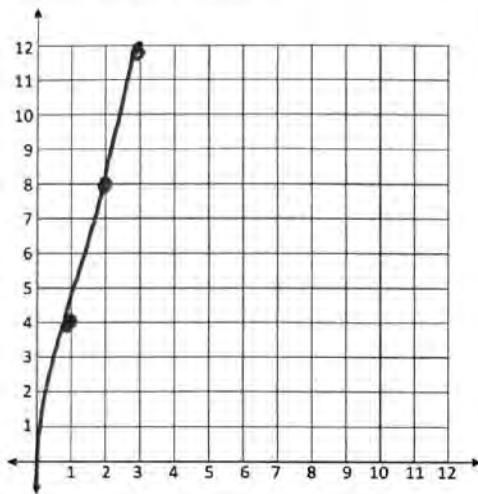
- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 1.
- (c) The constant of proportionality is 3.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .

Remember: Each row of the table is a coordinate pair.

Graph the table or use the equation to make a table and then graph.

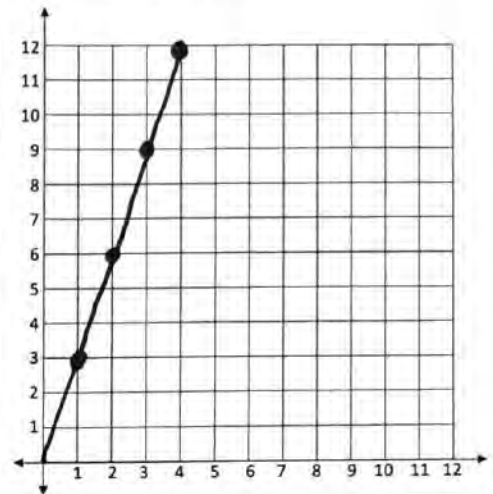
1. Graph the table shown below.

x	y
1	4
2	8
3	12
4	16



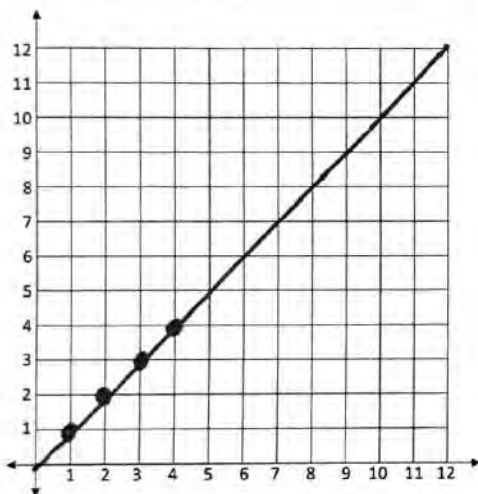
2. Graph the equation:  $y = 3x$

x	y
1	3
2	6
3	9
4	12



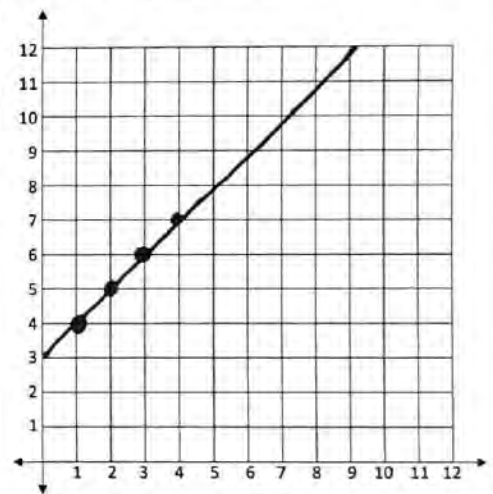
3. Graph the table shown below.

x	y
1	1
2	2
3	3
4	4



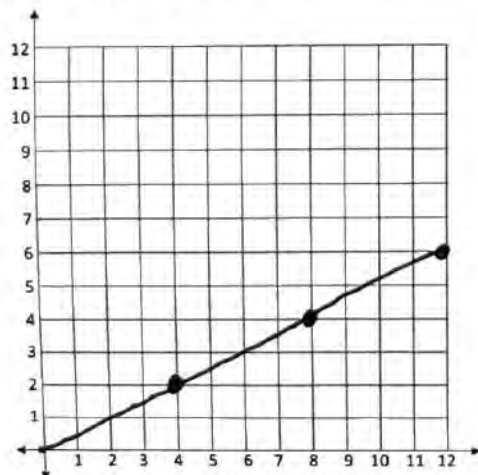
4. Graph the equation:  $y = x + 3$

x	y
1	4
2	5
3	6
4	7



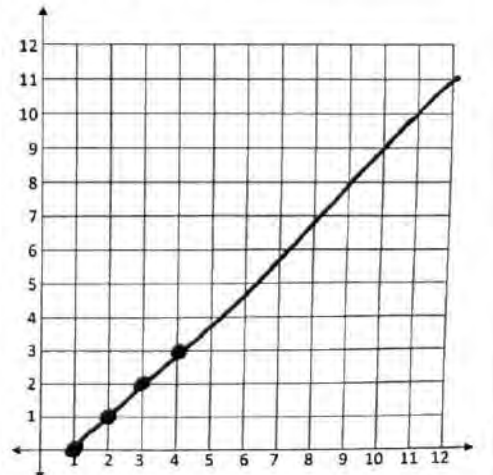
5. Graph the table shown below.

x	y
4	2
8	4
12	6
16	8



6. Graph the equation:  $y = x - 1$

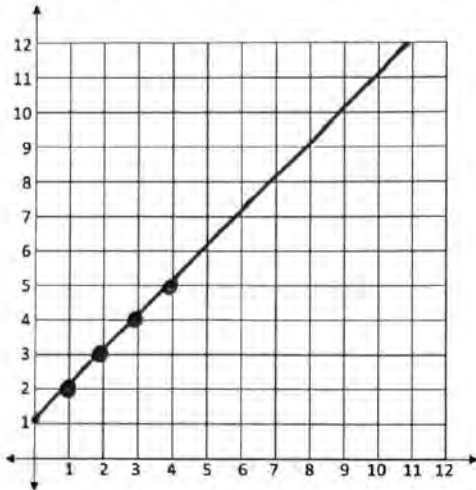
x	y
1	0
2	1
3	2
4	3



Graph the table or use the equation to make a table and then graph.

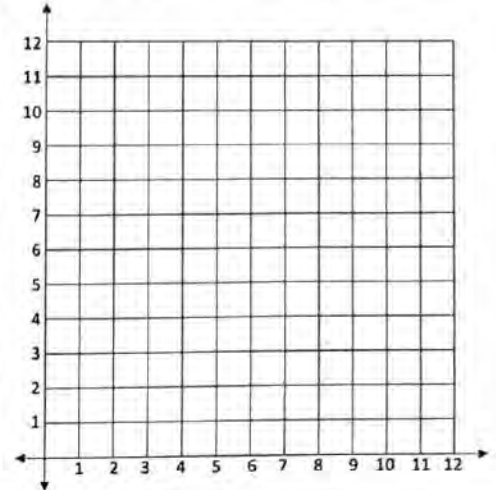
7. Graph the table shown below.

x	y
1	2
2	3
3	4
4	5



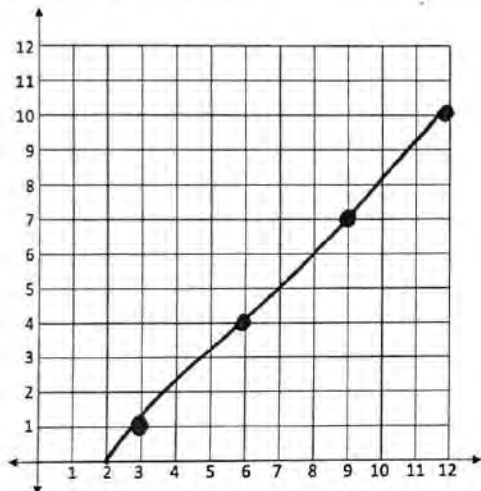
8. Graph the equation:  $y = 2x$

x	y
1	
2	
3	
4	



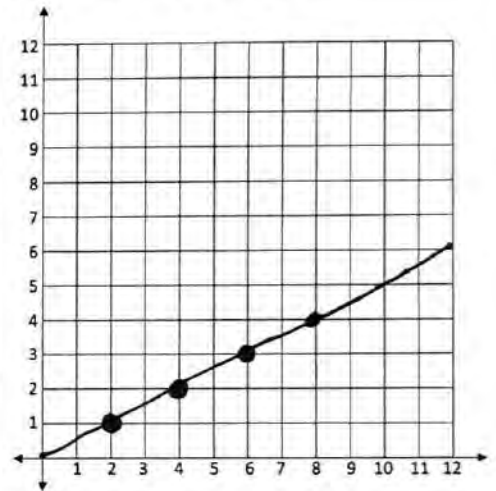
9. Graph the table shown below.

x	y
3	1
6	4
9	7
12	10



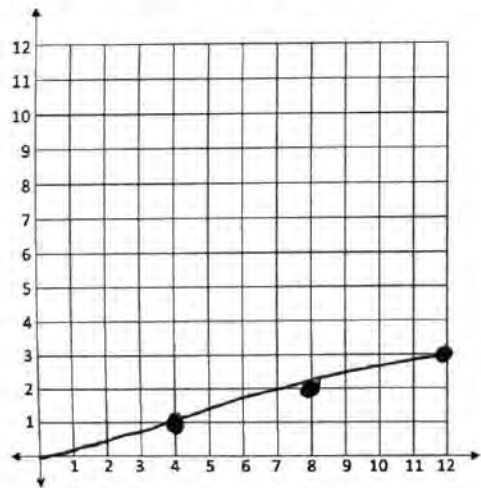
10. Graph the equation:  $y = x \div 2$

x	y
2	1
4	2
6	3
8	4



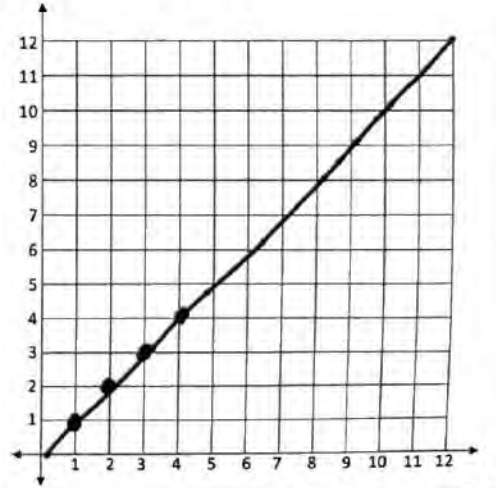
11. Graph the table shown below.

x	y
4	1
8	2
12	3
16	4



12. Graph the equation:  $y = x$

x	y
1	1
2	2
3	3
4	4



# **G7 U2 Lesson 11**

Interpret points on the graph of a proportional relationship, and identify the constant of proportionality from the graph of a proportional relationship.



**G7 U2 Lesson 11 - Today we will represent a story with a table, equation and graph.**

**Warm Welcome (Slide 1):** Tutor choice

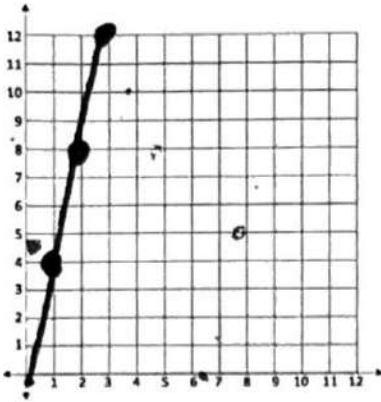
**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will represent a story with a table, equation and graph. You have already worked on all of these things so you just have to put them together.

Use  $y = 4x$  to fill in the table. Then graph.

$y = 4x$   
 $y = 4 \cdot 1$   
 $y = 4$   
 $y = 4x$   
 $y = 4 \cdot 2$   
 $y = 8$

x	y
1	4
2	8
3	12

$y = 4x$   
 $y = 4 \cdot 3$   
 $y = 12$



**Let's Review (Slide 3):** We know how to graph coordinates from a table and equation. This says, "Use  $y = 4x$  to fill in the table. Then graph." First, I am going to plug in each value of  $x$ . I write  $y = 4x$  then with  $x = 1$ , it becomes  $y = 4$  times 1. I do the math and write it underneath,  $y$  equals 4. I'll write that on my table. Let's do it again. I write  $y = 4x$  then with  $x = 2$ , it becomes  $y = 4$  times 2. I do the math and write it underneath,  $y$  equals 8. I'll write that on my table. Let's do it again. I write  $y = 4x$  then with  $x = 3$ , it becomes  $y = 4$  times 3. I do the math and write it underneath,  $y$  equals 12. I'll write that on my table.

Now that I have complete rows, I can think of each row as a coordinate pair. I have (1,4). We always do side to side before we go up and down. I circle the 1 on this line. I circle the 4 on this line and then I follow them both until I see where they meet up. The next row is (2,8). I circle the 2 on this line. I circle the 8 on this line and then I follow them both until I see where they meet up. The next row is (3,12). I circle the 3 on this line. I circle the 12 on this line and then I follow them both until I see where they meet up. And look, I can use a straight edge to draw a line through these points.

x	y
Cups	tblsp
1	
2	
3	
4	

**Let's Talk (Slide 4):** We can get coordinates from stories too. We know this because we have the acronym, "GETS." Where G stands for graph, E stands for equation, T stands for table and S stands for story. Let's read this story together. Follow along with your eyes while I read out loud. "In order to make hot chocolate, Jeb needs 2 tablespoons of cocoa for every cup of milk. Let  $x$  represent the cups of milk. Let  $y$  represent the tablespoons of cocoa." First, let's label the  $x$  and  $y$  on the table.

x	y
Cups	tblsp
1	2
2	
3	
4	



We've done story problems like this before and I know you remember that you can always draw a picture. Here's a cup of milk, and we need 2 tablespoons. So I know to put a 2 here.

x	y
Cups	tblsp
1	2
2	4
3	
4	



And we can keep going, right? Another cup needs another 2 tablespoons. Now we see that 2 cups has 4 tablespoons. I put that on the table.

In order to make hot chocolate, Jeb needs 2 tablespoons of cocoa for every cup of milk. Let  $x$  represent the cups of milk. Let  $y$  represent the tablespoons of cocoa.

Write an equation:

$$y = 2x$$

x	y
Cups	Tbsp
1	2
2	4
3	6
4	8

By now, I'm noticing a pattern but I can keep drawing if I need to. It's "times 2." Let me write that on every line.

This helps me understand what the equation is. We multiply  $x$  by 2 to get  $y$  so I will write  $y$  equals  $2x$ .



All the rest is very familiar, just like we did on the last slide. But before we plot the points, we want to label the graph. The horizontal axis is first so it's  $x$ . I am going to label that cups of milk. The vertical axis is next so it's  $y$ . I am going to label that tablespoons of cocoa.



Now I can think of each row as a coordinate pair. I have (1,2). We always do side to side before we go up and down. I circle the 1 on this line. I circle the 2 on this line and then I follow them both until I see where they meet up. The next row is (2,4). I circle the 2 on this line. I circle the 8 on this line and then I follow them both until I see where they meet up. The next row is (3,6). I circle the 3 on this line. I circle the 6 on this line and then I follow them both until I see where they meet up. And again, we've made a line.

x	y
Workout min	total min
1	
2	
3	
4	

**Let's Think (Slide 5):** Let's do one more example. "Jason always needs 5 minutes to cool down from a workout no matter how long it is. Let  $x$  represent the length of Jason's workout in minutes. Let  $y$  represent the total number of minutes." I am going to put the words on table.  $X$  is workout minutes.  $Y$  is total minutes.

x	y
Workout min	total min
1	6
2	
3	
4	



This is a bit hard to draw but let's think of clock. Jason works out for one minute. Then he does a 5 minute cooldown. That's 6 minutes all together. I write that on the table.

x	y
Workout min	total min
1	6
2	7
3	
4	



Now, Jason does 2 minutes instead. I'm going to do a new drawing because he would workout for 2 minutes. Then he would do a 5 minutes cooldown. That's 7 minutes all together. I write that on the table.

Jason always needs 5 minutes to cool down from a workout no matter how long it is. Let  $x$  represent the length of Jason's workout in minutes. Let  $y$  represent the total number of minutes.

x Workout min	y total min
1	6
2	7
3	8
4	9

Write an equation:

$$y = x + 5$$



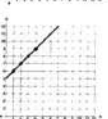
Now I start to see the pattern. It was 1 plus the 5 minute cooldown. Then 2 plus the 5 minute cooldown. I will write in all the plus fives. So then it would be 3 plus 5 is 8 and 4 plus 5 is 9. It is easy to see what my equation would be,  $y$  equals  $x$  plus 5.

Time to graph! But it is super important that before we plot the points, we label the graph. The horizontal axis is first so it's  $x$ . I am going to label that workout minutes. The vertical axis is next so it's  $y$ . I am going to label that total minutes.

We start with (1,6). I circle the 1 on this line. I circle the 6 on this line and then I follow them both until I see where they meet up. The next row is (2,7). I circle the 2 on this line. I circle the 7 on this line and then I follow them both until I see where they meet up. The next row is (3,8). I circle the 3 on this line. I circle the 8 on this line and then I follow them both until I see where they meet up. The next row is (4,9). I circle the 4 on this line. I circle the 9 on this line and then I follow them both until I see where they meet up. And again, we've made a line. Now, this graph isn't proportional like our last one. But we still see the acronym, GETS. We can still make a graph, equation, table and story as all equivalent representations.

In order to make hot chocolate, Jen needs 2 tablespoons of cocoa for every cup of milk. Let  $x$  represent the cups of milk. Let  $y$  represent the tablespoons of cocoa. Write an equation:  $y = 2x$

cups	tblsp
1	2
2	4
3	6
4	8

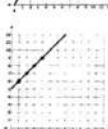


Is it proportional? **YES!**

**Let's Think (Slide 6):** We've done three different equations with tables and graphs. For all of those, "we still determine if a relationship is proportional the same way we know." Here are pictures from the last two slides we just did. Let's figure out if they are proportional. We know they need to have a constant of proportionality and they need to have an equation in the form,  $y = kx$ . I can see that here with  $y = 2x$ . So is it proportional? YES!

In order to make hot chocolate, Jen needs 2 tablespoons of cocoa for every cup of milk. Let  $x$  represent the cups of milk. Let  $y$  represent the tablespoons of cocoa. Write an equation:  $y = 2x$

cups	tblsp
1	2
2	4
3	6
4	8



Is it proportional? **YES!**

$$\begin{array}{l} \frac{2}{1} \\ \frac{2}{2} \\ \frac{2}{3} \\ \frac{2}{4} \end{array}$$

We can also divide each row,  $y$  divided by  $x$ . If they have the same unit rate then it's a constant of proportionality. I am going to do this super quickly. 2 divided by 1 is 2. 4 divided by 2 is 2. 6 divided by 3 is 2. 8 divided by 4 is 2. So again, is it proportional? YES!

Jason always needs 5 minutes to cool down from a workout no matter how long it is. Let  $x$  represent the length of Jason's workout in minutes. Let  $y$  represent the total number of minutes. Write an equation:  $y = x + 5$

x	y
1	6
2	7
3	8
4	9



Is it proportional?

In order to make the chocolate, Jan needs 2 tablespoons of cocoa for every cup of milk. Let's represent the relationship of cocoa and milk. Let  $x$  represent the tablespoons of cocoa. Write an equation:  $y = 2x$

cups of milk	tblsp
1	2
2	4
3	6
4	8

Is it proportional? **YES!**

$$\begin{array}{r} 2 \\ 1 \overline{) 2} \\ \underline{-2} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 2 \overline{) 4} \\ \underline{-4} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 3 \overline{) 6} \\ \underline{-6} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 4 \overline{) 8} \\ \underline{-8} \\ 0 \end{array}$$

Let's look at the next one. First of all, the equation is  $y = x + 5$ . That is not in  $y = kx$  form. So, is it proportional? **NO!**

Janet always needs 8 minutes to cool down from a workout no matter how long it is. Let's represent the length of Janet's workout in minutes. Let  $x$  represent the total number of minutes. Write an equation:  $y = x + 5$

min	tblsp
1	6
2	7
3	8
4	9

Is it proportional? **NO!**

$$\begin{array}{r} 2 \\ 1 \overline{) 2} \\ \underline{-2} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 2 \overline{) 4} \\ \underline{-4} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 3 \overline{) 6} \\ \underline{-6} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 4 \overline{) 8} \\ \underline{-8} \\ 0 \end{array}$$

Let's divide each row to check. 6 divided by 1 is 6. But 7 divided by 2 is 3 and then I have a remainder. I do not get the same number so I do not have a constant of proportionality. And so the answer to that question, "Is it proportional?" is still **NO!**

**Let's Try It (Slide 7):** Now we will graph from tables and equations together. I will take you through step by step.

# WARM WELCOME



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**Today we will represent a story with a table, equation and graph.**

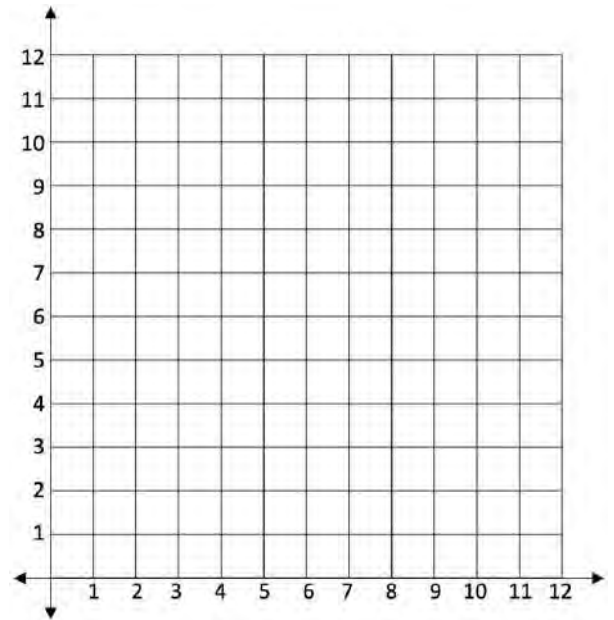
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## Let's Review:

We know how to graph coordinates from a table and equation.

Use  $y = 4x$  to fill in the table. Then graph.

x	y
1	
2	
3	



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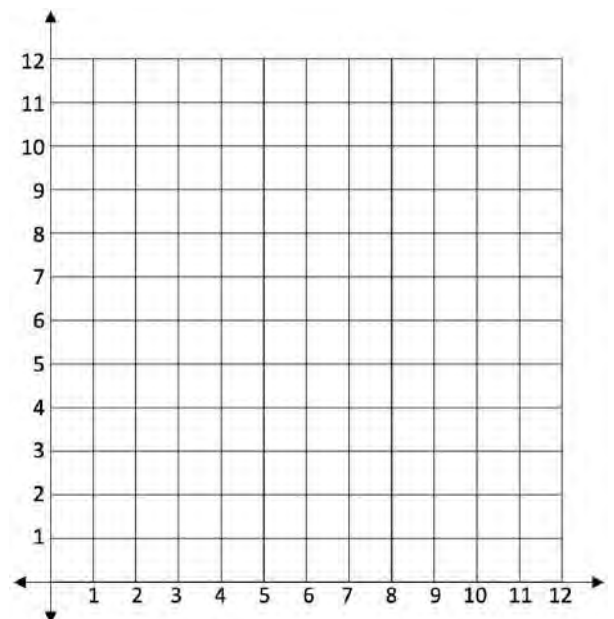
## Let's Talk:

We can get coordinates from stories.

In order to make hot chocolate, Jeb needs 2 tablespoons of cocoa for every cup of milk. Let  $x$  represent the cups of milk. Let  $y$  represent the tablespoons of cocoa.

Write an equation:

x	y
1	
2	
3	
4	



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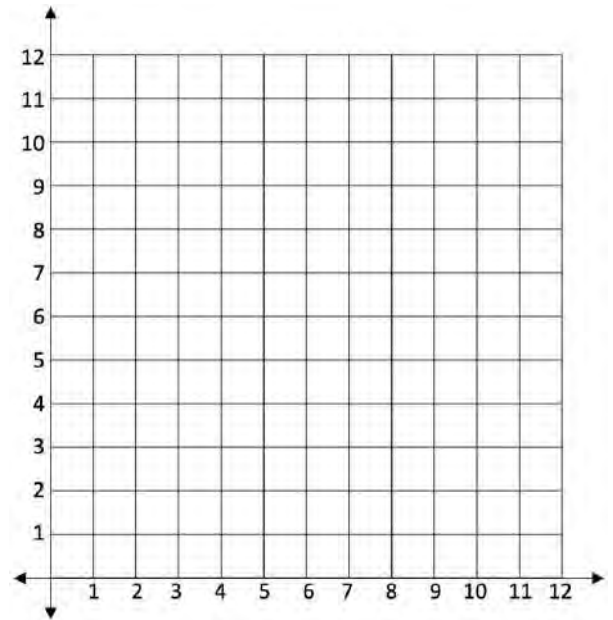


# Let's Think: We can get coordinates from equations.

Jason always needs 5 minutes to cool down from a workout no matter how long it is. Let  $x$  represent the length of Jason's workout in minutes. Let  $y$  represent the total number of minutes.

Write an equation:

$x$	$y$
1	
2	
3	
4	



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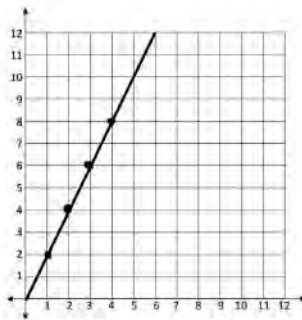
# Let's Think: We still determine if a relationship is proportional the same way we know.

In order to make hot chocolate, Jeb needs 2 tablespoons of cocoa for every cup of milk. Let  $x$  represent the cups of milk. Let  $y$  represent the tablespoons of cocoa.

Write an equation:

$$y = 2x$$

$x$	$y$
cups	tbsp
1	2
2	4
3	6
4	8



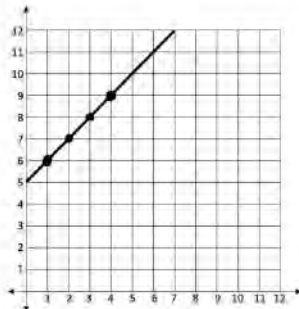
Is it proportional?

Jason always needs 5 minutes to cool down from a workout no matter how long it is. Let  $x$  represent the length of Jason's workout in minutes. Let  $y$  represent the total number of minutes.

Write an equation:

$$y = x + 5$$

$x$	$y$
work out	total
1	6
2	7
3	8
4	9



Is it proportional?

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# Let's Try It:

We will do it together step by step.

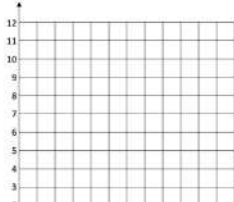
Name: \_\_\_\_\_ G7 U2 Lesson 11 - Let's Try It

**Brian gets paid \$5 for each window he washes. Let  $x$  represent the number of windows he washes. Let  $y$  represent the number of dollars Brian earns. Find the amount of money that Brian earns for washing 0 windows then 2 windows then 4 windows then 6 windows.**

1. Complete the table shown below.

x	y
0	
2	
4	
6	

2. Label the axes of the graph.



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# On your Own:

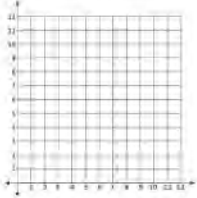
Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 11 - Independent Work

Use the story to complete the table then write an equation and graph it.

1. Rose is making a bracelet. Every time she puts on one gold bead, she follows it with three pink beads. Let  $x$  represent the number of gold beads. Let  $y$  represent the number of pink beads. Find the number of pink beads when there is 1 gold bead then 2 gold beads then 3 gold beads.

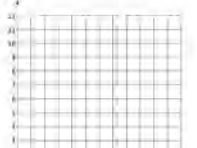
x	y
1	
2	
3	



Write an equation to match the story and table. Then graph.

2. Miles is always 2 years younger than his brother, and that is how it will be for the rest of his life. Let  $x$  represent his brother's age. Let  $y$  represent Miles' age. Find out how old Miles is when his brother is 4 years old then 5 years old then 6 years old.

x	y
4	
5	
6	



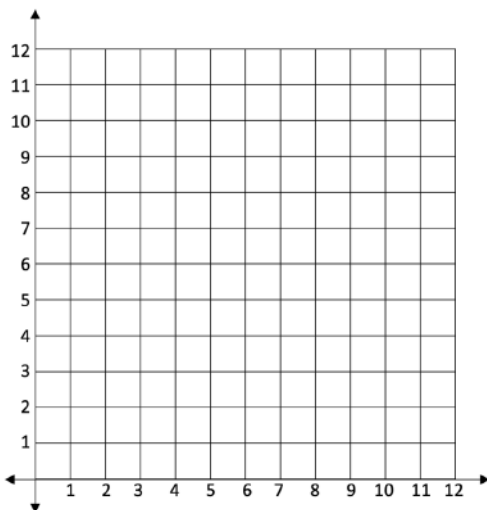
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Brian gets paid \$5 for each window he washes. Let  $x$  represent the number of windows he washes. Let  $y$  represent the number of dollars Brian earns. Find the amount of money that Brian earns for washing 0 windows then 2 windows then 4 windows then 6 windows.

1. Complete the table shown below.

$x$	$y$
0	
2	
4	
6	

2. Label the axes of the graph.



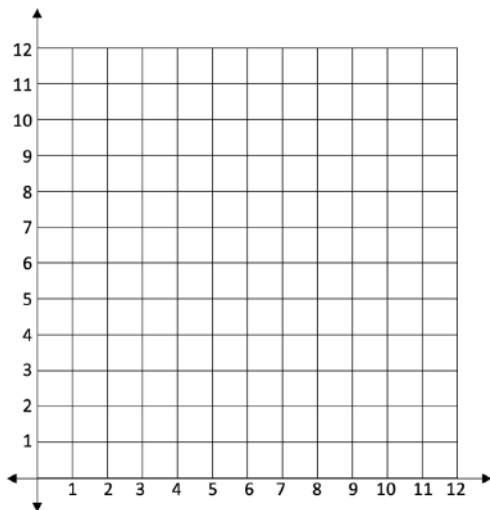
3. Graph the points from the table.
4. Write an equation to match the table: \_\_\_\_\_
5. Is the relationship a proportion? \_\_\_\_\_
6. Circle ALL the reasons for your answer:
- (a) There is no constant of proportionality.
  - (b) The constant of proportionality is 5.
  - (c) The constant of proportionality is 10.
  - (d) The equation is not in the form  $y = kx$ .
  - (e) The equation is in the form  $y = kx$ .

Whenever Lisa works at the cafe, her boss always gives her an extra \$5 on top of whatever money is in the tip jar. Let  $x$  represent the number of dollars in the tip jar. Let  $y$  represent the total number of dollars that Lisa gets. Find the total number of dollars Lisa gets when there are 0 dollars in the tip jar then 2 dollars then 4 dollars then 6 dollars.

7. Complete the table shown below.

$x$	$y$
0	
2	
4	
6	

8. Label the axes of the graph.



9. Graph the points from the table.

10. Write an equation to match the table: \_\_\_\_\_

11. Is the relationship a proportion? \_\_\_\_\_

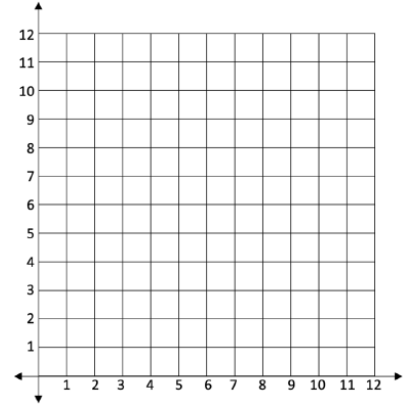
12. Circle ALL the reasons for your answer:

- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 5.
- (c) The constant of proportionality is 10.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .

Use the story to complete the table then write an equation and graph it.

1. Rose is making a bracelet. Every time she puts on one gold bead, she follows it with three pink beads. Let  $x$  represent the number of gold beads. Let  $y$  represent the number of pink beads. Find the number of pink beads when there is 1 gold bead then 2 gold beads then 3 gold beads.

$x$	$y$
1	
2	
3	

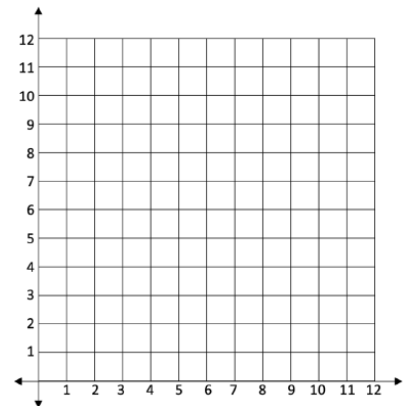


Write an equation to match the story and table. Then graph.

\_\_\_\_\_

2. Miles is always 2 years younger than his brother, and that is how it will be for the rest of his life. Let  $x$  represent his brother's age. Let  $y$  represent Miles' age. Find out how old Miles is when his brother is 4 years old then 5 years old then 6 years old.

$x$	$y$
4	
5	
6	

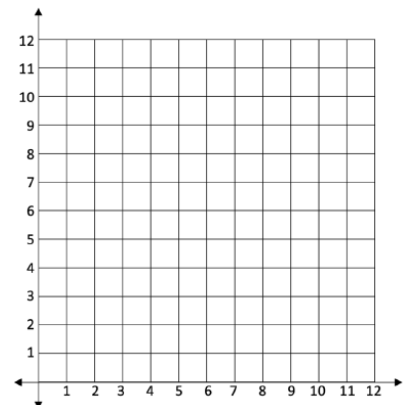


Write an equation to match the story and table. Then graph.

\_\_\_\_\_

3. Nathaniel always tries to get twice as many gems on Roblox as his brother. Let  $x$  represent his brother's gems. Let  $y$  represent Nathaniel's gems. Find the number of gems Nathaniel has when his brother has 1 gem then 2 gems then 3 gems.

$x$	$y$
1	
2	
3	



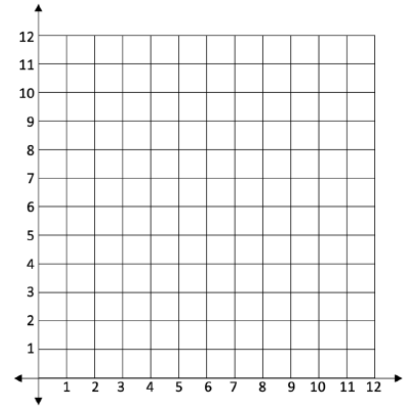
Write an equation to match the story and table. Then graph.

\_\_\_\_\_

Use the story to complete the table then write an equation and graph it.

4. Leo likes to practice for two hours for every piano lesson that he has. Let  $x$  represent the number of piano lessons. Let  $y$  represent the hours of practice. Find the number of hours of practice for 1 piano lesson then 2 piano lessons then 3 piano lessons.

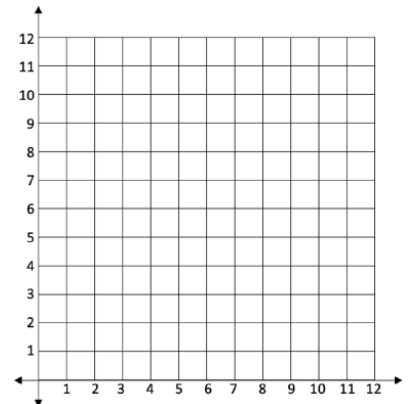
$x$	$y$
1	
2	
3	



Write an equation to match the story and table. Then graph.

5. Martin gets \$1 for every cookie he sells. Let  $x$  represent the number of cookies. Let  $y$  represent the number of dollars. Find the amount of money Martin gets for 4 cookies then 5 cookies then 6 cookies.

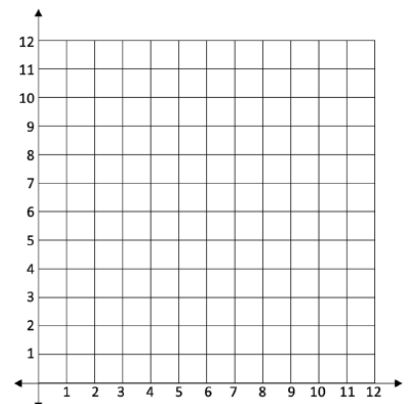
$x$	$y$
4	
5	
6	



Write an equation to match the story and table. Then graph.

6. Delish Donut shop always adds one extra donut to its customers orders. Let  $x$  represent the number of donuts a customer orders. Let  $y$  represent the number of donuts the shop gives. Find the total number of donuts that the shop packs when a customer order 4 donuts then 5 donuts then 6 donuts.

$x$	$y$
4	
5	
6	



Write an equation to match the story and table. Then graph.



Name: ANSWER KEY

Brian gets paid \$5 for each window he washes. Let  $x$  represent the number of windows he washes. Let  $y$  represent the number of dollars Brian earns. Find the amount of money that Brian earns for washing 0 windows then 2 windows then 4 windows then 6 windows.

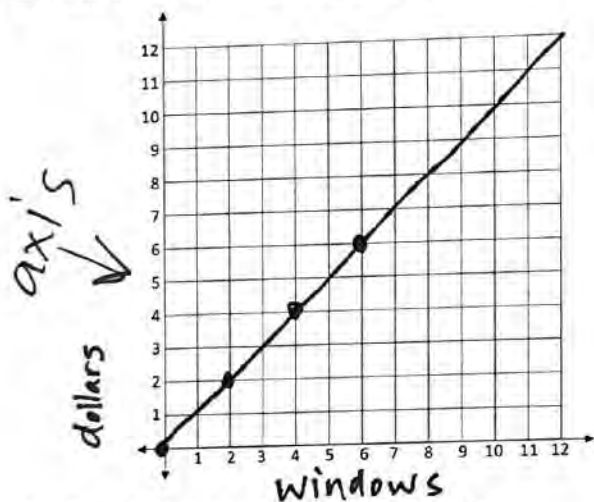
1. Complete the table shown below.

windows

x	y
0	0
2	10
4	20
6	30



2. Label the axes of the graph.



3. Graph the points from the table.

4. Write an equation to match the table:  $y = 5x$

5. Is the relationship a proportion? yes

6. Circle ALL the reasons for your answer:

- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 5.
- (c) The constant of proportionality is 10.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .

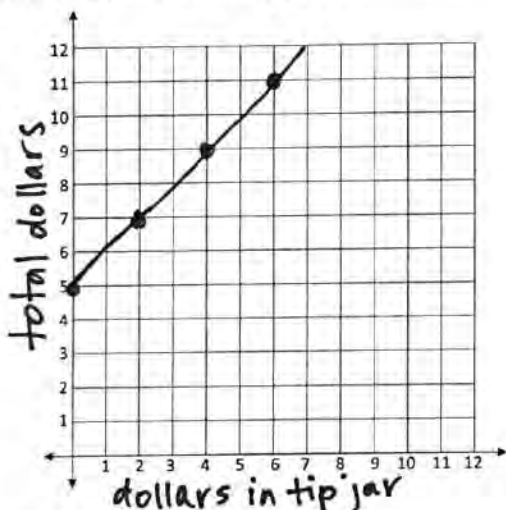
Whenever Lisa works at the cafe, her boss always gives her an extra \$5 on top of whatever money is in the tip jar. Let  $x$  represent the number of dollars in the tip jar. Let  $y$  represent the total number of dollars that Lisa gets. Find the total number of dollars Lisa gets when there are 0 dollars in the tip jar then 2 dollars then 4 dollars then 6 dollars.

7. Complete the table shown below.

dollars in tip jar	total dollars
$x$	$y$
0	5
2	7
4	9
6	11



8. Label the axes of the graph.



$$0 \overline{)5}$$

$$2 \overline{)7} \begin{array}{r} 3\frac{1}{2} \\ -6 \\ \hline 1 \end{array}$$

$$4 \overline{)9} \begin{array}{r} 2\frac{1}{4} \\ -8 \\ \hline 1 \end{array}$$

9. Graph the points from the table.

10. Write an equation to match the table:  $y = x + 5$

11. Is the relationship a proportion? NO

12. Circle ALL the reasons for your answer:

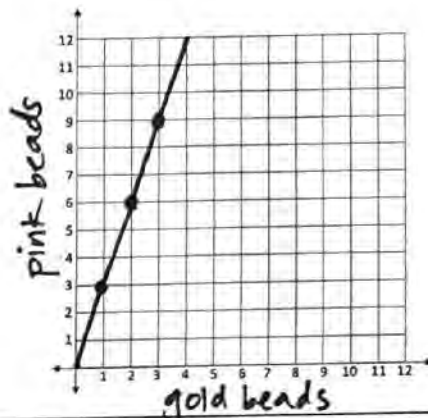
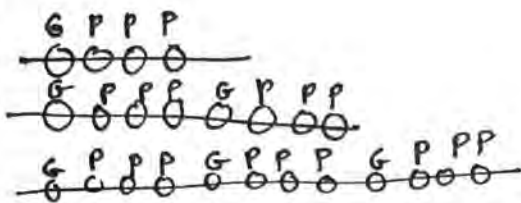
- (a) There is no constant of proportionality.
- (b) The constant of proportionality is 5.
- (c) The constant of proportionality is 10.
- (d) The equation is not in the form  $y = kx$ .
- (e) The equation is in the form  $y = kx$ .

# Name: ANSWER KEY

Use the story to complete the table then write an equation and graph it.

1. Rose is making a bracelet. Every time she puts on one gold bead, she follows it with three pink beads. Let  $x$  represent the number of gold beads. Let  $y$  represent the number of pink beads. Find the number of pink beads when there is 1 gold bead then 2 gold beads then 3 gold beads.

gold beads	pink beads
$x$	$y$
1	3
2	6
3	9

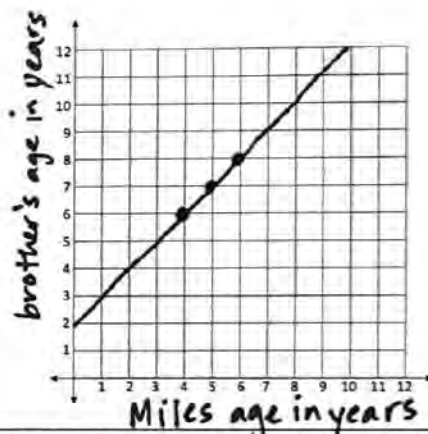
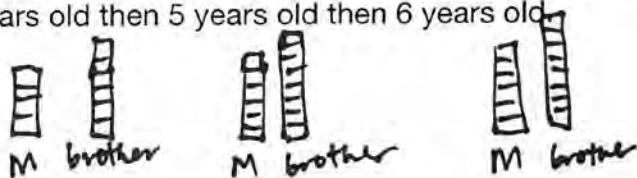


Write an equation to match the story and table. Then graph.

$$y = 3x$$

2. Miles is always 2 years younger than his brother, and that is how it will be for the rest of his life. Let  $x$  represent his brother's age. Let  $y$  represent Miles' age. Find out how old Miles is when his brother is 4 years old then 5 years old then 6 years old.

Miles age	brother's age
$x$	$y$
4	6
5	7
6	8

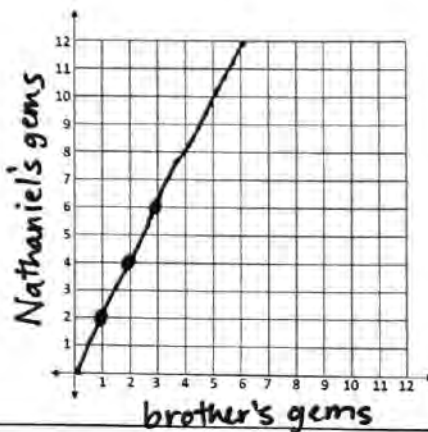


Write an equation to match the story and table. Then graph.

$$y = x + 2$$

3. Nathaniel always tries to get twice as many gems on Roblox as his brother. Let  $x$  represent his brother's gems. Let  $y$  represent Nathaniel's gems. Find the number of gems Nathaniel has when his brother has 1 gem then 2 gems then 3 gems.

brother's gems	Nathaniel's gems
$x$	$y$
1	2
2	4
3	6



Write an equation to match the story and table. Then graph.

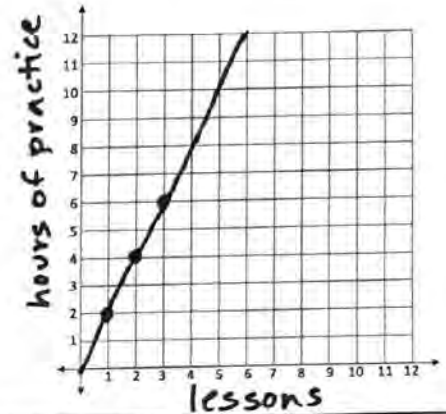
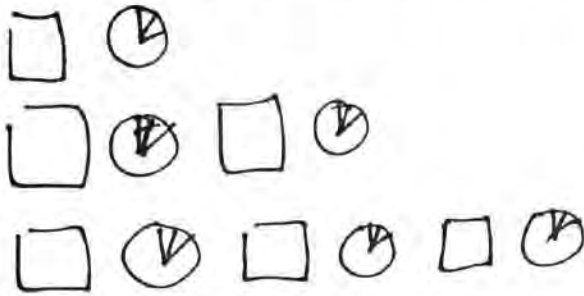
$$y = 2x$$

Use the story to complete the table then write an equation and graph it.

4. Leo likes to practice for two hours for every piano lesson that he has. Let  $x$  represent the number of piano lessons. Let  $y$  represent the hours of practice. Find the number of hours of practice for 1 piano lesson then 2 piano lessons then 3 piano lessons.

lessons hours

x	y
1	2
2	4
3	6



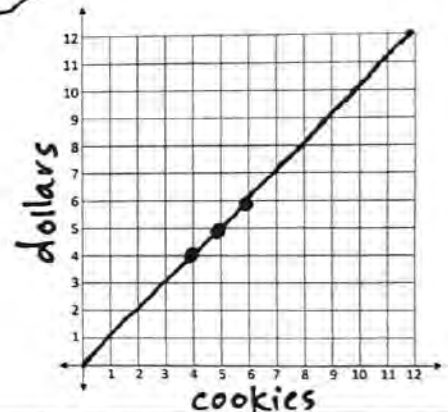
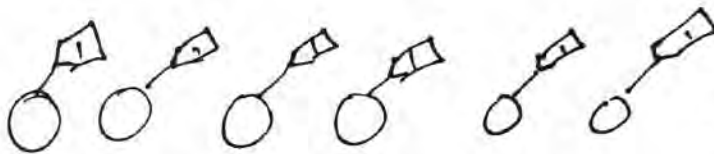
Write an equation to match the story and table. Then graph.

$$y = 2x$$

5. Martin gets \$1 for every cookie he sells. Let  $x$  represent the number of cookies. Let  $y$  represent the number of dollars. Find the amount of money Martin gets for 4 cookies then 5 cookies then 6 cookies.

cookies dollars

x	y
4	4
5	5
6	6



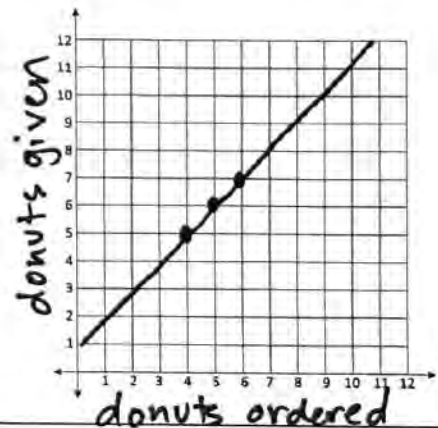
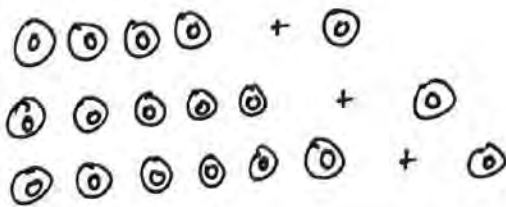
Write an equation to match the story and table. Then graph.

$$y = 1x$$

6. Delish Donut shop always adds one extra donut to its customers orders. Let  $x$  represent the number of donuts a customer orders. Let  $y$  represent the number of donuts the shop gives. Find the total number of donuts that the shop packs when a customer order 4 donuts then 5 donuts then 6 donuts.

donuts ordered donuts given

x	y
4	5
5	6
6	7



Write an equation to match the story and table. Then graph.

$$y = x + 1$$

# **G7 U2 Lesson 12**

Interpret and compare two related proportional relationships on the same graph.

## G7 U2 Lesson 12 - Today we will make a generalization about graphs of proportional relationships.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will make a generalization or notice a pattern in the graph of proportional relationships versus not proportional relationships. Eventually we will be able to just look at a graph and tell if it is a proportion. It's going to be very cool! Let's do this!

**Let's Review (Slide 3):** A relationship is proportional if it has a constant of proportionality. We already know this from previous lessons. These are two stories and graphs from our last lesson. We worked on them already together but let's read them again. Follow along with your eyes while I read it out loud. "In order to make hot chocolate, Jeb needs 2 tablespoons of cocoa for every cup of milk." We see the equation,  $y = 2x$ . And the question is, "Is it proportional? How do you know?" What do you think?

**Possible Student Answers, Key Points:**

- I think it is proportional because the equation is in the form,  $y = kx$ .
- I think it is proportional because the equation is a one step multiplication equation.
- I think it is proportional because it is "times 2" for each row of the table.
- I think it is proportional because the constant of proportionality is 2.
- I think it is proportional because 2 divided by 1 is 2 and 4 divided by 2 is 2 and 6 divided by 3 is 2 and 8 divided by 4 is 2.

Is it proportional? How do you know?  
It is proportional because the equation is in  $y=kx$  form and the constant of proportionality is 2.

You all had a lot of great ideas. It is proportional! I am going to write an explanation using the best math vocabulary I know. I will write, "It is proportional because the equation is in  $y = kx$  form and the constant of proportionality is 2."

Let's look at the next one. It said, "Jason always needs 5 minutes to cool down from a workout no matter how long it is." We see the equation,  $y = x + 5$ . And the question is, "Is it proportional? How do you know?" What do you think?

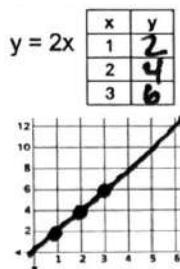
**Possible Student Answers, Key Points:**

- I think it is NOT proportional because the equation is not in the form,  $y = kx$ .
- I think it is NOT proportional because the equation is a one step addition equation.
- I think it is NOT proportional because it is "plus 5" for each row of the table.
- I think it is NOT proportional because there isn't a constant of proportionality.
- I think it is NOT proportional because 6 divided by 1 is 6 and 7 divided by 2 is 2 and a little bit.

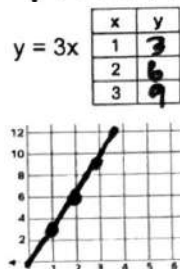
Is it proportional? How do you know?  
It is NOT proportional because the equation is not in  $y=kx$  form and there isn't a constant of proportionality.

You all had a lot of great ideas. It is NOT proportional! I am going to write an explanation using the best math vocabulary I know. I will write, "It is not proportional because the equation is not in  $y = kx$  form and there isn't constant of proportionality." Now we've reviewed equations and tables. Let's see what we can learn about graphs.

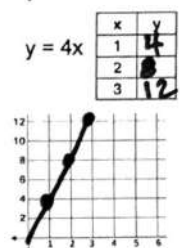




**Let's Talk (Slide 4):** We can notice a pattern in the graphs of proportions. We can see that all these equations are in the form  $y = kx$  so these are all proportions. Let's graph them and see what we notice. First, I will plug each number into  $y = 2x$ . That would be 2 times 1 is 2, 2 times 2 is 4 and 2 times 3 is 6. I am going to graph each of these, (1,2) and (2,4) and (3,6).



Next I will plug each number into  $y = 3x$ . That would be 3 times 1 is 3, 3 times 2 is 6 and 3 times 3 is 9. I am going to graph each of these, (1,3) and (2,6) and (3,9).



Finally, I will plug each number into  $y = 4x$ . That would be 4 times 1 is 4, 4 times 2 is 8 and 4 times 3 is 12. I am going to graph each of these, (1,4) and (2,8) and (3,12).

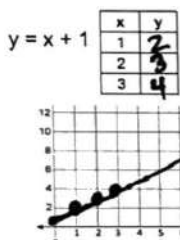
What do you notice about all of these graphs? **Possible Student Answers, Key Points:**

- They all make straight lines.
- They all go up diagonally.
- They all start in the bottom corner.

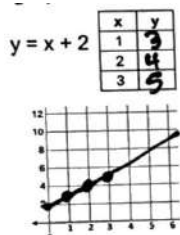
All of these are straight lines. They go up diagonally. But the important thing we are going to pay attention is that they all go through this bottom corner here. That bottom corner has a special name called the "origin." Origin means start. This is called the origin because the axes that make the graph all start here. The coordinates of the origin are (0,0).

Graphs of proportions always *go through the origin (0,0)*

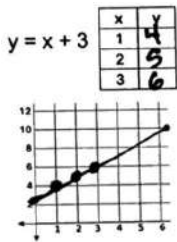
This first graph starts at (0,0). This middle graph starts at (0,0) and the last graph starts at (0,0). Now we can fill in the blanks. Graphs of proportions always go through the origin, (0,0).



**Let's Think (Slide 5):** The pattern we just saw is NOT there for graphs that are NOT proportions. Let's explore. First, I will plug each number into  $y = x + 1$ . That would be 1 plus 1 is 2 and 2 plus 1 is 3 and 3 plus 1 is 4. I am going to graph each of these, (1,2) and (2,3) and (3,4).



Next I will plug each number into  $y = x + 2$ . That would be 1 plus 2 is 3 and 2 plus 2 is 4 and 3 plus 2 is 5. I am going to graph each of these, (1,3) and (2,4) and (3,5).



Next I will plug each number into  $y = x + 3$ . That would be 1 plus 3 is 4 and 2 plus 3 is 5 and 3 plus 3 is 6. I am going to graph each of these, (1,4) and (2,5) and (3,6).

What do you notice about all of these graphs? **Possible Student Answers, Key Points:**

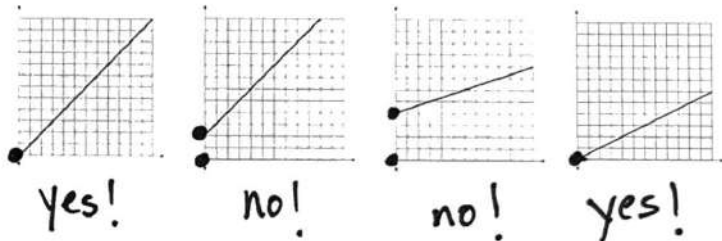
- They all make straight lines.
- They all go up diagonally.
- They do NOT all start in the bottom corner.

All of these are straight lines. They go up diagonally. But the important thing we are going to pay attention is that they do NOT go through this bottom corner like on the last slide. Now we can fill in the blanks. Lines that don't go through the origin (0,0) are not proportions.

Lines that don't go through (0,0) are not proportions.  
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**Let's Think (Slide 6):** Now we know the important origin trick! We should be able to tell which graph is proportional just by looking. The graphs below don't even have numbers. Let's do a quick thumbs up and thumbs down. Thumbs up means yes and thumbs down means no. Do you think this first graph is proportional? *Give some think time for kids to show their thumbs up or down.* Yes, this is proportional because the line goes through the origin (0,0) right here. Let's do the next one. Do you think this first graph is proportional? *Give some think time for kids to show their thumbs up or down.* No, this is NOT proportional because the line does NOT go through the origin (0,0) right here.

Do you expect the relationship to be proportional?



the next one. Do you think this first graph is proportional? *Give some think time for kids to show their thumbs up or down.* No, this is NOT proportional because the line does NOT go through the origin (0,0) right here. Let's do the next one. Do you think this first graph is proportional? *Give some think time for kids to show their thumbs up or down.* Yes, this is proportional because the line goes through the origin (0,0) right here.

**Let's Try It (Slide 7):** Now we will look at some more graphs together and I will walk you through checking if they are proportions step by step.

# WARM WELCOME



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**Today we will make a generalization  
about graphs of proportional  
relationships.**

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# Let's Review:

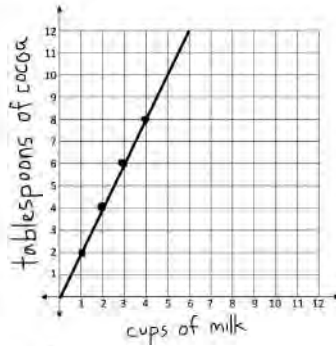
A relationship is proportional if it has a constant of proportionality.

In order to make hot chocolate, Jeb needs 2 tablespoons of cocoa for every cup of milk. Let  $x$  represent the cups of milk. Let  $y$  represent the tablespoons of cocoa.

Write an equation:

$$y = 2x$$

x	y
cups	tblsp
1	2
2	4
3	6
4	8



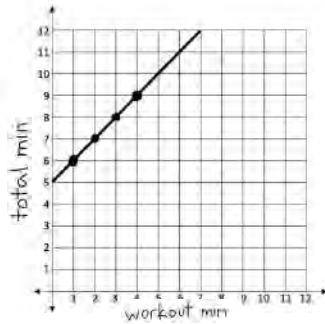
Is it proportional? How do you know?

Jason always needs 5 minutes to cool down from a workout no matter how long it is. Let  $x$  represent the length of Jason's workout in minutes. Let  $y$  represent the total number of minutes.

Write an equation:

$$y = x + 5$$

x	y
work out	total
1	6
2	7
3	8
4	9



Is it proportional? How do you know?

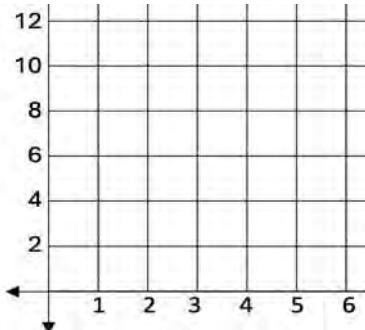
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# Let's Talk:

We can notice a pattern in the graphs of proportions.

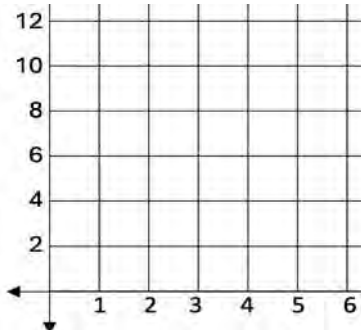
$$y = 2x$$

x	y
1	
2	
3	



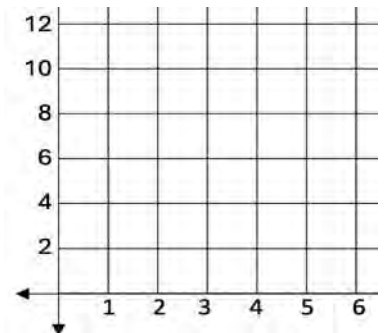
$$y = 3x$$

x	y
1	
2	
3	



$$y = 4x$$

x	y
1	
2	
3	



Graphs of proportions always \_\_\_\_\_

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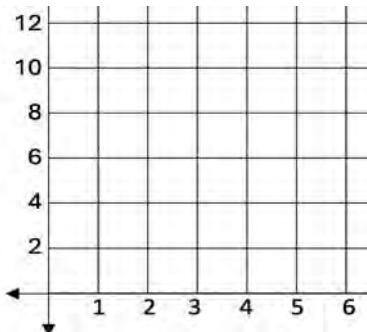


Let's Think:

The pattern we just saw is not there for graphs that are NOT proportions.

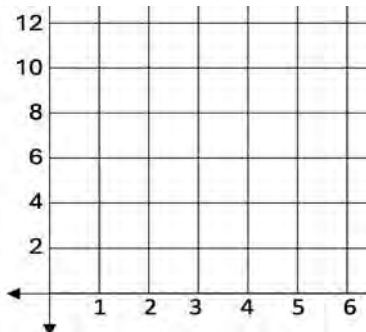
$y = x + 1$

x	y
1	
2	
3	



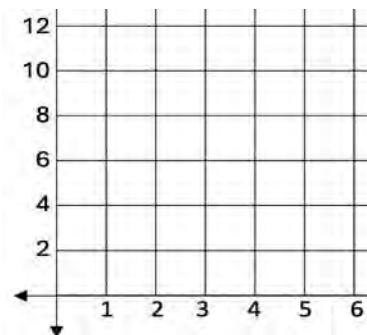
$y = x + 2$

x	y
1	
2	
3	



$y = x + 3$

x	y
1	
2	
3	



Lines that \_\_\_\_\_ are not proportions.

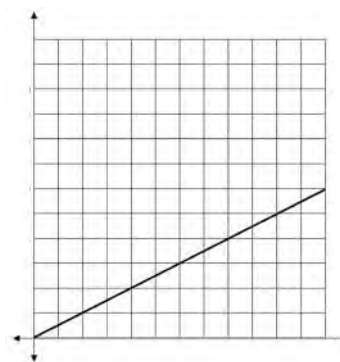
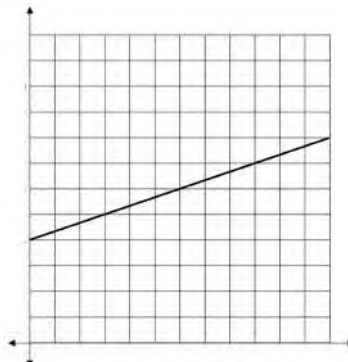
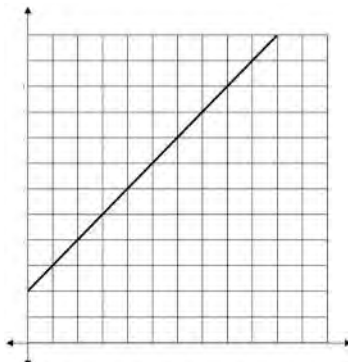
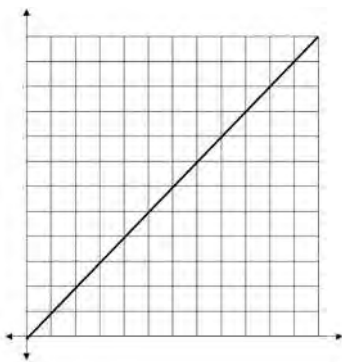
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Let's Think:

We should be able to tell which graph is proportional just by looking.

Do you expect the relationship to be proportional?



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# Let's Try It: We will do it together step by step.

Name: \_\_\_\_\_ G7 U2 Lesson 12 - Let's Try It

Determine if the graph is proportional. Use the table and equation to check.

1. Does the graph below intercept the origin (0,0)? \_\_\_\_\_

2. Based on your answer to #1, do you expect the relationship to be proportional? \_\_\_\_\_

3. Use the points on the graph to fill in the table.

x	y

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# On your Own: Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 12 - Independent Work

Determine if the graph is proportional. Use the table and equation to check.

1. Based on the graph, is the relationship proportional? \_\_\_\_\_

Complete the table.

x	y

Does it have a constant of proportionality? \_\_\_\_\_

Is it in the form,  $y = kx$ ? \_\_\_\_\_

Write an equation. \_\_\_\_\_

---

2. Based on the graph, is the relationship proportional? \_\_\_\_\_

Complete the table.

x	y

Does it have a constant \_\_\_\_\_

Is it in the form, \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

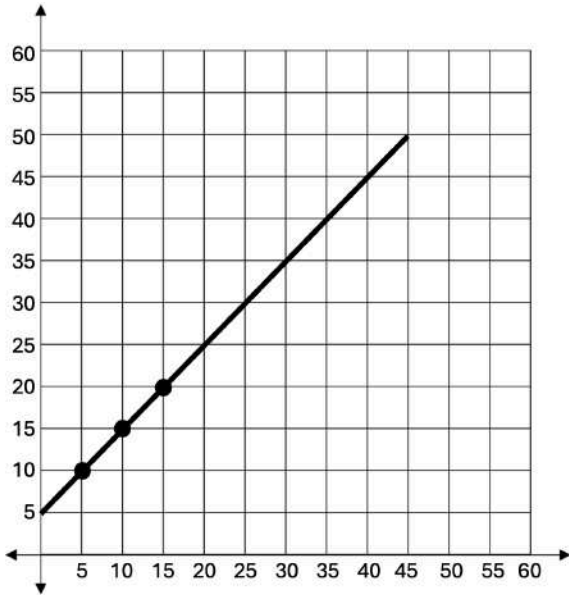
Write an equation. \_\_\_\_\_

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**Determine if the graph is proportional. Use the table and equation to check.**

1. Does the graph below intercept the origin (0,0)? \_\_\_\_\_



2. Based on your answer to #1, do you expect the relationship to be proportional? \_\_\_\_\_

3. Use the points on the graph to fill in the table.

<b>x</b>	<b>y</b>

4. Does the table have a constant of proportionality? If so, what is it? \_\_\_\_\_

5. Based on your answer to #4, do you expect the relationship to be proportional? \_\_\_\_\_

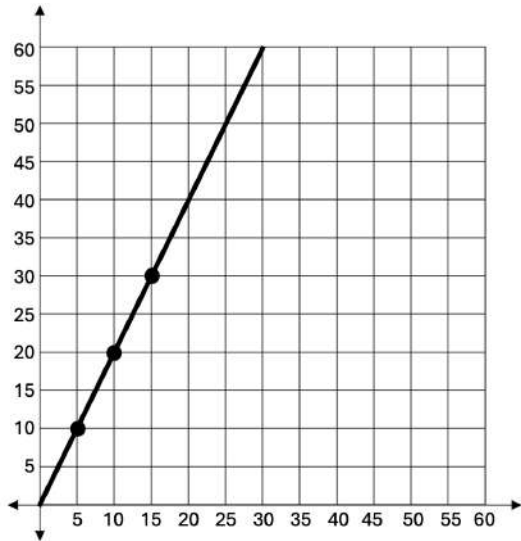
6. Make an equation to match the table. \_\_\_\_\_

7. Is the equation in  $y = kx$  form? \_\_\_\_\_

8. Based on your answer to #7, do you expect the relationship to be proportional? \_\_\_\_\_

Determine if the graph is proportional. Use the table and equation to check.

9. Does the graph below intercept the origin (0,0)? \_\_\_\_\_



10. Based on your answer to #9, do you expect the relationship to be proportional? \_\_\_\_\_

11. Use the points on the graph to fill in the table.

<b>x</b>	<b>y</b>

12. Does the table have a constant of proportionality? If so, what is it? \_\_\_\_\_

13. Based on your answer to #12, do you expect the relationship to be proportional? \_\_\_\_\_

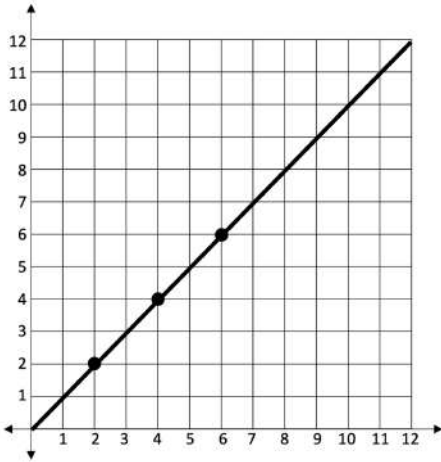
14. Make an equation to match the table. \_\_\_\_\_

15. Is the equation in  $y = kx$  form? \_\_\_\_\_

16. Based on your answer to #15, do you expect the relationship to be proportional? \_\_\_\_\_

Determine if the graph is proportional. Use the table and equation to check.

1. Based on the graph, is the relationship proportional? \_\_\_\_\_



Complete the table.

x	y

Write an equation.

\_\_\_\_\_

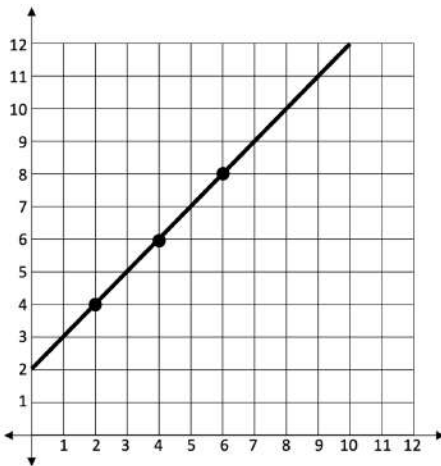
Does it have a constant of proportionality? \_\_\_\_\_

Is it in the form,  $y = kx$ ? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

2. Based on the graph, is the relationship proportional? \_\_\_\_\_



Complete the table.

x	y

Write an equation.

\_\_\_\_\_

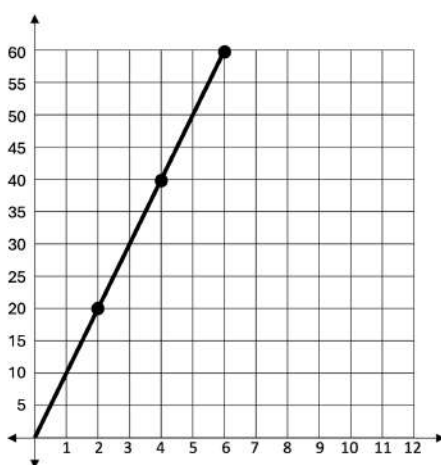
Does it have a constant of proportionality? \_\_\_\_\_

Is it in the form,  $y = kx$ ? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

3. Based on the graph, is the relationship proportional? \_\_\_\_\_



Complete the table.

x	y

Write an equation.

\_\_\_\_\_

Does it have a constant of proportionality? \_\_\_\_\_

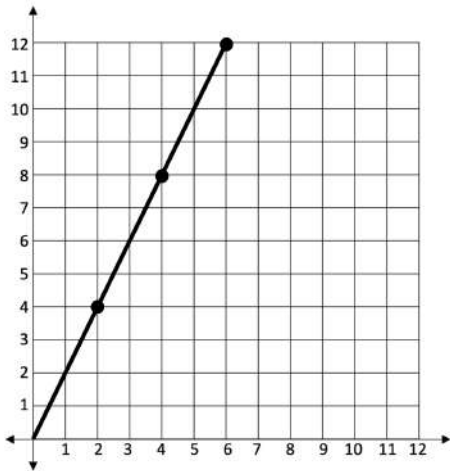
Is it in the form,  $y = kx$ ? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Determine if the graph, table and equation are proportional.

1. Based on the graph, is the relationship proportional? \_\_\_\_\_



Complete the table.

x	y

Write an equation.

\_\_\_\_\_

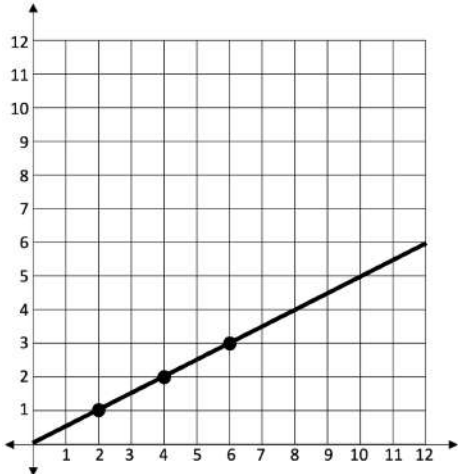
Does it have a constant of proportionality? \_\_\_\_\_

Is it in the form,  $y = kx$ ? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

2. Based on the graph, is the relationship proportional? \_\_\_\_\_



Complete the table.

x	y

Write an equation.

\_\_\_\_\_

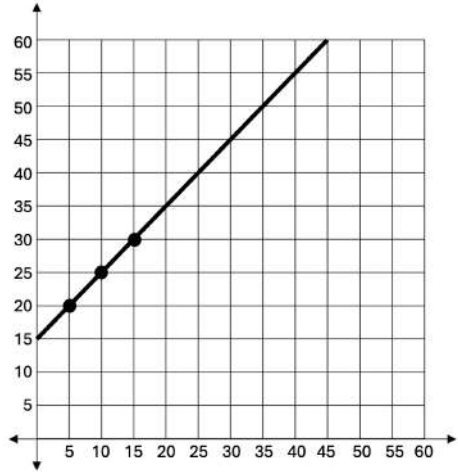
Does it have a constant of proportionality? \_\_\_\_\_

Is it in the form,  $y = kx$ ? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

3. Based on the graph, is the relationship proportional? \_\_\_\_\_



Complete the table.

x	y

Write an equation.

\_\_\_\_\_

Does it have a constant of proportionality? \_\_\_\_\_

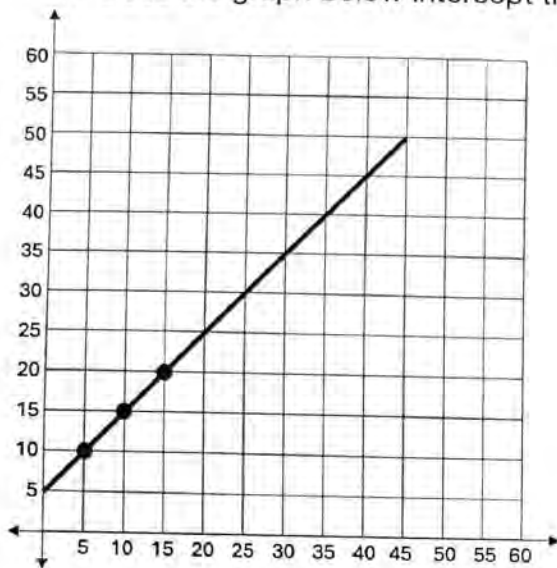
Is it in the form,  $y = kx$ ? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Is it a proportion? \_\_\_\_\_

Determine if the graph is proportional. Use the table and equation to check.

1. Does the graph below intercept the origin (0,0)? NO



2. Based on your answer to #1, do you expect the relationship to be proportional? NO

3. Use the points on the graph to fill in the table.

x	y
5	10
10	15
15	20

$$\begin{array}{r} 02 \\ 5 \overline{)10} \\ \underline{-10} \\ 00 \end{array}$$

$$\begin{array}{r} 01\frac{5}{10} \\ 10 \overline{)15} \\ \underline{-10} \\ 5 \end{array}$$

4. Does the table have a constant of proportionality? If so, what is it? NO

5. Based on your answer to #4, do you expect the relationship to be proportional? NO

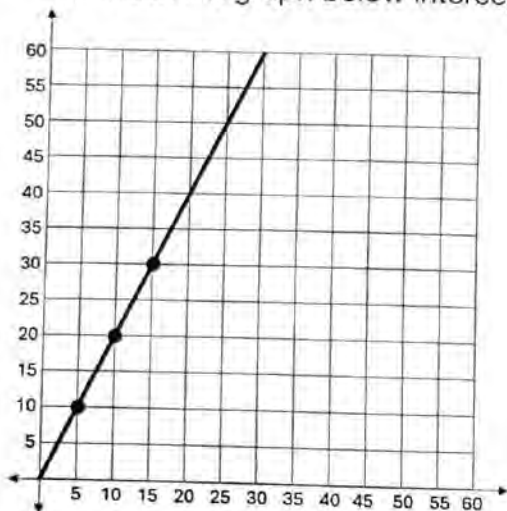
6. Make an equation to match the table.  $y = x + 5$

7. Is the equation in  $y = kx$  form? NO

8. Based on your answer to #7, do you expect the relationship to be proportional? NO

Determine if the graph is proportional. Use the table and equation to check.

9. Does the graph below intercept the origin (0,0)? YES



10. Based on your answer to #9, do you expect the relationship to be proportional? YES

11. Use the points on the graph to fill in the table.

x	y
5	10
10	20
15	30

12. Does the table have a constant of proportionality? If so, what is it? Yes, it is 2.

13. Based on your answer to #12, do you expect the relationship to be proportional? Yes

14. Make an equation to match the table.  $y = 2x$

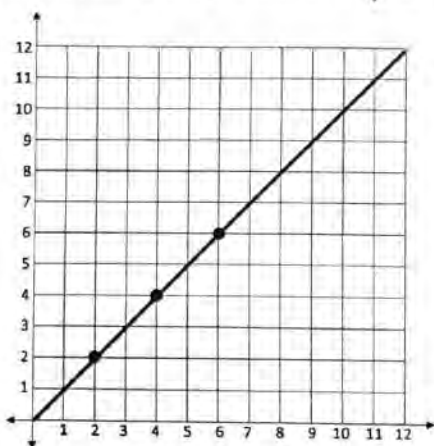
15. Is the equation in  $y = kx$  form? yes

16. Based on your answer to #15, do you expect the relationship to be proportional? yes



Determine if the graph is proportional. Use the table and equation to check.

1. Based on the graph, is the relationship proportional? yes



Complete the table.

x	y
2	2
4	4
6	6

$\frac{1}{2} \sqrt{2} = \frac{1}{2}$   
 $\frac{1}{4} \sqrt{4} = \frac{1}{4}$   
 $\frac{1}{6} \sqrt{6} = \frac{1}{6}$

Write an equation.

$y = 1x$

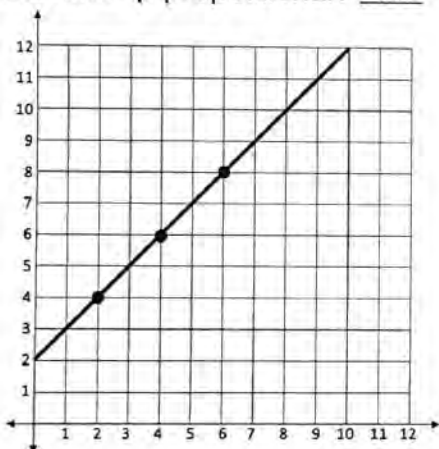
Does it have a constant of proportionality? yes

Is it in the form,  $y = kx$ ? yes

Is it a proportion? yes

Is it a proportion? yes

2. Based on the graph, is the relationship proportional? no



Complete the table.

x	y
2	4
4	6
6	8

$\frac{2}{2} \sqrt{4} = 2$   
 $\frac{4}{4} \sqrt{6} = 1.5$   
 $\frac{6}{6} \sqrt{8} = 2$

Write an equation.

$y = x + 2$

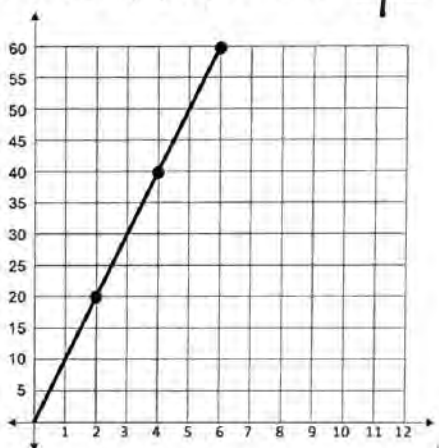
Does it have a constant of proportionality? no

Is it in the form,  $y = kx$ ? no

Is it a proportion? no

Is it a proportion? no

3. Based on the graph, is the relationship proportional? yes



Complete the table.

x	y
2	20
4	40
6	60

$\frac{10}{2} \sqrt{20} = 10$   
 $\frac{10}{4} \sqrt{40} = 10$   
 $\frac{10}{6} \sqrt{60} = 10$

Write an equation.

$y = 10x$

Does it have a constant of proportionality? yes

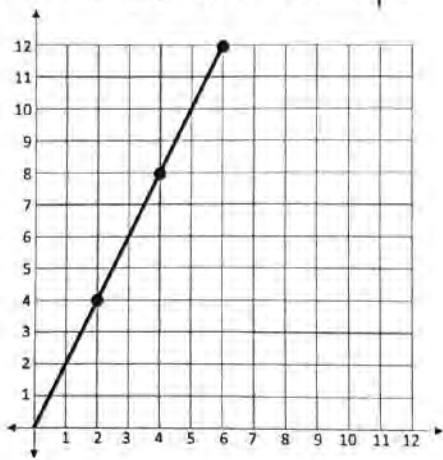
Is it in the form,  $y = kx$ ? yes

Is it a proportion? yes

Is it a proportion? yes

Determine if the graph, table and equation are proportional.

1. Based on the graph, is the relationship proportional? Yes



Complete the table.

x	y
2	4
4	8
6	12

$$\begin{array}{r} 2 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array} \quad \begin{array}{r} 0.2 \\ 6 \overline{)12} \\ \underline{-12} \\ 0 \end{array}$$

Write an equation.

$$y = 2x$$

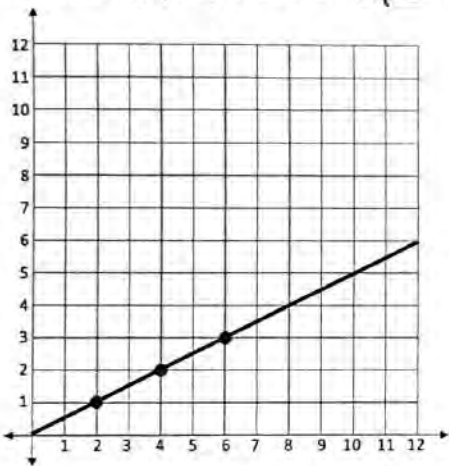
Does it have a constant of proportionality? Yes

Is it in the form,  $y = kx$ ? Yes

Is it a proportion? Yes

Is it a proportion? Yes

2. Based on the graph, is the relationship proportional? Yes



Complete the table.

x	y
2	$\frac{1}{2}$
4	2
6	3

$$\begin{array}{r} 0.5 \\ 2 \overline{)1} \\ \underline{-1} \\ 0 \end{array} \quad \begin{array}{r} 0.33 \\ 4 \overline{)2} \\ \underline{-1} \\ 1 \end{array}$$

Write an equation.

$$y = \frac{1}{2}x$$

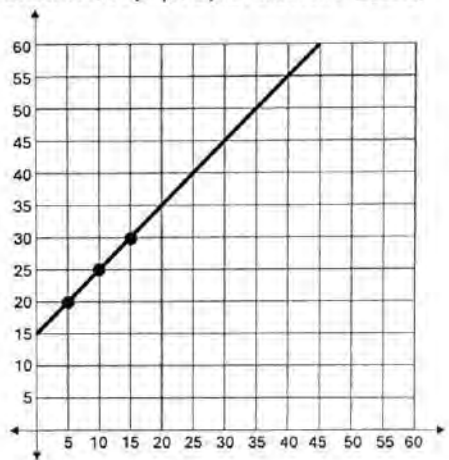
Does it have a constant of proportionality? Yes

Is it in the form,  $y = kx$ ? Yes

Is it a proportion? Yes

Is it a proportion? Yes

3. Based on the graph, is the relationship proportional? No



Complete the table.

x	y
5	20
10	25
15	30

$$\begin{array}{r} 0.4 \\ 5 \overline{)20} \\ \underline{-20} \\ 0 \end{array}$$

$$\begin{array}{r} 0.25 \\ 10 \overline{)25} \\ \underline{-20} \\ 5 \end{array}$$

Write an equation.

$$y = x + 15$$

Does it have a constant of proportionality? No

Is it in the form,  $y = kx$ ? No

Is it a proportion? No

Is it a proportion? No

# **G7 U2 Lesson 13**

Interpret and compare the same proportional relationship using two different sets of tables, graphs, and equations.

## G7 U2 Lesson 13 - Today we will interpret points on the graph with the context of a story.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will interpret points on the graph with the context of a story. You already know all the math parts of this. Now you just need to make sure you pay really close attention to the words. It's easy to jump in and just crunch numbers. But you could do it incorrectly if you aren't paying attention to the words.

**Let's Review (Slide 3):** We know that a graph can also be represented as a table and equation. This says, "Write an equation for the graph shown below." This thought bubble is reminding me to stop and think. Because just like I said, we don't want to just jump to answer-getting. We stop and think and if we try to show work on our paper for every problem. So, what work can we show for this before jumping to an answer? [Possible Student Answers, Key Points:](#)



- We could look at what operation is being done to x to get y.
- We could see if we notice a pattern.
- We could draw a table.

To write the equation, we will need an operation and maybe you can just see it with your eyeballs. But the very easiest thing would be to make a table.

x	y
2	4
4	6
6	8

I am going to draw one here with an x and y. Now let's start putting points. This first dot is above the 2 and next to the 4. I am going to write 2 and 4 in my table. Now, I put the 2 in the x column because that is on the horizontal axis. The 4 goes in the y column because that is on the vertical axis. If I switch these and put the 4 and then the 2, I will get the wrong answer. Let's do the next point. It is above the 4 so 4 is x. It is next to the 6 so 6 is y. Let's do the next point. It is above the 6 so 6 is x. It next to the 8 so 8 is y.

x	y
2	4
4	6
6	8

Now I can look at my table and find the operation. I can even put circles here to write it down. At first, I must think it is "times 2" because 2 times 2 makes 4. But that doesn't work for the next row because 4 times 2 would be 8. Let me try "plus 2." 2 plus 2 is 4. 4 plus 2 is 6. 6 plus 2 is 8. That's it!

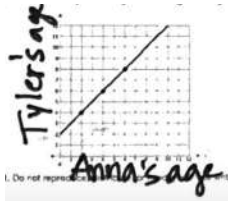
$$y = x + 2$$

Now that I've found the operation, I can write the equation. I see that x is getting added by 2 so I write  $y = x + 2$ . The big idea here is making a table from a graph is really really helpful in writing an equation. And guess what? It is going to be really help in answering story problem questions with a graph too. So we have to make a promise here and now that we are always going to show our work by drawing a table.

**Let's Talk (Slide 4):** Because a graph can be represented with a story, we can use it to answer questions. This is the big idea for today so I am going to write it down. Every time we want to answer a question about graph, we will use a TABLE with WORDS.

Every time we want to answer a question about a graph, we will use a table with words.

Remember, the words are very important. The words tell us if we're looking for x or y.



Let's try this problem. Read along silently with your eyes while I read out loud. "The graph below shows the relationship between Anna's age and Tyler's age. Let  $x$  be Anna's age in years. Let  $y$  be Tyler's age in years. How old will Tyler be when Anna is 9 years old?" The first thing I am going to do is label my graph. X is Anna's age in years. Y is Tyler's age in years.

Anna	Tyler
2	4
4	6
6	8

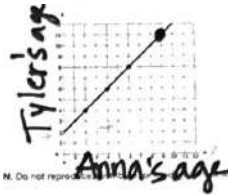
Now, we said we would always make a table with words. This is the same graph as before. So we know the numbers. That's not the important part. The important part is which side is Anna's age and which side is Tyler's age. It matters because one person is younger and one is older, right? Here are the numbers from before.

Anna	Tyler
2 + 2	4
4 + 2	6
6 + 2	8
9 + 2	11

Now,  $x$  is Anna's age so the left hand column is Anna.  $y$  is Tyler's age so the right hand column is Tyler. We already talked about how the operation is "plus 2." When I go to answer this equation, "how old will Tyler be when Anna is 9 years old," the most important thing is asking myself, is the 9 an  $x$  or a  $y$ ? In this case, the 9 is an  $x$ . Why is that? **Possible Student Answers, Key Points:**

- Anna is 9 years old and it said let  $x$  be Anna's age in years.
- We labeled the left column as Anna's age so the 9 goes there.

The story said "let  $x$  be Anna's age in years" so if the 9 is Anna's age then it is  $x$  and it goes in the left column. Now it is easy to see that we add 2 and Tyler's age would be 11.



And look, if I put a dot on that point, it is on my line so I know I am right.

**Let's Think (Slide 5):** It is really important that we don't mix up the words on the  $x$ -axis and  $y$ -axis. Let

me know you why. We know from before that every time we want to answer a question about a graph, we will use a TABLE with WORDS.

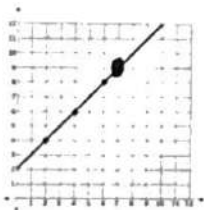
Every time we want to answer a question about a graph, we will use a Table with words.

Anna's age	Tyler's age
$x$	7
2 + 4	4
4 + 4	6
6 + 4	8
9	

Now we have the exact same problem as before but look! The words in the the question were crossed out and changed. Now it says, "how old will Anna be when Tyler is 9 years old?" It's a totally different question with that little change. Because remember we said we have to ask ourselves, is 9 an  $x$  or a  $y$ ? This time the 9 is the  $y$  because Tyler is 9 years old and  $y$  is Tyler's age. When I put that 9 in the table, I have to put it on the right hand side now. That is why we have to pay attention to the words and not just the numbers.

Anna's age	Tyler's age
$x$	7
2 + 4	4
4 + 4	6
6 + 4	8
7	9

Now I'm not going to add 2. I am asking what plus 2 makes 9. That is 7. So if Tyler is 9 then Anna is 7. I got a different answer because there were different words.



And look, if I put a dot on that point, it is on my line so I know I am right.

**Let's Try It (Slide 6):** Now here's what's really cool. Even if the point is not shown on the graph, we can find the relationship to answer a question. We still remember that every time we want to answer a

question about a graph, we will use a TABLE with WORDS.

Every time we want to answer a question about a graph, we will use a Table with Words.

Anna's Tylers

age	age
x	y
2	4
4	6
100	8

$100 + 2 = 102$

We have the same problem as before but let's look at the question. It says, "how old will Tyler be when Anna is 100 years old?" There's no 100 on my graph but I can still figure it out. Now, we still ask ourselves that key question, "Is 100 the x or the y?" It is the x because it is Anna's age so it goes on the left hand side of my table. Now I can see that I need to do 100 plus 2 equals 102. So when Anna is 100 then Tyler is 102.

**Let's Try It (Slide 7):** Now we will graph from tables and equations together. I will take you through step by step.



# WARM WELCOME



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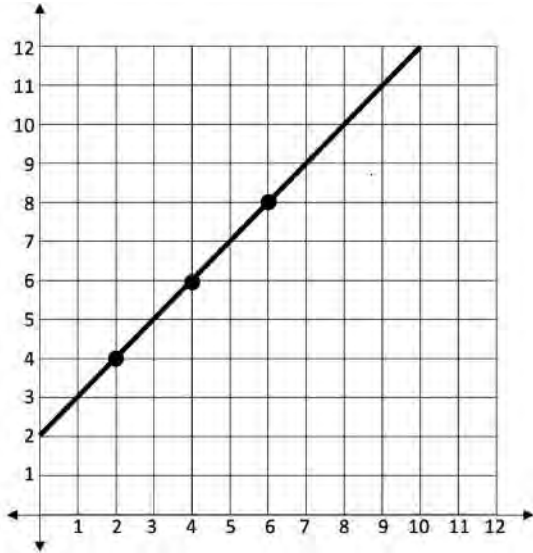
**Today we will interpret points on the graph with the context of a story.**

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## Let's Review:

We know that a graph can also be represented as a table and equation.

Write an equation for the graph shown below.



What work can we show before jumping to an answer?

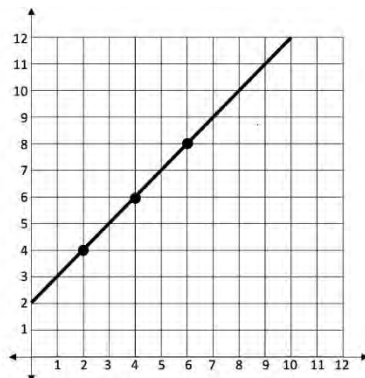
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## Let's Talk:

Because a graph can be represented with a story, we can use it to answer questions.

Every time we want to answer a question about a graph, we will use a \_\_\_\_\_ with \_\_\_\_\_.

The graph below shows the relationship between Anna's age and Tyler's age. Let  $x$  be Anna's age in years. Let  $y$  be Tyler's age in years. How old will Tyler be when Anna is 9 years old?



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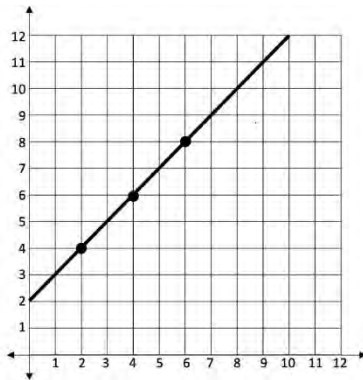


## Let's Think:

It is really important that we don't mix up the words on the x-axis and y-axis.

Every time we want to answer a question about a graph, we will use a \_\_\_\_\_ with \_\_\_\_\_.

The graph below shows the relationship between Anna's age and Tyler's age. Let  $x$  be Anna's age in years. Let  $y$  be Tyler's age in years. How old will ~~Tyler~~ Anna be when Anna is 9 years old?  
Tyler



Anna's age	Tyler's age
$x$	$y$
2	4
4	6
$b$	8

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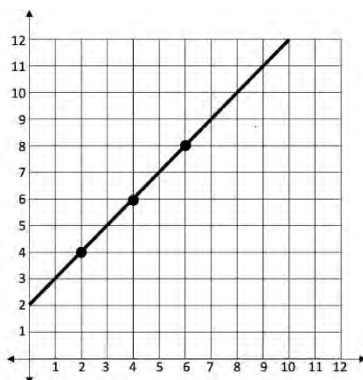


## Let's Think:

Even if the point is not shown on the graph, we can find the relationship to answer a question.

Every time we want to answer a question about a graph, we will use a \_\_\_\_\_ with \_\_\_\_\_.

The graph below shows the relationship between Anna's age and Tyler's age. Let  $x$  be Anna's age in years. Let  $y$  be Tyler's age in years. How old will Tyler be when Anna is 100 years old?



Anna's age	Tyler's age
$x$	$y$
2	4
4	6
$b$	8

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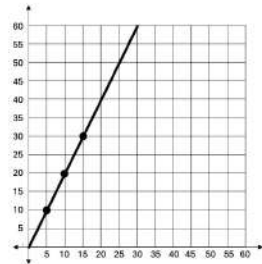
# Let's Try It:

## We will do it together step by step!

Name: \_\_\_\_\_

G7 U2 Lesson 13 - Let's Try It

Kathy's Kitty Shop uses the graph below to determine how many pounds of kitty litter they will need weekly based on the number of kitties available for adoption. Let  $x$  represent the number of kitties. Let  $y$  represent the number of pounds of kitty litter.



(a) How many pounds of kitty litter would need to be ordered for 20 kitties?

1. Every time we want to answer a question about a graph, we will use a \_\_\_\_\_ with \_\_\_\_\_. Draw one.

2. What operation do you observe in your table? \_\_\_\_\_ Draw it in circles on each row.
3. Is the number given in the question represented by  $x$  or  $y$ ? \_\_\_\_\_ Put it on your table.
4. Solve for the other variable.

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# On your Own:

## Now it's time for you to do it on your own.

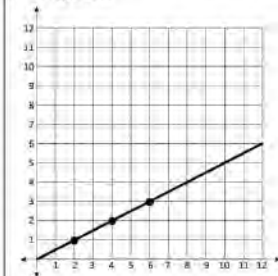
Name: \_\_\_\_\_

G7 U2 Lesson 13 - Independent Work

Remember: You must make a table with words to answer questions about graphs.

Use the graph to answer the questions below. Show your work.

The graph below shows how much Whitney waters her garden based on how much it rains in a particular week. Let  $x$  represent the inches of rain and  $y$  represent the gallons of water Whitney uses.



1. How many gallons will Whitney use if it rained 8 inches that week?

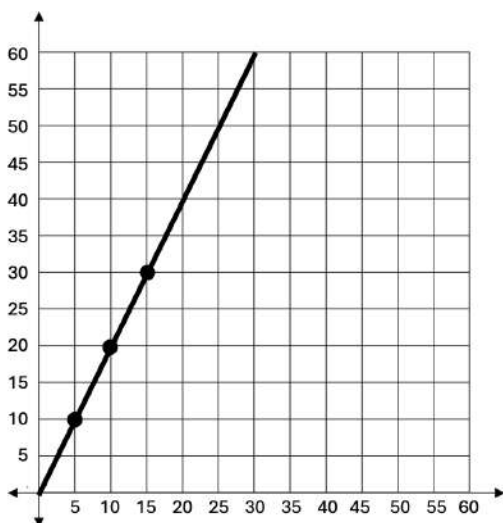
2. How many inches must it have rained if Whitney uses 10 gallons of water?

Martin uses the graph below to determine how much to pay his employees based on how long they work. Let  $x$  represent the number of hours they work. Let  $y$  represent the number of dollars

3. How much would Meryl need to pay the kid for 10 hours of raking?

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Kathy's Kitty Shop uses the graph below to determine how many pounds of kitty litter they will need weekly based on the number of kitties available for adoption. Let  $x$  represent the number of kitties. Let  $y$  represent the number of pounds of kitty litter.



(a) How many pounds of kitty litter would need to be ordered for 20 kitties?

1. Every time we want to answer a question about a graph, we will use a \_\_\_\_\_ with \_\_\_\_\_. Draw one.

2. What operation do you observe in your table? \_\_\_\_\_ Draw it in circles on each row.
3. Is the number given in the question represented by  $x$  or  $y$ ? \_\_\_\_\_ Put it on your table.
4. Solve for the other variable.
5. Write your answer in a complete sentence using words from the story.

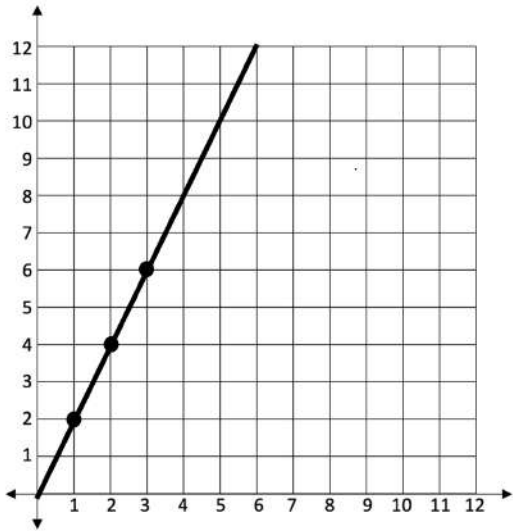
(b) How many kitties must there be if 80 pounds of kitty litter were ordered?

6. Is the number given in the question represented by  $x$  or  $y$ ? \_\_\_\_\_ Put it on your table.
7. Solve for the other variable.
8. Write your answer in a complete sentence using words from the story.

Remember: You must make a table with words to answer questions about graphs.

Use the graph to answer the questions below. Show your work.

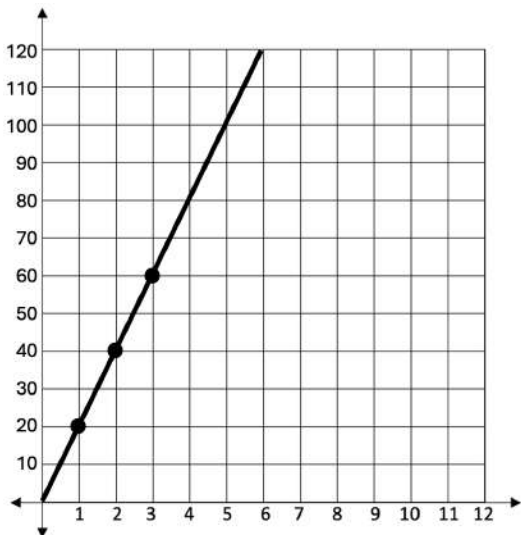
The graph below shows how much Whitney waters her garden based on the number of days it does not rain. Let  $x$  represent the days it does not rain and  $y$  represent the gallons of water Whitney uses.



1. How many gallons will Whitney use if it did not rain for 8 days?

2. How many days must it have not rained if Whitney uses 10 gallons of water?

Martin uses the graph below to determine how much to pay his employees based on how long they work. Let  $x$  represent the number of hours they work. Let  $y$  represent the number of dollars they get paid.



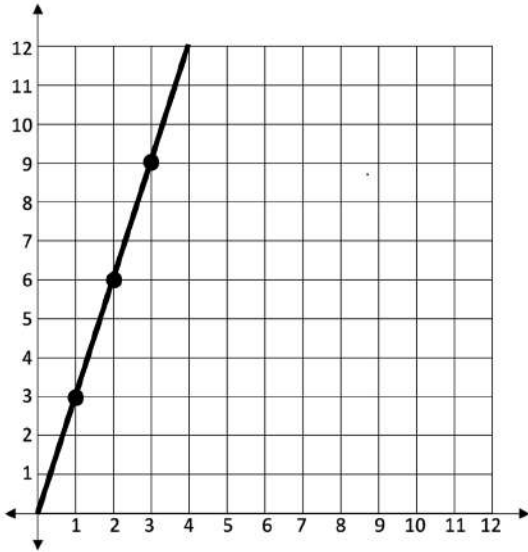
3. How much would Meryl need to pay the kid for 10 hours of raking?

4. How many hours of raking must have been done if Meryl paid \$80?



Use the graph to answer the questions below. Show your work.

Charlie's Hat Shop uses the graph below to determine how many hats to buy for the teams in the Superbowl. Let  $x$  represent the number of boxes of away team hats. Let  $y$  represent the number of home team hats.

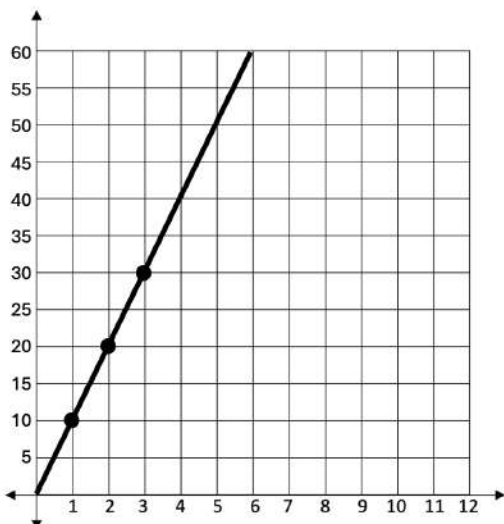


5. How many boxes of home team hats would Charlie buy if he planned to buy 5 boxes of away team hats?

6. How many boxes of away team hats would Charline buy if he planned to buy 4 boxes of home team hats?

7. How many boxes of away team hats would Charline buy if he planned to buy 100 boxes of home team hats?

The graph below shows how many months Julia must train based on the length of her upcoming race. Let  $x$  represent the length of the race in miles. Let  $y$  represent the number of days Julia must train.

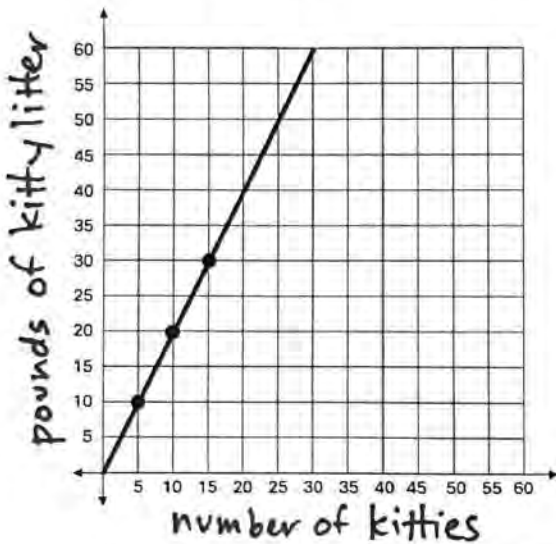


8. How long must the race be if Julia trained for 60 days?

9. How many days would Julia train for a 5 mile race?

10. How many days would Julia train for a 10 mile race?

Kathy's Kitty Shop uses the graph below to determine how many pounds of kitty litter they will need weekly based on the number of kitties available for adoption. Let  $x$  represent the number of kitties. Let  $y$  represent the number of pounds of kitty litter.



(a) How many pounds of kitty litter would need to be ordered for 20 kitties?

1. Every time we want to answer a question about a graph, we will use a table with words. Draw one.

kitties	pounds of kitty litter
5	10
10	20
15	30
20	40

- What operation do you observe in your table?  $\times 2$  Draw it in circles on each row.
- Is the number given in the question represented by  $x$  or  $y$ ?  $x$  Put it on your table.
- Solve for the other variable.

$$y = 2x$$

$$y = 2 \cdot 20$$

$$y = 40$$

5. Write your answer in a complete sentence using words from the story.

20 kitties will need 40 pounds of kitty litter.

(b) How many kitties must there be if 80 pounds of kitty litter were ordered?

- Is the number given in the question represented by  $x$  or  $y$ ?  $y$  Put it on your table.
- Solve for the other variable.

$$y = 2x$$

$$\frac{80}{2} = \frac{2x}{2}$$

$$40 = x$$

8. Write your answer in a complete sentence using words from the story.

If 80 pounds of kitty litter were ordered, there must be 40 kitties.

# Name: ANSWER KEY

Remember: You must make a table with words to answer questions about graphs.

Use the graph to answer the questions below. Show your work.

The graph below shows how much Whitney waters her garden based on the number of days it does not rain. Let  $x$  represent the days it does not rain and  $y$  represent the gallons of water Whitney uses.



1. How many gallons will Whitney use if it did not rain for 8 days?

days of no rain	gallons of water
1	2
2	4
3	6
8	16

$$y = 2x$$

$$y = 2 \cdot 8$$

$$y = 16$$

2. How many days must it have not rained if Whitney uses 10 gallons of water?

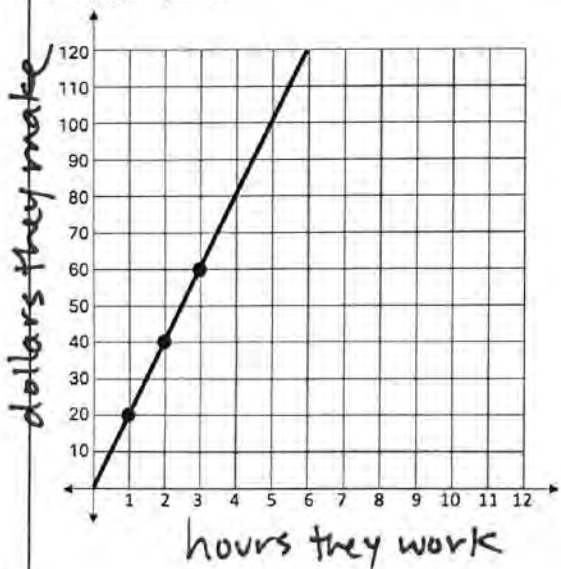
5	10
---	----

$$y = 2x$$

$$10 = \frac{2x}{2}$$

$$5 = x$$

Martin uses the graph below to determine how much to pay his employees based on how long they work. Let  $x$  represent the number of hours they work. Let  $y$  represent the number of dollars they get paid.



3. How much would Meryl need to pay the kid for 10 hours of raking?

hours	dollars
1	20
2	40
3	60
10	200

$$y = 20x$$

$$y = 20 \cdot 10$$

$$y = 200$$

4. How many hours of raking must have been done if Meryl paid \$80?

4	80
---	----

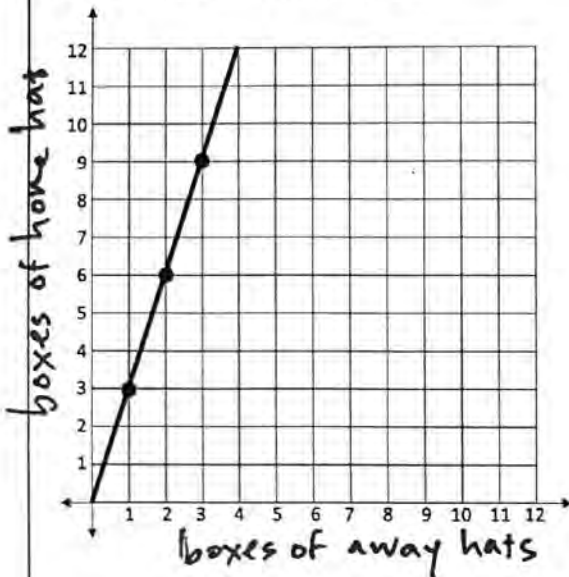
$$y = 20x$$

$$80 = \frac{20x}{20}$$

$$4 = x$$

Use the graph to answer the questions below. Show your work.

Charlie's Hat Shop uses the graph below to determine how many hats to buy for the teams in the Superbowl. Let  $x$  represent the number of boxes of away team hats. Let  $y$  represent the number of home team hats.



5. How many boxes of home team hats would Charlie buy if he planned to buy 5 boxes of away team hats?

boxes of away hats	boxes of home hats
1 x 3	3
2 x 3	6
3 x 3	9
5 x 3	15

$$y = 3x$$

$$y = 3 \cdot 5$$

$$y = 15$$

6. How many boxes of away team hats would Charline buy if he planned to buy 4 boxes of home team hats?

$$1\frac{1}{3} \times 3 = 4$$

$$y = 3x$$

$$\frac{4}{3} = \frac{3x}{3}$$

$$1\frac{1}{3} = x$$

7. How many boxes of away team hats would Charline buy if he planned to buy 100 boxes of home team hats?

$$33\frac{1}{3} \times 3 = 100$$

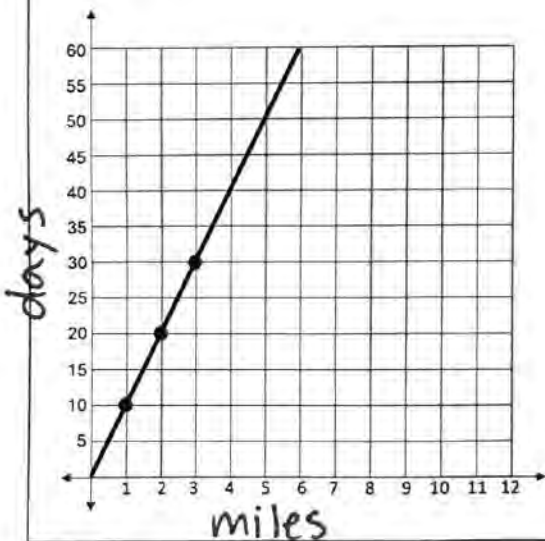
$$y = 3x$$

$$100 = \frac{3x}{3}$$

$$33\frac{1}{3} = x$$

$$\begin{array}{r} 33 \\ 3 \overline{)100} \\ \underline{-96} \\ 40 \\ \underline{-39} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

The graph below shows how many months Julia must train based on the length of her upcoming race. Let  $x$  represent the length of the race in miles. Let  $y$  represent the number of days Julia must train.



8. How long must the race be if Julia trained for 60 days?

miles	days
1 x 10	10
2 x 10	20
3 x 10	30
6 x 10	60

$$y = 10x$$

$$60 = \frac{10x}{10}$$

$$6 = x$$

9. How many days would Julia train for a 5 mile race?

$$5 \times 10 = 50$$

$$y = 10x$$

$$y = 10 \cdot 5$$

$$y = 50$$

10. How many days would Julia train for a 10 mile race?

$$10 \times 10 = 100$$

$$y = 10x$$

$$y = 10 \cdot 10$$

$$y = 100$$

# **G7 U2 Lesson 14**

Represent a proportional relationship in four different ways.

**G7 U2 Lesson 14 - Today we will use the constant of proportionality for more complicated proportion problems.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will use the constant of proportionality for more complicated proportion problems. These ones are going to involve fractions. But that's no problem. We're still going to use the same ideas we've been using for this unit.

**Let's Review (Slide 3):** We will need to multiply, divide and simplify fractions for our work today. I want to show you the correct work for each of these. First it says, "Given the equation y equals one half x, what is y when x equals 2?" I am going to recopy the equation with a 2 in place of x so it is y equals one half times 2.

$$y = \frac{1}{2} \cdot 2$$

$$y = \frac{1}{2} \cdot \frac{2}{1}$$

$$y = \frac{2}{2} \quad \begin{array}{r} \frac{1}{2} \\ 2 \cancel{\frac{2}{2}} \\ 0 \end{array}$$

$$\boxed{y = 1}$$

When I am multiplying a fraction by a whole number, I can help myself see which numbers to multiply by putting a 1 under the whole number.

Now I multiply across like any normal fractions. 1 x 2 is 2 in the numerator and 2 x 1 is 2 in the denominator. I write it as y = 2 over 2, which is the same as 1 whole. That's because it's like saying, I have 2 pieces and it takes 2 pieces to make a whole pie. Hopefully this is reminding you of what you learned in 6th grade.

Let's do the next one. I will copy it over with 2 in place of the y this time. It would be 2 equals one half x. Now I want to get x by itself so I have to do the opposite operations to both sides. The opposite operation is division but another way to do the opposite is to do what division secretly is, multiplying by the reciprocal. Hopefully you learned that in sixth grade too.

$$2 = \frac{1}{2} \cdot x$$

$$\begin{array}{r} \times \frac{2}{1} \\ \times \frac{2}{1} \end{array}$$

$$\boxed{4 = x}$$

So to get rid of 1 over 2, I am going to multiply each side by 2 over 1.

The right side cancels out and on the left side, we are back to think of 2 as 2 over 1. That gives us 2 times 2 in the numerator and 1 times 1 in the denominator. 4 over 1 is 4.

$$\frac{1\frac{2}{4}}{2} = 1\frac{1}{2}$$

$$\begin{array}{r} 4 \overline{)6} \\ -4 \\ \hline 2 \end{array}$$

Let's do the next one. It says, "how do we simplify 6 over 4?" This is called an improper fraction because the top is bigger than the bottom. The fraction sign is like a secret division symbol. So this is like 6 divided by 4. It goes in 1 time. Subtract 4 and there is 2 left. So we get 1 and 2 fourths. We can simplify this even more by dividing the top and bottom by the same number. In this case, I'll divide by 2 on the top and divide by 2 on the bottom. That's 1 and 1 half.

$$y = 1\frac{1}{3} \cdot 2$$

$$y = \frac{4}{3} \cdot \frac{2}{1}$$

Okay, two more, this time we have mixed numbers because there is a whole number and a fraction. I rewrite the problem with 2 in place of x and get y equals one and one third times 2. But I can't really multiply this yet. I have to turn the mixed number into something more manageable. I think of 1 whole as a group of 3 or 1 times 3 so really there is 3 plus 1 on top. That's 4 thirds.

$$y = \frac{8}{3}$$

Now I can multiply like normal by putting that 1 under the 2. I multiply 4 x 2 is 8 for the top and 3 x 1 is 3 for the bottom. Y equals 8 thirds.



$$\begin{array}{r} 2\frac{2}{3} \\ 3 \overline{)8} \\ \underline{-6} \\ 2 \end{array}$$

Or I can divide this and then I will get 2 and 2 thirds.

$$2 = \frac{1}{3}x$$

Last one, I copy the equation with y equals 2. So 2 equals one and one third x. I can't do anything until I change this for a mixed number. That would be 2 equals 4 thirds x.

$$2 = \frac{4}{3}x$$

Now just like last time, I wanted to get rid of the fraction, I multiply by the reciprocal. I write times 3 fourths on this side and times 3 fourths on this side.

$$\frac{6}{4} = x$$

To multiply 3 fourths times 2, I put a 1 under the 2 and now I have 3 times 2 is 6 on top and 4 times 1 is 4 on the bottom. I get 6 fourths.

$$\boxed{1\frac{1}{2} = x}$$

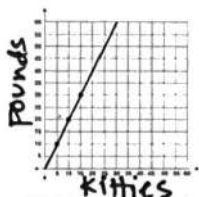
$$1\frac{2}{4} = 1\frac{1}{2}$$

$$\begin{array}{r} 1\frac{2}{4} \\ 4 \overline{)6} \\ \underline{-4} \\ 2 \end{array}$$

Or I can divide this and then I will get 1 and 2 fourths.

This is actually the hardest part of our work. The rest are ideas that you've already learned. This stuff is also review but it might be a little rusty because it has been a while. That's okay. I can remind you if you get stuck.

**Let's Review (Slide 4):** One other thing we can review is that we use the constant of proportionality to



find other pairs on a table or graph. This is a problem from your practice page in our last lesson with a different question. Let's review. Read silently with your eyes while I read out loud. "Kathy's Kitty Shop uses the graph below to determine how many pounds of kitty litter they will need weekly based on the number of kitties available for adoption. Let x represent the number of kitties. Let y represent the number of pounds of kitty litter. How many pounds of kitty litter would we buy for 4 kitties?" We always make a table for this kind of problem. I am going to write x and y with kitties for x and pounds for y.

Kitties	pounds
5	10
10	20
15	30

I see 5 for kitties and 10 for pounds. I see 10 for kitties and 20 for pounds. I see 15 for kitties and 30 for pounds.

Kitties	pounds
5 × 2	10
10 × 2	20
15 × 2	30

Now here's the thing, for this problem and the other problems we've been doing up to now, the operation really jumps out at you. It is kind of obvious that it is "times 2" because 5 times 2 is 10 and 10 times 2 is 20 and so on. But if I didn't know what number to do, that's okay. I can divide to find the constant of proportionality. Even though we know it's true, let me know you because you're going to need it when the numbers get harder.

$$\begin{array}{r} 02 \\ 5 \overline{)10} \\ \underline{-10} \\ 00 \end{array}$$

$$\begin{array}{r} 02 \\ 10 \overline{)20} \\ \underline{-20} \\ 00 \end{array}$$

I will do y divided by x so 10 divided by 5. That's 2. That's what we said it would be! Let's do another. 20 divided by 10. That's 2.

Kitties	pounds
5 × 2	10
10 × 2	20
15 × 2	30
4 × 2	8

So when I put 4 kitties on the graph, I am going to use that same constant of proportionality. 4 times 2 is 8.

$$y = 2x$$

$$y = 2 \cdot 4$$

$$y = 8$$

Or if I need to, I can think of it like an equation,  $y = 2x$ . Then I plug in 4 to get  $y = 2$  times 4 and  $y$  equals 8.

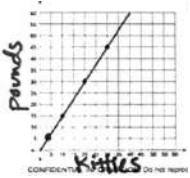
This is maybe the only brand new idea for today, and it is something I want you to pay attention to. When we find the constant of proportionality, it is actually another point on the graph. The constant of proportionality is  $y$  when  $x$  is 1. That's because

The constant of proportionality is  $y$  when  $x$  is 1. we divided  $y$  by the number of  $x$  and now we just have 1 group of  $x$ .

Kitties	pounds
5 × 2	10
10 × 2	20
15 × 2	30
4 × 2	8
1 × 2	2

In this case, we can put another line on our table.  $x$  is 1 and  $y$  is the constant of proportionality, 2.

**Let's Talk (Slide 5):** Now we're going to explore this same idea with fractional answers. This says, "When the constant of proportionality is not obvious, we have to divide to find it." This is what we just reviewed on the last slide. Sometimes the operation on the graph isn't going to be obvious and we'll have to do some number crunching to figure it out. Here's an example. Read along with my silently while I read out loud. "Let's imagine the numbers were a little different... Kathy's Kitty Shop decides to this new graph. Let  $x$  represent the number of kitties. Let  $y$  represent the number of pounds of kitty litter. How many pounds of kitty litter would we buy for 4 kitties?" Let's label our axes again.



Kitties	pounds
10	15
20	30
30	45

We know we need to make a table. I see 10 on  $x$  and 15 on  $y$ . Then I see 20 on  $x$  and 30 on  $y$ . Then I see 30 on  $x$  and 45 on  $y$ . By the way, I can also put  $(0,0)$  on here. But there isn't an obvious operation for each row here. I can't think of 10 times what to make 15 or 20 times what to make 30. So it is going to be really hard to figure out 4 kitties.

$$0 \frac{15}{10} = 1 \frac{1}{2}$$

$$\begin{array}{r} 10 \overline{)15} \\ -10 \\ \hline 5 \end{array}$$

I am going to divide to find the constant of proportionality. 15 divided by 10 is 1 then subtract 10 and have 5 left over. So I get 1 and 5 tenths. I am going to divide the top by 5 and the bottom by 5. This is really 1 and 1 half.

The constant of proportionality is  $y$  when  $x$  is 1. Now remember, the constant of proportionality is  $y$  when  $x$  is 1.

Kitties	pounds
10 × 1½	15
20 × 1½	30
30 × 1½	45
1 × 1½	1½

So I can put this right here on my table and I can fill in the circles for each row now.

Kitties	Pounds
10	15
20	30
30	45
1	$1\frac{1}{2}$
4	$6$

And now I can put 4 on my table and see that I have to multiply 4 times 1 and 1 half.

$$y = \frac{1}{2}x$$

$$y = 1\frac{1}{2} \cdot 4$$

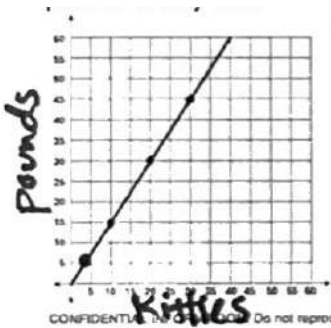
$$y = \frac{3}{2} \cdot 4$$

$$y = \frac{12}{2} = 6$$

I also can make an equation if I want to. It would be y equals 1 and 1 half times x. So when I plug in 4, I see I have to do 1 and 1 half times 4.

Let's remember what we said at the beginning about multiplying by a mixed number. We have to change 1 and 1 half into 3 halves. That's because we have 1 whole which is 1 group of 2 halves plus 1 half. That's 3 halves. So this is really y equals 3 halves times 4. Now I can do the math with a 1 under the 4. 3 times 4 is 12. 1 times 1 is 2. I get 12 over 2 which is like 12 divided by 2, which is 6.

Kitties	Pounds
10	15
20	30
30	45
1	$1\frac{1}{2}$
4	$6$

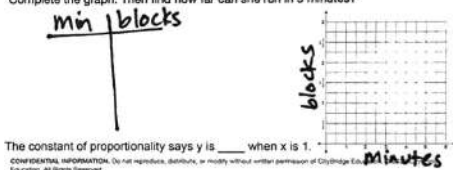


Let's put that on the table and graph.

**Let's Think (Slide 6):**

If the graph isn't provided for us, we can use the origin to make a graph from a story. Read along silently with your eyes while I read out loud.

Jamie ran 3 blocks in 6 minutes at a constant rate. Draw a graph of the relationship between the time she takes in minutes, x, and the distance she runs in blocks, y. Complete the graph. Then find how far can she run in 5 minutes?



The constant of proportionality says y is \_\_\_\_\_ when x is 1.

“Jamie ran 3 blocks in 6 minutes at a constant rate. Draw a graph of the relationship between the time she takes in minutes, x, and the distance she runs in blocks, y. Complete the graph. Then how far can she run in 5 minutes?” Let's start with the meaning of x and y. I can put minutes on the graph for x and blocks on the graph for y. On my table, it's the same minutes for x and blocks for y.

min	blocks
0	0

Now since this is a “constant rate,” I know it's a proportion so I can put 0 and 0 on the table for the origin and I can make that a point on my graph too.

min	blocks
0	0
6	3

The story says 3 blocks in 6 minutes. So I am going to put 6 for x and 3 for y, and I can make that point on my graph too.

$$\begin{array}{r} 0\frac{3}{6} = \frac{1}{2} \\ 6 \overline{) 3} \\ \underline{-0} \\ 3 \end{array}$$

Now, remember that we have to divide to find the constant of proportionality. That's 3 divided by 6. 6 doesn't go into 3. So we get 0 with a remainder of 3. That's 3 sixths. I can simplify that to 1 half.

Remember, the constant of proportionality is y when x is 1.

The constant of proportionality is y when x is 1.

min	blocks
0	0
6	3
1	$\frac{1}{2}$

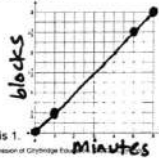
So we can put that on our table. It's 1 and 1 half. I can graph that too.

And now I know that the equation for this table is  $y = 1\frac{1}{2}x$ . I am finally ready to answer the question. Notice how much work I had to do before jumping into solving the problem. There was this secret hidden constant of proportionality that I had to find before I even dealt with the 5 minutes. Okay, so 5 minutes goes on the graph under  $x$ . Now I can plug it into the equation. I get  $y = 1\frac{1}{2} \times 5$ . I put a 1 under the 5. I get  $y = \frac{5}{2}$ . I do some division on the side. 2 goes into 5 two times. I subtract 4 and have 1 left. So it is 1 and 1 half.

min	blocks
0	0
6	3
1	$\frac{1}{2}$
5	$2\frac{1}{2}$

Jamie ran 3 blocks in 6 minutes at a constant rate. Draw a graph of the relationship between the time she takes in minutes,  $x$ , and the distance she runs in blocks,  $y$ . Complete the graph. Then find how far can she run in 5 minutes?

min	blocks
0	0
6	3
1	$\frac{1}{2}$
5	$2\frac{1}{2}$



Now I can put that on the table and I can graph that too.

The constant of proportionality says  $y$  is \_\_\_\_\_ when  $x$  is 1.

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**Let's Try It (Slide 7):** Now let's try another one of these together. I will lead you through step by step.

# WARM WELCOME



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**Today we will use the constant of proportionality for more complicated proportion problems.**

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## Let's Review:

We will need to multiply, divide and simplify fractions for our work today.

Given the equation  $y = \frac{1}{2}x$ , what is  $y$  when  $x = 2$ ?

Given the equation  $y = \frac{1}{2}x$ , what is  $x$  when  $y = 2$ ?

How do we simplify  $\frac{6}{4}$ ?

Given the equation  $y = 1\frac{1}{3}x$ , what is  $y$  when  $x = 2$ ?

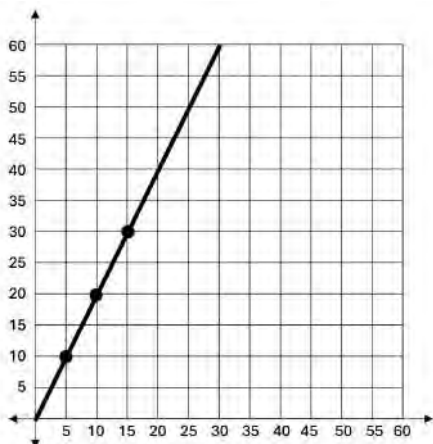
Given the equation  $y = 1\frac{1}{3}x$ , what is  $x$  when  $y = 2$ ?

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## Let's Review:

We use the constant of proportionality to find other pairs on a table or graph.

Kathy's Kitty Shop uses the graph below to determine how many pounds of kitty litter they will need weekly based on the number of kitties available for adoption. Let  $x$  represent the number of kitties. Let  $y$  represent the number of pounds of kitty litter.



How many pounds of kitty litter would we buy for 4 kitties?

The constant of proportionality is  $y$  when  $x$  is \_\_\_\_\_.

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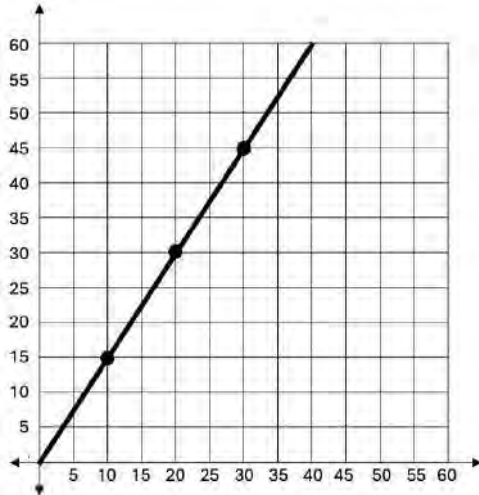




## Let's Talk:

**When the constant of proportionality is not obvious, we have to divide to find it.**

Let's imagine the numbers were a little different.... Kathy's Kitty Shop decides to this new graph. Let  $x$  represent the number of kitties. Let  $y$  represent the number of pounds of kitty litter.



How many pounds of kitty litter would we buy for 4 kitties?

The constant of proportionality is  $y$  when  $x$  is \_\_\_\_.

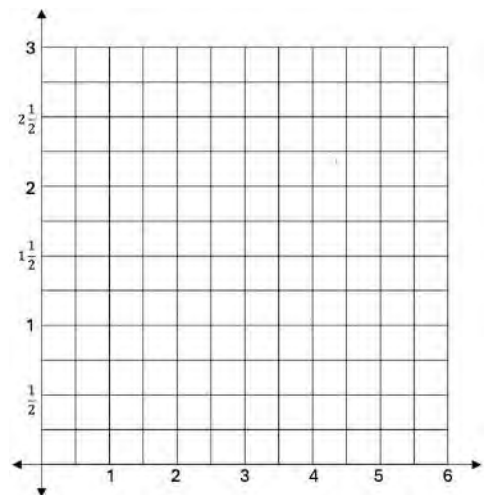
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## Let's Think:

**We can use the origin to make a graph from a story.**

Jamie ran 3 blocks in 6 minutes at a constant rate. Draw a graph of the relationship between the time she takes in minutes,  $x$ , and the distance she runs in blocks,  $y$ . Complete the graph. Then how far can she run in 5 minutes?



The constant of proportionality is  $y$  when  $x$  is \_\_\_\_.

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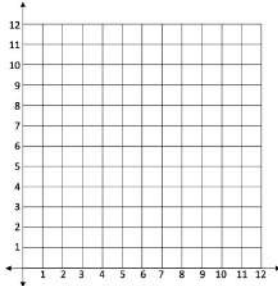


# Let's Try It:

## We will do it together step by step!

Name: \_\_\_\_\_ G7 U2 Lesson 14 - Let's Try It

To make her gumbo, Jonel uses 1 stick of butter for 3 cups of chopped vegetables. Let  $x$  represent the cups of chopped vegetables. Let  $y$  represent the sticks of butter. Make a graph.



1. Label the axes of the graphs with the correct words.
2. Put the information from the story on a table with words.
3. Make a row for when  $x$  is 0 on your table.
4. Graph the two rows you have so far.
4. Find the constant of proportionality. \_\_\_\_\_

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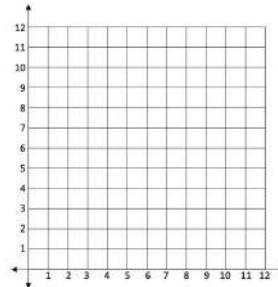


# On your Own:

## Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 14 - Independent Work

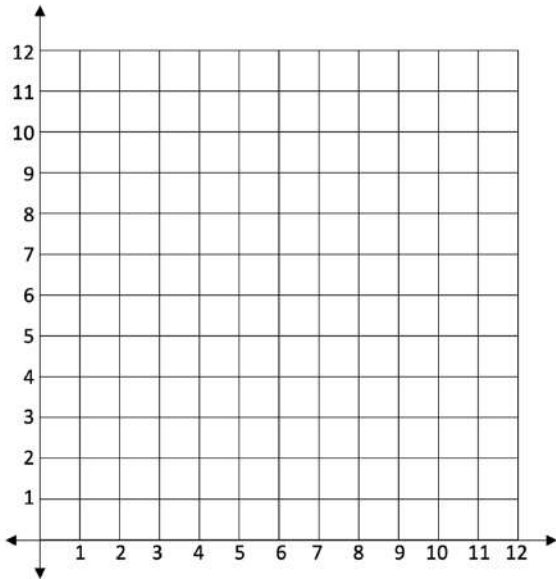
Remember: Proportions go through the origin and the constant of proportionality is  $y$  when  $x$  is 1.  
Make a graph for the story and answer the questions using the constant of proportionality.

<p>Dan's mom says he has to eat 2 apples before he can eat 1 cookie. Make a graph where <math>x</math> represents the number of apples and <math>y</math> represents the number of cookies Dan can eat.</p>	<p>1. How many apples does Dan need to eat to have 3 cookies?</p>
	<p>2. How many cookies can Dan eat if he has 3 apples?</p>
<p>Lisa's sink is dripping at the constant rate of 6 mL every 4 minutes. Let <math>x</math> represent the number of minutes. Let <math>y</math> represent the number of mL.</p>	<p>3. How many minutes will cause a leak of 8 mL?</p>

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Name: \_\_\_\_\_

To make her gumbo, Jonel uses 1 stick of butter for 3 cups of chopped vegetables. Let  $x$  represent the cups of chopped vegetables. Let  $y$  represent the sticks of butter. Make a graph.



1. Label the axes of the graphs with the correct words.
2. Put the information from the story on a table with words.

3. Make a row for when  $x$  is 0 on your table.

4. Graph the two rows you have so far.

4. Find the constant of proportionality. \_\_\_\_\_

5. The constant of proportionality is  $y$  when  $x$  equals \_\_\_\_\_. Put the constant of proportionality on the table.

6. Write an equation with your constant of proportionality. \_\_\_\_\_

**How many sticks of butter will Jonel need for 5 cups of vegetables?**

7. Is the number given in the question represented by  $x$  or  $y$ ? \_\_\_\_\_ Put it on your table.

8. Solve for the other variable.

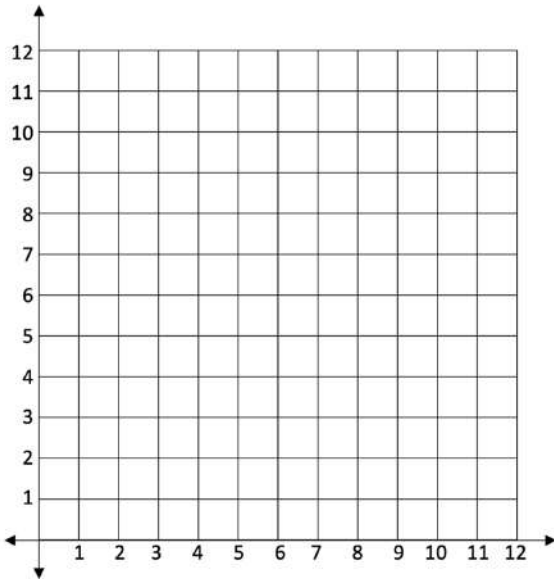
9. Write your answer in a complete sentence using words from the story.

Name: \_\_\_\_\_

Remember: Proportions go through the origin and the constant of proportionality is  $y$  when  $x$  is 1.

Make a graph for the story and answer the questions using the constant of proportionality.

Dan's mom says he has to eat 2 apples before he can eat 1 cookie. Make a graph where  $x$  represents the number of apples and  $y$  represents the number of cookies Dan can eat.



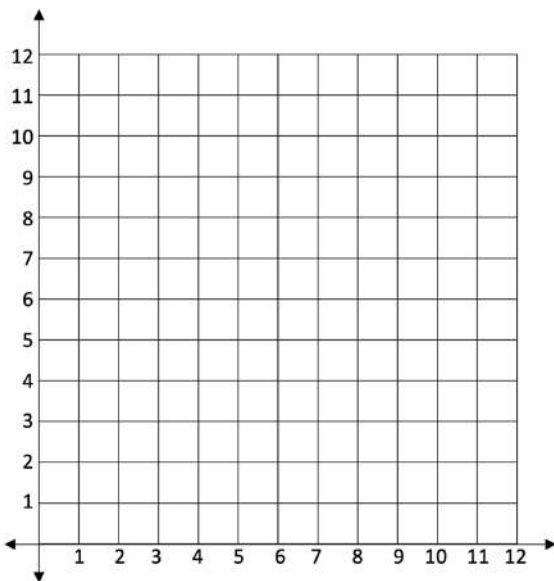
Make a table:

Make an equation:

1. How many apples does Dan need to eat to have 3 cookies?

2. How many cookies can Dan eat if he has 3 apples?

Lisa's sink is dripping at the constant rate of 6 mL every 4 minutes. Let  $x$  represent the number of minutes. Let  $y$  represent the number of mL.



Make a table:

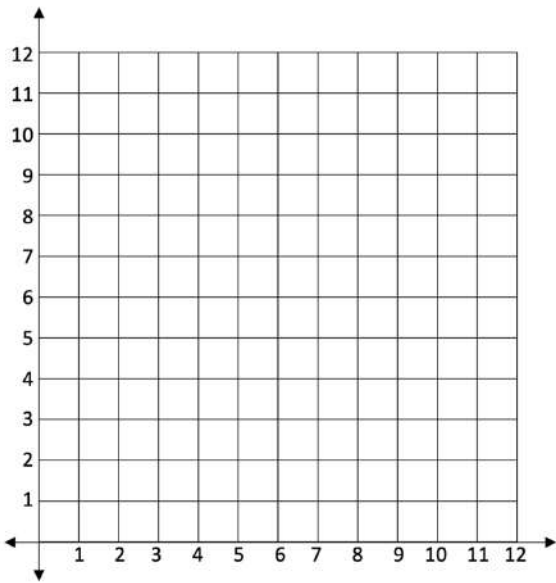
Make an equation:

3. How many minutes will cause a leak of 8 mL?

4. How much water will drip over 5 minutes?

Make a graph for the story and answer the questions using the constant of proportionality.

Dan uses 1 jar of sauce for every 2 pounds of pasta that he cooks. Let  $x$  represent the number of pounds of pasta. Let  $y$  represent the number of jars of sauce.



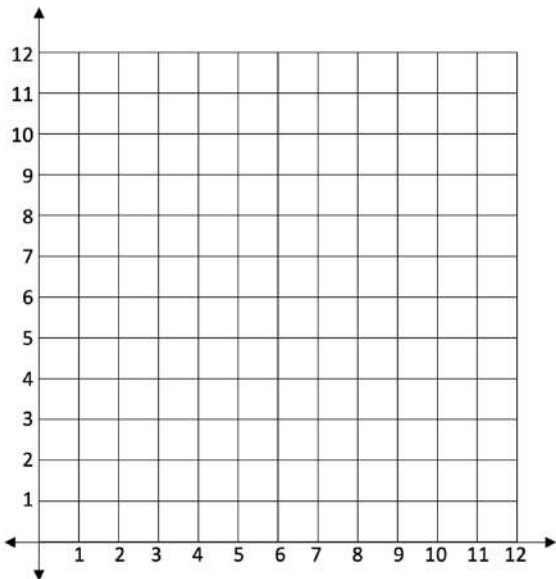
Make a table:

Make an equation:

5. How many jars of sauce will Dan use for 5 pounds of pasta?

6. How many pounds of pasta would Dan use for 3 jars of sauce?

Lisa spends 5 hours to edit 4 pages of her writing. Let  $x$  represent the number of pages. Let  $y$  represent the number of hours Lisa spends editing.



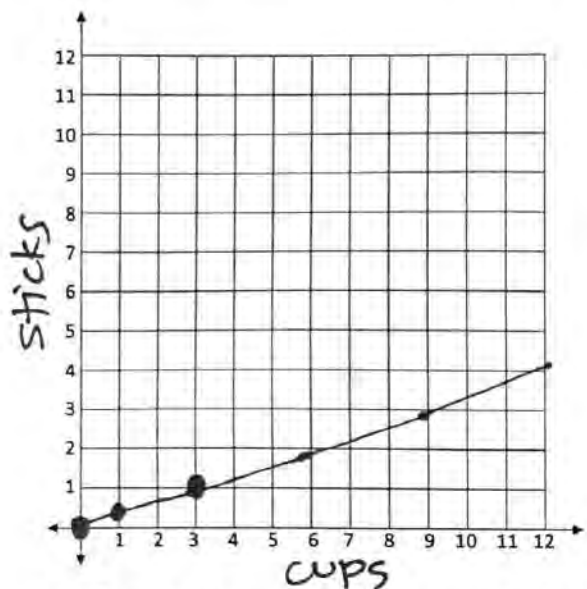
Make a table:

Make an equation:

7. How long will Lisa spend editing 10 pages?

8. How many pages can Lisa edit in 4 hours?

To make her gumbo, Jonel uses 1 stick of butter for 3 cups of chopped vegetables. Let  $x$  represent the cups of chopped vegetables. Let  $y$  represent the sticks of butter. Make a graph.



1. Label the axes of the graphs with the correct words.
2. Put the information from the story on a table with words.

cups	sticks
$0 \times \frac{1}{3}$	0
$3 \times \frac{1}{3}$	1
$1 \times \frac{1}{3}$	$\frac{1}{3}$

3. Make a row for when  $x$  is 0 on your table.

4. Graph the two rows you have so far.

4. Find the constant of proportionality.  $\frac{1}{3}$

$$\begin{array}{r} 0\bar{3} \\ 3 \overline{)1} \\ \underline{-0} \\ 1 \end{array}$$

5. The constant of proportionality is  $y$  when  $x$  equals 1. Put the constant of proportionality on the table.

6. Write an equation with your constant of proportionality.  $y = \frac{1}{3}x$

**How many sticks of butter will Jonel need for 5 cups of vegetables?**

7. Is the number given in the question represented by  $x$  or  $y$ ? X Put it on your table.

8. Solve for the other variable.

$$\begin{array}{r} 1\frac{2}{3} \\ 3 \overline{)5} \\ \underline{-3} \\ 2 \end{array}$$

$$\begin{aligned} y &= \frac{1}{3}x \\ y &= \frac{1}{3} \cdot 5 \\ y &= \frac{5}{3} = 1\frac{2}{3} \end{aligned}$$

9. Write your answer in a complete sentence using words from the story.

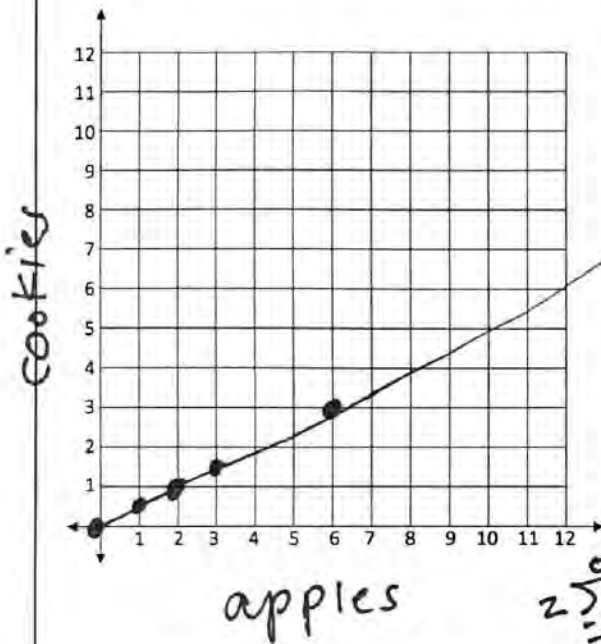
If Jonel cooks 5 cups of vegetables, she will need  $1\frac{2}{3}$  sticks of butter.



Remember: Proportions go through the origin and the constant of proportionality is y when x is 1.

Make a graph for the story and answer the questions using the constant of proportionality.

Dan's mom says he has to eat 2 apples before he can eat 1 cookie. Make a graph where x represents the number of apples and y represents the number of cookies Dan can eat.



Make a table:

apples		cookies
0		0
2	$\times \frac{1}{2}$	1
1	$\times \frac{1}{2}$	$\frac{1}{2}$
6	$\times \frac{1}{2}$	3
3	$\times \frac{1}{2}$	$1\frac{1}{2}$

Make an equation:

$$y = \frac{1}{2}x$$

$$\begin{array}{r} 0\frac{1}{2} \\ 2 \overline{)1} \\ \underline{-0} \\ 1 \end{array}$$

1. How many apples does Dan need to eat to have 3 cookies?

$$y = \frac{1}{2}x$$

$$3 = \frac{1}{2}x$$

$$\frac{2}{1} \cdot \frac{2}{1}$$

$$\boxed{\frac{6}{1} = x}$$

2. How many cookies can Dan eat if he has 3 apples?

$$y = \frac{1}{2}x$$

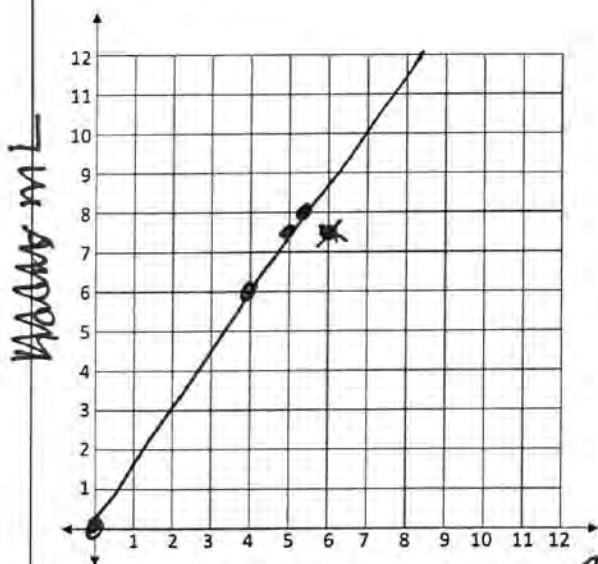
$$y = \frac{1}{2} \cdot 3$$

$$y = \frac{3}{2}$$

$$\boxed{y = 1\frac{1}{2}}$$

$$\begin{array}{r} 1\frac{1}{2} \\ 2 \overline{)3} \\ \underline{-2} \\ 1 \end{array}$$

Lisa's sink is dripping at the constant rate of 6 mL every 4 minutes. Let x represent the number of minutes. Let y represent the number of mL.



Make a table:

min		mL
4	$\times \frac{1}{2}$	6
0	$\times \frac{1}{2}$	0
$5\frac{1}{3}$	$\times \frac{1}{2}$	8
5	$\times \frac{1}{2}$	$7\frac{1}{2}$

Make an equation:

$$y = \frac{1}{2}x$$

$$\begin{array}{r} 1\frac{1}{2} \\ 4 \overline{)6} \\ \underline{-4} \\ 2 \end{array}$$

3. How many minutes will cause a leak of 8 mL?

$$y = \frac{1}{2}x$$

$$8 = \frac{3}{2}x$$

$$\frac{2}{3} \cdot \frac{2}{3}$$

$$\frac{16}{3} = x$$

$$\boxed{5\frac{1}{3} = x}$$

$$\begin{array}{r} 5\frac{1}{3} \\ 3 \overline{)16} \\ \underline{-15} \\ 1 \end{array}$$

4. How much water will drip over 5 minutes?

$$y = \frac{1}{2}x$$

$$y = \frac{1}{2} \cdot 5$$

$$y = \frac{5}{2}$$

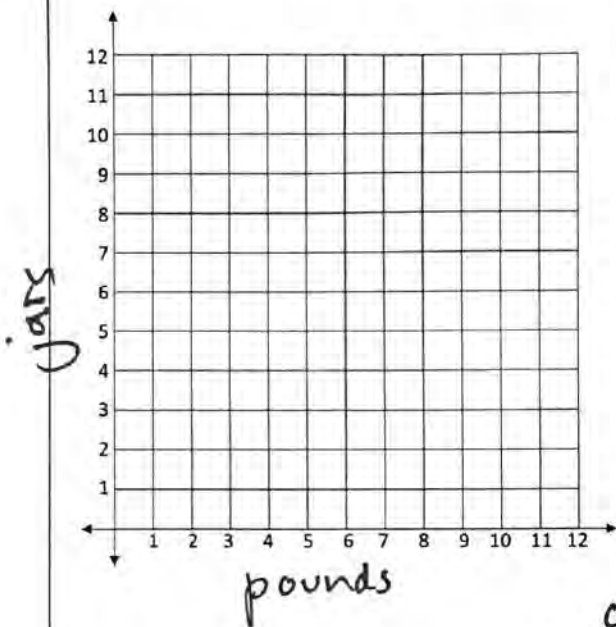
$$y = 2\frac{1}{2}$$

$$\boxed{y = 2\frac{1}{2}}$$

$$\begin{array}{r} 2\frac{1}{2} \\ 2 \overline{)5} \\ \underline{-4} \\ 1 \end{array}$$

Make a graph for the story and answer the questions using the constant of proportionality.

Dan uses 1 jar of sauce for every 2 pounds of pasta that he cooks. Let  $x$  represent the number of pounds of pasta. Let  $y$  represent the number of jars of sauce.



Make a table:

pounds	jars
$0 \times \frac{1}{2}$	0
$2 \times \frac{1}{2}$	1
$5 \times \frac{1}{2}$	$2\frac{1}{2}$
$6 \times \frac{1}{2}$	3

Make an equation:

$$y = \frac{1}{2}x$$

5. How many jars of sauce will Dan use for 5 pounds of pasta?

$$y = \frac{1}{2}x$$

$$y = \frac{1}{2} \cdot \frac{5}{1}$$

$$y = \frac{5}{2}$$

$$y = 2\frac{1}{2}$$

6. How many pounds of pasta would Dan use for 3 jars of sauce?

$$y = \frac{1}{2}x$$

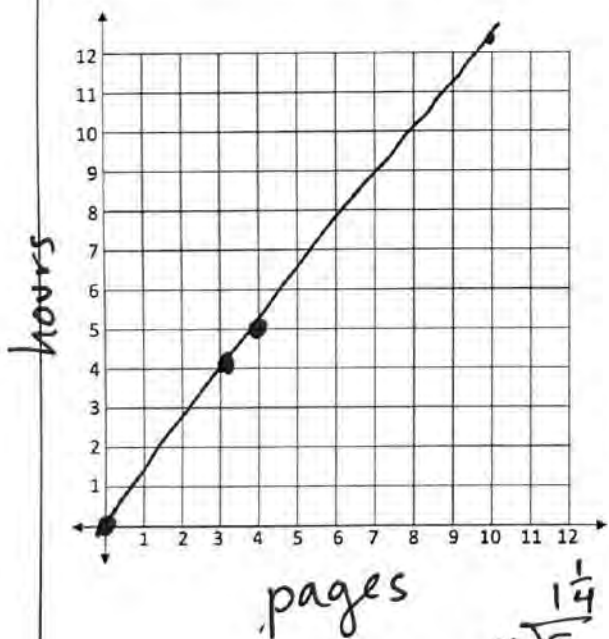
$$3 = \frac{1}{2}x$$

$$\cdot \frac{2}{1} \quad \cdot \frac{2}{1}$$

$$\frac{6}{1} = x$$

$$6 = x$$

Lisa spends 5 hours to edit 4 pages of her writing. Let  $x$  represent the number of pages. Let  $y$  represent the number of hours Lisa spends editing.



Make a table:

pages	hours
$4 \times \frac{1}{4}$	5
$0 \times \frac{1}{4}$	0
$10 \times \frac{1}{4}$	$12\frac{1}{2}$
$3\frac{1}{2} \times \frac{1}{4}$	4

Make an equation:

$$y = \frac{1}{4}x$$

7. How long will Lisa spend editing 10 pages?

$$y = \frac{1}{4}x$$

$$y = \frac{5}{4} \cdot \frac{10}{1}$$

$$y = \frac{50}{4}$$

$$y = 12\frac{2}{4} = 12\frac{1}{2}$$

8. How many pages can Lisa edit in 4 hours?

$$y = \frac{1}{4}x$$

$$4 = \frac{5}{4}x$$

$$\cdot \frac{4}{5} \quad \cdot \frac{4}{5}$$

$$\frac{16}{5} = x$$

$$3\frac{1}{5} = x$$

# **G7 U2 Lesson 15**

Interpret and compare the same proportional relationship using two different sets of tables, graphs, and equations.

**G7 U2 Lesson 15 - Today we will find and compare rates.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will find and compare rates. We are going to make sure we use the correct language to describe the rate and then we just put together all the things we've learned in the rest of the unit. Let's go!

**Let's Review (Slide 3):** We know the constant of proportionality is the unit rate for each proportion on a graph, table or equation. We learned that at the very beginning of this unit when we were learning what proportions even were. This says, "Jeff the chef made a graph to show how quickly he can chop onions. Steph the chef uses the equation,  $y = 2x$ , to show her rate. What is each chef's rate?" Let's start by finding the rate or constant of proportionality for Jeff. You already know how to do this. We make a table to start. X is minutes and y is onions. I will fill it in with the numbers. The first point is 2 minutes and 1 onion. The next point is 4 minutes and 2 onions. The next point is 6 minutes and 3 onions.

minutes	onions
2	1
4	2
6	3

Jeff the chef made a graph to show how quickly he can chop onions. Steph the chef uses the equation,  $y = 2x$ , to show her rate where x is the number of minutes and y is the number of onions. What is each chef's rate?

x	y
2	1
4	2
6	3

Handwritten calculations:  $\frac{0.5}{1} = \frac{1}{2}$  and  $\frac{0.5}{2} = \frac{1}{4}$

To find the constant of proportionality, we divide each row, y divided by x. 1 divided by 2 is really 0.5. But then I take that remainder and turn it into a fraction. It's one half.

Jeff the chef made a graph to show how quickly he can chop onions. Steph the chef uses the equation,  $y = 2x$ , to show her rate where x is the number of minutes and y is the number of onions. What is each chef's rate?

x	y
2	1
4	2
6	3

Handwritten calculations:  $\frac{0.5}{1} = \frac{1}{2}$  and  $\frac{0.5}{2} = \frac{1}{4}$

**$\frac{1}{2}$  onion per min**

All these rows should be the same. That's why it is called a CONSTANT. But let's just check the next row. 2 divided by 4 is also zero. The remainder becomes two fourths. I can simplify that fraction by dividing the top and bottom by two and I see it is equivalent to one half. Now, this is the big idea of today's lesson - the words that go with this rate are really important so that we know what we're talking about. Y divided by x means onions divided by minutes so this means Jeff can chop half an onion per minute.

Jeff the chef made a graph to show how quickly he can chop onions. Steph the chef uses the equation,  $y = 2x$ , to show her rate where x is the number of minutes and y is the number of onions. What is each chef's rate?

x	y
2	1
4	2
6	3

Handwritten calculations:  $\frac{0.5}{1} = \frac{1}{2}$  and  $\frac{0.5}{2} = \frac{1}{4}$

**$\frac{1}{2}$  onion per min**  
**2 onions per min**

Let's do Steph's rate. She uses the equation  $y = 2x$ . This is actually soooo easy because the constant of proportionality is right there in the problem. It is 2. But again, the words matter. Y is onions divided by x, which is minutes. So she can chop 2 onions per minute. It is helpful to know the two people's rate because now we can figure out who is faster. Jeff can chop half an onion every minute. Steph can chop two onions every minute. Steph is faster.

Draw a new graph and table if the meaning of x and y were switched.

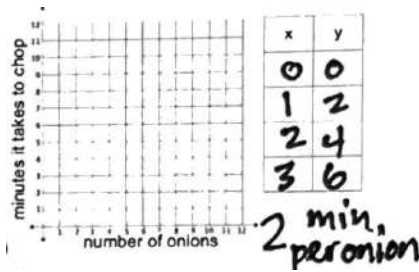
x	y
0	0
2	1
4	2
6	3

Handwritten label:  **$\frac{1}{2}$  onion per min**

**Let's Talk (Slide 4):** If we switch the meaning of the x and y axis, we switch the meaning of the constant of proportionality or unit rate. So we have to be super duper careful that we pay attention to what is y and what is x because the rate is y divided by x or y per x. Let me show you what I mean. We just used this graph on the previous slide so the table is already filled in. We found the unit rate

was half an onion per minute.

Now it says, "Draw a new graph and table if the meaning of x and y were switched. And you can see on the second graph that now x is the number of onions and y is the number of minutes. I am just going to flip the order of the numbers in each row. Let's find the new unit rate. I know I divide the row. This time it's 2 divided by 1, which is 2. The next row is 4 divided by 2, which is 2. So the unit rate or the constant of proportionality here is 2. Our final question asks, "What is the new meaning of the unit rate?" It's not 2 onions per minute. That was when we divided onions by minutes on the last graph. In this graph we divided minutes by onions. So it is 2 minutes per onion.



You can hear how we get different pieces of information. The first graph tells us how many onions if we were to set a timer for 1 minute - like a race against the clock. The second graph tells us how many minutes for 1 onion. So I can plan how long 1 onion will take. This brings up back to the main idea for today: the words we use for a rate are very important. But now we see it is not just the words but the order of the words. Onions per minute is not the same as minutes per onion. We're going to have to be very careful moving forward.

**Let's Think (Slide 5):** We can compare rates if they are set up with the same units. So now that we understand that means the words AND the order of the words. Let's solve this problem together. Read along with me while I read the problem out loud. *Read the word problem.* Let's start by finding the rate for the table. We know we can divide each row. We will divide 16 by 2 and get 8. We will divide \$32 by 4 and get 8. You see what's happening here.

Miles has two options for paying for Roblox Premium. The table below shows the cost if he pays month by month. His other option is to pay \$84 for the year. Which option has the cheaper rate?

Months	Cost in dollars
2	\$16
4	\$32
6	\$48
8	\$64

$$\begin{array}{r} 08 \\ 2 \overline{)16} \\ \underline{-16} \\ 00 \end{array} \quad \begin{array}{r} 08 \\ 4 \overline{)32} \\ \underline{-32} \\ 00 \end{array}$$

Miles has two options for paying for Roblox Premium. The table below shows the cost if he pays month by month. His other option is to pay \$84 for the year. Which option has the cheaper rate?

Months	Cost in dollars
2	\$16
4	\$32
6	\$48
8	\$64

$$\begin{array}{r} 08 \\ 2 \overline{)16} \\ \underline{-16} \\ 00 \end{array} \quad \begin{array}{r} 08 \\ 4 \overline{)32} \\ \underline{-32} \\ 00 \end{array}$$

8 dollars per month

The rate is 8. But 8 what?!?!? It's 8 dollars per month because y was dollars and x was months and we divided y by x.

Miles has two options for paying for Roblox Premium. The table below shows the cost if he pays month by month. His other option is to pay \$84 for the year. Which option has the cheaper rate?

Months	Cost in dollars
2	\$16
4	\$32
6	\$48
8	\$64

$$\begin{array}{r} 08 \\ 2 \overline{)16} \\ \underline{-16} \\ 00 \end{array} \quad \begin{array}{r} 08 \\ 4 \overline{)32} \\ \underline{-32} \\ 00 \end{array}$$

8 dollars per month

84 dollars for 12 months

Now let's figure out the other option. It says that "his other option is to pay \$84 for the year." If our first rate is dollars per month, it is super helpful to have another rate in dollars per year. We need this one to be in dollars per month too. I am going to translate this to \$84 for 12 months because that's how many months are in a year.

84 dollars for 12 months

$$\begin{array}{r} 07 \\ 12 \overline{)84} \\ \underline{-84} \\ 00 \end{array} \quad \begin{array}{r} 12 \\ +12 \\ \hline 24 \\ +12 \\ \hline 36 \end{array} \quad \begin{array}{r} 36 \\ +12 \\ \hline 48 \\ +12 \\ \hline 60 \end{array} \quad \begin{array}{r} 60 \\ +12 \\ \hline 72 \\ +12 \\ \hline 84 \end{array}$$

Now I can find the unit rate with division. 84 divided by 12. 12 doesn't go into 8. If I don't know how many times it goes into 84, I can add it up on the side of my paper. 12 plus 12 is 24. 24 plus 12 is 36. 36 plus 12 is 48. 48 plus 12 is 60. 60 plus 12 is 72. 72 plus 12 is 84. There we go! I count those up and I have 7 twelves.



84 dollars for 12 months

$$\begin{array}{r} 07 \\ 12 \overline{)84} \\ \underline{-84} \\ 00 \end{array} \quad \begin{array}{r} 12 \\ +12 \\ \hline 24 \\ +12 \\ \hline 36 \end{array} \quad \begin{array}{r} 36 \\ +12 \\ \hline 48 \\ +12 \\ \hline 60 \end{array} \quad \begin{array}{r} 60 \\ +12 \\ \hline 72 \\ +12 \\ \hline 84 \end{array}$$

7 dollars per month

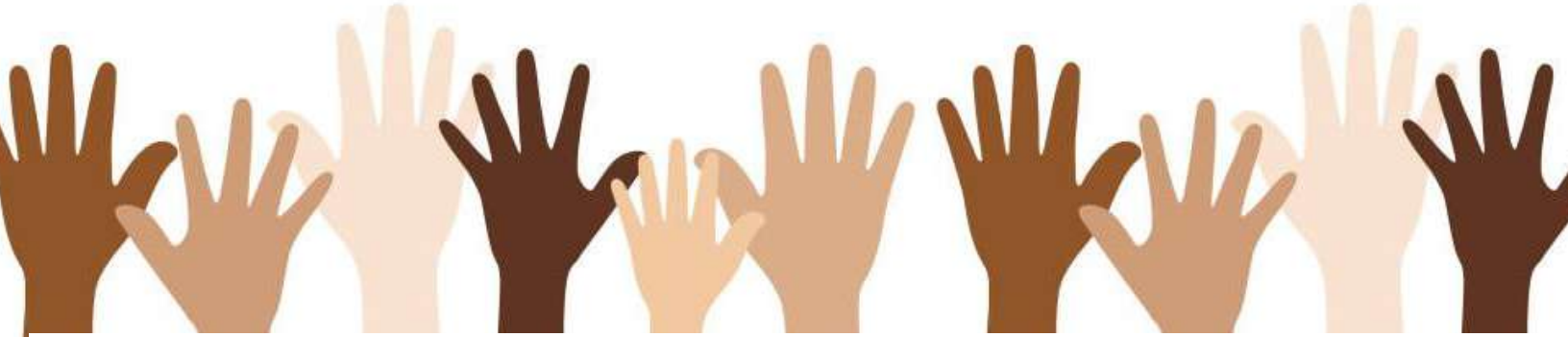
The rate if Miles pays for the whole year is 7. But we need words! We divided dollars by months so it's 7 dollars per month.

We can compare these because BOTH of them are now written in dollars per month. We see that paying for the whole year is cheaper than paying month by month like on the table. That is often the case, by the way. Companies will often give a discount when people commit to paying for a longer amount of time.

**Let's Try It (Slide 6):** Now we will work through some problems together. I will take you through step by step.



# WARM WELCOME



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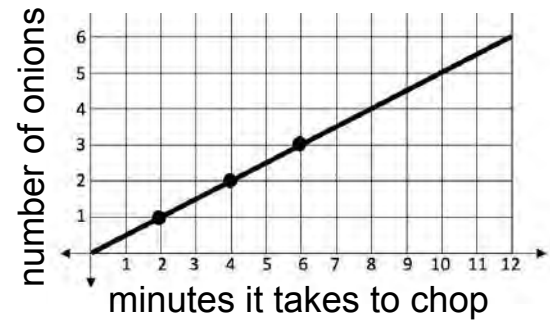
## Today we will find and compare rates.

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## Let's Review:

We know the constant of proportionality is the unit rate for each proportion on a graph, table or equation.

Jeff the chef made a graph to show how quickly he can chop onions. Steph the chef uses the equation,  $y = 2x$ , to show her rate where  $x$  is the number of minutes and  $y$  is the number of onions. What is each chef's rate?

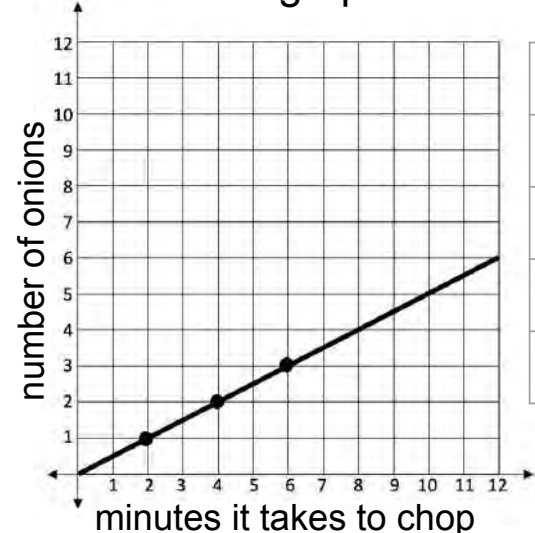


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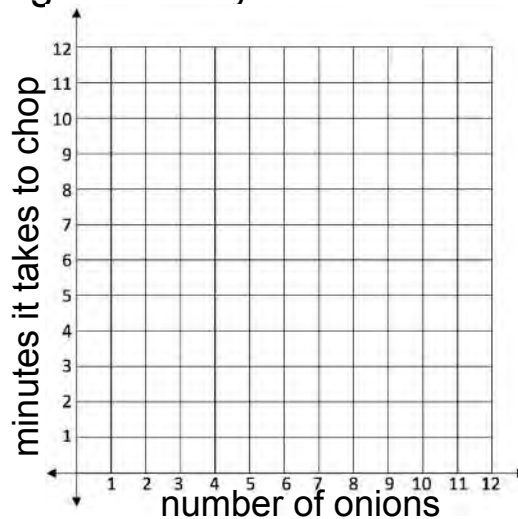
## Let's Talk:

If we switch the meaning of the  $x$  and  $y$  axis, we switch the meaning of the constant of proportionality or unit rate.

Draw a new graph and table if the meaning of  $x$  and  $y$  were switched.



x	y
0	0
2	1
4	2
6	3



x	y

What is the new meaning of the unit rate?

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## Let's Think:

We can compare rates if they are set up with the same units.

Miles has two options for paying for Roblox Premium. The table below shows the cost if he pays month by month. His other option is to pay \$84 for the year. Which option has the cheaper rate?

Months	Cost in dollars
2	\$16
4	\$32
6	\$48
8	\$64

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## Let's Try It:

We will do it together step by step!

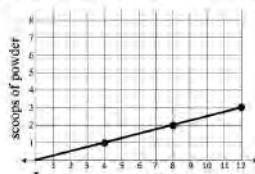
Name: \_\_\_\_\_ G7 U2 Lesson 15 - Let's Try It

The table below shows the amount of powder Susannah uses to make Kool-aid based on the amount of water.

- How many scoops of powder does Susannah use for every cup of water?
- How many cups of water does Susannah use for every scoop of Kool-aid?

Cups of water	Scoops of powder
4	3
6	$4\frac{1}{2}$
8	6
10	$7\frac{1}{2}$

The graph below shows the ratios used to make Kool-aid for Sunnyside Little League games. Sweetness can be determined by the rate of scoops of powder per cups of water. Is Susannah's Kool-aid sweeter than the Kool-aid at Sunnyside Little League?



3. Make a table.

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## On your Own:

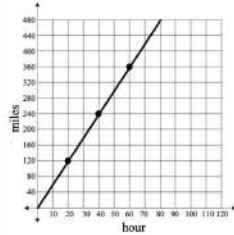
Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 15 - Independent Work

Solve the proportion problems using a graph, table or equation. Be sure to show your work.

1. A store sells 3 t-shirts for \$15. What is the cost per t-shirt?

2. The graph below shows the distance that the Gianni family traveled on their trip. Assuming they went at a constant rate, what was their speed in miles per hour?



3. Complete the table so that the rates are equivalent.

Dollars		24	18	22
---------	--	----	----	----

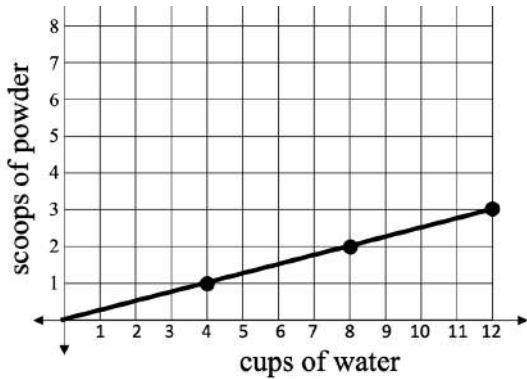
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The table below shows the amount of powder Susannah uses to make Kool-aid based on the amount of water.

- How many scoops of powder does Susannah use for every cup of water?
- How many cups of water does Susannah use for every scoop of Kool-aid?

Cups of water	Scoops of powder
4	3
6	$4\frac{1}{2}$
8	6
10	$7\frac{1}{2}$

The graph below shows the ratios used to make Kool-aid for Sunnyside Little League games. Sweetness can be determined by the rate of scoops of powder per cups of water. Is Susannah's Kool-aid sweeter than the Kool-aid at Sunnyside Little League?



3. Make a table.

4. Find the rate of scoops of powder per cups of water on the graph.

5. Write your answer in a complete sentence.

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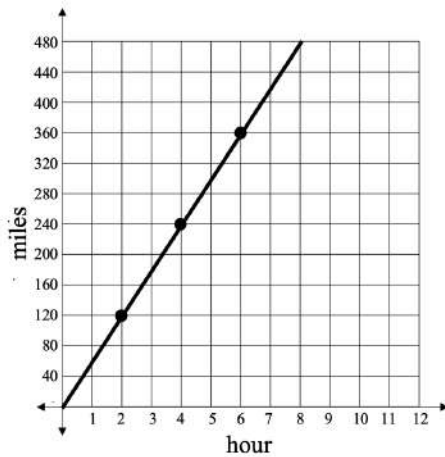
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Name: \_\_\_\_\_

Solve the proportion problems using a graph, table or equation. Be sure to show your work.

1. A store sells 3 t-shirts for \$15. What is the cost per t-shirt?

2. The graph below shows the distance that the Gianni family traveled on their trip. Assuming they went at a constant rate, what was their speed in miles per hour?



3. Complete the table so that the rates are equivalent.

Dollars		24	18		33
Hours	1	4		7	

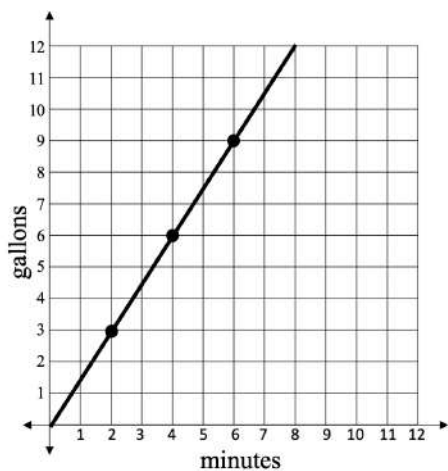
4. Lisa answered 30 math facts in 2 minutes. Dan wrote 12 math facts in  $\frac{1}{2}$  minute. Who solves math facts at a faster rate?



Solve the proportion problems using a graph, table or equation. Be sure to show your work.

5. Because of the different amounts of gravity on different planets, a person who weighs 100 pounds on Earth will feel like they weigh 38 pounds on Mercury. Write an equation using the variables,  $x$  and  $y$ , to calculate the weight of a person on each planet. Let  $x$  represent the weight in pounds on Mercury. Let  $y$  represent the weight in pounds on Earth.

6. John uses the equation,  $y = 2x$ , to find the flow of the hose at his house in gallons per minute. Sammy used the graph below to find the flow of the hose at his house. Whose hose flows at a faster rate?



7. The table below shows the amount of sugar used in a bread recipe based on the amount of flour.

Cups of sugar	Cups of flour
2	5
3	$7\frac{1}{2}$
4	10
5	$12\frac{1}{2}$

a. How many cups of sugar are used for every cup of flour?

b. How many cups of flour are used for every cup of sugar?

The table below shows the amount of powder Susannah uses to make Kool-aid based on the amount of water.

1. How many scoops of powder does Susannah for every cup of water?

$$\begin{array}{r} 0\frac{3}{4} \\ 4 \overline{)3} \\ \underline{-0} \\ 3 \end{array}$$

$\frac{3}{4}$  scoop per cup

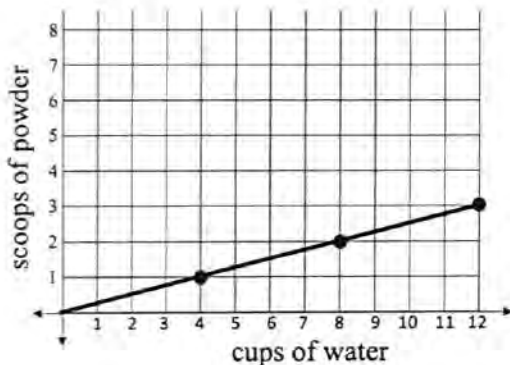
Cups of water	Scoops of powder
4	3
6	$4\frac{1}{2}$
8	6
10	$7\frac{1}{2}$

2. How many cups of water does Susannah for every scoop of Kool-aid?

$$\begin{array}{r} 1\frac{1}{3} \\ 3 \overline{)4} \\ \underline{-3} \\ 1 \end{array}$$

$1\frac{1}{3}$  cup per scoop

The graph below shows the ratios used to make Kool-aid for Sunnyside Little League games. Sweetness can be determined by the rate of scoops of powder per cups of water. Is Susannah's Kool-aid sweeter than the Kool-aid at Sunnyside Little League?



3. Make a table.

cups	scoops
4	1
8	2
12	3

4. Find the rate of scoops of powder per cups of water on the graph.

$$\begin{array}{r} 0\frac{1}{4} \\ 4 \overline{)1} \\ \underline{-0} \\ 1 \end{array}$$

$\frac{1}{4}$  scoop per cup

5. Write your answer in a complete sentence.

Susannah's Kool-aid is sweeter because she does  $\frac{3}{4}$  scoops per cup and the little league does  $\frac{1}{4}$  scoops per cup.

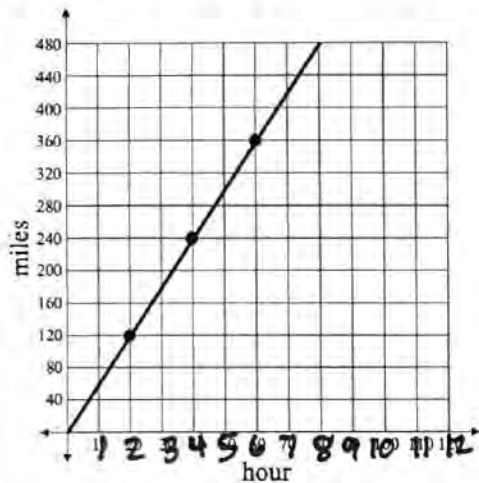
Solve the proportion problems using a graph, table or equation. Be sure to show your work.

1. A store sells 3 t-shirts for \$15. What is the cost per t-shirt?

$$\begin{array}{r} 05 \\ 3 \overline{)15} \\ \underline{-15} \\ 00 \end{array}$$

\$5 per t-shirt

2. The graph below shows the distance that the Gianni family traveled on their trip. Assuming they went at a constant rate, what was their speed in miles per hour?



hour	miles
<del>2</del>	120
<del>4</del>	240
<del>6</del>	360

$$\begin{array}{r} 060 \\ 20 \overline{)120} \\ \underline{-120} \\ 000 \end{array}$$

60 miles per hour

3. Complete the table so that the rates are equivalent.

Dollars	6	24	18	42	33
Hours	1	4	3	7	5½

$$\begin{array}{r} 06 \\ 4 \overline{)24} \\ \underline{-24} \\ 00 \end{array}$$

$$05\frac{3}{6} = 5\frac{1}{2}$$

$$\begin{array}{r} 6 \overline{)33} \\ \underline{-30} \\ 3 \end{array}$$

4. Lisa answered 30 math facts in 2 minutes. Dan wrote 12 math facts in  $\frac{1}{2}$  minute. Who solves math facts at a faster rate?

Lisa:

$$\begin{array}{r} 15 \\ 2 \overline{)30} \\ \underline{-20} \\ 10 \\ \underline{-10} \\ 00 \end{array}$$

Dan:

$$\frac{1}{2} \overline{)12}$$

$$12 \div \frac{1}{2}$$

$$12 \times \frac{2}{1}$$

24 facts per min

15 facts per min

Solve the proportion problems using a graph, table or equation. Be sure to show your work.

5. Because of the different amounts of gravity on different planets, a person who weighs 100 pounds on Earth will feel like they weigh 38 pounds on Mercury. Write an equation using the variables,  $x$  and  $y$ , to calculate the weight of a person on each planet. Let  $x$  represent the weight in pounds on Mercury. Let  $y$  represent the weight in pounds on Earth.

Mercury -  $X$  |  $Y$  - Earth  
38 | 100

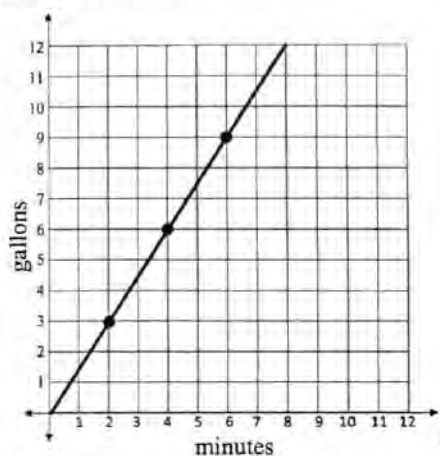
$$y = 2\frac{24}{38}x \text{ or } y = 2.6x$$

$$\begin{array}{r} 002.6 \\ 38 \overline{) 100.0} \\ \underline{-76} \phantom{0} \\ 240 \\ \underline{-228} \\ 12 \end{array}$$

$$\begin{array}{r} 38 \\ +38 \\ \hline 76 \\ +38 \\ \hline 114 \\ +38 \\ \hline 152 \end{array}$$

$$\begin{array}{r} 152 \\ \overline{) 38} \\ \underline{190} \\ 228 \end{array}$$

6. John uses the equation,  $y = 2x$ , to find the flow of the hose at his house in gallons per minute. Sammy used the graph below to find the flow of the hose at his house. Whose hose flows at a faster rate?



John: 2 gallons per min

Sammy:

min	gallons
2	3
4	6
6	9

$$\begin{array}{r} 1\frac{1}{2} \\ 2 \overline{) 3} \\ \underline{-2} \\ 1 \end{array}$$

$1\frac{1}{2}$  gallons per min

John's hose is faster.

7. The table below shows the amount of sugar used in a bread recipe based on the amount of flour.

Cups of sugar	Cups of flour
2	5
3	$7\frac{1}{2}$
4	10
5	$12\frac{1}{2}$

a. How many cups of sugar are used for every cup of flour?

$$\begin{array}{r} 0\frac{2}{5} \\ 5 \overline{) 2} \\ \underline{-0} \\ 2 \end{array}$$

$\frac{2}{5}$  cups of sugar per cup of flour

b. How many cups of flour are used for every cup of sugar?

$$\begin{array}{r} 2\frac{1}{2} \\ 2 \overline{) 5} \\ \underline{-4} \\ 1 \end{array}$$

$2\frac{1}{2}$  cups of flour per cup of sugar



# G7 Unit 3:

Proportional Relationships and Percentages

# **G7 U3 Lesson 1**

Calculate the percentage of a rectangular area that is covered by another region, and explain why the percentage is the same in scaled copies of the same figure.

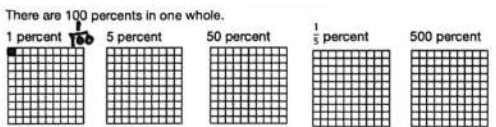


**G7 U3 Lesson 1 - Today we will convert between a fraction, decimal and percent then draw a picture.**

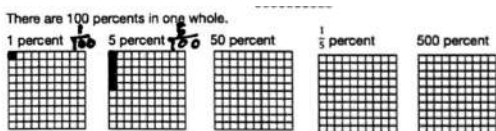
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will convert between a fraction, decimal and percent then draw a picture. This is some review from 6th grade that will let us solve 7th grade percent problems in future lessons.

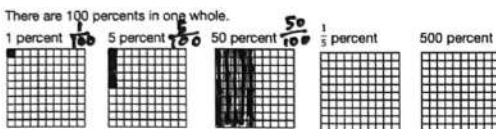
**Let's Review (Slide 3):** Percent means out of 100. Per means “for every” or “out of.” Cent should make you think of money. This says, “We’ve heard of cents when we are talking about 100 pennies in a dollar.” This is like 100 cents in a dollar. *Point to the 100 pretend pennies and the dollar.* Just like there 100 cents in a dollar, there are 100 percents in one whole. And it can be one whole anything. On a test, you can get up to 100 percent of the whole test. On a sale, you can pay up to 100 percent of the whole price. At the bottom of this slide, we have hundreds grids. *Point to a square.* This square has 100 little squares in it. So, let’s shade the percents we see.



1 percent just means 1 out of 100. It can help to write that out as a fraction since that’s what I’m saying with words. To shade it, I just color in one little square. This a percent. Like 1 cent in a dollar.

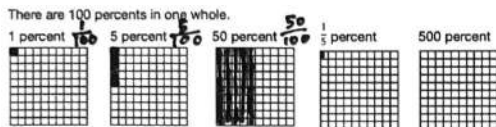


5 percent is just as easy. This means 5 out of 100. I am going to write that as a fraction. Now I shade 5 little squares out of the 100 squares in the whole.

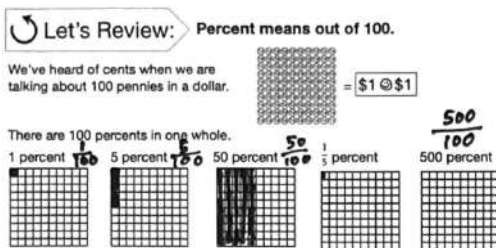


50 percent is just as easy. This means 50 out of 100. I am going to write that as a fraction. Now, I could count this as 1, 2, 3, 4 and on... But I can see this is a row of ten so I am just going to shade 10, 20, 30, 40, 50. That’s 50 little squares out of 100 squares in the whole. That’s 50%

Now these are a little challenging but as long as you slow down and switch out the word “percent” for “out of 100” in your mind, you will figure this out. 1 fifth means 1 out of 5. So this is 1 out of 5 out of the 100. That’s like 1 fifth of something that was already in 100 pieces. That’s 1 fifth of 1 square. We’re



going to turn fractions into percents in a few more slides. The thing I want you to see here is 1 fifth is not the same as 1 fifth percent. 1 fifth means I cut a whole into 5 pieces and shade 1. 1 fifth percent means I cut 1 whole into 100 pieces and share 1 fifth of the little pieces.



This is another one that is really challenging because 500 has the word “hundred” in it so people don’t think they have to say “out of 100” for the percent as well. But they do. This is 5 hundred out of a 100. I can write that as a fraction.

**Let's Review:** Percent means out of 100.

We've heard of cents when we are talking about 100 pennies in a dollar.

There are 100 percents in one whole.

1 percent  $\frac{1}{100}$     5 percent  $\frac{5}{100}$     50 percent  $\frac{50}{100}$     1 percent  $\frac{1}{100}$     50 percent  $\frac{50}{100}$

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100 out of 100 would be the whole thing. So 500 out of 100 would be this whole 100 and then another whole hundred then another and another and another. I actually can't even fit the drawing on this slide because it would be so big. *You will want to start drawing another whole big square then another but this won't fit on your slide and that is okay. Just explain why it won't fit.* Great job. So now we know how to draw what percents look like. Let's think about fractions and decimals.

**Let's Talk (Slide 4):** When we think about turning decimals into percents, we use place value. I bet you can figure out what place value we use. Help me fill in the blanks here. Since percent means OUT OF 100, we look in the HUNDREDTHS place. Let's try it.

TH	0.08
H	
T	
O	
N	
E	

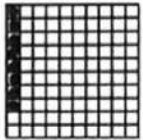
 I have zero point zero eight. But we don't really read it that way. We can use a place value chart and see that this place after the decimal is the tenths and this next place is the hundredths.

TH	0.08
H	1
T	0
O	0
N	
E	

 Another trick that I use is to put a 1 under the decimal point and then put two zeros right under the digits. Now I can see what it is out of 100.

TH	0.08
H	1
T	0
O	0
N	
E	

 $\frac{8}{100}$ 
 This is 8 out of 100. Let me write that as a fraction. Now I am going to write that as a percent.



We can shade this easily just like on the last slide. 8 out of 100 is 8 little squares out of the total 100 squares. It's 8 percent.

TH	0.80
H	
T	
O	
N	
E	

 Let's do the next one. If I use a place value chart, I have tenths then hundredths. This is 80 hundredths. I am going to write that as a fraction - 80 over 100.

TH	0.80
H	1
T	0
O	0
N	
E	

 $\frac{80}{100}$ 
 Let's use the 1 under the decimal trick. I write 1 then zero under the 8 and zero under the zero. I can see the 80 over 100 right in front of me. Now we know we need to shade 80 little squares. I will do it in tens - 10, 20, 30, 40, 50, 60, 70 and 80. It's 80 percent.



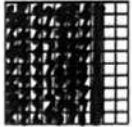
Are 0.08 and 0.8 the same? No! One was 8 hundredths and the other was 80 hundredths. The 8 is in a different place from the decimals so it has a different value. Kind of like pennies and dimes. 0.08 is 8 pennies and 0.80 is 80 pennies.

T  
0.8

Let's do the next one. If I use a place value chart, this is only tenths. We don't want tenths, do we? We want hundredths because percent means out of 100. I am going to add a zero so now it's 80 hundredths.

TH  
0.80  
100

$\frac{80}{100}$



Let's use the 1 under the decimal trick. I put one zero under the 8. That's not out of 100 so I put another zero. This is 80 hundredths or 80 out of 100. Let's write that fraction down. And now we know we need to shade 80 little squares. I will do it in tens - 10, 20, 30, 40, 50, 60, 70 and 80. It's 80 percent.

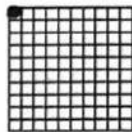
If you look at the ones we've done so far, 0.8 is the same as the one we just did, 0.80. That's because the 8 is right next to the decimal for both. But they are NOT the same as 0.08 where the zero pushes the 8 into a different place values. That reminds us that we don't change the value by adding zeros on this side. It is just an equivalent form.

TH  
0.008

Let's do zero point zero zero eight. Of course we don't really read a decimal that way. We use a place value chart. I have tenths, hundredths and thousands. Should I just assume this is 8 since I see an 8? NO! We are doing percents and percent means out of 100. So I still need to look for just hundredths and not let this trick me.

TH  
0.008  
100

$\frac{0}{100}$



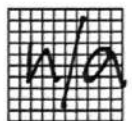
This is where the 1 under the decimal trick is super handy. I will write one, zero, zero and then I'm going to stop. I just want this out of 100. And look, it's ZERO out of 100. Then we just have these little pieces even smaller than hundredths. I will put the decimal there to show it's 00.8%. When I shade this, I am NOT going to shade the whole square. I am going to shade 8 tenths of square.

TH  
8.80

Last one! It is eight point eighth. Or if we use a place value chart, it is 8 and 8 tenths. If I look in the hundredths place, I need to annex a zero. It is 880 hundredths. I am starting to see this is more than 80 hundredths. It is 8 whole then the 80 hundredths.

TH  
8.80  
100

$\frac{880}{100}$



If I put the 1 under the decimal and then put 2 zeros, I still see, it is a big number of wholes, 8, plus 80 hundredths. This is another one that I can't really draw because I would need to draw 8 whole boxes. I am just going to let this one go. But we know it is big. It would be 100 - 200 - 300 - 400 - 500 - 600 - 700 - 800 then another 80! It is 880%.

That was great practice. Let's just review the big idea for this slide: If we want to change a decimal to a percent... *Read the top line.* "Since percent means out of one hundred, we look in the hundredths place."

**Let's Think (Slide 5):** Now let's work on fractions. When we think about turning a fraction to a percent, we think about equivalent forms. It would be nice to find an equivalent fraction with 100 in the denominator.

It would be nice to find an equivalent fraction with 100 in the denominator.

$$\frac{3}{5} \times 2 = \frac{6}{10} \times 10 = \frac{60}{100}$$

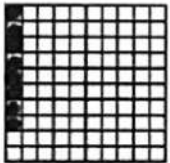
Let's think about 3 fifths. I can do "times 2" on the top and "times 2" on the bottom. Then I get 6 tenths. Now it is more obvious how to get to 100. I am going to do "times 10" on the top and "times 10" on the bottom. This is 60 hundredths.



That's the same as 0.60 with 60 in the hundredths place or 60%. I just shade 60 little squares.

$$\frac{2}{25} \times 4 = \frac{8}{100}$$

Let's think about 2 twenty-fifths. I know that 4 twenty-fives make 100 so I am going to do "times 4" on the top and "times 4" on the bottom. That makes 8 hundredths. That's 8 in the hundredths place of a decimal, which is zero point zero eight. Or 8 percent.

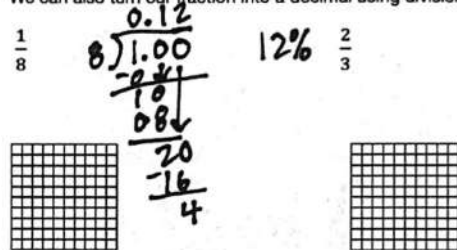


I shade 8 little squares.

Now, this won't work every time. These fractions have friendly numbers that are easy to turn into 100. There is one way to turn fractions into decimals that will work every time. We'll explore that on the next slide.

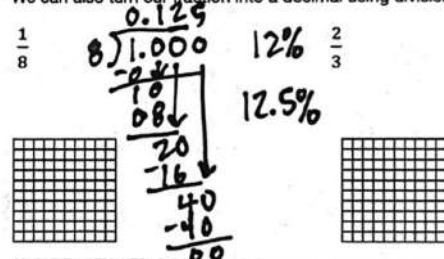
**Let's Think (Slide 6):** We still have this one main idea. When we think about turning fractions into percents, we think about equivalent forms. If it isn't easy to find equivalent fractions, we can use

We can also turn our fraction into a decimal using division.

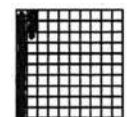


equivalent decimals. It isn't easy to look at 1 eighth and figure out how to multiply it to make 100. We could get close but there isn't anything times 8 to make 10 or 100. But we can also turn our fraction into a decimal using division. The top goes in the division symbol. 8 doesn't go into 1 so I put a 0. I am going to add a decimal and put two zeros so that gets me to the hundredths place. 8 goes into 10 one time. I subtract 8 and get 2. Pull down the 0. 8 goes into 20 two times. I subtract 16 and get 4. This is my percent: 12%.

We can also turn our fraction into a decimal using division.



But let's see happens if we keep dividing. I will put another zero. I pull it down and now 8 goes into 40 five times. Subtract 40 and get zero. I could think of this as 12.5% if I wanted to. Once I get into the thousandths and ten thousandths, our pieces are getting so small that they are insignificant.



We shade the 12 squares. If we want we can shade 5 tenths of another square.

Let's do one more. It isn't easy to look at  $\frac{2}{3}$  and figure out how to multiply it to make 100. We could get close but there isn't anything times 3 to make 10 or 100. But we can also turn our fraction into a decimal using division. The top goes in the division symbol. 3 doesn't go into 2. So I put a 0. I

We can also turn our fraction into a decimal using division.

Handwritten work for  $\frac{1}{8}$ :  $8 \overline{)1.000}$  with steps showing  $125$  and  $1000$ . A 10x10 grid has 12.5% shaded (12 full columns and 5 squares in the 13th column). The percentage  $12.5\%$  is written next to it.

Handwritten work for  $\frac{2}{3}$ :  $3 \overline{)2.00}$  with steps showing  $66$  and  $200$ . A 10x10 grid has 66% shaded (6 full columns and 6 squares in the 7th column). The percentage  $66\%$  is written next to it.

am going to add a decimal and put two zeros so that gets me to the hundredths place. 3 goes into 20 six times. I subtract 18 and get 2. I pull down the zero. 3 goes into 20 six times. I subtract 18 and get 2. We can see where this is going. It's going to repeat on and on. But I have what I need to get a percent, right? It is 66 hundredths so I will shade 66 little squares.

**Let's Try It (Slide 7):** Now we will practice converting between all these forms together. I will lead you through step by step.



# WARM WELCOME



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**Today we will convert between a fraction, decimal and percent then draw a picture..**

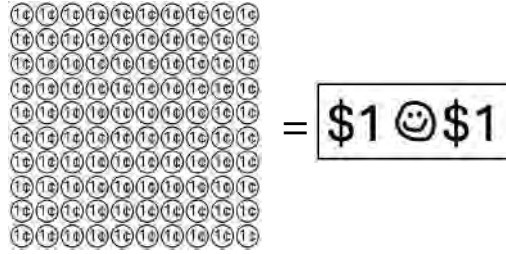
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# Let's Review:

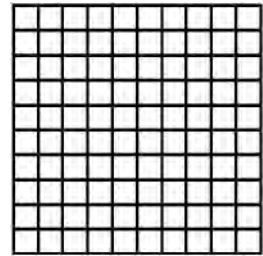
**Percent means out of 100.**

We've heard of cents when we are talking about 100 pennies in a dollar.

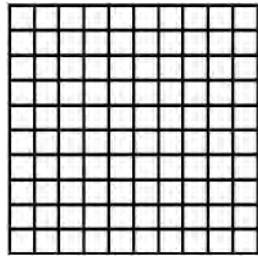


There are 100 percents in one whole.

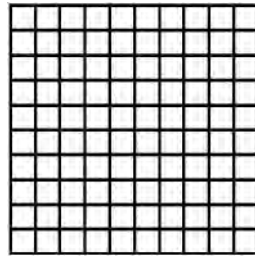
1 percent



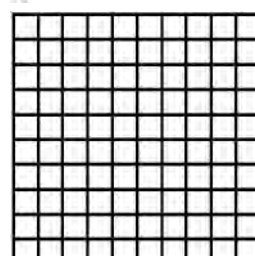
5 percent



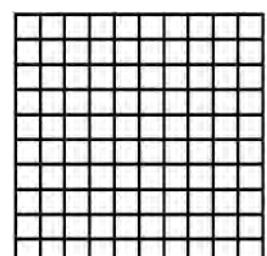
50 percent



$\frac{1}{5}$  percent



500 percent



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# Let's Talk:

**When we think about turning decimals into percents, we use the place value.**

Since percent means \_\_\_\_\_, we look in the \_\_\_\_\_ place.

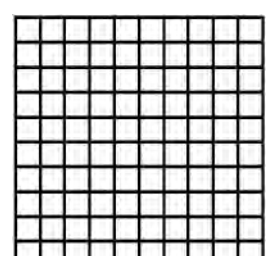
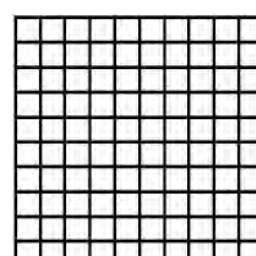
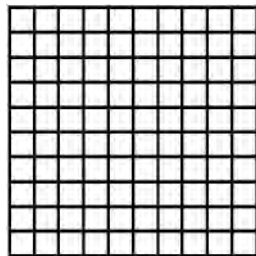
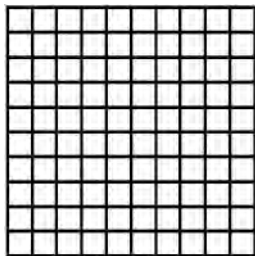
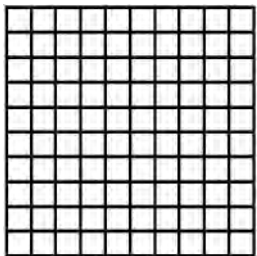
0.08

0.80

0.8

0.008

8.8



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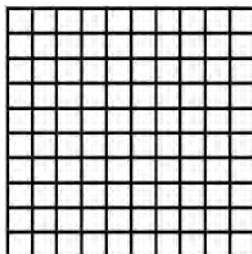
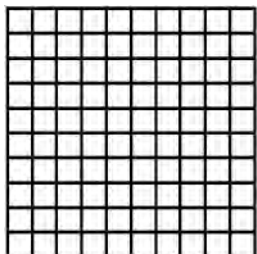
## Let's Think:

When we think about turning fractions into percents, we think about equivalent forms.

It would be nice to find an equivalent fraction with \_\_\_\_\_ in the denominator.

$$\frac{3}{5}$$

$$\frac{2}{25}$$



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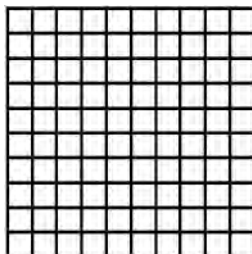
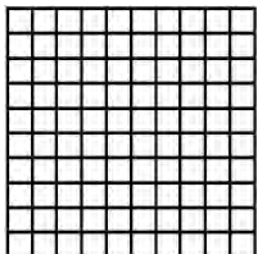
## Let's Think:

When we think about turning fractions into percents, we think about equivalent forms.

We can also turn our fraction into a decimal using division.

$$\frac{1}{8}$$

$$\frac{2}{3}$$



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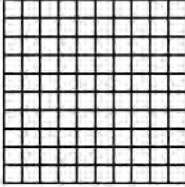
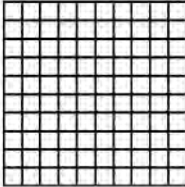
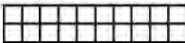


# Let's Try It:

We will do it together step by step.

Name: \_\_\_\_\_ G7 U3 Lesson 1 - Let's Try It

Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{1}{5}$				
	0.345			
		6%		

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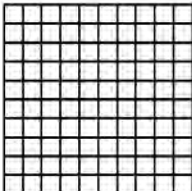
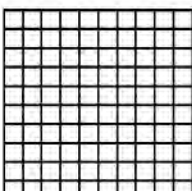
# On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 13 - Independent Work

Remember: Percent means out of 100.

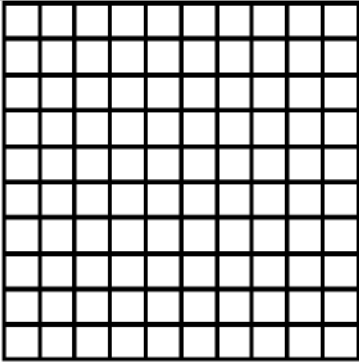
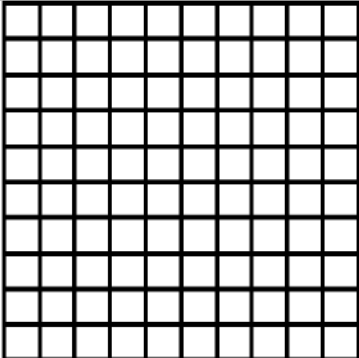
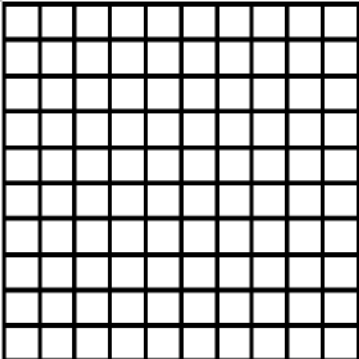
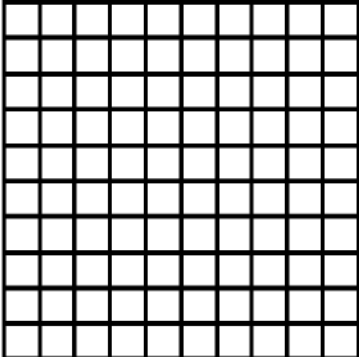
Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{1}{2}$				
	0.565			

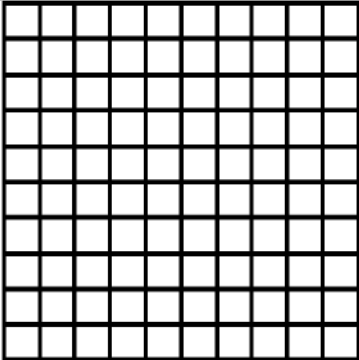
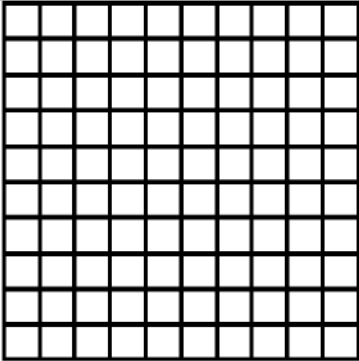
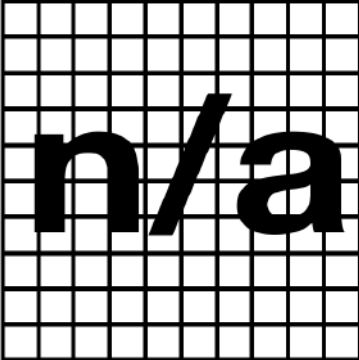
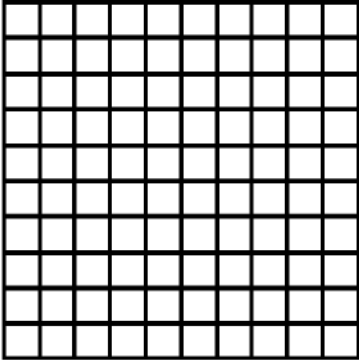
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Name: \_\_\_\_\_

Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{1}{5}$				
	0.345			
		6%		
		60%		

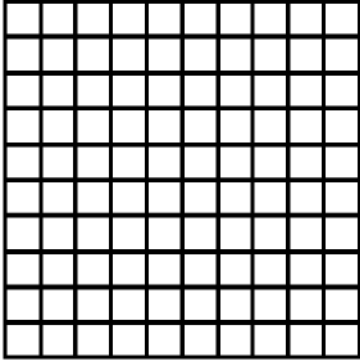
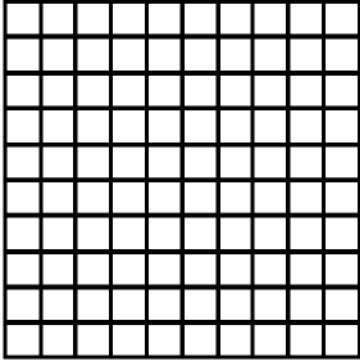
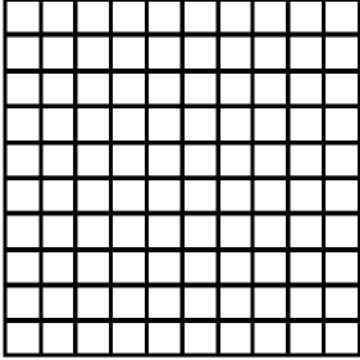
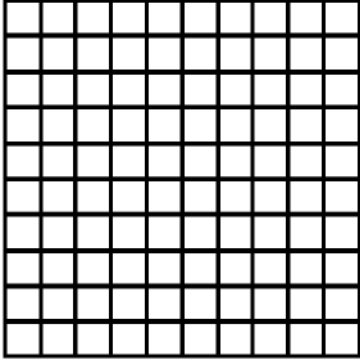
Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{4}{9}$				
	0.2			
		500%		
		$\frac{3}{4}\%$		

Name: \_\_\_\_\_

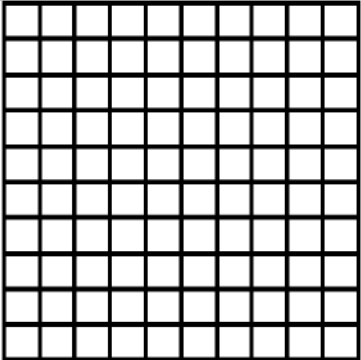
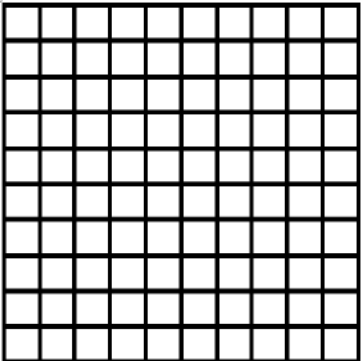
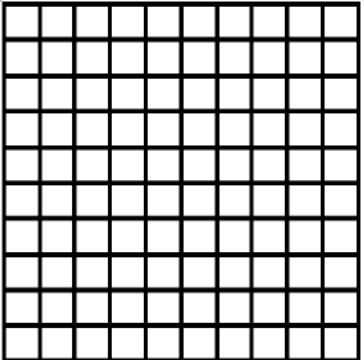
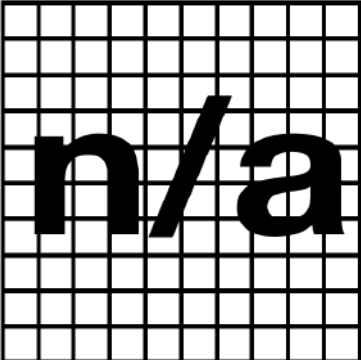
Remember: Percent means out of 100.

Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{1}{2}$				
	0.565			
		2%		
		20%		



Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{3}{7}$				
	1.3			
		$\frac{9}{10}\%$		
		200%		

Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{1}{5}$	0.2 or 0.20	20%		$\begin{array}{r} 0.20 \\ 5 \overline{)1.00} \\ \underline{-1.0} \\ 00 \end{array}$
$\frac{345}{1000}$	0.345	34.5%		$\begin{array}{r} 0.345 \\ 1000 \end{array}$
$\frac{6}{100}$ or $\frac{3}{50}$	0.06	6%		$\begin{array}{r l l} & T & H \\ 0 & 0 & 6 \\ \hline & & \end{array}$
$\frac{60}{100}$ or $\frac{30}{50}$ or $\frac{3}{5}$	0.60 or 0.6	60%		

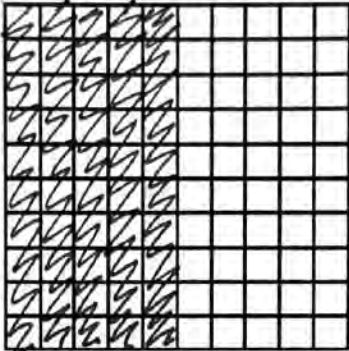
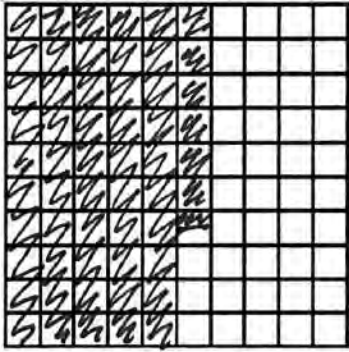
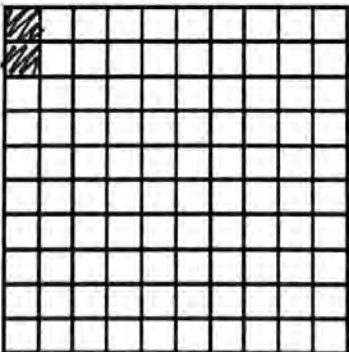
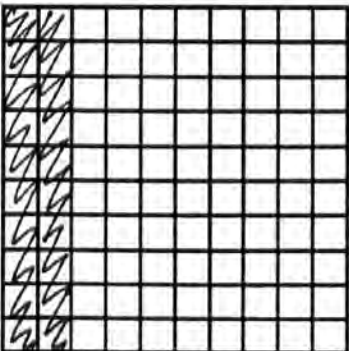
Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{4}{9}$	0.44	44%		$\begin{array}{r} 0.44 \\ 9 \overline{)4.00} \\ \underline{-36} \phantom{0} \\ 40 \\ \underline{-36} \\ 4 \end{array}$
$\frac{2}{10}$ or $\frac{1}{5}$	0.2	20%		$0.20$
$\frac{500}{100}$ or $\frac{5}{1}$	5.00	500%	n/a	$\begin{array}{r} 5.00 \\ 100 \end{array}$
$\frac{75}{10000}$	0.0075	$\frac{3}{4}\%$		$0.\overline{100}^{\frac{3}{4}}$ $\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \\ \underline{-28} \phantom{0} \\ 20 \\ \underline{-20} \\ 00 \end{array}$

# Name: ANSWER KEY

Remember: Percent means out of 100.

Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{1}{2}$	0.5 or 0.50	50%		$\begin{array}{r} 0.50 \\ 2 \overline{)1.00} \\ \underline{-10} \\ 000 \end{array}$
$\frac{565}{1000}$	0.565	56.5%		$0.565$
$\frac{2}{100}$ or $\frac{1}{50}$	0.02	2%		$0.\underline{\underline{2}}_{100}$
$\frac{20}{100}$ or $\frac{2}{10}$ or $\frac{1}{5}$	0.20 or 0.2	20%		$.\underline{\underline{20}}_{100}$

Complete each row to show equivalent fraction, decimal and percent forms. Then shade the picture. Show any scratch work in the space provided.

Fraction	Decimal	Percent	Picture	Work shown:
$\frac{3}{7}$	0.42	42%		$\begin{array}{r} 0.42 \\ 7 \overline{)3.000} \\ \underline{-28} \phantom{0} \\ 20 \\ \underline{-14} \\ 6 \end{array}$
$\frac{13}{10}$ & $1\frac{3}{10}$	1.3	130%		$\begin{array}{r} 1.30 \\ 10 \overline{)130} \\ \underline{-100} \\ 30 \\ \underline{-30} \\ 0 \end{array}$
$\frac{9}{1000}$	.009	$\frac{9}{10}\%$		$\begin{array}{r} 0.009 \\ 10 \overline{)9.000} \\ \underline{-90} \\ 00 \end{array}$
$\frac{200}{100}$ or $\frac{2}{1}$ or 2	2.00 or 2	200%		$\begin{array}{r} 2.00 \\ 10 \overline{)200} \\ \underline{-200} \\ 00 \end{array}$

# **G7 U3 Lesson 2**

Solve problems involving equivalent ratios with fractional quantities.



**G7 U3 Lesson 2 - Today we will relate percents to proportions to find a fraction of a given amount.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will relate percents to proportions to find a fraction of a given amount. I think it is going to blow your mind to see how you can use everything you've learned about tables and equations to solve percent problems.

Table:

total pieces	shaded pieces
2	1

**Let's Review (Slide 3):** A constant of proportionality shows when a relationship is proportional. We spent the whole last unit learning all about that. *Read from the slide.* "Let's draw a picture of an easy proportion and see what we notice about the constant of proportionality and the percent." Our table says "total pieces" and "shaded pieces." We'll just start with what we see in the top tape diagram. There are 2 pieces total and 1 piece is shaded. I am going to write that in the table.

Tape diagram:



Table:

total pieces	shaded pieces
2	1
4	2

Now, we aren't going to shade any more or any less. It is going to stay this same area shaded but I am going to turn this into an equivalent fraction shaded by cutting each piece into 2. Now we have 4 total pieces and 2 pieces are shaded. This is the same picture with all equal pieces. We can see they are proportional. But also we could look across the rows. 2 divided by 1 is 2. 4 divided by 2 is 2.

Tape diagram:



Table:

total pieces	shaded pieces
2	1
4	2
6	3

Let's do one more. This time I'll cut each piece into three pieces. Now I have 6 total pieces and 3 shaded pieces. I am going to write that in the graph. And look, we can still think of the row as 6 divided by 3 equals 2. So this is a proportion and we know there's a constant of proportionality.

But BEFORE we find the constant of proportionality let's turn each of these fractions into percents, which we learned in our last lesson. If you remember, we said you divide the top by the bottom so 1 half is 1 divided by 2. 2 doesn't go into 1. So I write 0 and I add a decimal with two zeros to bring me

Tape diagram:



Table:

total pieces	shaded pieces
2	1
4	2
6	3

Find the percent:

$$\frac{1}{2} \overset{0.50}{2 \overline{)1.00}} \begin{array}{r} 0.50 \\ -1.00 \\ \hline 000 \end{array} 50\%$$

to the hundredths place. 2 goes into 10 five times. Subtract 10 and nothing is left. So the next place is zero. This is 50%. But wait a minute! We just divided to find the percent. But don't we divide to find the constant of proportionality?!?!? We do! We do the exact same dividing to find the constant of proportionality as we do to find the percent.

Tape diagram:



Table:

total pieces	shaded pieces
2	1
4	2
6	3

Find the percent:

$$\frac{2}{4} \overset{0.50}{2 \overline{)1.00}} \begin{array}{r} 0.50 \\ -1.00 \\ \hline 000 \end{array} 50\%$$

$$\frac{2}{4} \overset{0.50}{4 \overline{)2.00}} \begin{array}{r} 0.50 \\ -2.00 \\ \hline 00 \end{array} 50\%$$

Let's do the next row. The fraction is 2 out of 4 so I divide 2 by 4. That's how I find the percent but that's also the same thing I would do to find the constant of proportionality! 4 doesn't go into 2 so I write 0 and I add a decimal with two zeros to get to the hundredths place. 4 goes into 20 five times. Subtract 20 and nothing is left. So the next place is zero. This is 0.50 or 50%.

$$\frac{3}{6} \overset{0.50}{6 \overline{)3.00}} \begin{array}{r} 0.50 \\ -3.00 \\ \hline 00 \end{array} 50\%$$

Let's do the next row. The fraction is 3 out of 6 so I divide 3 by 6. That's how I find the percent but that's also the same thing I would do to find the constant of proportionality! 6 doesn't go into 3 so I write 0 and I add a decimal with two

zeros to get to the hundredths place. 6 goes into 30 five times. Subtract 30 and nothing is left. So the next place is zero. This is 0.50 or 50%.

Table:

total pieces	shaded pieces
2 x 0.5	1
4 x 0.5	2
6 x 0.5	3
x 0.5	

All we were doing was reviewing things you've already learning but we've already seen the most important idea of today: just like we divide to find the constant of proportionality, we divide to find the percent. So the percent IS THE SAME as the constant of proportionality. I can put "times zero point five" in each of these circles to see the proportion.

Table:

total pieces	shaded pieces
2 x 0.5	1
4 x 0.5	2
6 x 0.5	3
100 x 0.5	50

And if I put 50% on the table. That would be 100 total pieces time 0.5 gets us 50 shaded pieces. That 50 is the percent and it is the same as the constant of proportionality.

**Let's Talk (Slide 4):** A percent is a constant of proportionality that relates a part to a whole. Imagine we want to continue the table from the last slide with a different total. This is filled in with the numbers from the last slide. Just like for the proportions from our last unit, we could write a multiplication equation, we can write one here. It will be TOTAL x PERCENT = PART. Or in this case, TOTAL x 0.5 =

Write an equation using 50%:  
**TOTAL x PERCENT = PART**  
**TOTAL x 0.5 = PART**

PART. Again, we can do that because we just saw that we did the same thing to find the constant of proportionality that we did to turn these fractions into percents. The percent IS the constant of proportionality.

Table:

total pieces	shaded pieces
2 x 0.5	1
4 x 0.5	2
6 x 0.5	3
100 x 0.5	50
24 x 0.5	

So, now we want to know, "How many pieces would be shaded out of 24 pieces?" I am going to put 24 on my table.

How many pieces would be shaded out of 24 pieces?  
**24 x 0.5 = ?**

I can see that I just multiply 24 x 0.5 equals question mark.

$$\begin{array}{r} 2 \\ \times 24 \\ \times 0.5 \\ \hline 12.0 \end{array}$$

I think of it as 24 x 5. Then we do 5 times 4 is 20. Carry the 2. 5 times 2 is 10 plus the 2 is 12. We get 120 but now we have to put in the decimal. There was one place in the problem so I have one place in my answer. It's 12!

Table:

total pieces	shaded pieces
2 x 0.5	1
4 x 0.5	2
6 x 0.5	3
100 x 0.5	50
24 x 0.5	12

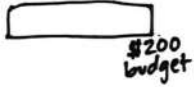
Let's put that on our table! It makes a lot of sense with everything else we see.

Let's draw a picture. There would be twelve shaded pieces then twelve other pieces. That's 24 pieces altogether and 12 shaded. We just found 50% of 24. The good news is that we don't have to go through all these steps every time. Now that we see the percent is the constant of proportionality, we can just jump to this equation. Let's do one more example.

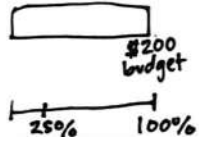


**Let's Think (Slide 5):** Since percents are just proportions in disguise, we can use a table diagram, a table or an equation to solve them. Read the problem silently with your eyes while I read the problem out loud. "Lisa has a budget of \$200 for her garden this year. So far, she has spent 25% of her budget on dirt! How much of her budget has Lisa spent so far?" It is always a good idea to draw a picture for a story problem so let's start there. It says, "Lisa has a budget of \$200 for her garden" so I am going to draw a rectangle and label it \$200 budget.

Draw a tape diagram:

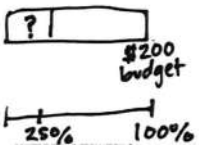


Draw a tape diagram:



Next it says, "she has spent 25% of her budget on dirt." I am going to draw that underneath what we just drew and I am going to line it up really carefully. We know 25 percent means 25 out of 100. So the whole amount is 100. And I think 25 goes about here.

Draw a tape diagram:



When the question asks, "how much of her budget has Lisa spent so far?" It is asking for the part of the \$200 budget so I can mark the part that lines up with 25% with a question mark. And I will write dollars for dirt. You can see how this is a proportion just like the fractions we just drew. And I'll warn you now that for today's problems the drawings might seem a bit tedious. But in our very next lesson, they can get complicated so we will want to draw a picture for every story problem no matter what.

Make a table:

total budget	\$ for dirt
200	?
100	25

Now let's make a table. To help us see how the percent is the constant of proportionality, I am going to make the first column the total. In this case, that's the total budget. And then the next column is the part, dollars for dirt. For right now, it is going to feel obvious where to put the numbers. But again, in our next lesson it gets more complicated so don't skip any steps.

Make a table:

total budget	\$ for dirt
200	?
100	25

I know that 200 is the total budget and we don't know the money for dirt. 100 is the total percent and 25 is part of the percent.

Make a table:

total budget	\$ for dirt
200 x .25	?
100 x .25	25

Now I can see why it is so useful to know the percent is the constant of proportionality in disguise. The secret operation happening here is  $\times 0.25$ .

Write an equation:

$$\text{TOTAL} \times \text{PERCENT} = \text{PART}$$

So that's the operation I am going to use to find the part of the \$200 too! Let's write this as an equation before we do any number crunching. On our last slide, we said, "TOTAL x PERCENT = PART."

Write an equation:

$$\begin{aligned} \text{TOTAL} \times \text{PERCENT} &= \text{PART} \\ \text{TOTAL} \times 0.25 &= \text{PART} \\ 200 \times 0.25 &= ? \end{aligned}$$

I am going to put in the \$200 x 0.25 equals question mark.

$$\begin{array}{r} 0.25 \\ \times 200 \\ \hline 50.00 \end{array}$$

Now we can do the math to the side. 200 x 25 is 5000. Now I need to put in the decimal spaces. There are two spaces so this is 50.00.

The question said, how much of her budget has Lisa spent so far. So far, Lisa has spent \$50. We used the percent like a constant of proportionality to get our answer.

**Let's Try It (Slide 6):** Now we will practice using the percent as the proportionality to solve some problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will relate percents to proportions to find a fraction of a given amount.**

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## Let's Review:

**A constant of proportionality shows when a relationship is proportional.**

Let's draw a picture of an easy proportion and see what we notice about the constant of proportionality and the percent.

Tape diagram:

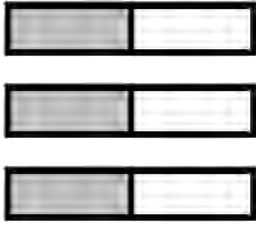


Table:

total pieces	shaded pieces
	○
	○
	○
	○

Find the percent:

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## Let's Talk:

**A percent is a constant of proportionality that relates a part to a whole.**

Imagine we want to continue the table from the last side with a different total.

Tape diagram:

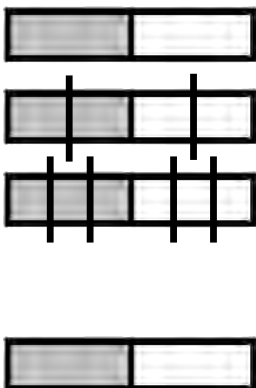


Table:

total pieces		shaded pieces
2	○ x 0.5	1
4	○ x 0.5	2
6	○ x 0.5	3
100	○ x 0.5	50
	○ x 0.5	

Write an equation using 50%:

How many pieces would be shaded out of 24 pieces?

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## Let's Think:

Since percents are just proportions in disguise, we can use a tape diagram, a table or an equation to solve them.

Lisa has a budget of \$200 for her garden this year. So far, she has spent 25% of her budget on dirt! How much of her budget has Lisa spent so far?

Draw a tape diagram:

Make a table:

Write an equation:

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## Let's Try It:

We will do it together step by step!

Name: \_\_\_\_\_

G7 U3 Lesson 2 - Let's Try It

**This is a true fact: Michael Jordan took nearly 1,800 free throws in his basketball career. And this is another true fact: He made about 30% of those free throws. How many free throws did Michael Jordan make in his career?**

1. Draw a tape diagram. Be sure to label your diagram with words.
2. Make a table. Be sure to label your table with words.
3. What is the multiplier that will work across the rows of the table? \_\_\_\_\_
4. Write an equation using words and the multiplier you named in #3 above.  
\_\_\_\_\_
5. Use the equation to find the missing value.

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U2 Lesson 13 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

1. There are 24 kids in class today. 80% of them were on time. How many kids were on time today?  Tape diagram:	Table:  Equation:
2. Lisa wants to buy jeans for \$150. She has a 25% off coupon. What discount will she get?  Tape diagram:	Table:  Equation:
3. Audrey did her HW for 90 minutes. She spent 35% of the time on math. How long did Audrey spend on math?	Table:

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Name: \_\_\_\_\_

**This is a true fact: Michael Jordan took nearly 1,800 free throws in his basketball career. And this is another true fact: He made about 30% of those free throws. How many free throws did Michael Jordan make in his career?**

1. Draw a tape diagram. Be sure to label your diagram with words.

2. Make a table. Be sure to label your table with words.

3. What is the constant of proportionality for this table? \_\_\_\_\_

4. Write an equation using the constant of proportionality:

\_\_\_\_\_

5. Use the equation to find the missing value.

6. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_

Name: \_\_\_\_\_

Solve each problem using a tape diagram, a table and an equation.

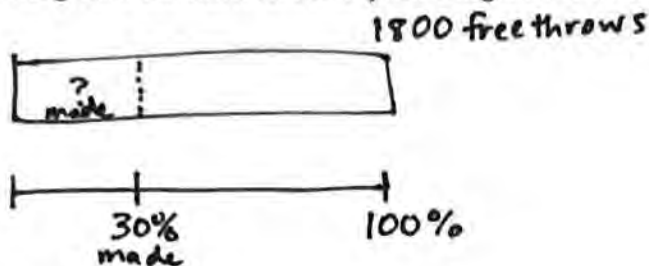
<p>1. There are 24 kids in class today. 80% of them were on time. How many kids were on time today?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>2. Lisa wants to buy jeans for \$150. She has a 25% off coupon. What discount will she get?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>3. Audrey did her HW for 90 minutes. She spent 35% of the time on math. How long did Audrey spend on math?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>4. Hudson needs 50 gallons of paint for his mural project. He has 75% of what he needs. How much paint does Hudson have?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>

Solve each problem using an equation and a table.

<p>5. The math test today had 25 questions. Emanda got 80% of the questions right. How many questions did Emanda get right?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>6. The chemical reaction takes 8 minutes to complete. So far it is 32% through. How many minutes must have passed so far?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>7. The movie is 120 minutes long. Imagine you have watched 30% of it. How many minutes have you watched?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>8. Jason saved \$80. The bank gave him 1% interest on his savings. How much money did the bank give Jason?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>

This is a true fact: Michael Jordan took nearly 1,800 free throws in his basketball career. And this is another true fact: He made about 30% of those free throws. How many free throws did Michael Jordan make in his career?

1. Draw a tape diagram. Be sure to label your diagram with words.



2. Make a table. Be sure to label your table with words.

total free throws	free throws made
1800	?
100	30

3. What is the constant of proportionality for this table? 0.30 or 0.3

4. Write an equation using the constant of proportionality:

$$\underline{\text{TOTAL} \times 0.3 = \text{PART}}$$

5. Use the equation to find the missing value.

$$1800 \times 0.3 = ?$$

$$540 = ?$$

$$\begin{array}{r} 1800 \\ \times 3 \\ \hline 5400 \end{array}$$

6. Write your answer in a complete sentence.

Michael Jordan made 540 out of the 1800 free throws.



# Name: ANSWER KEY

Solve each problem using a tape diagram, a table and an equation.

<p>1. There are 24 kids in class today. 80% of them were on time. How many kids were on time today?</p> <p>Tape diagram:</p>	<p>Table:</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; padding: 5px;">class kids</th> <th style="padding: 5px;">kids on time</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">24</td> <td style="text-align: center; padding: 5px;"><math>\times 0.8</math> ?</td> </tr> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">100</td> <td style="text-align: center; padding: 5px;"><math>\times 0.8</math> 80</td> </tr> </tbody> </table> <p>Equation:</p> $\begin{array}{r} 24 \\ \times 0.8 \\ \hline 19.2 \end{array}$ <p style="text-align: center;">TOTAL <math>\times 0.8 =</math> PART  <math>24 \times 0.8 = ?</math>  <span style="border: 1px solid black; padding: 2px;">19.2 = ?</span></p>	class kids	kids on time	24	$\times 0.8$ ?	100	$\times 0.8$ 80
class kids	kids on time						
24	$\times 0.8$ ?						
100	$\times 0.8$ 80						
<p>2. Lisa wants to buy jeans for \$150. She has a 25% off coupon. What discount will she get?</p> <p>Tape diagram:</p>	<p>Table:</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; padding: 5px;">\$ for jeans</th> <th style="padding: 5px;">\$ off</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">150</td> <td style="text-align: center; padding: 5px;"><math>\times 0.25</math> ?</td> </tr> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">100</td> <td style="text-align: center; padding: 5px;"><math>\times 0.25</math> 25</td> </tr> </tbody> </table> <p>Equation:</p> $\begin{array}{r} \$150 \\ \times .25 \\ \hline 37.50 \end{array}$ <p style="text-align: center;"><math>150 \times .25 = ?</math>  <span style="border: 1px solid black; padding: 2px;">37.5 = ?</span></p>	\$ for jeans	\$ off	150	$\times 0.25$ ?	100	$\times 0.25$ 25
\$ for jeans	\$ off						
150	$\times 0.25$ ?						
100	$\times 0.25$ 25						
<p>3. Audrey did her HW for 90 minutes. She spent 35% of the time on math. How long did Audrey spend on math?</p> <p>Tape diagram:</p>	<p>Table:</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; padding: 5px;">min on HW</th> <th style="padding: 5px;">min on math</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">90</td> <td style="text-align: center; padding: 5px;"><math>\times 0.35</math> ?</td> </tr> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">100</td> <td style="text-align: center; padding: 5px;"><math>\times 0.35</math> 35</td> </tr> </tbody> </table> <p>Equation:</p> $\begin{array}{r} 90 \\ \times 0.35 \\ \hline 31.50 \end{array}$ <p style="text-align: center;"><math>90 \times 0.35 = ?</math>  <span style="border: 1px solid black; padding: 2px;">31.5 = ?</span></p>	min on HW	min on math	90	$\times 0.35$ ?	100	$\times 0.35$ 35
min on HW	min on math						
90	$\times 0.35$ ?						
100	$\times 0.35$ 35						
<p>4. Hudson needs 50 gallons of paint for his mural project. He has 75% of what he needs. How much paint does Hudson have?</p> <p>Tape diagram:</p>	<p>Table:</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; padding: 5px;">gallons he needs</th> <th style="padding: 5px;">gallons he has</th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">50</td> <td style="text-align: center; padding: 5px;"><math>\times 0.75</math> ?</td> </tr> <tr> <td style="border-right: 1px solid black; text-align: center; padding: 5px;">100</td> <td style="text-align: center; padding: 5px;"><math>\times 0.75</math> 75</td> </tr> </tbody> </table> <p>Equation:</p> $\begin{array}{r} 50 \\ \times 0.75 \\ \hline 37.50 \end{array}$ <p style="text-align: center;"><math>50 \times 0.75 = ?</math>  <span style="border: 1px solid black; padding: 2px;">37.5 = ?</span></p>	gallons he needs	gallons he has	50	$\times 0.75$ ?	100	$\times 0.75$ 75
gallons he needs	gallons he has						
50	$\times 0.75$ ?						
100	$\times 0.75$ 75						

Solve each problem using an equation and a table.

5. The math test today had 25 questions. Emanda got 80% of the questions right. How many questions did Emanda get right?

Tape diagram:

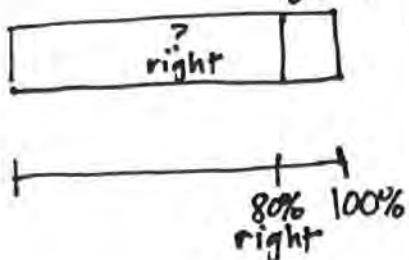


Table:

questions on the test	questions right
25	$\times 0.8$ ?
100	$\times 0.8$ 80

Equation:

$$\begin{array}{r} 25 \\ \times 0.8 \\ \hline 20.0 \end{array}$$

$$25 \times 0.8 = ?$$

$$\boxed{20 = ?}$$

6. The chemical reaction takes 8 minutes to complete. So far it is 32% through. How many minutes must have passed so far?

Tape diagram:

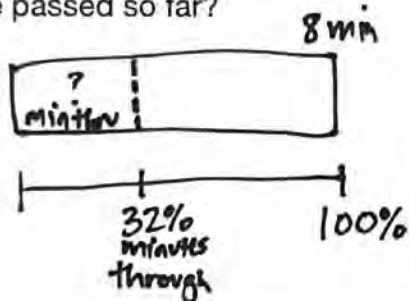


Table:

min to complete	min through
8	$\times 0.32$ ?
100	$\times 0.32$ 32

Equation:

$$\begin{array}{r} 0.32 \\ \times 8 \\ \hline 2.56 \end{array}$$

$$8 \times 0.32 = ?$$

$$\boxed{2.56 = ?}$$

7. The movie is 120 minutes long. Imagine you have watched 30% of it. How many minutes have you watched?

Tape diagram:

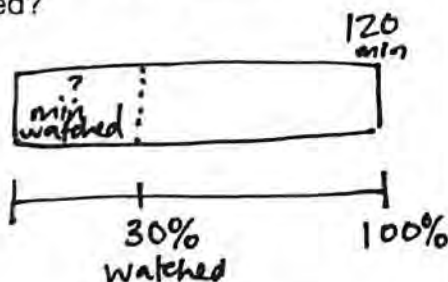


Table:

min of movie	min watched
120	$\times 0.3$ ?
100	$\times 0.3$ 30

Equation:

$$\begin{array}{r} 120 \\ \times 0.3 \\ \hline 36.0 \end{array}$$

$$120 \times 0.3 = ?$$

$$\boxed{36 = ?}$$

8. Jason saved \$80. The bank gave him 1% interest on his savings. How much money did the bank give Jason?

Tape diagram:

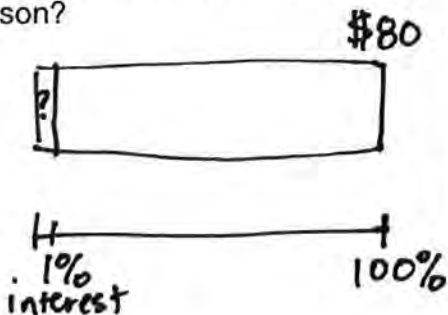


Table:

\$ saved	\$ interest
80	$\times 0.01$ ?
100	$\times 0.01$ 1

Equation:

$$\begin{array}{r} 80 \\ \times 0.01 \\ \hline 0.80 \end{array}$$

$$80 \times 0.01 = ?$$

$$\boxed{0.80 = ?}$$

## **G7 U3 Lesson 3**

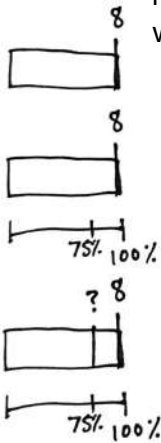
Calculate and interpret the scale factor and constant of proportionality for a proportional relationship.

**G7 U3 Lesson 3 - Today we will find the part, the total or the percent in story problems.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will find the part, the total or the percent in story problems. That's a lot of different kind of story problems so we're going to have to read carefully. Let's dive in!

**Let's Review (Slide 3):** We learned in our last class that we can use the percent like a constant of proportionality. Read this problem silently along with me while I read out loud. "Lisa is at work for 8 hours. While she is gone, her kitties spend 75% of the time watching the birds out the window. How long do the kitties spend watching the birds?" Let's draw a tape diagram first. It says, "Lisa is at work for 8 hours" so I will draw a bar and call it 8 hours at work.



Then it says, "While she is gone her kitties spend 75% of the time watching the birds" so I will draw a line for the percent and I will mark 100 altogether because percent means out of 100. And let's estimate that 75 is around here.

Then it says, "How long do the kitties spend watching the birds?" Now I have to ask myself a key question here and I am going to be asking a version of it for every problem today. The question is, "Is the kitties watching the birds, part of the 8 hours or the whole amount of hours?" The kitties spend part of the time watching the birds so this question is asking for part. I will draw that with a question mark here.

Table:

Whole	part
8	?
100	75

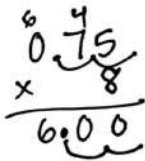
Now we can set up a table with the whole amount in the first column and the part in the second column. The whole column is hours gone. The part column is hours watching birds. I have 8 whole hours gone. I don't know the part watching birds. For the percent, 100 is always the whole amount and the part is 75.

Find the percent:

$$\text{TOTAL} \times \text{PERC} = \text{PART}$$

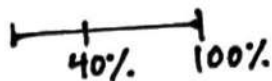
$$8 \times 0.75 = ?$$

I can fill in the percent as the constant of proportionality. I can see this bottom row is  $\times 0.75$  so the top line must be  $\times 0.75$  too. But let's write our equation:  $\text{TOTAL} \times \text{PERCENT} = \text{PART}$ . That is going to be  $8 \times 0.75$  equals question mark.



I am going to multiply on the side here. I can just do  $8 \times 75$ . 8 times 5 is 40. Carry the 4. 8 x 7 is 56 plus 4 is 60. My answer is 600. But I left off two decimal places so I have to put those back in. I get 6.00. If Lisa is at work for 8 hours, then the kitties watch the birds for 6 hours. That sounds very sweet.

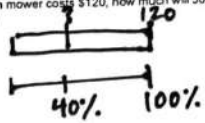
**Let's Talk (Slide 4):** There are two other kinds of problems we could be asked to answer. It says here, "We need to read carefully to be sure about whether we are looking for the part or the whole or the percent. Draw a tape diagram to determine if these two problems are the same." Read the first problem silently along with me while I read it out loud. "John wants to buy a lawn mower for a summer



job. His dad said he would pay for 40% of the lawnmower. If the lawn mower costs \$120, how much will John's dad pay?" Let's reread and draw it sentence by sentence WITH WORDS. The first sentence doesn't give us a lot of numerical information. Let's look at the next sentence. It says, "His dad says he would pay for 40% of the lawnmower." We can draw a line for that.

We mark 40% but we also mark 100%. And let's put some words. 40% is the part that the dad will pay and 100% is the whole amount for the lawnmower.

John wants to buy a lawnmower for a summer job. His dad said he would pay for 40% of the lawnmower. If the lawnmower costs \$120, how much will John's dad pay?



$$120 \times 0.4 = ?$$

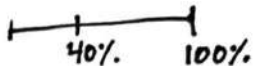
$$\begin{array}{r}
 120 \\
 \times 0.4 \\
 \hline
 480
 \end{array}$$

Then it says, "If the lawn mower costs \$120..." That's the whole cost of the lawn mower so I'll draw a rectangle for that with \$120 for the whole cost. And I have a question mark for the part that the dad will pay.

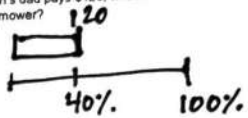
Now, we know we have the equation:  $\text{WHOLE} \times \text{PERCENT} = \text{PART}$ . In this case, 120 is the whole times 0.40 which is the percent equals question mark. I multiply 120 times 0.4 on the side. I get 4 times 0 is 0. 4 times 2 is 8. 4 times 1 is 4. So, 480 but there is one decimal place so I'll put that in. My answer is \$48. John's dad will pay \$48.

Let's see if that's the same as the next problem. Read along with me silently while I read out loud.

"John wants to buy a lawn mower for a summer job. His dad said he would pay for 40% of the lawnmower. If John's dad pays \$120, what is the total cost of the lawn mower?" Now we will draw. The dad still says he'll pay 40%. So we will draw the line with 40% and 100%.



John wants to buy a lawn mower for a summer job. His dad said he would pay for 40% of the lawnmower. If John's dad pays \$120, what is the total cost of the lawn mower?



But look at the next line. This time it says, "If John's dad pays \$120..." This is different! It says what John's dad pays, which is the part not the total whole amount. When I draw this picture, I still have a rectangle but this time the 120 goes as the part that the dad pays. The question mark is the whole amount because the question asked, "What is the total cost of the lawn mower?"

$$\begin{array}{r}
 ? \times 0.4 = 120 \\
 \hline
 0.4 \quad 0.4
 \end{array}$$

We can still use the equation:  $\text{WHOLE} \times \text{PERCENT} = \text{PART}$ . But now the location of the numbers is different because it's a different story. We don't know the whole. I'll write a question mark there. That question is times 0.40, which is the percent. And that equals 120, which is the part this time. I can't figure out the question mark just by looking at it. I am going to divide by 0.4 on each side.

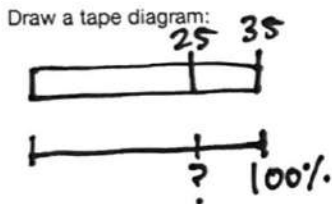
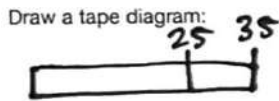
Then over on the side of my paper, I do 120 divided by 0.4. Hopefully from sixth grade you remember that you can divide by a decimal so you shift the decimal point one place in the divisor which means you shift the decimal point one place in the dividend. Now I can do the math. 4 doesn't go into 1 so I

$$\begin{array}{r}
 0300 \\
 0.4 \overline{)1200} \\
 \underline{-12} \\
 00
 \end{array}$$

put a zero. 4 goes into 12 three times. That's minus 12 with a remainder of 0. It is really tempting to stop here but it's really important that I keep the place value. I have two more spaces after that twelve so I need two more zeros. My answer is 300. And that makes sense, right? If the dad pays \$120 then the total cost of the lawn mower must be more than that. It will cost \$300 altogether.

These two problems were NOT the same. In the first problem, the 120 was the total amount. In the second problem, the 120 was the partial amount. We have to draw a picture carefully to represent the story before we can jump into number-crunching.

**Let's Think (Slide 5):** Let's do one more example. This says, "We need to identify what is the part and what is the whole in any percent story problem." Read the problem silently with me while I read out loud. "Rose surveyed 35 of her classmates about whether they were Taylor Swift fans. 25 of the classmates said yes. What percent of the people surveyed are Taylor Swift fans?" We're going to draw, of course! When I read that Rose surveyed 35 people and 25 said yes to being Taylor Swift fans, I can hear the part and the whole. So on my rectangle, I will draw 35 people and then mark 25 as part of the 35.



Write an equation:

$$35 \times ? = \frac{25}{35}$$

The question says, "What percent of the people surveyed are Taylor Swift fans?" Even though there are no obvious numbers in that question, I can still draw a percent line. I know the whole percent is 100% and I can mark a question mark to show that I'm looking for the percent out of 100.

We still have our equation:  $\text{WHOLE} \times \text{PERCENT} = \text{PART}$ . This time I know the whole but I don't know the percent. So I will have 35 times question mark equals 25. To solve for the question mark, I divide by 35 on both sides.

I get some tricky division here that I'm going to have to do on the side. It's actually not that different than when I divide a fraction to find the percent. This is a fraction actually. 25 kids like Taylor Swift out of 35 total kids. So, 35 doesn't go into 25. I put two zeros. Then I add a decimal and annex 2 zeros so I

can divide to the hundredths place. My percent is out of 100 so I will be looking to the hundredths place. This is the first time we've divided by such a big number in this unit. It's not a big deal except you probably can't skip count by 35. I am just going to add up 35's on the side of my paper to see how many times it goes into 250. 35 plus 35 is 70. 70 plus 35 is 105. 105 plus 35 is 140. 140 plus 35 is 175. 175 plus 35 is 210. 210 plus 35 is 245. *Point to each 35 that you added as you count.* I added 1 - 2 - 3 - 4 - 5 - 6 - 7 thirty-fives. So I put

7 on my division. That's minus 245. That leaves me with 5. Now I pull down the zero. 35 goes into 50 one time. I subtract 35. There's going to be a remainder of 15 but I've got what I need. This is 71%. 71% of the people that Rose surveyed said they like Taylor Swift. Alright, Taylor!

**Let's Try It (Slide 6):** Now we will practice solving more percent problems. I will lead you through step by step.



# WARM WELCOME



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**Today we will find the part, the total or the percent in story problems.**

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## Let's Review:

**We can use the percent like a constant of proportionality.**

Lisa is at work for 8 hours. While she is gone, her kitties spend 75% of the time watching the birds out the window. How long do the kitties spend watching the birds?

Tape diagram:

Table:

Find the percent:

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## Let's Talk:

**We need to read carefully to be sure about whether we are looking for the part or the whole or the percent.**

Draw a tape diagram to determine if these two problems are the same.

John wants to buy a lawn mower for a summer job. His dad said he would pay for 40% of the lawnmower. If the lawn mower costs \$120, how much will John's dad pay?

John wants to buy a lawn mower for a summer job. His dad said he would pay for 40% of the lawnmower. If John's dad pays \$120, what is the total cost of the lawn mower?

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 3 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

1. If a student answers 15 out of 20 questions on a quiz, what percentage of questions did they answer correctly?  Tape diagram:	Table:  Equation:
2. Annaleah wants to buy a Lego set that costs \$400. So far she has saved 45% of the price. How much has Annaleah saved?  Tape diagram:	Table:  Equation:
3. The FDA recommends that a person eat 70 grams of protein in a day. So far, Roxanna has eat 50 grams of protein. What percent of the recommendation has Roxanna eaten?	Table:

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Name: \_\_\_\_\_

Solve each problem using a tape diagram, a table and an equation.

<p>1. If a student answers 15 out of 20 questions on a quiz, what percentage of questions did they answer correctly?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>2. Annaleah wants to buy a Lego set that costs \$400. So far she has saved 45% of the price. How much has Annaleah saved?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>3. The FDA recommends that a person eat 70 grams of protein in a day. So far, Roxanna has eat 50 grams of protein. What percent of the recommendation has Roxanna eaten?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>4. Julia's phone loses 15% of its charge in 3 hours. If the phone loses its charge at a constant rate, how long would we expect it to take to lose all its charge?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>

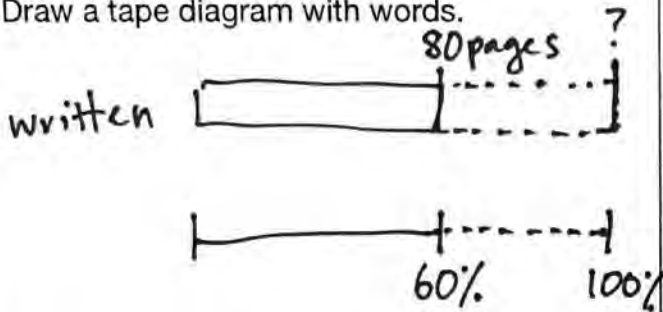


Solve each problem using an equation and a table.

<p>5. The flight from California to Peru is 10 hours. So far, Nick has been on the flight for 3 hours. What percent of the trip has Nick completed?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>6. The directions on Patrick's fish tank says that 40% of water should be salt water and the rest should be fresh water. If Patrick puts in 12 gallons of salt water, how much water must the tank hold?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>7. Tony recycles 10% of his household waste. If he has 25 pounds of waste this week, how many pounds would we expect to be recycled?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>
<p>8. Benny took 40 free throws this year. He made 15 of them. What percent of free throws did Benny make this year?</p> <p>Tape diagram:</p>	<p>Table:</p>          <p>Equation:</p>

Jesse has written 80 pages of her senior paper. She estimates that she is 60% done. How many pages must Jesse be planning to write?

1. Draw a tape diagram with words.



2. Make a table with words.

planning	written
?	80
100	60

3. Write an equation to represent the problem:

whole × percent = part

4. Use the equation to find the missing value.

$$\frac{? \times 0.6}{0.6} = \frac{80}{0.6}$$

$$0.6 \overline{) 80.00}$$

$$\begin{array}{r} 133 \\ - 60 \\ \hline 20 \\ - 18 \\ \hline 20 \\ - 18 \\ \hline 20 \\ - 18 \\ \hline 20 \end{array}$$

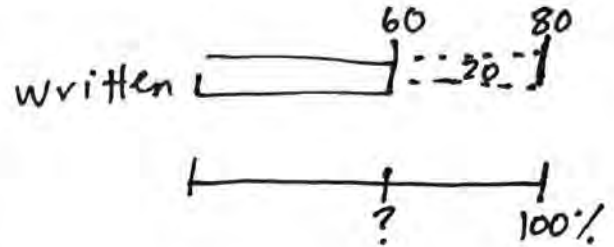
**133**

5. Write your answer in a complete sentence.

Jesse must be planning to write about 133 pages.

Jesse has written 60 pages of her senior paper. She estimates that the total paper will be 80 pages. What percent of the paper has Jesse written?

6. Draw a tape diagram with words.



7. Make a table with words.

whole	part
80	60
100	?

8. Write an equation to represent the problem:

whole × percent = part

9. Use the equation to find the missing value.

$$\frac{80 \times ?}{80} = \frac{60}{80}$$

$$80 \overline{) 60.00}$$

$$\begin{array}{r} 00.75 \\ - 56.00 \\ \hline 400 \\ - 400 \\ \hline 0 \end{array}$$

**75%**

10. Write your answer in a complete sentence.

Jesse has written 75% of the paper.

# Name: ANSWER KEY

Solve each problem using a tape diagram, a table and an equation.

1. If a student answers 15 out of 20 questions on a quiz, what percentage of questions did they answer correctly?

Tape diagram:

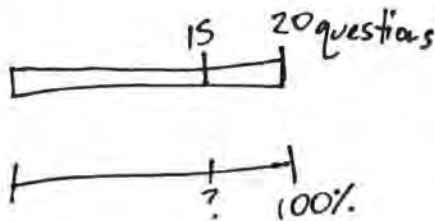


Table:

total	part
20	15
100	?

Equation:

$$\frac{20 \times ?}{20} = \frac{15}{20}$$

**75%**

$$\begin{array}{r} 00.75 \\ 20 \overline{) 15.00} \\ \underline{-1400} \\ 100 \\ \underline{-100} \\ 000 \end{array}$$

2. Annaleah wants to buy a Lego set that costs \$400. So far she has saved 45% of the price. How much has Annaleah saved?

Tape diagram:

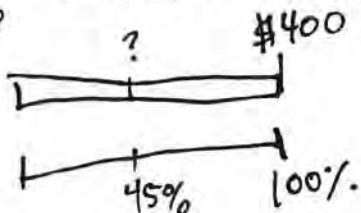


Table:

total	part
400	?
100	45

Equation:

$$400 \times 0.45 = ?$$

**\$180**

$$\begin{array}{r} 0.45 \\ \times 400 \\ \hline 18000 \end{array}$$

3. The FDA recommends that a person eat 70 grams of protein in a day. So far, Roxanna has eat 50 grams of protein. What percent of the recommendation has Roxanna eaten?

Tape diagram:

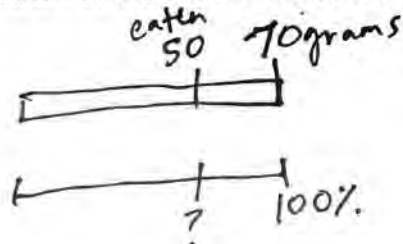


Table:

total	part
70	50
100	?

Equation:

$$\frac{70 \times ?}{70} = \frac{50}{70}$$

**71%**

$$\begin{array}{r} 00.71 \\ 70 \overline{) 50.00} \\ \underline{4900} \\ 100 \\ \underline{-70} \\ 30 \end{array}$$

4. Julia's phone loses 15% of its charge in 3 hours. If the phone loses its charge at a constant rate, how long would we expect it to take to lose all its charge?

Tape diagram:

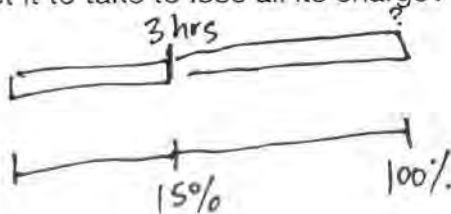


Table:

total	part
?	3
100	15

Equation:

$$\frac{? \times 0.15}{0.15} = \frac{3}{0.15}$$

**20 hours**

$$\begin{array}{r} 0.20 \\ 0.15 \overline{) 3.00} \\ \underline{-300} \\ 00 \end{array}$$

Solve each problem using an equation and a table.

5. The flight from California to Peru is 10 hours. So far, Nick has been on the flight for 3 hours. What percent of the trip has Nick completed?

Tape diagram:

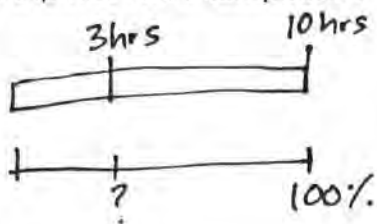


Table:

whole	part
10	3
100	?

$$\begin{array}{r} 0.30 \\ 10 \overline{) 3.00} \\ \underline{-30} \phantom{0} \\ 000 \end{array}$$

Equation:

$$\frac{10 \times ?}{10} = \frac{3}{10}$$

$$\boxed{30\%}$$

6. The directions on Patrick's fish tank says that 40% of water should be salt water and the rest should be fresh water. If Patrick puts in 12 gallons of salt water, how much water must the tank hold?

Tape diagram:

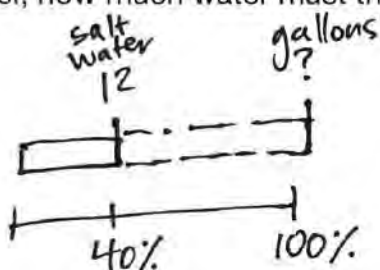


Table:

whole	part
?	12
100	40

$$\begin{array}{r} 0.30 \\ 0.4 \overline{) 12.0} \\ \underline{-12} \phantom{0} \\ 000 \end{array}$$

Equation:

$$\frac{? \times 0.4}{0.4} = \frac{12}{0.4}$$

$$\boxed{30 \text{ gallons}}$$

7. Tony recycles 10% of his household waste. If he has 25 pounds of waste this week, how many pounds would we expect to be recycled?

Tape diagram:

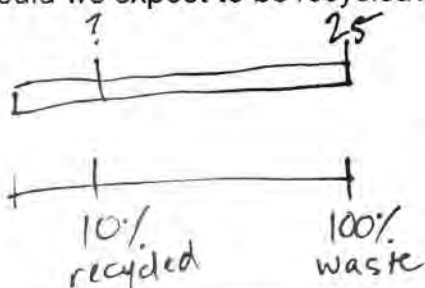


Table:

whole	part
25	?
100	10%

$$\begin{array}{r} 25 \\ \times 0.1 \\ \hline 2.5 \end{array}$$

Equation:

$$25 \times 0.1 = ?$$

$$\boxed{2.5 \text{ pounds}}$$

8. Benny took 40 free throws this year. He made 15 of them. What percent of free throws did Benny make this year?

Tape diagram:

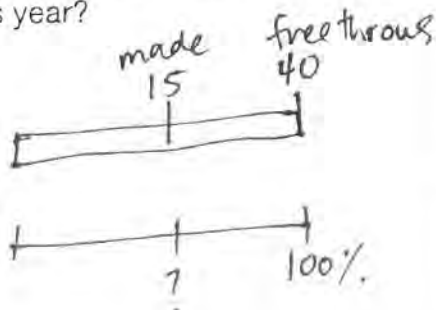


Table:

whole	part
40	15
100	?

$$\begin{array}{r} 00.37 \\ 40 \overline{) 15.00} \\ \underline{12} \phantom{0} \\ 300 \\ \underline{280} \\ 20 \end{array}$$

Equation:

$$\frac{40 \times ?}{40} = \frac{15}{40}$$

$$37\%$$

# **G7 U3 Lesson 4**

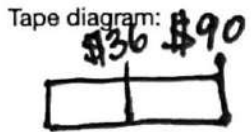
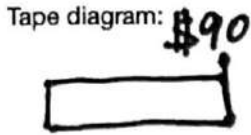
Use fractions to describe increases and decreases.

**G7 U3 Lesson 4 - Today we will use percents to compare quantities.**

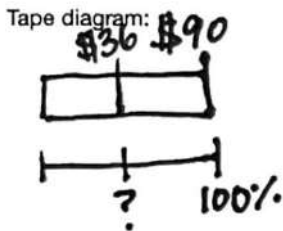
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will use percents to compare quantities. We are going to use the same steps we used in our last lesson to a new idea.

**Let's Review (Slide 3):** We already know we can use a multiplication equation to solve percent problems. Read along silently with eyes while I read out loud, "Lisa has saved \$90. She is going to spend \$36 on a hat. What percent of Lisa's money does she plan to spend?" We know that we always draw a tape diagram for any word problem. That helps us figure out what is the whole and what is the part. It says, "Lisa has saved \$90." So I am going to draw a box and call it \$90.



Now it says, "She is going to spend \$36 on a hat." I know that the money she is going to spend is part of the money she has saved so I am going to draw it as a part.



Now the question is asking, "What percent of Lisa's money does she plan to spend?" So I have to draw a percent line. I know 100 is the whole percent but I don't know the percent that represents the part. So I will put a question mark.

17

whole saved	part for hat
90	36
100	?

When I make a table with this information, it is super interesting because the question mark is in a different spot. I know 90 is the money saved and 36 is the money to spend. 100 is the whole. I don't know the percent. Which also means I don't know the constant of proportionality. In other words, I don't know what to multiply each row in my table by.

Equation:

$$\text{whole} \times \text{percent} = \text{part}$$

$$90 \times ? = 36$$

$$\frac{90 \times ?}{90} = \frac{36}{90}$$

Luckily I have the equation that relates all these numbers: WHOLE x PERCENT = PART. I put 90 as the whole times question mark equals 36. Then to solve for the question mark, I have to divide by 90 on each side.

$$90 \overline{) 36.00}$$

$$\underline{-360}$$

$$000$$

Let me draw my division box to the side. It's 36 divided by 90. That's zero but then I add a decimal and annex two zeros. I can do 90 into 360 just like 9 into 36. That's 4. So, I get 4 and subtract 360. There's zero so the next digit is zero. And there's my answer in the hundredths place. Lisa is going to spend 40% of her money. That is helpful to hear as a percent. For example, we can hear that it is less than half. Percents are super helpful like that. So let's see another way we can use them.

**Let's Talk (Slide 4):** We can also use percents to compare two quantities. In this case, we aren't going to have a part and a whole like one number is part of another. But we can think of them that way to solve the problem. Let me show you what I mean. Read the problem silently while I read out loud.



“Lisa has saved \$90. Her sister has saved \$36. What percent of Lisa’s money does Lisa’s sister have?” This problem has the exact same numbers, 90 and 36. We know that percent is 40% because we just did the problem. The point here is that Lisa has save money and her sister has saved money. Her sister’s money isn’t PART of Lisa’s money. There is no part and whole. But we can still compare them and percents are still really helpful to have a sense of how their size compares to each other. Let’s draw a picture. It says, “Lisa has saved \$90.” I am going to draw a rectangle and label it \$90.

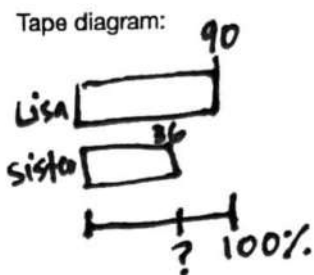
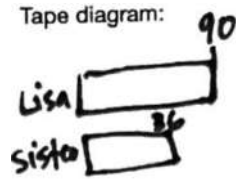
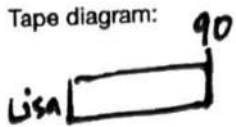


Table:

Lisa	sister
90	36
100	?

Next it says, “Her sister has saved \$36.” We already said that the sister’s money is not part of Lisa’s money. So, I am not going to draw it as a part. I am going to have to draw a whole new rectangle that is lined up right at the beginning. It’s shorter, right?

But the question is still a percent question so I still need a percent line. Here’s the very most important thing. I have to be clear on the percent OF WHAT to know where the 100% is. It says what percent OF LISA’S MONEY. So we are going to use Lisa’s money as if it’s the whole because it’s what we want to find the percent of. We call that the referent because it’s the number that the percent is referring to. That number is like 100% and now I can mark a question mark for where the other number is a percent of it.

The table looks the same as it did before. I have 90 compare to 36. Then I have 100 and I don’t know the percent compared to 100. But we can see how this is still like a proportion even though it’s not a part and a whole.

So, I am going to take our old equation that was WHOLE x PERCENT = PART and I am just going to think of it as REFERENT x PERCENT = QUANTITY. There isn’t a part and whole. But there are two numbers and one is the referent, the number that we’re referring or comparing to. The other is just the quantity we’re comparing to the referent. I plug in the numbers like always. 90 x question mark equals 36. We divide by 90 on both sides.

$$\text{referent} \times \text{percent} = \text{quantity}$$

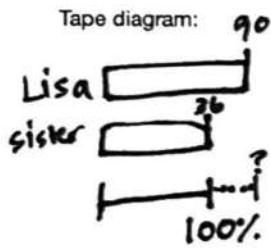
$$\frac{90 \times ?}{90} = \frac{36}{90}$$

$$0.40 = 40\%$$

$$\begin{array}{r} 90 \overline{) 36.00} \\ \underline{-360} \\ 000 \end{array}$$

And we already know from the last slide that this is 0.40. So 40%. Lisa’s sister has saved 40% of what Lisa has saved.

**Let’s Think (Slide 5):** The most important thing is that we must be very careful to identify what quantity is the referent. Because that’s the quantity that goes where the whole normally goes in the equation. Look at how the problem below has been changed. Instead of it saying “What percent of Lisa’s money does Lisa’s sister have?” Now it says, “What percent of Lisa’s sister’s money does Lisa have?” It is a percent of Lisa’s sister’s money. So Lisa’s sister’s money is the referent now and we’re comparing Lisa’s money to it.



The tape diagram would look the same to start. Lisa has \$90. Lisa's sister has saved \$36. But now when we draw the percent, it's a percent of Lisa's sister's money so \$36 is like the whole total amount. That is where I will put 100%. Lisa's money is going to be more than 100% then. Just like Lisa saved more than what her sister did. I will put the question mark here.

Table:

sister	Lisa
36	90
100	?

My table will have to look different because now the sister is the referent so I put it like this.

Equation:

$$\frac{36 \times ?}{36} = \frac{90}{36}$$

Let's write this as an equation. It is still REFERENT x PERCENT = QUANTITY. But this time the referent is 36 times question mark equal 90. I have to divide by 36 on each side.

$$\begin{array}{r} 1 \\ 36 \\ +36 \\ \hline 72 \\ +36 \\ \hline 108 \\ +36 \\ \hline 144 \\ +36 \\ \hline 180 \end{array}$$

$$\begin{array}{r} 02.50 \\ 36 \overline{)90.00} \\ \underline{-72} \phantom{00} \\ 180 \\ \underline{-180} \\ 000 \end{array}$$

Let me write 90 divided by 36 on the side of my paper. I don't know how to multiply or divide 36 so I write the addition on the side of my paper. 36 plus 36 is 72. If I add another 36, I get 108. So that means that 36 goes into 90 two times. I subtract 72 and get 18. I need hundredths for percents. So I will add a decimals and annex two zeros.

I will pull down one zero. Now I need 36 into 180. I will keep adding 36s. 108 plus 36 is 144. 144 plus 36 is 180. Yay! I will count up my 36s. *Point to the 36s that you added as you count them up.* There are 1 - 2 - 3 - 4 - 5! I get 5 and subtract 180. We're done.

Remember that we look in the hundredths place for a percent. So this is 2.50, which means that Lisa saved 250% of what her sister saved. Do you see how powerful percents can be? If you tell me Lisa saved 250% of what her sister saved, I realize that Lisa saved a lot more than her sister.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will use percents to compare quantities.**

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 **Let's Review:**

**We can use a multiplication equation to solve percent problems.**

Lisa has saved \$90. She is going to spend \$36 on a hat. What percent of Lisa's money does she plan to spend?

Tape diagram:

Table:

Equation:

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 **Let's Talk:**

**We can also use percents to compare two quantities.**

Lisa has saved \$90. Her sister has saved \$36. What percent of Lisa's money does Lisa's sister have?

Tape diagram:

Table:

Equation:

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## Let's Think:

**We must be very careful to identify what quantity is the referent.**

Lisa has saved \$90. Her sister has saved \$36. What percent of ~~Lisa's~~ <sup>Lisa's sister's</sup> money does ~~Lisa's sister~~ have?  
Lisa

Tape diagram:

Table:

Equation:

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_ G7 U3 Lesson 4|- Let's Try It

<p><b>Bob ran for 75 minutes. Joe ran for 66% of the time that Bob ran for. How long did Bob run?</b></p>	<p><b>Tom ran for 60 minutes. Joe ran for 36 minutes. What percent of Tom's time did Joe run?</b></p>
<p>1. Draw a tape diagram with words.</p>	<p>6. Draw a tape diagram with words.</p>
<p>2. Make a table with words.</p>	<p>7. Make a table with words.</p>
<p>3. Write an equation to represent the problem:</p> <p>_____</p>	<p>8. Write an equation to represent the problem:</p> <p>_____</p>
<p>4. Use the equation to find the missing value.</p>	<p>9. Use the equation to find the missing value.</p>

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 US Lesson 4 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

<p>1. In Ms. Ramish's class there are 24 kids who do soccer after school, 15% as many kids who do fencing after school. How many kids do fencing after school?</p> <p>Tape diagram:</p>	<p>Table:</p> <p>Equation:</p>
<p>2. Alex scored 40 points in a game. Ben scores 50 points. What percent of Ben's score did Alex score?</p> <p>Tape diagram:</p>	<p>Table:</p> <p>Equation:</p>
<p>3. Sammy's Bake Shop sells 20% as many pastries as loaves of bread. If Sammy sold 12 pastries, how many loaves of bread did he sell?</p> <p>Tape diagram:</p>	<p>Table:</p> <p>Equation:</p>

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**Bob ran for 75 minutes. Joe ran for 66% of the time that Bob ran for. How long did Bob run?**

1. Draw a tape diagram with words.

2. Make a table with words.

3. Write an equation to represent the problem:

\_\_\_\_\_

4. Use the equation to find the missing value.

5. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Tom ran for 60 minutes. Joe ran for 36 minutes. What percent of Tom's time did Joe run?**

6. Draw a tape diagram with words.

7. Make a table with words.

8. Write an equation to represent the problem:

\_\_\_\_\_

9. Use the equation to find the missing value.

10. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Name: \_\_\_\_\_

Solve each problem using a tape diagram, a table and an equation.

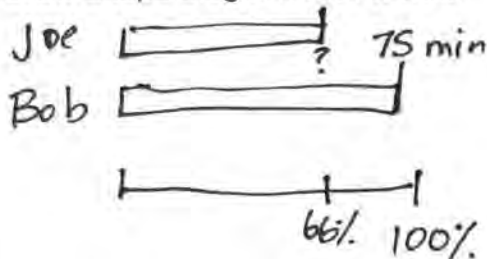
<p>1. In Ms. Ramish's class there are 24 kids who do soccer after school. 25% as many kids who do fencing after school. How many kids do fencing after school?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>2. Alex scored 40 points in a game. Ben scores 50 points. What percent of Ben's score did Alex score?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>3. Sammy's Bake Shop sells 20% as many pastries as loaves of bread. If Sammy sold 12 pastries, how many loaves of bread did he sell?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>4. Sarah solved 12 math problems. Michael solved 15 math problems. What percent of Sarah's problems did Michael solve?</p> <p>Tape diagram:</p>	<p>Equation:</p>

Solve each problem using an equation and a table.

<p>5. A company has 80 employees who work from home. It has 40% as many employees who work in the office. How many employees work in the office?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>6. Mary read a 200 page book. Lucy read a 220 page book. What percent of Mary's read did Lucy read?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>7. Sadie ran 5 miles. Robbie ran 10 miles. What percent of Robbie's distance did Sadie run?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>8. The library has 30 nonfiction series in its collection. This is 10% as many fiction series as it has. How many fiction series must it have in its collection?</p> <p>Tape diagram:</p>	<p>Equation:</p>

Bob ran for 75 minutes. Joe ran for 66% of the time that Bob ran for. How long did Bob run?

1. Draw a tape diagram with words.



2. Make a table with words.

Bob	Joe
75	?
100	66

3. Write an equation to represent the problem:

referent × percent = quantity

4. Use the equation to find the missing value.

$$75 \times 0.66 = ?$$

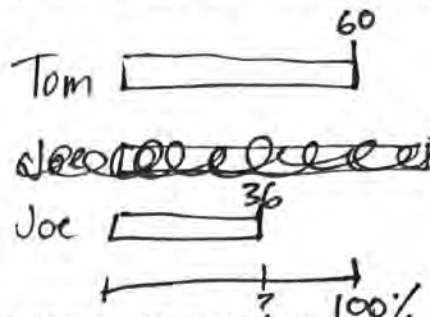
$$\begin{array}{r} 0.66 \\ \times 75 \\ \hline 330 \\ 4620 \\ \hline 49.55 \end{array}$$

5. Write your answer in a complete sentence.

Bob ran for 49.55  
minutes.

Tom ran for 60 minutes. Joe ran for 36 minutes. What percent of Tom's time did Joe run?

6. Draw a tape diagram with words.



7. Make a table with words.

Tom	Joe
60	36
100	?

8. Write an equation to represent the problem:

referent × percent = quantity

9. Use the equation to find the missing value.

$$60 \times ? = \frac{36}{60}$$

$$\begin{array}{r} 00.60 \\ 60 \overline{) 36.00} \\ \underline{-360} \\ 000 \end{array}$$

10. Write your answer in a complete sentence.

Joe ran 60% of the time  
Tom ran.

# Name: ANSWER KEY

Solve each problem using a tape diagram, a table and an equation.

<p>1. In Ms. Ramish's class there are 24 kids who do soccer after school. 75% as many kids who do fencing after school. How many kids do fencing after school?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent whole</th> <th>quantity part</th> </tr> </thead> <tbody> <tr> <td>24</td> <td>?</td> </tr> <tr> <td>100</td> <td>75</td> </tr> </tbody> </table> <p>Equation: <math>24 \times 0.75 = ?</math></p> <p><b>6</b></p> <p><i>Handwritten calculations:</i>  <math display="block">\begin{array}{r} 24 \\ \times 0.75 \\ \hline 120 \\ 1680 \\ \hline 1800 \\ \hline 1800 \\ - 1200 \\ \hline 600 \end{array}</math></p>	referent whole	quantity part	24	?	100	75
referent whole	quantity part						
24	?						
100	75						
<p>2. Alex scored 40 points in a game. Ben scores 50 points. What percent of Ben's score did Alex score?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent</th> <th>quantity</th> </tr> </thead> <tbody> <tr> <td>50</td> <td>40</td> </tr> <tr> <td>100</td> <td>?</td> </tr> </tbody> </table> <p>Equation: <math>50 \cdot ? = 40</math></p> <p><b>80%</b></p> <p><i>Handwritten calculations:</i>  <math display="block">\begin{array}{r} 0.80 \\ 50 \overline{)40.00} \\ \underline{-400} \\ 000 \end{array}</math></p>	referent	quantity	50	40	100	?
referent	quantity						
50	40						
100	?						
<p>3. Sammy's Bake Shop sells 20% as many pastries as loaves of bread. If Sammy sold 12 pastries, how many loaves of bread did he sell?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent</th> <th>quantity</th> </tr> </thead> <tbody> <tr> <td>?</td> <td>12</td> </tr> <tr> <td>100</td> <td>20</td> </tr> </tbody> </table> <p>Equation: <math>? \times 0.2 = \frac{12}{0.2}</math></p> <p><b>60 loaves</b></p> <p><i>Handwritten calculations:</i>  <math display="block">\begin{array}{r} 0.60 \\ 0.2 \overline{)12.00} \\ \underline{-120} \\ 00 \end{array}</math></p>	referent	quantity	?	12	100	20
referent	quantity						
?	12						
100	20						
<p>4. Sarah solved 12 math problems. Michael solved 15 math problems. What percent of Sarah's problems did Michael solve?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent</th> <th>quantity</th> </tr> </thead> <tbody> <tr> <td>12</td> <td>15</td> </tr> <tr> <td>100</td> <td>?</td> </tr> </tbody> </table> <p>Equation: <math>\frac{12 \times ?}{12} = \frac{15}{12}</math></p> <p><b>125%</b></p> <p><i>Handwritten calculations:</i>  <math display="block">\begin{array}{r} 0.125 \\ 12 \overline{)15.00} \\ \underline{-120} \\ 30 \\ \underline{-24} \\ 60 \end{array}</math></p>	referent	quantity	12	15	100	?
referent	quantity						
12	15						
100	?						

Solve each problem using an equation and a table.

<p>5. A company has 80 employees who work from home. It has 40% as many employees who work in the office. How many employees work in the office?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent</th> <th>quantity</th> </tr> </thead> <tbody> <tr> <td>80</td> <td>?</td> </tr> <tr> <td>100</td> <td>40</td> </tr> </tbody> </table> <p>Equation:</p> $80 \times 0.4 = ?$ <p><b>32 employees</b></p>	referent	quantity	80	?	100	40
referent	quantity						
80	?						
100	40						
<p>6. Mary read a 200 page book. Lucy read a 220 page book. What percent of Mary's read did Lucy read?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent</th> <th>quantity</th> </tr> </thead> <tbody> <tr> <td>200</td> <td>220</td> </tr> <tr> <td>100</td> <td>?</td> </tr> </tbody> </table> <p>Equation:</p> $\frac{200 \times ?}{200} = \frac{220}{200}$ <p><b>110%</b></p>	referent	quantity	200	220	100	?
referent	quantity						
200	220						
100	?						
<p>7. Sadie ran 5 miles. Robbie ran 10 miles. What percent of Robbie's distance did Sadie run?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent</th> <th>quantity</th> </tr> </thead> <tbody> <tr> <td>10</td> <td>5</td> </tr> <tr> <td>100</td> <td>?</td> </tr> </tbody> </table> <p>Equation:</p> $\frac{10 \times ?}{10} = \frac{5}{10}$ <p><b>50%</b></p>	referent	quantity	10	5	100	?
referent	quantity						
10	5						
100	?						
<p>8. The library has 30 nonfiction series in its collection. This is 10% as many fiction series as it has. How many fiction series must it have in its collection?</p> <p>Tape diagram:</p>	<p>Table:</p> <table border="1"> <thead> <tr> <th>referent</th> <th>quantity</th> </tr> </thead> <tbody> <tr> <td>?</td> <td>30</td> </tr> <tr> <td>100</td> <td>10</td> </tr> </tbody> </table> <p>Equation:</p> $\frac{? \times 0.1}{0.1} = \frac{30}{0.1}$ <p><b>300 series</b></p>	referent	quantity	?	30	100	10
referent	quantity						
?	30						
100	10						



## **G7 U3 Lesson 5**

Use decimals to describe increases and decreases Lesson 6 Find percent increases and decreases when given an original amount.

**G7 U3 Lesson 5 - Today we will find percent increase and decrease.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will find percent increase and decrease. This is a really common way to use percents because people are really familiar with what an amount out of 100 means. 90% is a lot out of 100 and 10% is a lot out of 100. So it's like a kind of shortcut for talking about size. And we really like to talk about the size of an increase, which is something going up. Or the size of a decrease, which is something going down.

**Let's Review (Slide 3):** We know a table helps us set up our percent equation. It's about to get REALLY important to label your table. You'll see what I mean. Read the problem below silently along with me while I read out loud. "There was a pair of jeans that cost \$80. The owner said they would decrease the price by \$10. What was the percent decrease in price?" We're going to draw. First thing, the jeans cost \$80. I will draw a rectangle and label it \$80. Now I am going to right one more really important word and that is "original." I am writing that because all our work about increase and decrease is going to be dependent on the ORIGINAL number we started with, meaning the first number we started with. That number is the referent. It is the foundation of our comparison. It is what all of our increases and decreases and percents REFER to. Back to drawing. The owner said they would decrease the price by \$10. \$10 is part of the \$80. We want to find the percent so we draw the percent line.

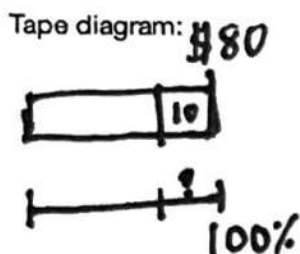


Table:

referent	decrease
80	10
100	?

On my table, I know the referent and the decrease. They are \$80 and \$10. I know 100% is the original price. But I don't know the percent. I don't know the multiplier for my table.

Equation:

$$\frac{80 \times ?}{80} = \frac{10}{80}$$

So in my equation, REFERENT x PERCENT = QUANTITY, I get 80 times question mark equals 10. I divide both sides by 80.

$$\begin{array}{r} 00.12 \\ 80 \overline{)10.00} \\ \underline{-80} \phantom{0} \\ 200 \\ \underline{-160} \\ 40 \end{array}$$

10 divided by 80 is zero. So I add a decimal annex two zeros to the hundredths place. 80 goes into 100 one time. I subtract 80 and get 20. Now I pull down a zero. 80 goes into 200 two times. That's 160 so I subtract 160. I get 40. I could keep going if I want to but I see 0.12, which is 12%.

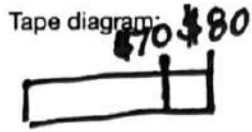
$$00.125 = 12.5\%$$

$$\begin{array}{r} 00.125 \\ 80 \overline{)10.000} \\ \underline{-80} \phantom{00} \\ 200 \phantom{0} \\ \underline{-160} \phantom{0} \\ 400 \\ \underline{-400} \\ 000 \end{array}$$

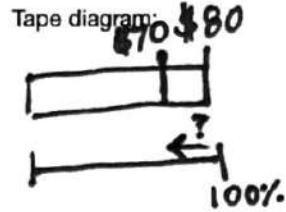
I like these even numbers so let's keep going. I annex one more zero and bring it down so I have 80 into 400. That's five since 8 times 5 is 40. I subtract 400 and have no remainder. Now I see 0.125 so the percent is 12.5%. This is just a more exact answer.

Notice what we did here because it is going to be important for the next problem. We had the referent and the decrease on the table. That's how we found the percent decrease. Now let's see if we can make that kind of table for the next problem.

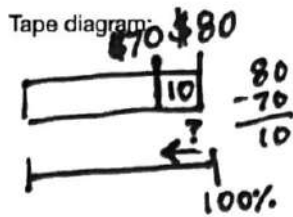
**Let's Talk (Slide 4):** This says, "Sometimes there is an extra step to find a percent increase or decrease." And that's what's going to happen here. It's the same story we just read only written a little differently. Read along with me while I read it out loud, "There was a pair of jeans that cost \$80. The owner said they would decrease the price to \$70. What was the percent decrease in price?"



Let's draw first. I have a rectangle labeled \$80.



Now the owner said they would decrease the price to \$70 so I will draw that in the rectangle. When I go to draw the percent, we have something special. I can mark 100% as the original price, which is the referent. But now I'm not looking for the new percent of the original price. I am looking for the percent decrease. I am looking for the percent that it went down. This is sooooo important for me to notice when I read. I am going to put a question mark here on my percent line because I want the percent decrease.



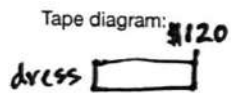
If I try to make a table with referent and decrease this time, I don't have enough information yet. I have 80 as the referent. I have 100% and I want to find the percent. But I don't know the decrease in price. I just know what the new price was. This is the extra step that we said we might have to do. I am going to find a value for the decrease, which is this shaded part. I do 80 minus 70 equals 10. This piece is 10 and NOW I can put the decrease on the table, which is 10.

Table:

ref	dec
80	10
100	?

Now this problem is just like the one we just did. It is the same actually, where we would write the equation 80 times question mark equals 10. The big idea is that if we want to find the percent increase or decrease we might have to do an extra step.

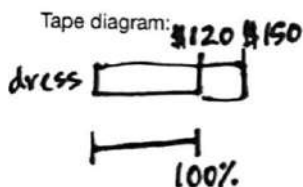
**Let's Think (Slide 5):** It is the same thing for this next slide. Sometimes there is an extra step to find a percent increase or decrease. Read the problem silently along with me while I read out loud. "There was a dress that Amelia really wanted that cost \$120. When she went back to the store, they had raised the price to \$150. What was the percent increase in price?" Let's draw.



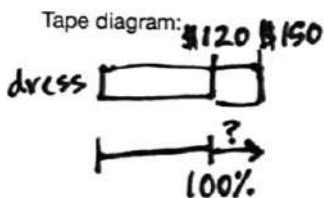
First, I have the original price of the dress. Remember, that's my referent. That's the whole that I am basing all my percents on. It is super important that I write the word original here.



Now, it says, "When she went back to the store, they had raised the price to \$150." I am going to have to draw that as the bar getting longer. It is going up to \$150. I will even write new so we know that's the new price.



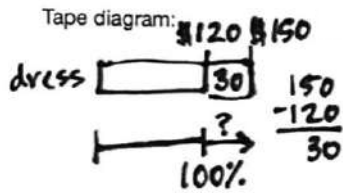
When I go to draw my percent line. The 100% goes to the \$120. That's the original amount, the referent.



And I want to find the percent increase. So I want to find an imaginary extra amount here.

ref	inc
120	
100	?

I can already see that the value of this increase is something I'm going to need to know. We can check on our table. I have the referent and the increase columns. The referent is \$120. I haven't figured out the increase yet. I only know that 100% corresponds to the referent. And I am trying to find this percent increase.



Let's go back and find this part of my picture. In other words, I need to know the actual increase in dollars before I can find the percent increase. That's where that extra step comes in. That is the most important thing. So, I will do 150 minus 120, which is 30.

ref	inc
120	30
100	?

I will write that in the box and in the table.

Equation:

$$\frac{120 \times ?}{120} = \frac{30}{120}$$

Now I have what I need to write an equation. It is REFERENT x PERCENT = QUANTITY. I know the referent is 120 x question mark equals 30. To find the percent, I have to divide each side by 120.

$$\begin{array}{r} 00.2 \\ 120 \overline{)30.00} \\ \underline{-240} \phantom{0} \\ 60 \end{array}$$

Let's do the math over to the side. 30 divided by 120 is zero so I will add a decimal and annex two zeros. I have no idea how to divide by 120 so I will add up some 120's on the side of my paper. 120 plus 120 is 240. I can't add another 120 without going over. That was two 120s so I write a 2 and subtract 240. That leaves 60.

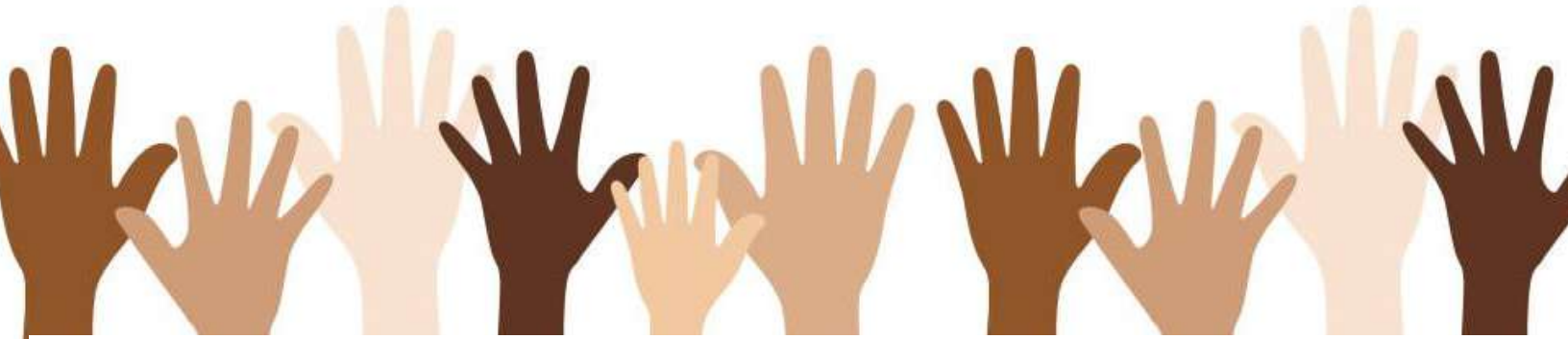
$$\begin{array}{r} 00.25 \\ 120 \overline{)30.00} \\ \underline{-240} \phantom{0} \\ 600 \\ \underline{-600} \\ 000 \end{array}$$

Pull down the zero. I am going to have to add some more 120s. 240 plus 120 is 360. 360 plus 120 is 480. 480 plus 120 is 600. Hooray, that is going to work. Let me count up these 120s. There are five! I will write down the 5 and subtract 600. That leaves zero. My decimal is 0.25 so 25%. There was a 25% increase in the price.

The most important thing is that we aren't going to take short cuts here. When we draw a picture and write a table with words, we can see if there is an extra step and do the math to get the right answer. The equation we need is always the multiplication equation so that's not really the hard part.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will find percent increase and decrease.**

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 **Let's Review:**

**We know a table helps us set up our percent equation.**

There was a pair of jeans that cost \$80. The owner said they would decrease the price by \$10. What was the percent decrease in price?

Tape diagram:

Table:

Equation:

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 **Let's Talk:**

**Sometimes there is an extra step to find a percent increase or decrease.**

There was a pair of jeans that cost \$80. The owner said they would decrease the price to \$70. What was the percent decrease in price?

Tape diagram:

Table:

Equation:

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## Let's Think:

**Sometimes there is an extra step to find a percent increase or decrease.**

There was a dress that Amelia really wanted that cost \$120. When she went back to the store, they had raised the price to \$150. What was the percent increase in price?

Tape diagram:

Table:

Equation:

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_ G7 U3 Lesson 5 - Let's Try It

<p>Emily has 200 books in her home library. She donated some books and now she has 160 books. What is the percent decrease in her collection?</p>	<p>The scientist was studying a tomato plant that had 12 leaves. After one week, it had 18 leaves. What was the percent increase in the number of leaves?</p>
<p>1. Draw a tape diagram with words.</p>	<p>6. Draw a tape diagram with words.</p>
<p>2. Make a table with words.</p>	<p>7. Make a table with words.</p>
<p>3. Write an equation to represent the problem:</p> <p>_____</p>	<p>8. Write an equation to represent the problem:</p> <p>_____</p>
<p>4. Use the equation to find the missing value.</p>	<p>9. Use the equation to find the missing value.</p>

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 US Lesson 5 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

1. A car's original price was 20K. Now it is 16K. What was the percent decrease in the car's price?  Tape diagram:	Equation:
2. Today is 64 degrees. Yesterday it was 80 degrees. What was the percent decrease in temperature from yesterday to today?  Tape diagram:	Equation:
3. A restaurant had 90 customers on Monday. On Tuesday, they had 120 customers. What was the percent increase in customers?  Tape diagram:	Equation:

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**Emily has 200 books in her home library. She donated some books and now she has 160 books. What is the percent decrease in her collection?**

1. Draw a tape diagram with words.

2. Make a table with words.

3. Write an equation to represent the problem:

\_\_\_\_\_

4. Use the equation to find the missing value.

5. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**The scientist was studying a tomato plant that had 12 leaves. After one week, it had 18 leaves. What was the percent increase in the number of leaves?**

6. Draw a tape diagram with words.

7. Make a table with words.

8. Write an equation to represent the problem:

\_\_\_\_\_

9. Use the equation to find the missing value.

10. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Name: \_\_\_\_\_

Solve each problem using a tape diagram, a table and an equation.

<p>1. A car's original price was 20K. Now it is 16K. What was the percent decrease in the car's price?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>2. Today is 64 degrees. Yesterday it was 80 degrees. What was the percent decrease in temperature from yesterday to today?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>3. A restaurant had 90 customers on Monday. On Tuesday, they had 120 customers. What was the percent increase in customers?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>4. A store had 50 apples in stock. It sold 10 apples. What is the percent decrease in stock?</p> <p>Tape diagram:</p>	<p>Equation:</p>

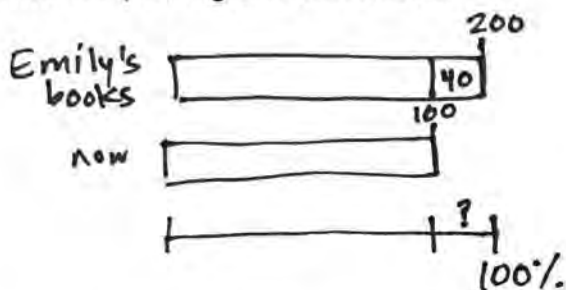
Solve each problem using an equation and a table.

<p>5. Sarah ran 6 miles this week. Last week she ran 5 miles. What is the percent increase in the distance she ran?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>6. Sarai had 200 followers on social media. After gaining 50 new followers, what is the percent increase?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>7. A store sold 450 items last month. This month, they sold 500 items. What was the percent increase in sales?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>8. A student got 40 points on their math test. After retaking the test, they got 44 points. What was the percent increase in their score?</p> <p>Tape diagram:</p>	<p>Equation:</p>

# Name: ANSWER KEY

Emily has 200 books in her home library. She donated some books and now she has 160 books. What is the percent decrease in her collection?

1. Draw a tape diagram with words.



2. Make a table with words.

whole	decrease
200	40
100	?

3. Write an equation to represent the problem:

$$\text{referent} \times \frac{\text{percent}}{100} = \text{decrease}$$

4. Use the equation to find the missing value.

$$200 \times \frac{?}{100} = 40$$

$$\frac{200}{20} \times \frac{?}{200} = \frac{40}{200}$$

$$200 \overline{) 40.00}$$

$$\underline{-200}$$

$$200$$

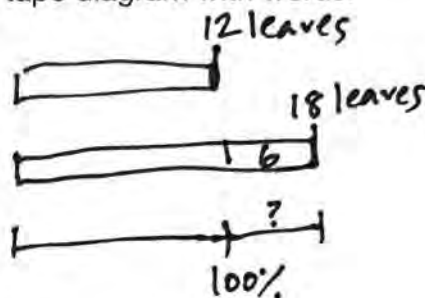
20%

5. Write your answer in a complete sentence.

The percent decrease in her collection was 20%.

The scientist was studying a tomato plant that had 12 leaves. After one week, it had 18 leaves. What was the percent increase in the number of leaves?

6. Draw a tape diagram with words.



7. Make a table with words.

original	new increase
12	6
100	?

8. Write an equation to represent the problem:

$$\text{referent} \times \frac{\text{percent}}{100} = \text{decrease}$$

9. Use the equation to find the missing value.

$$12 \times \frac{?}{100} = 6$$

$$\frac{12}{12} \times \frac{?}{100} = \frac{6}{100}$$

$$12 \overline{) 6.00}$$

$$\underline{-60}$$

$$00$$

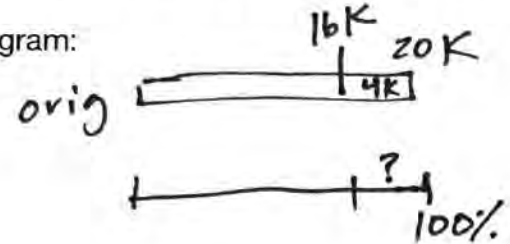
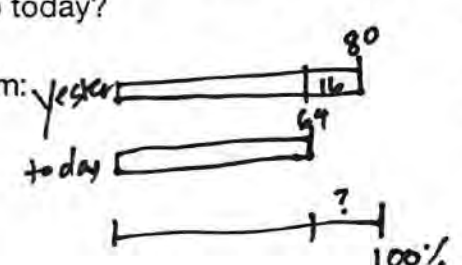
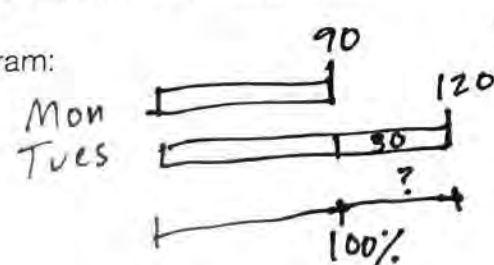
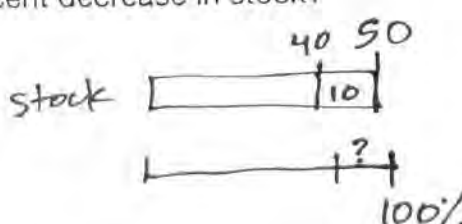
50%

10. Write your answer in a complete sentence.

The percent increase in the leaves was 50%.



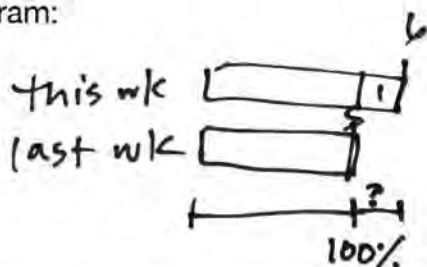
Solve each problem using a tape diagram, a table and an equation.

<p>1. A car's original price was 20K. Now it is 16K. What was the percent decrease in the car's price?</p> <p>Tape diagram:</p> 	<p>Equation:</p> $20 \times \frac{?}{100} = \frac{4}{100}$ $\frac{20 \times ?}{20} = \frac{4}{20}$ $? = \frac{4}{20}$ $? = 0.20$ $? = 20\%$
<p>2. Today is 64 degrees. Yesterday it was 80 degrees. What was the percent decrease in temperature from yesterday to today?</p> <p>Tape diagram:</p> 	<p>Equation:</p> $80 \times \frac{?}{100} = \frac{16}{100}$ $\frac{80 \times ?}{80} = \frac{16}{80}$ $? = \frac{16}{80}$ $? = 0.20$ $? = 20\%$
<p>3. A restaurant had 90 customers on Monday. On Tuesday, they had 120 customers. What was the percent increase in customers?</p> <p>Tape diagram:</p> 	<p>Equation:</p> $90 \times \frac{?}{100} = \frac{30}{100}$ $\frac{90 \times ?}{90} = \frac{30}{90}$ $? = \frac{30}{90}$ $? = 0.33$ $? = 33\%$
<p>4. A store had 50 apples in stock. It sold 10 apples. What is the percent decrease in stock?</p> <p>Tape diagram:</p> 	<p>Equation:</p> $50 \times \frac{?}{100} = \frac{10}{100}$ $\frac{50 \times ?}{50} = \frac{10}{50}$ $? = \frac{10}{50}$ $? = 0.20$ $? = 20\%$

Solve each problem using an equation and a table.

5. Sarah ran 6 miles this week. Last week she ran 5 miles. What is the percent increase in the distance she ran?

Tape diagram:



Equation:

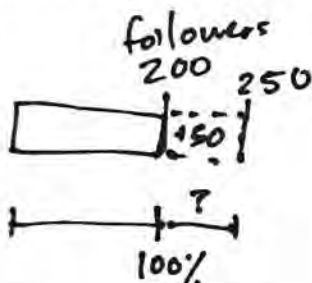
$$\frac{6 \times ?}{6} = \frac{1}{6}$$

$$16\%$$

$$\begin{array}{r} 0.16 \\ 6 \overline{) 1.00} \\ \underline{40} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

6. Sarai had 200 followers on social media. After gaining 50 new followers, what is the percent increase?

Tape diagram:



Equation:

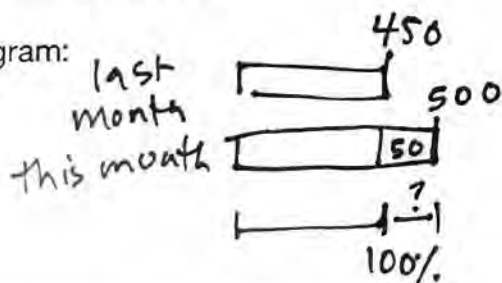
$$\frac{200 \times ?}{200} = \frac{50}{200}$$

$$25\%$$

$$\begin{array}{r} 0.25 \\ 200 \overline{) 50.00} \\ \underline{400} \\ 1000 \\ \underline{1000} \\ 0000 \end{array}$$

7. A store sold 450 items last month. This month, they sold 500 items. What was the percent increase in sales?

Tape diagram:



Equation:

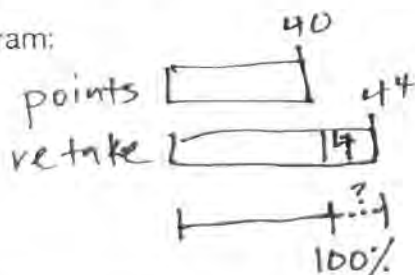
$$\frac{450 \times ?}{450} = \frac{50}{450}$$

$$11\%$$

$$\begin{array}{r} 0.11 \\ 450 \overline{) 50.00} \\ \underline{450} \\ 500 \\ \underline{450} \\ 50 \end{array}$$

8. A student got 40 points on their math test. After retaking the test, they got 44 points. What was the percent increase in their score?

Tape diagram:



Equation:

$$\frac{40 \times ?}{40} = \frac{4}{40}$$

$$10\%$$

$$\begin{array}{r} 0.10 \\ 40 \overline{) 4.00} \\ \underline{40} \\ 00 \end{array}$$

# **G7 U3 Lesson 6**

Use double number lines to solve problems about percent increases and decreases.

## G7 U3 Lesson 6 - Today we will find the new amount given the percent increase or decrease.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will find the new amount given the percent increase or decrease. We are going to have the same steps that we've always had. We're just going to need to be sure to read the problem very carefully.

**Let's Review (Slide 3):** We know how to draw a picture for percent increase or decrease problems.

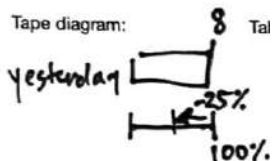
Tape diagram:



Read this one with me silently while I read it out loud, "Dan worked 8 hours yesterday. Today he worked 25% less. How much did Dan decrease his work?"

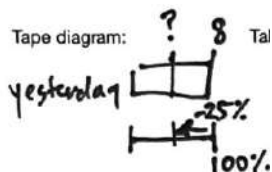
We start with a picture with words. I am going to draw a bar and label it as 8 hours yesterday.

Tape diagram:



It says he worked 25% less. I will draw a line to represent the percent. I have 100% and I have 25%.

Tape diagram:



I want to know the part that is 25% here. I am going to mark it as part of the hours from yesterday with a question mark.

whole	part
8	?
100	25

I can label a table with whole and part in this case because we have the whole amount he worked and then the part that he worked less. I have 8 hours total and I don't know the part. I have 100% and 25% is the part.

Equation:

$$8 \times 0.25 = ?$$

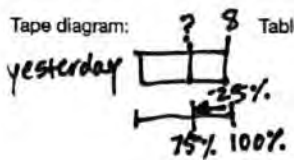
I can see the multiplier because I've been given the percent. It is 0.25. I will write my equation: WHOLE x PERCENT = PART. So  $8 \times 0.25 =$  question mark.

$$\begin{array}{r} 4 \\ 0.25 \\ \times 8 \\ \hline 2.00 \end{array}$$

I am going to multiply  $8 \times 0.25$  off to the side. 8 times 5 is 40; I carry the 4. 8 times 2 is 16 plus 4 is 20. I get 200. But I have two decimal places in my problem so I put two decimal places in my answer. My final answer is 2.00 or two.

For my final answer sentence, I can say, "Dan decreased his work by 2 hours." Keep this problem in mind because we are going to change the question and see how it changes our problem.

**Let's Talk (Slide 4):** Here's our new problem. It says at the top, "If the problem is asking for a new quantity after a percent increase or decrease, we will need to add or subtract." Let's see how this goes. Some words are crossed out. Read along with me silently while I read out loud, "Dan worked 8 hours yesterday. Today he worked 25% shorter. How much did Dan work?" Interesting! So now, it doesn't want to know how much Dan decreased his work. But how much he worked - as in, how much did he work after the decrease. A lot of this is the same problem, right? I am still going to find 25% of 8. That was 2 on the last slide. But there's going to be another step. Let's draw a picture to see. I draw a rectangle for the 8 hours. That's the original time, the referent. I want to draw my percent line and I know 100% lines up with the 8 whole hours. But this time, just to show that is decreasing which means going down, I am going to draw an arrow going back this way and label it 25%.



Let's think for a minute. If I am going down 25% from 100, what is going to be left? It will be 75% left. I can mark that here. I could even solve this problem by finding 75% of 8! We can put this on a table now. But remember we had to do that extra step. And the question marks helped us see that.

whole	part
8	?
100	75

On my table, I have the whole and the part. I put 8. I don't know the part. But the whole percent is 100% and the part is 75%.

Equation:  $8 \times 0.75 = ?$

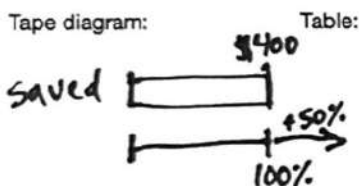
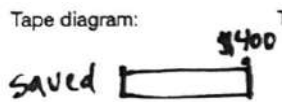
Now I can write an equation. It is whole x percent equals part.  $8 \times 0.75$  equals question mark.

$$\begin{array}{r} 0.75 \\ \times 8 \\ \hline 600 \end{array}$$

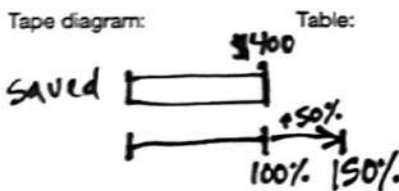
Now I will multiply  $8 \times 0.75$  off to the side.  $8 \times 5$  is 40 and I carry the 4.  $8$  times 7 is 56 plus 4 is 60. So I get 600. I put in my decimal point for the two place values and I get 6.00 or 6.

So, here's the big idea... On the last slide, if I just need to know what the percent decrease or increase was, I can just do my regular multiplication equation. But on this slide, if I need to find the NEW final quantity AFTER the increase or decrease, I am going to have an extra step of adding or subtracting to find a new percent before I set up my table or equation.

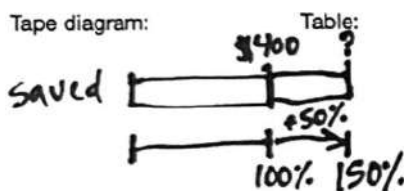
**Let's Think (Slide 5):** Let's try another one just to be clear. Remember, "If the problem is asking for a new quantity after a percent increase or decrease, we will need to add or subtract." Read this problem silently along with me while I read it out loud. "Audrey has saved \$400. She needs 50% more money to buy her new computer. What must the cost of the new computer be?" Let's draw our diagram with words. I will start with a rectangle that I label \$400 saved.



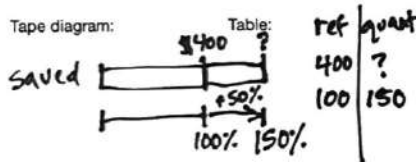
She needs 50% more. Okay, I will start with the 100% line. But now I need 50% more. That means I am going to extend my line. I can draw an arrow and label it 50%.



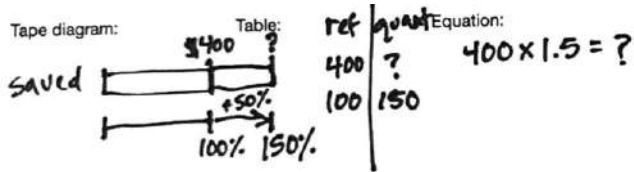
Now, here is where the most important thinking needs to come in. The question is not asking how much more money she needs. It is asking for the final cost. So I need to know the final percent. I will add 100 plus 50 and get 150%.



That means that I will be extending my rectangle too. The question is asking for the final price. So I will my question mark here at the end where the final price would be. We can see how that question mark corresponds to 150% not 50%. That's the percent we're going to need to use.



Now I can make a table. This time I don't have a whole and a part. I have my referent and my quantity. 400 is the referent. I'm trying to find the new final quantity. 100% is the referent and 150% is the new final.



I can see that the multiplier will be 1.5. Now I am going to write my equation: REFERENT x PERCENT = QUANTITY so  $400 \times 1.5$  equals question mark.

$$\begin{array}{r} 1.5 \\ \times 400 \\ \hline 600.0 \end{array}$$

Let's multiply  $400 \times 1.5$  on the side. I'm just going to do  $1.5 \times 4$  and pop the two zeros back on later.  $4 \times 5$  is 20 and I carry the 2. Then  $4 \times 1$  is 4 plus 2 is 6. That's 60 with two more zeros is 6000. But now I need my decimal which is one place value over. So my final answer is 600.0 or six hundred. That makes sense as a final answer because she is going to save more money.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.



# WARM WELCOME



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**Today we will find the new amount given the percent increase or decrease.**

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 **Let's Review:**

**We know how to draw a picture for percent increase or decrease problems.**

Dan worked 8 hours yesterday. Today he worked 25% less. How much did Dan decrease his work?

Tape diagram:

Table:

Equation:

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 **Let's Talk:**

**If the problem is asking for a new quantity after a percent increase or decrease, we will need to add or subtract.**

Dan worked 8 hours yesterday. Today he worked 25% shorter. How much did Dan ~~decrease his~~ work?

Tape diagram:

Table:

Equation:

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## Let's Think:

If the problem is asking for a new quantity after a percent increase or decrease, we will need to add or subtract.

Audrey has saved \$400. She needs 50% more money to buy her new computer. What must the cost of the new computer be?

Tape diagram:

Table:

Equation:

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## Let's Try It:

We will do it together step by step!

Name: \_\_\_\_\_ G7 U3 Lesson 6 - Let's Try It

<p>Last year, the revenue of ABC Company was 15 million dollars. This year, the revenue increased by 20%. What was the increase in revenue?</p> <p>1. Draw a tape diagram with words.</p>   <p>2. Make a table with words.</p>   <p>3. Write an equation to represent the problem:</p> <p>_____</p> <p>4. Use the equation to find the missing value.</p>	<p>Last year, the revenue of ABC Company was 15 million dollars. This year, the revenue increased by 20%. What was the revenue of ABC Company this year?</p> <p>6. Draw a tape diagram with words.</p>   <p>7. Make a table with words.</p>   <p>8. Write an equation to represent the problem:</p> <p>_____</p> <p>9. Use the equation to find the missing value.</p>
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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 6 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

1. The price of a laptop was \$800. After a 15% discount, what was the new price?  Tape diagram:	Equation:
2. Sarah saved \$200 in January. In February she saved 25% less. How much less money did she save in February than January?  Tape diagram:	Equation:
3. Tom drank 5 cups of water yesterday. Today he drank 60% more water. How many cups of water did Tom drink today?  Tape diagram:	Equation:

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**Last year, the revenue of ABC Company was 15 million dollars. This year, the revenue increased by 20%. What was the increase in revenue?**

1. Draw a tape diagram with words.

2. Make a table with words.

3. Write an equation to represent the problem:

\_\_\_\_\_

4. Use the equation to find the missing value.

5. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Last year, the revenue of ABC Company was 15 million dollars. This year, the revenue increased by 20%. What was the revenue of ABC Company this year?**

6. Draw a tape diagram with words.

7. Make a table with words.

8. Write an equation to represent the problem:

\_\_\_\_\_

9. Use the equation to find the missing value.

10. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Name: \_\_\_\_\_

Solve each problem using a tape diagram, a table and an equation.

<p>1. The price of a laptop was \$800. After a 15% discount, what was the new price?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>2. Sarah saved \$200 in January. In February she saved 25% less. How much less money did she save in February than January?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>3. Tom drank 5 cups of water yesterday. Today he drank 60% more water. How many cups of water did Tom drink today?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>4. When Georgio first learned to type, he could type 90 words per minute. Now he can type 120 words per minute. What is the percent increase in Georgio's typing speed?</p> <p>Tape diagram:</p>	<p>Equation:</p>

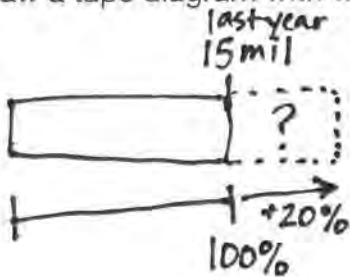


Solve each problem using an equation and a table.

<p>5. The price of a concert ticket was \$50. After a 10% service fee, what was the final price of the ticket?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>6. Emily usually uses 4 cups of sugar in her cookie recipe. But now she has decided to increase the sugar by 25%. How much extra sugar is Emily going to add to her recipe?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>7. So far, the Jones family has driven 200 miles on their road trip. Tomorrow, they will drive 20% farther. What is the total distance they will have driven by the end of the day tomorrow?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>8. Last week, Lisa used her cell phone for 12 hours. This week, her usage was down 25%. How many hours was she on her cell this week?</p> <p>Tape diagram:</p>	<p>Equation:</p>

Last year, the revenue of ABC Company was 15 million dollars. This year, the revenue increased by 20%. What was the increase in revenue?

1. Draw a tape diagram with words.



2. Make a table with words.

original	increase
15	$\times 0.2$ ?
100	$\times 0.2$ 20

3. Write an equation to represent the problem:

$$\text{original} \times \text{percent increase} = \text{percent increase}$$

~~15 \times 0.2 = ?~~

4. Use the equation to find the missing value.

$$15 \times 0.2 = ?$$

$$3.0 = ?$$

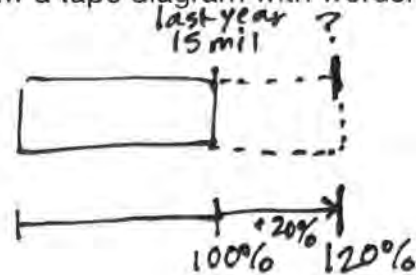
$$\begin{array}{r} 15 \\ \times 0.2 \\ \hline 3.0 \end{array}$$

5. Write your answer in a complete sentence.

The revenue increased  
by 3 million dollars.

Last year, the revenue of ABC Company was 15 million dollars. This year, the revenue increased by 20%. What was the revenue of ABC Company this year?

6. Draw a tape diagram with words.



7. Make a table with words.

original	new
15	$\times 1.2$ ?
100	$\times 1.2$ 120

8. Write an equation to represent the problem:

$$\text{original} \times \text{percent} = \text{new}$$

9. Use the equation to find the missing value.

$$15 \times 1.2 = ?$$

$$\begin{array}{r} 15 \\ \times 1.2 \\ \hline 30 \\ 150 \\ \hline 18.0 \end{array}$$

10. Write your answer in a complete sentence.

The new revenue was  
18 million dollars.

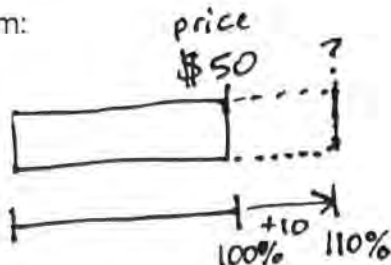
Solve each problem using a tape diagram, a table and an equation.

<p>1. The price of a laptop was \$800. After a 15% discount, what was the new price?</p> <p>Tape diagram:</p>	<p>Equation:</p> $800 \times 0.85 = ?$ $\begin{array}{r} 0.85 \\ \times 800 \\ \hline 680.00 \end{array}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">\$680</div>
<p>2. Sarah saved \$200 in January. In February she saved 25% less. How much less money did she save in February than January?</p> <p>Tape diagram:</p>	<p>Equation:</p> $200 \times 0.25 = ?$ $\begin{array}{r} 0.25 \\ \times 200 \\ \hline 50.00 \end{array}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">\$50</div>
<p>3. Tom drank 5 cups of water yesterday. Today he drank 60% more water. How many cups of water did Tom drink today?</p> <p>Tape diagram:</p>	<p>Equation:</p> $5 \times 1.6 = ?$ $\begin{array}{r} 1.6 \\ \times 5 \\ \hline 8.0 \end{array}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">8 cups</div>
<p>4. When Georgio first learned to type, he could type 90 words per minute. Now he can type 120 words per minute. What is the percent increase in Georgio's typing speed?</p> <p>Tape diagram:</p> $\begin{array}{r} 0.33 \\ - 90 \\ \hline 30 \end{array}$	<p>Equation:</p> $\frac{90 \times ?}{90} = \frac{30}{90}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; font-size: 2em;">33%</div> $\begin{array}{r} 0.33 \\ 90 \overline{) 30.00} \\ \underline{270} \phantom{0} \\ 300 \\ \underline{-270} \\ 300 \\ \underline{-270} \\ 300 \end{array}$

Solve each problem using an equation and a table.

5. The price of a concert ticket was \$50. After a 10% service fee, what was the final price of the ticket?

Tape diagram:



Equation:

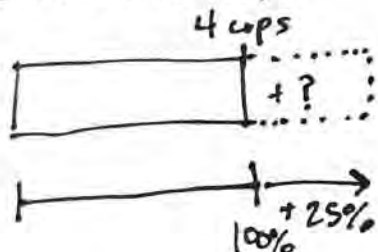
$$50 \times 1.1 = ?$$

$\boxed{\$55}$

$$\begin{array}{r} 1.1 \\ \times 50 \\ \hline 55.0 \end{array}$$

6. Emily usually uses 4 cups of sugar in her cookie recipe. But now she has decided to increase the sugar by 25%. How much extra sugar is Emily going to add to her recipe?

Tape diagram:



Equation:

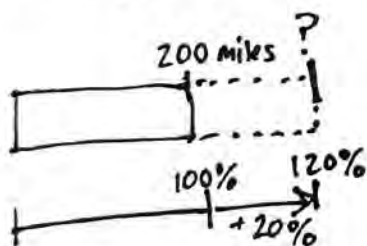
$$4 \times 0.25 = ?$$

$\boxed{1 \text{ cup}}$

$$\begin{array}{r} 0.25 \\ \times 4 \\ \hline 1.00 \end{array}$$

7. So far, the Jones family has driven 200 miles on their road trip. Tomorrow, they will drive 20% farther. What is the total distance they will have driven by the end of the day tomorrow?

Tape diagram:



Equation:

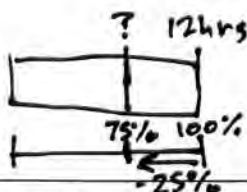
$$200 \times 1.2 = ?$$

$\boxed{240 \text{ miles}}$

$$\begin{array}{r} 1.2 \\ \times 200 \\ \hline 240.0 \end{array}$$

8. Last week, Lisa used her cell phone for 12 hours. This week, her usage was down 25%. How many hours was she on her cell this week?

Tape diagram:



Equation:

$$12 \times 0.75 = ?$$

$\boxed{9 \text{ hours}}$

$$\begin{array}{r} 12 \\ \times 0.75 \\ \hline 9.00 \end{array}$$

# **G7 U3 Lesson 7**

Use equations to represent percent increases and decreases.

**G7 U3 Lesson 7 - Today we will find the original amount given the percent increase or decrease.**

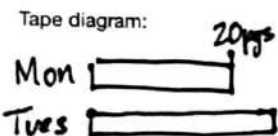
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will find the original amount given the percent increase or decrease.

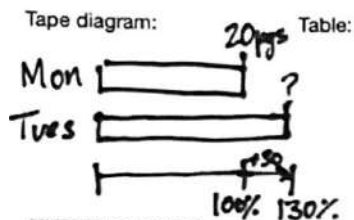
**Let's Review (Slide 3):** We know how to find a new amount when we are given the percent change.



Read the story silently with me while I read it out loud, "Lelac wrote 20 pages of her novel on Monday. The next day, she was able to write 30% more than she did the day before. How much did Lelac write on Tuesday?" I am going to draw a rectangle and call it 20 pages and write Monday.



It says the next day she wrote 30% more. I don't know how many pages that is but I can draw a rectangle and put a question mark to find out how much it is. I will write Tuesday.



I also need to draw my percent line. 100% is the original amount on Monday and then this extra amount is 30% so I will draw an arrow showing a 30% increase.

Mon	Tues
20	?
100	130

I can either find out what 30% is and add it or find 130%. Let's do it that way today. In my table, I will have Monday and Tuesday. 20 pages is Monday. I don't know Tuesday. But for percentages, I have 100% and 130%.

Equation:

$$20 \times 1.3 = ?$$

I can see my multiplier must be  $\times 1.30$  because I turn 130% into a decimal. Now my equation is clear: REFERENT  $\times$  PERCENT = QUANTITY so  $20 \times 1.3$  equals question mark.

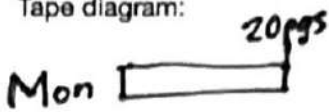
$$\begin{array}{r} 1.3 \\ \times 20 \\ \hline 26.0 \end{array}$$

Let's do the math to the side. I will do  $1.3 \times 2$ . 2 times 3 is 6. 2 times 1 is 2. I need to put that zero back on. So I get 260. But now I need to put that decimal point in there with one space so my final answer is 26.0 or twenty six.

On this slide, we were given the original amount and we had to find the new amount. Now let's think about what we should do if we were given the new amount and we had to find the original amount.

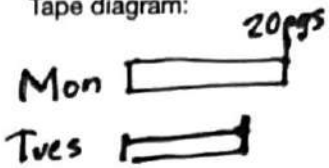
**Let's Talk (Slide 4):** We can imagine that to find the original amount, we will need to work backwards. It's kind of like we are doing the opposite of the slide we just did. This can maybe feel confusing. We might worry, "When do we go forward and when do we go backwards?" The good news is that as long as we are using labels to keep the numbers organized then when we plug them into the equation with a question, it will be easier to see what number crunching to do. Let's try this example. The problem might sound the same but it will be different. Read along silently with me while I read out loud, "Lelac wrote 20 pages of her novel on Monday. This was up 25% from the day before. How

Tape diagram:



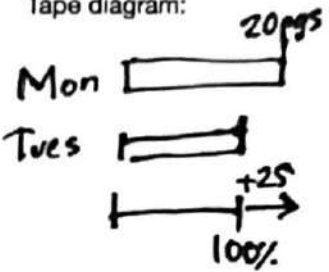
much did Lelac write on Sunday?" Let's draw! I am going to make a rectangle and label it 20 pages for Monday.

Tape diagram:



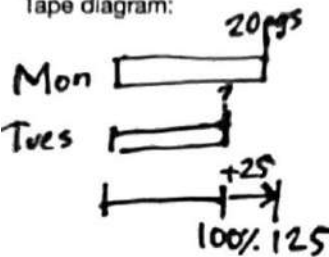
It is says, "This was up 25% from the day before." So, this rectangle is 25% higher than the day before. I will draw a smaller rectangle and label it the day before.

Tape diagram:



I need a percent line and here's where it gets tricky. It said Monday was 25% up. That means it's up from 100%. So I'm not going to draw a line for 100% up to Monday's total. I am going to draw a line for 100% up to the day before. Then I can draw an arrow to show, it goes 25% up. This requires super careful reading. "This was up" makes me think, "What was up?" And I have to remember that I had just read about Monday. So Monday was up from another day. Monday isn't the referent. Monday is the new amount. The day before is the referent. The day before is the original amount.

Tape diagram:



So, if I am going to make a table, I have to choose what bits I want to solve for but it's probably easiest to just put Sunday and Monday. I don't know Sunday's amount. I know Monday was 20 pages. I know Sunday was the 100%. Monday's is not 25%. It's 25% UP from 100%. So I have to put 125%. Remember that there's always at least two steps with percent increase problems and here was one of those steps.

Table:

Tues	Mon
?	20
100	125

I will put Tuesday and Monday on the table with the percents.

Equation:

$$\frac{? \times 1.25}{1.25} = \frac{20}{1.25}$$

Now I have what I need to write the equation. The multiplier is  $\times 1.25$  because that's turning the percent to a decimal. Now I can write REFERENT  $\times$  PERCENT = QUANTITY and this time it is question mark times 1.25 equals 20. Now this is interesting! Now I can see that we can't just multiply like we have been because it is not clear what to multiply by. Now in the setup of the equation itself, the working backwards that we need to do is obvious. I am going to divide by 1.25 on each side.

$$1.25 \overline{) 2000}$$

Let's write out this division in a separate place. I have 20 divided by 1.25. I can't divide by a decimal very easily so I will shift the decimal two places to the right of my divisor and my dividend. Now it is 2000 divided by 125.

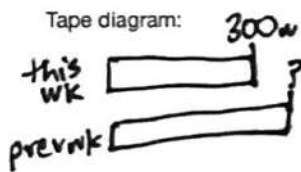


$$\begin{array}{r} 0016 \\ 125 \overline{) 2000} \\ \underline{-125} \phantom{0} \\ 750 \\ \underline{-750} \\ 000 \end{array}$$

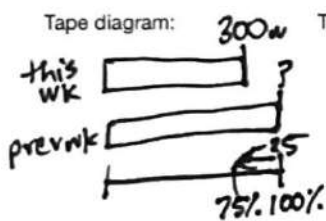
$$\begin{array}{r} 125 \cdot 500 \\ +125 \cdot 625 \\ \hline 250 \phantom{00} \\ +125 \cdot 750 \\ \hline 375 \phantom{00} \\ +125 \cdot 750 \\ \hline 500 \phantom{00} \end{array}$$

125 doesn't go into 2 so I'll put a zero. It doesn't go into 20 so I'll put a zero. It goes into 200, one time. I subtract 125 and get 75. I pull down a zero and get 750. This is pretty big to divide so I will add up some 125s on the side of my paper. 125 plus 125 is 250. 250 plus 125 is 375. 375 plus 125 is 500. 500 plus 125 is 625. 625 plus 125 is 750. That's as high as we need to go. Let's count all those 125s. *Point as you count them up.* 1 - 2 - 3 - 4 - 5 - 6! I write 6 and subtract 750. There's none left and I have my answer - 16!

Let's look back at our drawing and see if that makes sense. If Lelac was up 25% from the day before, might she have read 16 the day before? Sounds reasonable!



**Let's Think (Slide 5):** These ideas can apply to percent decrease as well. Read along with me silently while I read out loud. "The widget machine pumped out 300 widgets per hour this week. That is down 25% from the previous week. How many widgets must it have pumped out last week?" Let's draw! I am going to make a rectangle and call it 300 widgets this week.



It says, "That is down 25% from the previous week." So this rectangle is down from another rectangle. That means I need to draw a bigger rectangle for it to be down from and label it "last week." To be 25% down, that means there needs to be a 100% to be down from. So I have to draw that here and then I can draw in my 25% down from there. I want to figure out this percent. So I do  $100 - 25 = 75$ . And now I have the information I need for my table and equation.

Table:

prev	this
?	300
100	75

I am going to put "previous week" for the 1st column and "this week" in the next column. I don't know this previous week. I know this week is 300. The previous week is the 100% and this week is 75%.

Equation:

$$\frac{? \times 0.75}{0.75} = \frac{300}{0.75}$$

I can see my multiplier will be  $\times 0.75$ . I am going to set up my equation. REFERENT  $\times$  PERCENT = QUANTITY. When I substitute, I get question mark times 0.75 equals 300. And I can work backwards to solve by dividing each side by 0.75.

$$\begin{array}{r} 00400 \\ 0.75 \overline{) 30000} \\ \underline{-300} \phantom{00} \\ 00000 \end{array}$$

Let's write that division off to the side. 300 divided by 0.75. I can't divide by a decimal so I shift it for both numbers and it becomes 30000 divided by 75. 75 doesn't go into 3 so I put a zero. 75 doesn't go into 30 so I put a zero. To find how many times it goes into 300, I am going to add on the side of my paper. 75 + 75 is 150. 150 plus 75 is 225. 225 plus 75 is 300. That's our answer. Let's count up the 75s. I see four of them. So I put 4 in my division. I subtract 300. I have zero left. But I can't just stop at 4. There is zero in the next place and zero in the next place and my answer is 400. We want to look back in our picture and think about the story to see if that makes sense. It said the 300 widgets is down from the previous week. 300 is down from 400 so that seems reasonable. Great work!

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will find the original amount given the percent increase or decrease.**

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 **Let's Review:**

**We know how to find a new amount when we are given the percent change.**

Lelac wrote 20 pages of her novel on Monday. The next day, she was able to write 30% more than she did the day before. How much did Lelac write on Tuesday?

Tape diagram:

Table:

Equation:

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 **Let's Talk:**

**We can imagine that to find the original amount, we will need to work backwards.**

Lelac wrote 20 pages of her novel on Monday. This was up 25% from the day before. How much did Lelac write on Sunday?

Tape diagram:

Table:

Equation:

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## Let's Think:

**These ideas can apply to percent decrease as well.**

The widget machine pumped out 300 widgets per hour this week. That is down 25% from the previous week. How many widgets must it have pumped out last week?

Tape diagram:

Table:

Equation:

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_ G7 U3 Lesson 7 - Let's Try It

<p><b>Bob's salary is 10% higher than Mark's salary. If Bob's salary is \$25 per hour, what must Mark's salary be?</b></p> <p>1. Draw a tape diagram with words.</p>   <p>2. Make a table with words.</p>   <p>3. Write an equation to represent the problem:</p> <p>_____</p> <p>4. Use the equation to find the missing value.</p>	<p><b>Bob's salary is 10% lower than Ken's salary. If Bob's salary is \$25 per hour, what must Ken's salary be?</b></p> <p>6. Draw a tape diagram with words.</p>   <p>7. Make a table with words.</p>   <p>8. Write an equation to represent the problem:</p> <p>_____</p> <p>9. Use the equation to find the missing value.</p>
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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 7 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

1. The height of a tree was 25% higher than its height the previous year. If the tree is 30 feet tall, how tall was it in the previous year?  Tape diagram:	Equation:
2. Jennifer picked 15% more apples today than yesterday. If she picked 100 apples yesterday, how many did she pick today?  Tape diagram:	Equation:
3. The shirt is on sale for \$25. The price was decreased 50% from the original price. What was the original price?  Tape diagram:	Equation:

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Name: \_\_\_\_\_

Solve each problem using a tape diagram, a table and an equation.

<p>1. The height of a tree was 25% higher than its height the previous year. If the tree is 30 feet tall, how tall was it in the previous year?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>2. Jennifer picked 15% more apples today than yesterday. If she picked 100 apples yesterday, how many did she pick today?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>3. The shirt is on sale for \$25. The price was decreased 50% from the original price. What was the original price?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>4. Lisa got 30 mosquito bites last week. After she sprayed her garden this week, the bites decreased 10%. How many fewer bites did Lisa get this week than last week?</p> <p>Tape diagram:</p>	<p>Equation:</p>

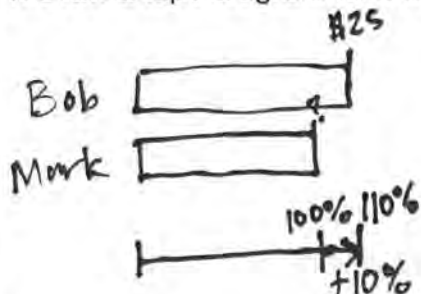


Solve each problem using an equation and a table.

<p>5. The choir director is expecting 50% as many people to attend this year's concert as last year's concert. If she is expecting 150 people, how many people attended last year?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>6. The choir director is expecting 50% more people to attend this year's concert as last year's concert. If she is expecting 150 people, how many people attended last year?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>7. Emily baked 40 cupcakes for the school bake sale last year. They sold out so fast that she has decided to bake 50% more this year. How many additional cupcakes is Emily planning to bake?</p> <p>Tape diagram:</p>	<p>Equation:</p>
<p>8. Emily baked 40 cupcakes for the school bake sale last year. They sold out so fast that she has decided to bake 50% more this year. How many cupcakes is Emily planning to bake?</p> <p>Tape diagram:</p>	<p>Equation:</p>

Bob's salary is 10% higher than Mark's salary. If Bob's salary is \$25 per hour, what must Mark's salary be?

1. Draw a tape diagram with words.



2. Make a table with words.

Bob	Mark
25	?
110	100

Mark	Bob
?	25
100	110

3. Write an equation to represent the problem:

Referent × percent = Quantity

4. Use the equation to find the missing value.

$$\frac{? \times 1.1}{1.1} = \frac{25}{1.1}$$

$$? = 22.72$$

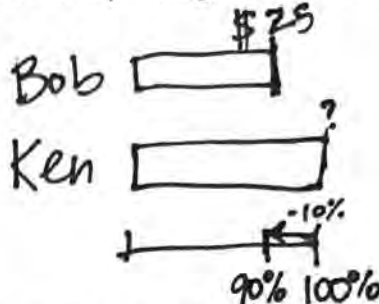
$$\begin{array}{r} 0.2272 \\ 1.1 \overline{) 25.0000} \\ \underline{-22} \phantom{00} \\ 030 \phantom{00} \\ \underline{-22} \phantom{00} \\ 80 \phantom{00} \\ \underline{-77} \phantom{00} \\ 30 \phantom{00} \end{array}$$

5. Write your answer in a complete sentence.

Mark's salary must be  
 \$22.72

Bob's salary is 10% lower than Ken's salary. If Bob's salary is \$25 per hour, what must Ken's salary be?

6. Draw a tape diagram with words.



7. Make a table with words.

Ken	Bob
?	25
100	90

8. Write an equation to represent the problem:

Referent × percent = Quantity

9. Use the equation to find the missing value.

$$\frac{? \times 0.9}{0.9} = \frac{25}{0.9}$$

$$? = 27.77$$

$$\begin{array}{r} 0.2777 \\ 0.9 \overline{) 25.0000} \\ \underline{-18} \phantom{00} \\ 70 \phantom{00} \\ \underline{-63} \phantom{00} \\ 70 \phantom{00} \\ \underline{-63} \phantom{00} \\ 70 \phantom{00} \\ \underline{-63} \phantom{00} \\ 70 \phantom{00} \\ \underline{-63} \phantom{00} \\ 70 \phantom{00} \end{array}$$

10. Write your answer in a complete sentence.

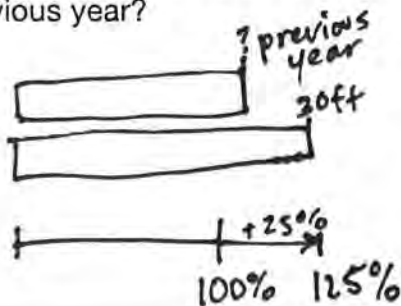
Ken's salary must be  
 \$27.77

# Name: ANSWER KEY

Solve each problem using a tape diagram, a table and an equation.

1. The height of a tree was 25% higher than its height the previous year. If the tree is 30 feet tall, how tall was it in the previous year?

Tape diagram:



Equation:

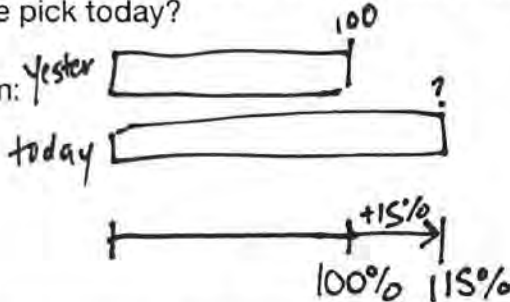
$$\begin{array}{r} 0024 \\ 125 \overline{) 3000} \\ \underline{-250} \phantom{0} \\ 500 \\ \underline{-500} \\ 000 \end{array}$$

$$\begin{array}{r} ? \times 1.25 = 30 \\ \hline 1.25 \quad 1.25 \end{array}$$

24 feet

2. Jennifer picked 15% more apples today than yesterday. If she picked 100 apples yesterday, how many did she pick today?

Tape diagram:



Equation:

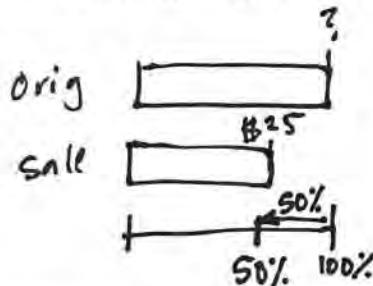
$$\begin{array}{r} 1.15 \\ \times 100 \\ \hline 11500 \end{array}$$

$$100 \times 1.15 = ?$$

115 apples

3. The shirt is on sale for \$25. The price was decreased 50% from the original price. What was the original price?

Tape diagram:



Equation:

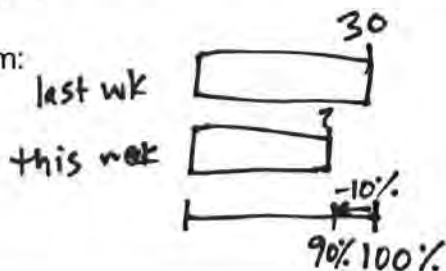
$$\begin{array}{r} 050 \\ 05 \overline{) 2500} \\ \underline{-25} \phantom{00} \\ 00 \end{array}$$

$$\begin{array}{r} ? \times 0.5 = 25 \\ \hline 0.5 \quad 0.5 \end{array}$$

\$50

4. Lisa got 30 mosquito bites last week. After she sprayed her garden this week, the bites decreased 10%. How many fewer bites did Lisa get this week than last week?

Tape diagram:



Equation:

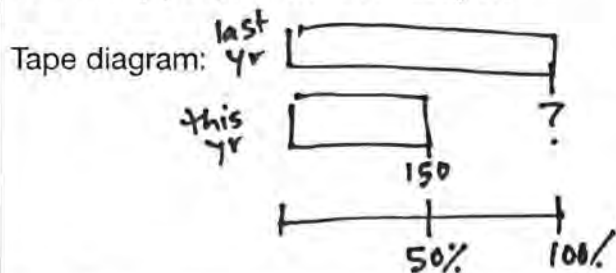
$$\begin{array}{r} 30 \\ \times .9 \\ \hline 27.0 \end{array}$$

$$30 \times .9 = ?$$

27 bites

Solve each problem using an equation and a table.

5. The choir director is expecting 50% as many people to attend this year's concert as last year's concert. If she is expecting 150 people, how many people attended last year?



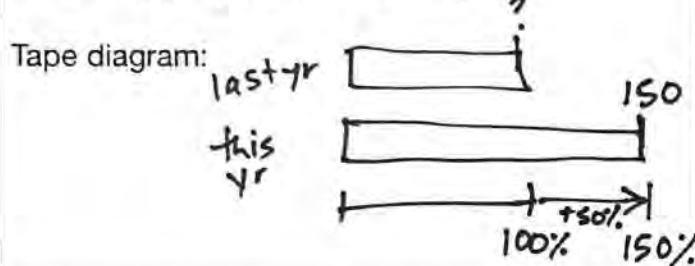
Equation:

$$\begin{array}{r} ? \times 0.5 = 150 \\ \hline 0.5 \quad 0.5 \end{array}$$

$$\begin{array}{r} 0.5 \overline{) 0300} \\ \underline{1500} \\ 00 \end{array}$$

300 people

6. The choir director is expecting 50% more people to attend this year's concert as last year's concert. If she is expecting 150 people, how many people attended last year?



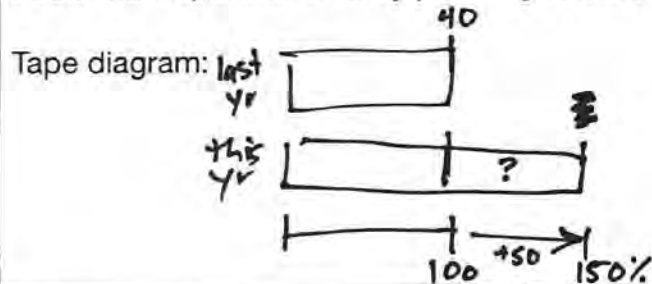
Equation:

$$\begin{array}{r} ? \times 1.5 = 150 \\ \hline 1.5 \quad 1.5 \end{array}$$

$$\begin{array}{r} 0100 \\ 1.5 \overline{) 1500} \\ \underline{1500} \\ 00 \end{array}$$

100 people

7. Emily baked 40 cupcakes for the school bake sale last year. They sold out so fast that she has decided to bake 50% more this year. How many additional cupcakes is Emily planning to bake?



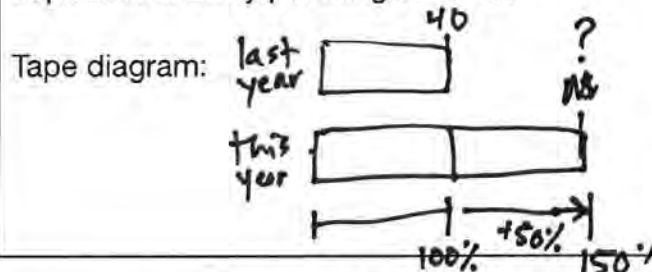
Equation:

$$40 \times 0.5 = ?$$

$$\begin{array}{r} 40 \\ \times 0.5 \\ \hline 20.0 \end{array}$$

20 more cupcakes

8. Emily baked 40 cupcakes for the school bake sale last year. They sold out so fast that she has decided to bake 50% more this year. How many cupcakes is Emily planning to bake?



Equation:

$$40 \times 1.5 = ?$$

$$\begin{array}{r} 40 \\ \times 1.5 \\ \hline 200 \\ 600 \\ \hline 600 \end{array}$$

60 cupcakes

# **G7 U3 Lesson 8**

Find percentages of quantities that are not whole numbers.

**G7 U3 Lesson 8 - Today we will solve markup and markdown problems.**

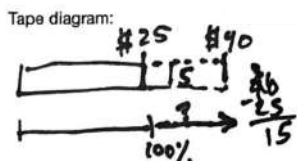
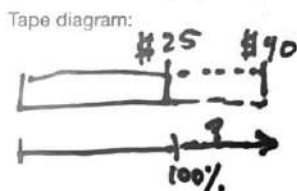
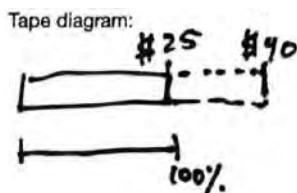
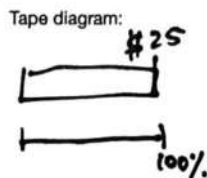
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will solve markup and markdown problems. You are going to be great at this!

**Let's Review (Slide 3):** The first thing to know is that there are a lot of special words for buying and selling. You all have seen buying and selling so these should be kind of familiar. But let's spell it out. The first word to know is mark up as in "The shirt had a \$5 markup." The markup is the amount the price went up. The next word to know is markdown as in "There was a \$2 markdown on the jeans." The markdown is the amount the price went down. Another word you might hear is "discount" or "dollars off." The next word to know is markup rate as in "The markup rate was 10%." Since this is a rate, it is represented with a percent. And in this case, it is a percent increase. The next word to know is markdown rate as in "There was a 25% markdown rate." Since this is a rate, it is represented with a percent. And in this case, it is a percent decrease. Sometimes this is called the "discount rate." The next word to know is original price as in "The original price was \$20." The original price is the starting price. This is always going to be our referent or the thing we're finding the percent of. Sometimes this will be called the "cost price" or the "wholesale price." And finally, we need to know the meaning of selling price as in "It had a \$10 selling price." Sometimes this is called the "sales price" or "discount price." Now let's look at these words in some problems.

The shirt had a \$5 markup. *amount price went up*  
 There was a \$2 markdown on the jeans. *amount price went down*  
 The markup rate was 10%. *percent increase*  
 There as a 25% markdown rate. *percent decrease*  
 The original price was \$20. *starting price/cost price/wholesale price*  
 It had a \$10 selling price. *Sales price/discount price*

**Let's Talk (Slide 4):** Markup and markdown problems are just percent change problems. Read this problem silently with me while I read it out loud, "John had a bookbag that he was going to sell for \$25. But then everyone wanted to buy it so he marked the price up to \$40. What was the markup rate?" Let's draw pictures of the problem. I am going to start with a rectangle and label it with \$25. Now, it said that John WAS going to sell the bookbag for \$25. So this is like the original price. That means it is our referent. It is the number we are going to refer to as 100% so we can figure out the percent increase or decrease.



Then it said he marked the price up to \$40. So I am going to draw more rectangle to show that the price increased to \$40 and I am going to label that sale. Since that is what it sold for.

Then the question said, "What was the markup rate?" That's like asking, "What was the percent increase?" So I am going to draw a percent line. Remember that the original price is the 100% so we start there. The price when up so I will draw an arrow up. That's the question mark. That's what the question is asking for.

But we know from our last lesson that we often have to do an extra step to find the increase from 25 to 40, especially if we want to find the percent increase from 100% to a new percent. Let's figure that out. It is the difference between the selling price and the original price so  $40 - 25$ , which is 15.



orig price	increase
25	15
100	?

Now, we know we're going to set up a table and equation. I will make a table with the original price and the decrease. Let's fill these in. For the original and the markdown table, we would put 25 and 15. We know the original is 100%, and we can find this other value here.

Equations:

$$\frac{25 \times ?}{25} = \frac{15}{25}$$

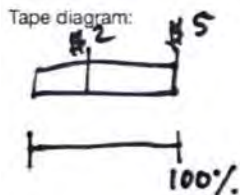
Let's write the equation. They are always based on REFERENT x PERCENT = QUANTITY. I am going to fill in 25 x question mark equals 15. We don't know the percent multiplier so we divide each side by 25.

$$25 \overline{) 15.00} = 60\%$$

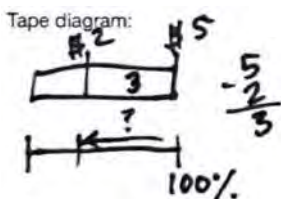
I am going to do that division quickly on the side just to move us along. At this point, you've seen how we put the decimal and two zeros. You've seen how we can add up 25s on the side if we need to. 15 divided by 25 is 0.60. That's a markup rate of 60%.

Now, if the question had asked what percent of the old price is the new price, I would have wanted to find the total final percent, which now we can see is 160%. But the point is we have to be very careful to know exactly what the question is asking for. Then we can put a question mark and make sure we find the number that corresponds to that question mark. It wasn't 40 this time. It was 40-25 this time.

**Let's Think (Slide 5):** The point of this is that we can always make at least two tables. We need words to make sense of them. This is especially important for markdown problems because it won't be as obvious which percent is for the selling price because it won't be over 100 like it was on the last slide. Since it's a markdown, the percent will be under 100. We really need words on the table to be clear on whether the percent represents the final percent or the percent of the markdown. I'll show you what I mean. Read the problem along with me while I read it out loud, "Ilana makes beautiful bracelets. She originally listed their price as \$5 on her website. But she ended up marking them down to \$2. What was the final markdown rate?" We'll start with our picture, which is even more important than ever because we can get two different percents and we need to keep track of what they mean. Okay, it said Ilana listed the price as \$5. That's the original price. I am going to draw a rectangle and label it as \$5 original.



We've been saying for lots of lessons now that the original price is the referent. So let me go ahead and draw my percent line and label it as 100% original.



Now, the question is asking, "What was the final markdown rate?" It wants the markdown rate not the rate of the final price compared to the original. So my question mark is the arrow not the final price. That helps me see that I don't yet have the information I need to set up a table. I don't have a number that corresponds to the question mark. I have to find it. I will do \$5 minus \$2 is \$3.

Tables:

orig	decrease
5	3
100	?

Let's draw the table. I will put the original price and the decrease so I can find the markdown rate. That's 5 and 3. Not the 2.



$$5 \times ? = \frac{3}{5}$$

Now I can write an equation using REFERENT x PERCENT = QUANTITY. That will be 5 x question mark equals 3. I need to divide by 5 on each side to solve for the question mark.

$$\begin{array}{r} 0.60 \\ 5 \overline{) 3.00} \\ \underline{-30} \phantom{0} \\ 0 \phantom{0} \end{array}$$

I will do the division quickly for us on the side. 3 divided by 5 is 0.60 which is 60%. When I look back at my picture, that 60% was found using the markdown so it corresponds to the markdown rate. It is this arrow pointing back. We can say, "The markdown rate was 60%." The main idea is that we need words and you need to come back to your picture to make sure that your final answer corresponds to what the question is asking.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will solve markup and  
markdown problems.**

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 **Let's Review:**

**There are a lot of special words for buying and selling.**

The shirt had a \$5 markup.

There was a \$2 markdown on the jeans.


The markup rate was 10%.

There as a 25% markdown rate.

The original price was \$20.

It had a \$10 selling price.

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 **Let's Talk:**

**Markup and markdown problems are just percent change problems.**

John had a bookbag that he was going to sell for \$25. But then everyone wanted to buy it so he marked the price up to \$40. What was the markup rate?

Tape diagram:

Table:

Equation:

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## Let's Think:

**We can always make at least two tables.  
We need words to make sense of them.**

Ilana makes beautiful bracelets. She originally listed their price as \$5 on her website. But she ended up marking them down to \$2. What was the final markdown rate?

Tape diagram:

Table:

Equation:

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_ G7, U3 Lesson B - Let's Try It

<p>The selling price of the shirt was \$25 after a \$15 mark down. What was the markdown rate?</p>	<p>The jeans were marked up 5% when there was only 1 pair left. If the original price was \$75, what was the selling price?</p>
<p>1. Draw a tape diagram with words.</p>	<p>6. Draw a tape diagram with words.</p>
<p>2. Make a table with words.</p>	<p>7. Make a table with words.</p>
<p>3. Write an equation to represent the problem:</p> <p>_____</p>	<p>8. Write an equation to represent the problem:</p> <p>_____</p>
<p>4. Use the equation to find the missing value.</p>	<p>9. Use the equation to find the missing value.</p>

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 US Lesson 8 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

1. A store marked up the price of a shirt by 30% and sold it for \$65. What was the original price of the shirt?  Tape diagram:	Equation:
2. The furniture store applied a 15% markup to a table. If the original price was \$75. What was the selling price of the table?  Tape diagram:	Equation:
3. A bakery marked down the price of a cake by 20% and Lisa bought it for \$24. What was the original price of the cake?  Tape diagram:	Equation:

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**The selling price of the shirt was \$25 after a \$15 mark down. What was the markdown rate?**

1. Draw a tape diagram with words.

2. Make a table with words.

3. Write an equation to represent the problem:

\_\_\_\_\_

4. Use the equation to find the missing value.

5. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**The jeans were marked up 5% when there was only 1 pair left. If the original price was \$75, what was the selling price?**

6. Draw a tape diagram with words.

7. Make a table with words.

8. Write an equation to represent the problem:

\_\_\_\_\_

9. Use the equation to find the missing value.

10. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

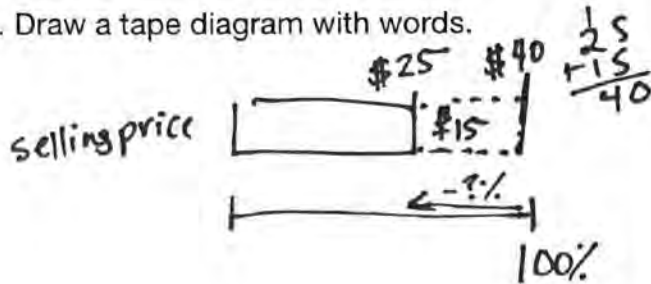






The selling price of the shirt was \$25 after a \$15 mark down. What was the markdown rate?

1. Draw a tape diagram with words.



2. Make a table with words.

original	markdown
40	15
100	?

3. Write an equation to represent the problem:

original x percent = markdown

4. Use the equation to find the missing value.

$$\frac{40 \times ?}{40} = \frac{15}{40}$$

37.5%

$$40 \overline{) 15.000}$$

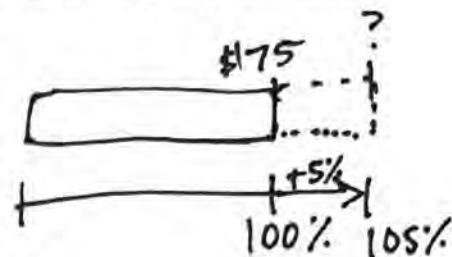
Handwritten long division showing 0.375.

5. Write your answer in a complete sentence.

The shirt was marked down 37.5%.

The jeans were marked up 5% when there was only 1 pair left. If the original price was \$75, what was the selling price?

6. Draw a tape diagram with words.



7. Make a table with words.

original	sale
75	?
100	105

8. Write an equation to represent the problem:

original x percent = sale

9. Use the equation to find the missing value.

$$75 \times 1.05 = ?$$

\$78.75

$$\begin{array}{r} 1.05 \\ \times 75 \\ \hline 525 \\ 7350 \\ \hline 78.75 \end{array}$$

10. Write your answer in a complete sentence.

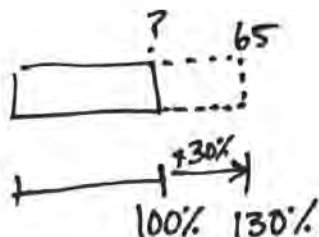
The selling price was \$78.75.

# Name: ANSWER KEY

Solve each problem using a tape diagram, a table and an equation.

1. A store marked up the price of a shirt by 30% and sold it for \$65. What was the original price of the shirt?

Tape diagram:



Equation:

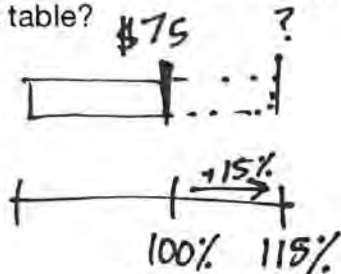
$$\frac{? \times 1.3}{1.3} = \frac{65}{1.3}$$

**\$50**

$$\begin{array}{r} 050 \\ 13 \overline{) 650} \\ \underline{65} \\ 000 \end{array}$$

2. The furniture store applied a 15% markup to a table. If the original price was \$75. What was the selling price of the table?

Tape diagram:



Equation:

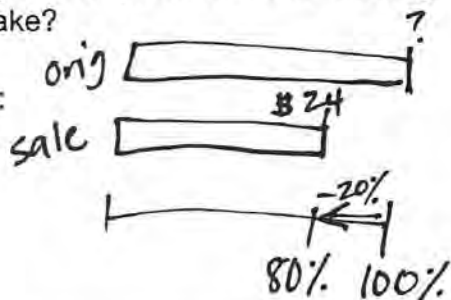
$$75 \times 1.15 = ?$$

**\$86.25**

$$\begin{array}{r} 1.15 \\ \times 75 \\ \hline 575 \\ 8050 \\ \hline 86.25 \end{array}$$

3. A bakery marked down the price of a cake by 20% and Lisa bought it for \$24. What was the original price of the cake?

Tape diagram:



Equation:

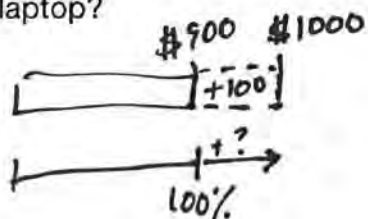
$$\frac{? \times 0.8}{0.8} = \frac{24}{0.8}$$

**\$30**

$$\begin{array}{r} 0.8 \overline{) 24.00} \\ \underline{24} \\ 00 \end{array}$$

4. A tech store was going to sell a laptop for \$900. But ended up selling it for \$1000. What was the markup rate on the laptop?

Tape diagram:



Equation:

$$\frac{900 \times ?}{900} = \frac{1000}{900}$$

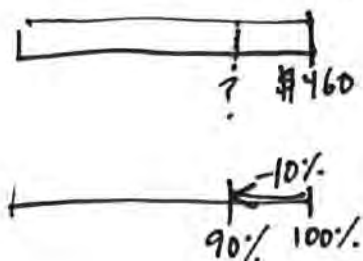
**11%**

$$\begin{array}{r} 1000.00 \\ 900 \overline{) 1000.00} \\ \underline{900} \\ 1000 \\ \underline{900} \\ 100 \end{array}$$

Solve each problem using an equation and a table.

5. A jewelry store applied a 10% discount to a \$460 watch. What was the selling price of the watch?

Tape diagram:



Equation:

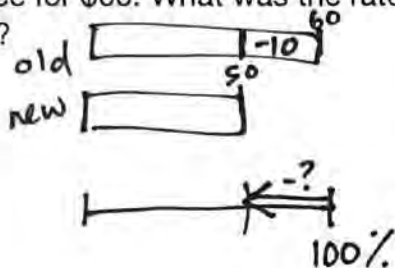
$$460 \times .9 = ?$$

$$\begin{array}{r} \$460 \\ \times .9 \\ \hline 414.0 \end{array}$$

$$\boxed{\$414}$$

6. A toy store sold a game for \$50 after setting the original price for \$60. What was the rate of the markdown?

Tape diagram:



Equation:

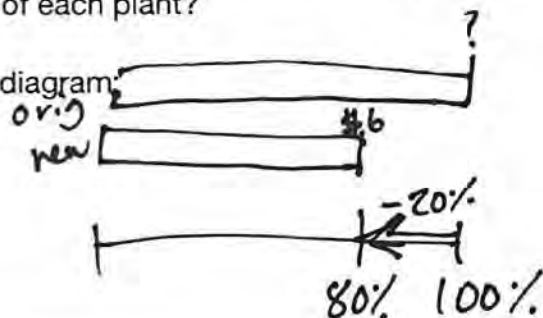
$$60 \times ? = \frac{10}{60}$$

$$\begin{array}{r} 60 \overline{) 10.000} \\ \underline{60} \phantom{00} \\ 400 \\ \underline{360} \phantom{0} \\ 400 \\ \underline{360} \phantom{0} \\ 40 \end{array}$$

$$\boxed{16.\overline{6}\%}$$

7. A gardening store marked down their tomato plants by 20% at the end of the season. If each plant was sold for \$6, what was the original price of each plant?

Tape diagram:



Equation:

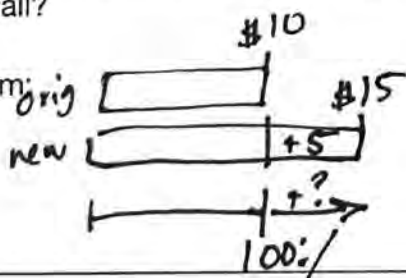
$$? \times 0.8 = 6$$

$$\begin{array}{r} 0.8 \overline{) 60.0} \\ \underline{56} \phantom{0} \\ 40 \end{array}$$

$$\boxed{\$7.50}$$

8. The price of a basketball was \$10. But then it was sold for \$15. What was the markup rate of the basketball?

Tape diagram:



Equation:

$$\frac{10 \times ?}{10} = \frac{5}{10}$$

$$\begin{array}{r} 10 \overline{) 5.00} \\ \underline{50} \phantom{0} \\ 00 \end{array}$$

$$\boxed{50\%}$$

# **G7 U3 Lesson 9**

Understand and solve problems about sales tax.

**G7 U3 Lesson 9 - Today we will solve percent error problems.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will solve percent error problems. These are just percent change problems in disguise. You're going to do great with them because you'll use the same picture, table and equation.

**Let's Review (Slide 3):** Sometimes we want to be able to describe the size of an error. An error is like a mistake. Imagine that we are guessing Beyonce's height. In real life, she is 66 inches tall. Serai guessed she was 76 inches tall. Terry guessed she was 4 inches tall. They are both wrong. But TERRY is way more wrong than SERAI. It's kind of funny actually. I'm not sure that Terry knows how big an inch is. Anyway...

Imagine that we are guessing Beyonce's height. In real life, she is 66 inches tall. Serai guessed she was 76 inches tall. Terry guessed she was 4 inches tall. They are both wrong. But Terry is way more wrong than Serai.

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Imagine that we are guessing Beyonce's height. In real life, she is 66 inches tall. Serai guessed she was 76 inches tall. Terry guessed she was 4 inches tall. They are both wrong. But Terry is way more wrong than Serai. The error is the difference between the correct number and incorrect number.

The error is the DIFFERENCE between the correct number and incorrect number.

Imagine that we are guessing Beyonce's height. In real life, she is 66 inches tall. Serai guessed she was 76 inches tall. Terry guessed she was 4 inches tall. They are both wrong. But Terry is way more wrong than Serai. The error is the difference between the correct number and incorrect number.

To find the difference, we subtract. And in this case, we don't care if the error is higher or lower so we just subtract the bigger number minus the smaller number. So, for Serai, the error is 76 minus 66, which is 10 inches. Not too bad. For Terry, the error is 66 minus 4, which is 62 inches. That's way WAY off.

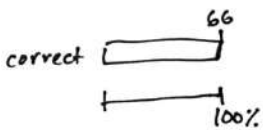
$$\begin{array}{r} 76 \\ - 66 \\ \hline 10 \end{array} \qquad \begin{array}{r} 66 \\ - 4 \\ \hline 62 \end{array}$$

We can describe how big the error each person made is by finding what percent of the right answer the error is. This is called the: percent error

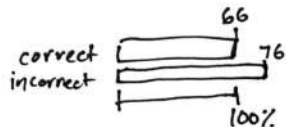
We can describe how big the error each person made is by finding what percent of the right answer the error is. This is called the PERCENT ERROR. I'm going to show you the math we do using the same tools we've been using.

**Let's Talk (Slide 4):** The most important thing to remember is that the correct number will always be the referent, which corresponds to 100%. That's just like how the original price was the referent in the

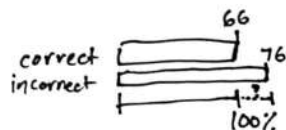
last lesson. We always need a baseline that we are comparing to, and in this case we are comparing error to the correct number. Let's draw a picture. I will draw a rectangle for the correct number, 66. And I better write the word, correct, to make sure I know that is the referent. In fact, I am going to put the percent line right now and label it 100%.

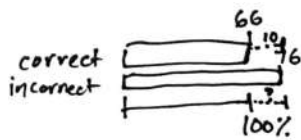


Okay, Serai guessed 76 inches. Let me draw that in.



I have to extend my percent line and I can mark the percent here with a question mark. But that is not the percent error. That's the percent that the wrong answer is out of the right answer.





I just want the percent of the error, the difference between the correct and incorrect answer here. So, there is always going to be a key first step on this picture and that is finding the error. I need to find the error to find the percent error. I always subtract the biggest number minus the smallest number. We already did 76 minus 66 is 10. If we go back to our picture, this block is 10. This is the error.

correct	error
66	10
100	?

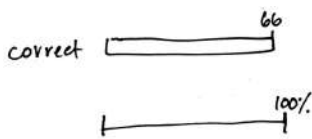
Now we can set up our table. I am going to always have the correct number and the error. I use the incorrect number to find the error but it doesn't do me any good to put it on my table. The correct number was 66 and the error was 6. We know 66 corresponds to 100% and now I can set up my equation.

$$\frac{66 \times ?}{66} = \frac{10}{66}$$

My equation is REFERENT x PERCENT = QUANTITY. In this case, it is like CORRECT NUMBER x PERCENT ERROR = ERROR. We have 66 times question mark equals 10. To find this amount, I divide by 66 on each side of the equation.

Handwritten long division: 66 into 10.00. 66 goes into 100 one time (66), leaving a remainder of 34. Bring down a 0 to get 340. 66 goes into 340 five times (330), leaving a remainder of 10. Bring down another 0 to get 100. 66 goes into 100 one time (66), leaving a remainder of 34. The result is 0.1515... with a 15% written below.

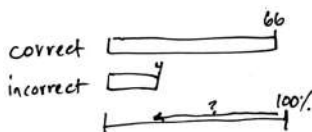
I am going to do 10 divided by 66 quickly over to the side. I might have to add up 66s on the side too. I get 0.15 with a little leftover but I will leave it here for now. That's 15%. Now we can say Serai's answer was 15% away from the correct answer.



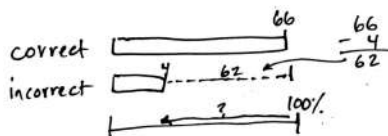
**Let's Think (Slide 5):** Now let's figure out Terry. We need to make sure we calculate the percent error not just the percent comparison so we're going to draw that picture very carefully and do the subtraction to find the error just like we did on our last slide. I am going to draw a picture. We start with 66 again. That is 100%.



Now Terry guessed Beyonce was 4 inches tall. That's hilarious. Let's mark 4 inches. We can make a line for the percent. But that's not the percent we want.



We want the percent error, which is this difference.



So we need to find the difference between our numbers. We already did 66 minus 4 earlier. The difference is 62. The error is 62. I am going to mark that on my picture.

correct	error
66	62
100	?

Now we can set up our table. I am going to always have the correct number and the error. I use the incorrect number to find the error but it doesn't do me any good to put it on my table. The correct number was 66 and the error was 62. We know 66 corresponds to 100% and now I can set up my equation.



$$\frac{66 \times ?}{66} = \frac{62}{66}$$

My equation is REFERENT x PERCENT = QUANTITY. In this case, it is like CORRECT NUMBER x PERCENT ERROR = ERROR. We have 66 times question mark equals 62. To find this amount, I divide by 66 on each side of the equation.

$$\begin{array}{r} 00.93 \\ 66 \overline{) 62.00} \\ \underline{-594} \phantom{0} \\ 260 \\ \underline{-198} \\ 62 \end{array}$$

93%

I am going to do 62 divided by 66 quickly over to the side. I might have to add up 66s on the side too. I get 0.93 with a little leftover but I will leave it here for now. That's 93%. Now we can say Serai's answer was 93% away from the correct answer.

You can see how finding the percent error helps us see that someone was wrong but also HOW wrong. Terry's answer was 93% away from the correct answer.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will solve percent error problems.**

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## Let's Review:

**Sometimes we want to be able to describe the size of an error.**

Imagine that we are guessing Beyonce's height. In real life, she is 66 inches tall. Serai guessed she was 76 inches tall. Terry guessed she was 4 inches tall.

They are both wrong. But \_\_\_\_\_ is way more wrong than \_\_\_\_\_.

The error is the \_\_\_\_\_ between the correct number and incorrect number.

We can describe how big the error each person made is by finding what percent of the right answer the error is. This is called the:

\_\_\_\_\_.

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## Let's Talk:

**The correct number will always be the referent, which corresponds to 100%.**

Beyonce is 66 inches tall. Serai guessed she was 76 inches tall. What is the percent error?

Tape diagram:

Table:

Equation:

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## Let's Think:

**We need to make sure we calculate the percent error not the percent comparison.**

Beyonce is 66 inches tall. Terry guessed she was 4 inches tall. What is the percent error?

Tape diagram:

Table:

Equation:

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_ G7 U3 Lesson 9 - Let's Try It

<p><b>On the Price is Right, the cost of a new coffeemaker is \$75. Margie guessed the price was \$50. What is the percent error in Margie's guess?</b></p>	<p><b>On the Price is Right, the cost of a new coffeemaker is \$75. Margie guessed the price was \$100. What is the percent error in Margie's guess?</b></p>
<p>1. Draw a tape diagram with words.</p>	<p>6. Draw a tape diagram with words.</p>
<p>2. Make a table with words.</p>	<p>7. Make a table with words.</p>
<p>3. Write an equation to represent the problem:</p> <p>_____</p>	<p>8. Write an equation to represent the problem:</p> <p>_____</p>
<p>4. Use the equation to find the missing value.</p>	<p>9. Use the equation to find the missing value.</p>

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## On your Own:

Now it's time for you to do it on your own.

Name: _____		G7 U3 Lesson 9 - Independent Work
Solve each problem using a tape diagram, a table and an equation.		
1. In Science class, Robbie was asked to weigh a 200 kg mass. She wrote 220 kg on her paper. What was Robbie's percent error?	Equation:	
Tape diagram:		
	Final answer: _____	
2. The teacher expected that 24 students would be in school but only 18 students were present. What was the percent error in the teacher's prediction?	Equation:	
Tape diagram:		
	Final answer: _____	
3. Ruby said the length of her sprint was 9.5 minutes. The official coach's report recorded it as 10 minutes. What was the percent error in what Ruby said?	Equation:	
Tape diagram:		

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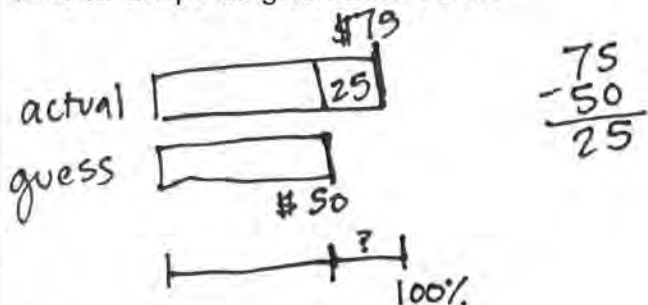


Solve each problem using an equation and a table.

<p>5. Lisa's brother measured her height and said it was 54 inches. But her real height is 60 inches. What was the brother's percent error?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>6. The weatherman said that it was going to be 85 degrees. It was actually 90 degrees. What was the percent error in the weatherman's prediction?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>7. A textbook was priced at \$45. Sarah thought it was going to be \$50. What was the percent error in her estimation of the book's price?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>8. The speed limit was 60 miles per hour. Nathaniel thought it was 70 miles per hour. What was Nathaniel's percent error?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>

On the Price is Right, the cost of a new coffeemaker is \$75. Margie guessed the price was \$50. What is the percent error in Margie's guess?

1. Draw a tape diagram with words.



2. Make a table with words.

actual	error
75	25
100	?

3. Write an equation to represent the problem:

$$\text{actual} \times \text{percent} = \text{error}$$

4. Use the equation to find the missing value.

$$75 \times ? = \frac{25}{75}$$

$$33\%$$

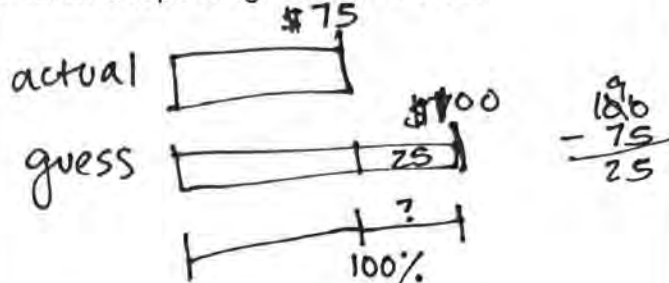
$$\begin{array}{r} 00.33 \\ 75 \overline{)25.00} \\ \underline{225} \phantom{0} \\ 250 \\ \underline{225} \\ 25 \end{array}$$

5. Write your answer in a complete sentence.

Margie's guess was  
33% off of the  
real price.

On the Price is Right, the cost of a new coffeemaker is \$75. Margie guessed the price was \$100. What is the percent error in Margie's guess?

6. Draw a tape diagram with words.



7. Make a table with words.

actual	error
75	25
100	?

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$$\text{actual} \times \text{percent} = \text{error}$$

9. Use the equation to find the missing value.

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$$33\%$$

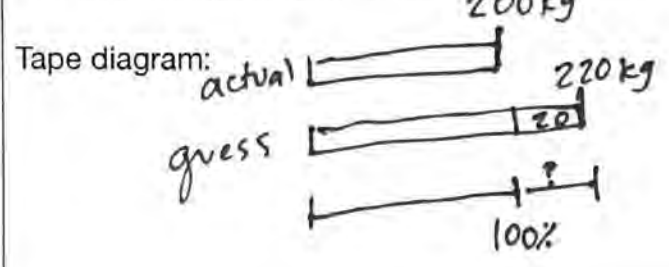
same!

10. Write your answer in a complete sentence.

Margie's guess was  
33% off of the  
real price.

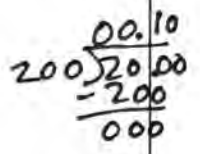
Solve each problem using a tape diagram, a table and an equation.

1. In Science class, Robbie was asked to weigh a 200 kg mass. She wrote 220 kg on her paper. What was Robbie's percent error?



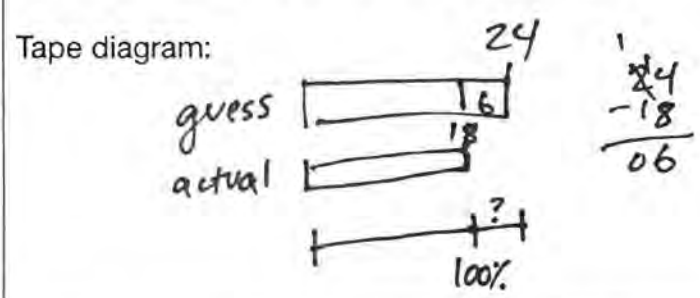
Equation:

$$\frac{200 \times ?}{200} = \frac{20}{200}$$



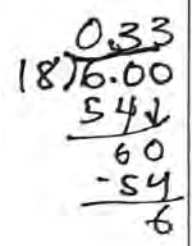
Final answer: 10%

2. The teacher expected that 24 students would be in school but only 18 students were present. What was the percent error in the teacher's prediction?



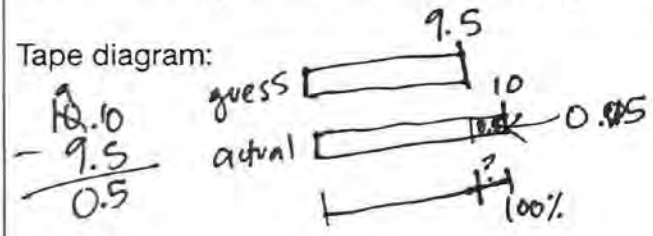
Equation:

$$\frac{18 \times ?}{18} = \frac{6}{18}$$



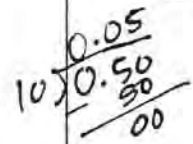
Final answer: 33%

3. Ruby said the length of her sprint was 9.5 minutes. The official coach's report recorded it as 10 minutes. What was the percent error in what Ruby said?



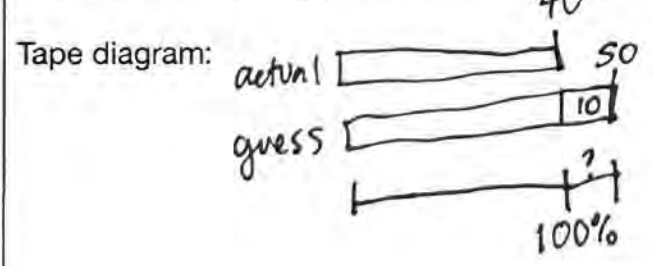
Equation:

$$\frac{10 \times ?}{10} = \frac{0.05}{10}$$



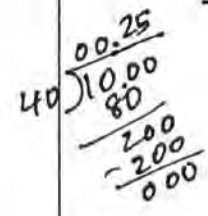
Final answer: 5%

4. The real age of a tree was 40 years. Tom incorrectly estimated it to be 50 years. What was the percent error in his estimation?



Equation:

$$\frac{40 \times ?}{40} = \frac{10}{40}$$

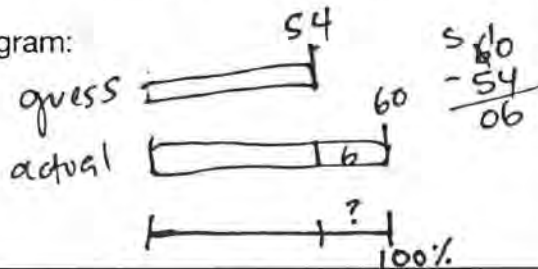


Final answer: 25%

Solve each problem using an equation and a table.

5. Lisa's brother measured her height and said it was 54 inches. But her real height is 60 inches. What was the brother's percent error?

Tape diagram:



Equation:

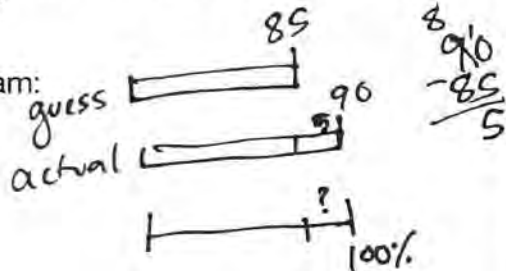
$$\frac{60 \times ?}{60} = \frac{6}{60}$$

$$\begin{array}{r} 0.10 \\ 60 \overline{) 6.00} \\ \underline{-60} \\ 00 \end{array}$$

Final answer: 10%

6. The weatherman said that it was going to be 85 degrees. It was actually 90 degrees. What was the percent error in the weatherman's prediction?

Tape diagram:



Equation:

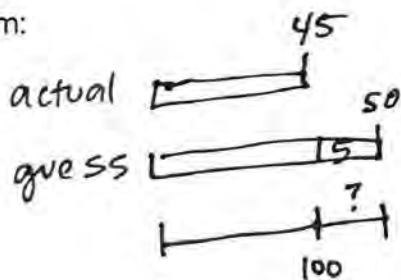
$$\frac{90 \times ?}{90} = \frac{5}{90}$$

$$\begin{array}{r} 0.05 \\ 90 \overline{) 5.00} \\ \underline{-45} \\ 50 \end{array}$$

Final answer: 5%

7. A textbook was priced at \$45. Sarah thought it was going to be \$50. What was the percent error in her estimation of the book's price?

Tape diagram:



Equation:

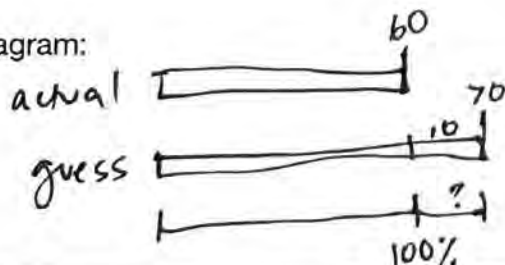
$$\frac{45 \times ?}{45} = \frac{5}{45}$$

$$\begin{array}{r} 0.11 \\ 45 \overline{) 5.00} \\ \underline{-45} \\ 50 \\ \underline{-45} \\ 5 \end{array}$$

Final answer: 11%

8. The speed limit was 60 miles per hour. Nathaniel thought it was 70 miles per hour. What was Nathaniel's percent error?

Tape diagram:



Equation:

$$\frac{60 \times ?}{60} = \frac{10}{60}$$

$$\begin{array}{r} 0.16 \\ 60 \overline{) 10.00} \\ \underline{-60} \\ 400 \\ \underline{-360} \\ 40 \end{array}$$

16%

# **G7 U3 Lesson 10**

Understand and solve problems about commission, markups, and discounts.

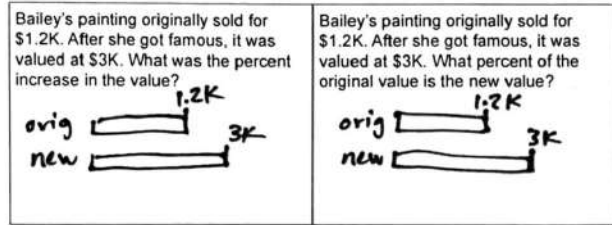
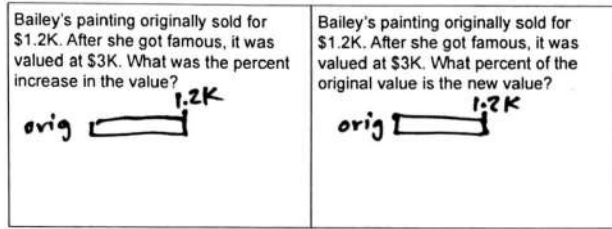
**G7 U3 Lesson 10 - Today we will solve a variety of problems for review.**

**Warm Welcome (Slide 1):** Tutor choice

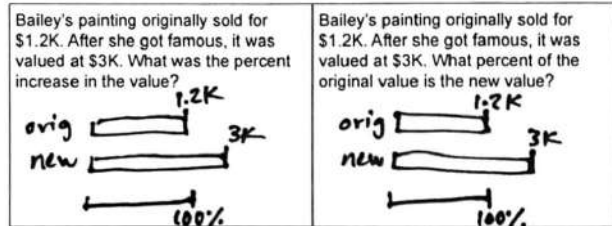
**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will solve a variety of problems for review. We're especially going to focus on when the problems seem really similar with only a small difference in wording. We are going to have to read very closely to see what the question is really asking for.

**Let's Review (Slide 3):** We know how to write a picture, table and equation to solve percent problems. This is asking, "How are the problems the same or different?" Read the first problem along with me while I read it out loud, "Bailey's painting originally sold for \$1.2K. After she got famous, it was valued at \$3K. What was the percent increase in the value?"

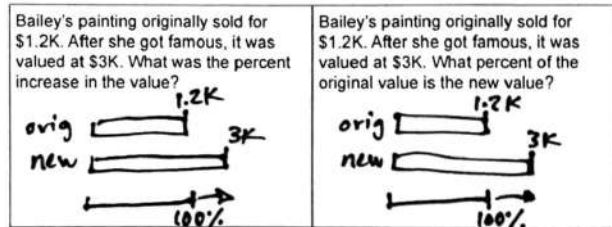
Now let's read the next problem. It starts off the same. It says, "Bailey's painting originally sold for \$1.2K. After she got famous, it was valued at \$3K. What percent of the original value is the new value?" So, let's draw a picture. I know I am going to have 1.2K original for both.



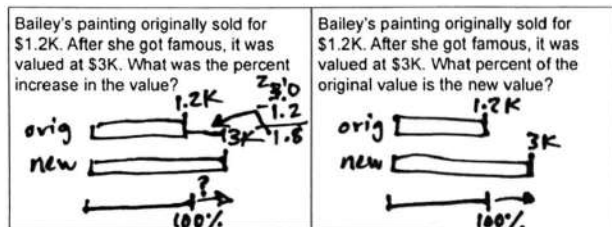
I know that she got famous and it increased to 3K for both. I'll write new for that price.



In both cases, when I draw the percent, the 1.2K is the original so it is the referent. It is the 100%.



And in both cases, the value increased so I can draw an arrow and mark the new value. Wow, this really seems like the same problem!



But here's where it gets interesting. This first problem wants to know the percent increase, which is this arrow here. I'll put a question mark. That is where it's increasing, going up. So if I'm going to find the percent increase. I need to find this dollar increase first. I will have to do 3 minus 1.2. I am going to do this quickly because this isn't a subtraction lesson. But I'll need to put a decimal on that 3 with a zero so I can borrow. I get 1.8.

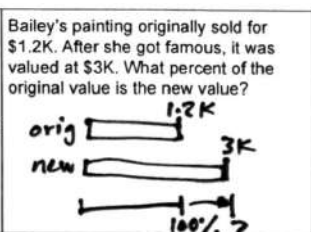
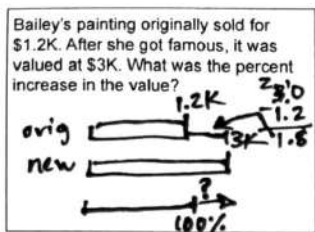


$$\frac{1.2 \times ?}{1.2} = \frac{1.8}{1.2}$$

So now I can see that when I write my table, I will use 1.2 and 1.8. And when I write the equation, it will be 1.2 x question mark equals 1.8.

$$1.2 \overline{) 1.80} = 1.5 = 150\%$$

I am going to do this math for you because number-crunching is not really the point right now. I divide by 1.2 on each side and do 1.8 divided by 1.2. That's really 18 divided by 12. I get 1.50. That's 150%. If we go back and look at our picture, it makes sense. It went up more than 100% because it went up more than 1.2.



Now for the second problem, It wants to know, "What percent of the original value is the new value?" Essentially, it wants me to compare the new value to the original value. The value increased but it's not a percent increase problem. The question mark for the percent comparison would be here.

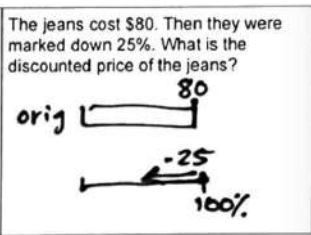
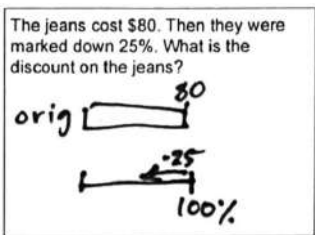
$$\frac{1.2 \times ?}{1.2} = \frac{3}{1.2}$$

And then when I set up my table, I am comparing original value and new value, and my equation is 1.2 x question mark equals 3 and I need to divide each side by 1.2.

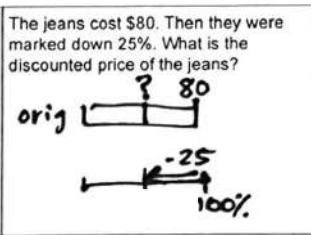
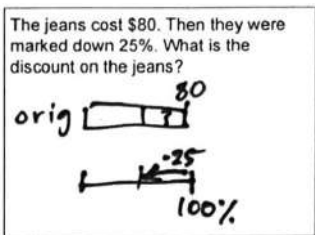
$$1.2 \overline{) 3.00} = 2.5 = 250\%$$

I will do that math quickly for us. 3 divided by 1.2 is the same as 30 divided by 12. That is 2.50 which is 250%. That makes sense because 100% plus the 150% of the last problem is 250%, and we can mark it on our picture here. Even though these were very similar problems, the final answer was different because of what the question was asking.

**Let's Talk (Slide 4):** The problems below might also seem similar but "There is a difference between asking for the discount and the discounted price."



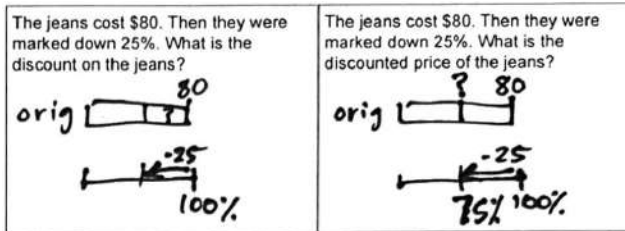
Let's read these problems and figure out, "Which problem requires an extra step?" *Read both problems out loud while the kids read along silently.* I am going to draw a rectangle for both problems and label it \$80 original. I am going to draw a percent line with 100% and draw my arrow down to show the 25% markdown.



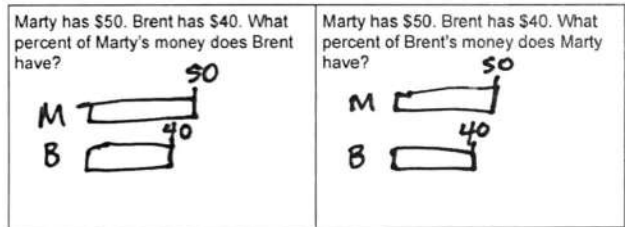
Now let's mark what the question of each problem is asking for. This first problem says, "What is the discount on the jeans?" It wants to know the amount of the discount or the amount of the decrease. In other words, it wants to know how much was subtracted. I can shade that here. It is the 25%. So in my table I would do original and discount and use 25% to find it.



In the other problem, it says, "What is the discounted price of the jeans?" It wants to know the new price AFTER the discount is subtracted. Do you see how this requires an extra step? I can either subtract the discount I find in the last problem. Or I can subtract the 25% and find this percent here.

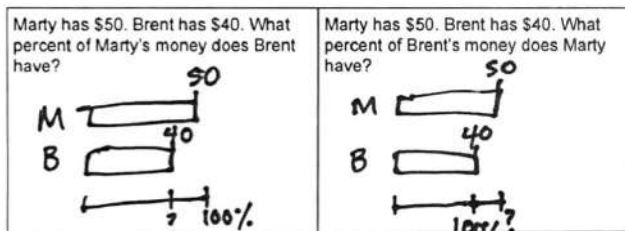


Let's do that last one. 100 minus 25 is 75. I will mark 75% on my picture. Now when I have my table I will use the original price and the new price. I will use 100% and 75%. We are not going to find these problems all the way right now. The point is that they are not the same. I really have to pay attention to what the question is asking.



**Let's Think (Slide 5):** We are going to think of one more set of problems where we really have to make sure we keep in mind this tip: "The referent is always the amount we're finding the percent of." We are asked here, "What is the referent in each problem?" Read along silently with me while I read each problem out loud. *Read each problem.* In both these problems, Marty has \$50 and Brent has \$40. So we can draw two rectangles lined up at the start.

Next we will need percent lines. But here is the key. This first problem is asking, "What percent of Marty's money does Brent have?" But this other question is asking, "What percent of Brent's money does Marty have?" For the first problem the key words are "percent of" because the words right after those words are the referent. It says, "What percent of Marty's money" so Marty's money is the



referent. I will put the 100% lined up with Marty and find the other percent from there. It is going to be less than 100%. For the second problem, it says, "What percent of Brent's money" so Brent's money is the referent. I will put the 100% lined up with Brent and find the other percent from there. It is going to be more than 100%.

$$\frac{50 \times ?}{50} = \frac{40}{50}$$

$$50 \overline{) 40.00} = 80\%$$

The first table would be referent then quantity with 50 then 40. And the 50 corresponds to 100%. When I set up my equation it is 50 x question mark equals 40. To solve, I divide by 50 on both sides. 40 divided by 50 is 0.80 which is 80%.

$$\frac{40 \times ?}{40} = \frac{50}{40}$$

$$40 \overline{) 50.00} = 125\%$$

The second table would be referent then quantity with 40 then 50. And the 40 corresponds to 100%. When I set up my equation it is 40 x questions equals 50. I divide by 40 on both sides. 50 divided by 40 is 1.25 which is 125%. Those are totally different answers. We are going to have to draw really carefully and pay close attention to what the question is asking today.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will solve a variety of  
problems for review.**

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## Let's Review:

**We know how to write a picture, table and equation to solve percent problems.**

How are the problems the same or different?

Bailey's painting originally sold for \$1.2K. After she got famous, it was valued at \$3K. What was the percent increase in the value?

Bailey's painting originally sold for \$1.2K. After she got famous, it was valued at \$3K. What percent of the original value is the new value?

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## Let's Talk:

**There is a difference between asking for the discount and the discounted price.**

Which problem requires an extra step?

The jeans cost \$80. Then they were marked down 25%. What is the discount on the jeans?

The jeans cost \$80. Then they were marked down 25%. What is the discounted price of the jeans?

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## Let's Think:

The referent is always the amount we're finding the percent of.

Which number is the referent in each problem?

Marty has \$50. Brent has \$40. What percent of Marty's money does Brent have?

Marty has \$50. Brent has \$40. What percent of Brent's money does Marty have?

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## Let's Try It:

We will do it together step by step!

Name: \_\_\_\_\_ G7 U3 Lesson 10 - Let's Try It

<p>150 attended the concert last year. This year, Marshall expects to see a 40% increase in attendance. How many additional people does Marshall expect to attend the concert?</p> <p>1. Draw a tape diagram with words.</p>   <p>2. Make a table with words.</p>   <p>3. Write an equation to represent the problem:</p> <p>_____</p> <p>4. Use the equation to find the missing value.</p>	<p>150 attended the concert last year. This year, Marshall expects to see a 40% increase in attendance. How many people does Marshall expect to attend the concert?</p> <p>6. Draw a tape diagram with words.</p> <p> </p>   <p>7. Make a table with words.</p>   <p>8. Write an equation to represent the problem:</p> <p>_____</p> <p>9. Use the equation to find the missing value.</p>
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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 10 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

<p>1. A company reduced its workforce by 12% due to budget cuts. If the original number of employees was 200, how many employees were let go?</p> <p>Tape diagram:</p>	<p>Equation:</p>   <p>Final answer: _____</p>
<p>2. A company reduced its workforce by 12% due to budget cuts. If the original number of employees was 200, how many employees does it have now?</p> <p>Tape diagram:</p>	<p>Equation:</p>   <p>Final answer: _____</p>
<p>3. Janice has earned \$20 mowing lawns. Kelsey earned \$25 babysitting. What percent of the money that Kelsey earned did Janice earn?</p> <p>Tape diagram:</p>	<p>Equation:</p>

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**150 attended the concert last year. This year, Marshall expects to see a 40% increase in attendance. How many additional people does Marshall expect to attend the concert?**

1. Draw a tape diagram with words.

2. Make a table with words.

3. Write an equation to represent the problem:

\_\_\_\_\_

4. Use the equation to find the missing value.

5. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**150 attended the concert last year. This year, Marshall expects to see a 40% increase in attendance. How many people does Marshall expect to attend the concert?**

6. Draw a tape diagram with words.

7. Make a table with words.

8. Write an equation to represent the problem:

\_\_\_\_\_

9. Use the equation to find the missing value.

10. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Name: \_\_\_\_\_

Solve each problem using a tape diagram, a table and an equation.

<p>1. A company reduced its workforce by 12% due to budget cuts. If the original number of employees was 200, how many employees were let go?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>2. A company reduced its workforce by 12% due to budget cuts. If the original number of employees was 200, how many employees does it have now?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>3. Janice has earned \$20 mowing lawns. Kelsey earned \$25 babysitting. What percent of the money that Kelsey earned did Janice earn?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>4. Janice has earned \$20 mowing lawns. Kelsey earned \$25 babysitting. What percent of the money that Janice earned did Kelsey earn?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>

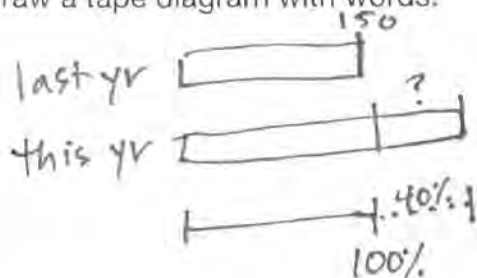


Solve each problem using an equation and a table.

<p>5. The price of a laptop was \$900. Then it was marked down 10%. What is the new cost of the laptop?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>6. The price of a laptop was \$900. Then it was marked down 10%. How much was the laptop marked down?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>7. The watch was initially valued at \$120. After 10 years, its value fell to \$100. What was the percent decrease in its value?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>
<p>8. The watch was initially valued at \$120. After 10 years, its value fell to \$100. What percent of the original value is the new value?</p> <p>Tape diagram:</p>	<p>Equation:</p>          <p>Final answer: _____</p>

150 attended the concert last year. This year, Marshall expects to see a 40% increase in attendance. How many additional people does Marshall expect to attend the concert?

1. Draw a tape diagram with words.



2. Make a table with words.

referent	increase
150	?
100	40

3. Write an equation to represent the problem:

$$\text{referent} \times \text{percent} = \text{increase}$$

4. Use the equation to find the missing value.

$$150 \times 0.4 = ?$$

$$\begin{array}{r} 150 \\ \times 0.4 \\ \hline 60.0 \end{array}$$

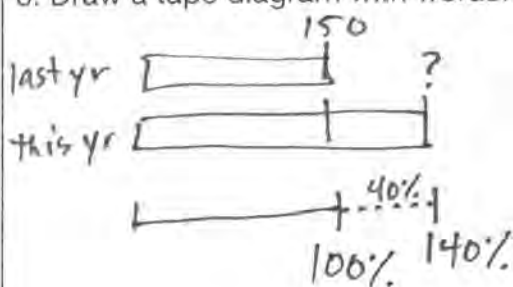
60

5. Write your answer in a complete sentence.

Marshall expects  
60 additional people.

150 attended the concert last year. This year, Marshall expects to see a 40% increase in attendance. How many people does Marshall expect to attend the concert?

6. Draw a tape diagram with words.



7. Make a table with words.

referent	new
150	?
100	140

8. Write an equation to represent the problem:

$$\text{referent} \times \text{percent} = \text{new}$$

9. Use the equation to find the missing value.

$$150 \times 1.4 = ?$$

$$\begin{array}{r} 150 \\ \times 1.4 \\ \hline 600 \\ 1500 \\ \hline 2100 \end{array}$$

210

10. Write your answer in a complete sentence.

Marshall expects  
210 people to attend  
this year.

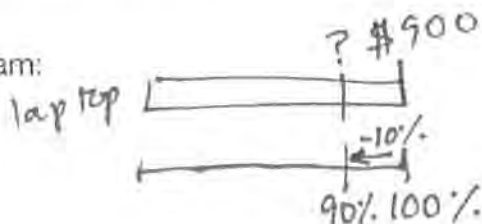
Solve each problem using a tape diagram, a table and an equation.

<p>1. A company reduced its workforce by 12% due to budget cuts. If the original number of employees was 200, how many employees were let go?</p> <p>Tape diagram:</p>	<p>Equation:</p> $200 \times 0.12 = ?$ $\begin{array}{r} 0.12 \\ \times 200 \\ \hline 24.00 \end{array}$ <p>Final answer: <u>24 employees</u></p>
<p>2. A company reduced its workforce by 12% due to budget cuts. If the original number of employees was 200, how many employees does it have now?</p> <p>Tape diagram:</p>	<p>Equation:</p> $200 \times 0.88 = ?$ $\begin{array}{r} 0.88 \\ \times 200 \\ \hline 176.00 \end{array}$ <p>Final answer: <u>176 employees</u></p>
<p>3. Janice has earned \$20 mowing lawns. Kelsey earned \$25 babysitting. What percent of the money that Kelsey earned did Janice earn?</p> <p>Tape diagram:</p>	<p>Equation:</p> $\frac{25 \times ?}{25} = \frac{20}{25}$ $\begin{array}{r} 0.80 \\ 25 \overline{)20.00} \\ \underline{-200} \\ 000 \end{array}$ <p>Final answer: <u>80%</u></p>
<p>4. Janice has earned \$20 mowing lawns. Kelsey earned \$25 babysitting. What percent of the money that Janice earned did Kelsey earn?</p> <p>Tape diagram:</p>	<p>Equation:</p> $\frac{20 \times ?}{20} = \frac{25}{20}$ $\begin{array}{r} 0.125 \\ 20 \overline{)25.00} \\ \underline{-200} \\ 50 \\ \underline{-40} \\ 100 \\ \underline{-100} \\ 000 \end{array}$ <p>Final answer: <u>125%</u></p>

Solve each problem using an equation and a table.

5. The price of a laptop was \$900. Then it was marked down 10%. What is the new cost of the laptop?

Tape diagram:



Equation:

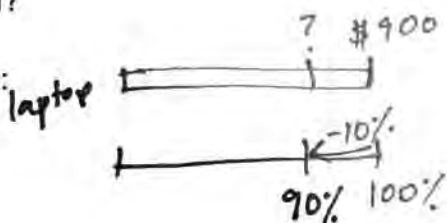
$$900 \times 0.9 = ?$$

$$\begin{array}{r} 900 \\ \times 0.9 \\ \hline 810.0 \end{array}$$

Final answer: \$810

6. The price of a laptop was \$900. Then it was marked down 10%. How much was the laptop marked down?

Tape diagram:



Equation:

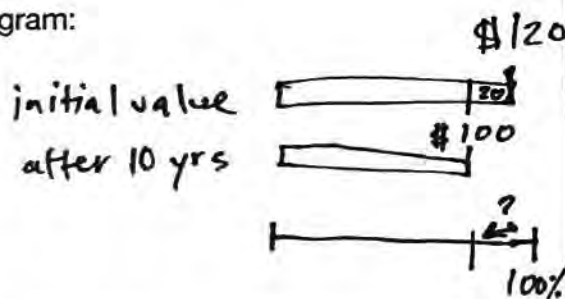
$$900 \times 0.1 = ?$$

$$\begin{array}{r} 900 \\ \times 0.1 \\ \hline 90.0 \end{array}$$

Final answer: \$90

7. The watch was initially valued at \$120. After 10 years, its value fell to \$100. What was the percent decrease in its value?

Tape diagram:



Equation:

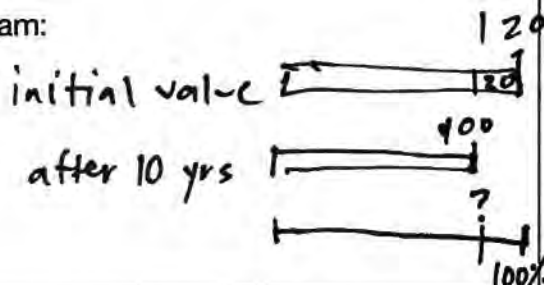
$$\frac{120 \times ?}{120} = \frac{20}{120}$$

$$\begin{array}{r} 00.16 \\ 120 \overline{) 20.00} \\ \underline{120} \phantom{00} \\ 800 \\ \underline{720} \\ 80 \end{array}$$

Final answer: 16%

8. The watch was initially valued at \$120. After 10 years, its value fell to \$100. What percent of the original value is the new value?

Tape diagram:



Equation:

$$\frac{120 \times ?}{120} = \frac{100}{120}$$

$$\begin{array}{r} 00.83 \\ 120 \overline{) 100.00} \\ \underline{960} \phantom{00} \\ 400 \end{array}$$

Final answer: 83%

# **G7 U3 Lesson 11**

Understand and solve problems involving percentage increase and percentage decrease using real world contexts such as tax, tip, and discount.

## G7 U3 Lesson 11 - Today we will solve simple interest problems.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will solve simple interest problems. You actually need to think about this in real life because it comes up. Let's dive in!

**Let's Review (Slide 3):** In real life, "people are paid interest on the money they save in a bank or they have to pay interest on loans they borrow." Basically, when we save in a bank, the bank can use the money while it's holding onto it. So the bank pays you a little bit of extra money called interest based on how much it is holding onto. Or if you are borrowing money and the bank is lending you money. They you pay interest to the bank as a way to pay them for letting you use the money. The interest is always a percent. And it is always based on the original amount you are starting with. It is also dependent on time. The longer you save, the more interest you get. The longer you borrow, the more interest you have to pay. Read along with me while I read this example, "Name the steps to make a table to find what happens if Lisa has a \$100 credit card balance and she has to pay 10% interest each month." I see that there is January. 10% of 100 is \$10 in interest. If we add that to the \$100 then she owes \$110 in January. The next month, February, will be \$10 for two months so that's \$20 interest. If we add that to the \$100 then it's \$120. You can see what's happening here. 3 months would be \$10 for 3 months. First, we find THE PERCENT OF THE ORIGINAL AMOUNT. That's always the original

First, we find the percent of the orig  
Then we multiply by the time  
Then we add that to the original

$$\text{original} \times \text{percent} \times \text{time} = \text{interest}$$

could also write: ORIGINAL X PERCENT X TIME + ORIGINAL = NEW. That is also the same as what we've been doing. We just add the original amount to find the new amount with interest.

times the percent like we've always done. But then it got multiplied by the number of months. So we'll have to do that too. So then we multiply by the time. And finally, if we wanted to find the balance, we had to add that to the original amount. So then we add the original amount.

I am going to write that as one big equation: ORIGINAL X PERCENT X TIME = INTEREST. This equation is very similar to what we've been using for the last 10 lessons. All we did was add time. We

**Let's Talk (Slide 4):** Sometimes the time we have to solve for might be a fractional amount. Let's explore these problems. *Read the first one.* I know I'm trying to find the interest so I will write the equation: ORIGINAL X PERCENT X TIME = INTEREST. I know the original amount is 1,000 times the interest, which is 0.02, times 2, which is the number of years. This is pretty straight forward. Let's do

A \$1,000 savings bond earns simple interest at the rate of 2% per year. The interest is paid at the end of each month. How much interest will it earn in two years?

$$\begin{aligned} \text{original} \times \text{percent} \times \text{time} &= \text{interest} \\ 1000 \times 0.02 \times 2 &= ? \\ 2000 \times 2 &= ? \\ 4000 &= ? \end{aligned}$$

the math to the side. 1000 x 0.02 is 2000 then we add the decimal point with two place values after it. We get 20. Now we do 20 x 2, which is 40. The interest it will earn in two years will be \$40.

Now let's read the next problem. *Read the second problem.* We still have the same equation. The original amount is still 1,000 times the interest rate, which is 0.2. But this time we don't have a full year. We have eight months. Months are a fraction of the year so I am going to have to multiply all this times a fraction. That is 8 out of 12 since it said 8 months and there are 12 months in a year.

A \$1,000 savings bond earns simple interest at the rate of 2% per year. The interest is paid at the end of each month. How much interest will it earn in two years?

$$\begin{aligned} \text{original} \times \text{percent} \times \text{time} &= \text{interest} \\ 1000 \times 0.02 \times 2 &= ? \\ 2000 \times 2 &= ? \\ 4000 &= ? \end{aligned}$$

A \$1,000 savings bond earns simple interest at the rate of 2% per year. The interest is paid at the end of each month. How much interest will it earn in eight months?

$$\begin{aligned} \text{original} \times \text{percent} \times \text{time} &= \text{interest} \\ 1000 \times 0.02 \times \frac{8}{12} &= ? \\ 20.00 \times \frac{8}{12} &= ? \quad \frac{160}{12} = ? \end{aligned}$$

$$\begin{array}{r} 013.33 \\ 12 \overline{)160.00} \\ \underline{-120} \phantom{00} \\ 40 \phantom{00} \\ \underline{-36} \phantom{00} \\ 40 \phantom{00} \\ \underline{-36} \phantom{00} \\ 040 \end{array}$$

And now I have to divide 160 divided by 12 is 13.33. The interest it will earn in eight months will be \$13.33.

**Let's Think (Slide 5):** "Sometimes we will be asked to find the time or the original amount or the interest." This is still as simple as using the same equation. We will plug in the values we know and solve for the values we don't know. We know we have two equations. Let's assume we don't know which one to use so we just go with this one: ORIGINAL X PERCENT X TIME = INTEREST. That's no problem but I might need to do some extra work to the side to find all the values.

John had \$450 in the bank. The balance earned interest every year for 10 years and now he has \$500. What was the yearly interest rate?

$$\begin{aligned} \text{original} \times \text{percent} \times \text{time} &= \text{interest} \\ 450 \times ? \times 10 &= \text{---} \leftarrow \begin{array}{r} 500 \\ -450 \\ \hline 050 \end{array} \end{aligned}$$

The original is \$450. We don't know the percent. We know the time is 10 years. We also don't know the interest. We only know the final balance was \$500. If I want to find the interest, I have to subtract the original amount. That's 500 minus 450, which is 50. \$50 is the interest I will plug into my equation.

$$\frac{4500 \times ?}{4500} = \frac{50}{4500}$$

Now I can solve. I can do 450 times 10. It is 4,500 so let me rewrite my equation as 4,500 x question mark equals 50. To solve this, I will divide by 4,500 on both sides.

$$4500 \overline{)50.00} = 0.0111\% \approx 1\%$$

That is kind of tricky division. 4500 doesn't go into 5 or 50. If I put a decimal and two zeros, it doesn't go into 500. It only goes into 5000 once. So I put a 1. I subtract 4500 and I get 500 left. So that means my answer is 0.01 or 1%.

That makes sense to me because it was only a smaller amount of interest over so many years.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.



# WARM WELCOME



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**Today we will solve simple interest problems.**

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## Let's Review:

**In real life, people are paid interest on money they save in a bank or they have to pay interest on loans they borrow.**

Name the steps to make a table to find what happens if Lisa has a \$100 credit card balance and she has to pay 10% interest each month.

Month	Interest	Balance
Jan	\$10	\$110
Feb	\$20	\$120
Jan	\$30	\$130

First, we find \_\_\_\_\_.

Then we \_\_\_\_\_.

Then we \_\_\_\_\_.

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## Let's Talk:

**Sometimes the time we have to solve for might be a fractional amount.**

A \$1,000 savings bond earns simple interest at the rate of 2% per year. The interest is paid at the end of each month. How much interest will it earn in two years?

A \$1,000 savings bond earns simple interest at the rate of 2% per year. The interest is paid at the end of each month. How much interest will it earn in eight months?

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## Let's Think:

Sometimes we will be asked to find the time or the original amount or the interest.

John had \$450 in the bank. The balance earned interest every year for 10 years and now he has \$500. What was the yearly interest rate?

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## Let's Try It:

We will do it together step by step!

Name: \_\_\_\_\_ G7 U3 Lesson 11 - Let's Try It

<p>Edna borrowed \$200 from her brother and she promised to pay him back with a simple interest rate of 5% per month. What is the total amount she will have to pay her brother after 6 months?</p>	<p>Sam's grandma gave him \$50 to start a savings account. It earned a 5% simple interest rate paid out yearly. Now he has \$100. How long must the money have stayed in the savings account?</p>
<p>1. Write an equation to represent the problem:</p>	<p>6. Write an equation to represent the problem:</p>
<p>2. Rewrite the equation with the values you know.</p>	<p>7. Rewrite the equation with the values you know.</p>
<p>3. Solve for the unknown amount in the equation.</p>	<p>8. Solve for the unknown amount in the equation.</p>
<p>4. Write your answer in a complete sentence.</p>	<p>9. Write your answer in a complete sentence.</p>
<p>_____</p>	<p>_____</p>

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_

G7 US Lesson 11 - Independent Work

Solve each problem using an equation.

1. If you borrow \$1500 at a simple interest rate of 6% for six months, how much interest will you owe when you pay back the loan?

2. Sarah deposits \$300 into a college saving account that offers 4% simple interest each year. How much money will be in the account at the end of 2 years?

3. Jake borrowed \$4000 to buy a car. When he paid back the loan three years later, he owed \$150 in interest. He had a simple annual interest rate. What must the interest rate have been on the money Jake borrowed?

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**Edna borrowed \$200 from her brother and she promised to pay him back with a simple interest rate of 5% per month. What is the total amount she will have to pay her brother after 6 months?**

1. Write an equation to represent the problem:

\_\_\_\_\_

2. Rewrite the equation with the values you know.

3. Solve for the unknown amount in the equation.

4. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

5. Does this amount seem reasonable? \_\_\_\_\_

**Sam's grandma gave him \$50 to start a savings account. It earned a 5% simple interest rate paid out yearly. Now he has \$100. How long must the money have stayed in the savings account?**

6. Write an equation to represent the problem:

\_\_\_\_\_

7. Rewrite the equation with the values you know.

8. Solve for the unknown amount in the equation.

9. Write your answer in a complete sentence.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

10. Does this amount seem reasonable? \_\_\_\_\_

Name: \_\_\_\_\_

Solve each problem using an equation.

1. If you borrow \$1500 at a simple interest rate of 6% for six months, how much interest will you owe when you pay back the loan?

2. Sarah deposits \$300 into a college saving account that offers 4% simple interest each year. How much money will be in the account at the end of 2 years?

3. Jake borrowed \$4000 to buy a car. When he paid back the loan three years later, he owed \$150 in interest. He had a simple annual interest rate. What must the interest rate have been on the money Jake borrowed?

4. Jane borrowed \$120 from her brother. She promised to pay it back with a monthly simple interest rate of 5%. When she paid it back, she owed her brother \$150. How many months must it have taken Jane to pay back her brother?

Solve each problem using an equation.

5. Lisa took out a 5 year loan with a yearly simple interest rate of 10%. When she paid back the loan, she owed \$400 in interest. What must the value of the original loan have been?

6. Burt deposits \$700 into a vacation fund that earns 3.5% simple interest each year. How much money will have in 3 years to go on his dream trip?

7. Micah borrows \$1,800 to pay for home renovations at an interest rate of 6% per month for 9 months. How much interest will he owe at the end of the loan?

8. Alex borrowed \$350 from his dad to cover a dentist bill. The loan has a monthly simple interest rate. After 4 months, Alex owes his dad \$400. What must the interest rate have been?



Edna borrowed \$200 from her brother and she promised to pay him back with a simple interest rate of 5% per month. What is the total amount she will have to pay her brother after 6 months?

1. Write an equation to represent the problem:

$$\text{Original} \times \text{percent} \times \text{time} = \text{interest}$$

2. Rewrite the equation with the values you know.

$$200 \times 0.05 \times 6 = ?$$

$$10 \times 6 = ?$$

$$60$$

$$\begin{array}{r} 200 \\ \times 0.05 \\ \hline 10.00 \end{array}$$

3. Solve for the unknown amount in the equation.

4. Write your answer in a complete sentence.

She will have to pay her brother \$60.

5. Does this amount seem reasonable? yes

Sam's grandma gave him \$50 to start a savings account. It earned a 5% simple interest rate paid out yearly. Now he has \$100. How long must the money have stayed in the savings account?

6. Write an equation to represent the problem:

$$\text{original} \times \text{percent} \times \text{time} = \text{interest}$$

7. Rewrite the equation with the values you know.

$$50 \times 0.05 \times ? = 50$$

$$\frac{2.5 \times ? = 50}{2.5 \quad 2.5}$$

$$20$$

$$\begin{array}{r} 50 \\ \times 0.05 \\ \hline 2.50 \end{array}$$

$$\begin{array}{r} 0.100 \\ - 50 \\ \hline 50 \end{array}$$

$$\begin{array}{r} 020 \\ 2.5 \overline{) 500} \\ \underline{500} \\ 000 \end{array}$$

8. Solve for the unknown amount in the equation.

9. Write your answer in a complete sentence.

The money must have stayed in the account for 20 years.

10. Does this amount seem reasonable? yes

Solve each problem using an equation.

1. If you borrow \$1500 at a simple interest rate of 6% for six months, how much interest will you owe when you pay back the loan?

$$\begin{array}{r} 3 \\ 1500 \\ \times 0.06 \\ \hline 90.00 \end{array}$$

$$\begin{array}{r} 90 \\ \times 6 \\ \hline 540 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$1500 \times 0.06 \times 6 = ?$$

$$90 \times 6 = ?$$

**\$540 interest**

2. Sarah deposits \$300 into a college saving account that offers 4% simple interest each year. How much money will be in the account at the end of 2 years?

$$\begin{array}{r} 300 \\ \times 0.04 \\ \hline 12.00 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$300 \times 0.04 \times 2 = ?$$

$$12 \times 2 = ?$$

$$24 = ?$$

$$\begin{array}{r} 300 \\ + 24 \\ \hline 324 \end{array}$$

**\$324 in 2 years**

3. Jake borrowed \$4000 to buy a car. When he paid back the loan three years later, he owed \$150 in interest. He had a simple annual interest rate. What must the interest rate have been on the money Jake borrowed?

$$\begin{array}{r} 4000 \\ \times 3 \\ \hline 12000 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$4000 \times ? \times 3 = 150$$

$$\frac{12000 \times ?}{12000} = \frac{150}{12000}$$

$$\begin{array}{r} 000.012 \\ 12000 \overline{) 150.000} \\ \underline{12000} \\ 30000 \\ \underline{24000} \end{array}$$

**1.2%**

4. Jane borrowed \$120 from her brother. She promised to pay it back with a monthly simple interest rate of 5%. When she paid it back, she owed her brother \$150. How many months must it have taken Jane to pay back her brother?

$$\begin{array}{r} 120 \\ \times 0.05 \\ \hline 6.00 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$120 \times 0.05 \times ? = 30$$

$$\frac{6 \times ?}{6} = \frac{30}{6}$$

$$\begin{array}{r} * \\ 150 \\ - 120 \\ \hline 30 \end{array}$$

**5 months**

Solve each problem using an equation.

5. Lisa took out a 5 year loan with a yearly simple interest rate of 10%. When she paid back the loan, she owed \$400 in interest. What must the value of the original loan have been?

$$\begin{array}{r} 0.1 \\ \times 5 \\ \hline 0.5 \end{array}$$

$$\begin{array}{r} 0800 \\ 0.5 \overline{)4000} \\ \underline{-4000} \\ 00 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$? \times 0.1 \times 5 = 400$$

$$\frac{? \times 0.5}{0.5} = \frac{400}{0.5}$$

$$\boxed{\$800}$$

6. Burt deposits \$700 into a vacation fund that earns 3.5% simple interest each year. How much money will have in 3 years to go on his dream trip?

$$\begin{array}{r} 0.035 \\ \times 700 \\ \hline 24.500 \end{array}$$

$$\begin{array}{r} 24.5 \\ \times 3 \\ \hline 73.5 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$700 \times 0.035 \times 3 = ?$$

$$24.5 \times 3 = ?$$

$$73.5 = ?$$

$$\begin{array}{r} * \\ 700.00 \\ + 73.50 \\ \hline 773.50 \end{array}$$

$$\boxed{\$773.50}$$

7. Micah borrows \$1,800 to pay for home renovations at an interest rate of 6% per month for 9 months. How much interest will he owe at the end of the loan?

$$\begin{array}{r} 1800 \\ \times 0.06 \\ \hline 11400 \end{array}$$

$$\begin{array}{r} 114 \\ \times 9 \\ \hline 1026 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$1800 \times 0.06 \times 9 = ?$$

$$114 \times 9 = ?$$

$$\boxed{\$1,026}$$

8. Alex borrowed \$350 from his dad to cover a dentist bill. The loan has a monthly simple interest rate. After 4 months, Alex owes his dad \$400. What must the interest rate have been?

$$\begin{array}{r} 350 \\ \times 4 \\ \hline 1400 \end{array}$$

$$\begin{array}{r} 00.03 \\ 1400 \overline{)50.00} \\ \underline{4200} \\ 800 \end{array}$$

original  $\times$  percent  $\times$  time = interest

$$350 \times ? \times 4 = 50$$

$$\frac{1400 \times ?}{1400} = \frac{50}{1400}$$

$$\boxed{3\%}$$

$$\begin{array}{r} * \\ 400 \\ - 350 \\ \hline 50 \end{array}$$

# **G7 U3 Lesson 12**

Understand problems about measurement error and use percentages to represent measurement error as percent error.

**G7 U3 Lesson 12 - Today we will solve problems about commissions, taxes, and fees.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will solve problems about commission, taxes, and fees.

**Let's Review (Slide 3):** There are a lot of ways that percents show up in real life. Here are a few examples we haven't talked about yet. It says, "Jason gave the waiter a 20% tip." A tip is the extra amount added to a bill that goes straight to a waiter or service worker. The next one says, "There was an automatic gratuity of 15% for large groups." Gratuity is another word for tip. The next one says, "Lori paid 5% sales tax on her shopping spree." The government charges money on top of what we buy in order to pay for public goods like schools and roads. There are taxes on sales and property and income. The next one says, "The salesman earned a 10% commission when the car was sold." A

- Jason gave the waiter a 20% tip.
- There was an automatic gratuity of 15% for large groups.
- Lori paid 5% sales tax on her shopping spree.
- The salesman earned a 10% commission when the car was sold.
- There was a 6% shipping fee for the purchase.

*referent x percent = quantity*

commission is something someone earns for selling something. It is how salespeople get paid because then how much they earn is based on how much they sell. The next one says, "There was a 6% shipping fee for the purchase." A fee is an extra charge that is added onto a purchase price. All of these are just percent problems so the equation we've always used applies: REFERENT x PERCENT = QUANTITY.

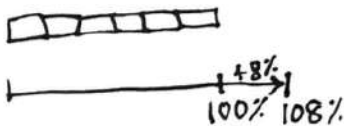
**Let's Talk (Slide 4):** We can draw a picture for these story problems just like we always do. Sometimes there might be multiple steps. Read along silently with me while I read out loud. *Read the story problem.* Let's draw a picture. It said, "Erin bought 6 new pairs of leggings." Let's draw 6 rectangles.

Tape diagram:



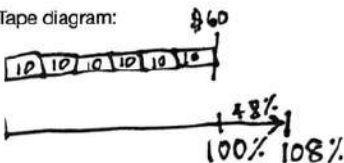
She was charged a shipping fee of 8%. Let's draw a percent line. We could call all these leggings 100% and then add a fee of 8% by drawing an arrow. That means this final point will be 108%. That's a little extra thinking that I'm doing for the problem but that's a good thing. I'm going to mark both of those numbers on my picture.

Tape diagram:



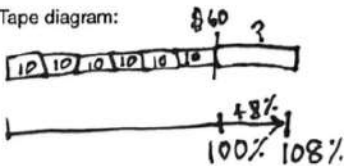
It says each pair of leggings cost \$10. So I can write that in each box. There's some extra math I can do there too, right? 10 times 6 is 60 so I will mark that as the original price before the tax. That's what corresponds to 100%.

Tape diagram:



Now, we've already done some extra steps to solve this problem. Deciding where to put the question mark is going to be the key because that will help us figure out what percent to use - 8% or 108%.

Tape diagram:



This problem asked how much Lisa paid for shipping so we will only want the shipping part not the whole final price. I will put the question mark here and I will use 8% which corresponds to it in the picture. If the question had asked for the final total then I would have put the question mark here. *Point to the total amount at the end of the bar.* And I would have used 108%.

Solution:

$$60 \times 0.08 = ?$$

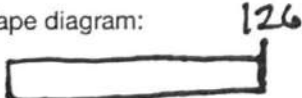
$$\$4.80$$

It's time for the equation: REFERENT x PERCENT = QUANTITY. That's  $60 \times 0.08$  equals question mark. I can just do this multiplication here to the side. 60 times 8 is 480 and now I add the decimal point with two spaces after it. Lisa paid \$4.80 for shipping.

**Let's Think (Slide 5):** When we need to work backwards to find an original value, we will need to make sure we use the percent for the whole final amount. Read this story along with me silently while I read it out loud. *Read the story.* If I draw a picture for this, I will start to see that I want to work backwards from the final price. Let me show you what I mean. It says, "Faith paid a 5% cleaning fee." So I will draw a 100% line and then show an arrow for 5% more, which is the fee.

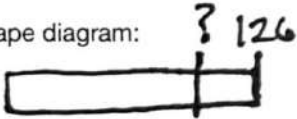


Tape diagram:



Now it also says, "she paid a final amount of \$126 per night." That's the final amount so that lines up with the end of my percent like this. It's not the 100%. It's the 100% plus the fee.

Tape diagram:



The question asked, "what was the cost per night before the fee?" I will put my question mark here to represent the cost before the fee. Now here's the key idea: I DON'T want to use the 5% because that will just represent the fee alone and I actually don't know what that fee is. I can't figure it out unless I know the original price. I am going to have to use a percent that represents the final whole amount. I am going to put a line where that is. I need THAT percent. It is 100 plus 5 so 105%. That is the percent that I will need to use because it corresponds to the information I do have, the final cost of \$126.

$$\begin{array}{r} ? \times 1.05 = 126 \\ \hline 1.05 \quad 1.05 \end{array}$$

Now we can do our equation, REFERENT x PERCENT = QUANTITY. I don't know the referent which is the original amount. That's what I'm trying to find. It will be question mark times 1.05 NOT 0.05 equals 126. To solve this, I will have to divide by 1.05.

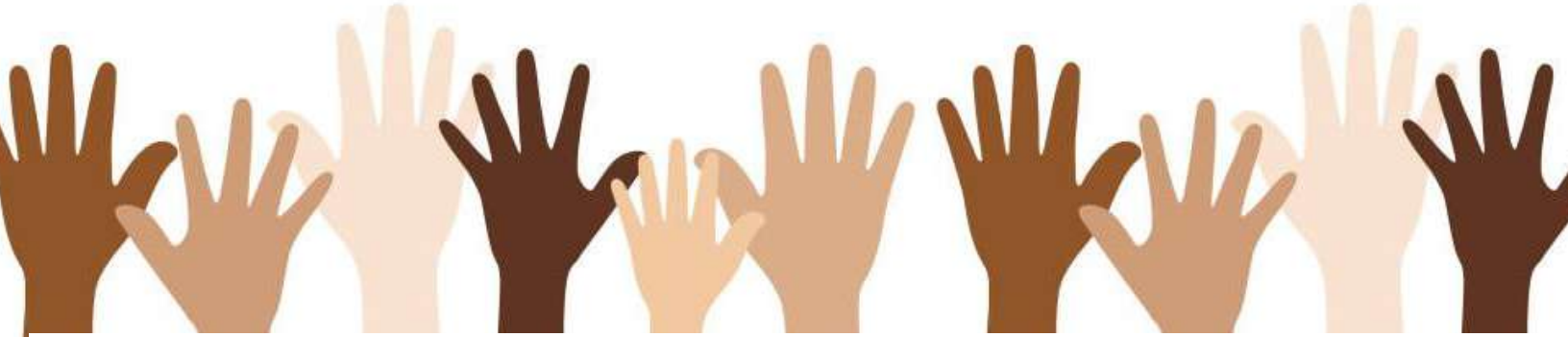
$$\begin{array}{r} 00120 \\ 105 \overline{)12600} \\ \underline{-105} \phantom{0} \\ 210 \\ \underline{-210} \\ 000 \end{array}$$

I am going to do that on the side of my paper. 126 divided by 1.05. I can't divide by a decimal so I have to shift the decimal for both numbers two spaces. So that's 12600 divided by 105. 105 doesn't go into 1 so I put a zero. It doesn't go into 12 so I put a zero. It's goes into 126 one time. I subtract 105 and get 21. Pull down a zero and it goes in two times. Subtract 210 and there's nothing left. This last space will be a zero. I get \$120. That makes sense. There was a lower price and then the fee was added.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.



# WARM WELCOME



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**Today we will solve problems about  
commissions, taxes, and fees.**

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 **Let's Review:**

**There are a lot of ways that percents show up in real life.**

Jason gave the waiter a 20% tip.

There was an automatic gratuity of 15% for large groups.

Lori paid 5% sales tax on her shopping spree.

The salesman earned a 10% commission when the car was sold.

There was a 6% shipping fee for the purchase.

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 **Let's Talk:**

**We can draw a picture for these story problems just like we always do.**

Sometimes there might be multiple steps. Try this one: Erin bought 6 new pairs of leggings. She was charged a shipping fee of 8%. If each pair of leggings cost \$10, how much did Lisa pay for shipping?

Tape diagram:

Solution:

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## Let's Think:

**When we need to work backwards to find an original value, we will need to make sure we use the percent for the whole final amount.**

Faith paid a 5% cleaning fee on top of the nightly cost for a home rental. If she paid a final amount of \$126 per night, what was the cost per night before the fee?

Tape diagram:

Solution:

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_ G7 U3 Lesson 12 - Let's Try It

<p><b>Elise wants to buy a bike for \$200. The sales tax is 5%. How much need to pay for the bike?</b></p>	<p><b>Penny bought a scooter and she was charged 4% sales tax. If the final price of the scooter was \$78, what must the original price of the scooter have been?</b></p>
<p>1. Draw a tape diagram with words.</p>	<p>6. Draw a tape diagram with words.</p>
<p>2. Make a table with words.</p>	<p>7. Make a table with words.</p>
<p>3. Write an equation to represent the problem:</p> <p>_____</p>	<p>8. Write an equation to represent the problem:</p> <p>_____</p>
<p>4. Use the equation to find the missing value.</p>	<p>9. Use the equation to find the missing value.</p>

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 12 - Independent Work

Solve each problem using a tape diagram, a table and an equation.

1. Alex sells his artwork through the gallery and pays a 20% fee on the selling price of \$500 per painting. How much money does he pay in fees if he sells 10 paintings?  Tape diagram:	Solution:    Final answer: _____
2. Beth had a \$40 restaurant bill but she paid \$48 because she added a tip. What percent tip did Beth leave for the waiter?  Tape diagram:	Solution:    Final answer: _____
3. There is an automatic gratuity of 20% added onto any restaurant bill with five or more people. If a table had a \$120 bill, what would be the final charge?  Tape diagram:	Solution:    Final answer: _____

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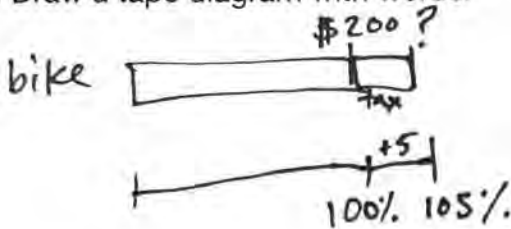






Elise wants to buy a bike for \$200. The sales tax is 5%. How much need to pay for the bike?

1. Draw a tape diagram with words.



2. Make a table with words.

price	price with tax
200	?
100	105

3. Write an equation to represent the problem:

price × percent = price w/ tax

4. Use the equation to find the missing value.

$$200 \times 1.05 = ?$$

$$210 = ?$$

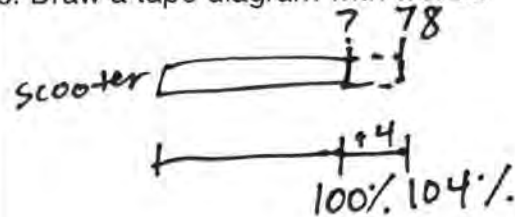
$$\begin{array}{r} 1.05 \\ \times 200 \\ \hline 21000 \end{array}$$

5. Write your answer in a complete sentence.

Elise will need to pay  
\$210 for the bike.

Penny bought a scooter and she was charged 4% sales tax. If the final price of the scooter was \$78, what must the original price of the scooter have been?

6. Draw a tape diagram with words.



7. Make a table with words.

price	price with tax
?	78
100	104

8. Write an equation to represent the problem:

price × percent = price w/ tax

9. Use the equation to find the missing value.

$$\frac{? \times 1.04 = 78}{1.04} = \frac{78}{1.04}$$

\$ 72

$$\begin{array}{r} 1.04 \overline{) 78.00} \\ \underline{72.80} \phantom{0} \\ 5.20 \\ \underline{5.12} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

10. Write your answer in a complete sentence.

The original price must  
have been about \$72.



# Name: ANSWER KEY

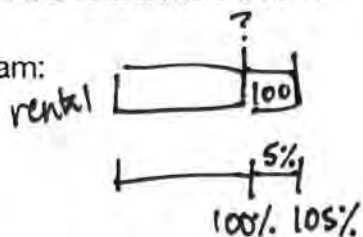
Solve each problem using a tape diagram, a table and an equation.

<p>1. Alex sells his artwork through the gallery and pays a 20% fee on the selling price of \$500 per painting. How much money does he pay in fees if he sells 10 paintings?</p> <p>Tape diagram:</p>	<p>Solution:</p> $500 \times 1.2 = ?$ $600 \times ?$ $600 \times 10 = 6000$ <p>Final answer: <u>\$6000</u></p>
<p>2. Beth had a \$40 restaurant bill but she paid \$48 because she added a tip. What percent tip did Beth leave for the waiter?</p> <p>Tape diagram:</p>	<p>Solution:</p> $40 \times ? = \frac{8}{40}$ <p>Final answer: <u>20%</u></p>
<p>3. There is an automatic gratuity of 20% added onto any restaurant bill with five or more people. If a table had a \$120 bill, what would be the final charge?</p> <p>Tape diagram:</p>	<p>Solution:</p> $120 \times 1.2 = ?$ <p>Final answer: <u>\$144</u></p>
<p>4. Meredith and her sister split the cost of a hotel room that cost \$200 per night. There was a 10% room fee in addition to the cost. What was the total amount that each girl paid?</p> <p>Tape diagram:</p>	<p>Solution:</p> $200 \times 1.1 = ?$ <p>Final answer: <u>\$110</u></p>

Solve each problem using an equation and a table.

5. Logan rents a car and is charged a 5% fee for a late return. If the fee was \$100, what must the total cost of the rental have been?

Tape diagram:



Solution:

$$\begin{array}{r} ? \times 0.05 = \frac{100}{0.05} \\ \hline 0.05 \end{array}$$

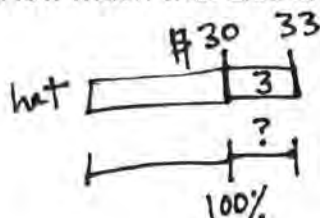
$$\begin{array}{r} 0.05 \overline{) 100.00} \\ \underline{-100} \\ 000 \end{array}$$

$$\begin{array}{r} 2000 \\ \underline{1000} \\ 2100 \end{array}$$

Final answer: \$2100

6. Lisa bought a hat for \$30. The final cost with taxes was \$33. How much was Lisa charged in taxes?

Tape diagram:



Solution:

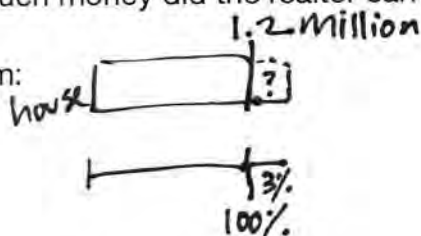
$$30 \times ? = \frac{3}{30}$$

$$\begin{array}{r} 0.10 \\ 30 \overline{) 3.00} \\ \underline{30} \\ 00 \end{array}$$

Final answer: 10%

7. Karen bought a house for 1.2 million dollars. The realtor earned a commission of 3% on the sale. How much money did the realtor earn?

Tape diagram:



Solution:

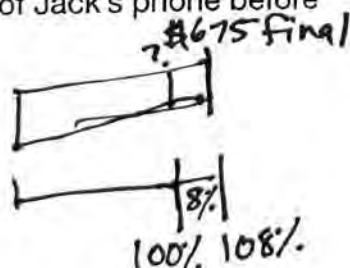
$$1.2 \times 0.03 = ?$$

$$\begin{array}{r} 1.2 \\ \times 0.03 \\ \hline 0.36 \end{array}$$

Final answer: .036 million

8. Jake paid a final price of \$675 for his new phone. This included an 8% sales tax. What was the selling price of Jack's phone before taxes?

Tape diagram:



Solution:

$$? \times 1.08 = \frac{675}{1.08}$$

$$\begin{array}{r} 0.0625 \\ 108 \overline{) 67500} \\ \underline{6480} \\ 2700 \\ \underline{1160} \\ 540 \\ \underline{540} \\ 000 \end{array}$$

Final answer: \$625

# **G7 U3 Lesson 13**

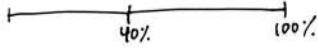
Solve and interpret problems that involve percent error by finding the correct amount, erroneous amount, or percent error.

**G7 U3 Lesson 13 - Today we will use mental math to solve percent problems.**

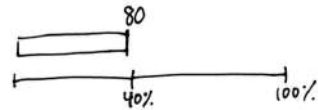
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will use mental math to solve percent problems.

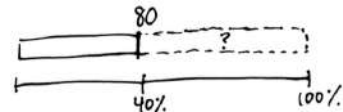
**Let's Review (Slide 3):** We have been using equation to solve percent problems for this whole unit. Read this problem silently while I read it out loud. *Read the problem out loud.* The first thing we do is draw a picture. We don't get any numerical information in the second sentence so I will draw the second sentence, "she has finished 40%."



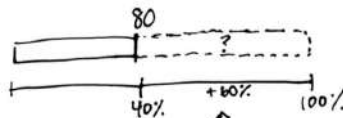
Read this problem silently while I read it out loud. *Read the problem out loud.* The first thing we do is draw a picture. We don't get any numerical information in the second sentence so I will draw the second sentence, "she has finished 40%."



Then it says, "Holly has read 80 pages" so that's 40% that she finished is also the 80 pages that she finished. I will draw a rectangle to correspond to 40%.



The question is asking, "how many more pages does she need to read?" So I don't want the total amount. I just want the amount that is more. I will mark that with a question mark here.



I don't have a percent for the amount that corresponds to that question mark so I have to subtract to find it. 100 minus 40 is 60% so the percent increase is 60%.

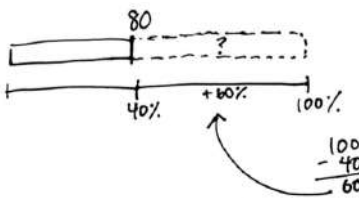


Even now, I don't have enough information to set up an equation for that question mark. I can set up a different equation to find the total amount. I'll mark that question mark. I would do question mark times 0.4 = 80. Divide by 0.4 on both sides and I get 200.

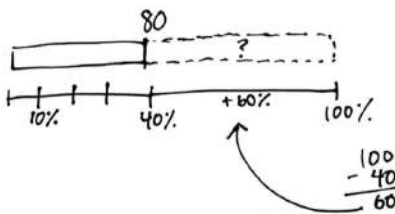
$$\begin{array}{l} ? \times 0.4 = 80 \\ \hline 0.4 \end{array} \quad \begin{array}{l} 200 \\ \times 0.4 \\ \hline 80 \end{array}$$

$$\begin{array}{r} 200 \\ - 80 \\ \hline 120 \end{array}$$

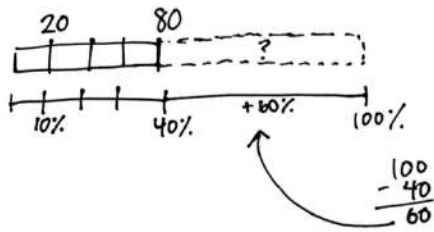
And now I can do 200 minus 80 to get this difference, which would be 120 pages. That's a lot of steps. Today we're trying to figure out how we can do some of the math in our heads. So let's try this problem again with mental math.



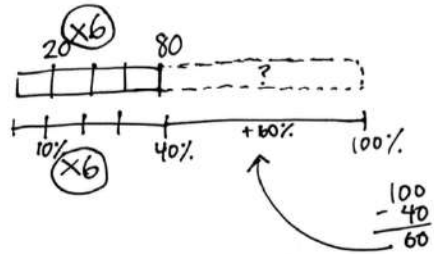
**Let's Talk (Slide 4):** If we think of our problem as a proportion, we can solve it in our heads by doing the same operation to both parts. This is the same problem that we just read. But now I'm going to draw a slightly larger picture to show our reasoning. And we're going to do all the calculations in our heads. First of all, it is the same word problem so it makes sense that the picture would start off the same.



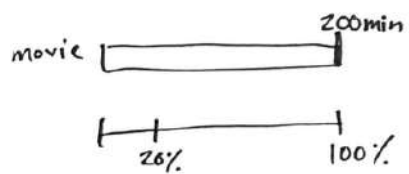
But now, instead of setting up an equation, I am going to look at these numbers and think about whether there is a common factor that I could figure out mentally for all these numbers. The vocabulary makes this seem more complicated than it really is. But basically I'm looking at 40 and 80 and 100 and thinking, "10 goes into all of those numbers." So, if I can figure out 10% then I can figure out all the other values. 40% divided by 4 is 10. Let me mark that on my picture.



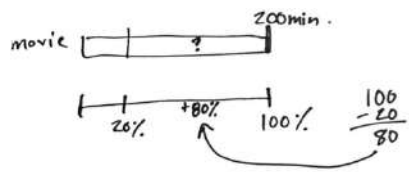
Since we are thinking of these as proportions then whatever I do to one line, I do to the other. So I am going to do 80 divided by 4 too. 80 divided by 4 is 20.



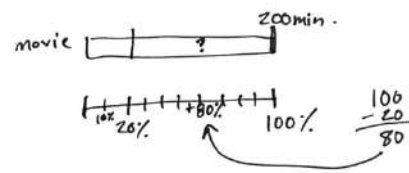
Now I can work my way up to 60%. 10% times 6 would make 60 so I have to do 20 pages x 6 which is 120.



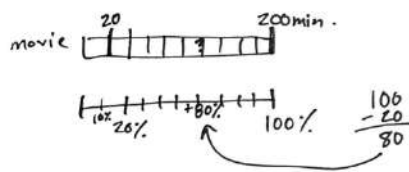
**Let's Think (Slide 5):** Let's do one more. This is the big idea we're going to keep in mind. We just operate on our quantities together to keep them in proportion. Read this problem silently while I read it out loud. *Read the problem.* Let's start by drawing 200 minutes. Next I am going to draw the 20% out of 100%.



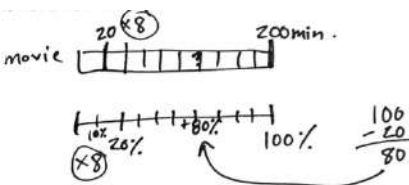
The question is asking for how much MORE Sue has to watch so I need the increase as a percent. 100 minus 20 is 80% so the increase is 80%. And I am going to draw a corresponding question mark on my bar where I want to find the number of additional pages.



Instead of setting up an equation, let's see if we can use mental math. 10% is still a really nice common factor. But this time, we don't know what the 20% corresponds to. We know what the 100% corresponds to. So let's divided that. 100% divided by 10 is 10%.



So I will do 200 minutes divided by 10 is 20 minutes. 10% corresponds to 20 minutes.



If I want to turn that into 80%, I will have to do "times 8. 10% times 8 is 80% so 20 pages x 8 is 160 pages. That's our answer.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will use mental math to solve percent problems.**

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 **Let's Review:**

**We have been using equations to solve percent problems for this whole unit.**

Holly is trying to finish a novel before her book club meeting. So far, she has finished 40%. If Holly has read 80 pages, how many more pages does she need to read?

Tape diagram:

Table:

Equation:

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 **Let's Talk:**

**If we think of our problem as a proportion, we can solve it in our heads by doing the same operation to both parts.**

Holly is trying to finish a novel before her book club meeting. So far, she has finished 40%. If Holly has read 80 pages, how many many pages does she need to read?

Let's draw a larger picture that we can mark up to show our reasoning:

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## Let's Think:

**We must operate on our quantities and percents together to keep them in proportion.**

Sue is watching a movie that is 200 minutes long. So far she's watched 20% of the movie. How much more of the movie does Sue have to watch?

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_

G7 U3 Lesson 13 - Let's Try It

**Lisa and Dan went out to eat. Their final dinner bill was \$120 dollars. They wanted to leave a 20% tip. How much of a tip will they leave?**

1. Draw a number line to represent the quantities including a question mark.
  
2. Draw a parallel number line for the corresponding percentages including a question mark.
3. Solve for a factor that will work for all the quantities and mark it on both lines.
4. Use the factor to solve for the question mark and record your work on both lines.
5. Write your answer in a complete sentence:

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**Audrey bought a \$200 dress for \$180. What percent discount did she get on the dress?**

6. Draw a number line to represent the quantities including a question mark.

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 13 - Independent Work

Solve each problem using mental math. Draw a double line to show your reasoning.

1. In Science class, Jerry measured a 50 cm piece of wood. He wrote 49 cm on his paper. What was Jerry's percent error?

2. Caitie paid a final price of \$88 for her new jeans. This included a 10% sales tax. What was the original selling price of the jeans?

3. The price of a shirt was \$10. But then it was sold for \$15. What was the markup rate of the shirt?

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Name: \_\_\_\_\_

**Lisa and Dan went out to eat. Their final dinner bill was \$120 dollars. They wanted to leave a 20% tip. How much of a tip will they leave?**

1. Draw a number line to represent the quantities including a question mark.
  
  
  
  
  
  
  
  
  
  
2. Draw a parallel number line for the corresponding percentages including a question mark.
3. Solve for a factor that will work for all the quantities and mark it on both lines.
4. Use the factor to solve for the question mark and record your work on both lines.
5. Write your answer in a complete sentence:

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---

**Audrey bought a \$200 dress for \$180. What percent discount did she get on the dress?**

6. Draw a number line to represent the quantities including a question mark.
  
  
  
  
  
  
  
  
  
  
7. Draw a parallel number line for the corresponding percentages including a question mark.
8. Solve for a factor that will work for all the quantities and mark it on both lines.
9. Use the factor to solve for the question mark and record your work on both lines.
10. Write your answer in a complete sentence:

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---

Name: \_\_\_\_\_

Solve each problem using mental math. Draw a double line to show your reasoning.

1. In Science class, Jerry measured a 50 cm piece of wood. He wrote 49 cm on his paper. What was Jerry's percent error?

2. Caitie paid a final price of \$88 for her new jeans. This included a 10% sales tax. What was the original selling price of the jeans?

3. The price of a shirt was \$10. But then it was sold for \$15. What was the markup rate of the shirt?

4. The height of a tree was 25% higher than its height the previous year. If the tree is 40 feet tall, how tall was it in the previous year?

Solve each problem using mental math. Draw a double line to show your reasoning.

5. An electronics store marked down the price of a TV by 20% and Lisa bought it for \$160. What was the original price of the TV?

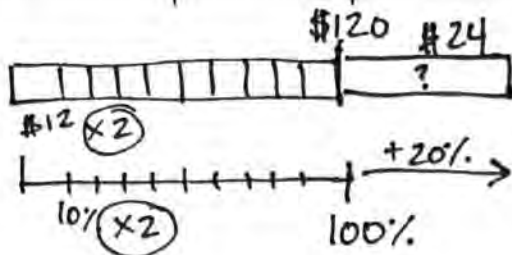
6. Lisa can ride her bike as fast as 8 miles per hour. Her goal is to be able to ride it as fast as 12 miles per hour. What percent increase does Lisa need to meet her goal?

7. Jojo wants to buy a Lego set that costs \$300. So far she has saved 60% of the price. How much has Jojo saved?

8. Sarah solved 20 math problems. Michael solved 24 math problems. What percent of Sarah's problems did Michael solve?

Lisa and Dan went out to eat. Their final dinner bill was \$120 dollars. They wanted to leave a 20% tip. How much of a tip will they leave?

1. Draw a number line to represent the quantities including a question mark.

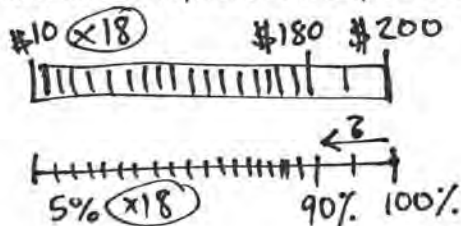


- Draw a parallel number line for the corresponding percentages including a question mark.
- Solve for a factor that will work for all the quantities and mark it on both lines.
- Use the factor to solve for the question mark and record your work on both lines.
- Write your answer in a complete sentence:

They will leave a \$24 tip.

Audrey bought a \$200 dress for \$180. What percent discount did she get on the dress?

6. Draw a number line to represent the quantities including a question mark.



$$200 \div 20 = 10$$

$$100 \div 20 = 5$$

$$\begin{array}{r} 4 \\ 18 \\ \times 5 \\ \hline 90 \end{array}$$

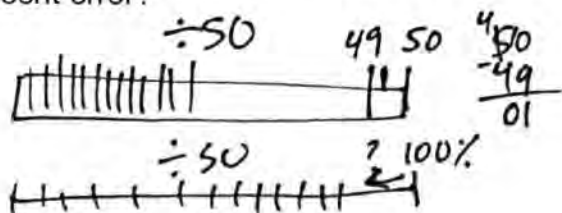
- Draw a parallel number line for the corresponding percentages including a question mark.
- Solve for a factor that will work for all the quantities and mark it on both lines.
- Use the factor to solve for the question mark and record your work on both lines.
- Write your answer in a complete sentence:

Lisa got a 10% discount.

# Name: ANSWER KEY

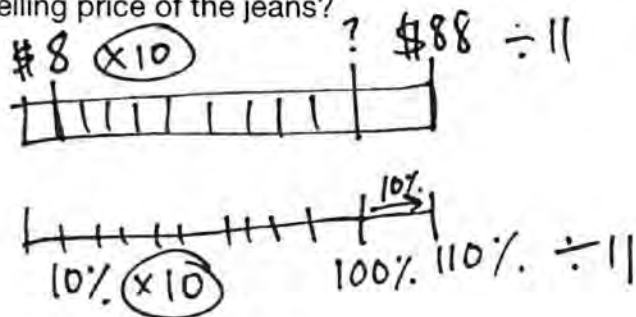
Solve each problem using mental math. Draw a double line to show your reasoning.

1. In Science class, Jerry measured a 50 cm piece of wood. He wrote 49 cm on his paper. What was Jerry's percent error?



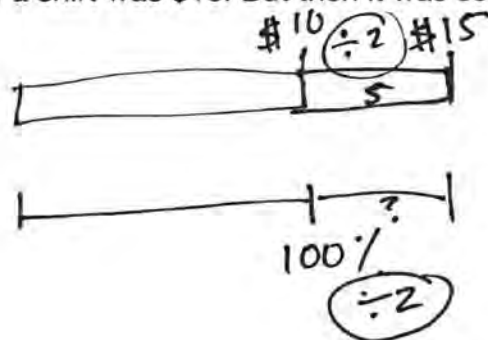
$$100\% \div 50 = \boxed{2\%}$$

2. Caitie paid a final price of \$88 for her new jeans. This included a 10% sales tax. What was the original selling price of the jeans?



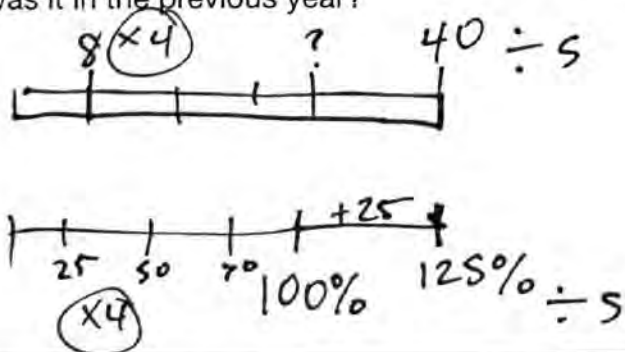
$$\boxed{\$80}$$

3. The price of a shirt was \$10. But then it was sold for \$15. What was the markup rate of the shirt?



$$100\% \div 2 = \boxed{50\%}$$

4. The height of a tree was 25% higher than its height the previous year. If the tree is 40 feet tall, how tall was it in the previous year?

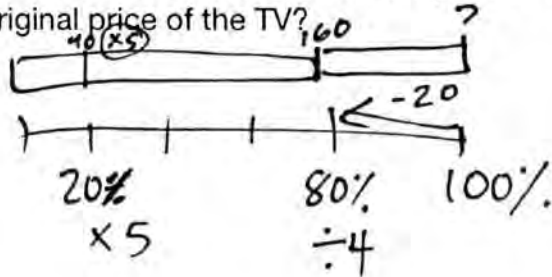


$$\boxed{32 \text{ feet tall}}$$



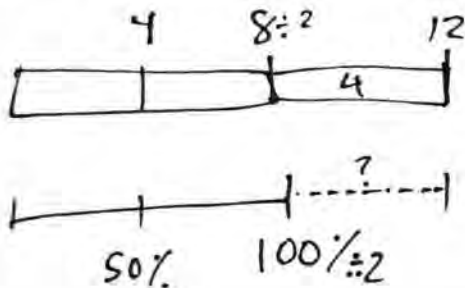
Solve each problem using mental math. Draw a double line to show your reasoning.

5. An electronics store marked down the price of a TV by 20% and Lisa bought it for \$160. What was the original price of the TV?



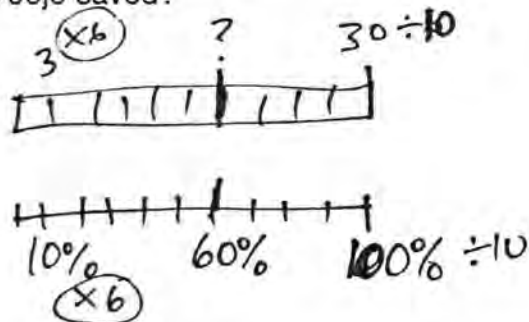
**\$ 200**

6. Lisa can ride her bike as fast as 8 miles per hour. Her goal is to be able to ride it as fast as 12 miles per hour. What percent increase does Lisa need to meet her goal?



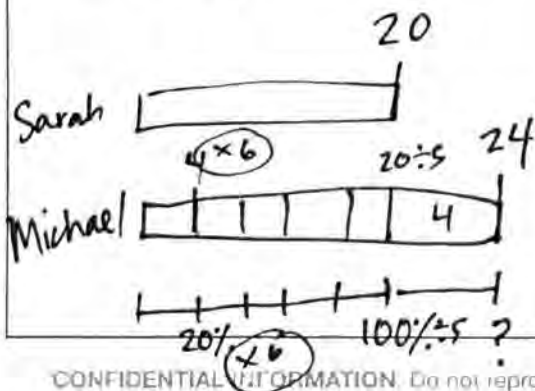
**50%**

7. Jojo wants to buy a Lego set that costs \$300. So far she has saved 60% of the price. How much has Jojo saved?



**\$ 18**

8. Sarah solved 20 math problems. Michael solved 24 math problems. What percent of Sarah's problems did Michael solve?



**120%**

## **G7 U3 Lesson 14**

Generate values that fall within the acceptable range for a measurement, given a maximum percent error and the correct value.

**G7 U3 Lesson 14 - Today we will work with percentages that are not whole numbers.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will work with percentages that are not whole numbers. We are going to use some related facts so we aren't doing a whole bunch of complicated place value multiplication. You might even be able to do it mentally like we did in our last lesson. I'll show you what I mean.

$$\begin{array}{l|l} \text{tbsp} & \text{mL} \\ 3 & 200 \end{array}$$

**Let's Review (Slide 3):** We already know that we multiply both parts of a proportion by the same number to preserve their relationship. Just like we multiply the top and bottom of a fraction to keep it equivalent. Read this problem along with me while I read it out. *Read the problem out loud.* The first thing that our table needs is words. And then I can put the numbers in place.

$$\begin{array}{l|l} \text{tbsp} & \text{mL} \\ 3 & 200 \\ \times 10 & \times 10 \\ 30 & 2,000 \end{array}$$

Now, the question is asking what she will need to make 10 servings. That means I'm going to need 10 times as many tablespoons and I'm also going to need 10 times as many milliliters. As long as I multiply both parts of this proportion, the relationship will stay the same. The cocoa will taste the same. So I need 30 tablespoons for 2,000 mL. We can apply this same idea to percents because they are proportions too.

**Let's Talk (Slide 4):** It is especially easy to multiply percentages by powers of ten because we know from our place value chart that each time we multiply by ten we just shift the digits one place to the left. Let me know you how we use this. We'll just do the first one the regular way. 3% of 200 means we have to do  $200 \times 0.03$ . We multiply that all out and get 6.

3% of 200 is 6 because  $200 \times 0.03$  is 6

$$\begin{array}{r} 200 \\ \times 0.03 \\ \hline 6.00 \end{array}$$

3% of 200 is 6 because  $200 \times 0.03$  is 6  
 $\times 10$      $\times 10$   
 30% of 200 is 60 because  $200 \times 0.3$  is 60

$$\begin{array}{r} 200 \\ \times 0.03 \\ \hline 6.00 \end{array}$$

Now, if we wanted to find 30% of 200 instead, we can see that 3% times 10 so I'll do the 6 times 10 too. 30% of 200 is 60.

3% of 200 is 6 because  $200 \times 0.03$  is 6  
 $\times 10$      $\times 10$   
 30% of 200 is 60 because  $200 \times 0.3$  is 60

$$\begin{array}{r} 200 \\ \times 0.03 \\ \hline 6.00 \end{array}$$

Let's do the multiplication on the side just to be sure. That would be  $200 \times 0.3$ . I will write all that out, and I really do get 60.

300% of 200 is \_\_\_\_\_ because  $200 \times 3$  is \_\_\_\_\_

$$\begin{array}{r} 200 \\ \times 0.3 \\ \hline 60.0 \end{array}$$

3% of 200 is 6 because  $200 \times 0.03$  is 6

$\times 10$      $\times 10$   
 30% of 200 is 60 because  $200 \times 0.3$  is 60

$$\begin{array}{r} 200 \\ \times 0.03 \\ \hline 6.00 \end{array}$$

Now we can see how to figure out 300%. It's just 30% times 10, which means 60 times 10. 300% of 200 is 600.

$\times 10$      $\times 10$   
 300% of 200 is 600 because  $200 \times 3$  is 600

$$\begin{array}{r} 200 \\ \times 0.3 \\ \hline 60.0 \end{array}$$

So, I can always take a percent I know and multiply to find a percent I don't know. We did that in our last lesson too. And guess what?!?! If we can use multiplication then we can also use division!

**Let's Talk (Slide 5):** Division is a great way to figure out percents less than 1 whole. This is going to be really useful because turning a super tiny percent into a decimal gets really messy and I don't feel like writing all that out anyway. So we're going to use division. And it is going to be easiest if we start with a nice big percent that is really easy to find. Believe it or not, in order to find 0.9%, I'm actually going to start with 900%. That's 900% is just 9. So if I want 900% of 200, I just do 200 times 9. I'm going to write that over to the side just so we know how we got this. But it's so easy - I can do that in my head. 200 times 9 is 1800.

900% of 200 is 1800  $(200 \times 9)$   
 90% of 200 is \_\_\_\_\_  
 9% of 200 is \_\_\_\_\_  
 0.9% of 200 is \_\_\_\_\_

900% of 200 is 1800  $(200 \times 9)$   
 $\div 10$  90% of 200 is 180  
 9% of 200 is \_\_\_\_\_  
 0.9% of 200 is \_\_\_\_\_

900% of 200 is 1800  $(200 \times 9)$   
 $\div 10$  90% of 200 is 180  
 $\div 10$  9% of 200 is 18  
 0.9% of 200 is \_\_\_\_\_

900% of 200 is 1800  $(200 \times 9)$   
 $\div 10$  90% of 200 is 180  
 $\div 10$  9% of 200 is 18  
 $\div 10$  0.9% of 200 is 1.8

Now we just keep dividing by powers of ten to get down to 0.9%. If I want 90%, that's 900% divided by 10. I have to divide 1800 by 10 too. 90% of 200 is 180.

Keep going! 90% divided by 10 is 9% so I do 180 divided by 10 which is 18.

We're almost there! 9% divided by 10 would be 0.9% so I do 18 divided by 10 which is 1.8.

We were able to find a very teeny time percent without doing a lot of really complicated multiplication.

$200 \times 0.009 = ?$

$$\begin{array}{r} 200 \\ \times 0.009 \\ \hline 1.800 \end{array}$$

It's much easier to think of it as shifts of place value from whole number multiplication than to change the percent into decimal and multiplying because otherwise we end up with lots of zeros and scoops and it's tricky. Let me just show what it would have to be. 0.9% is less than 1% so it's very little it wouldn't be 0.09. That's 9%. It would be 0.009%. 200 x 0.0009 is 9 times 0 and 9 times 0 and 9 times 2 and then count up the decimal places and I end up with 1.8. Same answer. So it works. But I can do this proportional reasoning we just did earlier in my head.

Let's do another example. We want to find  $\frac{1}{4}\%$ . That's very small, right?

100% of 200 is 200  $(200 \times 1)$   
 10% of 200 is \_\_\_\_\_  
 1% of 200 is \_\_\_\_\_

That's less than 1%. It's a fraction of a percent, which is already a hundredth. So it's a fraction of a hundredth. Phew! Let's start with something that really easy. 100% of 200 is the whole thing, right? It's 200. Or you could think of it as  $200 \times 1$  if you really wanted to multiply. I'll write that to the side just to keep track of what the thinking was there.

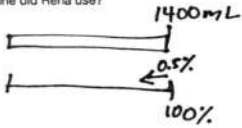
100% of 200 is 200  $(200 \times 1)$   
 $\div 10$  10% of 200 is 20  
 $\div 10$  1% of 200 is 2

Now we can work our way closer to  $\frac{1}{4}\%$ . Divide both of these by 10. Then 10% of 200 is 20. Divide both of these by 10 again. Then 1% of 200 is 2.

$$\begin{array}{l} 100\% \text{ of } 200 \text{ is } \frac{200}{\div 10} \\ 10\% \text{ of } 200 \text{ is } \frac{20}{\div 10} \\ 1\% \text{ of } 200 \text{ is } \frac{2}{\div 4} \\ \frac{1}{4}\% \text{ of } 200 \text{ is } \frac{2}{4} \end{array} \quad (200 \times 1)$$

Now, if we want to turn 1% into  $\frac{1}{4}\%$ , we just need to divide by 4. 2 divided by 4 is 2 fourths. You can imagine that if we want to do this as multiplication we would have had to turn  $\frac{1}{4}$  into 0.25 then done 0.25% which is 0.0025. It gets very cumbersome. It is better to start with a whole number and work our way down with division.

Rena was making a chemical solution that required 1,400 mL of Chlorine mixed in water. She accidentally used 0.5% less than she was supposed to. How much Chlorine did Rena use?



**Let's Think (Slide 6):** Now we can continue this proportional reasoning with percent word problems and understand how it really comes in hand. Read the problem silently while I read this out loud. *Read the problem.* We start with a picture. I will draw the 1,400 mL and mark the 100% and the 0.5% less. I need to find 0.5% of 1400.

$$500\% \rightarrow 1400 \times 5 = 7,000$$

That seems tricky so I'm going to start with 500% of 1400. That's just 1400 times 5 which is 7000.

$$500\% \rightarrow 1400 \times 5 = 7,000$$

$$50\% \rightarrow 700$$

$$5\% \rightarrow 70$$

$$\boxed{0.5\% \rightarrow 7}$$

Now I can work my way down with division. 500% divided by 10 is 50% while 7000 divided by 10 is 700. Then 50% divided by 10 is 5% while 700 divided by 10 is 70. Then 5% divided by 10 is 0.5% while 70 divided by 10 is 7. So 0.5% of 1400 is 7.

**Let's Try It (Slide 7):** Now we will practice some more problems. I will lead you through step by step.

# WARM WELCOME



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**Today we will work with percentages  
that are not whole numbers.**

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## Let's Review:

**We multiply both parts of a proportion by the same number to preserve their relationship.**

Lisa puts 3 tablespoons of cocoa in 200 mL of milk to make 1 serving of hot chocolate. What will she need to make 10 servings? Make a table to show how to increase her recipe while keeping the proportion.

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## Let's Talk:

**It is especially easy to multiply percentages by powers of ten.**

3% of 200 is \_\_\_\_\_ because  $200 \times 0.03$  is \_\_\_\_\_

30% of 200 is \_\_\_\_\_ because  $200 \times 0.3$  is \_\_\_\_\_

300% of 200 is \_\_\_\_\_ because  $200 \times 3$  is \_\_\_\_\_

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## Let's Talk:

**Division is a great way to figure out percents less than 1 whole.**

900% of 200 is \_\_\_\_\_

100% of 200 is \_\_\_\_\_

90% of 200 is \_\_\_\_\_

10% of 200 is \_\_\_\_\_

9% of 200 is \_\_\_\_\_

1% of 200 is \_\_\_\_\_

0.9% of 200 is \_\_\_\_\_

$\frac{1}{4}$ % of 200 is \_\_\_\_\_

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## Let's Think:

**We can continue this proportional reasoning with percent word problems.**

Rena was making a chemical solution that required 1,400 mL of Chlorine mixed in water. She accidentally used 0.5% less than she was supposed to. How much Chlorine was Rena short?

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## Let's Try It:

We will do it together step by step!

Name: \_\_\_\_\_ G7 U3 Lesson 14 - Let's Try It

**A factory that makes widgets expects that 0.8% of the widgets will be defective in any batch its machines make. How many widgets would it expect to be defective in a batch of 5,000 widgets?**

1. Draw a number line to represent the quantities including a question mark.
2. Draw a parallel number line for the corresponding percentages including a question mark.
3. Solve for a factor that will work for all the quantities and mark it on both lines.
4. Use the factor to solve for the question mark and record your work on both lines.
5. Write your answer in a complete sentence:

---



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**Francie was told by a scientist that 0.25% of a rock she found is quartz. If the mass of the quartz is 50 grams, what is the total weight of the rock?**

6. Draw a number line to represent the quantities including a question mark.

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 14 - Independent Work

Solve each problem using related percents. Show your work.

1. Katie measured the amount of chemical solution to 0.2% less than the true volume. If the chemical solution was 48 mL then what was her measurement?
2. At the supermarket, Lindsey finds 0.07% of the weight of the apple is 0.048 kg. What would be 7% of that weight?
3. There are 400 students at Burning Tree Elementary.  $\frac{3}{4}$  % of the students take a scooter home from school. How many students take a school home from school?

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Name: \_\_\_\_\_

**A factory that makes widgets expects that 0.8% of the widgets will be defective in any batch its machines make. How many widgets would it expect to be defective in a batch of 5,000 widgets?**

1. What percent would be easiest to find? \_\_\_\_\_
2. Draw a tape diagram to show the relationships in the problem with a question mark.

3. Which approach makes sense for this problem?

- (a) Find a related percent and divide down
- (b) Multiply the percent up.

4. Show your work.

5. Write your answer in a complete sentence:

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**Francie was told by a scientist that 0.25% of a rock she found is quartz. If the mass of the quartz is 50 grams, what is the total weight of the rock?**

6. What percent would be easiest to find? \_\_\_\_\_
7. Draw a tape diagram to show the relationships in the problem with a question mark.
  
8. Which approach makes sense for this problem?
  - (a) Find a related percent and divide down
  - (b) Multiply the percent up.
9. Show your work.

10. Write your answer in a complete sentence:

---

---

Name: \_\_\_\_\_

Solve each problem using related percents. Show your work.

1. Katie measured the amount of chemical solution to 0.2% less than the true volume. If the chemical solution was 48 mL then what was her measurement?

2. At the supermarket, Lindsey finds 0.07% of the weight of the apple is 0.048 kg. What would be 7% of that weight?

3. There are 400 students at Burning Tree Elementary.  $\frac{1}{2}$  % of the students take a scooter home from school. How many students take a school home from school?

4. Cleveland was told that his newborn puppy was 0.8% of the total weight he would be when he was fully grown. If the dog weighs 6 ounces now, how much will it weigh when it is fully grown?

Solve each problem using related percents. Show your work.

5. In the entire 800 person population of the school, only  $\frac{1}{4}\%$  chose fencing as their favorite sport. How many students chose fencing as their favorite sport?

6. Jenny can drive 1 mile on 0.2% of the gas in the gas tank of her car. How far can Jenny drive on a full tank of gas?

7. An investment portfolio of \$1,000 is expected to yield 0.3% annually. How much money would one expect the portfolio to earn in a year?

8. A swimming pool loses water at a rate of 0.75% per day due to evaporation. How much water will the pool lose if it has 50,000 liters of water?

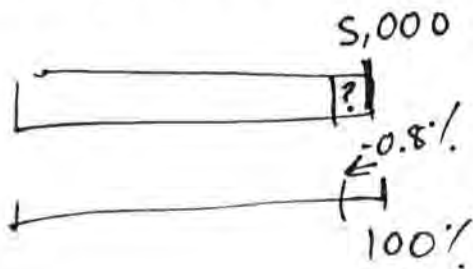
Name:

ANSWER KEY

G7 U3 Lesson 14 - Let's Try It

A factory that makes widgets expects that 0.8% of the widgets will be defective in any batch its machines make. How many widgets would it expect to be defective in a batch of 5,000 widgets?

1. What percent would be easiest to find? 800%
2. Draw a tape diagram to show the relationships in the problem with a question mark.



0.8% of 5,000

3. Which approach makes sense for this problem?

- (a) Find a related percent and divide down  
(b) Multiply the percent up.

4. Show your work.

800% of 5,000 is 40,000  
80% of 5,000 is 4,000  
8% of 5,000 is 400  
0.8% of 5,000 is 40

5. Write your answer in a complete sentence:

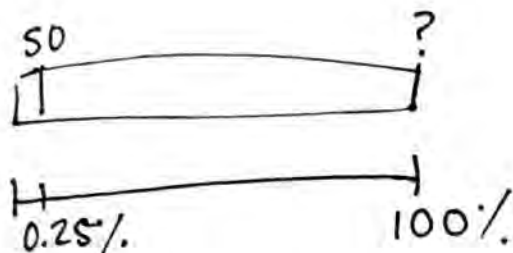
The factory should expect 40 widgets to be defective.



Francie was told by a scientist that 0.25% of a rock she found is quartz. If the mass of the quartz is 50 grams, what is the total weight of the rock?

6. What percent would be easiest to find? 25%

7. Draw a tape diagram to show the relationships in the problem with a question mark.



8. Which approach makes sense for this problem?

(a) Find a related percent and divide down

(b) Multiply the percent up.

9. Show your work.

$$\begin{array}{l} 0.25\% \text{ is } 50 \\ 2.5\% \text{ is } 500 \\ 25\% \text{ is } 5,000 \\ \times 4 \\ 100\% \text{ is } \boxed{20,000} \end{array}$$

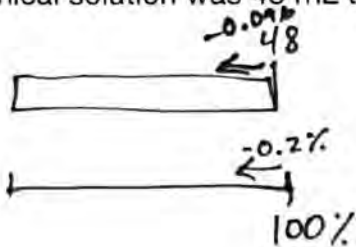
10. Write your answer in a complete sentence:

The total weight of the rock is 20,000 grams.

# Name: ANSWER KEY

Solve each problem using related percents. Show your work.

1. Katie measured the amount of chemical solution to 0.2% less than the true volume. If the chemical solution was 48 mL then what was her measurement?



$$0.2\% \text{ of } 48$$

$$\begin{array}{r} 799 \\ 48.000 \\ - 0.096 \\ \hline 47.904 \end{array}$$

$$200\% \text{ of } 48 \text{ is } 96$$

$$20\% \text{ of } 48 \text{ is } 9.6$$

$$2\% \text{ of } 48 \text{ is } 0.96$$

$$0.2\% \text{ of } 48 \text{ is } 0.096$$

2. At the supermarket, Lindsey finds 0.07% of the weight of the apple is 0.048 kg. What would be 7% of that weight?

$$0.07\% \text{ is } 0.048$$

$$0.7\% \text{ is } 0.48$$

$$7\% \text{ is } 4.8 \text{ kg}$$

3. There are 400 students at Burning Tree Elementary.  $\frac{1}{2}\%$  of the students take a scooter home from school. How many students take a school home from school?

$$\frac{1}{2}\% \text{ of } 400$$

$$100\% \text{ of } 400 \text{ is } 400$$

$$10\% \text{ of } 400 \text{ is } 40$$

$$1\% \text{ of } 400 \text{ is } 4$$

$$\frac{1}{4}\% \text{ of } 400 \text{ is } 1$$

4. Cleveland was told that his newborn puppy was 0.5% of the total weight he would be when he was fully grown. If the dog weighs 6 ounces now, how much will it weigh when it is fully grown?

$$0.5\% \text{ is } 6$$

$$5\% \text{ is } 60$$

$$50\% \text{ is } 600$$

$$100\% \text{ is } 1200 \text{ ounces}$$

Solve each problem using related percents. Show your work.

5. In the entire 800 person population of the school, only  $\frac{1}{4}\%$  chose fencing as their favorite sport. How many students chose fencing as their favorite sport?

$$\frac{1}{4}\% \text{ of } 800$$

$$\begin{aligned} 100\% \text{ of } 800 & \text{ is } 800 \\ 10\% \text{ of } 800 & \text{ is } 80 \\ 1\% \text{ of } 800 & \text{ is } 8 \\ \frac{1}{4}\% \text{ of } 800 & \text{ is } \boxed{2} \end{aligned}$$

6. Jenny can drive 1 mile on 0.2% of the gas in the gas tank of her car. How far can Jenny drive on a full tank of gas?

$$\begin{aligned} 0.2\% & \text{ is } 1 \text{ mile} \\ 2\% & \text{ is } 10 \text{ miles} \\ 20\% & \text{ is } 100 \text{ miles} \\ 100\% & \text{ is } \boxed{500 \text{ miles}} \end{aligned}$$

7. An investment portfolio of \$1,000 is expected to yield 0.3% annually. How much money would one expect the portfolio to earn in a year?

$$0.3\% \text{ of } 1,000$$

$$\begin{aligned} 300\% \text{ of } 1,000 & \text{ is } 3,000 \\ 30\% \text{ of } 1,000 & \text{ is } 300 \\ 3\% \text{ of } 1,000 & \text{ is } 30 \\ 0.3\% \text{ of } 1,000 & \text{ is } \boxed{3} \end{aligned}$$

8. A swimming pool loses water at a rate of 0.75% per day due to evaporation. How much water will the pool lose if it has 50,000 liters of water?

$$0.75\% \text{ of } 50,000$$

$$\begin{aligned} 100\% \text{ of } 50,000 & \text{ is } 50,000 \\ 25\% \text{ of } 50,000 & \text{ is } 12,500 \\ 75\% \text{ of } 50,000 & \text{ is } 37,500 \\ 7.5\% \text{ of } 50,000 & \text{ is } 3,750 \\ 0.75\% \text{ of } 50,000 & \text{ is } \boxed{375} \end{aligned}$$

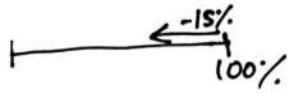
# **G7 U3 Lesson 15**

Solve problems about real-world situations that involve percent increase and decrease.

**G7 U3 Lesson 15 - Today we will solve multi-step percent problems.**

**Warm Welcome (Slide 1):** Tutor choice

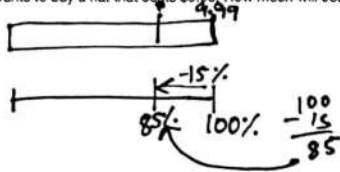
**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we will solve multi-step percent problems. This is the last lesson of this unit so we are just seeing how we can take all the awesome work we've done to the next level.



**Let's Review (Slide 3):** We already know use a diagram, table and equation to solve percent problems. Read the problem silently along with me while I read the problem out loud. *Read the problem out loud.* It says there's a 15% discount so I'll start by drawing a number line for 15% out of 100%.

It says, "Joshua wants to buy a hat that costs \$9.99." So I will draw that. When I mark the question mark for the new price, I see I have to find the new percent.

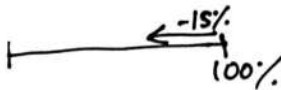
Rita's Hat Shop is having a sale where all hats are discounted by 15%. Joshua wants to buy a hat that costs \$9.99. How much will Joshua spend?



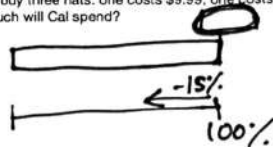
$$9.99 \times 0.85 = ?$$

So I am going to subtract  $100 - 15$  which is 85%. Now I can set up a table or an equation. We would do  $9.99 \times 0.85$  equals question mark. I am not going to do all that number-crunching right now. We just want to review the ideas we already know. Now let's think about how we will use these same ideas for multi-step problems.

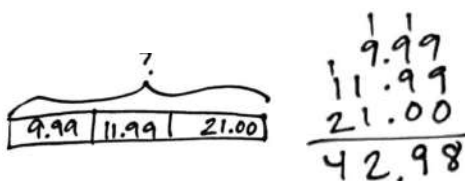
**Let's Talk (Slide 4):** For multi-step word problems, it is easier to calculate any sums before solving for the percent. Read the problem silently while I read the problem out loud. *Read the problem out loud.* We know to draw a picture. I will start with that first sentence and it is the same as the problem we just did on the last slide.



Rita's Hat Shop is having a sale where all hats are discounted by 15%. Cal wants to buy three hats: one costs \$9.99, one costs \$11.99, and one costs \$21.00. How much will Cal spend?

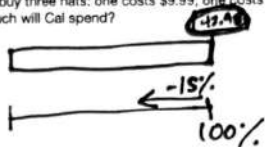


But now when I go to draw the rectangle, I realize that I am going to have a lot of math to do before I can fill that in.



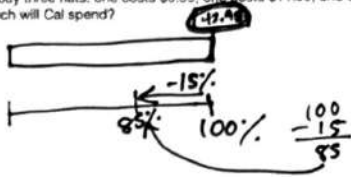
That's no problem. All this means is that I need to do some work off to the side of my paper. I might even need to draw a whole other tape diagram to figure that rectangle out and then I can go back. So let's see, Cal wants to buy 3 hats. So this rectangle is really 9.99 and 11.99 and 21.00. I am going to have to do that math first. I'm not going to spend much time talking that through since I know you know how to add. We get \$42.98.

Rita's Hat Shop is having a sale where all hats are discounted by 15%. Cal wants to buy three hats: one costs \$9.99, one costs \$11.99, and one costs \$21.00. How much will Cal spend?



Now, I can go back to my percent work. \$42.98 is that total amount.

Rita's Hat Shop is having a sale where all hats are discounted by 15%. Cal wants to buy three hats: one costs \$9.99, one costs \$11.99, and one costs \$21.00. How much will Cal spend?



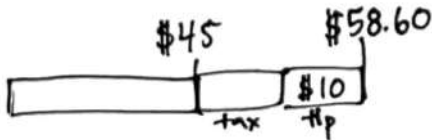
And now I do all the work I did before.  $100 - 15$  is 85%.

$$\begin{array}{r}
 37 \\
 42.98 \\
 \times 0.85 \\
 \hline
 21490 \\
 343840 \\
 \hline
 365330
 \end{array}$$

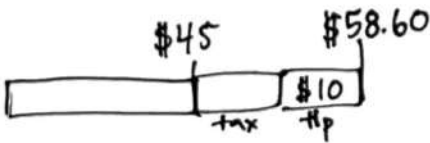
And I do  $42.98 \times 0.85$ . Let me show you that work. That is a lot! Imagine if I had found the percent for each separate hat. That would be a lot of number-crunching. So it makes sense to get the total bill and then find the final percent. So from now on, as we read, we might need to draw more than one tape diagram.



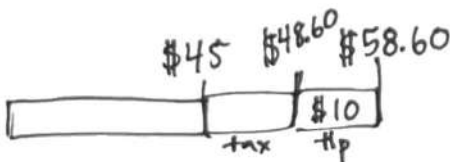
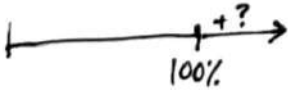
**Let's Think (Slide 5):** Let's do another one. This time we'll see that sometimes we will need to do a final step AFTER we find the percent. Read the problem silently while I read the problem out loud. *Read the problem out loud.* Let's draw! First, we represent the \$45 dinner.



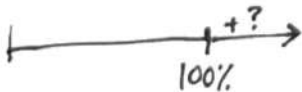
But it says, "she ended up paying \$58.60 with the tax and tip." I am going to make a rectangle for tax and a rectangle for tip. All of that makes \$58.60. Now it says the tip was \$10.



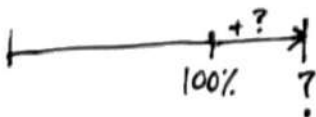
The question is asking for the tax percentage on the dinner. So now I will draw a percent line.



I can see in my picture that the first thing I need to do is get rid of the tip. That's \$48.60.

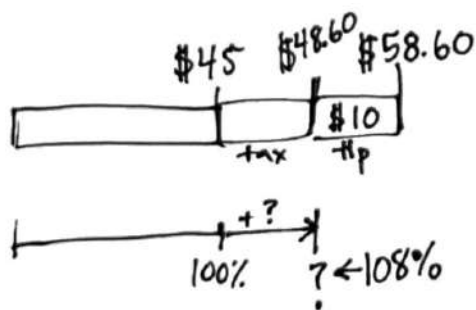


But I still can't figure out this tax until I figure out the percent that corresponds to the \$48.60. I'm going to put a question mark there too. Hopefully this is helping you see that we need to work in pieces.



$$\begin{array}{r}
 01.08 \\
 45 \overline{) 48.60} \\
 \underline{-45} \downarrow \\
 36 \downarrow \\
 \underline{-00} \downarrow \\
 360 \\
 \underline{-360} \\
 000
 \end{array}$$

So, let's just figure out that larger percent. 45 times question mark equals \$48.60. Divide by 45 on each side. I will do the math over to the side. I get 1.08 which is 108%.



That's not my final answer though. I have to do 108% minus 100% to see that it's 8% tax.

**Let's Try It (Slide 6):** Now we will practice some more problems. I will lead you through step by step.



# WARM WELCOME



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**Today we will solve multi-step percent problems.**

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 **Let's Review:**

**We already know use a diagram, table and equation to solve percent problems.**

Rita's Hat Shop is having a sale where all hats are discounted by 15%. Joshua wants to buy a hat that costs \$9.99. How much will Joshua spend?

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 **Let's Talk:**

**For multi-step word problems, it is easier to calculate any sums before solving for the percent.**

Rita's Hat Shop is having a sale where all hats are discounted by 15%. Cal wants to buy three hats: one costs \$9.99, one costs \$11.99, and one costs \$21.00. How much will Cal spend?

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## Let's Think:

**Sometimes we will need to do a final step after we find the percent.**

Jenna had a \$45 dinner. But she ended up paying \$58.60 with the tax and tip. If the tip was \$10, what was tax percentage that Jenna was charged on the dinner?

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## Let's Try It:

**We will do it together step by step!**

Name: \_\_\_\_\_ G7 U3 Lesson 15 - Let's Try It

Francie was told by a scientist that 0.25% of a rock she found is quartz and 3% is feldspar. The rest of the rock is calcite. If the volume of the quartz is 50 cubic cm, what is the volume of the calcite?

1. Draw a diagram with words to represent the problem.
2. Be sure to include as many question marks as you need in your diagram for #1.
3. Solve for one question mark.
4. Redraw your diagram after you have solved for some of the missing information.
5. Continue to solve.

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## On your Own:

Now it's time for you to do it on your own.

Name: \_\_\_\_\_ G7 U3 Lesson 15 - Independent Work

Draw a tape diagram to represent each problem then solve. Show your work.

1. Barney wants to order two egg sandwiches and a coffee at Kelly's Cafe. Egg sandwiches cost \$6. The coffee costs \$2. Barney must pay 7% of the balance in sales tax. He wants to leave 20% of the balance in a tip. How much does Barney need?

2. Francie, Julie and Karen collected 15 pounds of shells. 18% were shark-eyed moon snail shells. If the girls split each type of shell evenly, how much shark-eyed moon snail shells did each girl get?

3. Cody works at an electronics store. He earns a \$1000 weekly salary as well as a commission on

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Name: \_\_\_\_\_

Draw a tape diagram to represent each problem then solve. Show your work.

1. Barney wants to order two egg sandwiches and a coffee at Kelly's Cafe. Egg sandwiches cost \$6. The coffee costs \$2. Barney must pay 7% of the balance in sales tax. He wants to leave 20% of the balance in a tip. How much tip does Barney need?

2. Francie, Julie and Karen collected 15 pounds of shells. 18% were shark-eyed moon snail shells. If the girls split each type of shell evenly, how much shark-eyed moon snail shells did each girl get?

3. Cody works at an electronics store. He earns a \$1000 weekly salary as well as a commission on whatever he sells. This week, Cody sold \$45,000 in electronics. His paycheck was \$5,500. What rate of commission must Cody receive on his sales?

Draw a tape diagram to represent each problem then solve. Show your work.

5. Bethany and her two brothers decided to split the cost of a Mother's Day gift for their mom. They decided to buy her a bouquet of flowers. The cost of the flowers was \$150. There was 6% sales tax on the cost of the flowers as well as a \$20 shipping fee. How much money did each person need to contribute for the gift?

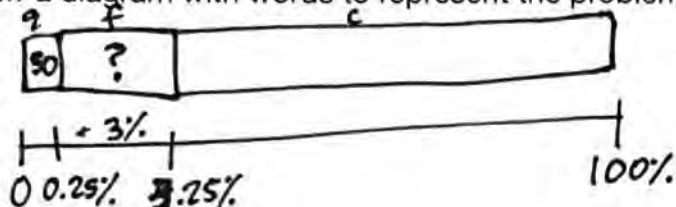
6. A \$200 lawn mower was marked up by 10% at Home Depot. Keira has a 10% coupon. What was the final price of the lawn mower?

7. In Ms. Perlo's 6th grade class, 25% of students got a C on the final exam. 40% of students got a B on the final exam. 30% of students got an A on the final exam. The rest of the students got a perfect score. There are 40 kids in Ms. Perlo's class. How many kids got a perfect score?



Francie was told by a scientist that 0.25% of a rock she found is quartz and 3% is feldspar. The rest of the rock is calcite. If the volume of the quartz is 50 cubic cm, what is the volume of the calcite?

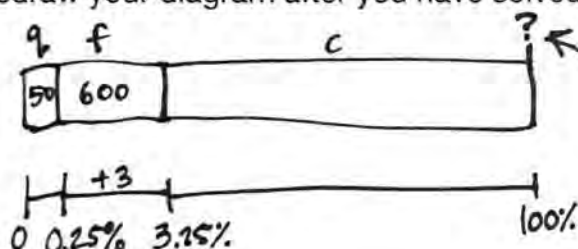
1. Draw a diagram with words to represent the problem.



0.25% is 50  
 0.50% is 100  
 1.00% is 200  
 3% is 600

2. Be sure to include as many question marks as you need in your diagram for #1.  
 3. Solve for one question mark.

4. Redraw your diagram after you have solved for some of the missing information.



0.25% is 50  
 2.5% is 500  
 25% is 5,000  
 ×4                      ×4  
 100% is 20,000

5. Continue to solve.

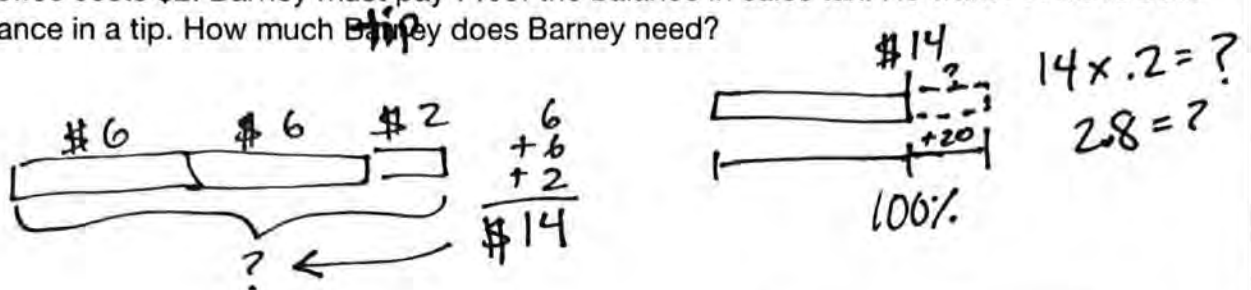
$$\begin{array}{r}
 20,000 \\
 - 600 \\
 \hline
 19,400 \\
 - 50 \\
 \hline
 19,350
 \end{array}$$

6. Write your final answer as a complete sentence.

The volume of the calcite is 19,350 cubic cm.

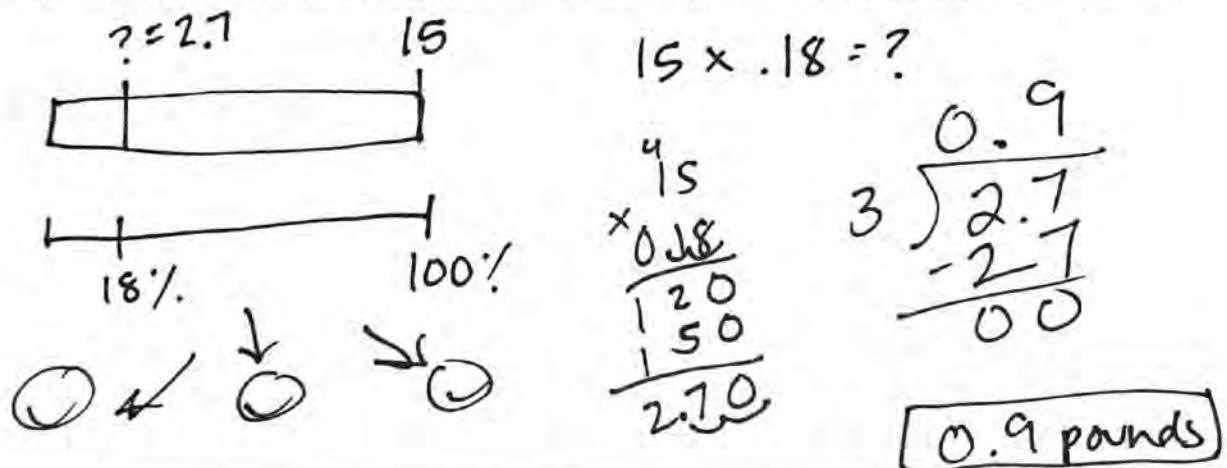
Draw a tape diagram to represent each problem then solve. Show your work.

1. Barney wants to order two egg sandwiches and a coffee at Kelly's Cafe. Egg sandwiches cost \$6. The coffee costs \$2. Barney must pay 7% of the balance in sales tax. He wants to leave 20% of the balance in a tip. How much ~~Barney~~ tip does Barney need?



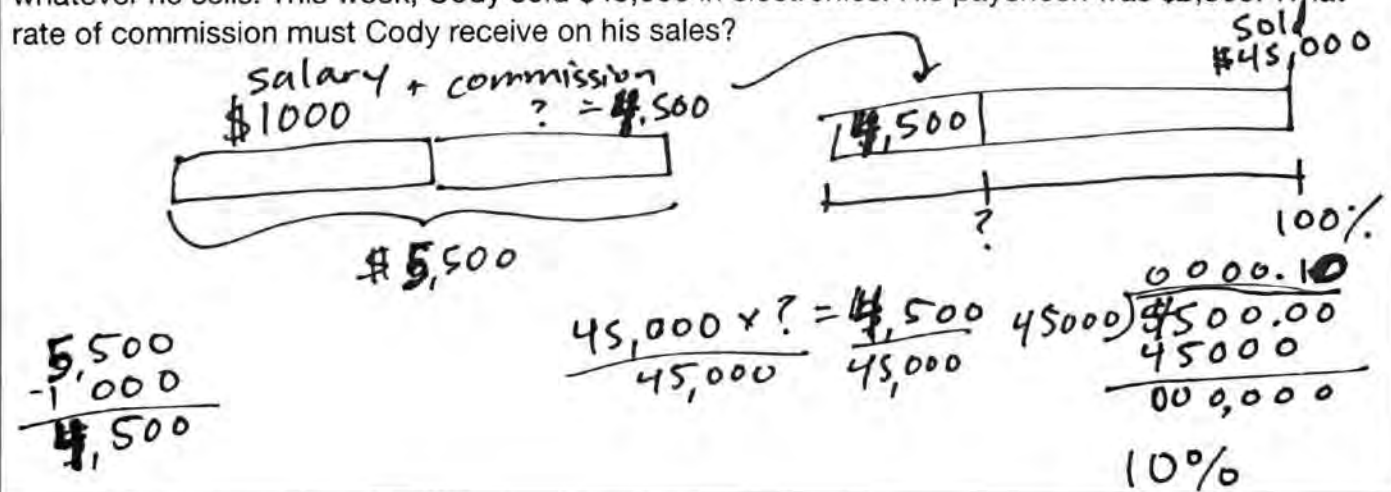
**\$2.80 tip**

2. Francie, Julie and Karen collected 15 pounds of shells. 18% were shark-eyed moon snail shells. If the girls split each type of shell evenly, how much shark-eyed moon snail shells did each girl get?



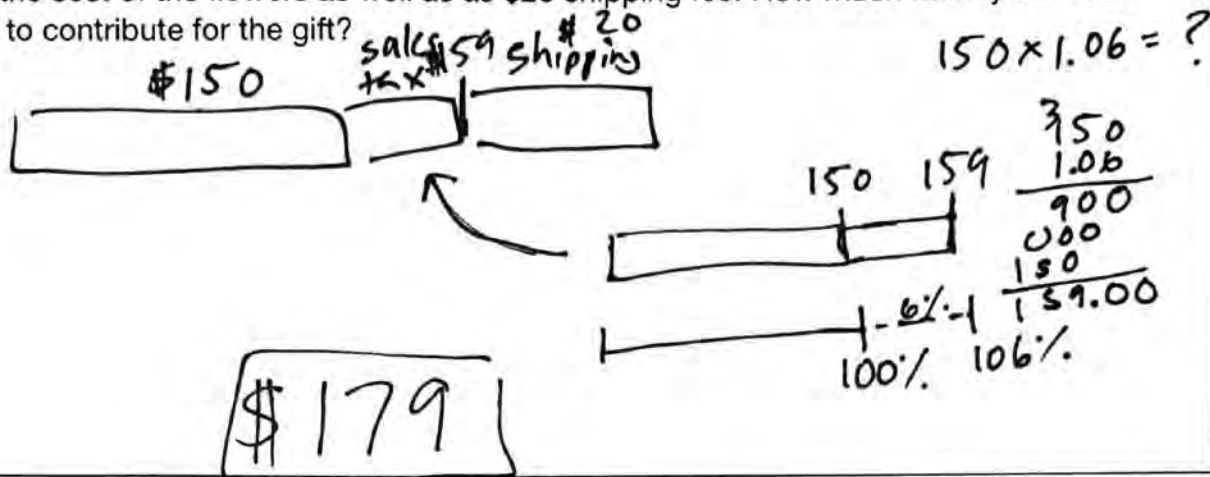
**0.9 pounds**

3. Cody works at an electronics store. He earns a \$1000 weekly salary as well as a commission on whatever he sells. This week, Cody sold \$45,000 in electronics. His paycheck was \$5,500. What rate of commission must Cody receive on his sales?

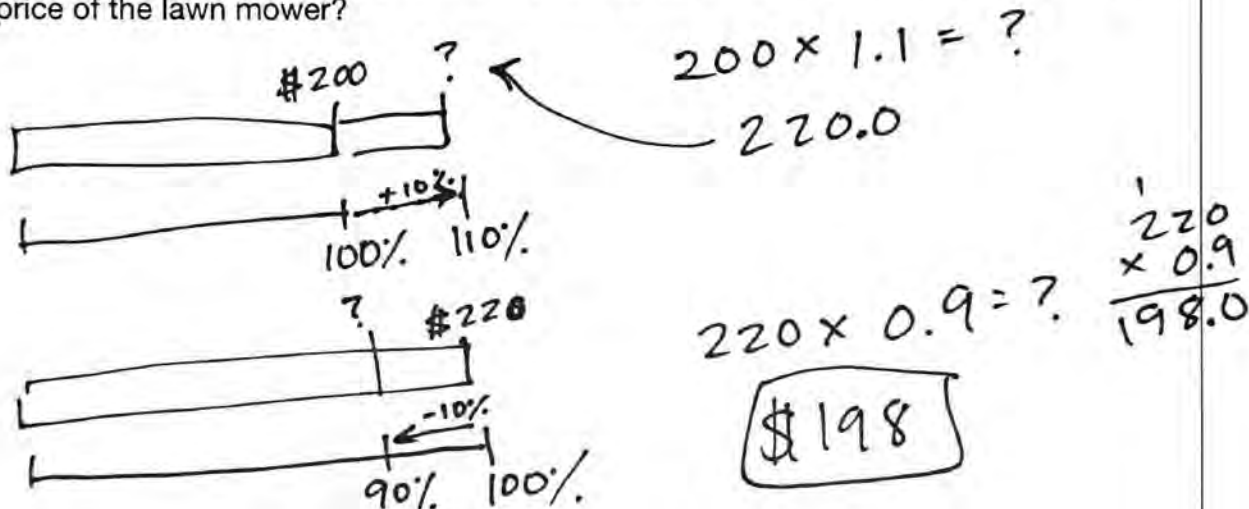


Draw a tape diagram to represent each problem then solve. Show your work.

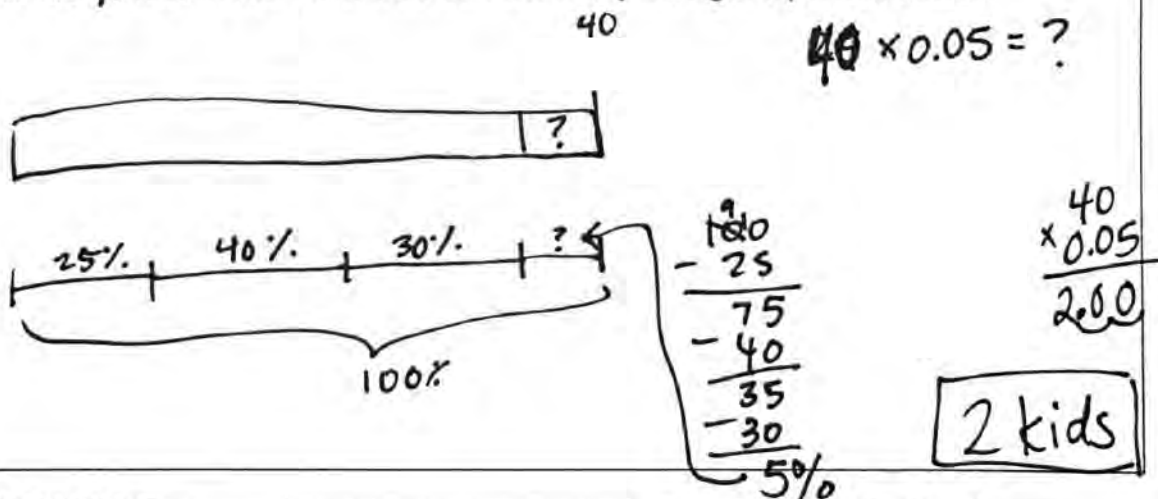
5. Bethany and her two brothers decided to split the cost of a Mother's Day gift for their mom. They decided to buy her a bouquet of flowers. The cost of the flowers was \$150. There was 6% sales tax on the cost of the flowers as well as a \$20 shipping fee. How much money did each person need to contribute for the gift?



6. A \$200 lawn mower was marked up by 10% at Home Depot. Keira has a 10% coupon. What was the final price of the lawn mower?



7. In Ms. Perlo's 6th grade class, 25% of students got a C on the final exam. 40% of students got a B on the final exam. 30% of students got an A on the final exam. The rest of the students got a perfect score. There are 40 kids in Ms. Perlo's class. How many kids got a perfect score?





# G7 Unit 4:

Rational Number Arithmetic

# **G7 U4 Lesson 1**

Interpret signed numbers in the context of temperature and elevation.

## G7 U4 Lesson 1 - Students will interpret signed numbers in the context of temperature and elevation.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today is our first lesson of a new unit all about rational numbers. A rational number just means any number that can be written as a fraction.  $\frac{1}{2}$  and  $\frac{2}{3}$  are rational numbers, because they're fractions. Rational numbers also include decimals and whole numbers, because we know we can represent them as fractions. Today, and for most of this unit, we'll look closely at a particular kind of rational number...negative numbers!

Today, we're going to interpret signed numbers in the context of temperature and elevation. Let's get going.

**Let's Talk (Slide 3):** Take a moment and think about these two questions. Why do we need negative numbers? Where do we see negative numbers in the world around us? When you have an idea or two, share what comes to mind. **Possible Student Answers, Key Points:**

- I see negative numbers in a video game when somebody loses points.
- I see negative numbers when it's really cold outside. The temperature can be negative.
- If I spend money, my account might show a negative number.

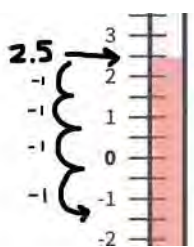
When we first learn about numbers, we tend to focus only on positive numbers, but there are times where negative numbers come in handy. Today, we'll look at two particularly common contexts: temperature and elevation.

**Let's Think (Slide 4):** Before we read this problem, take a look at the image of the two thermometers. What do you notice or wonder about the visual? **Possible Student Answers, Key Points:**

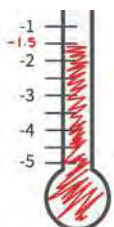
- I notice there are two thermometers labeled afternoon and night. I notice the temperature in the afternoon is between 2 and 3 degrees. I notice the second thermometer is blank. I notice some negative numbers.
- I wonder where these temperatures were take. I wonder if it was snowing, because it seems cold.

Interesting. Let's use the picture of the thermometers to help us solve these problems. (*read the problem aloud*) The first prompt wants us to figure out what the new temperature will be if the temperature drops 4 degrees. What temperature does the afternoon thermometer show? How do you know? **Possible Student Answers, Key Points:**

- It's halfway between the 2 and the 3. The temperature in the afternoon is 2.5 or  $2\frac{1}{2}$  degrees.



(*label 2.5 with an arrow*) If the temperature drops 4 degrees, I can picture the red on the thermometer going down 4 degrees. (*draw and label hops on the thermometer as -1 while you narrate the temperature decreasing*) It was 2.5 degrees. If it decreases 1 degree, I see on the thermometer it's now 1.5 degrees. If it decreases 1 degree again, I see it's 0.5 degrees. Again, it's -0.5 degrees. And if it decreases one more time, we're now at -1.5 degrees. So if the temperature starts at 2.5 degrees and drops 4 degrees, the thermometer shows us that it is now, what? (*-1.5 degrees*)



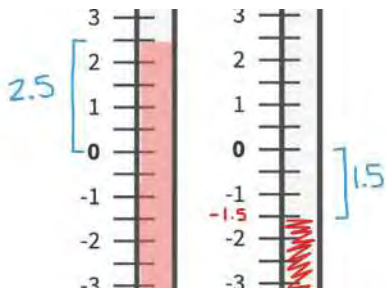
How could I show that on the blank thermometer? (*shade and label thermometer after student explains*) **Possible Student Answers, Key Points:**

- -1.5 is in the middle of -1 and -2. You can shade to the tick mark that is halfway between -1 and -2.

$$2.5 > -1.5$$

$$-1.5 < 2.5$$

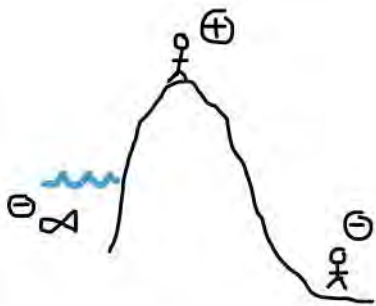
The afternoon temperature was 2.5 degrees. We just figured out that the night temperature is -1.5 degrees. The problem now asks us to write an inequality comparing the afternoon and night temperatures. We can look at the thermometer to easily see that 2.5 is greater than -1.5. (*write  $2.5 > -1.5$* ) The afternoon thermometer's red section is higher up along the thermometer than the night's thermometer. Also, positive numbers are always greater than negative numbers. We can write this comparison another way. What would the inequality look like if we started with -1.5? (*-1.5 is less than 2.5*) (*write  $-1.5 < 2.5$* ) Either way we wrote this comparison is correct.



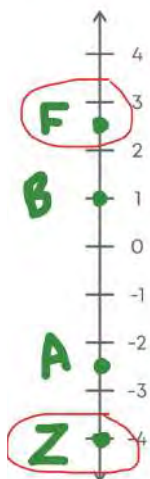
For the last question, they want us to consider how far each temperature is from freezing if zero degrees represents freezing. We can use the thermometer like a number line and mark the distance from 0 to each value. (*draw bracket from 0 to 2.5 on the first thermometer*) How far is 2.5 from 0? (*2.5*) (*write 2.5 next to bracket*) Great, now let's look at the night time temperature. (*draw bracket from 0 to -1.5*) I know the temperature is -1.5. If I look at the intervals from 0 to -1.5, I see that -1.5 is 1.5 away from 0. I might be tempted to say it is *negative* 1.5 from 0, but the question is just asking for the distance. We can say -1.5 is 1.5 degrees from 0.

We just thought about negative numbers in the context of temperature. Let's look at one more context for negative numbers.

**Let's Think (Slide 5):** This question involves negative numbers in the context of elevation. We'll read it in a moment.



(*sketch a simple picture as you explain*) Elevation refers to the height of something, usually in reference to sea level. We tend to use 0 to represent sea level. Positive elevations refer to heights *above* sea level. If I'm standing on top of a hill, I'd have a positive elevation. Negative elevations refer to heights *below* sea level. A fish swimming underwater will often be represented with a negative elevation. It's important to note that negative elevations can sometimes refer to objects or people on land too. If you're standing in a deep valley, you might be below sea level, so you'd have a negative elevation.



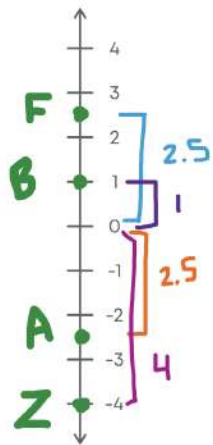
(*read the problem aloud*) The first question wants us to think about whose elevation is highest and lowest. Before we answer, let's mark each person's elevation on the vertical number line so we can visualize the situation. I'll mark Bailey and Zack first, since those numbers are already labeled clearly on the number line. (*mark 1 and -4 with B and Z, respectively*) Where should I mark Frank and Aya's elevations? **Possible Student Answers, Key Points:**

- 2.5 should go halfway between the 2 and the 3. -2.5 should go halfway between the -2 and the -3.

(*label each point*) The vertical number line makes it very easy to see whose elevation is highest and lowest. Frank's elevation is highest, since he's closest to the top of the number line. Zack's elevation is lowest, since he's closest to the bottom of the number line.

The next prompt wants us to consider how far each person is from sea level. Remember, elevations at sea level are typically represented as 0. This prompt is similar to the temperature problem, when it asked us to name how far each temperature was from 0. Let's look at our number line to help us.





Frank's elevation is 2.5 yards. I know that is 2.5 spaces from 0 on the number line. We can say Frank is 2.5 yards from sea level. *(draw and label bracket from 2.5 to 0)*

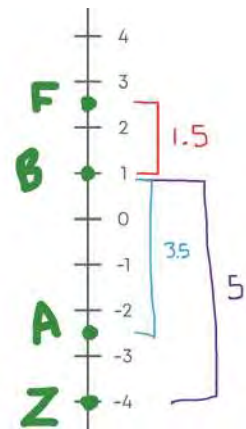
*(continue drawing and labeling brackets as you narrate)*

Bailey's elevation is 1 yard. How far is Bailey from sea level? **(He is 1 yard from sea level)**

The other two people have negative elevations. This question is just asking us how far each person is from sea level, or how far they are from 0, so we can respond simply using positive values to name how many yards away they are. How far are Aya and Zack from sea level? How do you know? **Possible Student Answers, Key Points:**

- Aya is 2.5 yards from sea level, because she is 2 and a half tick marks below 0.
- Zack is 4 yards from sea level, because he is 4 tick marks below 0.

When a question asks us how far people are from 0, we can use a number line to count the intervals between their location to 0.



Let's wrap this up by now finding how far Bailey is from each friend. We know Bailey is at an elevation of +1 yard. Instead of finding the friends' distances from 0, we can now just count how far they are away from 1 on the number line. *(draw and label brackets as you count intervals during the next sequence of dialogue)*

Frank is at positive 2.5 yards. I can move my finger from 1 to 2.5, and see that Bailey is 1.5 yards from Frank. Aya is at negative 2.5 yards. I can move my finger from 1 to -2.5. I see that Bailey is *(whisper count as you move your finger 1, 2, 3, 3.5...)* 3.5 yards from Aya. How far is Bailey from Zack? How do you know? **Possible Student Answers, Key Points:**

- Bailey is 5 yards from Zack. If I count the intervals from 1 to -4 on the number line, she is 5 spaces away from Zack.

Our answers just now were all positive values, because we were just asked to name how far they are from Bailey. When asked for a distance, it's most common to respond with a positive value, regardless of an object or person's position.

Nice job! We just answered questions involving negative values related to elevation.

**Let's Try it (Slides 6 - 7):** Now let's work with some more problems where we have to interpret signed numbers in the context of temperature and elevation. As we work, it will be helpful to visualize the position of values by using a vertical number line. We'll also want to keep in mind whether it makes more sense to answer the given question using a negative value or a positive value. Depending on what is being asked, one type of signed number may be more appropriate than another. I know you're going to do great!

# WARM WELCOME



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**Today we will interpret signed numbers  
in the context of temperature and  
elevation.**

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## Let's Talk:

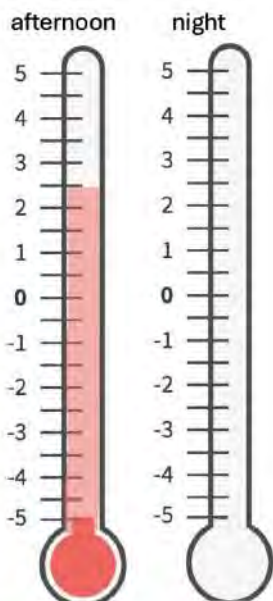
**Why do we need negative numbers?**

**Where do we see negative numbers in the world around us?**

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## Let's Think:

**The temperature in the afternoon is shown in the first thermometer.**

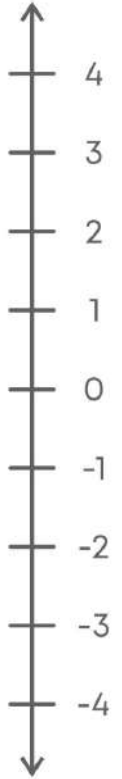


- At night, the temperature drops 4 degrees. Shade the thermometer. What is the new temperature?
- Write an inequality to compare the afternoon and night temperatures.
- Zero degrees is considered freezing. How far is each temperature from freezing?

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# Let's Think:

NAME	ELEVATION (yards)
Frank	2.5
Aya	-2.5
Bailey	1
Zack	-4



- Whose elevation is highest? Lowest?
- How far is each person from sea level?
- How far is Bailey from each of her friends?

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# Let's Try It:

**Let's explore interpreting signed numbers in the context of temperature and elevation together.**

Name: \_\_\_\_\_ G7 U4 Lesson 1 - Let's Try It

The first thermometer represents the temperature in the town of Mathville yesterday.

- What Celsius temperature is shown on the first thermometer?
- A cold front moves in today, and the temperature drops 10 degrees. Shade the second thermometer to show the new temperature.
- The old temperature was \_\_\_\_\_ °, and the new temperature is \_\_\_\_\_ °.
  - above, above
  - above, below
  - below, below
  - below, above
- What is today's temperature in Celsius?

The thermometer below shows the temperature in Numbertown today.

- What is the temperature, in degrees Celsius, in Numbertown?
- Is it colder in Mathville or in Numbertown today?
- Write two inequalities to compare today's temperature in Mathville to today's temperature in Numbertown.

The table shows the elevation of several plants.

Plant	Elevation (meters)
Pondweed	-1
Fern	0.5
Sugar Kelp	-2.5
Cattail	0
Sunflower	3

- Plot and label each plant's elevation on the number line.
- Which plant is at the highest elevation?
- Which plant is at the lowest elevation?
- Write two inequalities to compare the elevation of the fern to the elevation of the pondweed.
- Complete each statement.
  - The sunflower is \_\_\_\_\_ feet from sea level.
  - The pondweed is \_\_\_\_\_ feet from sea level.
  - The sugar kelp is \_\_\_\_\_ feet from sea level.
- Complete each statement and circle above or below.
  - The sunflower is \_\_\_\_\_ feet (above/below) the cattail.
  - The pondweed is \_\_\_\_\_ feet (above/below) the sunflower.
  - The fern is \_\_\_\_\_ feet (above/below) the sugar kelp.

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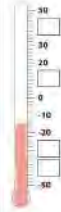


# On your Own:

## Now it's time to explore interpreting signed numbers in the context of temperature and elevation on your own.


Name: \_\_\_\_\_ 57.144 Lesson 1 - Independent Work

**1. Look at the thermometer shown here.**



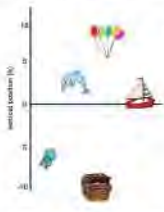
- What numbers go in the boxes?
- What temperature does the thermometer show?
- Jesse said the thermometer shows -24 degrees. Explain why that is incorrect.

**2. Label each thermometer with the temperature it shows.**



Which temperature is highest? Lowest?

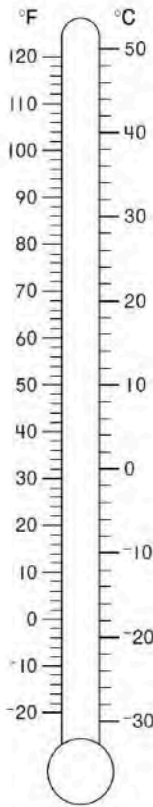
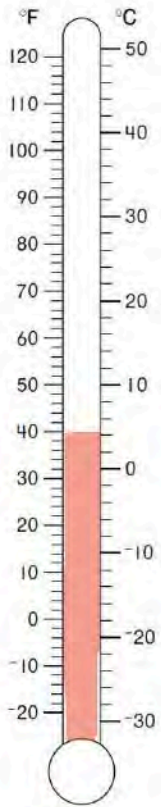
**3. Use the images and the number line to respond to the prompts.**



- How far above or below sea level is each object?
- How far is the jellyfish from the dolphin? How far are the balloons from the treasure?
- A seagull flies 2 feet above sea level. How does its distance from sea level compare with the vertical distance from each object?
- A shark swims 4 feet below sea level. How does its distance from sea level compare with the vertical distance from each object?
- A person is 1 floor from sea level. What two values could be used to represent the person's possible vertical position?

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The first thermometer represents the temperature in the town of Mathville yesterday.



1. What Celsius temperature is shown on the first thermometer?

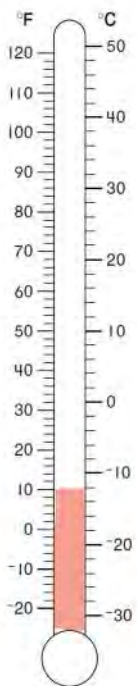
2. A cold front moves in today, and the temperature drops 10 degrees. Shade the second thermometer to show the new temperature.

3. The old temperature was \_\_\_\_\_ °, and the new temperature is \_\_\_\_\_ °.

- a. above, above
- b. above, below
- c. below, below
- d. below, above

4. What is today's temperature in Celsius?

The thermometer below shows the temperature in Numbertown today.



5. What is the temperature, in degrees Celsius, in Numbertown?

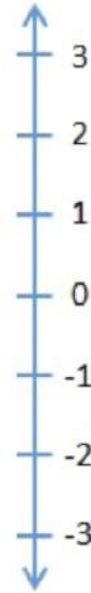
6. Is it colder in Mathville or in Numbertown today?

7. Write two inequalities to compare today's temperature in Mathville to today's temperature in Numbertown.

The table shows the elevation of several plants.

8. Plot and label each plant's elevation on the number line.

Plant	Elevation (meters)
Pondweed	-1
Fern	0.5
Sugar Kelp	-2.5
Cattail	0
Sunflower	3



9. Which plant is at the highest elevation?

10. Which plant is at the lowest elevation?

11. Write two inequalities to compare the elevation of the fern to the elevation of the pondweed.

12. Complete each statement.

The sunflower is \_\_\_\_\_ feet from sea level.

The pondweed is \_\_\_\_\_ foot from sea level.

The sugar kelp is \_\_\_\_\_ feet from sea level.

13. Complete each statement and circle above or below.

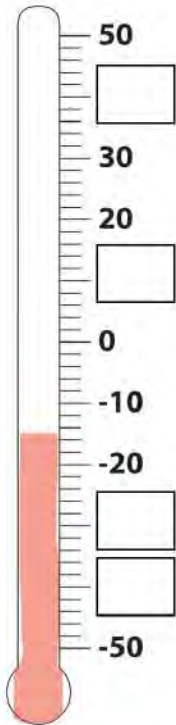
The sunflower is \_\_\_\_\_ feet (above/below) the cattail.

The pondweed is \_\_\_\_\_ feet (above/below) the sunflower.

The fern is \_\_\_\_\_ feet (above/below) the sugar kelp.



**1. Look at the thermometer shown here.**



a. What numbers go in the boxes?

b. What temperature does the thermometer show?

c. Jesse said the thermometer shows -24 degrees. Explain why that is incorrect.

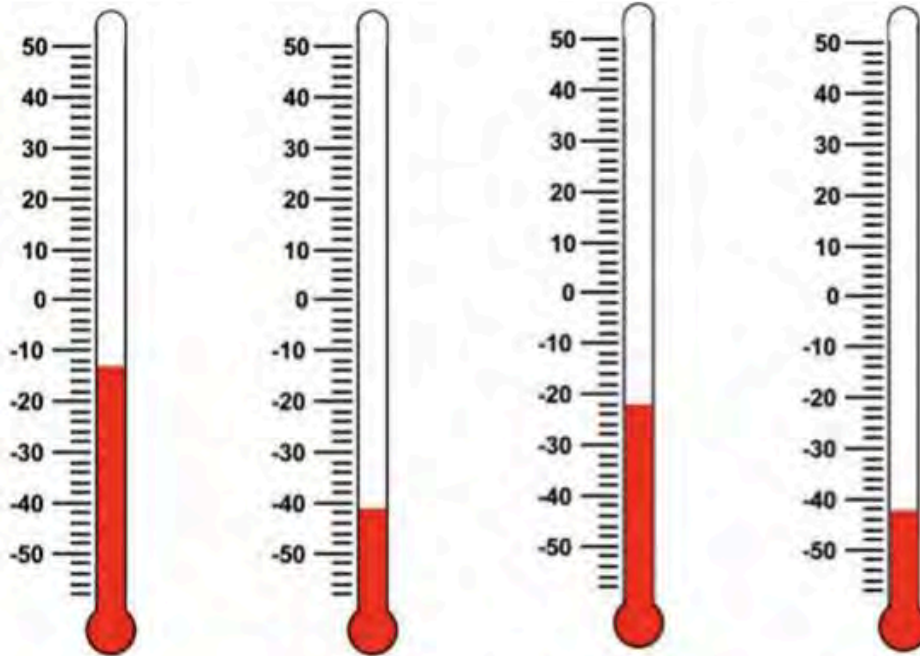
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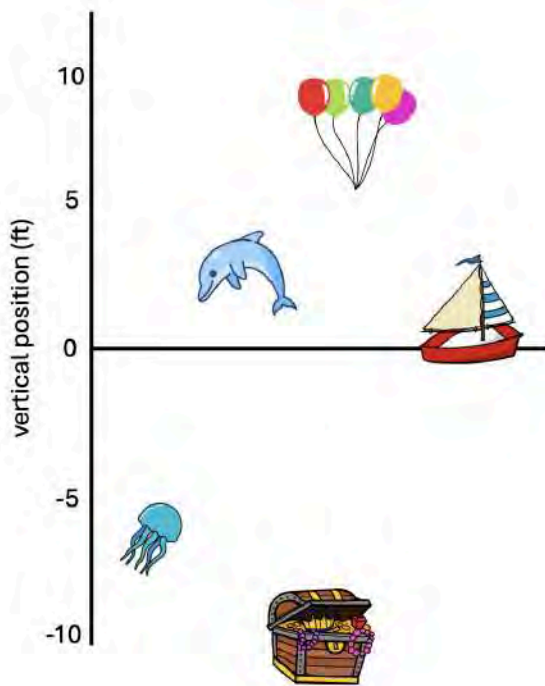
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**2. Label each thermometer with the temperature it shows.**



Which temperature is highest? Lowest?

3. Use the images and the number line to respond to the prompts.



a. How far above or below sea level is each object?

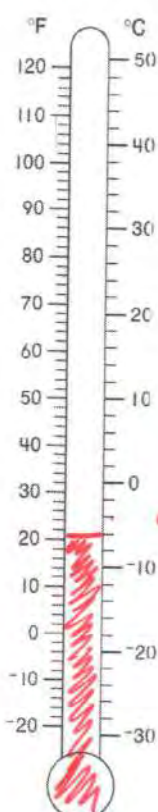
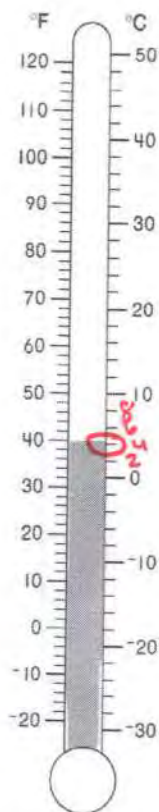
b. How far is the jellyfish from the dolphin? How far are the balloons from the treasure?

c. A seagull flies 5 feet above sea level. How does its distance from sea level compare with the vertical distance from each object?

d. A shark swims 4 feet below sea level. How does its distance from sea level compare with the vertical distance from each object?

e. A person is 1 foot from sea level. What two values could be used to represent the person's possible vertical position?

The first thermometer represents the temperature in the town of Mathville yesterday.



1. What Celsius temperature is shown on the first thermometer?

4°C

2. A cold front moves in today, and the temperature drops 10 degrees. Shade the second thermometer to show the new temperature.

-6°C

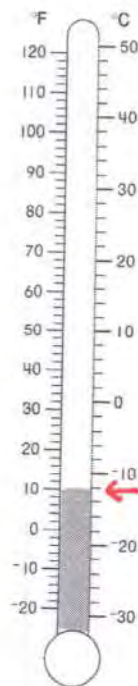
3. The old temperature was \_\_\_\_\_ 0, and the new temperature is \_\_\_\_\_ 0.

- a. above, above
- b. above, below
- c. below, below
- d. below, above

4. What is today's temperature in Celsius?

-6°C

The thermometer below shows the temperature in Numbertown today.



5. What is the temperature, in degrees Celsius, in Numbertown?

-12°C

6. Is it colder in Mathville or in Numbertown today?

It is colder in Numbertown.  
I know because  $-12 < -6$ .

7. Write two inequalities to compare today's temperature in Mathville to today's temperature in Numbertown.

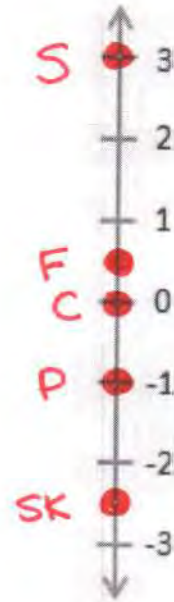
$-12 < -6$

$-6 > -12$

The table shows the elevation of several plants.

8. Plot and label each plant's elevation on the number line.

Plant	Elevation (meters)
Pondweed (P)	-1
Fern (F)	0.5
Sugar Kelp (SK)	-2.5
Cattail (C)	0
Sunflower (S)	3



9. Which plant is at the highest elevation?

The sunflower is at the highest elevation.

10. Which plant is at the lowest elevation?

The sugar kelp is at the lowest elevation.

11. Write two inequalities to compare the elevation of the fern to the elevation of the pondweed.

$$0.5 > -1$$

$$-1 < 0.5$$

12. Complete each statement.

The sunflower is 3 feet from sea level.

The pondweed is 1 foot from sea level.

The sugar kelp is 2.5 feet from sea level.

13. Complete each statement and circle above or below.

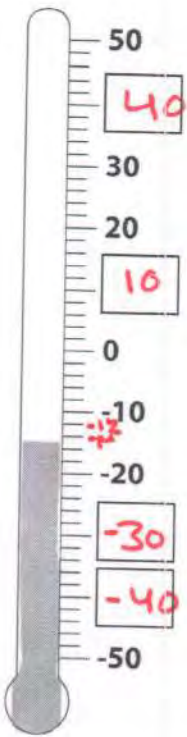
The sunflower is 3 feet (above/below) the cattail.

The pondweed is 4 feet (above/below) the sunflower.

The fern is 3 feet (above/below) the sugar kelp.



1. Look at the thermometer shown here.



a. What numbers go in the boxes?

40

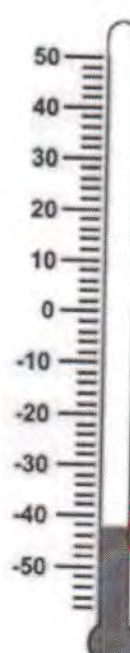
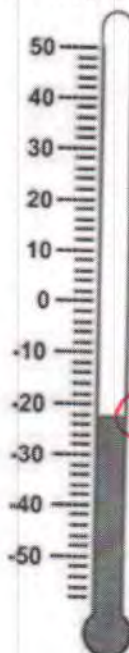
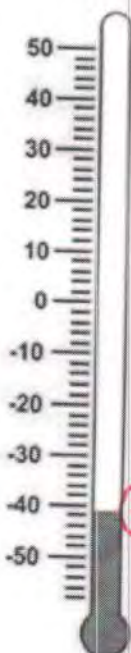
b. What temperature does the thermometer show?

about  $-15^{\circ}$  (or  $-14 / -16$ )

c. Jesse said the thermometer shows  $-24$  degrees. Explain why that is incorrect.

That is incorrect, because the temperature is between  $-10$  and  $-20$ .  $-24^{\circ}$  would be between  $-20$  and  $-30$ .

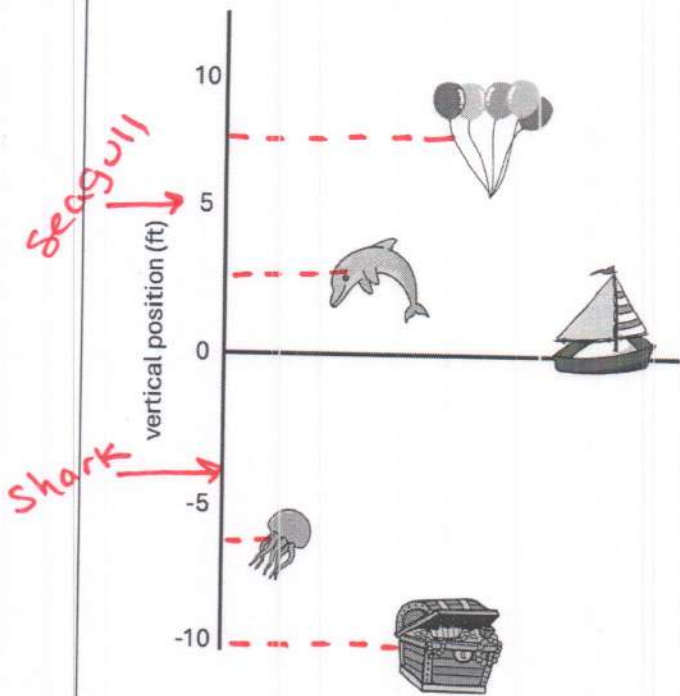
2. Label each thermometer with the temperature it shows.



Which temperature is highest? Lowest?

The highest is  $-14^{\circ}$ , and the lowest is  $-42^{\circ}$ .

3. Use the images and the number line to respond to the prompts.



a. How far above or below sea level is each object?

(answers may vary slightly)

- balloons: 8 feet above
- dolphin: 2 feet above
- boat: 0 feet above/below
- jellyfish: 6 feet below
- treasure: 10 feet below

b. How far is the jellyfish from the dolphin? How far are the balloons from the treasure?

8 feet

18 feet

c. A seagull flies 5 feet above sea level. How does its distance from sea level compare with the vertical distance from each object?

- It's 3 feet below the balloons.
- It's 3 feet above the dolphin.
- It's 5 feet above the boat.
- It's 11 feet above the jellyfish.
- It's 15 feet above the treasure.

d. A shark swims 4 feet below sea level. How does its distance from sea level compare with the vertical distance from each object?

- It's 12 feet below the balloons.
- It's 6 feet below the dolphin.
- It's 4 feet below the boat.
- It's 2 feet above the jellyfish.
- It's 6 feet above the treasure.

e. A person is 1 foot from sea level. What two values could be used to represent the person's possible vertical position?

+1 (above sea level)      -1 (below sea level)

## **G7 U4 Lesson 2**

Use a number line to add positive and negative numbers, generalize how to add positive and negative numbers



**G7 U4 Lesson 2 - Students will use a number line to add positive and negative numbers, and generalize how to add positive and negative numbers.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we spent time thinking about negative values in the contexts of temperature and elevation. What are some things that come to mind when you think about negative numbers in those contexts? **Possible Student Answers, Key Points:**

- We see negative numbers on thermometers. 0 degrees Celsius is freezing, and negative numbers are below freezing.
- Negative numbers in elevation mean you're below sea level. Sea level is often thought of as having an elevation of 0. Sometimes you can have a negative elevation, but not actually be under water.

Today, those contexts will come in handy as we learn about how to add with positive and negative numbers.

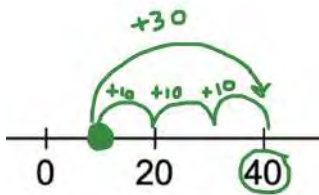
**Let's Talk (Slide 3):** Take a look at the two images here. What is similar? What is different? **Possible Student Answers, Key Points:**

- They both have positive and negative numbers. They both count by intervals of 20. They both show -20.
- One is a thermometer, and the other is a number line. One is oriented horizontally and the other is vertical. One uses a point to show the value, and the other uses a shaded scale.

You noticed a lot of important things. In our previous lesson we saw how a thermometer can help us think about positive and negative values. Since a thermometer is very similar to a number line, we can also use number lines to think about negative values. Whether they're horizontal or vertical, number lines can help us easily visualize negative numbers. Today, we'll use number lines to help us think about adding with positive and negative numbers.

**Let's Think (Slide 4):** Picture a thermometer in your mind. What happens to the thermometer when the temperature rises? (the red line goes up) What happens to the thermometer when the temperature falls? (the red line goes down) In this series of problems, we'll see how we can use a horizontal number line to represent the rise and fall of temperatures, including some values that are negative.

(read the problem aloud)

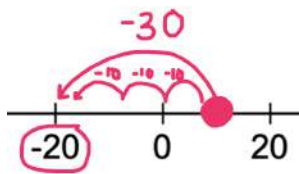


(sketch on number line as you narrate) Part A says the temperature is 10 degrees, so I'll mark a point on 10. It's not labeled on the number line, but I know 10 is halfway between 0 and 20. Since the temperature is *increasing* 30 degrees, I'll move up my number line 10, then 10, then 10. I know the temperature will be 40 degrees. I can also show that using one big hop of 30, if that's easier for me.

$$10 + 30 = 40$$

What equation could I write to represent this change in temperature, and how do you know? (write equation as student shares) **Possible Student Answers, Key Points:**

- We can write  $10 + 30 = 40$ . We started at 10 degrees, it went up 30 degrees, and we ended up at 40 degrees on the number line.

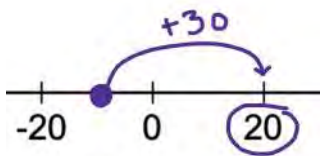


$$10 - 30 = -20$$

$$10 + (-30) = -20$$

Part B has a similar starting temperature, but the problem says the temperature decreased 30 degrees. This means I'll still mark the starting temperature at 10 degrees on my number line, but I'll move 30 degrees in the other direction. I can show my change in temperature in hops of 10 or one big hop of 30. What temperature is it now? (-20 degrees)

(write equations as you narrate) We can write two equations to represent this change. We can think of this as  $10 - 30 = -20$ , because we started at 10 degrees and went down 30 degrees, ending up at -20 on the number line. We can also think about this as  $10 + (-30) = -20$ . We can think of subtracting as adding a negative quantity. Both equations are equivalent, and depending on the situation, one might be more helpful to think about than the other.

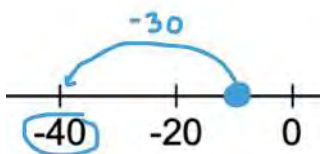


$$-10 + 30 = 20$$

Using what we just did in Part A and Part B, how would you represent Part C on the number line? (draw as student shares, supporting as needed) Possible Student Answers, Key Points:

- I would start at -10, between 0 and -20. Then I would draw a hop of 30 to the right, since the temperature is increasing. My answer would be 20 degrees.

(write equation) We can represent that change in temperature with the equation  $-10 + 30 = 20$ . The starting temperature was -10. We increased, or added, 30 degrees. We ended up at 20 degrees.



$$-10 - 30 = -40$$

$$-10 + (-30) = -40$$

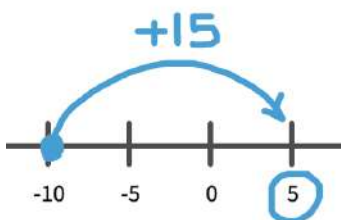
Lastly, Part D wants us show the temperature decreasing 30 degrees from -10 degrees. I'll start at -10 like before, but this time I'll move left 30 to show that the temperature is going down. It would be -40 degrees.

(write equations as student shares) How could I write a subtraction and an addition equation to represent this change? Possible Student Answers, Key Points:

- I can write  $-10 - 30 = -40$ , since we started at -10 and went back 30.
- I can also write  $-10 + (-30) = -40$ , since subtracting is the same as adding a negative value.

Excellent work. We just used a horizontal number line to model temperatures rising and falling. We were able to use equations to show the change in temperature. In the case of the temperature decreasing, we were able to write a subtraction equation and an equation that thought of the change as adding a negative value. Let's keep going.

**Let's Think (Slide 5):** (read the problem aloud) We'll model our thinking similarly for these problems, even though these are about elevation rather than temperature.

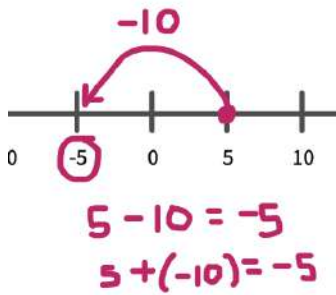


$$-10 + 15 = 5$$

Think about Part A. How could I use a number line to model that the fish starts below sea level, and then jumps out of the water? (sketch on number line as student explains) Possible Student Answers, Key Points:

- Mark a point at -10, since the fish starts 10 feet below sea level. We should draw a hop to the right 15, because the elevation increased 15 feet.

The number line shows us that the fish's new elevation is positive 5, so 5 feet above sea level. We can represent that with the equation  $-10 + 15 = 5$ . (write equation) Both the number line and the equation show that the fish started at -10 feet, increased 15 feet in elevation, and ended up with an elevation of +5 feet.



In Part B, we have a seagull diving into the water. How could you use the information from the prompt to model the elevation change on a number line? What words or phrases helped you think about the model? (*sketch on number line as student shares*) **Possible Student Answers, Key Points:**

- Mark a starting point of +5. The phrase “above sea level” tells me it should be positive 5 rather than -5.
- Draw an arrow to the left 10, since the seagull is “diving”. The word diving tells me the elevation is decreasing.

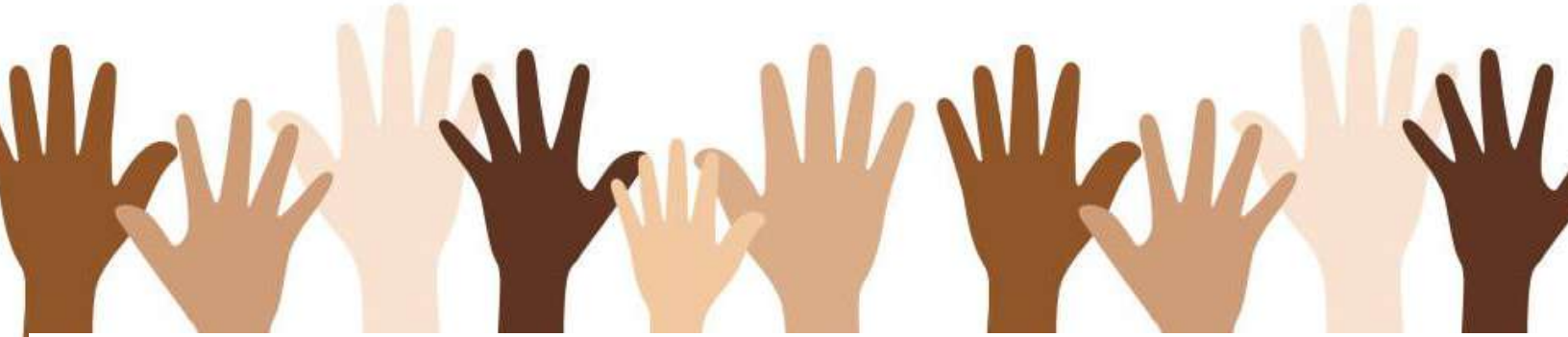
We can think of this as the equation  $5 - 10 = -5$ . The seagull started at 5 feet above sea level. The elevation decreased 10 feet. The seagull ended up at -5 feet. How else can we write this equation? **Possible Student Answers, Key Points:**

- We can write it as  $5 + (-10) = -5$ . We can always think of subtracting as adding a negative value.

The past several examples hopefully make it clear that number lines are incredibly helpful tools when thinking about increasing and decreasing temperature and elevation.

**Let's Try it (Slides 6 - 7):** Now let's try some more problems using a horizontal number line to represent changes in temperature and elevation. As we work, we'll want to carefully think about whether the values in a situation are best represented by positive or negative numbers. We'll also want to consider whether the problems involve increasing or decreasing, as that will tell us which direction to shift along the number line. When we write equations to show our thinking, remember that we can think of subtraction equations as addition equations if instead of subtracting, we add a negative value. Let's go for it!

# WARM WELCOME

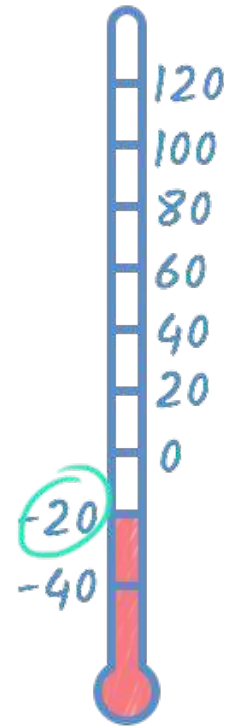
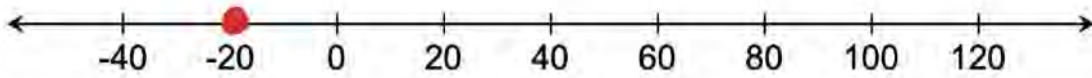


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**Today we will use a number line to add positive and negative numbers, and generalize how to add positive and negative numbers.**

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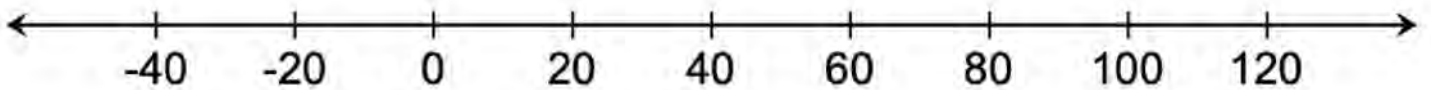
## Let's Talk:



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## Let's Think:

**Use the number line to represent each change in temperature.**



- The temperature is 10 degrees. It **increases** 30 degrees.
- The temperature is 10 degrees. It **decreases** 30 degrees.
- The temperature is -10 degrees. It **increases** 30 degrees.
- The temperature is -10 degrees. It **decreases** 30 degrees.

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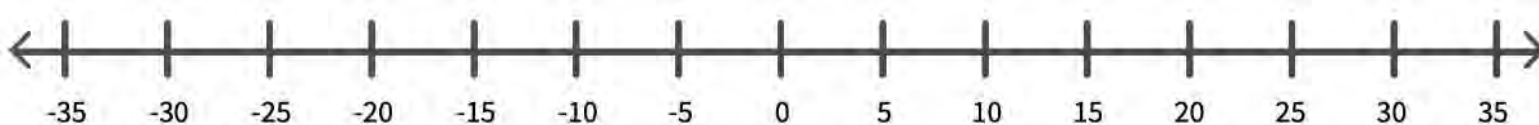


# Let's Think:

Use the number line to represent each change in elevation. Write an equation to match your work.

A fish swims 10 feet below sea level. It jumps out of the water, increasing its elevation by 15 feet.

A seagull flies 5 feet above sea level. It dives down 10 feet to catch a fish in the water.



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# Let's Try It:

Let's explore using a number line to add positive and negative numbers together.

Name: \_\_\_\_\_ 57 U4 Lesson 2 - Let's Try It

**Use the thermometers to answer the questions.**

1. Slide the first thermometer to show a temperature of 12 degrees.

2. Slide the second thermometer to show what the temperature would be if it increased 10 degrees. What temperature is it?

3. Show the starting temperature and the change in temperature using the horizontal number line.

4. Write an equation to show the increase in temperature.

**Use the thermometers to answer the questions.**

5. Shade the first thermometer to show a temperature of 25 degrees.

6. Shade the second thermometer to show what the temperature would be if it decreased 15 degrees. What temperature is it?

7. Show the starting temperature and the change in temperature using the horizontal number line.

8. Fill in the blanks to make two true equations that represent the change in temperature:  
 $25 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$        $25 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

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The temperature in Stockholm yesterday was -20 degrees. Today the temperature in Stockholm increased 15 degrees.

9. Use the number line to show the change in temperature.

10. What equation can represent this change in temperature?

The temperature in Montreal yesterday was -15 degrees. Today the temperature in Montreal decreased 15 degrees.

11. Use the number line to show the change in temperature.

12. Write two equations that can represent this change in temperature.

13. Rather than subtracting, we can represent a decrease in temperature as \_\_\_\_\_ a \_\_\_\_\_ number.

**Use a number line and an equation to represent the scenario.**

14. Mason was hiking... His hike began 10 feet below sea level. He hiked up 25 feet in elevation.

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# On your Own:

Now it's time to explore using a number line to add positive and negative numbers on your own.

Name: \_\_\_\_\_ G7 U4 Lesson 2 - Independent Work

1. The temperature in Providence this morning is 4 degrees. The temperature is expected to drop 10 degrees.

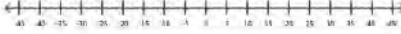
a. Use the number line to represent the change in temperature.



b. Write an addition and a subtraction equation that can be used to represent the change in temperature.


2. The temperature in Kansas City yesterday was 45 degrees. The temperature dropped 25 degrees today.

a. Use the number line to represent the change in temperature.




b. Write an addition and a subtraction equation that can be used to represent the change in temperature.

3. The temperature in Colorado Springs was -5 degrees. Later in the day, the temperature went up 20 degrees. Represent the change in temperature by using the number line grid by using an equation.




4. Alexia started hiking at an elevation 15 feet above sea level. By the end of her hike, her elevation had decreased 40 feet. Which equations correctly represent the situation? Select all that apply. Use the number line to help you.



a.  $15 + 40 = 55$   
 b.  $15 - 40 = 55$   
 c.  $15 + 40 = -25$   
 d.  $15 + (-40) = -25$   
 e.  $-15 + 40 = -25$   
 f.  $-15 - 40 = -25$

5. Taylor drew arrows on the number line below.



Write and solve a story problem that Taylor could have been representing. Write an equation to represent the change in temperature.

\_\_\_\_\_

\_\_\_\_\_

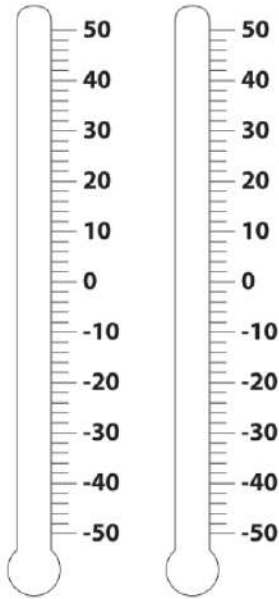
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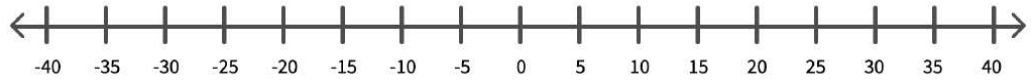
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**Use the thermometers to answer the questions.**

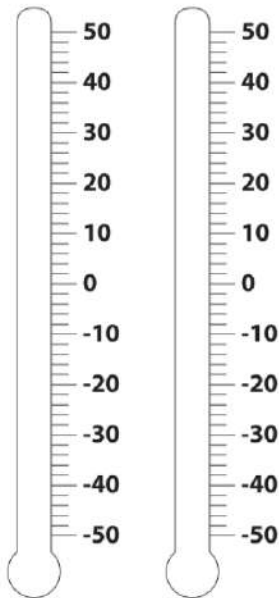


1. Shade the first thermometer to show a temperature of 12 degrees.
2. Shade the second thermometer to show what the temperature would be if it increased 10 degrees. What temperature is it?
3. Show the starting temperature and the change in temperature using the horizontal number line.

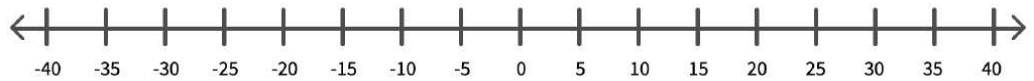


4. Write an equation to show the increase in temperature.

**Use the thermometers to answer the questions.**



5. Shade the first thermometer to show a temperature of 25 degrees.
6. Shade the second thermometer to show what the temperature would be if it decreased 15 degrees. What temperature is it?
7. Show the starting temperature and the change in temperature using the horizontal number line.



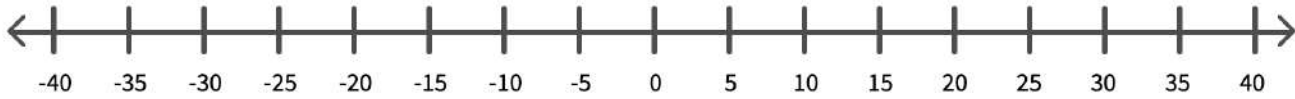
8. Fill in the blanks to make two true equations that represent the change in temperature.

$25 - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$25 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

The temperature in Stockholm yesterday was  $-20$  degrees. Today the temperature in Stockholm increased 15 degrees.

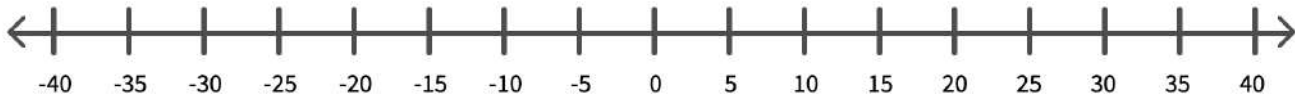
9. Use the number line to show the change in temperature.



10. What equation can represent this change in temperature?

The temperature in Montreal yesterday was  $-15$  degrees. Today the temperature in Montreal decreased 15 degrees.

11. Use the number line to show the change in temperature.

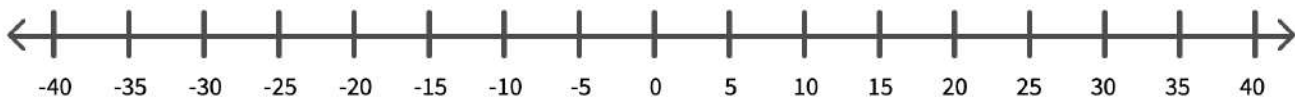


12. Write two equations that can represent this change in temperature.

13. Rather than subtracting, we can represent a decrease in temperature as \_\_\_\_\_ a \_\_\_\_\_ number.

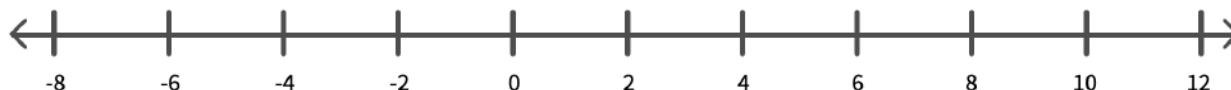
**Use a number line and an equation to represent the scenario.**

14. Mason was hiking.. His hike began 10 feet below sea level. He hiked up 25 feet in elevation.



1. The temperature in Providence this morning is 4 degrees. The temperature is expected to drop 10 degrees.

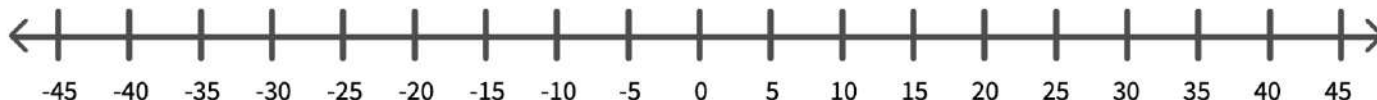
a. Use the number line to represent the change in temperature.



b. Write an addition and a subtraction equation that can be used to represent the change in temperature.

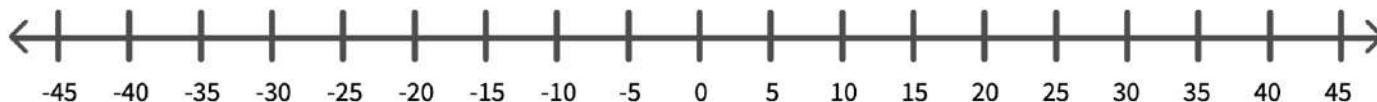
2. The temperature in Kansas City yesterday was 45 degrees. The temperature dropped 25 degrees today.

a. Use the number line to represent the change in temperature.

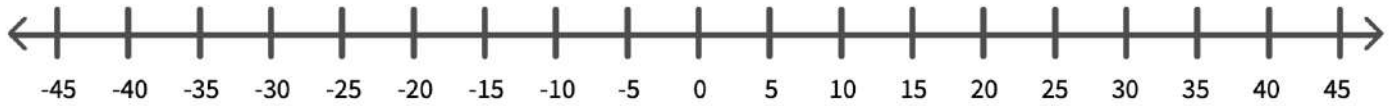


b. Write an addition and a subtraction equation that can be used to represent the change in temperature.

3. The temperature in Colorado Springs was -5 degrees. Later in the day, the temperature went up 20 degrees. Represent the change in temperature by using the number line and by using an equation.

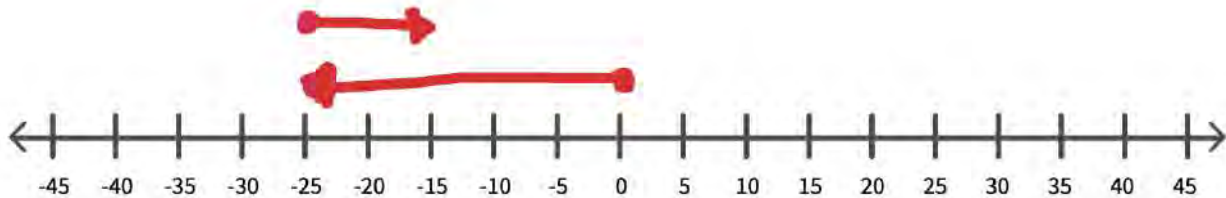


4. Alexia started hiking at an elevation 15 feet above sea level. By the end of her hike, her elevation had decreased 40 feet. Which equations correctly represent the situation? Select all that apply. Use the number line to help you.



- a.  $15 + 40 = 55$
- b.  $15 - 40 = 55$
- c.  $15 - 40 = -25$
- d.  $15 + (-40) = -25$
- e.  $-15 + 40 = -25$
- f.  $-15 - 40 = -25$

5. Taylor drew arrows on the number line below.



Write and solve a story problem that Taylor could have been representing. Write an equation to represent the change in temperature.

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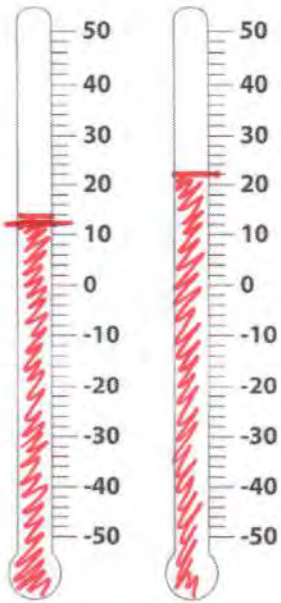
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Use the thermometers to answer the questions.



1. Shade the first thermometer to show a temperature of 12 degrees.

2. Shade the second thermometer to show what the temperature would be if it increased 10 degrees. What temperature is it?

22°

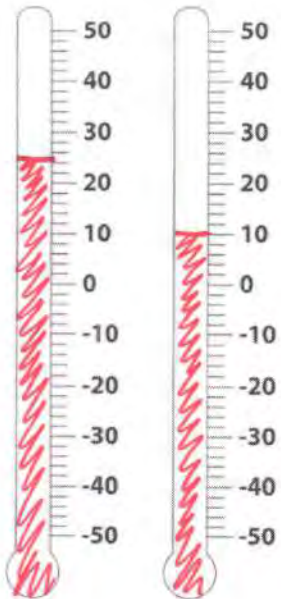
3. Show the starting temperature and the change in temperature using the horizontal number line.



4. Write an equation to show the increase in temperature.

$$12 + 10 = 22$$

Use the thermometers to answer the questions.

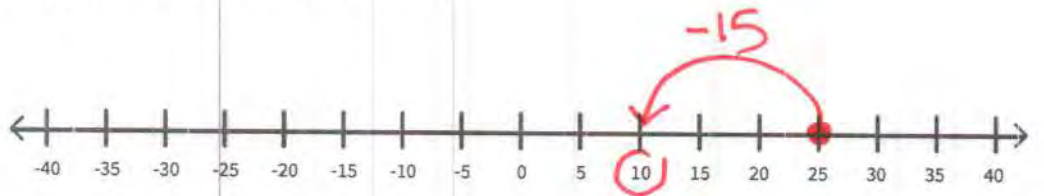


5. Shade the first thermometer to show a temperature of 25 degrees.

6. Shade the second thermometer to show what the temperature would be if it decreased 15 degrees. What temperature is it?

10°

7. Show the starting temperature and the change in temperature using the horizontal number line.



8. Fill in the blanks to make two true equations that represent the change in temperature.

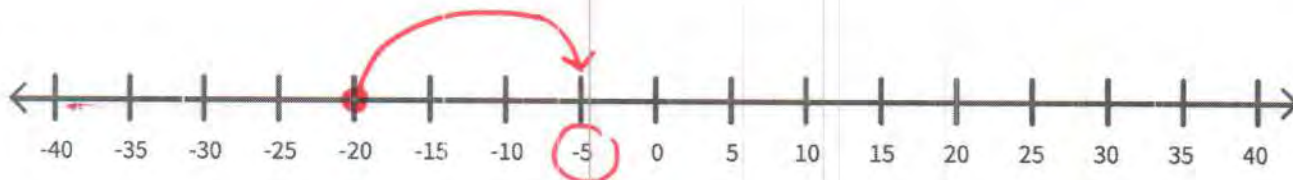
$$25 - 15 = 10$$

$$25 + -15 = 10$$



The temperature in Stockholm yesterday was -20 degrees. Today the temperature in Stockholm increased 15 degrees.

9. Use the number line to show the change in temperature.

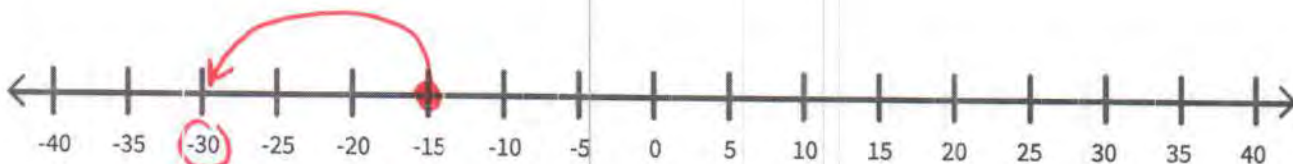


10. What equation can represent this change in temperature?

$$-20 + 15 = -5$$

The temperature in Montreal yesterday was -15 degrees. Today the temperature in Montreal decreased 15 degrees.

11. Use the number line to show the change in temperature.



12. Write two equations that can represent this change in temperature.

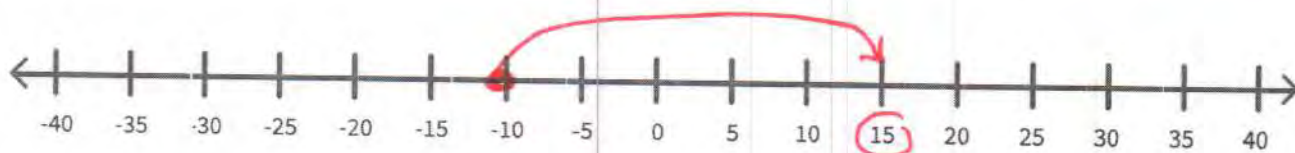
$$-15 - 15 = -30$$

$$-15 + -15 = -30$$

13. Rather than subtracting, we can represent a decrease in temperature as adding a negative number.

Use a number line and an equation to represent the scenario.

14. Mason was hiking.. His hike began 10 feet below sea level. He hiked up 25 feet in elevation.

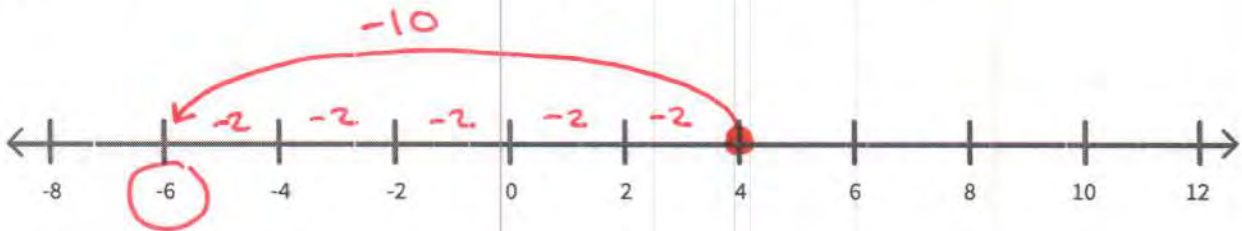


$$-10 + 25 = 15$$

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1. The temperature in Providence this morning is 4 degrees. The temperature is expected to drop 10 degrees.

a. Use the number line to represent the change in temperature.



b. Write an addition and a subtraction equation that can be used to represent the change in temperature.

$$4 + -10 = -6 \quad 4 - 10 = -6$$

2. The temperature in Kansas City yesterday was 45 degrees. The temperature dropped 25 degrees today.

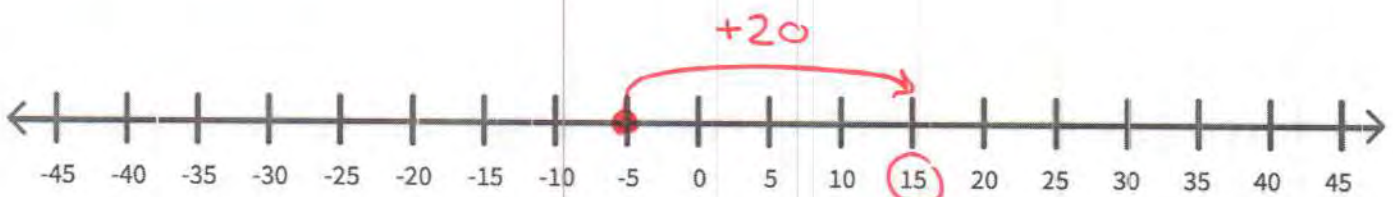
a. Use the number line to represent the change in temperature.



b. Write an addition and a subtraction equation that can be used to represent the change in temperature.

$$45 + -25 = 20 \quad 45 - 25 = 20$$

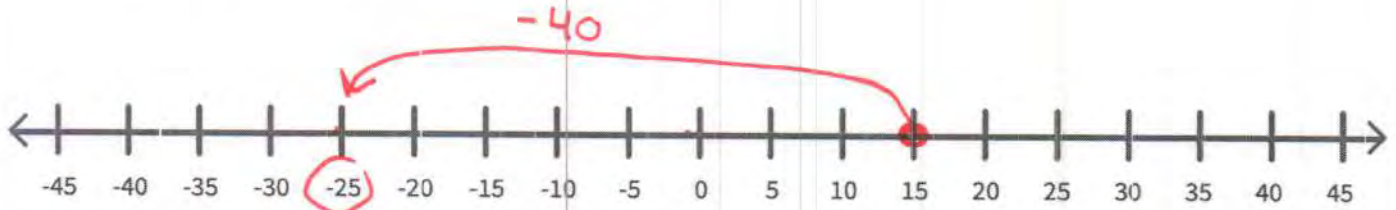
3. The temperature in Colorado Springs was -5 degrees. Later in the day, the temperature went up 20 degrees. Represent the change in temperature by using the number line and by using an equation.



$$-5 + 20 = 15$$

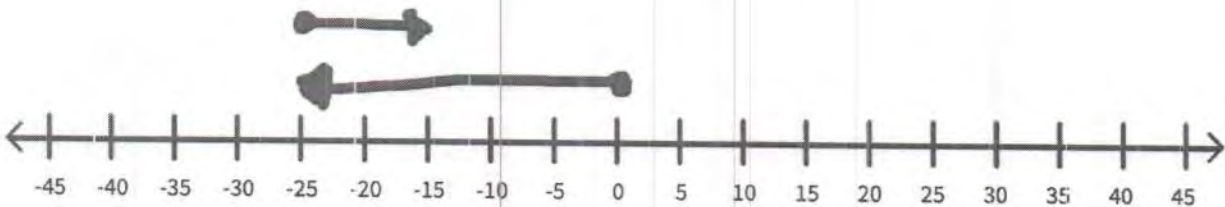


4. Alexia started hiking at an elevation 15 feet above sea level. By the end of her hike, her elevation had decreased 40 feet. Which equations correctly represent the situation? Select all that apply. Use the number line to help you.



- ~~a.  $15 + 40 = 55$~~
- ~~b.  $15 - 40 = 55$~~
- c.  $15 - 40 = -25$
- d.  $15 + (-40) = -25$
- ~~e.  $-15 + 40 = -25$~~
- ~~f.  $-15 - 40 = -25$~~

5. Taylor drew arrows on the number line below.



Write and solve a story problem that Taylor could have been representing. Write an equation to represent the change in temperature.

The temperature was  $-25^{\circ}$  this morning.  
 It increased  $10^{\circ}$  by midday. What is  
 the temperature now?

$$-25 + 10 = -15$$

## **G7 U4 Lesson 3**

Understand what positive and negative numbers mean in a situation involving money and calculate an account balance after a deposit or withdrawal.

**G7 U4 Lesson 3 - Students will understand what positive and negative numbers mean in a situation involving money and calculate an account balance after a deposit or a withdrawal.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We've been learning a lot about negative numbers. We've used number lines to think about negative numbers in the context of both temperature and elevation. Those contexts will continue to help us, but today, we'll focus on thinking about negative numbers in a new context: money! Let's get to work.

**Let's Talk (Slide 3):** Take a look at the two images here showing screenshots of mobile bank statements. What do you notice about these? What do you wonder? **Possible Student Answers, Key Points:**

- I notice the first person has a negative total. I notice the second picture shows more money. I notice the second bank statement shows some negative transactions and one positive transaction.
- I wonder how you end up with a negative balance. I wonder how the second amount got to be so much. I wonder if negative numbers mean that the person is spending money.

Negative numbers are very important when thinking about money.

When thinking about money, we typically use positive values to represent *earning* money. For instance, the second screenshot shows the person earned \$8,000, so they represent the value with positive 8,000. When money is put into your account like that, it's called a deposit.

You'll also notice their balance, or their bank total, is positive. That means they have \$41,682 in their account.

When we see negative numbers on a bank statement, it can mean that you owe money or that you're spending money. The first balance shows -\$230.70. This means the person doesn't have any money in their account, and actually owes the bank \$230.70. This can happen when you spend more money than you actually have using a credit card. We also see negative numbers in the second bank statement. For example, the person spent \$1.68 at Starbucks, so it's represented as negative \$1.68. When money is removed from a bank account, it's called a withdrawal.

Let's use positive and negative numbers to think about bank balances, deposits, and withdrawals.

**Let's Think (Slide 4):** This problem wants us to think about how Prince's account balance of \$4 would change if different events occurred. We'll use a number line and equations to show our thinking.



Part A wants us to think about how much Prince will have if he deposits \$2. I'll start by marking his beginning balance with a point at positive 4, because he has \$4. If he owed \$4 for some reason, I would start at -4 on the number line, but that's not the case

in this scenario. How can I show he is depositing \$2 on this number line? (*sketch as student shares*)

**Possible Student Answers, Key Points:**

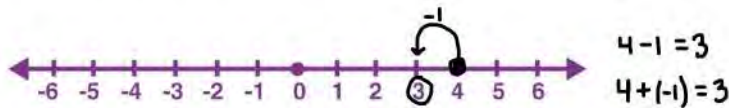
- You should draw an arrow to the right, because a deposit means he is earning money or putting money into his account.
- You should hop 2 spaces right to represent earning \$2.

$$4 + 2 = 6$$

He has \$4. He deposits \$2. Now he has \$6. We can see that on the number line. And we can use the equation  $4 + 2 = 6$  to represent what is happening. (*write equation*)

Part B is different, because it's asking about a withdrawal of \$1. What is happening if Prince is withdrawing money from his account? **Possible Student Answers, Key Points:**

- He's spending, or he's losing money. His account balance is getting smaller.

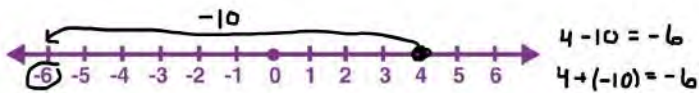


(sketch and label as you narrate) I'll mark his starting balance at \$4. I'll draw an arrow to the left to show that he withdrew one dollar. I'll label that with -1. We can see on the number line that he

now has \$3 in his account. We can represent this with subtraction by writing  $4 - 1 = 3$ . We can also think of subtracting as adding a negative value, so another equation that represents this scenario could be  $4 + (-1) = 3$ .

Part C wants us to think about another withdrawal. This time, he is taking withdrawing \$10. What do you notice about withdrawing that amount? **Possible Student Answers, Key Points:**

- He's spending more than he did in Part B.
- He doesn't have enough money in his account.
- I think his balance will end up being negative. He will owe money.



Let's show this on the number line. (sketch and label as you narrate) We'll start at 4, because that's his original balance. He is withdrawing, or losing, \$10. I'll model that with a big arrow going

back 10 spaces. I can see that his balance will now be -\$6. This means he actually owes the bank money.

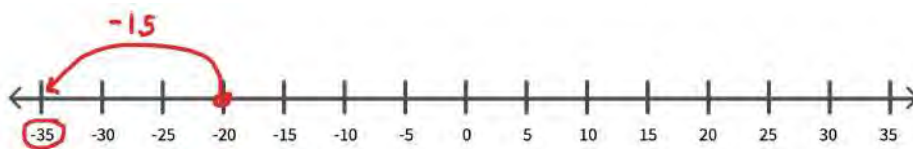
What two equations can I use to represent this situation? **Possible Student Answers, Key Points:**

- We can say  $4 - 10 = -6$ .
- Subtracting a number is the same as adding a negative, so we can also write  $4 + (-10) = -6$ .

Just like we can use a number line to work with positive and negative numbers in contexts of elevation and temperature, we can use similar thinking to help us tackle problems involving money and bank account balances. Nicely done.

**Let's Think (Slide 5):** Let's try one more series that's just a bit different. (read problem aloud) What do you notice is similar and different about this problem compared to the last one? **Possible Student Answers, Key Points:**

- It's still about depositing and withdrawing money. It still wants us to use a number line.
- She has a negative balance to start. The number line counts by fives.



Great, let's start by thinking about Part A. It wants to know her balance if she starts with a balance of \$20 and withdraws \$15. (sketch and label number line as you narrate) Where should I start on

the number line given her current account balance? (-20) I'll start at -20. She is withdrawing 15 dollars, so I know her balance is decreasing. I'll show that by labeling an arrow that goes back 15 dollars. Her balance, I can see, is now -\$35. This makes sense, because she owed money to start with, and then she kept spending. Her balance shows that she owes more now.

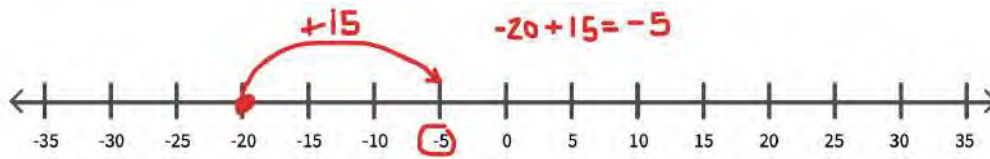
$$\begin{aligned} -20 - 15 &= -35 \\ -20 + (-15) &= -35 \end{aligned}$$

We can represent that change with the equation  $-20 - 15 = -35$ . We can also represent it using addition by adding a negative value. That would look like  $-20 + (-15) = -35$ . (write equations)

Part B is asking us to think about what her balance will be if she deposits, or gains, \$15. Describe what that change might look like on the number line. (*sketch and label while student shares*) [Possible Student Answers](#),

Key Points:

- Plot a point at -20 for her starting balance.
- Since she's earning \$15, draw an arrow from -20 to the right 15. You end up on -5.



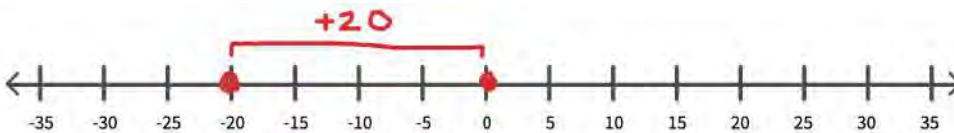
We ended up at -5. This means her new balance is -\$5. We can represent that with the equation  $-20 + 15 = -5$ . The balance started at -20, increased 15 dollars, and

ended up at -5. Why is it that she earned money, but our answer is still negative? [Possible Student Answers](#),

Key Points:

- She didn't earn enough money. She owed the bank more than she deposited, so she still owes the bank a little bit of money.

The last part of this problem asks us to think about what Jayla needs to do to get her balance to \$0. The number line can help me think about this situation. I know if her balance is negative, she definitely doesn't want to lose more money if her goal is to get back to \$0. I know she'll have to increase her amount. (*sketch a bracket from -20 to 0 on the number line*).



money. How much will she need to earn? (\$20) Jayla will need to earn, or deposit, \$20 to get her bank balance back to \$0 and not owe the bank any more money.

We just used a number line to help us think carefully about situations involving earning and losing money.

**Let's Try it (Slides 6 - 7):** Now let's practice a few more problems. As we read each problem, pay close attention to the language being used. We can think of a positive balance as the total money in an account and a negative balance as how much somebody owes the bank. We can think of deposits as earning money and withdrawals as losing money. The language in each problem can help us think about how best to model. Should a number be negative or positive? Should my arrow shift left or right? Let's work together to do a few more problems involving money.



# WARM WELCOME



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**Today we will understand what positive and negative numbers mean in a situation involving money and calculate an account balance after a deposit or a withdrawal.**

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Let's Talk:

What do you notice?  
What do you wonder?

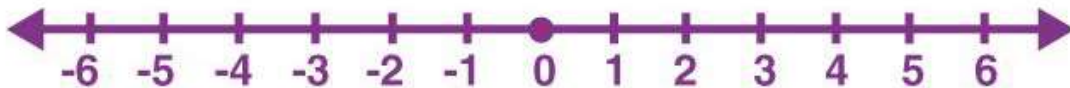


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Let's Think:

Prince has a bank balance of \$4.



- How much would Prince have if he deposits \$2?
- How much would Prince have if he withdraws \$1?
- How much would Prince have if he withdraws \$10?

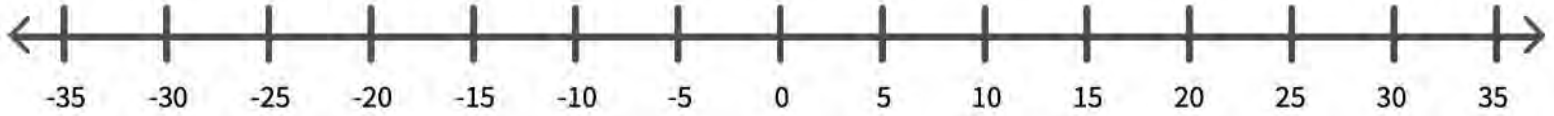
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## Let's Think:

Jayla's bank statement says that her balance is **-\$20**.



- What is Jayla's balance if she withdraws \$15?
- What is Jayla's balance if she deposits \$15?
- What does Jayla need to do to have a balance of \$0?

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## Let's Try It:

Let's explore what positive and negative numbers mean in a situation involving money together.

Name: \_\_\_\_\_ G7 U4 Lesson 3 - Let's Try It

- When you put money into a bank account, it's called a \_\_\_\_\_.
  - deposit
  - withdrawal
  - balance
- When you take money out of a bank account, it's called a \_\_\_\_\_.
  - deposit
  - withdrawal
  - balance
- To represent withdrawals, banks use \_\_\_\_\_ numbers. To represent deposits, banks use \_\_\_\_\_ numbers.
  - positive, positive
  - negative, negative
  - negative, positive
  - positive, negative

**Bernard has \$60 in his bank account. He wants to buy a video game that costs \$40.**

- Use the number line to represent Bernard's balance after buying the video game.
- Complete the subtraction equation and the addition equation to represent the situation.
 
$$60 - \underline{\quad} = \underline{\quad}$$

$$60 + \underline{\quad} = \underline{\quad}$$

**Bernard now has \$20 in his bank account. He withdraws \$35 to buy some groceries.**

- Use the number line to represent Bernard's balance after buying the groceries.
- Write an addition equation to represent this situation.
 
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$
- In your own words, what does it mean when a bank balance is expressed using a negative number?
 

\_\_\_\_\_

\_\_\_\_\_

**Reyanna has a balance of -\$15 in her account.**

- Show Reyanna's balance on the number line.
- How much money will Reyanna need to deposit to have a balance of \$0? Use the number line to help you.
 

\_\_\_\_\_
- Write an addition equation that represents this situation.
 
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$
- What would Reyanna's balance be if she started with -\$15 and withdrew \$5 to buy a sandwich? Use a number line to help you, and write an equation to match your work.
 

\_\_\_\_\_

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# On your Own:

## Now it's time to explore what positive and negative numbers mean in a situation involving money own.


Name: \_\_\_\_\_ G7 U4 Lesson 3 Independent Work

1. Logan's bank account has a balance of  $-\$15$ .

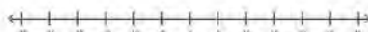
a. Use the number line to show his new balance if he deposited  $\$5$ . Write an equation to match the situation.



b. Use the number line to show his new balance if he withdrew  $\$5$ . Write an equation to match the situation.



c. Use the number line to show his new balance if he deposits  $\$20$ . Write an equation to match the situation.

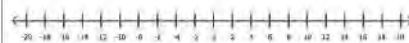


2. Daniella's bank statement shows an account balance of  $-\$100$ . How much does she need to deposit to have a balance of  $\$0$ ? How do you know?

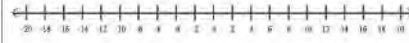
\_\_\_\_\_

\_\_\_\_\_


3. Tina has a balance of  $\$16$ . She spends  $\$2$ . Represent this situation using the number line and an addition equation.



4. Tina has a balance of  $\$15$ . She makes a  $\$20$  withdrawal. Represent this situation using the number line and an addition equation.



4. Write and solve a story problem that could be represented using the number line shown below. Include an equation as part of your work.



\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

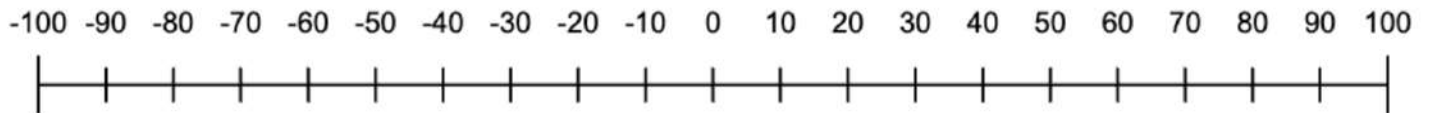
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Name: \_\_\_\_\_

1. When you put money into a bank account, it's called a \_\_\_\_\_.
  - a. deposit
  - b. withdrawal
  - c. balance
  
2. When you take money out of a bank account, it's called a \_\_\_\_\_.
  - a. deposit
  - b. withdrawal
  - c. balance
  
3. To represent withdrawals, banks use \_\_\_\_\_ numbers. To represent deposits, banks use \_\_\_\_\_ numbers.
  - a. positive, positive
  - b. negative, negative
  - c. negative, positive
  - d. positive, negative

**Bernard has \$60 in his bank account. He wants to buy a video game that costs \$40.**

4. Use the number line to represent Bernard's balance after buying the video game.



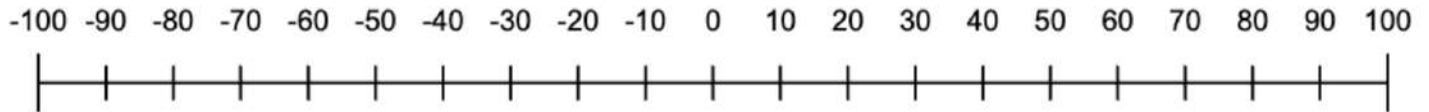
5. Complete the subtraction equation and the addition equation to represent the situation.

$$60 - \underline{\quad} = \underline{\quad}$$

$$60 + \underline{\quad} = \underline{\quad}$$

**Bernard now has \$20 in his bank account. He withdraws \$35 to buy some groceries.**

6. Use the number line to represent Bernard's balance after buying the groceries.



7. Write an addition equation to represent this situation.

8. In your own words, what does it mean when a bank balance is expressed using a negative number?

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**Reyanna has a balance of -\$15 in her account.**

9. Show Reyanna's balance on the number line.



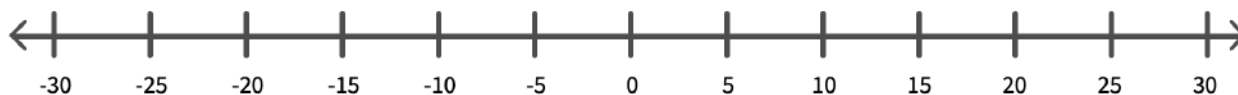
10. How much money will Reyanna need to deposit to have a balance of \$0? Use the number line to help you.

11. Write an addition equation that represents this situation.

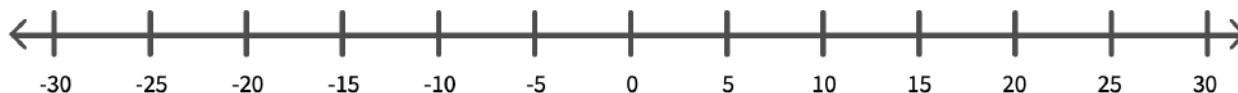
12. What would Reyanna's balance be if she started with -\$15 and withdrew \$5 to buy a sandwich? Use a number line to help you, and write an equation to match your work.

**1. Logan's bank account has a balance of -\$15.**

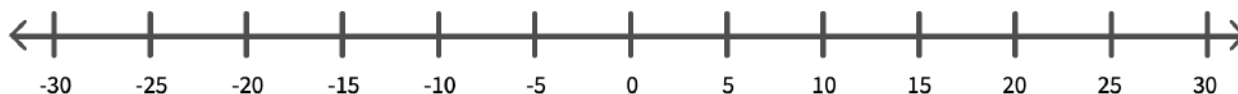
- a. Use the number line to show his new balance if he deposited \$5. Write an equation to match the situation.



- b. Use the number line to show his new balance if he withdrew \$5. Write an equation to match the situation.



- c. Use the number line to show his new balance if he deposits \$20. Write an equation to match the situation.

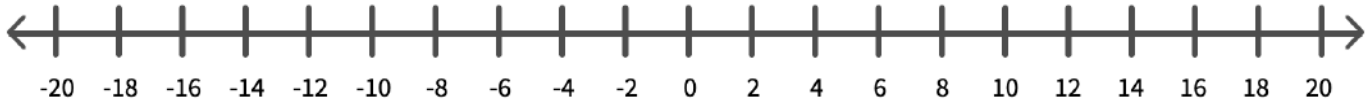
**2. Danielle's bank statement shows an account balance of -\$100. How much does she need to deposit to have a balance of \$0? How do you know?**

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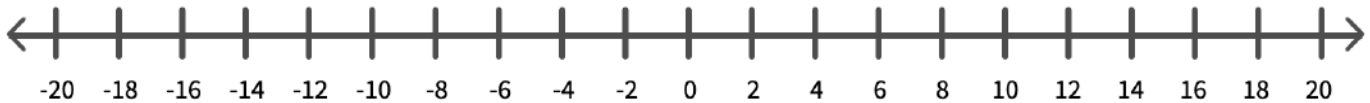
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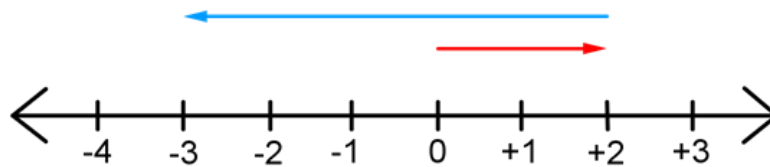
**3. Tina has a balance of \$16. She spends \$2.** Represent this situation using the number line and an addition equation.



**4. Tina has a balance of \$16. She makes a \$20 withdrawal.** Represent this situation using the number line and an addition equation.



**4. Write and solve a story problem that could be represented using the number line shown below. Include an equation as part of your work.**



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Name: KEY

1. When you put money into a bank account, it's called a \_\_\_\_\_.

- a. deposit
- b. withdrawal
- c. balance

2. When you take money out of a bank account, it's called a \_\_\_\_\_.

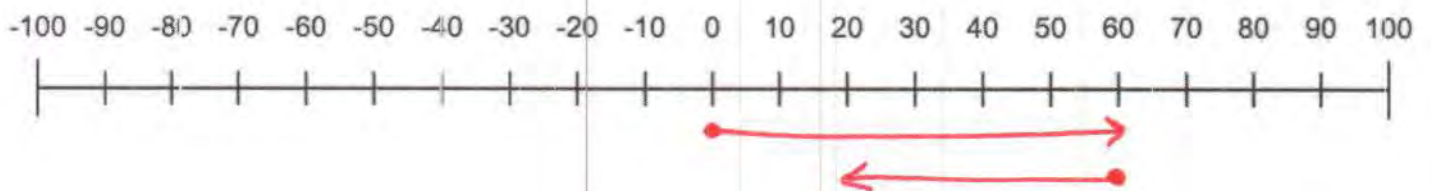
- a. deposit
- b. withdrawal
- c. balance

3. To represent withdrawals, banks use <sup>(-)</sup> \_\_\_\_\_ numbers. To represent deposits, banks use <sup>(+)</sup> \_\_\_\_\_ numbers.

- a. positive, positive
- b. negative, negative
- c. negative, positive
- d. positive, negative

**Bernard has \$60 in his bank account. He wants to buy a video game that costs \$40.**

4. Use the number line to represent Bernard's balance after buying the video game.



5. Complete the subtraction equation and the addition equation to represent the situation.

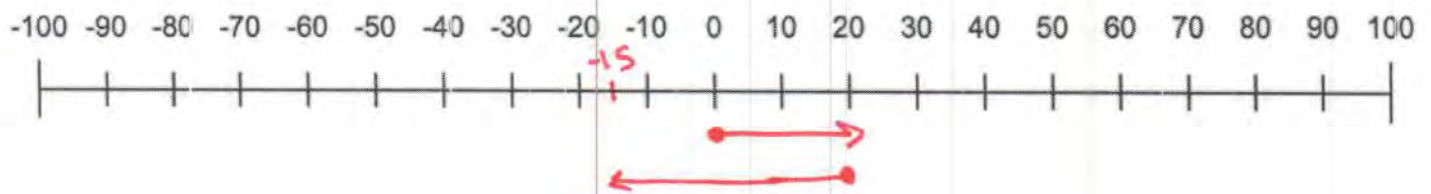
$$60 - \underline{40} = \underline{20}$$

$$60 + \underline{-40} = \underline{20}$$



Bernard now has \$20 in his bank account. He withdraws \$35 to buy some groceries.

6. Use the number line to represent Bernard's balance after buying the groceries.



7. Write an addition equation to represent this situation.

$$20 + -35 = -15$$

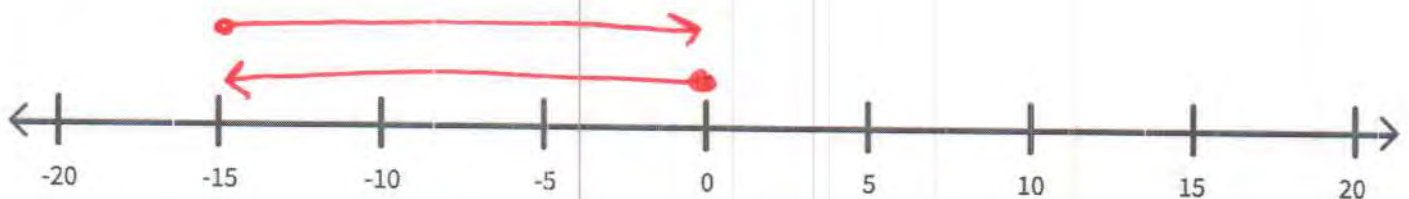
8. In your own words, what does it mean when a bank balance is expressed using a negative number?

It means you owe the bank money.

You took out more money than you have.

Reyanna has a balance of  $-\$15$  in her account.

9. Show Reyanna's balance on the number line.



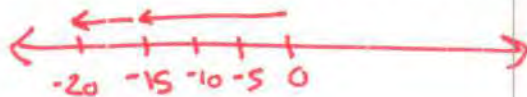
10. How much money will Reyanna need to deposit to have a balance of \$0? Use the number line to help you.

(\$15)

11. Write an addition equation that represents this situation.

$$-15 + 15 = 0$$

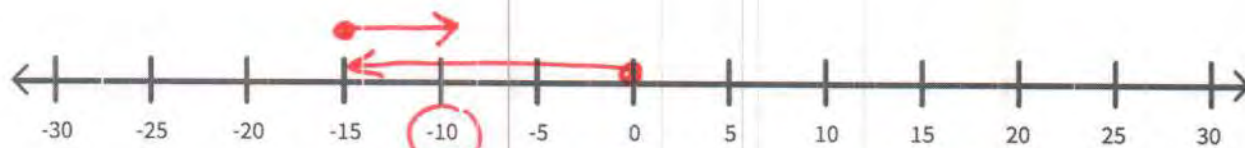
12. What would Reyanna's balance be if she started with  $-\$15$  and withdrew \$5 to buy a sandwich? Use a number line to help you, and write an equation to match your work.



$$\begin{aligned} -15 + -5 &= -20 \\ -15 - 5 &= -20 \end{aligned} \quad \text{(\$-20)}$$

**1. Logan's bank account has a balance of -\$15.**

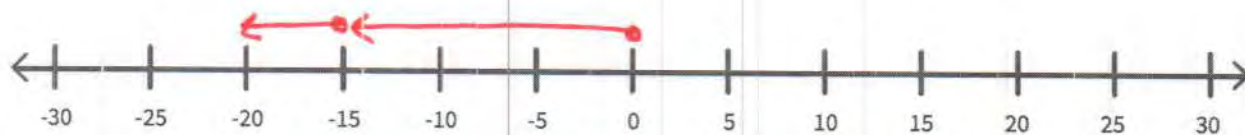
- a. Use the number line to show his new balance if he deposited \$5. Write an equation to match the situation.



$$-15 + 5 = -10$$

**-\$10**

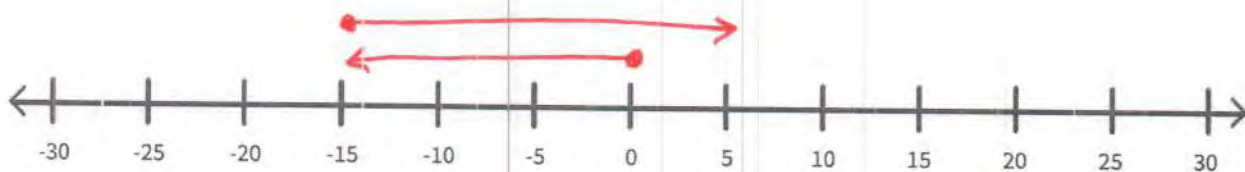
- b. Use the number line to show his new balance if he withdrew \$5. Write an equation to match the situation.



$$-15 - 5 = -20$$

**-\$20**

- c. Use the number line to show his new balance if he deposits \$20. Write an equation to match the situation.



$$-15 + 20 = 5$$

**\$5**

- 2. Danielle's bank statement shows an account balance of -\$100. How much does she need to deposit to have a balance of \$0? How do you know?**

She owes \$100 so she needs to deposit \$100 to break even and return to \$0.



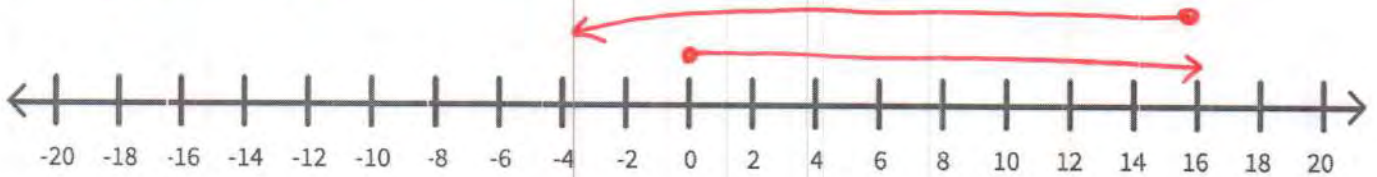
3. Tina has a balance of \$16. She spends \$2. Represent this situation using the number line and an addition equation.

$$16 + -2 = 14$$

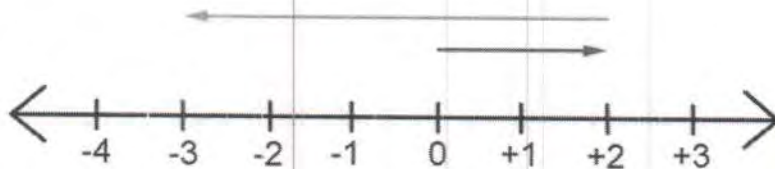


4. Tina has a balance of \$16. She makes a \$20 withdrawal. Represent this situation using the number line and an addition equation.

$$16 + -20 = -4$$



4. Write and solve a story problem that could be represented using the number line shown below. Include an equation as part of your work.



I start at an elevation of 2 feet.  
I descend 5 feet. What is my new  
elevation.

$$2 + -5 = -3$$

# **G7 U4 Lesson 4**

Use a number line to subtract positive and negative numbers.

## G7 U4 Lesson 4 - Students will use a number line to subtract positive and negative numbers.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** The past several lessons have been dedicated to using number lines to think about negative numbers in various contexts. What important ideas stand out to you so far when you think about negative numbers? **Possible Student Answers, Key Points:**

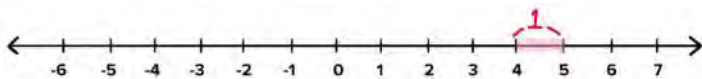
- Temperature, elevation, and money are all contexts where negative numbers are used in everyday life.
- Horizontal or vertical number lines can help us compare negative numbers and add with negative numbers.
- Subtracting a number can be thought of as adding a negative number.

Excellent! Today, we'll continue using some of that thinking to help us think about how to subtract with positive and negative numbers.

**Let's Talk (Slide 3):** When you first learned about subtraction back in elementary school, you probably learned about it in terms of taking away. For example, I have 6 oranges and I give 2 oranges to my friend. How many oranges do I have now? That's one interpretation, but we also know another interpretation where we can think of subtraction as finding the difference between two quantities. For example, I have 6 oranges and my friend has 2 oranges. What's the difference between our amounts of oranges? In either interpretation, the answer is 4 oranges.

Today, let's focus on the interpretation of subtraction as finding the difference between values. With that in mind, let's look at these three brief questions.

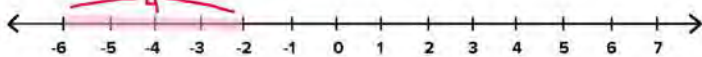
What would you say the difference is between 4 and 5? (1) The difference is 1. We can see that on the number line, because there is 1 space between 4 and 5. (*highlight interval between 4 and 5 and label with 1*)



What's the difference between 2 and -1? Count the spaces between 2 and -1 to help you. (3) The difference between 2 and -1 is 3. We see three spaces on the number line between -1 and 2. (*highlight intervals between -1 and 2 and label with 3*)



How could we use the number line to find the difference between -2 and -6? (*mark on the number line as student shares thinking*) **Possible Student**



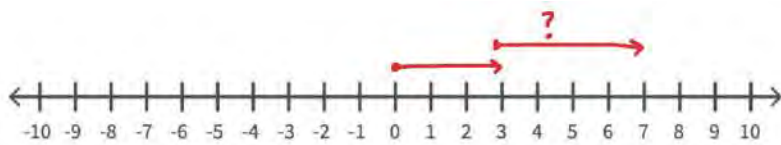
**Answers, Key Points:**

- We could mark -6 and -2 and count the spaces between the two numbers.
- The difference between -2 and -6 is 4 because there are 4 spaces between -2 and -6.

Nice work! The thinking you just did is all it takes to think about subtracting with positive and negative numbers. Let's keep using the idea of *difference* to help us subtract.

**Let's Think (Slide 4):** For this problem, we'll find a series of unknowns by using the number line.

For Part A, they want us to think about 3 plus an unknown number equals 7. You might already know the answer, but let's picture it on a number line so that we can use similar thinking when the problems get a little trickier.

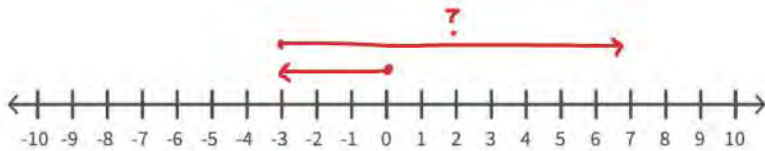


The first number is 3. (*sketch on number line as you narrate*) I'll draw a line from 0 to 3 to show that. The question wants me to find the value I can add to 3 to make 7. I'll draw a line from 3 to 7 and label it with a question mark, since that's the unknown in this problem.

What's the value of the unknown, and how can you tell by looking at the number line? **Possible Student Answers, Key Points:**

- The value is 4. The line from 3 to 7 is 4 spaces long.

Correct! The line from 3 to 7 is 4 units long. So, we can say  $3 + 4 = 7$ . The unknown is 4. I know it is positive 4, because of the direction of the arrow representing the unknown. Let's try the next one.



This question has a similar structure, but it's asking us to think about  $-3$  plus an unknown number that would result in 7. (*sketch on number line as you narrate*) I'll draw a line from 0 to  $-3$  to show I'm starting at  $-3$ . I'm trying to

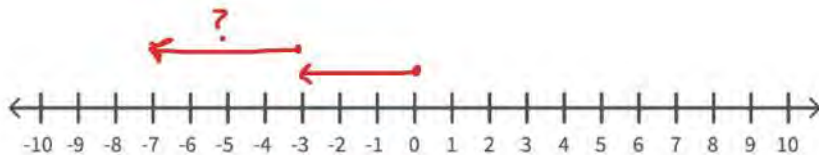
figure out what I can add to that value to make 7, so I'll draw another line from  $-3$  to 7 and label it with a question mark. What's the value of this unknown? How do you know? **Possible Student Answers, Key Points:**

- The value is 10. The line going from  $-3$  to 7 is 10 units long.

Nice. The line from  $-3$  to 7 is 10 units long, so I can say that  $-3 + 10 = 7$ . The unknown is 10. I know it is positive 10, because of the direction of the arrow representing the unknown. Let's try one more.

This one wants me to think about  $-3$  plus an unknown number that results in  $-7$ . How could I set up the number line to show this? Think about how we made the previous models. (*sketch as student shares, supporting as necessary*) **Possible Student Answers, Key Points:**

- The first number is  $-3$ , so you can draw a line from 0 to  $-3$ .
- The problem should result in a total of  $-7$ , so draw another line from  $-3$  to  $-7$  and label it with a question mark.



This unknown arrow goes across 4 spaces, but notice the direction of the arrow is different from our other two examples. This arrow points left. That means the value of this unknown isn't 4, it's *negative* 4. So  $-3$

$+ (-4) = -7$ . The unknown is  $-4$ .

- a.  $3 + ? = 7$       $7 - 3 = 4$
- b.  $-3 + ? = 7$       $7 - (-3) = 10$
- c.  $-3 + ? = -7$       $-7 - (-3) = -4$

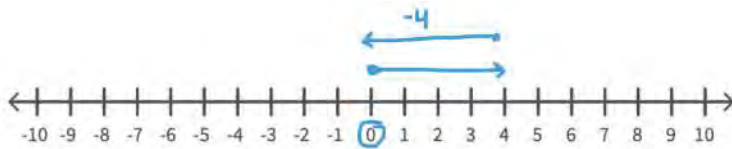
We just found three unknown parts using a number line to help us find the difference between two numbers. In a sense, even though the original equations showed addition, we were actually subtracting. We can rewrite each equation we just solved using subtraction. (*write equations as you narrate*) For part A, we were finding the difference between 3 and 7, which we can write as  $7 - 3 = 4$ . For part B, we were finding the difference between  $-3$  and 7, which we can write as  $7 - (-3) = 10$ .

What subtraction equation could we think about for part C, and why? Possible Student Answers, Key Points:

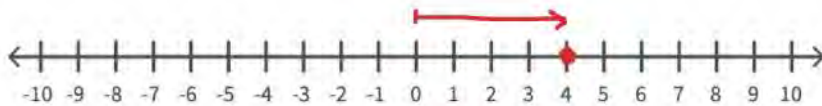
- We were finding the difference between -3 and -7, so we could write  $-7 - (-3) = -4$ .

Thinking of unknown parts as finding the difference between two numbers on a number line can be a useful way to find unknown values.

**Let's Think (Slide 5):** Let's solve a few more problems on a number line.



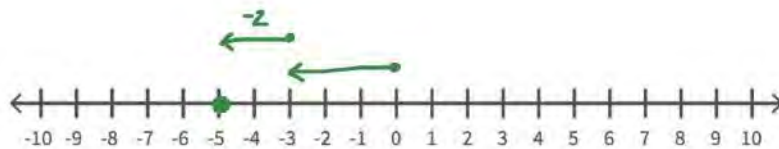
The first problem is  $4 + (-4)$ . Let's draw from 0 to 4 to show that value. (*draw line*) Now, we need to add -4. I'll use an arrow facing the left to show that I'm adding -4 to 4. (*draw line from 4 to 0 and label the line as -4*) If I add -4 to 4, where do we end up? (0) The number line shows us that the value of 4 plus -4 is 0.



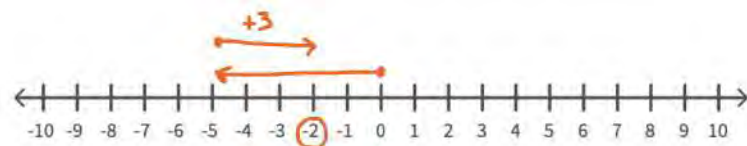
Part B asks us to find  $4 - 4$ . I can think of this subtraction as the difference between 4 and 4. (*sketch as you narrate*) I'll plot 4 on the number line to represent the first 4. Then, I'll draw an arrow from 0 to 4 to represent the other 4. What's the

difference or the distance between the point on the number line and where my arrow ended? It's 0! They're at the same point in this example.

Look at our last two examples.  $4 + (-4) = 0$  and  $4 - 4 = 0$ . They both ended up with the same answer. That makes sense, because we've previously learned that we can think of subtracting as adding a negative or adding the opposite value. So, subtracting 4 from 4 in the Part B is the same thing as adding -4 to 4 in Part A.



Part C is asking us to consider  $-5 - (-3)$ . It wants to know the difference between -5 and -3. I'll make a point at -5. Then I can draw an arrow from 0 to -3 to show that value. I want to draw an arrow to get from -3 to -5. When I do that, I can see the difference is -2. The direction of the arrow helps me see my answer is -2 instead of +2.



Let's wrap up with Part D. How could I use a number line to show  $-5 + 3$ ? (*sketch as student shares*) Possible Student Answers, Key Points:

- Draw an arrow from 0 to -5 to show our starting point. To add 3, we can draw an arrow from -5 up 3 spaces. We end up at -2.

Note how Part C and Part D have the same answer.  $-5 - (-3) = -2$  and  $-5 + 3 = -2$ . Once again, this goes back to the idea that subtracting is the same as adding the opposite value. In Part C, we were subtracting -3. Subtracting -3 is the same as adding its opposite, which is exactly what we did in Part D.

Being able to think flexibly about equations, and being able to model addition and subtraction clearly on a number line can help us carefully find add and subtract with negative numbers.



**Let's Try it (Slides 6 - 7):** Now let's work on some more problems together, before you get a chance to try some out independently. Number lines will likely be the best way to show our thinking today. If we see subtraction, we know that we can think of it as finding the difference between values. We also saw that subtracting can be thought of as adding the opposite of a number. Both ways of thinking can help us tackle problems, depending on what makes the most sense to us with the numbers we're given. Let's give it a shot!

# WARM WELCOME



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**Today we will use a number line to subtract positive and negative numbers.**

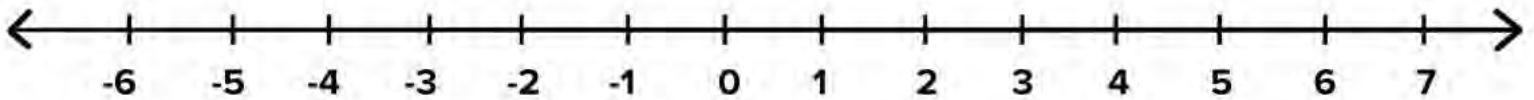
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 **Let's Talk:** What's the difference between...

**4 and 5?**

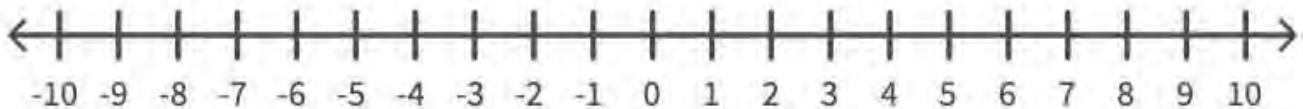
**2 and -1?**

**-2 and -6?**



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 **Let's Think:** Use the number line to find each unknown.



a.  $3 + ? = 7$

b.  $-3 + ? = 7$

c.  $-3 + ? = -7$

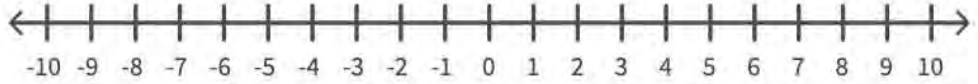
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# Let's Think:

Find the value of each expression in the table. What do you notice?

EXPRESSION	VALUE
$4 + (-4)$	
$4 - 4$	
$-5 - (-3)$	
$-5 + 3$	



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# Let's Try It:

Let's explore using a number line to subtract positive and negative numbers together.

Name: \_\_\_\_\_ G7 U4 Lesson 4 - Let's Try It

Consider the equation  $4 + 7 = 9$

- Use an arrow on the number line to show the first addend.
- Draw another arrow to represent the unknown.
- Rewrite the equation as a subtraction equation.
- The value of the unknown is \_\_\_\_\_.

Consider the equation  $7 + 7 = 5$

- Use an arrow on the number line to show the first addend.
- Draw another arrow to represent the unknown.
- Rewrite the equation as a subtraction equation.
- The value of the unknown is \_\_\_\_\_.

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Consider the equation  $3 - 10 = 7$ .

- Plot 3 on the number line.
- Draw an arrow to represent 10 starting at 0.
- Draw an arrow from 10 to 3 to represent the difference.
- The value of the unknown is \_\_\_\_\_.

Look at the table below.

EXPRESSION	VALUE
$3 + 5$	
$3 - 5$	
$3 + (-5)$	
$3 - (-5)$	

- Find the value of each expression. Sketch a number line if that's helpful for you.
- Which expressions are equivalent in value?
- Subtracting a number is always the same as adding its \_\_\_\_\_.
- Rewrite each expression below as an addition expression. Find the value of each.
 

$-7 - 3$	$10 - (-9)$
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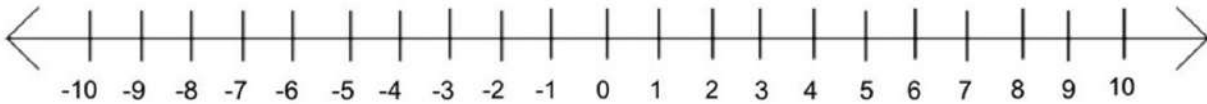
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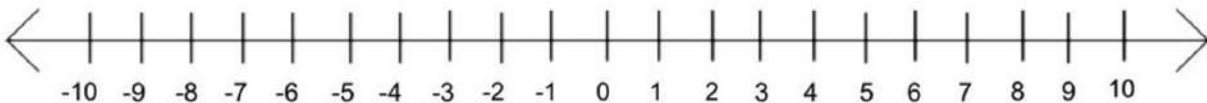
Name: \_\_\_\_\_

**Consider the equation  $4 + ? = 9$**



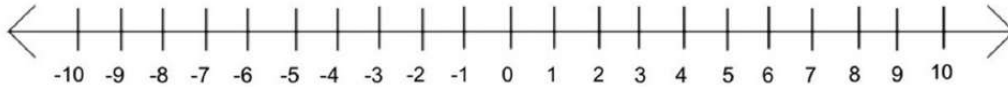
1. Use an arrow on the number line to show the first addend.
2. Draw another arrow to represent the unknown.
3. Rewrite the equation as a subtraction equation.
4. The value of the unknown is \_\_\_\_\_.

**Consider the equation  $7 + ? = 5$**



5. Use an arrow on the number line to show the first addend.
6. Draw another arrow to represent the unknown.
7. Rewrite the equation as a subtraction equation.
8. The value of the unknown is \_\_\_\_\_.

Consider the equation  $3 - 10 = ?$ .



9. Plot 3 on the number line.

10. Draw an arrow to represent 10 starting at 0.

11. Draw an arrow from 10 to 3 to represent the difference.

12. The value of the unknown is \_\_\_\_\_.

Look at the table below.

EXPRESSION	VALUE
$3 + 5$	
$3 - 5$	
$3 + (-5)$	
$3 - (-5)$	

13. Find the value of each expression. Sketch a number line if that's helpful for you.

14. Which expressions are equivalent in value?

15. Subtracting a number is always the same as adding its \_\_\_\_\_.

16. Rewrite each expression below as an addition expression. Find the value of each.

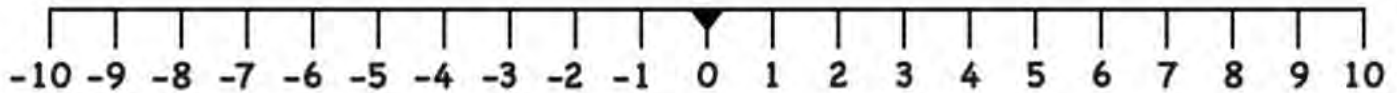
$$-7 - 3$$

$$10 - (-6)$$



1. Consider the equation  $-9 + ? = 6$ .

a. Represent the equation on the number line.

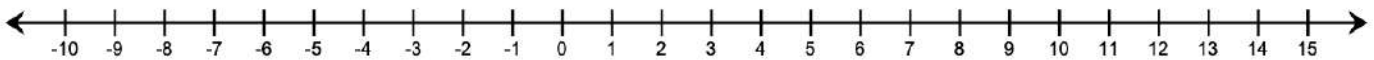


b. Rewrite the equation as a subtraction equation.

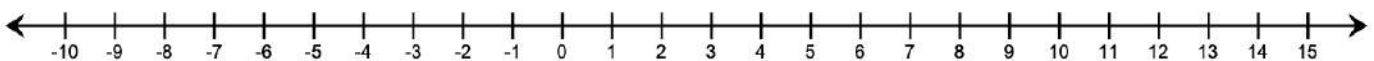
c. What is the value of the unknown?

2. Use the number lines to model how to find the value of each unknown.

a.  $7 - 13 = ?$



b.  $6 - (-6) = ?$



3. Peter was trying to find the value of  $5 - (-3)$ . Amanda said it's the same as  $5 + 3$ . Explain why Amanda is correct.

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4. Rewrite each expression below as an addition expression. Then find the value. Sketch a number line, if that will help you.

$$3 - 8$$

$$-7 - 7$$

$$4 - (-9)$$

Name: KEY

Consider the equation  $4 + ? = 9$

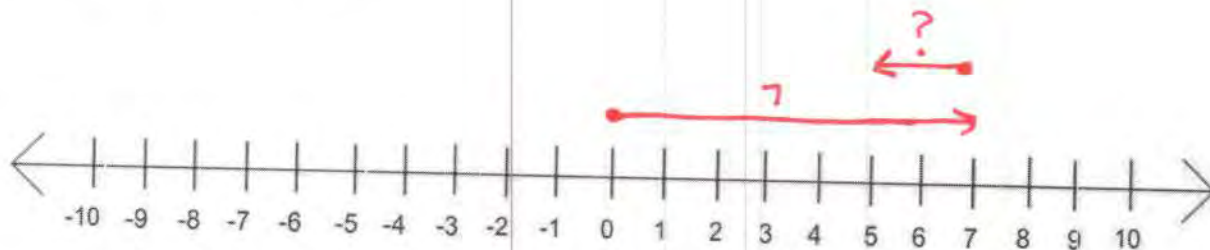


1. Use an arrow on the number line to show the first addend. ✓
2. Draw another arrow to represent the unknown. ✓
3. Rewrite the equation as a subtraction equation.

$$9 - 4 = ?$$

4. The value of the unknown is 5.

Consider the equation  $7 + ? = 5$

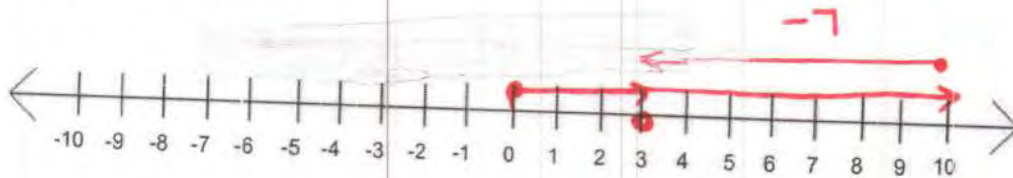


5. Use an arrow on the number line to show the first addend. ✓
6. Draw another arrow to represent the unknown. ✓
7. Rewrite the equation as a subtraction equation.

$$5 - 7 = ?$$

8. The value of the unknown is -2.

Consider the equation  $3 - 10 = ?$ .



9. Plot 3 on the number line. ✓
10. Draw an arrow to represent 10 starting at 0. ✓
11. Draw an arrow from 10 to 3 to represent the difference. ✓
12. The value of the unknown is -7.

Look at the table below.

EXPRESSION	VALUE
$3 + 5$	8
$3 - 5$	-2
$3 + (-5)$	-2
$3 - (-5)$	8

13. Find the value of each expression. Sketch a number line if that's helpful for you. ✓

14. Which expressions are equivalent in value?

$3 + 5 = 3 - (-5)$  and  $3 - 5 = 3 + (-5)$

15. Subtracting a number is always the same as adding its opposite.

16. Rewrite each expression below as an addition expression. Find the value of each.

$-7 - 3$	$10 - (-6)$
$-7 + (-3)$	$10 + 6$
$(-10)$	$(16)$

Name: KEY

1. Consider the equation  $-9 + ? = 6$ .

a. Represent the equation on the number line.



b. Rewrite the equation as a subtraction equation.

$$6 - (-9) = \text{?}$$

c. What is the value of the unknown?

15

2. Use the number lines to model how to find the value of each unknown.

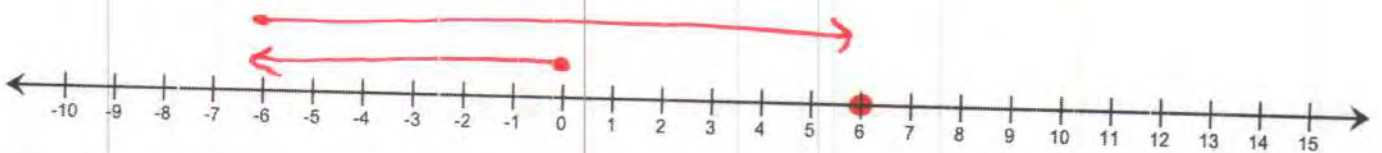
a.  $7 - 13 = ?$

$(-6)$



b.  $6 - (-6) = ?$

$(12)$





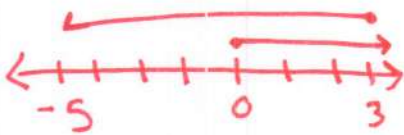
3. Peter was trying to find the value of  $5 - (-3)$ . Amanda said it's the same as  $5 + 3$ . Explain why Amanda is correct.

I can think of  $5 - (-3)$  as the difference between  $-3$  and  $5$ , which is  $8$ .  
 $5 + 3$  also has a value of  $8$ .

4. Rewrite each expression below as an addition expression. Then find the value. Sketch a number line, if that will help you.

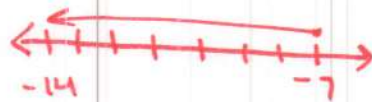
$3 - 8$

$$3 + (-8)$$
$$\textcircled{-5}$$



$-7 - 7$

$$-7 + (-7)$$
$$\textcircled{-14}$$



$4 - (-9)$

$$4 + 9$$
$$\textcircled{13}$$



## **G7 U4 Lesson 5**

Solve subtraction expressions that have the same numbers in the opposite order, and explain the relationship between their differences.



**G7 U4 Lesson 5 - Students will solve subtraction expressions that have the same numbers in the opposite order, and explain the relationship between their differences.**

**Warm Welcome (Slide 1):** Tutor choice

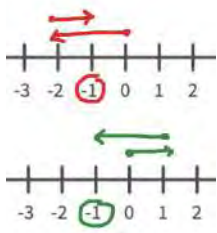
**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we thought about how we can use a number line to represent subtraction involving negative numbers. We also saw how it can be helpful to think about subtraction as finding the *difference* between two numbers.

Today, let's keep thinking about subtraction. We'll pay particular attention to the order of the numbers impacts our answer.

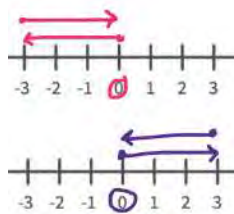
**Let's Talk (Slide 3):** Look at the pairs of addition problems here. What do you notice? What do you wonder?

**Possible Student Answers, Key Points:**

- I notice each pair has the same numbers in a different order. I notice some pairs have negative numbers and some do not.
- I wonder what some of the answers are. I wonder if we can model each one on the number line.



You already know that  $4 + 5$  and  $5 + 4$  will both total 9. When adding signed numbers, the order does not matter. Whether we're adding positive numbers, negative numbers, or both, the order of the addends will not affect the answer. Think about the second pair of problems. (*model on number line as you narrate*) To show  $-2 + 1$ , I can draw an arrow to -2 then add 1 with another arrow. The answer is -1. If I reverse the order of the addends and think about 1 plus -2, I can model that by first drawing an arrow to -1. Then, add an arrow showing -2. The answer is still -1. Switching the order of the addends didn't change a thing.



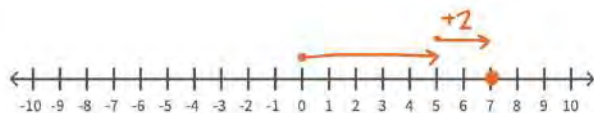
What happens if we add numbers in the opposite order in the third example? (*if necessary, sketch a model on the number line as student shares*) **Possible Student**

**Answers, Key Points:**

- I know that the answer will be the same. You can add in any order.
- $3 + (-3)$  would look like an arrow to +3 then we'd add an arrow showing -3, ending up at 0. To show  $-3 + 3$ , we'd draw an arrow to -3 then show another arrow adding 3. We end up with an answer of 0 both times.

When adding signed numbers, the order does not matter. Now let's keep this in mind as we think about what happens when we subtract in a different order.

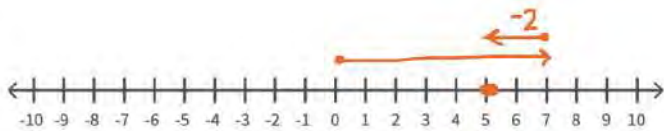
**Let's Think (Slide 4):** This example wants us to show  $7 - 5$  and then  $5 - 7$ . Let's start by modeling to find  $7 - 5$ . It's okay if you already know the answer, let's just picture what is happening.



(*sketch and label on number line as you narrate*) We can think of this as finding the difference between 7 and 5. I can mark 7 on my number line. I'll draw an arrow from 0 to 5 to represent 5. Then I'll draw an arrow from 5 to 7 to represent our unknown. In this case, the direction and length of the arrow make it clear that our answer is +2.

Now, we'll use a number line to model subtracting the same numbers, just in a different order. What do you predict will happen? **Possible Student Answers, Key Points:**

- Maybe we'll get the same answer like we did with the addition problems.
- I don't think we'll get the same answer. I think  $5 - 7$  will be negative, because 7 is bigger than 5.



Let's tackle part B and find out. For part B, we're finding the difference between 5 and 7. (*sketch and label on number line as you narrate*) I'll start by marking 5 on the number line. Next, I'll draw an arrow from 0 to 7 to represent 7. Then, I'll draw the arrow from 7 to 5 to represent the unknown value. What is the unknown

value in this case, and how do you know? **Possible Student Answers, Key Points:**

- The unknown value is -2. The arrow is 2 units long and it points left, so I know it is negative.

We found that  $7 - 5$  is positive 2 and that  $5 - 7$  is negative 2. Unlike with addition, we can see that the order of signed numbers matters when dealing with subtraction.

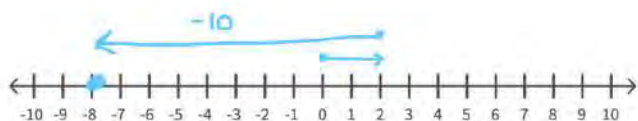
Our answers have the same magnitude, but opposite signs. We can think of the first problem as asking us to figure out how to get from 5 to 7. We move 2 up the number line, so the difference is +2. We can think of the second problem as asking us to figure out how to get from 7 to 5. In that example, we moved 2 down the number line, so the difference was -2.

Let's try another set to see if this conjecture holds true.

**Let's Think (Slide 5):** Notice how the order of numbers in each expression is switched. Let's model  $-8 - 2$  first.

We can think of  $-8 - 2$  as asking us to determine how to get from 2 to -8. How can I model  $-8 - 2$  on the number line? (*sketch and label on number line as student shares*) **Possible Student Answers, Key Points:**

- Mark -8 with a point on the number line. Then draw an arrow from 0 to 2 to show positive 2. Last, draw an arrow from 2 to -8 to represent the unknown.

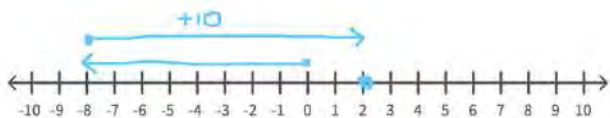


Great. Our unknown is this long line. If I look carefully, I see that the line is pointing to the left and is 10 units

long. This tells me that  $-8 - 2$  is -10. I can also think of this as showing me that to get from 2 to -8, I have to move 10 spaces left.

Let's try the other example with the numbers in a different order. What do you predict will happen based on the work we've done so far? **Possible Student Answers, Key Points:**

- I don't think the answer will be the same, because we're subtracting.
- Maybe our answer will be positive, since the answer we just found was negative.



We can think of  $2 - (-8)$  as finding the difference between 2 and -8. In other words, how can we get from -8 to 2. Let's model it. (*sketch and label the number line as you narrate*) I'll mark 2 with a point. I'll draw the first arrow from 0 to -8 to show -8. Then, I'll draw an arrow for our unknown

stretching from -8 to +2. What's the value of the unknown? (**positive 10**) Our answer is +10. The arrow shows that to get from -8 to 2, we have to move up the number line 10 spaces.

The distance between the numbers in each problem we did was the same, so our answers had the same magnitude. In this case, it was 10. The arrows representing the unknowns pointed in different directions, so one answer was -10 and the other was +10.

How is adding signed numbers in the opposite order different from subtracting signed numbers in the opposite order? [Possible Student Answers, Key Points:](#)

- When we add signed numbers in a different order, we end up with the same answer.
- When we subtract signed numbers in a different order, the sign on our answer is different.

**Let's Try it (Slides 6 - 7):** Now let's work on a few more examples where we think about subtraction problems that have the same numbers in a different order. We know that when we add, the order doesn't matter. We now know that when we subtract, the order does matter. When we subtract the same numbers in a different order, the magnitude of our answer stays the same but the sign on our answers is different. We'll carefully use number lines to help us think through a few more examples. Time to get started!

# WARM WELCOME



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**Today we will solve subtraction expressions that have the same numbers in the opposite order, and explain the relationship between the differences.**

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Let's Talk:

What happens when we add numbers in the opposite order?

$$5 + 4$$

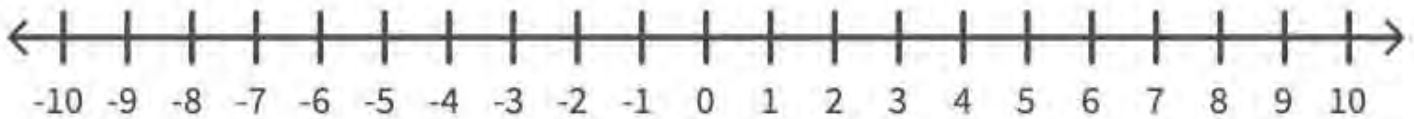
$$-2 + 1$$

$$3 + (-3)$$

$$4 + 5$$

$$1 + (-2)$$

$$-3 + 3$$

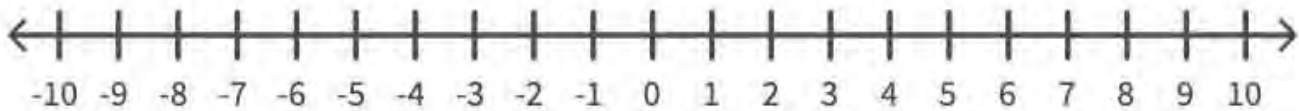


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Let's Think:

Use the number line to find each unknown.



a.  $7 - 5$

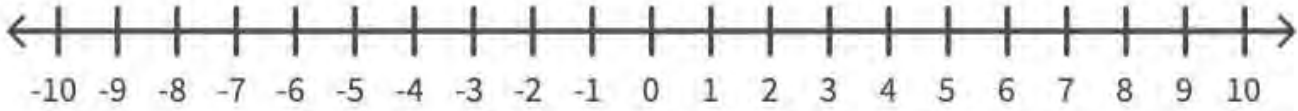
b.  $5 - 7$

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# Let's Think:

## Use the number line to find each unknown.



a.  $-8 - 2$

b.  $2 - (-8)$

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# Let's Try It:

## Let's explore solving subtraction problems that have the same numbers in the opposite order together.

Name: \_\_\_\_\_ 07 U4 Lesson 5- Let's Try It

Find the sum of each pair of addition expressions. A number line is provided, in case that helps you think about the values.

1.  $6 + 2$                        $2 + 6$   
 2.  $1 + -7$                        $-7 + 1$   
 3.  $-9 + 5$                        $5 + -9$   
 4.  $-4 + -2$                        $-2 + -4$

5. What do you notice about what happens when you switch the order of the addends?  
 \_\_\_\_\_  
 \_\_\_\_\_

Find the value of each pair of subtraction expressions. Notice how the numbers switch positions in each pair.

6.  $5 - 2$                        $2 - 5$   
 7.  $6 - (-3)$                        $-3 - 6$

8. What do you notice happens when you switch the order of numbers when subtracting?  
 \_\_\_\_\_  
 \_\_\_\_\_

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Consider the expression  $(-11) - 4$ .

9. Plot  $-11$  on the number line below.

10. Draw an arrow to represent  $4$ .

11. Draw an arrow to represent the difference between  $-11$  and  $4$ . What is the value?

Consider the expression  $4 - (-11)$ .

12. Plot  $4$  on the number line below.

13. Draw an arrow to represent  $-11$ .

14. Draw an arrow to represent the difference between  $4$  and  $-11$ . What is the value?

15. Why are the answers to #11 and #14 opposites? Use the number lines to help explain.  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

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# On your Own:

Now it's time to explore solving subtraction equations that have the same numbers in the opposite order on your own.


Name: \_\_\_\_\_ 67 U4 Lesson 5 - Independent Work

1. Show each expression on a number line.


a.  $-8 + 5$




b.  $5 + (-8)$



c.  $8 - 5$



d.  $5 - (-8)$




2. Find each sum.

$-1 + 6$        $6 + (-1)$


What happens when you change the order of numbers in an addition expression?

3. Find each difference using the number lines.

$10 - (-7)$



$-7 - 10$



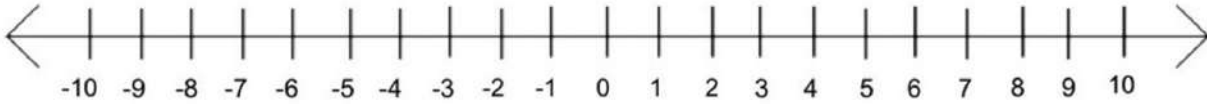
What happens when you change the order of numbers in a subtraction expression?

4. Marquise is trying to find the value of  $6 - 12$ . He says he can change the order of the numbers and find  $12 - 9$  to get his answer. Do you agree or disagree? Explain and include the correct difference in your response.

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Find the sum of each pair of addition expressions. A number line is provided, in case that helps you think about the values.



1.  $6 + 2$

$2 + 6$

2.  $1 + -7$

$-7 + 1$

3.  $-9 + 5$

$5 + -9$

4.  $-4 + -2$

$-2 + -4$

5. What do you notice about what happens when you switch the order of the addends?

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Find the value of each pair of subtraction expressions. Notice how the numbers switch positions in each pair.

6.  $5 - 2$

$2 - 5$

7.  $6 - (-3)$

$-3 - 6$

8. What do you notice happens when you switch the order of numbers when subtracting?

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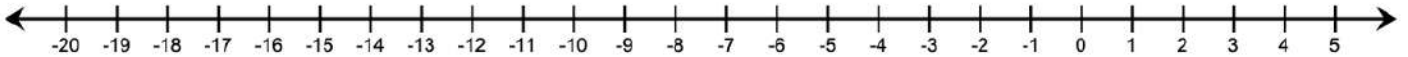
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Consider the expression  $(-11) - 4$ .

9. Plot -11 on the number line below.

10. Draw an arrow to represent 4.

11. Draw an arrow to represent the difference between -11 and 4. What is the value?

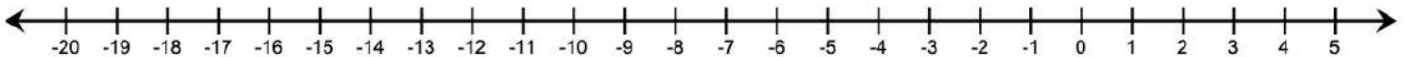


Consider the expression  $4 - (-11)$ .

12. Plot 4 on the number line below.

13. Draw an arrow to represent -11.

14. Draw an arrow to represent the difference between 4 and -11. What is the value?



15. Why are the answers to #11 and #14 opposites? Use the number lines to help explain.

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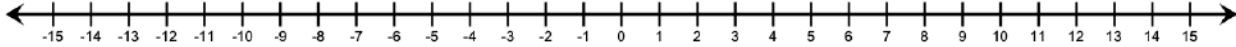
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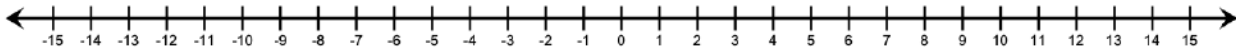
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**1. Show each expression on a number line.**

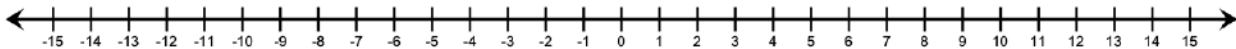
a.  $-8 + 5$



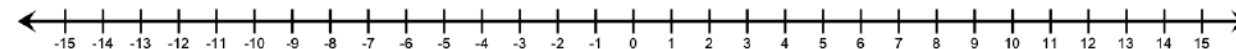
b.  $5 + (-8)$



c.  $8 - 5$



d.  $5 - (-8)$

**2. Find each sum.**

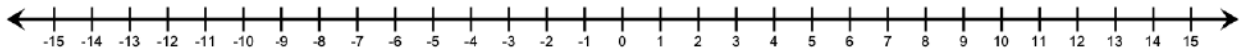
$-1 + 6$

$6 + (-1)$

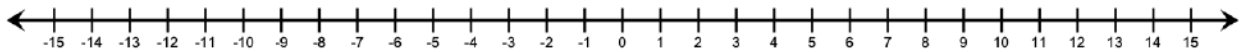
What happens when you change the order of numbers in an addition expression?

**3. Find each difference using the number lines.**

$10 - (-7)$



$-7 - 10$



What happens when you change the order of numbers in a subtraction expression?

**4. Marquise is trying to find the value of  $9 - 12$ . He says he can change the order of the numbers and find  $12 - 9$  to get his answer.** Do you agree or disagree? Explain and include the correct difference in your response.

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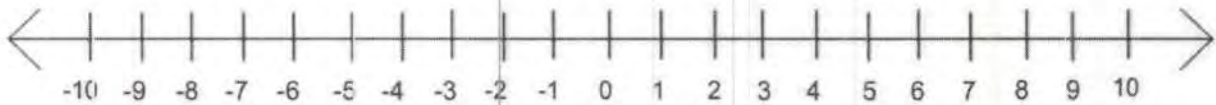
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Find the sum of each pair of addition expressions. A number line is provided, in case that helps you think about the values.



1.  $6 + 2$  8       $2 + 6$  8  
 2.  $1 + -7$  -6       $-7 + 1$  -6  
 3.  $-9 + 5$  -4       $5 + -9$  -4  
 4.  $-4 + -2$  -6       $-2 + -4$  -6

5. What do you notice about what happens when you switch the order of the addends?

The sum is the same when the addends are reversed.

Find the value of each pair of subtraction expressions. Notice how the numbers switch positions in each pair.

6.  $5 - 2$        $2 - 5$   
3      -3  
 7.  $6 - (-3)$        $-3 - 6$   
9      -9

8. What do you notice happens when you switch the order of numbers when subtracting?

The answers are not the same.  
They are opposites.

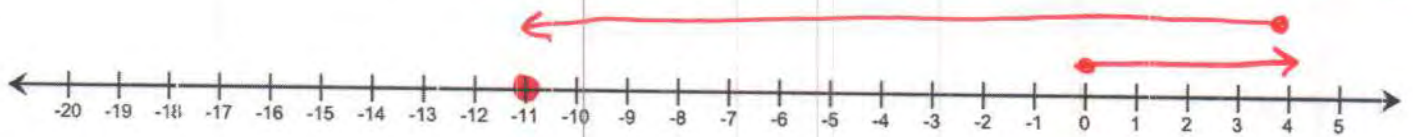
Consider the expression  $(-11) - 4$ .

9. Plot  $-11$  on the number line below.

10. Draw an arrow to represent  $4$ .

11. Draw an arrow to represent the difference between  $-11$  and  $4$ . What is the value?

$-15$



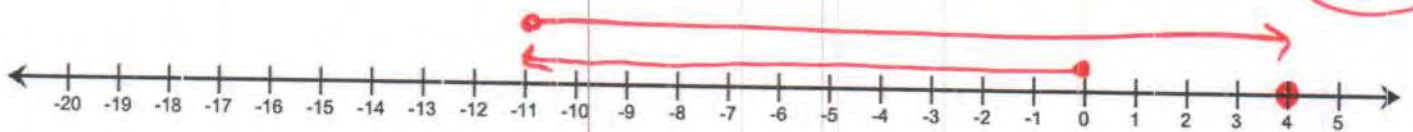
Consider the expression  $4 - (-11)$ .

12. Plot  $4$  on the number line below.

13. Draw an arrow to represent  $-11$ .

14. Draw an arrow to represent the difference between  $4$  and  $-11$ . What is the value?

$15$

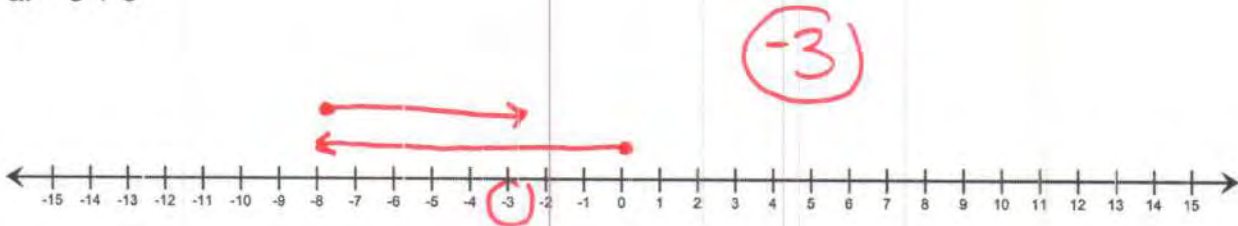


15. Why are the answers to #11 and #14 opposites? Use the number lines to help explain.

The first answer is  $-15$ , because the arrow from  $4$  to  $-11$  represents a value of  $-15$  since it points left. The second answer is  $15$ , because the arrow points right.

1. Show each expression on a number line.

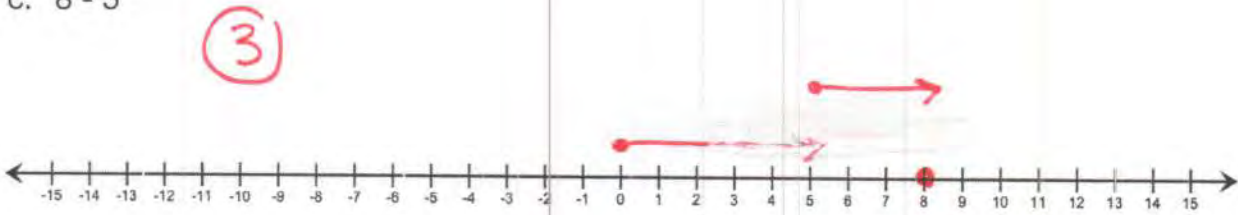
a.  $-8 + 5$



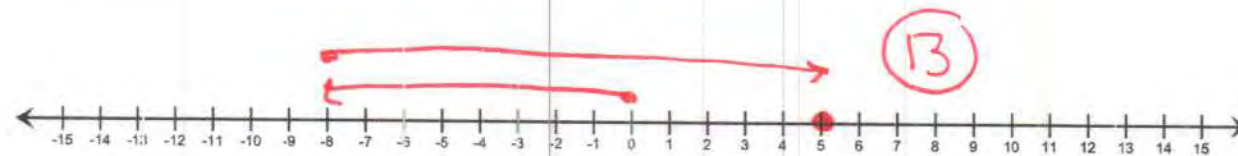
b.  $5 + (-8)$



c.  $8 - 5$



d.  $5 - (-8)$



2. Find each sum.

$-1 + 6$

5

$6 + (-1)$

5

What happens when you change the order of numbers in an addition expression?

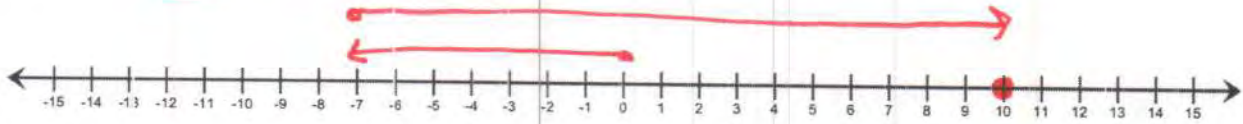
The sum stays the same.



3. Find each difference using the number lines.

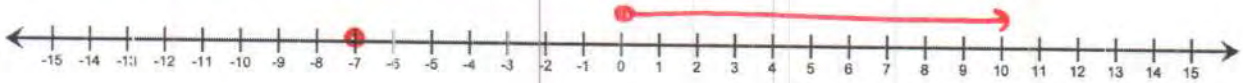
$10 - (-7)$

17



$-7 - 10$

-17



What happens when you change the order of numbers in a subtraction expression?

The answer is not the same.

The answers are opposites.

4. Marquise is trying to find the value of  $9 - 12$ . He says he can change the order of the numbers and find  $12 - 9$  to get his answer. Do you agree or disagree? Explain and include the correct difference in your response.

$9 - 12$

$9 + (-12) = -3$

No, Marquise is incorrect. You can't change the order of numbers in a subtraction problem.  $9 - 12$  is equal to  $-3$ , while  $12 - 9$  is equal to  $3$ .

# **G7 U4 Lesson 6**

Add and subtract signed numbers to represent gains and losses in different contexts.

## G7 U4 Lesson 6 - Students will add and subtract signed numbers to represent gains and losses in different contexts.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We have been working with adding and subtracting signed numbers for the past few lessons. When you think about everyday contexts that involve signed numbers, what comes to mind? **Possible Student Answers, Key Points:**

- Temperature involves signed numbers. Warmer temperatures are positive numbers, colder temperatures are often negative numbers.
- When I'm above sea level, my elevation is positive. If I'm below sea level, my elevation is negative.
- My bank account might show positive or negative numbers depending on if I'm earning or spending money. If I owe money to the bank, that can be represented with a negative value.

Today we'll wrap up this set of lessons by exploring adding and subtracting signed numbers in a variety of contexts. Some will be familiar and some might be new. Let's take a look!

**Let's Talk (Slide 3):** Check out the table shown here. Look it over for a moment, and when you're ready, share out some things you notice and some things you wonder. **Possible Student Answers, Key Points:**

- I notice the table shows weekdays, inventory, and change. I notice that the inventory goes down and up. I notice some of the changes are positive values and some are negative.
- I wonder what the empty box means. I wonder why Saturday and Sunday aren't included. I wonder what the inventory represents.

This table represents inventory at a mattress store. It can be useful to use signed numbers to represent inventory at stores to keep track of items you have in stock and to know when you need to order more items. We'll see this context and a few more throughout our work with signed numbers today.

**Let's Think (Slide 4):** Our first problem asks us to consider the table representing the mattress store's inventory.

Part A asks what the positive and negative numbers mean in this situation. If I put myself in the shoes of a mattress store owner, and I think about my inventory of mattresses, why might the number of mattresses I have in my inventory decrease like we see happening on Monday, Tuesday, and Wednesday? What could be happening with my business if the number of mattresses I have in inventory increases like we see on Thursday? **Possible Student Answers, Key Points:**

- The negative numbers probably represent mattresses that the store is selling. For every mattress the store sells, there inventory should decrease by 1 mattress.
- The positive numbers could be the store restocking mattresses. They don't want to run out, so they need to reorder every now and then.
- The positive numbers could also be returns, but I'm not sure 20 returns in one day makes sense given this context.

Great. From the table, we can assume that the mattress store sold 1 mattress on Monday, 6 on Tuesday, and 6 on Wednesday. Those changes were all noted with a negative number. Then on Thursday, the change was +20. This likely represents the store restocking 20 new mattresses.

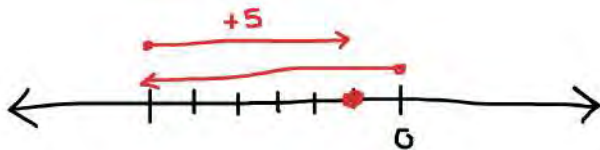
$$24 - 10 = 14$$

Part B wants us to complete the table if the store sells 10 mattresses on Friday. What number would represent the store *selling* 10 mattresses? (-10) Yes, if the store sells 10 mattresses, those are being removed from the inventory. (*write*  $24 - 10 = 14$ ) We could complete the table by writing -10 as the change.

The inventory on Saturday morning would be 14 mattresses.

$$-1 - (-6) = ?$$

In Part C, the problem asks us to find the difference between the change on Monday and the change on Tuesday. I can think of that difference by writing the equation  $-1 - (-6) = ?$ . (write equation)



How can I use a number line to represent this equation? (sketch a quick number line and draw/label as student shares)

Possible Student Answers, Key Points:

- Draw a number line. Mark -1 with a point on the line, then draw an arrow from 0 to -6 to represent that value. The unknown difference could be marked using an arrow from -6 to -1.

We can see that the difference between the change on Monday and the change on Tuesday is 5 mattresses. We used an equation and a number line to help us find that amount.

**Let's Think (Slide 5):** Let's work with some signed numbers in another context. Take a second to review the table shown here, so you can start to get a sense of what this problem is about. (pause) This table shows Ms. Han's account balance from April to October. Before we respond to the prompts, what do the positive and negative mean in this situation? Possible Student Answers, Key Points:

- The positive balances mean that she had money in her account. The negative balances mean that she owed the bank money that month.
- The positive changes mean she earned money or made deposits. The negative changes mean she spent money or made withdrawals.

DAY	BALANCE	CHANGE
Apr	\$50	\$150
May	\$200	-\$25
June	\$175	0
July	\$175	-\$100
Aug	\$75	-\$100
Sep	-\$25	\$250
Oct	\$225	\$500

Part A asks us to complete the missing values in the table using the information that we know. Start with the month of May, and let's think about the change. What do you notice happened between May and June? Look at the balances for those months. (The balance went down \$25.) Since the balance decreased 25 dollars, I'll mark that the change was negative \$25. (fill in table)

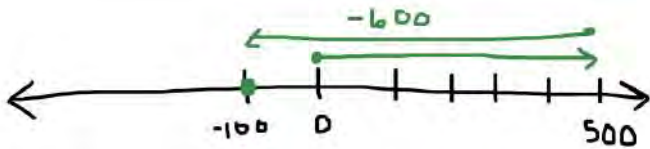
What about the July balance? What do you notice happened from June to July? (The change in June was \$0.) Since the change in June was \$0, that means the balance would not increase or decrease. Let's write \$175 for the July balance, since it stayed the same as the balance from June. (fill in table)

Our last missing value in the table is the October balance. The balance in September was -\$25, and the table shows us that it increased \$250 that month. I can think of that as  $-25 + 250$ . (write expression) We could use a number line to model this expression, but I think we can make this problem easier. I know I can switch the order of addends. (write  $250 + (-25)$ ) Now, I can just think of this as 250 plus -25 or 250 minus 25. That's a little easier to consider. What's the balance in October? (\$225) Well done.

$$-25 + 250$$

$$250 + (-25)$$

Part B wants us to find the difference between the change in August, -\$100, and the change in October, \$500. We can use the equation  $-100 - 500$  to help us think about this. Or, we can use a number line to help us think about this.



(sketch a number line with intervals of 100) I'll mark -100 on the number line, and I'll draw an arrow from 0 to 500 to show positive 500. (sketch as you narrate) The unknown difference would be the distance between 500 and -100. We can think of this as -600. Since the question just wants to know the amount of

the change, it's appropriate to say the difference between the change was simply \$600.

The last prompt is Part C. It asks us how much money Ms. Han will have at the start of the next month, November. What information do we know that can help us figure this out? [Possible Student Answers, Key Points:](#)

- We know the balance in October is \$225, because we found that earlier.
- We know the change in October is positive \$500, so her balance will be higher in November.

$$225 + 500 = ?$$

The October balance is \$225 and it increased \$500. We can think of that using the equation  $225 + 500 = ?$ . (write equation) What's the balance going to be in November? (\$725) Excellent!

We just solved a variety of problems with signed numbers in different contexts. I think you're ready to get some practice.

**Let's Try it (Slides 6 - 7):** Now let's work on a few more to further improve our skills. As we move through each problem, we'll want to pause and think about the context. We can ask ourselves what the positive and negative numbers mean in that situation to help us make sense of the problem. From there, we can use equations and/or number lines to help us add and subtract with signed numbers depending on what the question asks. We're ready!

# WARM WELCOME



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**Today we will add and subtract signed numbers to represent gains and losses in different contexts.**

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## Let's Talk:

**What do you notice? What do you wonder?**

DAY	INVENTORY	CHANGE
Mon	17	-1
Tue	16	-6
Wed	10	-6
Thu	4	+20
Fri	24	

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## Let's Think:

**The table below represents the inventory at a mattress store.**

DAY	INVENTORY	CHANGE
Mon	17	-1
Tue	16	-6
Wed	10	-6
Thu	4	+20
Fri	24	

- What do the positive and negative numbers mean in this situation?
- If they sell 10 mattresses on Friday, what number completes the table?
- What is the difference between the change on Monday and the change on Tuesday?

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# Let's Think:

The table below reflects Ms. Han's bank account balance.

DAY	BALANCE	CHANGE
Apr	\$50	\$150
May	\$200	
June	\$175	0
July		-\$100
Aug	\$75	-\$100
Sep	-\$25	\$250
Oct		\$500

- Use the information to complete the table.
- What is the difference between the change in August and the change in October?
- How much money will she have at the start of the next month?

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# Let's Try It:

Let's explore adding and subtracting signed numbers to represent gains and losses together.

Name: \_\_\_\_\_ G7 U4 Lesson 6 - Let's Try It

Clark owns a shoe store. He keeps track of how many pairs of shoes he has in stock by using the table below.

DAY	PAIRS IN STOCK	CHANGE
Mon	100	+25
Tue	75	-10
Wed	60	+120
Thu		
Fri		

- Which best describes what happened on Monday?
  - The store received 25 pairs of shoes.
  - The store sold 25 pairs of shoes.
  - The store received 120 pairs of shoes.
  - The store sold 100 pairs of shoes.
- In your own words, describe what happened on Tuesday.

3. The table shows that the store received a shipment of 120 pairs of shoes on Wednesday. How many pairs will the store have in stock at the start of the day on Thursday? Record your answer in the table.

4. Clark sold 40 pairs of shoes on Thursday. Show how he can record that in the table.

5. How many pairs will the store have in stock at the start of the day on Friday? Record your answer in the table.

6. Clark wants to find the difference between the change of inventory on Wednesday and the change of inventory on Thursday. Represent each change on the number line. Then write an equation to find the difference.

7. What is the difference between the change of inventory on Tuesday and the change of inventory on Thursday?

A bakery keeps track of their cake inventory in the table below. Some of the values are missing.

DAY	CAKES	CHANGE
Mon	40	-10
Tue		+20
Wed	10	
Thu	20	
Fri	15	-10
Sat	5	25
Sun		-5

- Find each missing value in the table.
- What is the difference between the change on Wednesday and the change on Saturday? Sketch a number line to show your thinking.
- What is the difference between the change on Monday and the change on Tuesday? Sketch a number line to show your thinking.
- What is the difference between the change on Saturday and the change on Sunday? Sketch a number line to show your thinking.

A team of researchers spent four weeks hiking in a canyon. Their elevations each week are shown in the table.

WEEK	ELEVATION (meters)	CHANGE
1	35	-3
2	-0.5	-4.5
3	-5	
4	0.7	2

- What does a negative elevation mean? A positive?
- What does a negative change in elevation mean? A positive?
- Fill in the missing value based on the information in the table.

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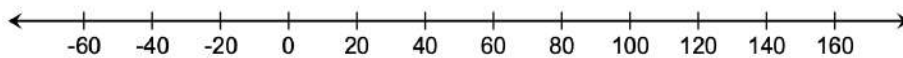


Clark owns a shoe store. He keeps track of how many pairs of shoes he has in stock by using the table below.

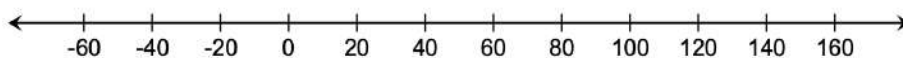
DAY	PAIRS IN STOCK	CHANGE
Mon	100	-25
Tue	75	-10
Wed	65	+120
Thu		
Fri		

- Which best describes what happened on Monday?
  - The store received 25 pairs of shoes.
  - The store sold 25 pairs of shoes.
  - The store received 100 pairs of shoes.
  - The store sold 100 pairs of shoes.
- In your own words, describe what happened on Tuesday.

- The table shows that the store received a shipment of 120 pairs of shoes on Wednesday. How many pairs will the store have in stock at the start of the day on Thursday? Record your answer in the table.
- Clark sold 40 pairs of shoes on Thursday. Show how he can record that in the table.
- How many pairs will the store have in stock at the start of the day on Friday? Record your answer in the table.
- Clark wants to find the difference between the change of inventory on Wednesday and the change of inventory on Thursday. Represent each change on the number line. Then write an equation to find the difference.



- What is the difference between the change of inventory on Tuesday and the change of inventory on Thursday?



A bakery keeps track of their cake inventory in the table below. Some of the values are missing.

DAY	CAKES	CHANGE
Mon	40	-10
Tue		-20
Wed	10	
Thu	20	
Fri	15	-10
Sat	5	25
Sun		-5

8. Find each missing value in the table.

9. What is the difference between the change on Wednesday and the change on Saturday? Sketch a number line to show your thinking.

10. What is the difference between the change on Monday and the change on Tuesday? Sketch a number line to show your thinking.

11. What is the difference between the change on Saturday and the change on Sunday? Sketch a number line to show your thinking.

A team of researchers spent four weeks hiking in a canyon. Their elevations each week are shown in the table.

WEEK	ELEVATION (m)	CHANGE
1	2.5	-3
2	-0.5	-4.5
3	-5	
4	0.7	2

12. What does a negative elevation mean? A positive?

13. What does a negative change in elevation mean? A positive?

14. Fill in the missing value based on the information in the table.

**1. A ski resort tracks the low temperature each month from January through May. Use the information to answer the questions.**

MONTH	LOW TEMPERATURE	CHANGE
Jan	-5	+7
Feb	2	-3
Mar	-1	+1
Apr	0	+6
May	6	+10

In January the temperature \_\_\_\_\_ degrees.

- a. increased 7
- b. decreased 7
- c. increased 5
- d. decreased 5

In February the temperature \_\_\_\_\_ degrees.

- a. increased 2
- b. decreased 2
- c. increased 3
- d. decreased 3

What is the difference between the change of temperature in February and the change of temperature in May?

- a. 3 degrees
- b. 7 degrees
- c. 10 degrees
- d. 13 degrees

**2. A hiker's smartwatch tracks her elevation each hour.**

PART A: Find the missing values in the table.  
Show how you know.

HOUR	ELEVATION (ft)	CHANGE (ft)
1st	10	-20
2nd	-10	
3rd	20	
4th	-20	10

PART B: What is the difference between the change in elevation the first hour and the change in elevation the fourth hour?

3. The robotics club is hosting a cookie sale this week to raise funds for a field trip. The table shows how many cookies they have in stock at the beginning of the day, and the change in inventory each day.

DAY	INVENTORY	CHANGE
Mon	500	-60
Tue	440	-100
Wed	340	+50
Thu	390	-110
Fri	280	

What does the value -60 mean in the context of this story?

What does the value +50 mean in the context of this story?

By the end of the week, the robotics team sold out of cookies. Complete the table to reflect that information.

What is the difference between the change on Monday and the change on Thursday?

Nneka and Juan are studying dolphins. They track the depth that a dolphin is swimming over the course of four hours, and they record their findings in the table. Nneka says the difference in elevation from Hour 1 to Hour 2 is 130 feet. Juan says the difference is 70 feet. Who is correct? Explain how you know.

HOUR	ELEVATION
1	-30
2	-100
3	-65
4	-60

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Clark owns a shoe store. He keeps track of how many pairs of shoes he has in stock by using the table below.

DAY	PAIRS IN STOCK	CHANGE
Mon	100	-25
Tue	75	-10
Wed	65	+120
Thu	185	-40
Fri	145	

- Which best describes what happened on Monday?
  - The store received 25 pairs of shoes.
  - The store sold 25 pairs of shoes.
  - The store received 100 pairs of shoes.
  - The store sold 100 pairs of shoes.
- In your own words, describe what happened on Tuesday.

The store sold 10 pairs of shoes.

3. The table shows that the store received a shipment of 120 pairs of shoes on Wednesday. How many pairs will the store have in stock at the start of the day on Thursday? Record your answer in the table.

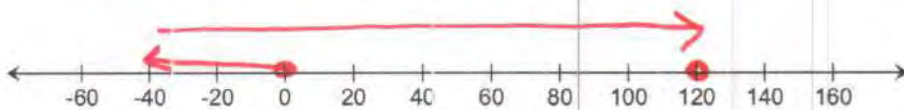
$$65 + 120 = 185$$

4. Clark sold 40 pairs of shoes on Thursday. Show how he can record that in the table.

5. How many pairs will the store have in stock at the start of the day on Friday? Record your answer in the table.

$$185 - 40 = 145$$

6. Clark wants to find the difference between the change of inventory on Wednesday and the change of inventory on Thursday. Represent each change on the number line. Then write an equation to find the difference.



$$120 - (-40) = ?$$

$$120 + 40$$

$$160$$

7. What is the difference between the change of inventory on Tuesday and the change of inventory on Thursday?



$$-10 - (-40)$$

$$-10 + 40$$

$$30$$

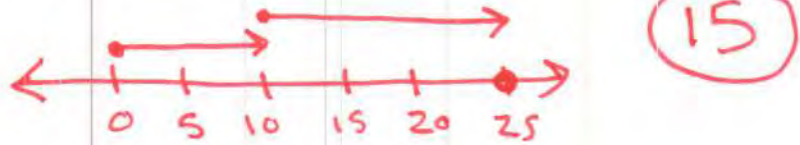


A bakery keeps track of their cake inventory in the table below. Some of the values are missing.

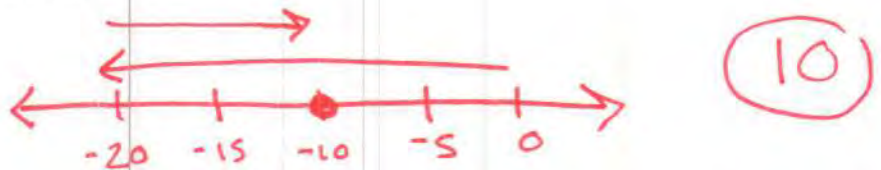
DAY	CAKES	CHANGE
Mon	40	-10
Tue	30	-20
Wed	10	+10
Thu	20	-5
Fri	15	-10
Sat	5	25
Sun	30	-5

8. Find each missing value in the table. ✓

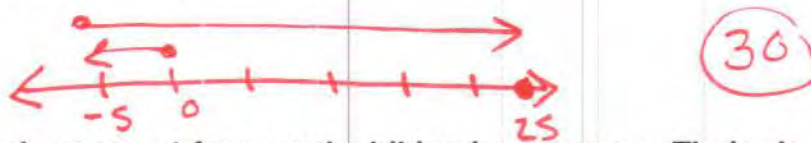
9. What is the difference between the change on Wednesday and the change on Saturday? Sketch a number line to show your thinking.



10. What is the difference between the change on Monday and the change on Tuesday? Sketch a number line to show your thinking.



11. What is the difference between the change on Saturday and the change on Sunday? Sketch a number line to show your thinking.



A team of researchers spent four weeks hiking in a canyon. Their elevations each week are shown in the table.

WEEK	ELEVATION (m)	CHANGE
1	2.5	-3
2	-0.5	-4.5
3	-5	+5.7
4	0.7	2

12. What does a negative elevation mean? A positive?

A negative elevation means the elevation decreases, whereas a positive elevation means the elevation increases.

13. What does a negative change in elevation mean? A positive?

A positive elevation is above sea level. A negative elevation is below sea level.

14. Fill in the missing value based on the information in the table.

$$-5 + ? = 0.7$$

1. A ski resort tracks the low temperature each month from January through May. Use the information to answer the questions.

MONTH	LOW TEMPERATURE	CHANGE
Jan	-5	+7
Feb	2	-3
Mar	-1	+1
Apr	0	+6
May	6	+10

In January the temperature \_\_\_\_\_ degrees.

- a. increased 7
- b. decreased 7
- c. increased 5
- d. decreased 5

In February the temperature \_\_\_\_\_ degrees.

- a. increased 2
- b. decreased 2
- c. increased 3
- d. decreased 3

What is the difference between the change of temperature in February and the change of temperature in May?

- a. 3 degrees
- b. 7 degrees
- c. 10 degrees
- d. 13 degrees

$$10 - (-3)$$

$$10 + 3$$

2. A hiker's smartwatch tracks her elevation each hour.

PART A: Find the missing values in the table. Show how you know.

HOUR	ELEVATION (ft)	CHANGE (ft)
1st	10	-20
2nd	-10	+30
3rd	20	-40
4th	-20	10

PART B: What is the difference between the change in elevation the first hour and the change in elevation the fourth hour?

$$10 - (-20)$$

$$10 + 20$$

$$(30)$$



3. The robotics club is hosting a cookie sale this week to raise funds for a field trip. The table shows how many cookies they have in stock at the beginning of the day, and the change in inventory each day.

DAY	INVENTORY	CHANGE
Mon	500	-60
Tue	440	-100
Wed	340	+50
Thu	390	-110
Fri	280	-280

What does the value -60 mean in the context of this story?

The team sold 60 cookies.

What does the value +50 mean in the context of this story?

The team baked 50<sup>more</sup> cookies.

By the end of the week, the robotics team sold out of cookies. Complete the table to reflect that information.

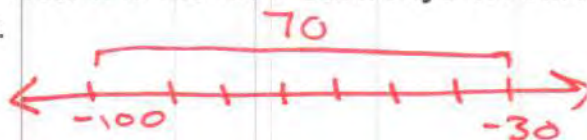
$$280 - 280 = 0$$

What is the difference between the change on Monday and the change on Thursday?

$$-110 - (-60) = -50$$

Nneka and Juan are studying dolphins. They track the depth that a dolphin is swimming over the course of four hours, and they record their findings in the table. Nneka says the difference in elevation from Hour 1 to Hour 2 is 130 feet. Juan says the difference is 70 feet. Who is correct? Explain how you know.

HOUR	ELEVATION
1	-30
2	-100
3	-65
4	-60



Juan is correct. The distance between -30 and -100 on a number line is 70.

## **G7 U4 Lesson 7**

Understand that the product of a negative number and positive number is negative and explain how signed numbers can be used to represent position and speed.

**G7 U4 Lesson 7 - Students will understand that the product of a negative number and a positive number is negative and explain how signed numbers can be used to represent position and speed.**

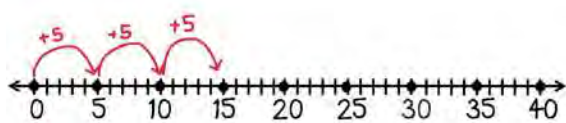
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Up until now, in our work with signed numbers, we've been focusing on addition and subtraction. Today is exciting, because we get to start thinking about what happens when we multiply with signed numbers. When you think about multiplication, what comes to mind? **Possible Student Answers, Key Points:**

- When we multiply we use the x or • symbol.
- We can think of multiplying as equal groups. It's like repeated addition.
- I know an algorithm to multiply big numbers.

We're going to take everything you already know about multiplication and everything you already know about positive and negative numbers, and we'll work to figure out what happens when you take the product of a negative number and a positive number.

**Let's Talk (Slide 3):** Let's warm up by thinking about a pot of water doing just that...warming up! Imagine there is a pot of water. You measure the temperature of the water every minute with a thermometer. The temperature of the water starts at 0 degrees. After 1 minute, you notice the temperature increased 5 degrees. What temperature is it now? (5 degrees)



(label a hop of +5 on the number line) After 1 minute, it's 5 degrees. Another minute goes by, and you notice the temperature increased 5 more degrees. What's the temperature now? (10 degrees) It's 10 degrees. (continue labeling hops on the number line as you narrate) What if after the next minute it increased 5 degrees again? What temperature would it be? (15 degrees) It would be 15 degrees, correct.

$$5 + 5 + 5 = 15$$
$$3 \cdot 5 = 15$$

We can represent this change in temperature using repeated addition. (write  $5 + 5 + 5 = 15$ ) We can also think of this in terms of multiplication. I see three groups of positive 5 on the number line, so I can write that as  $3 \cdot 5 = 15$ . (write equation)

What do you think would be different if instead of increasing, the temperature decreased by 5 degrees each minute? **Possible Student Answers, Key Points:**

- We'd still hop by groups of 5, but in the other direction along the number line.
- I think our answer would be negative. Maybe it would be -15 degrees.

Let's look at some problems to better understand how to multiply a positive number by a negative number.

**Let's Think (Slide 4):** This problem presents a different context. It wants us to show each car's final position on a number line and using an equation. Notice, each car is moving in a different direction along the line.

Let's start by thinking about the red car. I know the red car moves right 6 feet, and I know this happens each second for 2 seconds. How could I represent that on the number line? (sketch and label on number line as student share, supporting as needed) **Possible Student**



**Answers, Key Points:**

- Start at 0, then draw two arrows to the right. Each arrow should be 6 units long.
- Draw an arrow going to the 6, so three tick marks right. Then draw another the same length.

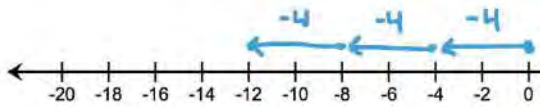
$$6 + 6 = 12$$

$$2 \cdot 6 = 12$$

We can draw two arrows moving right, with each arrow representing 6 feet. I have to be a little careful because the scale on this number line is counting by two. I can think of two equations to represent that. Repeated addition could work. (*write equations as you narrate*) That would look like  $6 + 6 = 12$ . Or, we see two groups of 6 on the number line. I can show that using multiplication by writing  $2 \cdot 6 = 12$ . The red car's final position is positive 12.

What's different about the movement of the blue car? [Possible Student Answers, Key Points:](#)

- It's pointing the other direction.
- It moves 4 feet each time.
- It moves for 3 seconds.



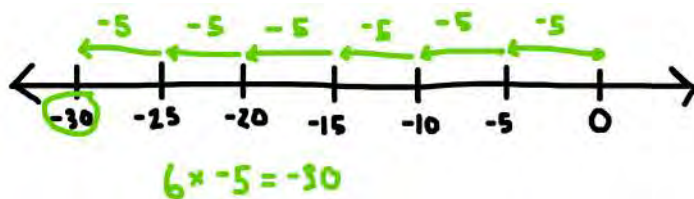
The blue car is moving left, toward the negative numbers on the number line. It moves 4 feet every second, and it does that for 3 seconds. I could represent that by drawing three arrows left, each showing -4. I should keep in mind that the number line is still counting by two. (*sketch and label number line*)

$$(-4) + (-4) + (-4) = -12$$

$$3 \cdot -4 = -12$$

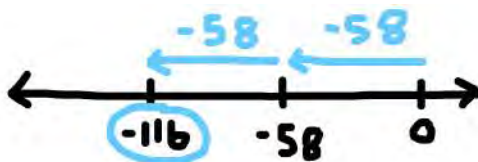
What repeated addition equation could I write to represent this? (*-4 plus -4 plus -4 equals -12*) (*write that*) We can also use multiplication to represent this, even though the groups are negative. It works the same way. I see on the number line that we have 3 groups, and each group is -4. I can think of 3 groups of -4 as being  $3 \cdot -4 = -12$ . We just multiplied a whole number by a negative number using a number line. Nicely done!

**Let's Think (Slide 5):** These three problems don't involve a context. Let's see if we can find the product of each.



For part A, it wants us to find the product of  $6 \cdot -5$ . We can think of that as 6 groups of -5. (*sketch a simple number line from 0 to -30 using a scale of 5*) I'm going to use a scale of 5. I don't need to draw every individual number when I make my own number line. Now, all I have to do is draw 6 groups of -5. I can use six arrows to represent the groups, and I'll make each arrow worth -5. (*draw and label what*

*you described*) Where did we end up on the number line? (*-30*) The product of  $6 \cdot -5$  is -30. (*write multiplication equation*) That makes sense, because I know 6 groups of positive 5 would be 30. So, 6 groups of negative 5 would be negative 30.



For part B, it wants us to find the product of  $-58 \cdot 2$ . Thinking of -58 groups is a little strange, because it's hard to wrap my head around the idea of negative groups. I know we can switch the order of factors in a multiplication problem, though. Let's rewrite  $-58 \cdot 2$  as  $2 \cdot -58$ . Now, I'm just thinking of 2 groups of -58. How could I represent this on a number line? (*sketch and label as student shares*)

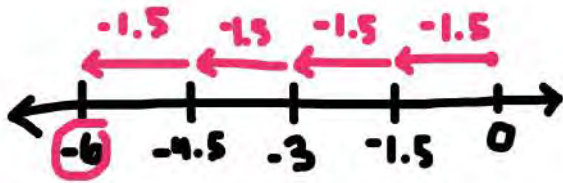
[Possible Student Answers, Key Points:](#)

- Show two arrows going left, since they are negative. Each arrow should represent -58.



$$2 \times -58 = -116$$

We could think of this as  $-58$  plus  $-58$ , or we can use multiplication to think of it as  $2 \cdot -58 = -116$ . Multiplying a positive times a negative number is not too different from multiplying two positive numbers, we just need to be mindful of the signs in our factors and product.



Let's do one more. Part C asks us to find the product of 4 and negative 1.5. How can I think of this equation in terms of a number line? **Possible Student Answers, Key Points:**

- We can think of it as 4 groups of  $-1.5$
- I can show 4 arrows to the left to represent the groups. Each arrow can be  $-1.5$ .
- I can use a scale of 0.5, 1, or 1.5 depending on what I think will be easiest.

$$4 \times -1.5 = -6$$

*(sketch as you narrate)* I'll use a scale of 1.5 to make my number line. If I show four groups of  $-1.5$ , my number line will need 4 arrows each representing negative 1.5. We end up with a product of  $-6$ . So, 4 times  $-1.5$  is equal to  $-6$ . *(write equation)*

How is multiplying a positive and negative number the same as and different than multiplying a positive and a positive number? **Possible Student Answers, Key Points:**

- When multiplying a positive by a negative, your arrows go left. Your answer ends up being negative.
- We can think of both situations as being \_\_\_ groups of \_\_\_\_\_. We can use arrows to represent the equal groups.
- We kind of use the same facts, just the sign changes.  $4 \times 2 = 8$  and  $4 \times -2 = -8$ , for example.

Great thinking.

**Let's Try it (Slides 6 - 7):** Now let's practice a little more together before you try some on your own. As we work, we'll think about each multiplication problem as being equal groups of a given quantity. If we need to rewrite our multiplication in a different order, we absolutely can. Number lines, repeated addition, and multiplication equations can all help us think about a problem and represent our thinking. Let's give it a try.



# WARM WELCOME



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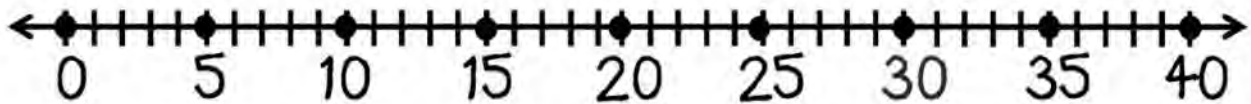
**Today we will understand that the product of a negative number and a positive number is negative and explain how signed numbers can be used to represent position and speed.**

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## Let's Talk:

**The thermometer in a pot of water measures 0 degrees.**

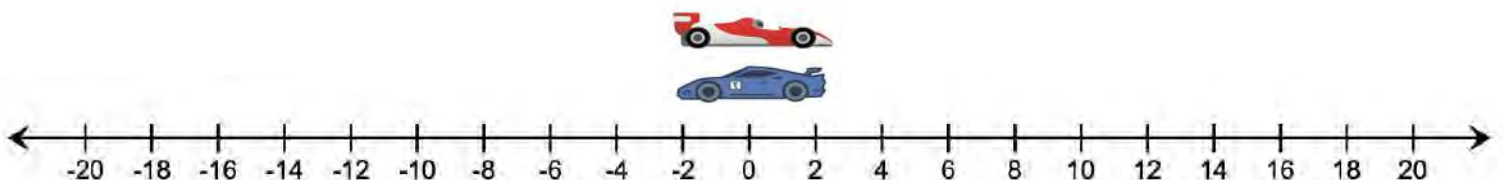
- After 1 minute, the temperature increases 5 degrees.
- After another minute, the temperature increases 5 degrees.
- After another minute, the temperature increases 5 degrees.



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## Let's Think:

The **red car** moves 6 feet right every second for 2 seconds. The **blue car** moves 4 feet left every second for 3 seconds. Write equations to show each car's final position on the number line.



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## Let's Think:

Sketch a number line to find each product.

a.  $6 \cdot -5$

b.  $-58 \cdot 2$

c.  $4 \cdot -1.5$

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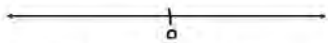
## Let's Try It:

Let's explore understanding that the product of a negative number and a positive number is negative together.

Name: \_\_\_\_\_ G7 U4 Lesson 7 – Let's Try It

**Alicia is keeping track of temperatures for a science project.**

1. The temperature at 8:00AM was 0 degrees. It increased at a constant rate of 5 degrees per hour for the next 3 hours. Draw and label arrows to represent the change each hour.



2. The number line shows \_\_\_\_\_ groups of \_\_\_\_\_ degrees.

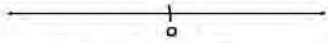
3. Represent the change in temperature using a repeated addition expression.

4. Represent the change in temperature using a multiplication expression.

5. What is the temperature after 3 hours? \_\_\_\_\_ degrees

**Alicia's friend Carlos lives in a different state, but is also keeping track of temperatures to help Alicia with her science project. He records the temperature as 0 degrees one evening.**

6. Carlos notices the temperature drops 2 degrees every hour for the next 4 hours. Draw and label arrows to represent the change each hour.



7. The number line shows \_\_\_\_\_ groups of \_\_\_\_\_ degrees.

8. Represent the change in temperature using a repeated addition expression and a multiplication expression.

9. What is the temperature after 4 hours? \_\_\_\_\_ degrees

**Fill in the blanks.**

10. A positive number multiplied by a positive number will always result in a \_\_\_\_\_ number, because we're repeatedly counting right along the number line.

11. A positive number multiplied by a negative number will always result in a \_\_\_\_\_ number, because we're repeatedly counting left along the number line.

**Consider the expression  $-3 \cdot 7$ .**

12. Rewrite the expression using the commutative property.

13. Now, we can think of this as \_\_\_\_\_ groups of \_\_\_\_\_.

14. Sketch a number line to represent this relationship.

15. Write and solve a corresponding repeated addition equation and a multiplication equation.

**For each expression below, find the value using any strategy or representation. Show your thinking.**

16.  $-8 \cdot 2$

17.  $9 \cdot -1.5$

18.  $-22 \cdot 6$

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## On your Own:

Now it's time to explore understanding that the product of a negative number and a positive number is negative on your own.

Name: \_\_\_\_\_ 07 U4 Lesson 7 – Independent Work

1. For each expression sketch a number line to find the product, write a repeated addition expression, and write a multiplication expression.

a.  $4 \times -6$

b.  $-2 \times 7$

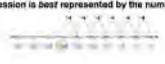
c. Choose one of the problems to write a story problem that could be solved using the expression.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

2. What multiplication expression is best represented by the number line model?



a.  $4 \times 6$   
b.  $-4 \times 6$   
c.  $-6 \times 4$   
d.  $-6 \times -4$

EXPLAIN:

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

3. A turtle is swimming at sea level. The turtle begins to dive 7 meters every minute for 5 minutes. Represent the turtle's dive with a number line, repeated addition, and multiplication.

NUMBER LINE:

\_\_\_\_\_

REPEATED ADDITION:

\_\_\_\_\_

MULTIPLICATION:

\_\_\_\_\_

The turtle's elevation is \_\_\_\_\_ meters after 5 minutes.

4. Find each value using any strategy.

$-5 \times -16$

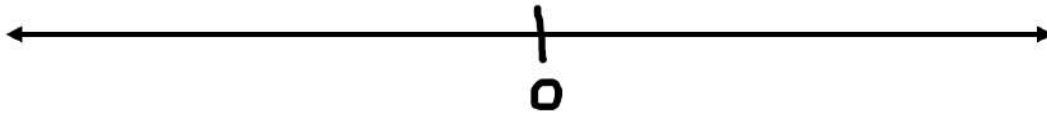
$-23 \times -5$

$-27 \times 9$

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**Alicia is keeping track of temperatures for a science project.**

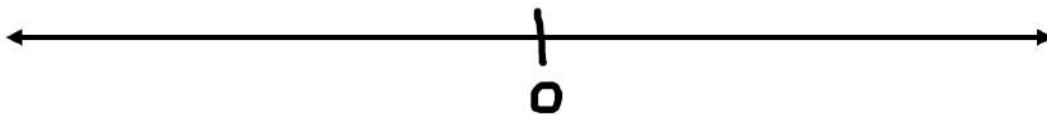
1. The temperature at 8:00AM was 0 degrees. It increased at a constant rate of 5 degrees per hour for the next 3 hours. Draw and label arrows to represent the change each hour.



2. The number line shows \_\_\_\_\_ groups of \_\_\_\_\_ degrees.
3. Represent the change in temperature using a repeated addition expression.
4. Represent the change in temperature using a multiplication expression.
5. What is the temperature after 3 hours? \_\_\_\_\_ degrees

**Alicia's friend Carlos lives in a different state, but is also keeping track of temperatures to help Alicia with her science project. He records the temperature as 0 degrees one evening.**

6. Carlos notices the temperature drops 2 degrees every hour for the next 4 hours. Draw and label arrows to represent the change each hour.



7. The number line shows \_\_\_\_\_ groups of \_\_\_\_\_ degrees.
8. Represent the change in temperature using a repeated addition expression and a multiplication expression.
9. What is the temperature after 4 hours? \_\_\_\_\_ degrees

**Fill in the blanks.**

10. A positive number multiplied by a positive number will always result in a \_\_\_\_\_ number, because we're repeatedly counting right along the number line.
11. A positive number multiplied by a negative number will always result in a \_\_\_\_\_ number, because we're repeatedly counting left along the number line.

**Consider the expression  $-3 \cdot 7$ .**

12. Rewrite the expression using the commutative property.
13. Now, we can think of this as \_\_\_\_\_ groups of \_\_\_\_\_.
14. Sketch a number line to represent this relationship.
15. Write and solve a corresponding repeated addition equation and a multiplication equation.

**For each expression below, find the value using any strategy or representation. Show your thinking.**

16.  $-8 \cdot 2$

17.  $9 \cdot -1.2$

18.  $-22 \cdot 6$

1. For each expression sketch a number line to find the product, write a repeated addition expression, and write a multiplication expression.

a.  $4 \cdot -9$

b.  $-2 \times 7$

c. Choose one of the problems to write a story problem that could be solved using the expression.

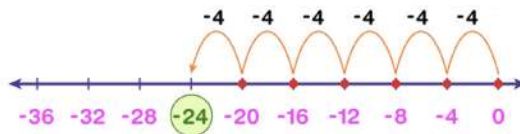
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2. What multiplication expression is *best* represented by the number line model?



- a.  $4 \times 6$
- b.  $-4 \times 6$
- c.  $-6 \times 4$
- d.  $-6 \times -4$

**EXPLAIN:**

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**3. A turtle is swimming at sea level. The turtle begins to dive 7 meters every minute for 5 minutes.** Represent the turtle's dive with a number line, repeated addition, and multiplication.

NUMBER LINE:

REPEATED ADDITION:

MULTIPLICATION:

The turtle's elevation is \_\_\_\_\_ meters after 5 minutes.

**4. Find each value using any strategy.**

$$5 \cdot -16$$

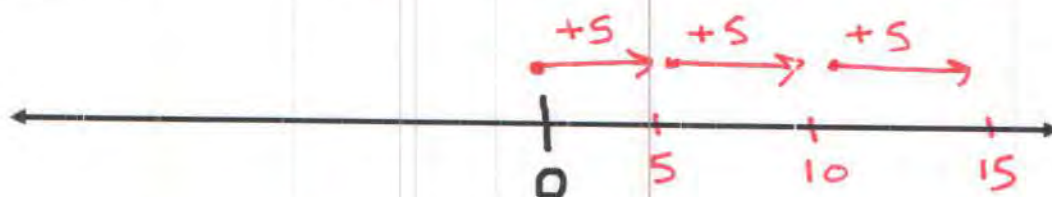
$$23 \cdot -5$$

$$-27 \cdot 9$$

Name: KEY

Alicia is keeping track of temperatures for a science project.

1. The temperature at 8:00AM was 0 degrees. It increased at a constant rate of 5 degrees per hour for the next 3 hours. Draw and label arrows to represent the change each hour.



2. The number line shows 3 groups of 5 degrees.
3. Represent the change in temperature using a repeated addition expression.

$$5 + 5 + 5$$

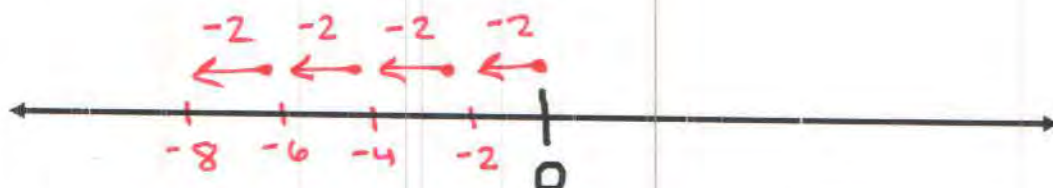
4. Represent the change in temperature using a multiplication expression.

$$3 \times 5$$

5. What is the temperature after 3 hours? (15) degrees

Alicia's friend Carlos lives in a different state, but is also keeping track of temperatures to help Alicia with her science project. He records the temperature as 0 degrees one evening.

6. Carlos notices the temperature drops 2 degrees every hour for the next 4 hours. Draw and label arrows to represent the change each hour.



7. The number line shows 4 groups of -2 degrees.
8. Represent the change in temperature using a repeated addition expression and a multiplication expression.

$$-2 + -2 + -2 + -2$$

$$4 \times -2$$

9. What is the temperature after 4 hours? (-8) degrees

Fill in the blanks.

10. A positive number multiplied by a positive number will always result in a positive number, because we're repeatedly counting right along the number line.
11. A positive number multiplied by a negative number will always result in a negative number, because we're repeatedly counting left along the number line.

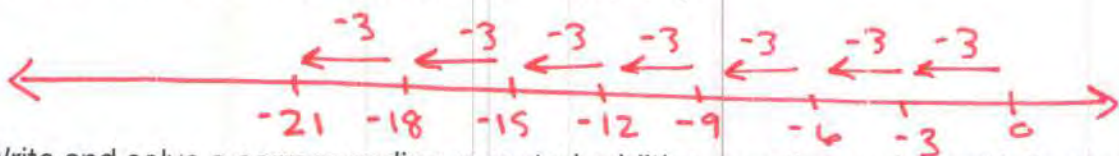
Consider the expression  $-3 \cdot 7$ .

12. Rewrite the expression using the commutative property.

$$7 \cdot -3$$

13. Now, we can think of this as 7 groups of -3.

14. Sketch a number line to represent this relationship.



15. Write and solve a corresponding repeated addition equation and a multiplication equation.

$$-3 + -3 + -3 + -3 + -3 + -3 + -3 = -21 \quad 7 \cdot -3 = -21$$

For each expression below, find the value using any strategy or representation. Show your thinking.

16.  $-8 \cdot 2$

$$2 \cdot -8 = (-16)$$

17.  $9 \cdot -1.2$

$$(-10.8)$$

$$12 \times 9 = 108$$
$$1.2 \times 9 = 10.8$$

18.  $-22 \cdot 6$

$$6 \cdot -22$$
$$(-132)$$

$$\begin{array}{r} 22 \\ \times 6 \\ \hline 132 \end{array}$$



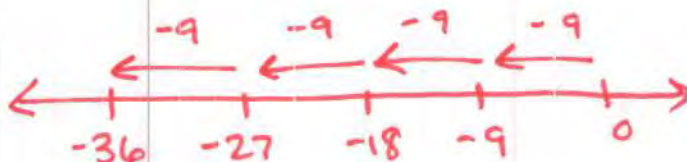
Name: KEY

1. For each expression sketch a number line to find the product, write a repeated addition expression, and write a multiplication expression.

a.  $4 \cdot -9$

$$-9 + -9 + -9 + -9 = -36$$

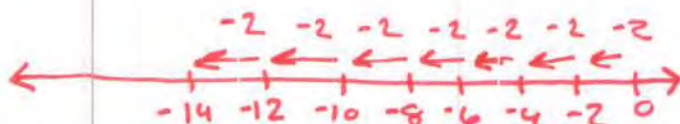
$$4 \cdot -9 = -36$$



b.  $-2 \times 7$

$$7 \times -2 = -14$$

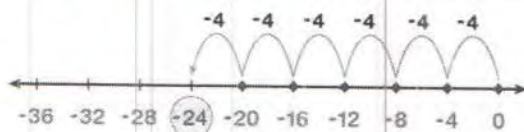
$$-2 + -2 + -2 + -2 + -2 + -2 + -2$$



c. Choose one of the problems to write a story problem that could be solved using the expression.

The temperature starts at  $0^\circ$  and drops 2 degrees every hour for 7 hours. What is the temperature now?

2. What multiplication expression is best represented by the number line model?



a.  $4 \times 6$

b.  $-4 \times 6$

c.  $-6 \times 4$

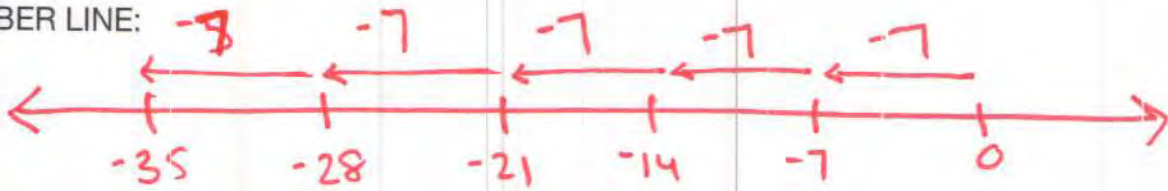
d.  $-6 \times -4$

EXPLAIN:

I see 6 groups of -4 which I can write as  $6 \times -4$  or  $-4 \times 6$ .

3. A turtle is swimming at sea level. The turtle begins to dive 7 meters every minute for 5 minutes. Represent the turtle's dive with a number line, repeated addition, and multiplication.

NUMBER LINE:



REPEATED ADDITION:

$$-7 + -7 + -7 + -7 + -7 = ?$$

MULTIPLICATION:

$$-7 \times 5 = ?$$

The turtle's elevation is -35 meters after 5 minutes.

4. Find each value using any strategy.

$$\begin{array}{r} 16 \\ \times 5 \\ \hline 80 \end{array}$$

$$5 \cdot -16$$

$$(-80)$$

$$\begin{array}{r} 23 \\ \times 5 \\ \hline 115 \end{array}$$

$$23 \cdot -5$$

$$(-115)$$

$$\begin{array}{r} 27 \\ \times 9 \\ \hline 243 \end{array}$$

$$-27 \cdot 9$$

$$(-243)$$

## **G7 U4 Lesson 8**

Interpret signed numbers when used to represent time in situations about speed and direction as well as understand that the product of two negative numbers is positive.

**G7 U4 Lesson 8 - Students will interpret signed numbers when used to represent time in situations about speed and direction as well as understand that the product of two negative numbers is positive.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** You've known how to multiply positive numbers by positive numbers for many years. When we were last together, we explored how how to multiply a positive number by a negative number. What do you remember about how multiplying positive numbers is similar to and different from multiplying a positive number by a negative number? [Possible Student Answers, Key Points:](#)

- No matter the signs on the numbers, we can think of multiplying as equal groups or repeated addition.
- When we model multiplying a positive number by a negative number, we draw arrows to the left rather than the right.
- Two positive numbers result in a product that is positive. When we multiply a positive number by a negative number, the product is negative.

Those are excellent points. Today, we'll explore what happens when we multiply a negative value by another negative value. Do you have any predictions as to what will happen? [Possible Student Answers, Key Points:](#)

- I think it will be negative, since the two factors are also negative.
- I think it will be positive, because a negative times a positive is negative. This must be different.
- I'm not sure. It's hard to think about two negative numbers as being equal groups of a value.

Let's find out...

**Let's Talk (Slide 3):** Before we dive in, let's practice some multiplication. Feel free to use the number line to help you solve or explain your thinking.

What would be the product of  $2 \cdot 5$ ? **(10)** Correct.  $2 \cdot 5 = 10$ . A positive number times a positive number is always positive.

What would be the product of  $2 \cdot -5$ ? **(-10)**  $2 \cdot -5$  can be thought of as two groups of -5. The product is -10. The product of a positive and a negative number is always negative.

What would be the product of  $-2 \cdot 5$ ? **(-10)** We can reorder this expression, if that helps. If I think of it as  $5 \cdot -2$ , I can picture 5 hops of -2 on the number line. The product is -10. Again, the product of a negative and a positive number is a negative number.

The last example is a negative number times a negative number. We haven't learned that yet, so let's come back to this one.

**Let's Think (Slide 4):** Before we jump straight into multiplying two negative numbers together, let's consider this expression. We'll first think about it by evaluating what is in parentheses first. Then we'll do it again, but we'll distribute the -4 first. By the end of this process, we'll know how to think about the product of two negative numbers.

$$\begin{array}{l} -4 \cdot (7 + (-2)) \\ -4 \cdot 5 \\ -20 \end{array}$$

Let's begin by evaluating what is in parentheses first. What is  $7 + (-2)$ , and how do you know? [Possible Student Answers, Key Points:](#)

- I can think of  $7 + (-2)$  as  $7 - 2$ . 7 minus 2 is 5.
- I can picture a number line. If I plot a point at 7 and add -2 with an arrow going left 2 spaces, I'll end up at -5.



7 plus -2 is 5. (rewrite expression as  $-4 \cdot 5$ ) Now all we have left to do is multiply  $-4 \cdot 5$ . We can reorder the factors and think of it as  $5 \cdot -4$ , or 5 groups of -4. What is the value of 5 groups of -4? (-20) The value of the expression is -20. Keep that in mind, as we evaluate the expression another way.

$$-4 \cdot (7 + (-2))$$

$$(-4 \cdot 7) + (-4 \cdot -2)$$

$$-28 + ?$$

Let's use the distributive property first this time. We can distribute the -4 to each term in parentheses. Let's rewrite the expression. (rewrite as  $(-4 \cdot 7) + (-4 \cdot -2)$ ) I know  $-4 \cdot 7$  is -28, because I can think of it as being 7 groups of -4. We don't yet know how to multiply -4 times -2, so I'll write a question mark for that product. (write  $-28 + ?$ )

$$-28 + ? = -20$$

We already simplified the expression, so we know it has to equal -20. (write  $-28 + ? = -20$ ) So, based on this, does  $-4 \cdot -2$  equal positive or negative 8? How do you know? Possible Student Answers, Key Points:

- It can't equal -8, because  $-28 + (-8)$  would be -36.
- It has to equal +8. I know  $-28 + 8$  is -20.

Negative 4 times negative 2 must be positive 8. This example helps us see that the product of two negative numbers is always positive. Let's keep that in mind as we find a few more products.

**Let's Think (Slide 5):** Part A wants us to find the product of -9 and -14.

$$\begin{array}{r} 14 \\ \times 9 \\ \hline 126 \end{array}$$

$$-9 \times -14 = 126$$

We just learned that two negative numbers multiplied together will result in a positive product. So will this product be positive or negative? (positive) Let's multiply 14 by 9 without thinking about the sign, then we'll make sure to note that our product is positive. (write  $14 \times 9$  in vertical form as you narrate) I know  $4 \times 9$  is 36. I'll write a 6 in the ones place, then regroup 3 tens. 1 ten  $\times 9$  is 9 tens. 9 tens plus the 3 tens we regrouped, means we have 12 tens. The product of 14 and 9 is 126.

The product of  $-9 \times -14$  is positive 126. (write equation horizontally)

$$\frac{1}{2} \times 11 = \frac{11}{2} \text{ or } 5\frac{1}{2}$$

$$-\frac{1}{2} \times 11 = -5\frac{1}{2}$$

Let's try the next one. It wants us to find the product of  $-\frac{1}{2} \times 11$ . What do we know about the product of a negative number and a positive number? (It will be negative.) We know the product will be negative, so let's multiply the numbers without worrying about the signs. How can I multiply  $\frac{1}{2}$  by 11? (write equation as student explains, supporting as needed) Possible Student Answers, Key Points:

- One half times 11 will be 11 halves, or  $11/2$ .  $11/2$  is equal to  $5\frac{1}{2}$ .
- You can multiply the 1 in the numerator by 11 to get 11. The denominator will stay 2. The answer is  $11/2$  or  $5\frac{1}{2}$ .

We know  $\frac{1}{2} \times 11$  is equal to  $11/2$  or  $5\frac{1}{2}$ . We also know that a negative value times a positive value equals a negative value. So  $-\frac{1}{2} \times 11 = -5\frac{1}{2}$ . (write equation)

$$-7 \times -0.8 = 5.6$$

The last example wants us to find the product of -7 and -0.8, or negative 8 tenths. I know  $7 \times 8$  tenth is 56 tenths or 5.6. What will be the sign on the product? (A negative times a negative is a positive.) Negative 7 times negative 0.8 is equal to positive 5.6. (write equation)

No matter if our numbers are whole numbers, decimals, or fractions, the same multiplication rules apply.

$$\begin{array}{l}
 + \bullet + = + \\
 - \bullet + = - \\
 + \bullet - = - \\
 - \bullet - = +
 \end{array}$$

(write out symbols as shown as you summarize the multiplication rules) We know that two positive numbers multiplied together results in a positive product. A negative times a positive results in a negative number. A positive times a negative results in a negative number. And we just learned that a negative times a negative results in a positive number. We can use these rules to help us multiply any two numbers moving forward.

Earlier in our time together we looked at the expression  $-2 \bullet -5$ . At the time, we weren't able to find the value. Now we can! What is the value of  $-2 \bullet -5$ ? **(-10)** Great job!

**Let's Try it (Slides 6 - 7):** Now let's practice with some more problems. We must pay careful attention to the sign on each of our factors in order to carefully arrive at the correct product. We know two positive numbers will result in a positive product. Any combination of a positive and negative number will result in a negative product. And today we learned that two negative numbers will result in a positive product. Let's use everything we've learned to carefully respond to the next few questions.

# WARM WELCOME



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**Today we will understand that the product of two negative numbers is positive.**

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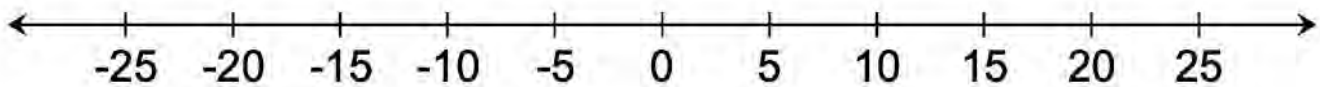
## Let's Talk:

$2 \cdot 5$

$2 \cdot -5$

$-2 \cdot 5$

$-2 \cdot -5$



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## Let's Think:

**Evaluate the expression by evaluating what is in parentheses first.**

$-4 \cdot (7 + (-2))$

**Evaluate the expression again using the distributive property.**

The product of two negative numbers is always \_\_\_\_\_.

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## Let's Think:

Find each product.

a.  $-9 \cdot -14$

b.  $-\frac{1}{2} \cdot 11$

c.  $-7 \cdot -0.8$

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## Let's Try It:

Let's explore understanding that the product of two negative numbers is positive together.

Name: \_\_\_\_\_ C37 U4 Lesson 8 - Let's Try It

Find each product. Sketch a number line, if that's helpful.

- $4 \cdot 3 =$  \_\_\_\_\_
- $4 \cdot -3 =$  \_\_\_\_\_
- $-4 \cdot 3 =$  \_\_\_\_\_

Consider the expression  $-3 \cdot (6 + (-4))$ .

- Find the value by evaluating what is in parentheses first.  

$$-3 \cdot (6 + (-4))$$
- Fill in the blanks to show how this expression can be evaluated using the distributive property.  

$$-3 \cdot (6 + (-4))$$

$$-3 \cdot \underline{\quad} + 3 \cdot \underline{\quad} =$$

$$\underline{\quad} + ? =$$
- To make sure the value of the expression matches the value you found in #4, what must be the value of  $-3 \cdot -4$ ?
- The product of a **negative** number and a **negative** number is always \_\_\_\_\_.

Find the value of each expression.

- $-4 \cdot -8$
- $4 \cdot -8$
- $-9 \cdot -2$
- $-9 \cdot 2$
- $-7 \cdot 5$
- $-7 \cdot -5$

Find the value of each expression.

- $\frac{1}{10} \cdot -30$
- $-\frac{1}{10} \cdot 30$
- $-\frac{1}{10} \cdot -30$
- $-\frac{1}{10} \cdot 30$

Summarize what we have learned so far about the product of signed numbers.

- POSITIVE  $\times$  POSITIVE = \_\_\_\_\_
- NEGATIVE  $\times$  POSITIVE = \_\_\_\_\_
- POSITIVE  $\times$  NEGATIVE = \_\_\_\_\_
- NEGATIVE  $\times$  NEGATIVE = \_\_\_\_\_

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## On your Own:

Now it's time to explore understanding that the product of two negative numbers is positive on your own.

Name: \_\_\_\_\_ G7.04 Lesson 8 – Independent Work

1. Find the value of each expression.

a.  $6 \cdot 6$

b.  $-6 \cdot 6$

c.  $-6 \cdot 0$

d.  $0 \cdot 6$

e.  $6 \cdot -6$

2. Determine whether each equation is TRUE or FALSE. If it's false, correct it.

$-4 \cdot -4 = -16$      $-3 \cdot 3 = 9$      $-6 \cdot -7 = -42$      $18 = 9 \cdot -3$

3. Find each solution.

a.  $\frac{1}{2} \times 14 =$

b.  $-\frac{1}{2} \times -14 =$

c.  $\frac{1}{2} \times -14 =$

d.  $2.4 \times -3 =$

e.  $-2.4 \times 3 =$

f.  $-2.4 \times -3 =$

4. Fill in the missing numbers in each equation.

a.  $(-2) \cdot (-49) = 7$

b.  $(-87) \cdot (-10) = 7$

c.  $(-7) \cdot 7 = 91$

d.  $? \cdot (-12) = 156$

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Find each product. Sketch a number line, if that's helpful.


1.  $4 \cdot 3 = \underline{\hspace{2cm}}$

2.  $4 \cdot -3 = \underline{\hspace{2cm}}$


3.  $-4 \cdot 3 = \underline{\hspace{2cm}}$

Consider the expression  $-3 \cdot (6 + (-4))$ .

4. Find the value by evaluating what is in parentheses first.

$$-3 \cdot (6 + (-4))$$


5. Fill in the blanks to show how this expression can be evaluated using the distributive property.

$$-3 \cdot (6 + (-4))$$


$$-3 \cdot \underline{\hspace{1cm}} + -3 \cdot \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{1cm}} + ? = \underline{\hspace{1cm}}$$

6. To make sure the value of the expression matches the value you found in #4, what must be the value of  $-3 \cdot -4$ ?
7. The product of a **negative** number and a **negative** number is always \_\_\_\_\_.



**Find the value of each expression.**

8.  $-4 \cdot -8$

9.  $4 \cdot -8$

10.  $-9 \cdot -2$

11.  $-9 \cdot 2$

12.  $-7 \cdot 5$

13.  $-7 \cdot -5$

**Find the value of each expression.**

14.  $-\frac{1}{10} \cdot -30$

15.  $\frac{1}{10} \cdot 30$

16.  $\frac{1}{10} \cdot -30$

17.  $-\frac{1}{10} \cdot 30$

**Summarize what we have learned so far about the product of signed numbers.**

18. POSITIVE x POSITIVE = \_\_\_\_\_

19. NEGATIVE x POSITIVE = \_\_\_\_\_

20. POSITIVE x NEGATIVE = \_\_\_\_\_

21. NEGATIVE x NEGATIVE = \_\_\_\_\_

**1. Find the value of each expression.**

a.  $6 \cdot 6$

b.  $-6 \cdot 6$

c.  $-6 \cdot -6$

d.  $6 \cdot -6$

**2. Determine whether each equation is TRUE or FALSE. If it's false, correct it.**

$4 \cdot -4 = =16$

$-3 \cdot 3 = 9$

$-6 \cdot -7 = -42$

$18 = -9 \cdot -3$

**3. Find each solution.**

a.  $\frac{1}{2} \times 14 =$

b.  $-\frac{1}{2} \cdot -14 =$

c.  $\frac{1}{2} \cdot -14 =$

d.  $2.4 \times -3 =$

e.  $-2.4 \times 3 =$

f.  $-2.4 \times -3 =$

**4. Fill in the missing numbers in each equation.**

a.  $(-2) \cdot (-45) = ?$

b.  $(-87) \cdot (-10) = ?$

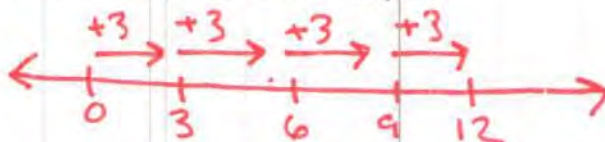
c.  $(-7) \cdot ? = 91$

d.  $? \cdot (-12) = 156$

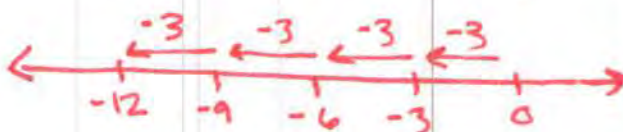
Name: KEY

Find each product. Sketch a number line, if that's helpful.

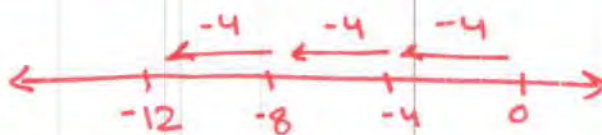
1.  $4 \cdot 3 = \underline{12}$



2.  $4 \cdot -3 = \underline{-12}$



3.  $-4 \cdot 3 = \underline{-12}$   
 $3 \cdot -4$



Consider the expression  $-3 \cdot (6 + (-4))$ .

4. Find the value by evaluating what is in parentheses first.

$$-3 \cdot (6 + (-4))$$

$$-3 \cdot (2)$$

$$\underline{-6}$$

5. Fill in the blanks to show how this expression can be evaluated using the distributive property.

$$-3 \cdot (6 + (-4))$$

$$-3 \cdot \underline{6} + -3 \cdot \underline{-4} = \underline{-6}$$

$$\underline{-18} + ? = \underline{-6}$$

6. To make sure the value of the expression matches the value you found in #4, what must be the value of  $-3 \cdot -4$ ?

$$\underline{+12}$$

7. The product of a **negative** number and a **negative** number is always positive.

Find the value of each expression.

8.  $-4 \cdot -8$  (32)

9.  $4 \cdot -8$  (-32)

10.  $-9 \cdot -2$  (18)

11.  $-9 \cdot 2$  (-18)

12.  $-7 \cdot 5$  (-35)

13.  $-7 \cdot -5$  (35)

Find the value of each expression.

14.  $-\frac{1}{10} \cdot -30$   $\frac{-30}{-10} =$  (3)

15.  $\frac{1}{10} \cdot 30$   $\frac{30}{10} =$  (3)

16.  $\frac{1}{10} \cdot -30$   $\frac{-30}{10} =$  (-3)

17.  $-\frac{1}{10} \cdot 30$   $-\frac{30}{10} =$  (-3)

Summarize what we have learned so far about the product of signed numbers.

18. POSITIVE x POSITIVE = positive

19. NEGATIVE x POSITIVE = negative

20. POSITIVE x NEGATIVE = negative

21. NEGATIVE x NEGATIVE = positive

Name: \_\_\_\_\_

KEY

G7 U4 Lesson 8 - Independent Work

1. Find the value of each expression.

a.  $6 \cdot 6$

(36)

b.  $-6 \cdot 6$

(-36)

c.  $-6 \cdot -6$

(36)

d.  $6 \cdot -6$

(-36)

2. Determine whether each equation is TRUE or FALSE. If it's false, correct it.

$4 \cdot -4 = 16$

FALSE

$4 \cdot -4 = -16$

$-3 \cdot 3 = 9$

FALSE

$-3 \cdot 3 = -9$

$-6 \cdot -7 = -42$

FALSE

$-6 \cdot -7 = 42$

$18 = -9 \cdot -3$

TRUE

3. Find each solution.

a.  $\frac{1}{2} \times 14 = \frac{14}{2} = 7$

b.  $-\frac{1}{2} \cdot -14 = \frac{-14}{-2} = 7$

c.  $\frac{1}{2} \cdot -14 = \frac{-14}{2} = -7$

d.  $2.4 \times -3 = -7.2$

e.  $-2.4 \times 3 = -7.2$

f.  $-2.4 \times -3 = 7.2$

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array} \rightarrow \begin{array}{r} 2.4 \\ \times 3 \\ \hline 7.2 \end{array}$$

4. Fill in the missing numbers in each equation.

a.  $(-2) \cdot (-45) = ?$   $90$

b.  $(-87) \cdot (-10) = ?$   $870$

c.  $(-7) \cdot ? = 91$

$-13$

d.  $? \cdot (-12) = 156$

$-13$

$$\begin{array}{r} 13 \\ 7 \overline{) 91} \\ \underline{-70} \\ 21 \\ \underline{-21} \\ 0 \end{array}$$

$$\begin{array}{r} 13 \\ 12 \overline{) 156} \\ \underline{-120} \\ 36 \\ \underline{-36} \\ 0 \end{array}$$



## **G7 U4 Lesson 9**

Use the relationship between multiplication and division to develop the rules for dividing rational numbers.

**G7 U4 Lesson 9 - Students will use the relationship between multiplication and division to develop rules for dividing rational numbers.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we spent time learning about multiplication with signed numbers. We already knew that when we multiply two positive numbers together, the product is positive. Then we learned that when we multiply a positive number by a negative number, the product was negative. We also learned that when we multiply two negative numbers together, the product is positive. Today, we'll focus on dividing with signed numbers. What do you predict will be the same or different when we divide signed numbers compared to when we multiply signed numbers? **Possible Student Answers, Key Points:**

- Maybe the rules will be the same, since multiplication and division are related.
- Maybe the rules will be different, because division is the opposite of multiplication.

Let's use what we know about multiplication to help us think about division.

**Let's Talk (Slide 3):** Let's think. How are multiplication and division related? Consider using the multiplication problem here to help explain what you mean. **Possible Student Answers, Key Points:**

- Multiplication and division are opposites.
- Multiplication and division both involve equal groups.
- If I know  $5 \times 2 = 10$ , then I know  $10 \div 2 = 5$  and  $10 \div 5 = 2$ .

$$\begin{aligned} 5 \times 2 &= 10 \\ 10 \div 5 &= 2 \\ 10 \div 2 &= 5 \end{aligned}$$

(write each equation as you narrate) Multiplication and division are opposites. So, if I know positive 5 times positive 2 is positive 10, I can use that to help me think about division. I can write related division facts that represent this same relationship. I know positive 10 divided by positive 5 is positive 2. I also know positive 10 divided by positive 2 is positive 5. We're going to use this thinking to help us think about the sign of numbers in a few more problems.

**Let's Think (Slide 4):** Take a look at the three division equations here. We haven't yet learned the rules for dividing with signed numbers, so let's use what we know about multiplication to help us find the unknown values in each equation.

$$\begin{aligned} 12 \times ? &= -36 \\ ? &= -3 \end{aligned}$$

Part A wants us to find negative 36 divided by positive 12. I can think of a related multiplication fact to help me. I'll think of this as  $12 \times ? = -36$ . (write equation) Now I can think about a number I can multiply by 12 to make -36. Does it make more sense for my unknown to be +3 or -3? **Possible Student Answers, Key Points:**

- The answer should be -3, because a positive times a negative would result in a negative.

A negative number divided by a positive number is a negative number.

A positive number divided by a negative number is a negative number.

A negative number divided by a negative number is a positive number.

Positive 12 times *negative* 3 would result in negative 36. The value of the unknown is -3. Based on this, we know that a negative number divided by a positive number, is a negative number. (fill in first blank)

$$\begin{aligned} -2 \times ? &= 16 \\ ? &= -8 \end{aligned}$$

The next problem, part B, wants us to find the quotient of positive 16 divided by -2. We haven't divided a positive dividend by a negative divisor yet, so let's think about a related multiplication fact. What related multiplication fact can I consider to help us think about this division problem? ( $-2 \times ? = 16$  or  $? \times -2 = 16$ ) (write equation)

Now I can think of  $-2$  times *something* results in positive 16. The unknown must be  $-8$ , not  $+8$ , because  $-2 \times -8$  would result in positive 16.

A negative number divided by a positive number is a negative number.

A positive number divided by a negative number is a negative number.

A negative number divided by a negative number is a positive number.

Based on this, we know that a positive number divided by a negative number is a negative number. (fill in second blank)

$$-5 \times ? = -25$$
$$? = 5$$

Let's try one more. Part C wants us to find the quotient of  $-25$  divided by  $-5$ . We haven't divided a negative dividend by a negative divisor before, so let's use multiplication to help us. I can write a related multiplication fact of  $-5 \times ? = -25$ . How can I use this fact to help me think about the unknown value? Possible Student Answers, Key Points:

- I know  $5 \times 5 = 25$ . So I can narrow the answer to our problem down to either 5 or  $-5$ .
- I know a negative times a negative is a positive, and a negative times a positive is a negative. That means, our unknown must be positive.

A negative number divided by a positive number is a negative number.

A positive number divided by a negative number is a negative number.

A negative number divided by a negative number is a positive number.

$-5$  times  $+5$  will equal  $-25$ . Our unknown has a value of  $+5$ . Based on this, we know that a negative number divided by a negative number is a positive number. (fill in third blank)

**Let's Think (Slide 5):** Let's use what we just found out to solve a few more division equations. What do you notice about the numbers in these problems? Possible Student Answers, Key Points:

- Some are positive, and some are negative.
- I see some decimals and some fractions.
- I notice that the first one has a bigger dividend than divisor.

$$-12 \div -24 = \left( \frac{12}{24} \text{ OR } \frac{1}{2} \right)$$

Whether we divide with whole numbers, fractions, or decimal numbers, the rules about signs stay the same. Let's look at Part A. It wants us to divide negative 12 by negative 24. Let's think about the problem without signs first. What is 12 divided by 24? ( $12/24$  or  $1/2$ ) It's  $12/24$  or  $1/2$ . Now, let's go back and think about the signs. A negative number divided by a negative number results in a positive number. Therefore our answer will be positive  $12/24$  or positive  $1/2$ . (write equation)

Look at Part B. Let's divide as if the numbers are just normal, positive numbers. Then we'll worry about the signs. How can I think about 3.6 divided by 4? Possible Student Answers, Key Points:

- I know 3.6 is 36 tenths. I can think 36 tenths divided by 4 is 9 tenths or 0.9.
- I can set up long division. I know 9 goes into 36 four times. So I can write 4 in my quotient, making sure my decimal point is in the correct place. 0.4 is the quotient.

$$-3.6 \div 4 = (-0.9)$$

3.6 divided by 4 is 0.9. Now, let's remember that the problem is a negative number divided by a positive number. Our answer should be *negative* 9 tenths, or  $-0.9$ . Even though we're working with decimal numbers, that doesn't change how we think about the signs of our numbers. (write equation)

Part C is our last one. It wants us to divide  $-1/4$  by 5. We can divide without thinking about the sign, then think about the sign once we have our numeric quotient. How can I think about  $1/4$  divided by 5? Possible Student Answers, Key Points:

- I can draw a model showing  $1/4$  and split it 5 ways. The answer is  $1/20$ .
- I can think of splitting  $1/4$  of a whole five ways.  $1/4$  divided by 5 is  $1/20$ .

$$-\frac{1}{4} \div 5 = \left(-\frac{1}{20}\right)$$

Great,  $\frac{1}{4}$  divided by 5 is  $\frac{1}{20}$ . What sign should our answer have, and how do you know? (write equation) Possible Student Answers, Key Points:

- Our answer should be  $-\frac{1}{20}$ . A negative divided by a positive is a negative.

$$- \div + = -$$

$$- \div - = +$$

$$+ \div - = -$$

$$+ \div + = +$$

Let's summarize our rules for dividing with signed numbers. (write symbols as shown as you prompt the student) A negative number divided by a positive number results in a...(negative number). A negative number divided by a negative number results in a...(positive number). A positive number divided by a negative number results in a...(negative number). And of course we know that a positive number divided by a positive number results in a...(positive number). The same rules that we use to multiply with signed numbers apply when we divide with signed numbers.

**Let's Try it (Slides 6 - 7):** Now let's work on some more division problems with signed numbers. We can divide any numbers as if they are positive, and then think about the sign after we do the division. We know that the rules for signs when we divide are the exact same as when we multiply with signed numbers. I know you're going to do great!


# WARM WELCOME



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**Today we will use the relationship between multiplication and division to develop the rules for dividing rational numbers.**


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 **Let's Talk:**

**How are  
multiplication  
and division  
related?**

$$5 \times 2 = 10$$

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 **Let's Think:**

**Use multiplication to find the unknown.**

$$-36 \div 12 = ?$$

$$16 \div -2 = ?$$

$$-25 \div -5 = ?$$

A negative number divided by a positive number is a \_\_\_\_\_ number.

A positive number divided by a negative number is a \_\_\_\_\_ number.

A negative number divided by a negative number is a \_\_\_\_\_ number.

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# Let's Think:

## Find each quotient.

a.  $-12 \div -24$

b.  $-3.6 \div 4$

c.  $-\frac{1}{4} \div 5$

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# Let's Try It:

## Let's explore using the relationship between multiplication and division to develop the rules for dividing rational numbers together.

Name: \_\_\_\_\_ G7 U4 Lesson 3 - Let's Try It

Fill in each statement using the term *positive* or *negative*.

- POSITIVE  $\times$  POSITIVE = \_\_\_\_\_
- NEGATIVE  $\times$  NEGATIVE = \_\_\_\_\_
- POSITIVE  $\times$  NEGATIVE = \_\_\_\_\_
- NEGATIVE  $\times$  POSITIVE = \_\_\_\_\_

Consider the equation below:  $6 \cdot 7 = 42$

- What is the missing value?
- Rewrite the equation as a division equation.
- A positive number divided by a positive number results in a \_\_\_\_\_ quotient.

Consider the equation below:  $-36 \div 4 = ?$

- Rewrite the equation as a multiplication equation.
- What is the missing value?
- A negative number divided by a positive number results in a \_\_\_\_\_ quotient.

Consider the equation below:  $24 \div -2 = ?$

- Rewrite the equation as a multiplication equation.
- What is the missing value?
- A positive number divided by a negative number results in a \_\_\_\_\_ quotient.

Consider the equation below:  $-32 + -8 = 7$

- Rewrite the equation as a multiplication equation.
- What is the missing value?
- A negative number divided by a negative number results in a \_\_\_\_\_ quotient.

Divide.

17:  $-35 \div -7 = ?$       $-35 \div 7 = ?$       $35 \div -7 = ?$       $35 \div 7 = ?$

18:  $-5 \div -15 = ?$       $-3 \div 4 = ?$       $-12.5 \div -5 = ?$       $2 \div -8 = ?$

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# On your Own:

Now it's time to explore using the relationship between multiplication and division to develop the rules for dividing rational numbers on your own.

Name: \_\_\_\_\_ G7 U4 Lesson 8 - Independent Work

1. Consider each equation below.

$-50 - -3 = ?$

a. Rewrite the equation as a multiplication equation.

b. Find the unknown factor.

$21 - -3 = ?$

c. Rewrite the equation as a multiplication equation.

d. Find the unknown factor.

$-72 \div \frac{4}{5} = ?$

e. Rewrite the equation as a multiplication equation.

f. Find the unknown factor.

$? \div -44 = -8$

g. Rewrite the equation as a multiplication equation.

h. Find the unknown factor.

2. Find the quotient.

$-55 \div -5 = ?$

a. -55  
b. 13  
c.  $-\frac{1}{13}$   
d.  $\frac{1}{13}$

3. Find the quotient.

$-4 \div \frac{1}{6} = ?$

a. -4  
b. 4  
c.  $-\frac{1}{4}$   
d.  $\frac{1}{4}$

4. Decide whether each statement is true or false. If it's true, explain how you know. If it's false, correct it.

a.  $-3 \div -10 = 10/3$

b.  $-3 \div 12 = \frac{1}{4}$

c.  $-3 \div -15 = 1/5$

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Name: \_\_\_\_\_

Fill in each statement using the term *positive* or *negative*.

1. POSITIVE x POSITIVE = \_\_\_\_\_
2. NEGATIVE x NEGATIVE = \_\_\_\_\_
3. POSITIVE x NEGATIVE = \_\_\_\_\_
4. NEGATIVE x POSITIVE = \_\_\_\_\_

Consider the equation below.

$$6 \cdot ? = 18$$

5. What is the missing value?
6. Rewrite the equation as a division equation.
7. A positive number divided by a positive number results in a \_\_\_\_\_ quotient.

Consider the equation below.

$$-36 \div 4 = ?$$

8. Rewrite the equation as a multiplication equation.
9. What is the missing value?
10. A negative number divided by a positive number results in a \_\_\_\_\_ quotient.

Consider the equation below.

$$24 \div -2 = ?$$

11. Rewrite the equation as a multiplication equation.
12. What is the missing value?
13. A positive number divided by a negative number results in a \_\_\_\_\_ quotient.

Consider the equation below.

$$-32 \div -8 = ?$$

14. Rewrite the equation as a multiplication equation.

15. What is the missing value?

16. A negative number divided by a negative number results in a \_\_\_\_\_ quotient.

**Divide.**

17.

$-35 \div -7 = ?$

$-35 \div 7 = ?$

$35 \div -7 = ?$

$35 \div 7 = ?$

18.

$-5 \div -15 = ?$

$-3 \div 4 = ?$

$-12.5 \div -5 = ?$

$\frac{1}{2} \div -8 = ?$

**1. Consider each equation below.**

$$-30 \div -3 = ?$$

- a. Rewrite the equation as a multiplication equation.
  
- b. Find the unknown factor.

$$21 \div -3 = ?$$

- c. Rewrite the equation as a multiplication equation.
  
- d. Find the unknown factor.

$$-72 \div 8 = ?$$

- e. Rewrite the equation as a multiplication equation.
  
- f. Find the unknown factor.

$$? = -44 \div -4$$

- g. Rewrite the equation as a multiplication equation.
  
- h. Find the unknown factor.

**2. Find the quotient.**

$$-65 \div -5 = ?$$

- a. -13
- b. 13
- c.  $-1/13$
- d.  $1/13$

**3. Find the quotient.**

$$-4 \div -16 = ?$$

- a. -4
- b. 4
- c.  $-1/4$
- d.  $1/4$

**4. Decide whether each statement is true or false. If it's true, explain how you know. If it's false, correct it.**

a.  $-3 \div -10 = 10/3$

---

---

b.  $-3 \div 12 = 1/4$

---

---

c.  $-3 \div -15 = 1/5$

---

---

Name: KEY

Fill in each statement using the term *positive* or *negative*.

1. POSITIVE x POSITIVE = positive
2. NEGATIVE x NEGATIVE = positive
3. POSITIVE x NEGATIVE = negative
4. NEGATIVE x POSITIVE = negative

Consider the equation below.

$$6 \cdot ? = 18$$

5. What is the missing value? 3

6. Rewrite the equation as a division equation.

$$18 \div 3 = ? \quad ? = \underline{6}$$

7. A positive number divided by a positive number results in a positive quotient.

Consider the equation below.

$$-36 \div 4 = ?$$

8. Rewrite the equation as a multiplication equation.

$$4 \cdot ? = -36$$

9. What is the missing value? -9

10. A negative number divided by a positive number results in a negative quotient.

Consider the equation below.

$$24 \div -2 = ?$$

11. Rewrite the equation as a multiplication equation.

$$-2 \cdot ? = 24$$

12. What is the missing value? -12

13. A positive number divided by a negative number results in a negative quotient.

Consider the equation below.

$$-32 \div -8 = ?$$

14. Rewrite the equation as a multiplication equation.

$$-8 \cdot ? = -32$$

15. What is the missing value?

(4)

16. A negative number divided by a negative number results in a positive quotient.

Divide.

17.

$$-35 \div -7 = ?$$

(5)

$$-35 \div 7 = ?$$

(-5)

$$35 \div -7 = ?$$

(-5)

$$35 \div 7 = ?$$

(5)

18.

$$-5 \div -15 = ?$$

( $\frac{1}{3}$ )

$$-3 \div 4 = ?$$

( $-\frac{3}{4}$ )

$$-12.5 \div -5 = ?$$

(2.5)

$$\frac{1}{2} \div -8 = ?$$

( $-\frac{1}{16}$ )

$$\begin{array}{r} 2.5 \\ 5 \overline{) 12.5} \\ \underline{-10} \phantom{0} \\ 2.5 \\ \underline{-2.5} \\ 0 \end{array}$$

$$\begin{array}{l} \frac{1}{2} \div -8 \\ \frac{1}{2} \times -\frac{1}{8} \end{array}$$



1. Consider each equation below.

$$-30 \div -3 = ?$$

a. Rewrite the equation as a multiplication equation.

$$-3 \cdot ? = -30$$

b. Find the unknown factor.

$$(10)$$

$$21 \div -3 = ?$$

c. Rewrite the equation as a multiplication equation.

$$-3 \cdot ? = 21$$

d. Find the unknown factor.

$$(-7)$$

$$-72 \div 8 = ?$$

e. Rewrite the equation as a multiplication equation.

$$8 \cdot ? = -72$$

f. Find the unknown factor.

$$(-9)$$

$$? = -44 \div -4$$

g. Rewrite the equation as a multiplication equation.

$$-4 \cdot ? = -44$$

h. Find the unknown factor.

$$(11)$$

2. Find the quotient.

$$-65 \div -5 = ?$$

- a. ~~13~~
- b. 13
- c. ~~1/13~~
- d. 1/13

$$\begin{array}{r} 13 \\ 5 \overline{)65} \\ \underline{-5} \phantom{0} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

3. Find the quotient.

$$-4 \div -16 = ?$$

- a. -4
- b. 4
- c.  $-\frac{1}{4}$
- d.  $\frac{1}{4}$

$$\frac{-4}{-16} = +\frac{4}{16}$$

4. Decide whether each statement is true or false. If it's true, explain how you know. If it's false, correct it.

a.  $-3 \div -10 = 10/3$

False.  $-3 \div -10 = \frac{3}{10}$

b.  $-3 \div 12 = \frac{1}{4}$

False.  $-3 \div 12 = -\frac{3}{12} = -\frac{1}{4}$

c.  $-3 \div -15 = 1/5$

True. A negative divided by a negative is a positive, and  $3 \div 15 = \frac{3}{15}$  or  $\frac{1}{5}$

# **G7 U4 Lesson 10**

Multiply and divide rational numbers to solve problems involving constant rates.

## G7 U4 Lesson 10 - Students will multiply and divide rational numbers to solve problems involving constant rate.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We have been learning a lot about multiplying and dividing with signed numbers lately. Today, we'll get the chance to apply some of that mathematical thinking to solve real-world problems about rates. When we refer to rates, we typically mean a ratio with two quantities that have different units. Miles per gallon. Kilometers per hour. Feet per second. We'll see a variety of rates in our work today, and we'll use what we know about multiplying and dividing signed numbers to help us tackle those problems.

Let's jump in.

**Let's Talk (Slide 3):** Take a moment to read both of these short problems. Do you think these problems deal with rates? How do you know? **Possible Student Answers, Key Points:**

- I think they do. Each problem has us thinking about two different units.
- Yes. The first problem is asking us about blueberries per minute. The second problem is asking us about chapters per day. Those are examples of rates.

We can think of both of these problems as having to do with rates. Let's think about the blueberry problem first. What is known in that problem? What is unknown? **Possible Student Answers, Key Points:**

- We know that she eats 5 blueberries every minute for 4 minutes.
- We don't know the total number of blueberries she eats in that time.

$$y = 5x$$
$$y = 5(4)$$
$$y = 20$$

I know she eats 5 blueberries every minute. 1 minute, 5 blueberries. 2 minutes, 10 blueberries. 3 minutes, 15 blueberries. I can represent this relationship with an equation. *(write as you narrate)* I know the total blueberries,  $y$ , is equal to 5 times the number of minutes,  $x$ . This equation makes it very easy for me to substitute in what I know, in this case 4 minutes, and answer the problem. I know  $y = 5(4)$ , because she eats for 4 minutes. So, the total number of blueberries is 20. We just used an equation to help solve that rate problem.

Think about the other problem. What is known? What is unknown? **Possible Student Answers, Key Points:**

- Ivan reads 6 chapters every day, and he wants to read 24 chapters total.
- We don't know how many days he'll take to read that many chapters.

$$y = 6x$$
$$24 = 6x$$
$$x = 4$$

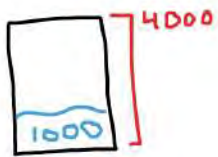
We can represent this problem with an equation too. I know he reads 6 chapters each day, so the total number of chapters he reads is equal to 6 times the number of days. In this case, I'll use  $y$  to represent the total number of chapters and  $x$  to represent the number of days Ivan reads. *(write equation  $y = 6x$ )* I know the total in this problem is 24 chapters, so I'll substitute that value in for  $y$ . Now I just need to think...24 is equal to 6 times what value? How could we solve this? **Possible Student Answers, Key Points:**

- The answer is  $x = 4$ , because I divided 24 by 6.
- The answer is  $x = 4$ , because I know  $6 \times 4$  is equal to 24.

Nice work. Equations can be used to represent situations with rates. In this case, these were positive rates. We didn't work with any negative numbers. As we'll see in some of our work today, there can be negative rates, and we can think about them similarly to how we think about positive rates. Let's look at our first official problem together.

**Let's Think (Slide 4):** I'm going to read this problem once through. (*read the problem aloud*) In your own words, what is this story about? **Possible Student Answers, Key Points:**

- It's about water in a swimming pool. There is some water being pumped into the tank, and some water is leaking out of the tank. It's asking how long it will take for the pool to overflow or to empty out.



I think it will help us if we picture the pool. I know there is 1,000 gallons in the pool at this moment, and that the pool could hold 4,000 gallons in all. (*sketch and label a simple drawing of the pool*)

$$+25 + (-5) = +20$$

The first question is asking us whether the water in the pool is rising or falling. I know the hose is spraying 25 gallons into the pool every minute. I can think of that as a positive rate of 25 gallons per minute. We also know there is water leaking out of the pool. It's leaking, which means it's leaving the pool, so we can think of that rate as -5 gallons per minute. (*write  $+25 + (-5) = ?$* ) Those two things are happening at the same time, so if I combine the positive rate of +25 and the negative rate of -5, at what rate is the water changing? How do you know?

**Possible Student Answers, Key Points:**

- $25 + (-5)$  is like subtracting  $25 - 5$ . The rate is 20.
- The rate would be positive 20. The water is filling at a rate of 20 gallons per minute.

If we combine the two rates, we can see that the water in the pool is rising at a rate of 20 gallons per minute. (*write +20 as solution to the equation*)

Part B now asks us to think how long it will take for the pool to overflow or empty completely. Which option makes the most sense based on what we know is happening to the pool? (**Overflowing, since the water is rising.**) There are already 1,000 gallons in the pool, and the pool can hold 4,000 gallons. That means, the pool has 3,000 gallons to go before it will overflow.

$$20x = 3000$$
$$x = 150$$

min.

I'm thinking that the water is rising at a rate of 20 gallons per minute, and the most water that can fit in the pool is 3,000 more gallons. I can think of that as the equation  $20x = 3,000$ . 20 times an unknown number of minutes,  $x$ , is equal to 3,000.

Take a second to do the math that makes sense to you to find the unknown. Then share out how you found the value of  $x$ . **Possible Student Answers, Key Points:**

- I divided 3,000 by 20. 3,000 divided by 20 is 150. It will take 150 minutes for the pool to overflow.
- I know  $20 \times 150 = 3,000$ . I know  $2 \times 15 = 30$ , so 2 tens  $\times$  15 tens is 30 hundreds or 3,000. The unknown is equal to 150 minutes.

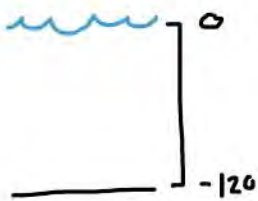
We can use division or a related multiplication fact to help us find that the unknown is 150. That means it will take 150 minutes for the pool to reach capacity and begin to overflow.

We just thought about positive and negative rates in the context of a swimming pool to help us solve this real-world problem. Let's try another with a different context.

**Let's Think (Slide 5):** Like last time, I'm going to read this problem once through. (*read the problem aloud*) In your own words, what is this story about? **Possible Student Answers, Key Points:**

- It's about a scuba diver who dives down to the bottom of the ocean, and we want to figure out how long it takes them to reach the bottom. Then they come back up, and we want to know at what rate.





Let's start with part A which asks us to think about the depth to which the person dives. We have seen several examples together in previous lessons where we consider elevation as a signed number. What signed number could represent this elevation, and how do you know? (*sketch and label a simple picture as shown while the student explains*) **Possible Student Answers, Key Points:**

- We could represent the elevation with  $-120$ . It's negative, because the elevation is below sea level. It's 120, because that's how far down the ocean floor is.

Now let's consider part B. The scuba diver is diving at a rate of 5 feet per minute. Since they're diving, I can think of the rate as being *negative* five feet per second. After 1 minute, they'd be at an elevation of  $-5$  feet.

After another minute, they'd be at  $-10$  feet. After another minute, they'd be at  $-15$  feet, and so on. Do you see a pattern? To represent this relationship, we can use an equation. (*write rate  $\times$  # of minutes = elevation*)

$$\text{rate} \times \# \text{ of min.} = \text{distance}$$

$$-5m = -120$$

$$m = 24 \text{ min}$$

I know the rate is  $-5$ . I don't know how many minutes, so I'll use  $m$ . Any variable would work. I also know that the diver ends up at an elevation of  $-120$  feet. (*substitute each value in the equation as you narrate*) All I need to do now is solve the equation to find the unknown number of minutes. How could I go about solving in this case? Take some time and use scratch paper if necessary.

**Possible Student Answers, Key Points:**

- I can divide both sides by  $-5$ .  $120$  divided by  $5$  is  $24$ .  $-120$  divided by  $-5$  is  $+24$ , because the quotient of two negative numbers is positive.
- I know  $5 \times 24 = 120$ . So,  $-5 \times 24 = -120$ . It couldn't be  $-24$ , because two negatives would multiply to get positive  $120$ .

Nicely done. It would take the scuba diver 24 minutes to reach the ocean floor at that rate. We substituted values we knew from the problem into an equation that represents the relationship. Then, we solved to find our unknown. Let's see if we can use similar thinking to answer part C.

(*re-read problem*) In our last problem, we knew the rate and the final elevation. What's different about this problem? **Possible Student Answers, Key Points:**

- It's still a rate problem, but the scuba diver is going up.
- We don't know the rate, but we do know the number of minutes. We're trying to find the unknown rate.

$$k \cdot 20 = 120$$

$$k = 6$$

$$6 \text{ Ft. per min}$$

We can still think of our situation as rate times number of minutes equals distance. Let's substitute in what we know. (*substitute values as you narrate*) I'll use  $k$  to represent rate, because that's commonly what mathematicians use, but any variable is acceptable. I know it takes the scuba diver 20 minutes, so I'll substitute 20 in for time. And the scuba diver is traveling 120 feet. I'll use a positive 120 since the scuba diver is going *up* 120 feet in this scenario. I can either use division divide both sides of this equation by 20, or I can use a multiplication fact to think about a number times 20 that equals 120. Either way, I end up with a solution of 6. The unknown rate is 6. In this context, that means the scuba

diver's elevation is increasing 6 feet per minute.

How can equations help us solve problems involving positive and negative rates? **Possible Student Answers, Key Points:**

- We can use an equation to represent problems involving rates. Once we have an equation that represents the relationship, we can substitute values and carefully solve to find unknowns.

**Let's Try it (Slides 6 - 7):** You've been working hard. Now let's try a couple more examples together before you have some time to work independently. We'll think about the values in rate problems and carefully consider whether values are best represented with positive or negative values. We'll also think about equations that can match the relationship in a given context. Like the scuba diver in our previous problem, let's "dive" in.



# WARM WELCOME



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**Today we will multiply and divide numbers to solve problems involving constant rate.**


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 **Let's Talk:**

**Maria eats 5 blueberries per minute for 4 minutes. How many blueberries does Maria eat?**

**Ivan reads 6 chapters per day. How many days will it take to read 24 chapters?**

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 **Let's Think:**

**A swimming pool contains 1,000 gallons of water. It can hold 4,000 gallons of water before overflowing. A hose sprays water into the pool at 25 gallons per minute, but there is also a leak in the pool that lets out 5 gallons of water per minute.**

- a. Is the water in the pool rising or falling?
- b. How long will it take before the pool overflows or empties?

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## Let's Think:

**The Great Barrier Reef is 120 feet below the ocean's surface. A scuba diver dives all the way to the ocean floor.**

- a. How can you represent the depth that the scuba diver dives?
- a. The scuba diver descends at a rate of -5 feet per minute. How much time will it take the diver to get to the bottom?
- b. It takes the scuba diver takes 20 minutes to swim back up to the surface. At what rate does the scuba diver ascend?

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## Let's Try It:

**Let's explore multiplying and dividing numbers to solve problems involving constant rate together.**

Name: \_\_\_\_\_ G7 U4 Lesson 10 - Let's Try It

Write an equation in the form  $y = kx$  to represent each relationship. Pay attention to whether it makes more sense to write the rate as positive or negative.

1. A hot air balloon rises 3 feet per second.
2. A plane descends 500 yards per minute for landing.
3. I take 100 steps backward per minute.
4. I take 100 steps forward per minute.
5. A dog loses 2 pounds per month.
6. A cat gains 1 pound every year.

A water tank currently holds 14 gallons of water, and it can hold a maximum 50.5 gallons. A pump is filling the tank at a rate of 8 gallons per minute. A gardener is using a hose that empties water from the tank at a rate of 2 gallons per minute.

7. Which rate best represents the pump filling the tank?
  - a. +8 gallons per minute
  - b. -8 gallons per minute
8. Which rate best represents the hose emptying water from the tank?
  - a. +2 gallons per minute
  - b. -2 gallons per minute
9. Is the water level in the tank rising or falling? Explain.
10. At what rate is the water level rising or falling? Explain.
11. Write an equation in the form  $y = kx$  to represent the scenario.

12. How long will it take the tank to become completely full or completely empty?

A hot air balloon ascends into the sky at a constant rate. It takes 6 hours to rise to a height of 21,672 feet.

13. Write an equation to represent the hot air balloons ascent from 0 feet.

14. It only took the hot air balloon 2 hours to descend. Write another relationship to represent the descent.

15. In your own words, explain how you know when a situation is best represented by a positive rate and when it is best represented by a negative rate?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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## On your Own:

Now it's time to explore multiplying and dividing numbers to solve problems with constant rate on your own.

Name \_\_\_\_\_ ID7 U4 Lesson 10 - Independent Work

1. Describe a situation or write a story problem where each of the following would be useful.

a.  $-10$  gallons per hour

b.  $-28$  inches per minute

c.  $-0.1$  liters per second

2. A submarine starts at an elevation of  $-240$  yards. For safety reasons, it can only rise toward the surface in  $50$ -meter intervals.

a. What will be its depth after the first interval?

b. How many intervals will it take for the submarine to reach sea level?

3. It takes  $16$  inches of yarn to make  $2$  friendship bracelets. At this rate, how much yarn will it take to make  $5$  friendship bracelets?

4. A whale is descending to the seafloor  $1,300$  feet below the surface. It takes the whale  $2$  hours to make this descent. Write an equation to represent the relationship between the whale's elevation and time.

5. Another whale's descent can be represented by the equation  $y = -150x$ , where  $y$  is the elevation and  $x$  is the time in hours. How long does it take this whale to make the descent?

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**Write an equation in the form  $y = kx$  to represent each relationship. Pay attention to whether it makes more sense to write the rate as positive or negative.**

1. A hot air balloon rises 3 feet per second.
2. A plane descends 500 yards per minute for landing.
3. I take 100 steps backward per minute.
4. I take 100 steps forward per minute.
5. A dog loses 2 pounds per month.
6. A cat gains 1 pound every year.

**A water tank currently holds 14 gallons of water, and it can hold a maximum 50.5 gallons. A pump is filling the tank at a rate of 8 gallons per minute. A gardener is using a hose that empties water from the tank at a rate of 2 gallons per minute.**

7. Which rate best represents the pump filling the tank?
  - a. +8 gallons per minute
  - b. -8 gallons per minute
8. Which rate best represents the hose emptying water from the tank?
  - a. +2 gallons per minute
  - b. -2 gallons per minute
9. Is the water level in the tank rising or falling? Explain.
10. At what rate is the water level rising or falling? Explain.
11. Write an equation in the form  $y = kx$  to represent the scenario.

12. How long will it take the tank to become completely full or completely empty?

**A hot air balloon ascends into the sky at a constant rate. It takes 6 hours to rise to a height of 21,672 feet.**

13. Write an equation to represent the hot air balloons ascent from 0 feet.

14. It only took the hot air balloon 2 hours to descend. Write another relationship to represent the descent.

15. In your own words, explain how you know when a situation is best represented by a positive rate and when it is best represented by a negative rate?

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**1. Describe a situation or write a story problem where each of the following would be useful.**

a. -10 gallons per hour

b. -28 inches per minute

c. -0.1 liters per second

**2. A submarine starts at an elevation of -240 yards. For safety reasons, it can only rise toward the surface in 50-meter intervals.**

a. What will be its depth after the first interval?

b. How many intervals will it take for the submarine to reach sea level?



**3. It takes 16 inches of yarn to make 2 friendship bracelets. At this rate, how much yarn will it take to make 5 friendship bracelets?**

**4. A whale is descending to the seafloor 1,800 feet below the surface. It takes the whale 3 hours to make this descent. Write an equation to represent the relationship between the whale's elevation and time.**

**5. Another whale's descent can be represented by the equation  $y = -150x$ , where  $y$  is the elevation and  $x$  is the time in hours. How long does it take this whale to make the descent?**

Write an equation in the form  $y = kx$  to represent each relationship. Pay attention to whether it makes more sense to write the rate as positive or negative.

1. A hot air balloon rises 3 feet per second.  $y = 3x$
2. A plane descends 500 yards per minute for landing.  $y = -500x$
3. I take 100 steps backward per minute.  $y = -100x$
4. I take 100 steps forward per minute.  $y = 100x$
5. A dog loses 2 pounds per month.  $y = -2x$
6. A cat gains 1 pound every year.  $y = 1x$  or  $y = x$

A water tank currently holds 14 gallons of water, and it can hold a maximum 50.5 gallons. A pump is filling the tank at a rate of 8 gallons per minute. A gardener is using a hose that empties water from the tank at a rate of 2 gallons per minute.

7. Which rate best represents the pump filling the tank?
  - a. +8 gallons per minute
  - b. -8 gallons per minute
8. Which rate best represents the hose emptying water from the tank?
  - a. +2 gallons per minute
  - b. -2 gallons per minute

9. Is the water level in the tank rising or falling? Explain.

It is rising. More water is being pumped in than emptied out.

10. At what rate is the water level rising or falling? Explain.

$$+8 + -2 = +6$$

The rate is +6 gallons per minute.

11. Write an equation in the form  $y = kx$  to represent the scenario.

$$y = 6x + 14$$

$$\text{OR } 6x + 14 = 50.5$$

12. How long will it take the tank to become completely full or completely empty?

$$\begin{array}{r} 4 \\ 50.5 \\ -14.0 \\ \hline 36.5 \end{array}$$

$$6x = 36.5$$

$$x = 6.083\dots$$

OR

$$x = 6\frac{1}{12} \text{ minutes}$$

A hot air balloon ascends into the sky at a constant rate. It takes 6 hours to rise to a height of 21,672 feet.

13. Write an equation to represent the hot air balloons ascent from 0 feet.

$$6x = 21,672$$



+21,672

14. It only took the hot air balloon 2 hours to descend. Write another relationship to represent the descent.

$$-2x = -21,672$$



-21,672

15. In your own words, explain how you know when a situation is best represented by a positive rate and when it is best represented by a negative rate?

Positive rates are used to represent increases, while negative rates best represent decreases.



1. Describe a situation or write a story problem where each of the following would be useful.

a. -10 gallons per hour

emptying water from a pool

b. -28 inches per minute

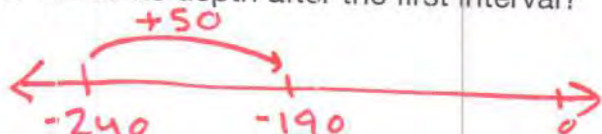
a bubble descending at a constant rate

c. -0.1 liters per second

pouring water out of a watering can

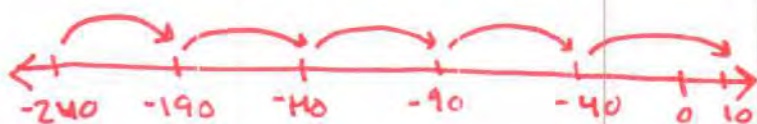
2. A submarine starts at an elevation of -240 yards. For safety reasons, it can only rise toward the surface in 50-meter intervals.

a. What will be its depth after the first interval?



-190 yd

b. How many intervals will it take for the submarine to reach sea level?



5 intervals

3. It takes 16 inches of yarn to make 2 friendship bracelets. At this rate, how much yarn will it take to make 5 friendship bracelets?

16 inches for 2 bracelets  $\rightarrow$  8" per bracelet

$$y = 8x$$

$$y = 8(5)$$

$$(40 \text{ inches})$$

4. A whale is descending to the seafloor 1,800 feet below the surface. It takes the whale 3 hours to make this descent. Write an equation to represent the relationship between the whale's elevation and time.

$$3 \cdot x = -1800$$

$$x = -600$$

-600  
ft per  
hour

5. Another whale's descent can be represented by the equation  $y = -150x$ , where  $y$  is the elevation and  $x$  is the time in hours. How long does it take this whale to make the descent?

$$-1800 = -150x$$

$$12 = x$$

$$\begin{array}{r} 12 \\ 150 \overline{) 1800} \\ \underline{150} \phantom{0} \\ 300 \\ \underline{-300} \\ 0 \end{array}$$

$$(12 \text{ hrs})$$

# **G7 U4 Lesson 11**

Use the relationship between addition and subtraction, and the relationship between multiplication and division, to evaluate expressions with all four operations on the rational numbers.

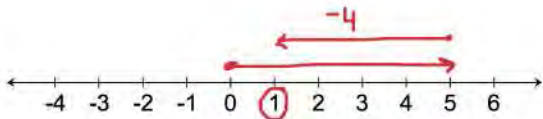
**G7 U4 Lesson 11 - Students will use the relationship between addition and subtraction, and the relationship between multiplication and division, to evaluate expressions with all four operations on rational numbers.**

**Warm Welcome (Slide 1):** Tutor choice

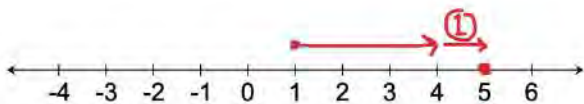
**Frame the Learning/Connect to Prior Learning (Slide 2):** We've been working hard to understand adding, subtracting, multiplying, and dividing with signed numbers, and we've been applying what we know to real world scenarios. Through this process, we've learned several different strategies to think about and tackle problems. Today is exciting, because our focus is all about choosing the *easiest* pathway toward a solution. Our goal is to be efficient. It's always nice when you can make the math a bit quicker or easier. I'll show you what I mean after we spend a couple minutes talking about inverses...

**Let's Talk (Slide 3):** We're going to look at two pairs of problems. Before we do any math, take a second and review both problem pairs. What do you notice? What do you wonder? [Possible Student Answers, Key Points:](#)

- I notice the red problems both start with 5. One adds  $-4$ , and the other subtracts  $+4$ .
- I notice the blue problems both start with 6. One multiplies by  $\frac{1}{2}$ , and the other divides by 2.
- I wonder what the answers are. I wonder if the answers are the same or different? I wonder which ones are easier to solve.



Let's focus on the red problem pair first. I could represent  $5 + (-4)$  on a number line. (*sketch and label as you narrate*) I can show an arrow going to 5, and then add another arrow pointing left representing  $-4$ . I see the answer is 1.



The other equation wants to think about  $5 - (+4)$ . I can think of this as the difference between 5 and 4. (*sketch and label as you narrate*) I'll mark 5 on the number line. I can draw an arrow to positive 4, and then drawn an arrow to show the difference between the two values. I can see that the

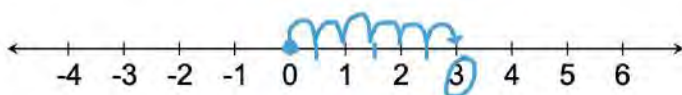
difference can be represented with an arrow representing  $+1$ .

These two equations are equivalent. The second, equivalent equation was created by using the additive inverse of the first equation. Instead of *adding a negative 4*, we can *subtract a positive 4*. Numbers are additive inverses if they can be added together to result in 0.

Either equation could be solved to find the value of  $+1$ , but sometimes certain equations make more sense with certain numbers or just feel easier for the person solving them. Which of these two equations feels easiest or most accessible to you? Why? [Possible Student Answers, Key Points:](#)

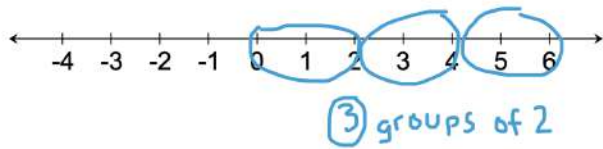
- I like the first equation, because it's easy for me to count back 4 from 5.
- I like the second equation, because I can picture the difference between 5 and 4 easily on the number line since they're close together.

There is no right or wrong choice to make, but it's always nice to consider your options before solving an equation. If you can make the math feel easier, why not do that?



Let's look at the blue equations now. For the first equation, I can model  $6 \times \frac{1}{2}$  by showing 6 groups of  $\frac{1}{2}$  on the number line. (*mark  $\frac{1}{2}$  tick marks, and show 6 hops to land at 3*) The product is 3.





For the second equation I can think of 6 divided into groups of 2. (sketch and label as you narrate) I can show hops or circles on my number line to make groups of 2. I see that I can make 3 groups of 2. The quotient is 3.

These equations involve what we call multiplicative inverses. Numbers are multiplicative inverses if you can multiply them together to get 1. In this case 2 and  $\frac{1}{2}$  are multiplicative inverses.  $2 \times \frac{1}{2} = 1$ . I can rewrite multiplication and division equations by using the multiplicative inverse and the opposite operation to write equivalent expressions.

Like in the red equations, we might notice that certain equivalent expressions feel better to solve than others. In the case of the blue equations, which version would prefer to think about and why? **Possible Student Answers, Key Points:**

- I prefer the first equation, because halves are easy for me to think about. I know 6 halves is 3.
- I prefer the second equation, because I don't have to think about fractions. I like when I can work with whole numbers.

We can use additive or multiplicative inverses to write equivalent expressions. Sometimes an equivalent expression might be easier to solve than the expression we're given. Let's look at some examples and think about which expressions work best for us.

**Let's Think (Slide 4):** Our first set of problems asks us to rewrite each expression using the additive inverse. Then, we get to pick which expression we prefer to help us find the value.

EXPRESSION	$12 + -8$	$7 - (-10)$	$-2 - 20$	$-3 + 15$
EQUIVALENT EXPRESSION	$12 - 8$	$7 + 10$	$-2 + (-20)$	$-3 - (-15)$
VALUE	4	17	-22	12

(fill in values in the table as shown as you narrate through each example)

The first expression is  $12 + -8$ . I know the additive inverse of  $-8$  is  $+8$ , because  $-8$  and  $+8$  combined have a value of 0. I can rewrite the

expression as  $12 - 8$ . These two expressions are equivalent. Which one would you prefer to use to solve, and why? **Possible Student Answers, Key Points:**

- I prefer the second one, because it just looks like a simple math fact from elementary school. I know  $12 - 8$  is 4 by heart.

The value of both expressions is 4. We can use either expression to arrive at that value.

(NOTE: Student preferences may vary. As they choose their preferred expressions in this lesson, circle or highlight the ones they name, and push them to justify their preference.)

The second expression is  $7 - (-10)$ . What is the additive inverse of  $-10$ ? ( $+10$ ) Instead of subtracting negative 10, I can add positive 10 to make an equivalent expression. Which one would you prefer to use to solve, and why? **Possible Student Answers, Key Points:**

- I prefer the second one, because the first one has two symbols next to each other which can sometimes be confusing to think about. I also just know  $7 + 10$  as a math fact.

The value of both expressions is 17. We can use either expression to arrive at that value.

The third expression is  $-2 - 20$ . How can I rewrite this expression using the additive inverse?  $(-2 + (-20))$   
 Instead of subtracting positive 20, I can add negative 20. Use either expression to think about the value, then justify why you used the expression you chose. ? Possible Student Answers, Key Points:

- The value is -22. I used the second expression because I could just combine both negative numbers. I could just think of it as adding 2 and 20, but they're negative.

Let's try one more. How could you rewrite the last problem using the additive inverse? ? Possible Student Answers, Key Points:

- Instead of adding positive 15, I can subtract -15. The equivalent expression is  $-3 - (-15)$ .

Either way we think about this problem, we should arrive at the same answer. I can think of the first expression as a number line. I'm picturing an arrow to -3, then an arrow going up 15 spaces to land at +12. The other problem, I can think about as the difference between -3 and -15. That may or may not be a little harder to picture in your mind. Whichever expression you choose to work with is fine as long as it is actually equivalent and it helps you think about the math in a way that is friendly for you.

**Let's Think (Slide 5):** For our last problem set together, we're going to follow the same sequence, but you'll notice these problems aren't addition and subtraction. This problem set wants us to use the

EXPRESSION	$-20 \cdot 1/5$	$20 \div -1/5$	$42 \times 1/7$	$7 \times 42$
EQUIVALENT EXPRESSION	$-20 \div 5$	$20 \times -5$	$42 \div 7$	$7 \div \frac{1}{42}$
VALUE	-4	-100	6	294

(fill in values in the table as shown as you narrate through each example)

I'll start by thinking about  $-20 \cdot \frac{1}{5}$ . I know the multiplicative inverse of  $\frac{1}{5}$  is 5, because  $\frac{1}{5} \cdot 5 = 1$ . Instead of  $-20 \cdot 1/5$ , I can write an

equivalent expression of -20 divided by 5. Either expression can work to help me find the value. Between these two, I would prefer to use -20 divided by 5, to avoid having to work with the fraction  $\frac{1}{5}$ . I know -20 divided by positive 5 is -4. Choosing the easier expression for me, helped me efficiently arrive at my answer.

Look at the second example. Instead of dividing by  $-\frac{1}{5}$ , I can multiply by the multiplicative inverse. What equivalent expression can I write?  $(20 \times -5)$   $20 \times -5$  is equivalent to 20 divided by  $-\frac{1}{5}$ . -5 and  $-\frac{1}{5}$  are multiplicative inverses, because if we multiply them together the product is 1. Which equivalent expression would you prefer to use to find the value, and why? Possible Student Answers, Key Points:

- The first one involves dividing by fractions. I know how to do that, but I think it'd be easier to multiply without the fraction. I'd choose the second equation.

The second equation avoids having to divide by fractions. I can easily think  $20 \times -5 = -100$ .

For the third expression, I can think of the multiplicative inverse of  $1/7$ . Instead of multiplying by  $1/7$ , I can divide 42 by 7. That feels a little easier to me than multiplying by a fraction. I know 42 divided by 6 is 7 without having to do too much work.

Take a moment to write an equivalent expression for the last expression. Pick one to use to find the value and justify your choice. Possible Student Answers, Key Points:

- Instead of multiplying 7 by 42, I can divide 7 by  $1/42$ . I know 42 and  $1/42$  are multiplicative inverses.
- I don't know  $7 \times 42$  by heart, but I think it would probably be easier to think about than dividing by  $1/42$ , so I'd use the first expression to find the value.  $7 \times 42$  is equal to 294.

An equivalent expression will give us the correct value. Quickly thinking about an equivalent expression with the additive or multiplicative inverse can help us consider potentially better options to help us arrive at the solution to a given problem.

**Let's Try it (Slides 6 - 7):** Now let's use this thinking to work through some more problems. When given an adding or subtracting expression, we know we can use the opposite operations and the additive inverse to find an equivalent expression. When given a multiplication or division expression, we can use the opposite operation and the multiplicative inverse to find an equivalent expression. When it comes time to evaluate or solve, it's up to our own preference to decide which solution pathway makes more sense. Personally, I prefer to avoid complicated sign combinations and unfriendly fraction operations, but everyone's preferences can be different. Let's see how efficient we can be...

# WARM WELCOME



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**Today we will use the relationship between addition and subtraction, and the relationship between multiplication and division to evaluate expressions with all four operations.**

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## Let's Talk:

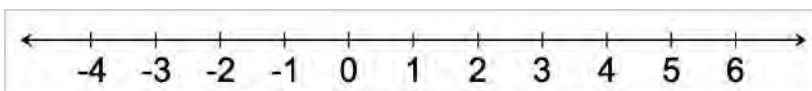
What do you notice? What do you wonder?

$$5 + (-4) = ?$$

$$5 - (+4) = ?$$

$$6 \cdot \frac{1}{2} = ?$$

$$6 \div 2 = ?$$



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## Let's Think:

Rewrite each expression to make an equivalent expression using the additive inverse. Then use either expression to find the value.

<b>EXPRESSION</b>	$12 + -8$	$7 - (-10)$	$-2 - 20$	$-3 + 15$
<b>EQUIVALENT EXPRESSION</b>				
<b>VALUE</b>				

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# Let's Think:

Rewrite each expression to make an equivalent expression using the multiplicative inverse. Then use either expression to find the value.

<b>EXPRESSION</b>	$-20 \cdot 1/5$	$20 \div -1/5$	$42 \times 1/7$	$7 \times 42$
<b>EQUIVALENT EXPRESSION</b>				
<b>VALUE</b>				

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# Let's Try It:

Let's explore using the relationship between addition and subtraction, and the relationship between multiplication and division to evaluate expressions with all four operations together.

Name: \_\_\_\_\_ Q7 U4 Lesson 11 - Let's Try It

Consider the expression  $6 + 7 = 0$ .

- Use an arrow to model the first addend.
- Draw another arrow to show how to reach a sum of 0.
- What is the unknown value? \_\_\_\_\_
- When a number is added to its opposite, the sum is always \_\_\_\_\_. We call these number pairs **additive inverses**.

Determine whether each pair of numbers are additive inverses or not.

5.  $-6$  and  $+2$                       YES // NO

6.  $-6$  and  $+6$                         YES // NO

7.  $8$  and  $16$                          YES // NO

8.  $-5$  and  $5$                          YES // NO

9.  $-1,288$  and  $-1,288$             YES // NO

Rewrite each expression below as an equivalent expression using the additive inverse of the second term. Find each value using the original or the rewritten expression.

10.  $3 - (-8)$       11.  $-4 + -15$       12.  $9 + -6$       13.  $7 - 10$

Adding a number always results in the same value as \_\_\_\_\_ its additive inverse.

Find the unknown in each equation.

14.  $4 + 7 = 1$

15.  $7 + -10 = 1$

16.  $1/5 + 7 = 1$

These factor pairs are called multiplicative inverses. When you multiply any pair of multiplicative inverses, the product is always \_\_\_\_\_.

Evaluate each pair of expressions:

17.  $8 \times 1/8$        $8 = 4$

18.  $8 \times 2$        $8 = 1/8$

Dividing by a number always results in the same value as multiplying by its \_\_\_\_\_.

Rewrite each expression below as an equivalent multiplication expression using the multiplicative inverse. Find each value using either the original or the rewritten expression.

19.  $24 \div 6$       20.  $6 \div 24$       21.  $-24 \div 8$       22.  $-24 \div -6$

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# On your Own:

Now it's time to explore using the relationship between addition and subtraction, and the relationship between multiplication and division to evaluate expressions with all four operations on your own.

Name: \_\_\_\_\_ G7 US Lesson 11 - Independent Work

1. Match each equation with an equivalent expression. Find the value of each pair using whichever expression is easiest for you.

$-7 + 8$	$10 - (-9)$
$-12 + (-12)$	$-7 - (-8)$
$-7 - 8$	$-12 - 12$
$-12 - (-12)$	$-12 + 12$
$(10) + 3$	$-7 + (-8)$
$-10 - 3$	$-10 + (-8)$

How did you decide which expressions were easier to evaluate?

\_\_\_\_\_

\_\_\_\_\_

2. Complete the table by writing an equivalent expression. Then find the value of both expressions in that column.

EXPRESSION	$-11 + -9$	$-11 - (-9)$	$32 + -8$	$32 + -8$
EQUIVALENT EXPRESSION				
VALUE				

3. Match each equation with an equivalent expression. Find the value of each pair using whichever expression is easiest for you.

$-20 + 4$	$-20 = 14$
$-20 + -4$	$-24 = -32$
$24 + -9$	$8 = 4$
$-24 + -6$	$6 = 14$
$8 = 14$	$24 = -32$
$8 = 4$	$-20 = 14$

How did you decide which expressions were easier to evaluate?

\_\_\_\_\_

\_\_\_\_\_

4. Complete each equation using an operation symbol to make the equation true.

$-18$  \_\_\_\_\_  $-9 = 27$

$12$  \_\_\_\_\_  $-13 = 25$

$12$  \_\_\_\_\_  $13 = 1$

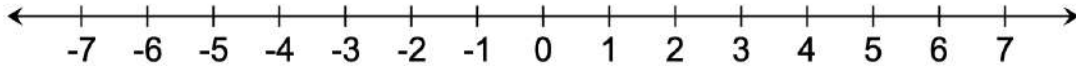
$-25$  \_\_\_\_\_  $+24 = 10$

$-25$  \_\_\_\_\_  $14 = -90$

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Consider the expression  $6 + ? = 0$ .



1. Use an arrow to model the first addend.
2. Draw another arrow to show how to reach a sum of 0.
3. What is the unknown value? \_\_\_\_\_
4. When a number is added to its opposite, the sum is always \_\_\_\_\_. We call these number pairs **additive inverses**.

Determine whether each pair of numbers are additive inverses or not.

- |                                      |           |
|--------------------------------------|-----------|
| 5. $-\frac{1}{2}$ and $+2$           | YES // NO |
| 6. $-\frac{1}{4}$ and $+\frac{1}{4}$ | YES // NO |
| 7. 8 and 16                          | YES // NO |
| 8. -5 and 5                          | YES // NO |
| 9. +1,288 and -1,288                 | YES // NO |

Rewrite each expression below as an equivalent expression using the additive inverse of the second term. Find each value using the original or the rewritten expression.

10.  $3 - (-8)$

11.  $-4 + -15$

12.  $9 + -6$

13.  $7 - 10$

Adding a number always results in the same value as \_\_\_\_\_ its additive inverse.

**Find the unknown in each equation.**

14.  $4 \cdot ? = 1$

15.  $? \cdot -10 = 1$

16.  $\frac{1}{2} \cdot ? = 1$

These factor pairs are called multiplicative inverses. When you multiply any pair of multiplicative inverses, the product is always \_\_\_\_\_.

**Evaluate each pair of expressions.**

17.  $8 \times \frac{1}{4}$        $8 \div 4$

18.  $6 \times 2$        $6 \div \frac{1}{2}$

Dividing by a number always results in the same value as multiplying by its \_\_\_\_\_  
\_\_\_\_\_.

**Rewrite each expression below as an equivalent multiplication expression using the multiplicative inverse. Find each value using either the original or the rewritten expression.**

19.  $24 \div 6$

20.  $6 \div 24$

21.  $-24 \div 6$

22.  $-24 \div -6$

1. Match each equation with an equivalent expression. Find the value of each pair using whichever expression is easiest for you.

$-7 + 8$

$-10 - (-3)$

$-12 + (-12)$

$-7 - (-8)$

$-7 - 8$

$-12 - 12$

$-12 - (-12)$

$-12 + 12$

$-10 + 3$

$-7 + (-8)$

$-10 - 3$

$-10 + (-3)$

How did you decide which expressions were easier to evaluate?

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2. Complete the table by writing an equivalent expression. Then find the value of both expressions in that column.

<b>EXPRESSION</b>	$-11 + -9$	$-11 - (-9)$	$32 \div -8$	$32 \cdot -8$
<b>EQUIVALENT EXPRESSION</b>				
<b>VALUE</b>				

**3. Match each equation with an equivalent expression. Find the value of each pair using whichever expression is easiest for you.**

$$-20 \div 4$$

$$-20 \cdot 4$$

$$24 \times -\frac{2}{3}$$

$$-24 \cdot -\frac{2}{3}$$

$$8 \cdot \frac{1}{4}$$

$$8 \cdot 4$$

$$-20 \div \frac{1}{4}$$

$$-24 \div -\frac{3}{2}$$

$$8 \div 4$$

$$8 \div \frac{1}{4}$$

$$24 \div -\frac{3}{2}$$

$$-20 \cdot \frac{1}{4}$$

How did you decide which expressions were easier to evaluate?

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**4. Complete each equation using an operation symbol to make the equation true.**

$$-18 \text{ \_\_\_\_\_\_ } -\frac{2}{3} = 27$$

$$12 \text{ \_\_\_\_\_\_ } -13 = 25$$

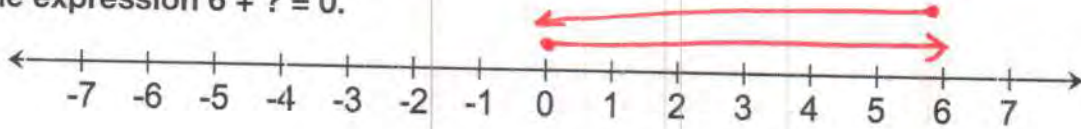
$$12 \text{ \_\_\_\_\_\_ } 13 = -1$$

$$-25 \text{ \_\_\_\_\_\_ } -\frac{2}{5} = 10$$

$$-25 \text{ \_\_\_\_\_\_ } \frac{1}{2} = -50$$

Name: KEY

Consider the expression  $6 + ? = 0$ .



1. Use an arrow to model the first addend.
2. Draw another arrow to show how to reach a sum of 0.
3. What is the unknown value? -6
4. When a number is added to its opposite, the sum is always 0. We call these number pairs **additive inverses**.

Determine whether each pair of numbers are additive inverses or not.

5.  $-\frac{1}{2}$  and  $+2$  YES // **NO**
6.  $-\frac{1}{4}$  and  $+\frac{1}{4}$  **YES** // NO
7. 8 and 16 YES // **NO**
8. -5 and 5 **YES** // NO
9. +1,288 and -1,288 **YES** // NO

Rewrite each expression below as an equivalent expression using the additive inverse of the second term. Find each value using the original or the rewritten expression.

- |  |   |   |  |
|--|---|---|--|
| 10. $3 - (-8)$<br><u><math>3 + 8</math></u><br><b>(11)</b> | 11. $-4 + -15$<br><u><math>-4 - 15</math></u><br><b>(-19)</b> | 12. $9 + -6$<br><u><math>9 - 6</math></u><br><b>(3)</b> | 13. $7 - 10$<br><u><math>7 + (-10)</math></u><br><b>(-3)</b> |
|--|---|---|--|

Adding a number always results in the same value as subtracting its additive inverse.

Find the unknown in each equation.

14.  $4 \cdot ? = 1$   $\left(\frac{1}{4}\right)$

15.  $? \cdot -10 = 1$   $\left(-\frac{1}{10}\right)$

16.  $\frac{1}{2} \cdot ? = 1$   $(2)$

These factor pairs are called multiplicative inverses. When you multiply any pair of multiplicative inverses, the product is always 1.

Evaluate each pair of expressions.

17.  $8 \times \frac{1}{4}$        $8 \div 4$   
 $\frac{8}{4} = 2$        $(2)$

18.  $6 \times 2$        $6 \div \frac{1}{2}$   
 $(12)$        $(12)$

Dividing by a number always results in the same value as multiplying by its multiplicative inverse.

Rewrite each expression below as an equivalent multiplication expression using the multiplicative inverse. Find each value using either the original or the rewritten expression.

19.  $24 \div 6$

$24 \times \frac{1}{6}$

$(4)$

20.  $6 \div 24$

$6 \times \frac{1}{24}$

$\frac{6}{24}$

$\left(\frac{1}{4}\right)$

21.  $-24 \div 6$

$-24 \times \frac{1}{6}$

$\frac{-24}{6}$

$(-4)$

22.  $-24 \div -6$

$-24 \times -\frac{1}{6}$

$\frac{-24}{-6}$

$(4)$



1. Match each equation with an equivalent expression. Find the value of each pair using whichever expression is easiest for you.

$-7 + 8$	$-10 - (-3)$ <b>(-7)</b>
$-12 + (-12)$	$-7 - (-8)$ <b>(1)</b>
$-7 - 8$	$-12 - 12$ <b>(-24)</b>
$-12 - (-12)$	$-12 + 12$ <b>(0)</b>
$-10 + 3$	$-7 + (-8)$ <b>(-15)</b>
$-10 - 3$	$-10 + (-3)$ <b>(-13)</b>

How did you decide which expressions were easier to evaluate? *(answers may vary)*

- *It helps me when the expressions are addition.*
- *It helps when there aren't <sup>too</sup> many + or - symbols.*

2. Complete the table by writing an equivalent expression. Then find the value of both expressions in that column.

EXPRESSION	$-11 + -9$	$-11 - (-9)$	$32 \div -8$	$32 \cdot -8$
EQUIVALENT EXPRESSION	$-11 - 9$	$-11 + 9$	$32 \times -\frac{1}{8}$	$32 \div -\frac{1}{8}$
VALUE	$-20$	$-2$	$-4$	$-256$

$\begin{array}{r} 32 \\ \times 8 \\ \hline 256 \end{array}$



3. Match each equation with an equivalent expression. Find the value of each pair using whichever expression is easiest for you.

$-20 \div 4$	$-20 \div \frac{1}{4}$	<b>(-80)</b>
$-20 \cdot 4$	$-24 \div -\frac{3}{2}$	<b>(16)</b>
$24 \times -\frac{2}{3}$	$8 \div 4$	<b>(2)</b>
$-24 \cdot -\frac{2}{3}$	$8 \div \frac{1}{4}$	<b>(32)</b>
$8 \cdot \frac{1}{4}$	$24 \div -\frac{3}{2}$	<b>(-16)</b>
$8 \cdot 4$	$-20 \cdot \frac{1}{4}$	<b>(-5)</b>

How did you decide which expressions were easier to evaluate? *(answers may vary)*

- I prefer expressions with whole numbers.*
- I prefer multiplying.*

4. Complete each equation using an operation symbol to make the equation true.

$-18 \underline{\div} -\frac{2}{3} = 27$        $-18 \times -\frac{3}{2} = \frac{54}{2} = 27$

$12 \underline{+} -13 = 25$        $12 + 13 = 25$

$12 \underline{+} 13 = -1$        $12 + -13 = -1$

$-25 \underline{\times} -\frac{2}{5} = 10$        $-25 \times -\frac{2}{5} = \frac{50}{5} = 10$

$-25 \underline{\div} \frac{1}{2} = -50$        $-25 \times 2 = -50$

# **G7 U4 Lesson 12**

Interpret situations involving rational numbers, including positive and negative values, and use rational numbers to represent and solve problems.

**G7 U4 Lesson 12 - Students will interpret situations involving rational numbers, including positive and negative values, and use rational numbers to represent and solve problems.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In some previous lessons we considered how signed numbers show up in problems about rates. You likely remember that a rate is a ratio with two quantities that have different units. Price per pound. Miles per gallon. Visits per year. Can you think of any other examples of rates you might see or hear in everyday life? **Possible Student Answers, Key Points:**

- I think about miles per hour or kilometers per hour when I think about the speed of a car or bus.
- If I work at a store, I might get paid hourly. Dollars per hour is a rate that would come in handy.
- Points per game is a rate that matters for football or basketball.

In today's lesson, we'll continue seeing how we can use positive and negative numbers to solve problems involving rates.

**Let's Talk (Slide 3):** Take a minute to look over the images here. Which images make you think of positive rates? Which images make you think of negative rates? Are there images that could represent positive *and* negative rates? **Possible Student Answers, Key Points:**

- The fire hydrant makes me think of a negative rate, because water is spraying out of it.
- The faucet and the gas pump make me think of positive rates, because they're filling things up.
- The money could be a positive or a negative rate. It depends on if they're depositing or withdrawing.
- The scuba diver and hot air balloon could be either. It depends on if they are going up or down.

Great thinking! As we work today, we'll want to carefully consider whether a given story involves a positive or negative rate based on the context. Then we'll use what we know about positive and negative numbers to help us answer questions.

**Let's Think (Slide 4):** I'm going to read through this first problem once. Then, I'll ask you to summarize what the problem is about in your own words. (*read problem*) What would you say this problem is about? What do we know? What is unknown? **Possible Student Answers, Key Points:**

- Adriana is saving money. She has some in her account already and she deposits the same amount each week for 9 weeks.
- We don't know the total after 9 weeks.

w	\$
0	25
1	39
2	53
3	67

I'm going to start with a table, just to make sure I'm clear on what is happening with the numbers in this story. (*draw t-chart labeled with weeks and \$, and write 0 - 3 in the column for weeks*) At the start of the story, 0 weeks, how much is in Adriana's account? (\$25) (*fill in dollar values as you and the student discuss*) One week later, I know she'll have \$14 more dollars based on the information in the story. After Week 1, I know that means she'll have \$39. She's going to keep depositing \$14 each week. How much will she have for Week 2 and Week 3? (\$53 and \$67) Let's pause there. From the table, we see that she starts with \$25, then she deposits \$14 per week as time goes on.

We can write this relationship using an equation. That might help us get to 9 weeks faster than continuing down our table. I know her total is equal to the starting amount plus her weekly deposits. (*write starting amount + \$14 per week = total*) I know her starting amount is \$25.

$$\text{starting amount} + \$14 \text{ per week} = \text{total}$$
$$\$25 + 14w = ?$$

She earns \$14 per week, which I'll represent as  $14w$ . I'll use a question mark to represent the unknown total. The equation that matches the story is  $25 + 14w = ?$ . (*write it*)

Why do you think I represented the weekly rate in the equation as  $14w$  instead of  $-14w$ ? [Possible Student Answers, Key Points:](#)

- She's gaining money, so it makes sense to think of it as a positive rate.
- $-14w$  would mean that she was losing or withdrawing money.

$$\begin{aligned}25 + 14(9) &= ? \\25 + 126 &= ? \\151 &= ? \\\$151\end{aligned}$$

Now, let's use our equation to find how much Adriana has in her account after 9 weeks. I'll substitute in a 9 for  $w$ . (*rewrite equation replacing  $w$  with 9*) Use any strategy to find  $14 \times 9$ , and let me know when you're ready. ( $14 \times 9 = 126$ ) So, she had \$25 in her account, and 14 dollars per week for 9 weeks is \$126 additional dollars. (*write  $25 + 126 = ?$* ) If I add those together, I know Adriana will have a total of \$151 in her account after 9 weeks.

We just solved a problem involving rates by using an equation. We started with a table. Tables can be helpful tools to think about how numbers are changing, but equations can often be more efficient. We could use this equation to quickly find any number of weeks simply by substituting for  $w$ . Want to know 14 weeks? Substitute 14 for  $w$ . Want to know 100 weeks? Substitute 100 for  $w$ . Equations are flexible tools that can help us represent relationships involving rates.

I think you're ready to try one more.

**Let's Think (Slide 5):** I'm going to read through this problem aloud. Like last time, I'll ask you to summarize what the problem is about in your own words. (*read problem*) What would you say this problem is about? What do we know? What is unknown? [Possible Student Answers, Key Points:](#)

- This problem is about elevation.
- A hiker starts at 50 feet and climbs down at 6 feet per minute.
- It's asking us to find the hiker's elevation at two different points.

We could represent this information in a table like the last problem, but let's see if we can just use an equation. To write an equation, it can be helpful to think of the story bit by bit and represent each part of the story as you go. In this case, I know the hiker started at an elevation, descended at a constant rate, and ended up at a different elevation. (*write starting elevation + feet per minute = final elevation*) Let's layer on specific information from the story now. What was the starting elevation? (50 feet) I'll start by writing 50. (*continue writing equation as you narrate*) Now I need to consider the rate. I'll use  $-6m$  to represent that the climber is *descending* 6 feet per minute, rather than  $+6m$  which would mean the hiker is ascending higher and higher. The equation  $50 - 6m = ?$  represents this scenario. It shows the hiker's starting height and the resulting change after climbing *down* at a rate of 6 feet per minute.

$$\begin{aligned}\text{starting elevation} + \text{ft per min.} &= \text{final elevation} \\50 + (-6)m &= ? \\50 - 6m &= ?\end{aligned}$$

$$\begin{aligned}50 - 6(2) &= ? \\50 - 12 &= 38 \text{ ft}\end{aligned}$$

Great! Now that we have the equation, we can use it to help us answer questions about the relationship. The first question asks us to find the hiker's elevation after 2 minutes. Let's substitute 2 in place of  $m$ . (*rewrite equation*) I know  $6 \times 2$  is 12 feet, so the hiker will have descended 12 feet during those 2 minutes.  $50 - 12$  means the final elevation after 2 minutes will be 38 feet.

What's will we do the same and what will we do different to find the hiker's elevation after 10 minutes?

[Possible Student Answers, Key Points:](#)

- We'll still substitute a value in for  $m$  and then do the math.
- We'll substitute 10 instead of 2 like we did before.

$$50 - 6(10) = ?$$
$$50 - 60 = -10 \text{ ft.}$$

(write equation substituting in 10 for  $m$ ) I know  $6 \times 10$  is 60. What is 50 minus 60? (-10) What does a solution of -10 mean in the context of this problem?

Possible Student Answers, Key Points:

- The hiker's elevation was 10 feet below sea level after hiking for 10 minutes.

Nice work. We just explored two contexts where we used rates involving positive or negative numbers.

**Let's Try it (Slides 6 - 7):** Now let's get a little more practice. Each context will be a bit different, so we'll want to consider carefully whether the rates involved should be positive or negative. We know tables can help us think about how our values change, but we saw today that equations can be particularly helpful in answering rate-related questions. Let's use all that we've just done to try a few more examples together before you show what you know independently.

# WARM WELCOME



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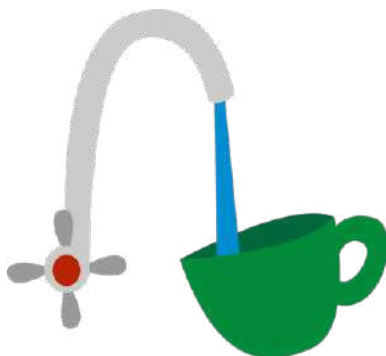
**Today we will interpret situations involving rational numbers, including positive and negative values, and use rational numbers to represent and solve problems.**

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## Let's Talk:

Which images might involve a positive rate?  
Which might involve a negative rate?



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## Let's Think:

Adriana has \$25 in her savings account. She deposits \$14 every week and does not make any withdrawals. How much money does she have in her savings after 9 weeks?

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## Let's Think:

A hiker is climbing at an elevation of 50 feet. If he descends 6 feet per minute, what will the hiker's elevation be after 2 minutes? 10 minutes?

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## Let's Try It:

Let's explore interpreting situations involving rational numbers and using rational numbers to represent and solve problems together.

Name: \_\_\_\_\_ 07 U4 Lesson 12 - Let's Try It

There are 225 gallons of gasoline in a tank at a gas station. The tank is being filled with a hose from a gasoline truck at a constant rate of 25 gallons per minute.

- Gasoline is being \_\_\_\_\_ the tank.
  - added to
  - drained from
- The hose is adding 25 gallons of water \_\_\_\_\_
  - every minute
  - one time
- How much gasoline will be in the gas tank after the hose has been filling it for 1 minute? Use an expression to show your thinking.
- How much gasoline will be in the tank after 2 minutes? Use an equation to show your thinking.
- Write an equation that can be used to find how much gasoline,  $y$ , will be in the tank after  $x$  minutes.
- Use your equation to find how much gasoline will be in the tank after 12 minutes.

The gas tank is now full at 525 gallons, so the truck with the hose drives away. Customers start buying gas, so the tank starts to lose gasoline at a rate of 5 meters per minute.

- Gasoline is being \_\_\_\_\_ the tank.
  - added to
  - drained from
- How much gasoline will be in the tank after 1 minute? Write an equation.
- How much gasoline will be in the tank after 2 minutes? Write an equation.

- Write an equation that can be used to find how much gasoline,  $g$ , will be in the tank after  $m$  minutes.
- Use your equation to find how much gasoline will be in the tank after 30 minutes.

DeJuan's bank account has \$305 in it. He spends \$12 to buy lunch at work every day. If he does not deposit or withdraw any more money, how much money will DeJuan have in his account after 5 days?

- How much will be in his account after 1 day? 2 days?
- Write an equation to represent the amount of money in the bank account,  $m$ , after  $d$  days.
- Use your equation to find how much money will be in the account after 10 days.
- Use your equation to find how much money will be in the account after 30 days.
- How much money will DeJuan need to deposit to bring his account balance back up to \$50?

An aquarium starts with 20,000 liters of water in it. A pump can either fill the water at a constant rate of 8 liters per minute, or it can drain the water at a rate of 8 liters per minute.

$$20,000 + 10 \cdot 8 \qquad 20,000 + 10 \cdot 8$$

- What does each expression represent in the context of the situation?

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## On your Own:

Now it's time to explore interpreting situations with rational numbers and using rational numbers to represent and solve problems on your own.

Name: \_\_\_\_\_ 57 U4 Lesson 12 - Independent Work

1. Harrison wants to buy a fancy dog bed for his puppy. Harrison currently has \$40 in his bank account and earns an allowance of \$25 per week. How much money will Harrison have after 7 weeks?

Was the rate in this scenario positive or negative? How do you know?

\_\_\_\_\_

\_\_\_\_\_

2. Patrice has \$66 in her account. She spends \$4 every day on coffee. If she does not make or spend any more money, how much will she have in her account after 19 weeks?

Was the rate in this scenario positive or negative? How do you know?

\_\_\_\_\_

\_\_\_\_\_

3. A clogged bathroom sink contains 90 ounces of water. Luisa unclogs the drain and water drains from the sink at a rate of 8 ounces per second. How many ounces are in the sink after 6 seconds?

Was the rate in this scenario positive or negative? How do you know?

\_\_\_\_\_

\_\_\_\_\_

4. A bank charges \$3.50 per month for a checking account. If Lucille's account has \$0, and no money is deposited or withdrawn, how many months will it take until her bank account is negative?

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**There are 225 gallons of gasoline in a tank at a gas station. The tank is being filled with a hose from a gasoline truck at a constant rate of 25 gallons per minute.**

1. Gasoline is being \_\_\_\_\_ the tank.
  - a. added to
  - b. drained from
  
2. The hose is adding 25 gallons of water \_\_\_\_\_.
  - a. every minute
  - b. one time
  
3. How much gasoline will be in the gas tank after the hose has been filling it for 1 minute? Use an expression to show your thinking.
  
4. How much gasoline will be in the tank after 2 minutes? Use an equation to show your thinking.
  
5. Write an equation that can be used to find how much gasoline,  $y$ , will be in the tank after  $x$  minutes.
  
6. Use your equation to find how much gasoline will be in the tank after 12 minutes.

**The gas tank is now full at 525 gallons, so the truck with the hose drives away. Customers start buying gas, so the tank starts to lose gasoline at a rate of 5 gallons per minute.**

7. Gasoline is being \_\_\_\_\_ the tank.
  - a. added to
  - b. drained from
  
8. How much gasoline will be in the tank after 1 minute? Write an equation.
  
9. How much gasoline will be in the tank after 2 minutes? Write an equation.

10. Write an equation that can be used to find how much gasoline,  $g$ , will be in the tank after  $m$  minutes.
11. Use your equation to find how much gasoline will be in the tank after 30 minutes.

**Dejuan's bank account has \$305 in it. He spends \$12 to buy lunch at work every day. If he does not deposit or withdraw any more money, how much money will Dejuan have in his account after 5 days?**

12. How much will be in his account after 1 day? 5 days?
13. Write an equation to represent the amount of money in the bank account,  $m$ , after  $d$  days.
14. Use your equation to find how much money will be in the account after 10 days.
15. Use your equation to find how much money will be in the account after 30 days.
16. How much money will Dejuan need to deposit to bring his account balance back up to \$0?

**An aquarium starts with 20,000 liters of water in it. A pump can either fill the water at a constant rate of 8 liters per minute, or it can drain the water at a rate of 8 liters per minute.**

$$20,000 + 10 \cdot -8$$

$$20,000 + 10 \cdot 8$$

17. What does each expression represent in the context of the situation?

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**1. Harrison wants to buy a fancy dog bed for his puppy. Harrison currently has \$42 in his bank account and earns an allowance of \$25 per week. How much money will Harrison have after 7 weeks?**

Was the rate in this scenario positive or negative? How do you know?

---

---

**2. Patrice has \$68 in her account. She spends \$4 every day on coffee. If she does not make or spend any more money, how much will she have in her account after 19 weeks?**

Was the rate in this scenario positive or negative? How do you know?

---

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**3. A clogged bathroom sink contains 90 ounces of water. Luisa unclogs the drain and water drains from the sink at a rate of 8 ounces per second. How many ounces are in the sink after 6 seconds?**

Was the rate in this scenario positive or negative? How do you know?

---

---

**4. A bank charges \$3.50 per month for a checking account. If Lucille's account has \$50, and no money is deposited or withdrawn, how many months will it take until her bank account is negative?**

Name: KEY

There are 225 gallons of gasoline in a tank at a gas station. The tank is being filled with a hose from a gasoline truck at a constant rate of 25 gallons per minute.

1. Gasoline is being \_\_\_\_\_ the tank.

- a. added to
- b. drained from

2. The hose is adding 25 gallons of water \_\_\_\_\_.

- a. every minute
- b. one time

3. How much gasoline will be in the gas tank after the hose has been filling it for 1 minute? Use an expression to show your thinking.

$$225 + 25$$

250 gallons

4. How much gasoline will be in the tank after 2 minutes? Use an equation to show your thinking.

5. Write an equation that can be used to find how much gasoline,  $y$ , will be in the tank after  $x$  minutes.

$$y = 225 + 25x$$

6. Use your equation to find how much gasoline will be in the tank after 12 minutes.

$$y = 225 + 25(12)$$
$$y = 225 + 300$$

525 gallons

The gas tank is now full at 525 gallons, so the truck with the hose drives away. Customers start buying gas, so the tank starts to lose gasoline at a rate of 5 <sup>gallons</sup> ~~gallons~~ per minute.

7. Gasoline is being \_\_\_\_\_ the tank.

- a. added to
- b. drained from

8. How much gasoline will be in the tank after 1 minute? Write an equation.

$$525 - 5 = 520$$

520 gallons

9. How much gasoline will be in the tank after 2 minutes? Write an equation.

$$525 - 2(5) = 515$$
$$525 - 10$$

515 gallons



10. Write an equation that can be used to find how much gasoline,  $g$ , will be in the tank after  $m$  minutes.

$$525 - 5m = g$$

11. Use your equation to find how much gasoline will be in the tank after 30 minutes.

$$525 - 5(30) = g$$

$$525 - 150 = g$$

$$\begin{array}{r} 45 \overline{) 525} \\ - 150 \\ \hline 375 \end{array}$$

375  
gallons

Dejuan's bank account has \$305 in it. He spends \$12 to buy lunch at work every day. If he does not deposit or withdraw any more money, how much money will Dejuan have in his account after 5 days?

12. How much will be in his account after 1 day? 2 days? 5 days?

$$305 - 12 = \$293$$

$$305 - 60 = \$245$$

13. Write an equation to represent the amount of money in the bank account,  $m$ , after  $d$  days.

$$305 - 12d = m$$

14. Use your equation to find how much money will be in the account after 10 days.

$$305 - 12(10) = m$$

$$305 - 120 = m$$

$$\$185 = m$$

15. Use your equation to find how much money will be in the account after 30 days.

$$305 - 12(30) = m$$

$$305 - 360 = m$$

$$-\$55$$

16. How much money will Dejuan need to deposit to bring his account balance back up to \$0?

$$-55 + 55 = 0$$

$$\$55$$

An aquarium starts with 20,000 liters of water in it. A pump can either fill the water at a constant rate of 8 liters per minute, or it can drain the water at a rate of 8 liters per minute.

$$20,000 + 10 \cdot -8$$

$$20,000 + 10 \cdot 8$$

17. What does each expression represent in the context of the situation?

The first equation represents the liters after draining for 10 minutes. The second equation represents liters after filling for 10 minutes.

1. Harrison wants to buy a fancy dog bed for his puppy. Harrison currently has \$42 in his bank account and earns an allowance of \$25 per week. How much money will Harrison have after 7 weeks?

$$42 + 25w = ?$$

$$42 + 25(7) = ?$$

$$42 + 175 = ?$$

$$(\$217)$$

$$\begin{array}{r} 175 \\ 42 \\ \hline 217 \end{array}$$

Was the rate in this scenario positive or negative? How do you know?

It is positive. Harrison is earning money.

2. Patrice has \$68 in her account. She spends \$4 every day on coffee. If she does not make or spend any more money, how much will she have in her account after 19 weeks?

$$68 - 4x = ?$$

$$68 - 4(19) = ?$$

$$68 - 76 = ?$$

$$(-\$8)$$

Was the rate in this scenario positive or negative? How do you know?

The rate was negative, because she spends money.



3. A clogged bathroom sink contains 90 ounces of water. Luisa unclogs the drain and water drains from the sink at a rate of 8 ounces per second. How many ounces are in the sink after 6 seconds?

$$90 - 8x = ?$$

$$90 - 8(6)$$

$$90 - 48$$

$$\boxed{42 \text{ oz}}$$

Was the rate in this scenario positive or negative? How do you know?

It was negative because the sink is losing water.

4. A bank charges \$3.50 per month for a checking account. If Lucille's account has \$50, and no money is deposited or withdrawn, how many months will it take until her bank account is negative?

$$50 - 3.5x = 0$$

$$50 \div 3.5 = ?$$

$$\begin{array}{r} 14 \\ 35 \overline{) 500} \\ \underline{35} \phantom{0} \\ 150 \\ \underline{-140} \\ 10 \end{array}$$

$$14 \frac{10}{35} \text{ or } 14 \frac{2}{7} \text{ months}$$

It would take 15 months to reach a negative value

# **G7 U4 Lesson 13**

Solve equations that involve negative numbers.

## G7 U4 Lesson 13 - Students will solve equations that involve negative numbers.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** You've been solving equations since 6th grade or earlier. Today, we're going to use everything we've been learning about working with positive and negative numbers to help solve one-step equations that involve negative numbers. There are many ways to solve one-step equations, but today we'll lean into writing related facts to help arrive at a solution.

Before we dive into negative numbers, let's refresh a bit by looking at equations with positive numbers.

**Let's Talk (Slide 3):** As I named, there are many ways to think about solving equations. Take a few moments, and see if you can determine what number could make each equation true. [Possible Student Answers, Key Points:](#)

- I know  $x = 4$ , because  $9 + 4$  is equal to 13.
- I know  $y = 11$ , because  $18 - 7 = 11$ .
- I know  $h = 3$ , because 3 groups of 2 makes 6.
- I know  $k = 4$ , because  $5 \times 4$  is the same as 20.

Great thinking! There are many ways to solve equations. I want us to think about related facts for a moment...

Let's consider how related facts could help us think about each of the solutions here. I'll use  $9 + x = 13$  as my first example. I know in this equation, 13 is my total and  $x$  and 4 are my parts that compose the total. Rather than write  $9 + x = 13$ , I can think of this as the total minus the part I know is equal to the part I don't know. I can write that as  $13 - 9 = x$ . (*write equation*) Rewriting this as a related fact, helps confirm that  $x$  must equal 4.

$$\begin{array}{l} 13 - 9 = x \\ 4 = x \end{array}$$

What about  $y + 7 = 18$ ? Can you think of a related subtraction fact that could help us prove that  $y = 11$ ? [Possible Student Answers, Key Points:](#)

- I know  $y$  and 7 are the parts in this relationship, and 18 is the total. I can write  $18 - 7 = y$  as a related fact.

$$\begin{array}{l} 18 - 7 = y \\ 11 = y \end{array}$$

(*write equation*) To help us solve this equation, we can use the related fact  $18 - 7 = y$ . We can see that  $y = 11$ .

Related facts can also help us solve multiplication equations. We can think of a related division fact that can help us arrive at a solution. Let's think about  $h \cdot 2 = 6$ . I know  $h$  groups of 2 is equal to 6. I can think of this as 6 divided by 2 is equal to something. (*write equation*) So  $h$  is equal to 3.

$$\begin{array}{l} 6 \div 2 = h \\ 3 = h \end{array}$$

How can I use similar thinking to think about finding  $k$ ? [Possible Student Answers, Key Points:](#)

- I know 5 times something is equal to 20. So I can think of 20 divided by 5 to help me find the unknown.

$$\begin{array}{l} 20 \div 5 = k \\ 4 = k \end{array}$$

(*write related fact*) 20 divided by 5 equals  $k$  can help us arrive at our solution. 20 divided by 5 is equal to 4.

Related facts can be a helpful way to solve various one-step equations. Let's see if that idea stays true when we consider negative numbers.

**Let's Think (Slide 4):** Let's start by solving two addition problems.

$$3 - 5 = x$$
$$-2 = x$$

We'll start by considering the solution to 5 plus  $x$  equals 3. In this example 5 and  $x$  are the parts that make 3. I can rewrite this as a related number sentence. (*write  $3 - 5 = x$* ) Now, all I have to do is think about the value of 3 minus 5. What is the value of  $3 - 5$ ? How do you know? **Possible Student Answers, Key Points:**

- 3 minus 5 is -2.
- I can think of this as 3 take away 5. I can also think of this as the difference between 3 and 5. Either way,  $x = -2$ .

$$y - 2 = 8.5$$
$$8.5 + 2 = y$$
$$10.5 = y$$

Let's look at the next example. This equation says  $y$  plus -2 is equal to 8.5. Let's start by rewriting this as  $y - 2 = 8.5$ , since adding -2 is the same as subtracting positive 2. (*write equation*) Based on this equation, we can still use a related fact to help us find the value of  $y$ . In the rewritten equation, I know that  $y$  is the total and 2 and 8.5 are the parts. So we can rewrite this as  $8.5 + 2 = y$ . (*rewrite equation*) Based on our related equation, we can add 8.5 and 2 to find the value of  $y$ . Take a moment, and let me know the value of  $y$  when you're ready. (10.5) Nice work.  $y = 10.5$

**Let's Think (Slide 5):** Now we'll use some similar thinking to consider two multiplication equations.

$$-5 \div \frac{1}{2} = m$$
$$-5 \times 2 = m$$
$$-10 = m$$

The first equation says  $\frac{1}{2}$  times an unknown number,  $m$ , equals -5. I know that I can write a related division fact to represent this multiplication equation. If  $\frac{1}{2} \times m = -5$ , then I know -5 divided by  $\frac{1}{2}$  is equal to  $m$ . (*write related fact*) To solve for the unknown, I can divide -5 by  $\frac{1}{2}$ . Thinking back to our last lesson, this might be easier to think about if we use the multiplicative inverse. Dividing by  $\frac{1}{2}$  is the same as multiplying by the multiplicative inverse of 2. What is -5 times 2? (-10) Correct. Negative 5 times positive 2 is equal to -10. Even though this original problem involved fractions, writing a related fact was helpful in terms of arriving at a solution efficiently.

$$-9 \div -2 = n$$
$$\frac{9}{2} = n$$
$$4\frac{1}{2} = n$$

Let's look at one more example. How could I use a related fact to think about this equation? **Possible Student Answers, Key Points:**

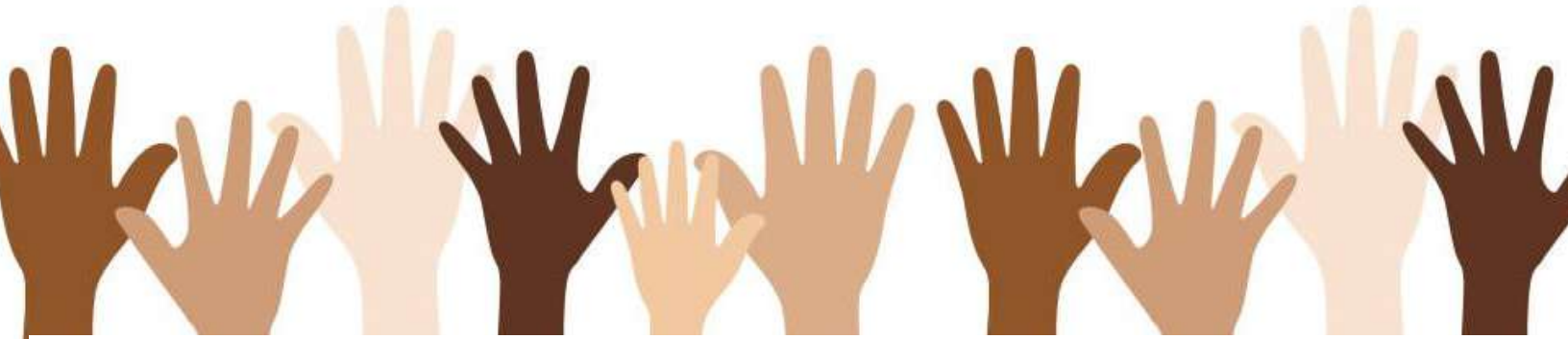
- We can write a related division fact to represent this multiplication equation.
- If -2 times a number equals -9, then I know -9 divided by -2 is equal to that number.

(*write -9 divided by -2 = n*) This related fact can help us solve the original multiplication equation. -9 divided by -2 aren't necessarily friendly numbers. Even before we consider solving, what do we know about a negative number divided by a negative number? (It will result in a positive number) So -9 divided by -2 will be a positive value. We can think of 9 divided by 2 as 9 over 2, or  $\frac{9}{2}$ . The solution to this equation is  $\frac{9}{2}$  or  $4\frac{1}{2}$  depending on whether you prefer a fraction greater than 1 or a mixed number.

Writing related facts can be immensely helpful when solving one-step equations.

**Let's Try it (Slides 6 - 7):** Now let's use what we've practiced to solve more one-step equations involving negative numbers. We've seen that writing related facts can help us arrive at an answer efficiently. We can write subtraction facts to think about addition equations. We can write division facts to think about multiplication equations. Let's apply what we've been practicing on a few more examples together.

# WARM WELCOME



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**Today we will solve equations that  
involve negative numbers.**

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Let's Talk:

What is each missing value? How do you know?

$$9 + x = 13$$

$$h \cdot 2 = 6$$

$$y + 7 = 18$$

$$5(k) = 20$$

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Let's Think:

Solve each equation.

$$5 + x = 3$$

$$y + (-2) = 8.5$$

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## Let's Think:

Solve each equation.

$$\left(\frac{1}{2}\right)m = -5$$

$$-9 = -2n$$

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## Let's Try It:

Let's explore solving equations that involve negative numbers together.

Name: \_\_\_\_\_ G7 U4 Lesson 13 - Let's Try It

Find the **additive inverse** of each number.

- 8
- 11.9
- 11
- 16

Find the **multiplicative inverse** of each number.

- 12
- 16
- $\frac{1}{10}$
- 9

Consider the equation  $x + 3 = 9$ .

- What is the value of  $x$ ?
- Write a related subtraction equation you could use to find  $x$ .
- Rewrite the related subtraction equation using the additive inverse.

Consider the equation  $x - (-3) = 9$ .

- Write a related subtraction equation you could use to find  $x$ .
- Determine the value of  $x$ .

Consider the equation  $10 + m = 6$ .

- Write a related subtraction equation you could use to find  $m$ . Determine the value of  $m$ .

Consider the equation  $y + (-6.4) = 13.4$ .

- Write a related subtraction equation you could use to find  $y$ . Determine the value of  $y$ .

Now let's think about multiplication. Consider the equation  $3 \cdot w = 18$ .

- What is the value of  $w$ ?
- Write a related division equation you could use to find  $w$ .
- Rewrite the related division equation using the multiplicative inverse.

Consider the equation  $x \cdot (-2) = -14$ .

- Write a related division equation you could use to find  $x$ .
- Determine the value of  $x$ .

Use the **multiplicative inverse** to rewrite and solve each equation.

$$\frac{1}{2}x = 30 \qquad n + (-0.8) = 5.4$$

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# On your Own:

Now it's time to solve equations that involve negative numbers on your own.

Name: \_\_\_\_\_ G7 (J4) Lesson 13 - Independent Work

1. Fill in the table to show the additive inverse and the multiplicative inverse of each number.

NUMBER	ADDITIVE INVERSE	MULTIPLICATIVE INVERSE
9		
$-4/5$		
-1.7		
$1/3$		

2. Rewrite each equation as a related subtraction equation. Then solve for  $x$ .

$5 + x = 19$        $x + (-7.2) = 10.5$

3. Rewrite each equation as a related subtraction equation. Then solve for  $y$ .

$-5 + y = 48$        $(-3)y = 6$

4. Gabriella was solving the equation below:

$$\frac{2}{3}x = -18$$

$$-18 = \frac{2}{3}x$$

$$-18 \times \frac{3}{2} = x$$

$$\frac{54}{2} = x$$

$$27 = x$$

Explain her mistake, and include the correct solution in your response.

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Name: \_\_\_\_\_

Find the additive inverse of each number.

1. -8
2. 11.9
3.  $\frac{1}{4}$
4.  $-\frac{1}{8}$

Find the multiplicative inverse of each number.

5. 12
6.  $-\frac{1}{5}$
7.  $\frac{1}{10}$
8. -9

Consider the equation  $x + 3 = 9$ .

9. What is the value of  $x$ ?
10. Write a related subtraction equation you could use to find  $x$ .
11. Rewrite the related subtraction equation using the additive inverse.

Consider the equation  $x + (-3) = 9$ .

12. Write a related subtraction equation you could use to find  $x$ .
13. Determine the value of  $x$ .

**Consider the equation  $10 + m = 6$ .**

14. Write a related subtraction equation you could use to find  $m$ . Determine the value of  $m$ .

**Consider the equation  $y + (-6.4) = 13.4$ .**

15. Write a related subtraction equation you could use to find  $y$ . Determine the value of  $y$ .

**Now let's think about multiplication. Consider the equation  $3 \cdot w = 18$ .**

16. What is the value of  $w$ ?

17. Write a related division equation you could use to find  $w$ .

18. Rewrite the related division equation using the multiplicative inverse.

**Consider the equation  $x \cdot (-2) = -14$ .**

19. Write a related division equation you could use to find  $x$ .

20. Determine the value of  $x$ .

**Use the multiplicative inverse to rewrite and solve each equation.**

$$\frac{2}{5}x = 30$$

$$n \cdot (-3.6) = 5.4$$

1. Fill in the table to show the additive inverse and the multiplicative inverse of each number.

NUMBER	ADDITIVE INVERSE	MULTIPLICATIVE INVERSE
9		
$-4/5$		
-1.7		
$1/3$		

2. Rewrite each equation as a related subtraction equation. Then solve for x.

$$6 + x = 19$$

$$x + (-7.5) = 10.5$$

3. Rewrite each equation as a related division equation. Then solve for y.

$$-6 \cdot y = 48$$

$$(-\frac{1}{4})y = 8$$

4. Gabriella was solving the equation below.

$$\frac{2}{3}x = -18$$

$$-18 \div \frac{2}{3} = x$$

$$-18 \times -\frac{3}{2} = x$$

$$\frac{54}{2} = x$$

$$27 = x$$

Explain her mistake, and include the correct solution in your response.

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Name: KEY

Find the additive inverse of each number.

1. -8

(8)

2. 11.9

(-11.9)

3.  $\frac{1}{4}$

( $-\frac{1}{4}$ )

4.  $-\frac{1}{8}$

( $\frac{1}{8}$ )

Find the multiplicative inverse of each number.

5. 12

( $\frac{1}{12}$ )

6.  $-\frac{1}{5}$

(-5)

7.  $\frac{1}{10}$

(10)

8. -9

( $-\frac{1}{9}$ )

Consider the equation  $x + 3 = 9$ .

9. What is the value of x?

( $x = 6$ )

10. Write a related subtraction equation you could use to find x.

$9 - 3 = x$

11. Rewrite the related subtraction equation using the additive inverse.

$9 + (-3) = x$

Consider the equation  $x + (-3) = 9$ .

12. Write a related subtraction equation you could use to find x.

$9 - (-3) = x$

13. Determine the value of x.

$9 + 3 = 12$      $x = (12)$

Consider the equation  $10 + m = 6$ .

14. Write a related subtraction equation you could use to find  $m$ . Determine the value of  $m$ .

$$6 - 10 = m$$
$$\boxed{-4 = m}$$

Consider the equation  $y + (-6.4) = 13.4$ .

15. Write a related subtraction equation you could use to find  $y$ . Determine the value of  $y$ .

$$13.4 - (-6.4) = y$$
$$13.4 + 6.4 = y$$
$$\boxed{y = 19.8}$$

Now let's think about multiplication. Consider the equation  $3 \cdot w = 18$ .

16. What is the value of  $w$ ?

$$\boxed{w = 6}$$

17. Write a related division equation you could use to find  $w$ .

$$18 \div 3 = w$$

18. Rewrite the related division equation using the multiplicative inverse.

$$18 \times \frac{1}{3} = \frac{18}{3} = 6$$
$$\boxed{w = 6}$$

Consider the equation  $x \cdot (-2) = -14$ .

19. Write a related division equation you could use to find  $x$ .

$$-14 \div -2 = x$$

20. Determine the value of  $x$

$$\boxed{x = 7}$$

Use the multiplicative inverse to rewrite and solve each equation.

$$\frac{2}{5}x = 30$$

$$30 \div \frac{2}{5} = x$$

$$30 \times \frac{5}{2} = \frac{150}{2}$$

$$\boxed{x = 75}$$

$$n \cdot (-3.6) = 5.4$$

$$5.4 \div -3.6 = n$$

$$5.4 \times -\frac{1}{3.6} = n$$

$$-\frac{5.4}{3.6} = n$$

$$\boxed{-1.5 = n}$$

$$\begin{array}{r} 36 \overline{) 540} \\ \underline{-360} \phantom{0} \\ 180 \end{array}$$

1. Fill in the table to show the additive inverse and the multiplicative inverse of each number.

NUMBER	ADDITIVE INVERSE	MULTIPLICATIVE INVERSE
9	$-9$	$\frac{1}{9}$
$-\frac{4}{5}$	$+\frac{4}{5}$	$-\frac{5}{4}$
-1.7	$+1.7$	$-\frac{1}{1.7}$
$\frac{1}{3}$	$-\frac{1}{3}$	3

2. Rewrite each equation as a related subtraction equation. Then solve for x.

$$6 + x = 19$$

$$19 - 6 = x$$

$$\boxed{13 = x}$$

$$x + (-7.5) = 10.5$$

$$10.5 - (-7.5) = x$$

$$10.5 + 7.5 = x$$

$$\boxed{18 = x}$$



multiplication  
↓  
division

3. Rewrite each equation as a related subtraction equation. Then solve for y.

$$-6 \cdot y = 48$$

$$48 \div -6 = y$$

$$\boxed{-8 = y}$$

$$(-\frac{1}{4})y = 8$$

$$8 \div -\frac{1}{4} = y$$

$$8 \times -4 = y$$

$$\boxed{-32 = y}$$

4. Gabriella was solving the equation below.

$$\frac{2}{3}x = -18$$

$$-18 \div \frac{2}{3} = x$$

$$-18 \times \frac{-3}{2} = x$$

$$\frac{54}{2} = x$$

$$\boxed{27 = x}$$

$$-18 \times \frac{3}{2} = \frac{-54}{2} = -27$$

Explain her mistake, and include the correct solution in your response.

She used the wrong multiplicative inverse.  $\frac{3}{2}$  is the correct multiplicative inverse of  $\frac{2}{3}$ . The correct answer is  $-27 = x$ .

# **G7 U4 Lesson 14**

Write and solve equations to represent situations that involve negative numbers.

**G7 U4 Lesson 14 - Students will write and solve equations to represent situations that involve negative numbers.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We're almost at the end of our unit that's been all about rational number arithmetic. We've learned how to add, subtract, multiply, and divide with negative values. We've learned how to use negative numbers in contexts like temperature, elevation, and money. Today we're going to keep pulling all these ideas we've been working with together to help us write and solve equations to represent situations involving negative numbers. Let's get started.

**Let's Talk (Slide 3):** Here we see two equations without a context. To get us warmed up, think about how you could solve the first equation. When you have an idea, share it. **Possible Student Answers, Key Points:**

- I can think about -9 plus what number would get me to positive 1 on a number line. I know it would take 10 hops to get to 1, so  $a = 10$ .
- I could rewrite the problem as a related subtraction fact and solve for the unknown.

$$\begin{aligned}1 - (-9) &= a \\1 + 9 &= a \\10 &= a\end{aligned}$$

Let's rewrite this addition equation as a related subtraction equation to help us solve. (*write as you narrate*) I know -9 and  $a$  are my parts that total positive 1. I can rewrite that as subtraction, starting with the total. 1 minus -9 is equal to  $a$ . Instead of minus -9, which can be tricky to think about, I'll use the additive inverse and add +9.  $1 + 9 = a$ , so  $a = 10$ .

Now think about how you could solve the second equation. When you have an idea, share it. **Possible Student Answers, Key Points:**

- I know -6 times -5 would equal +30, so  $c$  must equal -5.
- I can rewrite as a related division fact to help me solve the multiplication equation.

$$\begin{aligned}30 \div -6 &= c \\-5 &= c \\30 \times -\frac{1}{6} &= -5\end{aligned}$$

Let's again use a related fact to help us find the unknown value. Since the equation involves multiplication, I'll use division to write the related fact. (*write as you narrate*) I know +30 divided by -5 will equal  $c$ . That means  $c$  must equal -5.

I could also use the multiplicative inverse to help me solve. The multiplicative inverse of -6 is  $-\frac{1}{6}$ .  $-30 \times -\frac{1}{6}$  is equal to  $30/6$ .  $30/6$  is still equal to 5. Either way, dividing by -6 or multiplying by  $-\frac{1}{6}$ , will help us arrive at the correct solution.

Let's take this thinking and use it in contexts from the world around us.

**Let's Think (Slide 4):** I'll read this problem once through, then I want you to summarize what the story is about in your own words. (*read problem*) **Possible Student Answers, Key Points:**

- The story is about a hiker. She starts at an elevation of 0 feet and we want to know how long it will take her to reach -72 feet.
- Raquel is descending 8 feet every minute from sea level, and we need to figure out how long it will take her to arrive at an elevation of -72 feet.

$$-8m = -72$$

There are many ways to solve this problem, but we'll use an equation since that's what we've been working with a lot lately. I know from our previous work with rates, that I can represent Raquel's descent of 8 feet per minute as the expression  $-8m$ . I'll write  $-8m = -72$  to represent this problem. (*write equation*) Why do you think I used a

negative value to help represent the rate and a negative value to help represent the total in the equation?

Possible Student Answers, Key Points:

- The  $-8m$  makes sense because she is climbing down. If she was climbing up, we might use  $+8m$  instead.
- The  $-72$  makes sense, because she ended up at 72 feet below sea level. If she ended up at 72 feet above sea level, we would have used  $+72$ .

$$\begin{aligned} -72 \div -8 &= 9 \\ -72 \times \frac{-1}{8} &= \frac{72}{8} = 9 \end{aligned}$$

Let's solve this equation to find how long it will take Raquel to reach an elevation  $-72$  feet. We can use a related fact to help us solve. I'll rewrite the multiplication equation using division. I know  $-72$  divided by  $-8$  will equal  $m$ . (write equation) What will  $m$  equal? (positive 9) The solution is  $+9$ . I know  $72$  divided by  $8$  is  $9$ , and a negative value divided by a negative value, results in a positive value.

We could have also used the multiplicative inverse to solve this, depending on what we prefer. What's the multiplicative inverse of  $-8$ ? ( $-\frac{1}{8}$ ) Instead of  $-72$  divided by  $-8$ , I can multiply  $-72$  by the multiplicative inverse.  $-72$  times  $-\frac{1}{8}$  is equal to positive  $72/8$ , or positive  $9$ . Either solution pathway can help us arrive at our correct answer.

What does a solution of  $9$  represent in the context of this problem? Go back to the original question, if that helps. Possible Student Answers, Key Points:

- The question asked for how many minutes it will take Raquel to get to the elevation of  $72$  feet below sea level. A solution of  $9$  means it takes her  $9$  minutes to travel that distance.

Excellent work. Let's look at one more problem with a different context.

**Let's Think (Slide 5):** Both of these questions we see will involve thinking about changes in Tina's bank account. Let's start with part A. What is known? What is unknown? Possible Student Answers, Key Points:

- We know her bank balance was lower on Monday and higher on Tuesday. Her balance started at  $-10$  and increased up to  $+15$ .
- We don't know how much it changed. The unknown is the change in her balance.

$$-10 + x = 15$$

We can use an equation to represent this story. (write equation as you narrate) You noticed that she started with a balance of  $-10$  dollars. She added some money, but we're not sure how much. I'll use " $+x$ " to represent that unknown amount. Once she added that money, her bank

account total was  $+15$ . I'll write  $= 15$  in my equation to represent that.  $-10$  plus  $x$  equals  $15$  is one way to represent this situation using an equation. Let's solve!

$$\begin{aligned} 15 - (-10) &= x \\ 15 + 10 &= 25 \end{aligned}$$

I'll rewrite this addition equation as a related subtraction equation. I can take the total,  $15$ , and take away the part I know,  $-10$ , and that will give me the other part. I'll write that as  $15$  minus  $-10$  equals  $x$ . (write equation) Instead of subtracting  $-10$ , which can be a little tricky to wrap my head around, I'll add the additive inverse of  $+10$ .  $15 + 10 = x$ , so I know  $x = 25$ . What does a solution of  $25$  mean in the

context of this problem? Possible Student Answers, Key Points:

- The question was asking for the change in her balance. That means Tina deposited  $\$25$  to increase her balance from Monday to Tuesday.

Nice work. We'll keep thinking about Tina and her money as we look at part B. (read question) This problem is a little different. In this case, we don't know the starting amount. We know she deposited, or added,  $\$15$  into the account and ended up with a total of  $\$10$ . We can use an equation to represent this relationship. (write equation as you narrate) I'll use  $x$  to represent the unknown starting balance. I'll write  $+15$  to represent

$$x + 15 = 10$$



the amount she deposited into her account. Then I know the total equals 10 dollars, so I can write  $x + 15 = 10$ .  $x + 15 = 10$  represents the given story. How can we use a related equation to solve for  $x$ ? Possible Student

Answers, Key Points:

- If  $x$  and 15 are the parts and 10 is the total, I can take the total and subtract a part I know to find the other part.
- $10 - 15 = x$  is a related equation I can use.

I know 10 minus 15 is -5. I could also use the additive inverse and think  $10 + (-15)$  is -5. In either case, my solution is  $x = -5$ . What does a solution of -5 mean in the context of this story problem? Possible Student

Answers, Key Points:

- We were trying to find the unknown starting balance, so this means her balance was -5 to begin with. That means she owed the bank 5 dollars before she deposited the 15 dollars.

We've just used equations to solve a variety of real-world problems involving negative numbers. Excellent work!

**Let's Try it (Slides 6 - 7):** We'll tackle a few more collaboratively before you get a chance to try some on your own. Like we've been doing, after reading a word problem, we'll pause to think about what we know and what is unknown. From there, we'll use the information in the problem to write an equation, using a variable for our unknown. Then, as we saw today, it can be helpful to solve equations by writing a related fact and/or using the additive or multiplicative inverse to arrive at our solution. Throughout this, pay attention to whether it makes more sense to represent values as positive or negative based on the story. Let's give it a try.

# WARM WELCOME



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**Today we will write and solve equations to represent situations that involve negative numbers.**

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Let's Talk:

How can we solve each equation?

$$-9 + a = 1$$

$$-6(c) = 30$$

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Let's Think:

**Raquel begins hiking at sea level, and she descends 8 feet every minute. How many minutes will it take her to reach an elevation of -72 feet?**

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## Let's Think:

Write and solve an equation to represent each situation.

- A) Tina's bank balance on Monday was  $-\$10$ . On Tuesday, her bank balance was  $\$15$ . How did her account balance change?
- B) Tina deposited  $\$15$  into her bank account, and now her balance is  $\$10$ . What was her starting balance?

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## Let's Try It:

Let's explore writing and solving equations to represent situations that involve negative numbers together.

Name: \_\_\_\_\_ G7/U4 Lesson 14 – Let's Try It

A submarine is at sea level. It descends toward the ocean floor at a rate of 36 yards per minute. The submarine plans to cruise at a final elevation of  $-385$  yards.

- What is the submarine's starting elevation?
- The submarine's elevation is \_\_\_\_\_
  - increasing
  - decreasing
- Complete the equation below to represent the submarine's descent. The  $m$  represents the number of minutes.
 
$$\frac{\text{rate}}{\text{rate}} \cdot m = \frac{\text{final elevation}}{\text{final elevation}}$$
- Rewrite the equation as a related division equation and solve.
- Solve the equation another way. This time, use the multiplicative inverse of  $-35$ .
- How many minutes will it take the submarine to reach an elevation of  $-385$  yards?

Zaire starts her hike at an elevation of  $-150$  feet. The trail ends at an elevation of  $75$  feet. She wants to determine the change in elevation from the start of the trail to the end of the trail.

- What is Zaire's starting elevation?
- What is Zaire's ending elevation?
- What is the unknown in this story?
- Write an addition equation to represent the situation. Use  $d$  to represent the trail's distance.

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11. Rewrite the equation as a related subtraction equation. Then solve.

12. What does your answer represent?
 

- Zaire's final elevation
- The rate Zaire hiked
- The distance Zaire hiked

Match each equation to the situation to an equation. Then solve.

13. The temperature was  $-4$  degrees in the morning, and it rose to  $8$  degrees by the afternoon. What is the change in temperature?  $-4x = -20$

14. The temperature was  $9$  degrees in the morning, and it fell to  $-4$  degrees by the afternoon. What is the change in temperature?  $9 = x - 4$

15. A turtle dives down toward the ocean floor. After 4 minutes, it was  $20$  meters below the surface. At what rate was it diving?  $-4 + x = 9$

16. A barracuda dives toward the ocean floor at a rate of  $4$  meters per second. How long will it take the barracuda to get from an elevation of  $0$  meters to an elevation of  $-20$  meters?  $Ax = -20$

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# On your Own:

Now it's time to write and solve equations to represent situations that involve negative numbers on your own.

Name: \_\_\_\_\_ 67 U4 Lesson 14 - Independent Work

1. The temperature has been dropping 4 degrees Fahrenheit every hour. If the current temperature is -18 degrees, how many hours ago was the temperature at 0 Degree Fahrenheit? Write and solve an equation. Explain what your variable represents.

2. Mr. Angel is rock climbing. He begins at sea level and climbs up 7 feet every minute. How many minutes will it take Mr. Angel to reach an elevation of 140 feet?

a. Write an equation to represent the situation. Use  $x$  to represent the number of minutes.

b. Solve the equation.

c. Write an answer sentence explaining what your solution means in the context of the problem.

3. A falcon is soaring at an elevation of 300 feet. It descends to an elevation of -25 feet. What is the falcon's change in elevation? Write and solve an equation to solve the problem. Include an answer sentence to explain what your solution means in the context of the problem.

4. Match each equation to the situation to an equation. Then solve.

An octopus dives at a rate of 5 feet per second. How long will it take for the octopus to get from the surface to an elevation of -30 feet?  $-5t = -30$

A squid dives toward the ocean floor, starting at sea level. After 5 minutes, the squid is 30 meters below sea level. At what rate was the squid diving?  $-12 + k = 11$

The temperature was -12 degrees and rose to 1 degree. What was the change in temperature?  $1 + b = -12$

The temperature was 1 degree and fell to -12 degrees. What was the change in temperature?  $6n = -80$

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**A submarine is at sea level. It descends toward the ocean floor at a rate of 35 yards per minute. The submarine plans to cruise at a final elevation of -385 yards.**

1. What is the submarine's starting elevation?
2. The submarine's elevation is \_\_\_\_\_.
  - a. increasing
  - b. decreasing
3. Complete the equation below to represent the submarine's descent. The  $m$  represents the number of minutes.

$$\frac{\quad}{\text{rate}} \cdot m = \frac{\quad}{\text{final elevation}}$$

4. Rewrite the equation as a related division equation and solve.
5. Solve the equation another way. This time, use the multiplicative inverse of -35.
6. How many minutes will it take the submarine to reach an elevation of -385 yards?

**Zaire starts her hike at an elevation of -150 feet. The trail ends at an elevation of 75 feet. She wants to determine the change in elevation from the start of the trail to the end of the trail.**

7. What is Zaire's starting elevation?
8. What is Zaire's ending elevation?
9. What is the unknown in this story?
10. Write an addition equation to represent the situation. Use  $d$  to represent the trail's distance.

11. Rewrite the equation as a related subtraction equation. Then solve.

12. What does your answer represent?

- a. Zaire's final elevation
- b. The rate Zaire hiked
- c. The distance Zaire hiked

**Match each equation to the situation to an equation. Then solve.**

13. The temperature was  $-4$  degrees in the morning, and it rose to  $9$  degrees by the afternoon. What is the change in temperature?

$$-4x = -20$$

14. The temperature was  $9$  degrees in the morning, and it fell to  $-4$  degrees by the afternoon. What is the change in temperature?

$$9 + x = -4$$

15. A turtle dives down toward the ocean floor. After  $4$  minutes, it was  $20$  meters below the surface. At what rate was it diving?

$$-4 + x = 9$$

16. A barracuda dives toward the ocean floor at a rate of  $4$  meters per second. How long will it take the barracuda to get from an elevation of  $0$  meters to an elevation of  $-20$  meters?

$$4x = -20$$



1. **The temperature has been dropping 4 degrees Fahrenheit every hour. If the current temperature is -16 degrees, how many hours ago was the temperature at 0 degrees Fahrenheit?** Write and solve an equation. Explain what your variable represents.

2. **Mr. Angel is rock climbing. He begins at sea level and climbs up 7 feet every minute. How many minutes will it take Mr. Angel to reach an elevation of 140 feet?**

a. Write an equation to represent the situation. Use  $x$  to represent the number of minutes.

b. Solve the equation.

c. Write an answer sentence explaining what your solution means in the context of the problem.

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**3. A falcon is soaring at an elevation of 300 feet. It descends to an elevation of -25 feet. What is the falcon's change in elevation?** Write and solve an equation to solve the problem. Include an answer sentence to explain what your solution means in the context of the problem.

**4. Match each equation to the situation to an equation. Then solve.**

An octopus dives at a rate of 5 feet per second.  
How long will it take for the octopus to get from the surface to an elevation of -30 feet?

$$-5n = -30$$

A squid dives toward the ocean floor, starting at sea level. After 5 minutes, the squid is 30 meters below sea level. At what rate was the squid diving?

$$-12 + k = 1$$

The temperature was -12 degrees and rose to 1 degree. What was the change in temperature?

$$1 + k = -12$$

The temperature was 1 degree and fell to -12 degrees. What was the change in temperature?

$$5n = -30$$

A submarine is at sea level. It descends toward the ocean floor at a rate of 35 yards per minute. The submarine plans to cruise at a final elevation of -385 yards.

1. What is the submarine's starting elevation? (0 yd)
2. The submarine's elevation is \_\_\_\_\_.
  - a. increasing
  - b. decreasing
3. Complete the equation below to represent the submarine's descent. The  $m$  represents the number of minutes.

$$\frac{-35}{\text{rate}} \cdot m = \frac{-385}{\text{final elevation}}$$

4. Rewrite the equation as a related division equation and solve.

$$-385 \div -35 = m$$

$$\text{(11 = m)}$$

$$\begin{array}{r} 11 \\ 35 \overline{) 385} \\ \underline{35} \phantom{0} \\ 35 \\ \underline{-35} \\ 0 \end{array}$$

5. Solve the equation another way. This time, use the multiplicative inverse of -35.

$$-385 \times -\frac{1}{35} = \frac{385}{35} = 11$$

$$\text{(m = 11)}$$

6. How many minutes will it take the submarine to reach an elevation of -385 yards?

$$\text{(11 minutes)}$$

Zaire starts her hike at an elevation of -150 feet. The trail ends at an elevation of 75 feet. She wants to determine the change in elevation from the start of the trail to the end of the trail.

7. What is Zaire's starting elevation? -150 ft
8. What is Zaire's ending elevation? +75 ft
9. What is the unknown in this story? the change in elevation
10. Write an addition equation to represent the situation. Use  $d$  to represent the trail's distance.

$$-150 + d = 75$$

11. Rewrite the equation as a related subtraction equation. Then solve.

$$75 - (-150) = d$$

$$75 + 150 = d$$

$$225 = d$$

12. What does your answer represent?

a. Zaire's final elevation

b. The rate Zaire hiked

c. The distance Zaire hiked

Match each equation to the situation to an equation. Then solve.

13. The temperature was -4 degrees in the morning, and it rose to 9 degrees by the afternoon. What is the change in temperature?

$$\begin{aligned} -4x &= -20 \\ -20 \div -4 &= x \\ 5 &= x \end{aligned}$$

14. The temperature was 9 degrees in the morning, and it fell to -4 degrees by the afternoon. What is the change in temperature?

$$\begin{aligned} 9 + x &= -4 \\ -4 - 9 &= x \\ -13 &= x \end{aligned}$$

15. A turtle dives down toward the ocean floor. After 4 minutes, it was 20 meters below the surface. At what rate was it diving?

$$\begin{aligned} -4 + x &= 9 \\ 9 - (-4) & \\ 9 + 4 & \\ x &= 13 \end{aligned}$$

16. A barracuda dives toward the ocean floor at a rate of 4 meters per second. How long will it take the barracuda to get from an elevation of 0 meters to an elevation of -20 meters?

$$\begin{aligned} 4x &= -20 \\ -20 \div 4 &= x \\ -5 &= x \end{aligned}$$



1. The temperature has been dropping 4 degrees Fahrenheit every hour. If the current temperature is -16 degrees, how many hours ago was the temperature at 0 degrees Fahrenheit? Write and solve an equation. Explain what your variable represents.

$$-4x = -16$$

$x = \#$  of hours

$$-16 \div -4 = x$$

$$4 = x$$

2. Mr. Angel is rock climbing. He begins at sea level and climbs up 7 feet every minute. How many minutes will it take Mr. Angel to reach an elevation of 140 feet?

- a. Write an equation to represent the situation. Use  $x$  to represent the number of minutes.

$$7x = 140$$

- b. Solve the equation.

$$140 \div 7 = x$$

$$20 = x$$

- c. Write an answer sentence explaining what your solution means in the context of the problem.

It will take him 20 minutes to climb to an elevation of 140 feet.

3. A falcon is soaring at an elevation of 300 feet. It descends to an elevation of -25 feet. What is the falcon's change in elevation? Write and solve an equation to solve the problem. Include an answer sentence to explain what your solution means in the context of the problem.

$$300 + x = -25$$

$$-25 - 300 = x$$

$$\boxed{-325 = x}$$

The change in elevation is -325 feet.

4. Match each equation to the situation to an equation. Then solve.

An octopus dives at a rate of 5 feet per second. How long will it take for the octopus to get from the surface to an elevation of -30 feet?

$$-5n = -30$$

$$-30 \div -5 = n$$

$$\boxed{6 = n}$$

A squid dives toward the ocean floor, starting at sea level. After 5 minutes, the squid is 30 meters below sea level. At what rate was the squid diving?

$$-12 + k = 1$$

$$1 - (-12) = k$$

$$1 + 12 = k$$

$$\boxed{13 = k}$$

The temperature was -12 degrees and rose to 1 degree. What was the change in temperature?

$$1 + k = -12$$

$$-12 - 1 = k$$

$$\boxed{-13 = k}$$

The temperature was 1 degree and fell to -12 degrees. What was the change in temperature?

$$5n = -30$$

$$-30 \div 5 = n$$

$$\boxed{-6 = n}$$

# **G7 U4 Lesson 15**

Use positive and negative numbers to represent directed change.



## G7 U4 Lesson 15 - Students will use positive and negative numbers to represent directed change.

**NOTE:** It is appropriate to have students use a calculator for the computation in this lesson.

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** I'm excited to work with you today, because this will be our final lesson in our unit about rational numbers. What are some things that stand out to you about what we've learned so far? **Possible Student Answers, Key Points:**

- We can represent temperature, elevation, money and other contexts using signed numbers.
- We can add and subtract signed numbers by using a number line.
- A negative number multiplied or divided by a negative number is positive. A negative number multiplied or divided by a positive number is negative.

We've learned so much! Today we'll tie everything we've been doing together with a few problems with varied contexts.

**Let's Talk (Slide 3):** Before we jump into solving problems, I'm curious. What do you already know or understand about stocks or the stock market? If you're unsure, can you use the image or the table to make some educated guesses? **Possible Student Answers, Key Points:**

- I've heard of the stock market on the news. I know stock prices go up and down.
- I'm not sure what stocks are. I think based on the pictures they involve money.
- People buy stocks in companies and they can earn money from them.

A stock is just a piece of a company that somebody can buy. When you buy stocks you own a small portion of the business. The price of the stock goes up and down depending on how the business is doing. People who own stocks are called shareholders, and many shareholders like to keep an eye on stock prices to see how their investment in the company is doing.

If you look at the table, you can see some information about a particular stock price. What do you notice from the table? **Possible Student Answers, Key Points:**

- I notice the stock is for Apple, the tech company.
- I notice the stock price increased from Day 1 to Day 2.
- I notice the table shows the change in price as a dollar amount and as a percent.

Some of our problems today will involve stock prices. You don't need to be a stock expert to solve the problems, and I can help answer questions you might have as we go. Let's look at our first problems.

**Let's Think (Slide 4):** For this problem, we'll work to complete the table before answering two questions about the information in the table. Review the table for a moment, and tell me what you notice and wonder before we start doing any math. **Possible Student Answers, Key Points:**

- I notice the Apple information from the last slide. I also see Dell and Google.
- I notice Google shows a negative change, which makes me think their stock price went down.
- I notice the Dell stock price went down a dollar.

$$\begin{aligned} ? - 2.5 &= 114.29 \\ 114.29 + 2.5 &= ? \\ \$ 116.79 &= ? \end{aligned}$$

Let's start by finding the Day 1 price of the Google stock. I notice based on the other columns that the price of the stock decreased. The negative change in value makes that clear. That means the stock was higher, and it dropped a bit. *(write equation)* I can write the equation  $? - 2.5 = 114.29$  to represent that I don't know the starting price, but I know it dropped 2.5 dollars and ended up costing

\$114.29. To solve the subtraction equation, I can write a related addition equation. I'll rewrite the equation as  $114.29 + 2.5 = ?$  (write related equation)

Take a moment to add these two numbers. If you use vertical form, make sure to carefully line up each place value. Let me know when you find the unknown starting value. (\$116.79) The Day 1 price of the Google stock was \$116.79. (fill the price in the blank on the chart)

$$78.15 - 77.15 = ?$$

$$\$1 = ?$$

$$-1$$

What's left to figure out in the chart? (We need to find the change in price for the Dell stock.) Let's find the change in price for the Dell stock by subtracting. (write equation) What is 78.15 minus 77.15? (1) Since the price decreased 1 dollar, I'll note that the change in dollar value was -1, and I'll fill that in the chart. (write -1 in the corresponding blank on the chart)

1 is what % of 78.15?

$$1 = x \cdot 78.15$$

The last thing we need to determine is the percent change. We know the dollar value change was -1, so we need to determine what percent of the original price 1 represents. In other words I'm trying to think "1 is what percent of 78.15"? (write that question out as shown) We can write a matching equation by writing  $1 = x \cdot 78.15$ , where x is the unknown percent. (write equations with arrows to show the corresponding parts of the question and the equation)

We can write a related division equation to help us find the unknown in this multiplication equation. What related division equation can help us? How do you know? Possible Student Answers, Key Points:

$$1 \div 78.15 = x$$

- Instead of  $1 = x \cdot 78.15$ , I can write  $1 \div 78.15 = x$ .
- I can think of 1 as the total product of x and 78.15. I can divide the total by the factor I know to find the unknown factor.

$$0.0128 = x$$

$$1.28\%$$

$$-1.28\%$$

(write related division equation) When I divide 1 by 78.15, my calculator shows a long decimal number. We can think of x as being about 0.0128 if I round to the ten thousandths place. That's our percent change in decimal form. To think about this as a percent, I can multiply the decimal by 100 or think about each digit shifting two place values left. The price changed about 1.28%. I'll write -1.28% in the box on the chart, since I know the percent represents a decrease and not an increase.

COMPANY	DAY 1 VALUE (\$)	DAY 2 VALUE (\$)	CHANGE IN VALUE (\$)	PERCENT CHANGE IN VALUE
Apple	106.5	112.75	6.25	5.87%
Google	116.79	114.29	-2.5	-2.14%
Dell	78.15	77.15	-1	-1.28%

The table is complete. Let's wrap up this section by answering the two questions. Both questions ask about magnitude, which we've seen before. Magnitude is simply the distance a value is from 0, regardless if whether the value is positive or negative. (highlight all three values representing the change in dollars) Which company's change in value has the greatest magnitude? How do you know? Possible Student Answers, Key Points:

- Apple has the greatest change in value, because 6.25 is farther from 0 than -2.5 and -1. Their value changed the most.

Apple's change in value is the greatest, because their stock price changed the most. 6.25 is the farthest value away from 0 of the three values.

(highlight all three values representing the percent change) Which company's percent change shows the smallest magnitude? How do you know? Possible Student Answers, Key Points:

- Dell's percent change is the smallest.  $-1.28$  is closest to  $0$  compared to the other values.

Dell's percent change has the smallest magnitude, because it's closer to  $0$  than the other percent change values. In other words, Dell's stock price changed the least.

We're becoming stock market experts! We just solved multiple problems involving the stock price of three major tech corporations.

**Let's Think (Slide 5):** (read the problem aloud) We have some information about a stock price that we can now use to respond to a few prompts. The first question wants us to find the new price of a stock. We know the stock increased  $2.4\%$  from  $\$107.75$ . That means the price will be  $100\%$  of  $\$107.75$  plus  $2.4\%$  more. I can think of the new price as being  $100\% + 2.4\%$ , or  $102.4\%$ . How can I write  $102.4\%$  as a decimal, so that I can work with it in an equation? Possible Student Answers, Key Points:

- I can divide  $102.4$  by  $100$  to get  $1.024$ .
- To convert a percent into a decimal, I can shift every digit two place values to the right.

$$107.75(1.024) = ?$$

$$? = \$110.34$$

If I want to find  $102.4\%$  of  $\$107.75$ , I can multiply  $107.75$  by  $1.024$ , which is the decimal equivalent to  $102.4\%$ . (write equation) If I plug that into my calculator, I get a new stock price of about  $\$110.34$ . I did a little rounding to the nearest penny, just to make my answer look like a typical dollar amount.

We just thought of an increase of  $2.4\%$  as  $100\%$  plus  $2.4\%$ . From there, we were able to use a decimal equivalent and multiplication to arrive at a stock price that was  $2.4\%$  more than the original.

$$110.34 \times 96 = ?$$

$$\$10,592.64$$

Part B wants us to find the total cost of  $96$  shares of the new stock price. Assuming we have access to a calculator, how could I find the total cost of  $96$  shares? (write and solve equation as student shares, supporting as needed) Possible Student Answers, Key Points:

- I can multiply  $110.34$  by  $96$ , since I'm buying  $96$  shares at that same share price. The product comes out to be  $10592.64$ , which is  $\$10,592.64$ .

If we multiply the cost of  $1$  new share,  $\$110.34$ , by  $96$ , we end up with a grand total of  $\$10,592.64$ .

$$86.54(0.985) = ?$$

$$\$85.24$$

The last question wants us to consider a stock that decreases in value by  $1.5\%$ . So, the new stock won't be  $100\%$  of the price of the old stock. It will be  $1.5\%$  less. What is  $1.5\%$  less than  $100\%$ ? ( $98.5\%$ ) The new stock price will be  $98.5\%$  of the old stock price. Let's use an equation to figure out the new stock price. What decimal value is equivalent to  $98.5\%$ ? ( $0.985$ ) I'll multiply the old price of the stock,  $\$86.54$ , by the decimal equivalent of  $98.5\%$ . (write equation) If I use my calculator to compute, I see  $85.2419$ .

Why might that be an unusual answer? How can I make it better fit the context of this problem? Possible Student Answers, Key Points:

- It's unusual because money is only represented to the hundredths place, since the hundredths digit represents pennies.
- You can just round the answer to the nearest hundredth. A more reasonable answer would be  $\$85.24$ .

We used our understanding of signed numbers along with our understanding of percents and decimal equivalents to respond to a variety of stock-related questions. Great work!

**Let's Try it (Slides 6 - 7):** Now let's try solving problems representing positive and negative change together. Later you're get a chance to try some on your own. Not every problem we see today will be about stocks, so we'll want to read the problem carefully to think about whether the change involved is best represented by a positive or a negative value. We'll also be dealing with percents in some problems, so we'll want to make sure we're careful when converting from decimal values to percents or vice versa. Let's try a few more out as we bring this unit to a close.

# WARM WELCOME



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**Today we will use positive and negative numbers to represent directed change.**

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## Let's Talk:

What do you know about stocks and the stock market?



COMPANY	DAY 1 VALUE (\$)	DAY 2 VALUE (\$)	CHANGE IN VALUE (\$)	PERCENT CHANGE IN VALUE
Apple	106.5	112.75	6.25	5.87%

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## Let's Think:

Complete the table.

COMPANY	DAY 1 VALUE (\$)	DAY 2 VALUE (\$)	CHANGE IN VALUE (\$)	PERCENT CHANGE IN VALUE
Apple	106.5	112.75	6.25	5.87%
Google		114.29	-2.5	-2.14%
Dell	78.15	77.15		

- Which company's change in value has the greatest magnitude?
- Which company's percent change has the smallest magnitude?

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## Let's Think:

A company's old stock price was \$107.75. The stock price increased +2.4%.

- What is the new price of the stock?
- What is the value of 96 shares at the new price?
- Another company's old stock price was \$86.54. The company experienced a -1.5% change in stock price. What is the new stock price?

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## Let's Try It:

Let's explore using positive and negative numbers to represent directed change together.

Name: \_\_\_\_\_ G7 U4 Lesson 15 - Let's Try It

Hana noticed that the price of groceries at her local store was fluctuating, so she started to keep track of the changes in price of items she buys regularly.

GROCERY ITEM	OLD PRICE	NEW PRICE
Box of cereal	\$1.50	\$1.60
Pineapple	\$1.98	\$1.48
Shredded cheese	\$2.06	\$1.99

- Which items increased in price? Which items decreased in price?
- Find each item's change in price. Represent the change in price using a positive or a negative value.
- Which item's change in dollars had the largest magnitude? How do you know?
- Which item's change in dollars had the smallest magnitude? How do you know?
- Find the percent change in price for each item. Represent this percent change in price using a positive or negative sign.
- Which item's change in percentage had the largest magnitude? Smallest magnitude?

Four friends tracked how far they could throw a paper airplane. Each person threw their paper airplane twice. Some of their information is shown in the table below.

Student	Throw #1 (meters)	Percent change	Throw #2 (seconds)
Benjamin	5.8	+10%	
Edward	10	-20.5%	1.25
Bella	2.3		8
Julie	3		8

- Whose distances increased? How do you know?
- Whose distances decreased? How do you know?
- Write and solve an equation to find Benjamin's second throw. Start by thinking about how to write 10% as a decimal.
- Write and solve an equation to find Edward's second throw. Start by thinking about how to write 20.5% as a decimal.
- Write and solve an equation to find Bella and Julie's percent change. Include a + or - symbol to note whether each change was positive or negative.

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# On your Own:

Now it's time to use negative numbers to represent directed change on your own.

Name: \_\_\_\_\_ G7 U4 Lesson 15 - Independent Work

1. Use the table to answer the questions below.

Company	Old price	New price	Price change
Health Food Company	\$15.16	\$16.25	
Clothing Company	\$82.21	\$80.15	
Toy Company	\$25.00		+2%
Social Media Company	\$105		-9.5%

ii. Fill in the missing values. Show your work in the space below.

b. Which stock's price change has the largest magnitude? Which stock's price change has the smallest magnitude?

c. Based on the new price of each, which has a greater value?  
 15 shares of the Health Food Company      3 shares of the Clothing Company

2. The stock price for three companies is shown in the first table below. The second table shows the change in stock price for each of the three companies after three months.

Company	Stock Price
Solar Power Company	\$4.51
Hydrology Company	\$5.51
Power Wind Company	\$8.18

3 Months Later	
Company	Change
Solar Power Company	+11.8%
Hydrology Company	-\$4.26
Power Wind Company	-\$6.37

iii. Callie said the Solar Power Company's price change has the greatest magnitude, since it is the only stock price that increased. Do you agree or disagree? Explain.

iv. Determine each company's stock price after three months.

v. Determine the percent change for each company's stock.

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Hana noticed that the price of groceries at her local store was fluctuating, so she started to keep track of the changes in price of items she buys regularly.

GROCERY ITEM	OLD PRICE	NEW PRICE
Box of cereal	\$3.50	\$3.60
Pineapple	\$1.98	\$1.48
Shredded cheese	\$2.06	\$2.99

1. Which items increased in price? Which items decreased in price?
2. Find each item's change in price. Represent the change in price using a positive or a negative value.
3. Which item's change in dollars had the largest magnitude? How do you know?
4. Which item's change in dollars had the smallest magnitude? How do you know?
5. Find the percent change in price for each item. Represent the percent change in price using a positive or negative sign.
6. Which item's change in percentage had the largest magnitude? Smallest magnitude?

Four friends tracked how far they could throw a paper airplane. Each person threw their paper airplane twice. Some of their information is shown in the table below.

Student	Throw #1 (meters)	Percent change	Throw #2 (seconds)
Benjamin	5.6	+10%	
Edward	10	-20.5%	
Bella	2.5		1.25
Julio	3		8

- Whose distances increased? How do you know?
- Whose distances decreased? How do you know?
- Write and solve an equation to find Benjamin's second throw. Start by thinking about how to write 10% as a decimal.
- Write and solve an equation to find Edwards's second throw. Start by thinking about how to write 20.5% as a decimal.
- Write and solve an equation to find Bella and Julio's percent change. Include a + or - symbol to note whether each change was positive or negative.

**1. Use the table to answer the questions below.**

Company	Old price	New price	Price change
Health Food Company	\$15.16	\$16.25	
Clothing Company	\$82.21	\$80.15	
Toy Company	\$25.90		+2%
Social Media Company	\$105		-9.5%

a. Fill in the missing values. Show your work in the space below.

b. Which stock's price change has the largest magnitude? Which stock's price change has the smallest magnitude?

c. Based on the new price of each, which has a greater value?

15 shares of the Health Food Company

3 shares of the Clothing Company

2. The stock price for three companies is shown in the first table below. The second table shows the change in stock price for each of the three companies after three months.

### 3 Months Later

Company	Stock Price
Solar Power Company	\$4.34
Pet Supply Company	\$6.14
Home Goods Company	\$11.28

Company	Change
Solar Power Company	+\$1.58
Pet Supply Company	-\$0.36
Home Goods Company	-\$4.30

- a. Caitlyn said the Solar Power Company's price change has the greatest magnitude, since it is the only stock price that increased. Do you agree or disagree? Explain.

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- b. Determine each company's stock price after three months.

- c. Determine the percent change for each company's stock.

Hana noticed that the price of groceries at her local store was fluctuating, so she started to keep track of the changes in price of items she buys regularly.

GROCERY ITEM	OLD PRICE	NEW PRICE
Box of cereal	\$3.50	\$3.60
Pineapple	\$1.98	\$1.48
Shredded cheese	\$2.06	\$2.99

1. Which items increased in price? Which items decreased in price?

box of cereal  
shredded cheese

pineapple

2. Find each item's change in price. Represent the change in price using a positive or a negative value.

$$3.60 - 3.50 = ?$$

$$+\$0.10$$

cereal

$$1.98 - 1.48 = ?$$

$$-\$0.50$$

pineapple

$$2.99 - 2.06 = ?$$

$$+\$0.93$$

cheese

3. Which item's change in dollars had the largest magnitude? How do you know?

The cheese is the farthest from 0.

4. Which item's change in dollars had the smallest magnitude? How do you know?

The cereal is the closest to 0.

5. Find the percent change in price for each item. Represent the percent change in price using a positive or negative sign.

cereal

$$0.1 = x \cdot 3.50$$

$$0.1 \div 3.5 = x$$

$$0.0286 \approx x$$

$$+2.86\%$$

pineapple

$$0.5 = x \cdot 1.98$$

$$0.5 \div 1.98 = x$$

$$0.2525 \approx x$$

$$-25.25\%$$

cheese

$$0.93 = x \cdot 2.06$$

$$0.93 \div 2.06 = x$$

$$0.4515 \approx x$$

$$+45.15\%$$

6. Which item's change in percentage had the largest magnitude? Smallest magnitude?

cheese

cereal



Four friends tracked how far they could throw a paper airplane. Each person threw their paper airplane twice. Some of their information is shown in the table below.

Student	Throw #1 (meters)	Percent change	Throw #2 (seconds)
Benjamin	5.6	+10%	
Edward	10	-20.5%	
Bella	2.5		1.25
Julio	3		8

7. Whose distances increased? How do you know?

Ben → positive % change  
Julio → increase from 3 to 8

8. Whose distances decreased? How do you know?

Edward → negative % change  
Bella → decrease from 2.5 to 1.25

9. Write and solve an equation to find Benjamin's second throw. Start by thinking about how to write 10% as a decimal.

$$5.6 \times 1.10 = ?$$

$$(6.16 \text{ sec.})$$

10. Write and solve an equation to find Edwards's second throw. Start by thinking about how to write 20.5% as a decimal.

$$10 \times 0.795 = ?$$

$$(7.95 \text{ sec.})$$

11. Write and solve an equation to find Bella and Julio's percent change. Include a + or - symbol to note whether each change was positive or negative.

**B**

$$2.5 - 1.25 = 1.25$$

$$1.25 = x \cdot 2.5$$

$$1.25 \div 2.5 = x$$

$$0.5 = x$$

$$(-50\%)$$

**J**

$$8 - 3 = 5$$

$$5 = x \cdot 3$$

$$5 \div 3 = x$$

$$x \approx 1.67$$

$$(167\%)$$



1. Use the table to answer the questions below.

Company	Old price	New price	Price change
Health Food Company	\$15.16	\$16.25	+7.19%
Clothing Company	\$82.21	\$80.15	-2.51%
Toy Company	\$25.90	<del>\$26.42</del>	+2%
Social Media Company	\$105	<del>\$95.03</del>	-9.5%

a. Fill in the missing values. Show your work in the space below.

$$\begin{array}{r} 16.25 \\ -15.16 \\ \hline 1.09 \end{array}$$

$$1.09 = x \cdot 15.16$$

$$1.09 \div 15.16 = x$$

$$0.0719 \approx x$$

+7.19%

$$\begin{array}{r} 82.21 \\ -80.15 \\ \hline 2.06 \end{array}$$

$$2.06 = x \cdot 82.21$$

$$2.06 \div 82.21 = x$$

$$0.0251 \approx x$$

-2.51%

$$25.9 \times 1.02 = x$$

26.42 \approx x

---


$$105 \times 0.905 = x$$

95.03 \approx x

b. Which stock's price change has the largest magnitude? Which stock's price change has the smallest magnitude?

↓  
Toy Company

↓  
Social Media Company

c. Based on the new price of each, which has a greater value?

15 shares of the Health Food Company

$$15(16.25) = ?$$

\$243.75

3 shares of the Clothing Company

$$3(80.15) = ?$$

\$240.45

2. The stock price for three companies is shown in the first table below. The second table shows the change in stock price for each of the three companies after three months.

Company	Stock Price
Solar Power Company	\$4.34
Pet Supply Company	\$6.14
Home Goods Company	\$11.28

### 3 Months Later

Company	Change
Solar Power Company	+\$1.58
Pet Supply Company	-\$0.36
Home Goods Company	-\$4.30

- a. Caitlyn said the Solar Power Company's price change has the greatest magnitude, since it is the only stock price that increased. Do you agree or disagree? Explain.

I disagree. I can think of magnitude as distance from 0. The Home Good's company has the greatest magnitude.

- b. Determine each company's stock price after three months.

**SP**

$$\begin{array}{r} 4.34 \\ + 1.58 \\ \hline \end{array}$$

\$5.92

**PS**

$$\begin{array}{r} 6.14 \\ - 0.36 \\ \hline \end{array}$$

\$5.78

**HG**

$$\begin{array}{r} 11.28 \\ - 4.30 \\ \hline \end{array}$$

\$6.98

- c. Determine the percent change for each company's stock.

**SP**

$$\begin{aligned} 1.58 &= x \cdot 4.34 \\ 1.58 \div 4.34 &= x \\ 0.3641 &\approx x \end{aligned}$$

36.41%

**PS**

$$\begin{aligned} 0.36 &= x \cdot 6.14 \\ 0.36 \div 6.14 &= x \\ 0.0586 &\approx x \end{aligned}$$

-5.86%

**HG**

$$\begin{aligned} 4.3 &= x \cdot 11.28 \\ 4.3 \div 11.28 &= x \\ 0.3812 &\approx x \end{aligned}$$

-38.12%



# G7 Unit 5:

Expressions, Equations, and Inequalities

# **G7 U5 Lesson 1**

Find unknown values in relationships, and interpret them as proportional and not proportional.



**G7 U5 Lesson 1 - Students will find unknown values in relationships, and interpret them as proportional or not proportional.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we get to start a brand new unit. This unit is all about expressions, equations, and inequalities. What do you already know about expressions, equations, or inequalities? **Possible Student Answers, Key Points:**

- I know equations have an equal sign. Like  $2 + 2 = 4$  is an equation. Sometimes equations have unknowns we have to solve for.
- I know expressions are made up of terms separated by symbols like  $+$ ,  $-$ ,  $\cdot$ , or  $\div$ .
- I know inequalities use the less than or greater than symbol.

It sounds like you already know a lot that will help us as we work through the next several lessons. We'll get started today by finding unknown values in relationships and determining whether the relationships we're looking at are proportional or not proportional. Time to get started!

**Let's Talk (Slide 3):** Let's head to the circus! Consider a situation where one box of popcorn costs \$3. How could this group of 4 people figure out how much it would cost for each person to have their own box of popcorn? **Possible Student Answers, Key Points:**

- They could add  $3 + 3 + 3 + 3$ , since all four of them are buying popcorn.
- They could write an equation like  $3 \cdot p = ?$  to find the total.
- They could make a table to show the cost of 1 person buying popcorn, then 2 people, then 3, then 4...

#	\$
1	3
2	6
3	9
4	12

There are many ways to consider solving this problem. Let's draw a table. *(draw a simple t-chart labeled with number of people and cost)* Let's find the cost if 1, 2, 3, or 4 people purchase a box of popcorn. *(write 1 through 4 in the first column)* How much would it cost 1 person to buy popcorn? **(\$3)** *(fill in value, and continue filling in values as the student shares)* What would be the cost if 2 people bought popcorn? **(\$6)** What about 3 people? **(\$9)** What about 4 people? **(\$12)** The total cost for 4 people to each buy a box of popcorn would be \$12. We can see this clearly in the table.

#	\$
1	3
2	6
3	9
4	12

This relationship is considered proportional. The number of people is proportional to the total cost. The cost per box of popcorn stays constant. It's \$3 for 1 person. *(write  $3/1 = 3$  next to that row of the table)* It's \$6 for 2 people. *(write  $6/2 = 3$  next to that row of the table)* It's \$9 for 3 people. *(continue writing constant of proportionality as you narrate)* It's \$12 for 4 people. No matter what, we end up with 3. This 3 is called our constant of proportionality. *(highlight each 3)* When we divide the amount of money by the number of people, we always get the same quotient, or the same cost per person. In each row, we could multiply the number of people times the constant of proportionality, 3, to find the total cost. This relationship is proportional.

**Let's Think (Slide 4):** Now, let's think about a different set of relationships. I'll read this problem once through, then I want you to summarize what it is about. *(read problem)* What do you know? What is unknown? **Possible Student Answers, Key Points:**

- We know there are 2 bus companies. They charge different rates to rent buses.
- We don't know how much it costs to rent 1, 2, 3, or 4 buses from each company. We don't know which company is the better deal.

BB

D	\$
1	700
2	1400
3	2100
4	2800

Let's draw a table to represent each bus company's cost. I'll start by making a table to represent Barry's Bus Company. (sketch and label a t-chart and fill in 1, 2, 3, and 4 days) How much would it cost to rent from Barry's Bus Company for 1 day? 2 days? 3 days? 4 days? Possible Student Answers, Key Points:

- Each day, this company charges \$700. One day would be \$700, two days would be \$1,400, 3 days would be \$2,100, and 4 days would be \$2,800.

(fill in the table as student shares)

AA

D	\$
1	700
2	1200
3	1700
4	2200

Let's draw a table to represent Aaron's Auto Company next. Aaron's Auto Company charges their customers in a different way. They charge a one-time flat fee of \$200. Then, they charge \$500 per day on top of the flat fee. If I think about 1 day, that means I'll have to pay \$200 plus \$500 for the day. (fill in 700 on the chart) For 2 days, I'd still pay the \$200 fee then I'd have to pay \$500 for the first day and another \$500 for the second day. That's \$1,200 total. (fill 1,200 in the chart) How much does Aaron's Auto Company charge for 3 days? 4 days? (fill in the table as the student shares) Possible Student Answers, Key Points:

- For three days, I'd have to pay \$200 plus \$1,500. The total would be \$1,700.
- For four days, I'd have to pay \$200 plus \$2,000. The total would be \$2,200.

Great! We've done everything this problem asked us to do, but let's think about these two companies a bit more. One of these companies shows a proportional relationship, meaning the cost per day stays constant no matter what. One of these companies shows a relationship that is not proportional. Let's think about the cost per day in each scenario, starting with Barry's Bus Company.

D	\$	
1	700	$\frac{700}{1} = 700$
2	1400	$\frac{1400}{2} = 700$
3	2100	$\frac{2100}{3} = 700$
4	2800	$\frac{2800}{4} = 700$

To find the cost per day, I can divide the cost by the number of days. (as you narrate, write each constant of proportionality as a fraction then write the value next to each row) For 1 day, Barry's Bus Company charges \$700. I can think of the cost per day as  $700/1$ , which I know equals 700. For 2 days, I can think of the cost per day as  $1400/2$ , which I know equals 700. Help me find the cost per day for Day 3 and Day 4. Possible Student Answers, Key Points:

- For Day 3, I can write  $2100/3$ . I know 2100 divided by 3 is 700.
- For Day 4, the cost per day is also 700. I know  $2800/4 = 700$ .

Since each row of the table represents the same cost per day, we can say that 700 is the constant of proportionality. This relationship is proportional. What about Aaron's Auto?

D	\$	
1	700	$\frac{700}{1} = 700$
2	1200	$\frac{1200}{2} = 600$
3	1700	$\frac{1700}{3} \approx 567$
4	2200	$\frac{2200}{4} = 550$

(follow a similar process, labeling the cost per day next to each row as you narrate) The cost per day in the first row can be represented by  $700/1$ . The cost per day is \$700. What about the next three days? You're welcome to use a calculator if it's helpful. Possible Student Answers, Key Points:

- For Day 2, I can think of 1200 divided by 2. That's 600.
- For Day 3, I can write  $1700/3$ . 1700 divided by 3 is about 567.
- For Day 4, the cost per day is 550 dollars. 2200 divided by 4 is 550.

Notice how the cost per day does not remain constant when we look at Aaron's Auto Company. This means the relationship is NOT proportional.

To determine whether a relationship is proportional, we can use division to see if the relationship between values stays constant for each pair of values. Let's keep going.

**Let's Think (Slide 5):** This question wants us to find the cost per item at a store. The first table represents belts, and the second table represents scarves. Based on the cost per item, we'll determine which relationship is proportional.

Number of scarves	Cost in dollars
2	\$12
6	\$30
11	\$50

$\frac{12}{2} = 6$   
 $\frac{30}{6} = 5$   
 $\frac{50}{11} \approx 4.5$

Let's start by looking at the scarves. Let's find the cost per scarf by using the table. In the first row, I see 2 scarves cost \$12 total. I can find the cost per scarf by dividing the cost by the number of scarves. I know  $12/6$  is equal to 6. *(label next to the row in the table, and continue doing so for the other rows as you narrate)*

The second row shows that the store charges \$30 to buy 6 scarves. I can find the cost per scarf by dividing the cost by the number of scarves. I know  $30/6 = 5$ . Hm, it looks like maybe the cost per scarf is not constant across the table,

but let's keep going to be sure...

How could I find the cost per scarf looking at the third row? You're welcome to use a calculator if you find it helpful. [Possible Student Answers, Key Points:](#)

- I can find the cost per scarf by dividing the total cost by the number of scarves. I can divide 50 by 11. My calculator shows a long decimal number, but it's about 4.5 if I round to the nearest tenth.

I can see that the cost per scarf is not constant for each pair of values. That means, the relationship between cost and number of scarves is NOT proportional.

Number of belts	Cost in dollars
2	\$5
6	\$15
11	\$27.50

$\frac{5}{2} = 2.5$   
 $\frac{15}{6} = 2.5$   
 $\frac{27.50}{11} = 2.5$

Let's now look at the belts. To find the cost per belt in the first row of this table, I can divide \$5 by 2 belts.  $5/2$  is equal to 2.5, so I know each belt costs \$2.50. *(write  $5/2 = 2.5$  next to the first row, and continue labeling as you and the student discuss the other rows)*

How can I find the cost per belt in the second row and third row of this table? If you want, you may use a calculator. [Possible Student Answers, Key Points:](#)

- I can divide \$15 by 6. I get \$2.50 per belt.
- I can divide \$27.50 by 11. I get \$2.50 per belt again.

Is this relationship between cost and number of belts proportional? How do you know? [Possible Student Answers, Key Points:](#)

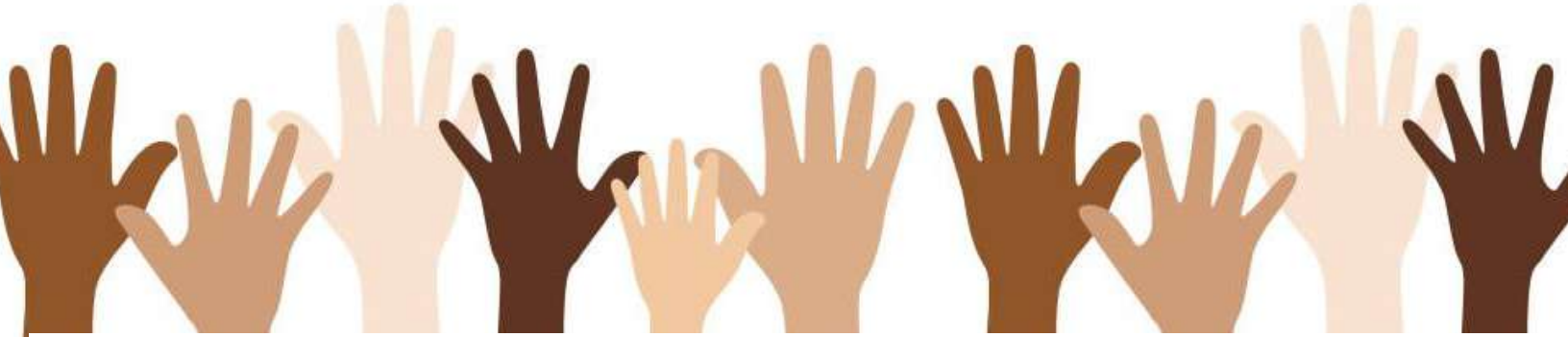
- The relationship is proportional. We found that 2.5 is the constant of proportionality. In every row, the cost per belt was \$2.50.

The scarves did not represent a proportional relationship because the cost per scarf depended on how many scarves you bought. The belts did represent a proportional relationship, because no matter how many belts you bought, the cost per belt remained constant. The cost per belt was always \$2.50.

**Let's Try it (Slides 6 - 7):** Now let's work through a few more problems together, then you'll get a chance to practice on your own. We'll look closely at each relationship to determine whether it is proportional or not proportional. We can use division to see if the constant of proportionality remains consistent across the entire relationship. Let's go for it!



# WARM WELCOME



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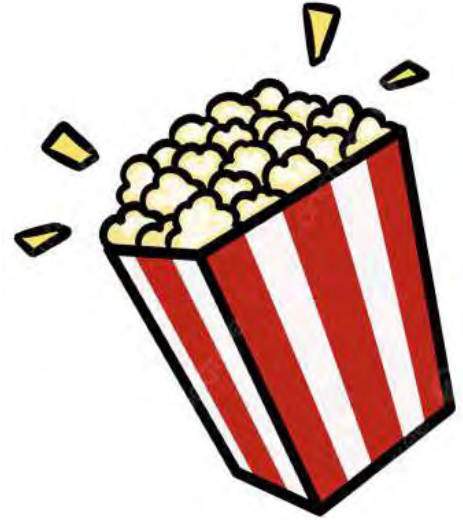
**Today we will find unknown values in relationships, and interpret them as proportional and not proportional.**

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## Let's Talk:



**Popcorn at the circus costs \$3.**



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## Let's Think:

**Barry's Bus Company charges \$700 each day to rent a bus. Aaron's Auto Company charges a flat fee of \$200 and \$500 per day to rent a bus.**

Sketch tables to represent the cost of renting 1, 2, 3, and 4 buses from each company.

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# Let's Think:

A clothing store sells belts and scarves. Use the tables to find the cost per item. Which relationship is proportional?

Number of scarves	Cost in dollars
2	\$12
6	\$30
11	\$50

Number of belts	Cost in dollars
2	\$5
6	\$15
11	\$27.50

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# Let's Try It:

Let's explore finding unknown values in relationships, and interpreting them as proportional or not proportional together.

Name: \_\_\_\_\_ G7 US Lesson 1 - Let's Try It

**A bike-share service charges \$4 per hour to rent a bike.**

- The \$2 in this story represents:
  - The cost to rent a bike once
  - The cost to rent a bike each hour
- How much would it cost to rent the bike for 2 hours?
- How much would it cost to rent the bike for 8 hours?
- How much would it cost to rent the bike for 24 hours?
- Divide each cost you found (Questions #2 - 4) by the number of hours to find the cost per hour.

6. The cost per mile...

- is the same, so this is a proportional relationship.
- is the same, so this is not a proportional relationship.
- is not the same, so this is a proportional relationship.
- is not the same, so this is not a proportional relationship.

The cost per hour in this problem is the **constant of proportionality**.

7. The constant of proportionality in this scenario is \_\_\_\_\_.

**A carnival charges an entry fee of \$5, and \$1 for every game or ride.**

- What does \$5 represent in this situation?
  - The cost for each game or ride.
  - The cost to enter the carnival.
- What does \$1 represent in this situation?
  - The cost for each game or ride.
  - The cost to enter the carnival.

10. Determine how much it would cost for one person to go to the carnival and visit 2 games or rides, 8 games or rides? 24 games or rides?

11. Divide each cost by the number of rides or games to find the cost per ride or game.

12. Is the relationship proportional? How do you know?

\_\_\_\_\_

\_\_\_\_\_

**Look at each relationship below. Determine whether each is proportional or not proportional. Show or explain how you know.**

13.

number of bagels	cost
2	\$1.80
7	\$6.30
10	\$9

14.

number muffins	cost
3	\$5
5	\$7
7	\$9

15.

number of donuts	cost
6	\$3.60
10	\$6.00
20	\$11.40

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# On your Own:

Now it's time to find unknown values in relationships, and interpret them as proportional and not proportional on your own.

Name: \_\_\_\_\_ G7 US Lesson 1 - Independent Work

1. A board game requires each player to start with \$10. Fill in the table to find how many dollars will be needed for 2, 3, 6, and 8 players. Then, fill in the column labeled dollars per player.

Number of players	Number of dollars	Dollars per player
2		
3		
6		
8		

Is the relationship proportional? How do you know?

\_\_\_\_\_

\_\_\_\_\_

2. An amusement park offers a season pass where guests pay a flat fee of \$20, and then they only pay \$5 per visit to the park.

Number of visits	Number of dollars
2	
3	
7	
10	

Is the relationship proportional? How do you know?

\_\_\_\_\_

\_\_\_\_\_

3. Patrick mows lawns over the summer. He charges his customers an \$80 fee for the summer, and \$22 per lawn mowing.

- How much does Patrick charge to mow 1 lawn?
- How much does Patrick charge to mow 2 lawns?
- How much does Patrick charge to mow 3 lawns?
- Patrick said this relationship is proportional, because the cost increases \$22 every time. Is he correct? How do you know?

\_\_\_\_\_

\_\_\_\_\_

4. Determine whether each table represents a relationship that could be proportional.

Number of sets	Cost in dollars
10	120
20	240
30	360
40	480

Number of bags	Cost in dollars
6	24
11	50
12	60

Number of fans	Cost in dollars
4	16
5	24
20	80

\_\_\_\_\_

\_\_\_\_\_

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**A bike-share service charges \$4 per hour to rent a bike.**

1. The \$4 in this story represents:
  - a. The cost to rent a bike once
  - b. The cost to rent a bike each hour.
  
2. How much would it cost to rent the bike for 2 hours?
  
3. How much would it cost to rent the bike for 8 hours?
  
4. How much would it cost to rent the bike for 24 hours?
  
5. Divide each cost you found (Questions #2 - 4) by the number of hours to find the cost per hour.
  
  
6. The cost per mile...
  - a. is the same, so this is a proportional relationship.
  - b. is the same, so this is not a proportional relationship.
  - c. is not the same, so this is a proportional relationship.
  - d. is not the same, so this is not a proportional relationship.

The cost per hour in this problem is the constant of proportionality.

7. The constant of proportionality in this scenario is \_\_\_\_\_.

**A carnival charges an entry fee of \$5, and \$1 for every game or ride.**

8. What does \$5 represent in this situation?
  - a. The cost for each game or ride.
  - b. The cost to enter the carnival.
  
9. What does \$1 represent in this situation?
  - a. The cost for each game or ride.
  - b. The cost to enter the carnival.

10. Determine how much it would cost for one person to go to the carnival and visit 2 games or rides. 8 games or rides? 24 games or rides?

11. Divide each cost by the number of rides or games to find the cost per ride or game.

12. Is the relationship proportional? How do you know?

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Look at each relationship below. Determine whether each is proportional or not proportional. Show or explain how you know.

13.

number of bagels	cost
2	\$1.80
7	\$6.30
10	\$9

14.

number muffins	cost
3	\$5
5	\$7
7	\$9

15.

number of donuts	cost
6	\$3.60
10	\$6.00
20	\$11.40

1. A board game requires each player to start with \$10. Fill in the table to find how many dollars will be needed for 2, 3, 6, and 8 players. Then, fill in the column labeled dollars per player

Number of players	Number of dollars	Dollars per player
2		
3		
6		
8		

Is the relationship proportional? How do you know?

---



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2. An amusement park offers a season pass where guests pay a flat fee of \$20, and then they only pay \$5 per visit to the park.

Number of visits	Total cost	Cost per visit
2		
5		
7		
10		

Is the relationship proportional? How do you know?

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**3. Patrick mows lawns over the summer. He charges his customers an \$80 fee for the summer, and \$22 per lawnmowing.**

- a. How much does Patrick charge to mow 1 lawn?
  
  
  
  
  
  
  
  
  
  
- b. How much does Patrick charge to mow 2 lawns?
  
  
  
  
  
  
  
  
  
  
- c. How much does Patrick charge to mow 3 lawns?
  
  
  
  
  
  
  
  
  
  
- d. Patrick said this relationship is proportional, because the cost increases \$22 every time. Is he correct? How do you know?

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**4. Determine whether each table represents a relationship that could be proportional.**

Number of hats	Cost in dollars
20	162
2	18
8	66

Number of hats	Cost in dollars
6	60
11	110
12	120

Number of hats	Cost in dollars
4	24
5	33
20	108

A bike-share service charges \$4 per hour to rent a bike.

1. The \$4 in this story represents:
- The cost to rent a bike once
  - The cost to rent a bike each hour.

hr	cost
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32

2. How much would it cost to rent the bike for 2 hours?
3. How much would it cost to rent the bike for 8 hours?
4. How much would it cost to rent the bike for 24 hours?

$24 \times 4 = \text{\$}96$

5. Divide each cost you found (Questions #2 - 4) by the number of hours to find the cost per hour.

$8 \div 2 = 4$   
 $32 \div 8 = 4$   
 $96 \div 24 = 4$

6. The cost per mile...
- is the same, so this is a proportional relationship.
  - is the same, so this is not a proportional relationship.
  - is ~~not the same~~, so this is a proportional relationship.
  - is ~~not the same~~, so this is not a proportional relationship.

The cost per hour in this problem is the constant of proportionality.

7. The constant of proportionality in this scenario is 4.

A carnival charges an entry fee of \$5, and \$1 for every game or ride.

8. What does \$5 represent in this situation?
- The cost for each game or ride.
  - The cost to enter the carnival.
9. What does \$1 represent in this situation?
- The cost for each game or ride.
  - The cost to enter the carnival.

10. Determine how much it would cost for one person to go to the carnival and visit 2 games or rides. 8 games or rides? 24 games or rides?

$$5 + 2(1)$$

$$\text{\$7}$$

$$5 + 8(1)$$

$$\text{\$13}$$

$$5 + 24(1)$$

$$\text{\$29}$$

11. Divide each cost by the number of rides or games to find the cost per ride or game.

$$7 \div 2 = \frac{7}{2} = 3\frac{1}{2}$$

$$13 \div 8 = \frac{13}{8} = 1\frac{5}{8}$$

$$29 \div 24 = \frac{29}{24} = 1\frac{5}{24}$$

12. Is the relationship proportional? How do you know?

No, this is not proportional. The cost per ride or game varies.

Look at each relationship below. Determine whether each is proportional or not proportional. Show or explain how you know.

13. *proportional*

number of bagels	cost
2	\\$1.80
7	\\$6.30
10	\\$9

$$1.80 \div 2 = 0.90 \checkmark$$

$$6.30 \div 7 = 0.90 \checkmark$$

$$9 \div 10 = 0.90 \checkmark$$

14. *not proportional*

number muffins	cost
3	\\$5
5	\\$7
7	\\$9

$$5 \div 3 = \frac{5}{3} = 1\frac{2}{3}$$

$$7 \div 5 = \frac{7}{5} = 1\frac{2}{5}$$

$$9 \div 7 = \frac{9}{7} = 1\frac{2}{7}$$

15. *not proportional*

number of donuts	cost
6	\\$3.60
10	\\$6.00
20	\\$11.40

$$3.60 \div 6 = 0.60 \checkmark$$

$$6 \div 10 = 0.60 \checkmark$$

$$11.40 \div 20 = 0.57 \times$$



1. A board game requires each player to start with \$10. Fill in the table to find how many dollars will be needed for 2, 3, 6, and 8 players. Then, fill in the column labeled dollars per player

Number of players	Number of dollars	Dollars per player
2	\$20	\$10
3	\$30	\$10
6	\$60	\$10
8	\$80	\$10

Is the relationship proportional? How do you know?

Yes. The dollars per player is consistent for any number of players.

2. An amusement park offers a season pass where guests pay a flat fee of \$20, and then they only pay \$5 per visit to the park.

Number of visits	Total cost	Cost per visit
2	\$30	\$15
5	\$45	\$9
7	\$55	$\$7\frac{2}{7}$
10	\$70	\$7

$$20 + 10 = 30$$

$$20 + 25 = 45$$

$$20 + 35 = 55$$

$$20 + 50 = 70$$

Is the relationship proportional? How do you know?

No. The cost per visit varies depending on the number of visits.



3. Patrick mows lawns over the summer. He charges his customers an \$80 fee for the summer, and \$22 per lawnmowing.

a. How much does Patrick charge to mow 1 lawn?

$$80 + 22 = \text{\$}102$$

$$102 \div 1 = 102$$

b. How much does Patrick charge to mow 2 lawns?

$$80 + 22 + 22$$

$$80 + 44 = \text{\$}124$$

$$124 \div 2 = 62$$

c. How much does Patrick charge to mow 3 lawns?

$$80 + 22(3)$$

$$80 + 66 = \text{\$}146$$

$$146 \div 3 = 48 \frac{2}{3}$$

d. Patrick said this relationship is proportional, because the cost increases \$22 every time. Is he correct? How do you know?

He is incorrect. It is not proportional, because the cost per lawn varies depending on the number of lawns.

4. Determine whether each table represents a relationship that could be proportional.

**NOT!**

Number of hats	Cost in dollars
20	162
2	18
8	66

$$162 \div 20 = 8 \frac{1}{10}$$

$$18 \div 2 = 9$$

$$66 \div 8 = 8 \frac{1}{4}$$

**PROPORTIONAL!**

Number of hats	Cost in dollars
6	60
11	110
12	120

$$60 \div 6 = 10$$

$$110 \div 11 = 10$$

$$120 \div 12 = 10$$

**NOT!**

Number of hats	Cost in dollars
4	24
5	33
20	108

$$24 \div 4 = 6$$

$$33 \div 5 = 6 \frac{3}{5}$$

$$108 \div 20 = 5 \frac{2}{5}$$

## **G7 U5 Lesson 2**

Interpret tape diagrams that represent word problems, and use them to find unknown values.

**G7 U5 Lesson 2 - Students will interpret tape diagrams that represent word problems, and use them to find unknown values.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we looked at relationships that were proportional and not proportional. We used information from a story or a table to find unknown values based on the relationships. Today's work will still have use considering how we can find unknown values, but our work will mostly focus on using tape diagrams to help us find unknown values. Tape diagrams, sometimes called bar models, are useful tools to help us visualize what is happening in math problems. We can use the visual of a tape diagram to help us consider how to find any potential unknowns. Let's get started.

**Let's Talk (Slide 3):** Here we see two different examples of tape diagrams. Take a moment to look at them, then tell me what you notice and wonder about each. **Possible Student Answers, Key Points:**

- I notice they both show a total of 9. I notice one is split into equal boxes and the other is split into different-sized boxes. I notice they're rectangular. I notice the unknown is labeled with x.
- I wonder what x equals in both tape diagrams. I wonder if x is the same in both. I wonder what equations or stories could match the equations.

Most our tape diagrams today will be related to a story or a context. Even though we don't know the story behind these two tape diagrams, we can still use the structure of the tape diagram to help us consider how to find each unknown value.

$$\begin{aligned}x + x + x &= 9 \\ 3 \cdot x &= 9 \\ 9 \div 3 &= x\end{aligned}$$

Let's look at the top tape diagram. There are many different ways I can look at this tape diagram to consider the value of the unknown. I see three boxes that each have the same value, x. I see the total of all three boxes is labeled as 9. If I wanted to think about this mathematically, I might think  $x + x + x = 9$ . I know if I add the three boxes together, that should give me the total. (*write equations as you narrate*) Or maybe my brain sees this as 3 groups of x equals 9, so I write  $3 \cdot x = 9$ . Or even still, I could look

at this tape diagram as a total of 9 split into 3 equal boxes, so I write the equation  $9 \div 3 = x$ . Regardless of how I think of the tape diagram, what is the value of x? ( $x = 3$ ) The tape diagram helps me visualize the solution pathway or pathways that most make sense to me.

$$\begin{aligned}x + 3 &= 9 \\ 9 - 3 &= x\end{aligned}$$

What about the other tape diagram? This one doesn't have equal groups like the last one. What different ways could somebody look at this one to determine what the value of x is?

(*write equations as student shares, supporting as necessary*) **Possible Student Answers, Key Points:**

- I see two parts, x and 3, and they total up to 9. I can write  $x + 3 = 9$  to represent the tape diagram.
- I can think of this as a total of 9. To find x, I could subtract out the part that is 3 to see what is left. So  $9 - 3 = x$  could help me.

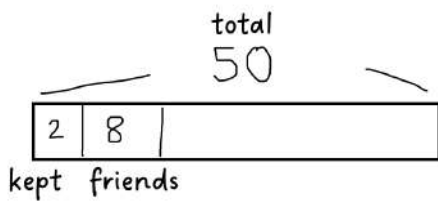
Great. Let's keep all this in mind as we look at some application problems where we can draw and then interpret tape diagrams to help us find unknowns.

**Let's Think (Slide 4):** I'll read this problem once through while you read it to yourself. After, I want you to summarize what the story is about in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- Ron made 50 flyers for his campaign. He kept some, gave some to his friends, and then split the rest up evenly to put in hallways in his school.

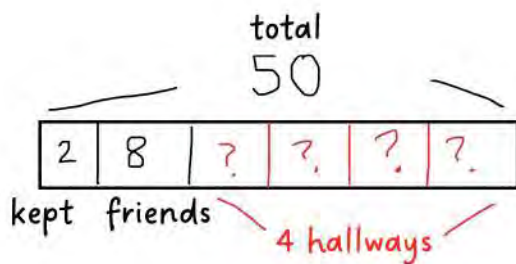


- We know how much he made. We know how much he kept for himself and gave to his friends. We don't know the number that he put in each of the four hallways.



Before we try to solve, let's visualize this story using a tape diagram. (*reread the first sentence*) I know Ron made a total of 50 flyers. (*draw as you narrate*) I can draw a rectangle to represent the total, and I'll label the entire rectangle as 50. (*reread the second sentence*) If he kept 2 for himself, I'll partition a small rectangle to show that out of the 50 flyers, he kept these 2. I'll put a 2 in the box and label it, so I don't forget what that 2 means in the story. I'll do the same thing for the

flyers he gives his friends. I'll partition, or cut, a slightly larger box to show that he gave a few more flyers to his friends. I'll put 8 in that box, and I'll label it as friends. (*point to empty space remaining*) This empty space in the rectangle represents the rest of the flyers.



I know Ron used the remaining flyers to hang equally along 4 hallways. How can I use this part of the tape diagram to show those 4 equal groups? (*partition the remaining section into 4 equal parts as student shares*) Possible Student Answers, Key Points:

- You can cut that part into fourths.
- You can partition the section into 4 equal-sized boxes to represent the amount in each hallway.

I'll label each box with a question mark since we don't know how many flyers he put in each hallway. I'll note that these four boxes represent the flyers that were put up in each of the four hallways. Now, we've represented the entire story. We can step back, look at the tape diagram, and start to think about how we can find those unknowns.

I see the total is 50, that's clear from the story and the model. I really only care about the flyers in the hallway, so I can remove the flyers that he kept and the flyers he gave to his friends. I could subtract  $50 - 2$ , and then subtract out the 8 he gave to his friends. Or, I also know that  $2 + 8$  is an easy 10. Let's just subtract the 10 flyers we know about from our total. (*write  $50 - 10 = 40$* ) Once we remove those flyers, we can see that he has 40 flyers to hang up equal among the 4 hallways. What math can we do to figure out how many flyers went up in each hallway? Possible Student Answers, Key Points:

$$50 - 10 = 40$$

$$40 \div 4 = 10$$

flyers

- I know  $4 \times ? = 40$ , so he has to put 10 up in each hallway.
- I know 40 divided into 4 equal groups is 10.
- I know  $10 + 10 + 10 + 10$  would be 40.

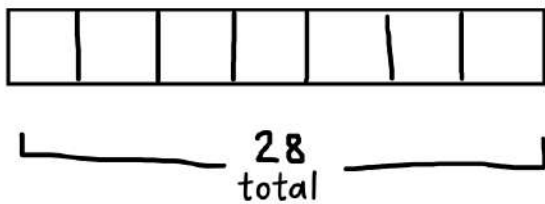
There are several ways to arrive at our answer. I'm going to think about 40 split into 4 equal groups. (*write equation*) I know 40 divided by 4 is equal to 10.

Ron hung 10 flyers up in each hallway. The tape diagram helped us visualize what was happening in the story so that we could think about the different parts in the story, the total, and what math we could do to eventually arrive at our unknown value. Let's try this thinking out with a different type of story.

**Let's Think (Slide 5):** I'll read this problem once through while you read it to yourself. After, I want you to summarize what the story is about in your own words. What is known? What is unknown? Possible Student Answers, Key Points:

- Jai's unpacking plates after a move, and we're trying to figure out how many plates were in each box before she started unpacking.

- I know she has 7 boxes. I know she has removed 5 plates from each, and there are 28 left to unpack. We don't know how many were in each box to start.



Let's start sketching our tape diagram. I know Jai has 7 boxes. I'll begin by drawing 7 identical boxes. I find it easiest to draw a big rectangle and then split it into 7 equal groups. (*sketch tape diagram as you narrate*) The total number of plates left in those 7 boxes now is 28, so I'll use a bracket to show that.



Now, let's think about what we know about each box. Based on the story, I know each box had some plates in it, and then Jai unpacked or removed 5 from each. I can use the expression  $p - 5$  to represent this. The variable  $p$  represents the number of plates to begin with, and I can subtract 5 from  $p$  to represent the plates she unpacked. (*label each box with  $p - 5$* )

That's all the information we know. Let's step back and look at the tape diagram to think about an easy way to find the value of our unknown.

$$28 \div 7 = 4$$

$$p - 5 = 4$$

$$p = 9 \text{ plates in each box originally}$$

I *could* add up " $p - 5$ " seven times, because I know all 7 boxes have a total of 28. Picturing that as an equation feels like a bit much, though. What if we started by dividing the known total of 28 by the number of boxes, since we know each box is identical? What is 28 divided by 7 boxes? (4) (*write 28 divided by 7 = 4*) Nice! This means each box currently has 4 plates in it. Thinking about just one box, I know she started with some plates,  $p$ , she took out 5 plates, and now she has 4 plates in each box. I can write  $p - 5 = 4$  to represent that. What math can

I consider to find  $p$  now? **Possible Student Answers, Key Points:**

- I know  $p$  is 9, because  $9 - 4 = 5$ .
- I can write a related addition equation to find the unknown.  $4 + 5 = p$  helps me see that  $p = 9$ .

The value of the unknown is 9. Jai had 9 plates in each box before she began unpacking. The tape diagram helped us picture the story. It helped us see the total and each equal box so that we could strategically solve the problem one step at a time.

**Let's Try it (Slides 6 - 7):** Now it's our time to try a couple more examples together. Like we've been doing, we'll read each problem while carefully considering what we know and don't know. Based on what we know, we'll sketch and label a tape diagram to represent the story. The visual of the tape diagram will help us consider which operation or operations will best support us in planning our solution pathway. Let's dive in together, then you'll have some time to practice independently.

# WARM WELCOME



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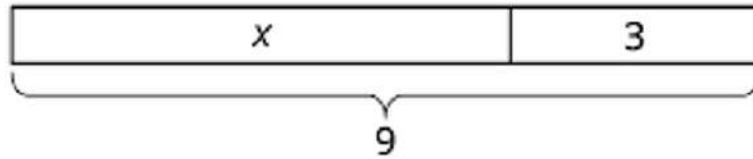
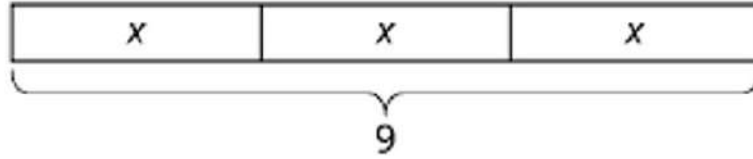
**Today we will interpret tape diagrams that represent word problems, and use them to find unknown values.**

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## Let's Talk:

What do you notice about the tape diagrams?  
What do you wonder?



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## Let's Think:

**Ron made 50 flyers for his campaign for student council president. He kept 2 for himself, gave 8 to friends, and divided the rest evenly along the 4 hallways of his school. How many flyers did he put in each hallway?**

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## Let's Think:

Jai packed  $p$  plates into each of 7 boxes when she moved. After unpacking 5 plates from each box, she has 28 plates left to unpack. How many plates did Jai originally pack in each box?

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## Let's Try It:

Let's explore interpreting tape diagrams that represent word problems, and use them to find unknown values together.

Name: \_\_\_\_\_ G7 US Lesson 2 - Let's Try It

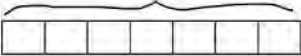
Marina's mom baked 72 cupcakes. She gave 30 cupcakes to Marina's teacher to give out to her class, and then divided the rest of them equally among 7 neighbors.

- What is happening with the cupcakes in this story?
  - They are baking cupcakes.
  - They are giving away cupcakes.
- How many total people did Marina's mom give cupcakes to? \_\_\_\_\_
- Did each person get the same amount of cards? Explain.  
\_\_\_\_\_
- Label the tape diagram with the values that are known from the story. Label the unknown(s) with  $c$  to represent cupcakes.
 


- What does  $c$  represent in this context?
  - The total number of cupcakes given away
  - The number of cupcakes the teacher got
  - The number of cupcakes each neighbor gets
- Use any strategy to find the value of  $c$ .

Xavier is making 7 identical gift bags to give his friends after his birthday party. He puts 6 stickers into each gift bag. He also puts some chocolate bars in each bag. In all, he uses 56 items.

- We know...
- We don't know...
- Use the tape diagram below to label what is known. Use  $x$  to represent any unknowns.
 


- Use the tape diagram to find the number of chocolate bars in each bag.

Nina has 5 racks of shoes with  $p$  pairs of shoes on each rack. She wants to donate 1 pair of shoes from each rack. After donating, she has 10 pairs of shoes left.

- What is known? What is unknown?
- Draw a tape diagram to represent the story. Use  $p$  to represent the number of pairs of shoes Nina originally has on each rack.
- Use the tape diagram to find the value of  $p$ .

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# On your Own:

Now it's time to interpret tape diagrams that represent word problems, and use them to find unknown values on your own.

Name: \_\_\_\_\_ G7 US Lesson 2 - Independent Work

1. Select ALL the stories that the tape diagram can represent.

a. Henry's spends \$47 on 4 tickets to the school play and \$27 at the concession stand.

b. Maria buys 4 bags of mints with 27 mints in each pack. She gives 47 mints to her brother.

c. Lance has a collection of 47 rocks. He gives 27 rocks to his best friend and puts the other rocks equally into 4 boxes.

d. Yusuf made 47 friendship bracelets. He sells 27 of them, and he splits the remaining bracelets up between 4 friends.

e. There are 47 coops and 27 cats in the animal shelter. The cats are split equally into 4 rooms.

2. Ms. Catalano is grading 109 essays. So far, she has graded 75 essays. How many essays does she have left to grade?

a. Draw a tape diagram to represent the story problem. Use  $x$  to represent the unknown.

b. Use the tape diagram to determine how many essays Ms. Catalano has left to grade.

3. Erin organizes her jewelry into boxes. She puts 3 bracelets and  $n$  necklaces into each box. After filling 6 boxes, she used a total of 42 pieces of jewelry.

a. Draw a tape diagram to represent the story problem.

b. Use the tape diagram to determine how many necklaces Erin puts in each box.

4. Mr. Guzman has 7 prize boxes for his students. Each box has  $p$  prizes. His students take 8 prizes from each box. Mr. Guzman had 21 prizes left in all. How many prizes were in each box to start with?

Draw a tape diagram as part of your solution pathway.

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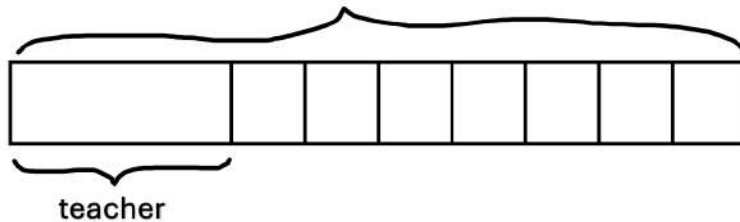
**Marina's mom baked 72 cupcakes. She gave 30 cupcakes to Marina's teacher to give out to her class, and then divided the rest of them equally among 7 neighbors.**

1. What is happening with the cupcakes in this story?
  - a. They are baking cupcakes.
  - b. They are giving away cupcakes.
2. How many total people did Marina's mom give cupcakes to? \_\_\_\_\_
3. Did each person get the same amount of cards? Explain.

---

---

4. Label the tape diagram with the values that are known from the story. Label the unknown(s) with  $c$  to represent cupcakes.



5. What does  $c$  represent in this context?
  - a. The total number of cupcakes given away
  - b. The number of cupcakes the teacher got
  - c. The number of cupcakes each neighbor gets
6. Use any strategy to find the value of  $c$ .

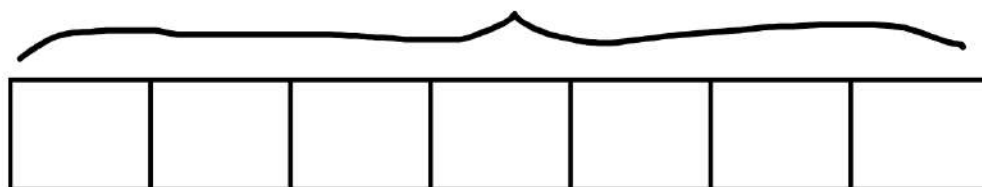


**Xavier is making 7 identical gift bags to give his friends after his birthday party. He puts 6 stickers into each gift bag. He also puts some chocolate bars in each bag. In all, he uses 56 items.**

7. We know...

8. We don't know...

9. Use the tape diagram below to label what is known. Use  $x$  to represent any unknowns.



10. Use the tape diagram to find the number of chocolate bars in each bag.

**Nina has 5 racks of shoes with  $p$  pairs of shoes on each rack. She wants to donate 1 pair of shoes from each rack. After donating, she has 10 pairs of shoes left.**

11. What is known? What is unknown?

12. Draw a tape diagram to represent the story. Use  $p$  to represent the number of pairs of shoes Nina originally had on each rack.

13. Use the tape diagram to find the value of  $p$ .



**3. Erin organizes her jewelry into boxes. She puts 3 bracelets and  $n$  necklaces into each box. After filling 6 boxes, she used a total of 42 pieces of jewelry.**

a. Draw a tape diagram to represent the story problem.

b. Use the tape diagram to determine how many necklaces Erin puts in each box.

**4. Mr. Guzman has 7 prize boxes for his students. Each box has  $p$  prizes. His students take 4 prizes from each box. Mr. Guzman had 21 prizes left in all. How many prizes were in each box to start with?**

Draw a tape diagram as part of your solution pathway.

Marina's mom baked 72 cupcakes. She gave 30 cupcakes to Marina's teacher to give out to her class, and then divided the rest of them equally among 7 neighbors.

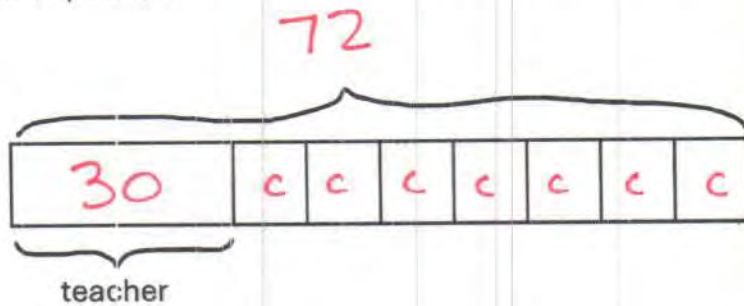
1. What is happening with the cupcakes in this story?
  - a. They are baking cupcakes.
  - b. They are giving away cupcakes.

2. How many total people did Marina's mom give cupcakes to? 8

3. Did each person get the same amount of cards? Explain.

The teacher gets 30 and the neighbors each get an unknown amount.

4. Label the tape diagram with the values that are known from the story. Label the unknown(s) with  $c$  to represent cupcakes.



5. What does  $c$  represent in this context?
  - a. The total number of cupcakes given away
  - b. The number of cupcakes the teacher got
  - c. The number of cupcakes each neighbor gets
6. Use any strategy to find the value of  $c$ .

$$\begin{array}{r} 72 \\ -30 \\ \hline 42 \end{array}$$

$$42 \div 7 = 6$$

$c = 6$

Xavier is making 7 identical gift bags to give his friends after his birthday party. He puts 6 stickers into each gift bag. He also puts some chocolate bars in each bag. In all, he uses 56 items.

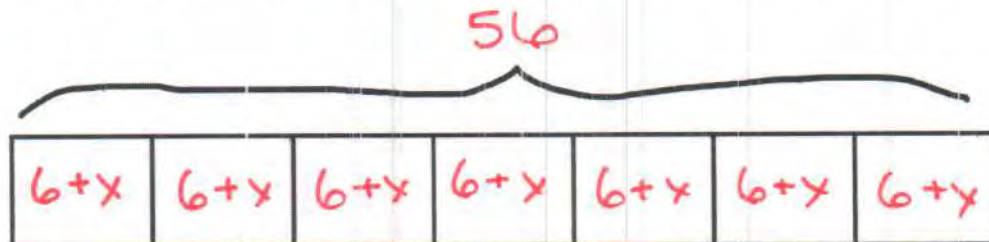
7. We know...

- there are 7 bags
- 6 stickers go in each
- there are 56 items in all

8. We don't know...

- how much chocolate goes in each

9. Use the tape diagram below to label what is known. Use  $x$  to represent any unknowns.



10. Use the tape diagram to find the number of chocolate bars in each bag.

$$56 \div 7 = 8$$

$$6 + x = 8$$

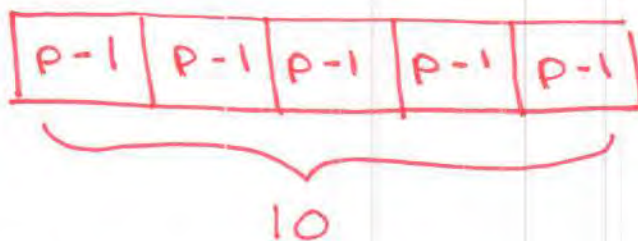
$$x = 2$$

Nina has 5 racks of shoes with  $p$  pairs of shoes on each rack. She wants to donate 1 pair of shoes from each rack. After donating, she has 10 pairs of shoes left.

11. What is known? What is unknown?

- 5 racks
- 10 left
- donates 1 per rack
- how many shoes started on each rack

12. Draw a tape diagram to represent the story. Use  $p$  to represent the number of pairs of shoes Nina originally had on each rack.



13. Use the tape diagram to find the value of  $p$ .

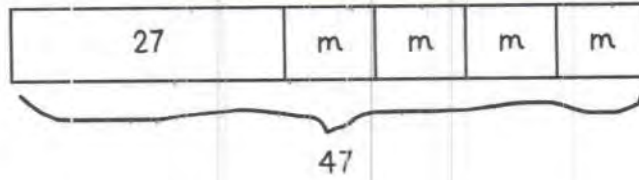
$$10 \div 5 = 2$$

$$p - 1 = 2$$

$$p = 3$$



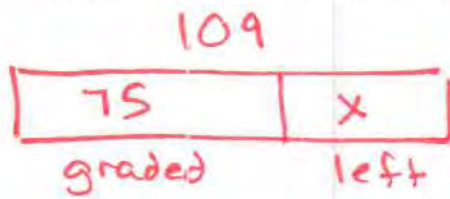
1. Select ALL the stories that the tape diagram can represent.



- a. Henry's spends \$47 on 4 tickets to the school play and \$27 at the concession stand.
- b. Maria buys 4 bags of mints with 27 mints in each pack. She gives 47 mints to her brother.
- c. Lance has a collection of 47 rocks. He gives 27 rocks to his best friend and puts the other rocks equally into 4 boxes.
- d. Yusef made 47 friendship bracelets. He sells 27 of them, and he splits the remaining bracelets up between 4 friends.
- e. There are 47 dogs and 27 cats in the animal shelter. The cats are split equally into 5 rooms.

2. Ms. Catalano is grading 109 essays. So far, she has graded 75 essays. How many essays does she have left to grade?

a. Draw a tape diagram to represent the story problem. Use x to represent the unknown.



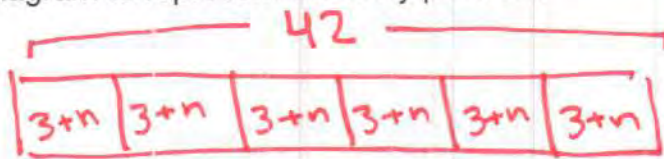
b. Use the tape diagram to determine how many essays Ms. Catalano has left to grade.

$$\begin{array}{r} 109 \\ - 75 \\ \hline 34 \end{array}$$

34 essays

3. Erin organizes her jewelry into boxes. She puts 3 bracelets and  $n$  necklaces into each box. After filling 6 boxes, she used a total of 42 pieces of jewelry.

a. Draw a tape diagram to represent the story problem.



b. Use the tape diagram to determine how many necklaces Erin puts in each box.

$$42 \div 6 = 7$$

$$3 + n = 7$$

$$n = 4$$

4. Mr. Guzman has 7 prize boxes for his students. Each box has  $p$  prizes. His students take 4 prizes from each box. Mr. Guzman had 21 prizes left in all. How many prizes were in each box to start with?

Draw a tape diagram as part of your solution pathway.



$$21 \div 7 = 3$$

$$p - 4 = 3$$

$$p = 7$$



## **G7 U5 Lesson 3**

Write and match equations and tape diagrams that represent the same situation.

**G7 U5 Lesson 3 - Students will write and match equations and tape diagrams that represent the same situation.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we thought about how we can use tape diagrams to help us think about solutions to math problems. How did tape diagrams help us do that? **Possible Student Answers, Key Points:**

- Tape diagrams help us picture what is happening in a story. I can easily see parts and totals and consider how to find any unknowns.
- Based on how I draw my tape diagram, I can think of different ways to arrive at the same correct answer.

Today we're going to continue working with tape diagrams. Similar to how we saw multiple solution pathways given the same tape diagram in our previous lesson, today we'll see how we can write multiple equations to represent the same tape diagram. Let's get started.

**Let's Talk (Slide 3):** Look at the two tape diagrams shown here. What is the same about them? What is different? **Possible Student Answers, Key Points:**

- They're both the same length. They both have a 14, at least one 6, and some x's. They both show a rectangle split into smaller rectangles.
- The first one has three boxes inside, and the second only has two boxes. The second one has equal-sized boxes. The first one only has one term inside each box.

I notice both tape diagrams show a total of 14, but there is some variation in terms of how each box is composed. The first shows a 6 and x and an x. The second shows two equal groups of 6 plus x. Our work today will involve writing two or more equations to represent the same tape diagram. The first question we work on together is going to ask us to think about these two tape diagrams.

**Let's Think (Slide 4):** For this problem, we're tasked with writing 2 different, yet equivalent, expressions to represent each tape diagram.

$$6 + x + x = 14$$

$$6 + 2x = 14$$

Let's start by looking at the first tape diagram and thinking about how we can represent the visual with an equation. We already noted that this tape diagram has a total of 14 and that the boxes that compose the large rectangle show 6, x, and x. I know that if I combine those values, the total will equal 14. We can write that as an equation by simply writing  $6 + x + x = 14$ . (*write equation*) Another way I can think about the tape diagram is to think about the two x's as being two *groups of x*. In that

case, I might think 6 plus two equal groups of x is equal to 14. I can write that using multiplication. 6 plus 2x equals 14. (*write equation*) That's it! We just used two different equations to represent the same tape diagram.

Before we look at the next tape diagram, think back to our previous lesson. Can you use the tape diagram, or maybe one of the equations, to figure out what x must be equal to? **Possible Student Answers, Key Points:**

- I know the total is 14 and one part is 6, so I can take out 6 from 14 which leaves me with 8. Then I know  $2x$  or  $x + x$  has to equal 8, so x has to be 4.

$$14 - 6 = 8$$

$$2x = 8$$

$$x = 4$$

(*write equations as you narrate*) We can subtract the 6 part from the total of 14. 14 minus 6 equals 8. I know the 2 x-values now have to be equal to 8.  $2x = 8$  or  $x + x = 8$ , so I know x has to be equal to 4.

Now let's do the same work with the other tape diagram. We already noted earlier that this tape diagram is a little different.

$$6 + x + 6 + x = 14$$

$$2(6 + x) = 14$$

I see two parts in this tape diagram:  $6 + x$  and another  $6 + x$ . They're both the same parts. I know if I combine those two parts together, the total should be 14. One way to write that as an equation is to simply add everything inside the tape diagram. 6 plus x plus the other 6 plus x equals 14. *(write equations as you narrate)* Since both groups in this tape diagram are the same, I can also think of this as 2 groups of " $6 + x$ " have a total of 14. I can write that as 2

relationships we see in the tape diagram.

I wonder if you can use the tape diagram or equations to help you find x. What do you think? **Possible Student Answers, Key Points:**

- I see 14 split into two equal groups, so each group has to be equal to 7. I know because  $7 + 7$  would equal 14. If each group is equal to 7, then I know x has to be 1. I know because  $6 + 1 = 7$ .

$$14 \div 2 = 7$$

$$6 + x = 7$$

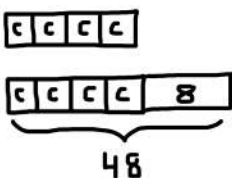
$$x = 1$$

We can easily see the equal groups in the tape diagram. If two equal groups have a total of 14, I know each group has a value of 7. There are a few ways to consider that, but I'll write 14 divided by 2, since my brain saw this as splitting 14 into 2 groups. *(write equations as you narrate)* If each group is worth 7, then I know  $6 + x$  has to equal 7. From there, x must be 1.  $6 + 1 = 7$ .

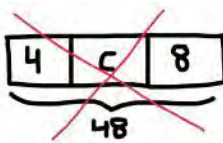
We just worked to write different, equivalent expressions to represent the first tape diagram. Then we did the same work with a different tape diagram.

Now let's see if we can create our own tape diagram based off of equations...

**Let's Think (Slide 5):** For this problem, we'll draw our own tape diagram to match the equation. Let's look at the first equation in red.



I can think of this equation as 4 groups of c plus 8 equals 48. I can start by drawing just that! I'll draw 4 groups of c by drawing four equal-sized rectangles. *(sketch as shown)* To that, I'll add 8 by drawing an adjacent rectangle and labeling that with the number 8. All four groups of c and the 8 should have a total of 48, so I'll use a bracket to show the total.



*(sketch a tape diagram showing three adjacent rectangles labeled 4, c, and 8)* Why would this tape diagram NOT represent the equation we were given? **Possible Student Answers, Key Points:**

- Our equation shows 4 groups of c. This equation only shows 1 group of c.
- This tape diagram shows  $4 + c + 8 = 48$  instead of  $4 \times c + 8 = 48$ .

When we make our own tape diagrams, it's important to think about what the operations in our equation mean and how we can represent them accurately.

$$48 - 8 = 40$$

$$40 \div 4 = 10$$

$$c = 10$$

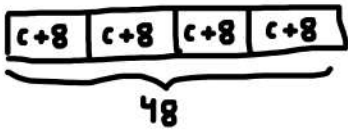
Before we look at our last equation, how could we use the correct tape diagram or the equation to find the solution? *(write equations as student shares, supporting as needed)*

**Possible Student Answers, Key Points:**

- I know the total is 48 and one part is 8.  $48 - 8 = 40$ , so I know all four equal groups of c must have a total of 40.  $40 \div 4 = 10$  or  $4 \times 10 = 40$  can help me see that  $c = 10$ .

Lastly, let's try sketching a tape diagram to represent the second equation in blue. What's the same and what's different about this equation? **Possible Student Answers, Key Points:**

- This equation also has a total of 48. The unknown is also labeled as  $c$ .
- This equation shows 4 groups of " $c + 8$ " instead of just 4 groups of  $c$ .



I can think of this equation as 4 groups of the value " $c + 8$ ". When I picture the tape diagram, I know that the 4 equal groups will have a total of 48. (*sketch a large rectangle, and partition it into 4 equal groups*) I'll use a bracket to show that the total is 48, and I can label  $c + 8$  inside each of the 4 groups. (*label as narrated*) Here is our tape diagram that represents the equation.

$$48 \div 4 = 12$$

$$c + 8 = 12$$

$$c = 4$$

Could we use the tape diagram or equation to help us think about the solution? How so? (*write equations as student shares, supporting as needed*) **Possible Student Answers, Key Points:**

- I know the 48 is split into 4 groups, so each group has to equal 12. 48 divided by 4 equals 12. If each group equals 12, I can think of  $c + 8 = 12$  to help me. I know  $c$  must be equal to 4.

Nicely done. Look at both tape diagrams we drew and their equations. What do you notice is different about the tape diagrams and why do you think that is? **Possible Student Answers, Key Points:**

- The first tape diagram has 4  $c$ 's and only 1 8, because we were just adding 8 to the four groups of  $c$ .
- The second tape diagram has 4  $c$ 's and 4 eights, because the equation shows 4 times the quantity of  $c + 8$ .

It's important to think carefully about what the operations in the equation mean, so that our tape diagram accurately reflects the problem.

**Let's Try it (Slides 6 - 7):** Now let's try out a few more similar problems. As we noticed today, whether we're writing an equation or a tape diagram, we want to carefully think about what operations mean. When we multiply, we're often thinking of it as equal groups. We can take equal groups of a single value or equal groups of an expression. Time to try out a few more before you get a chance to work on your own.

# WARM WELCOME



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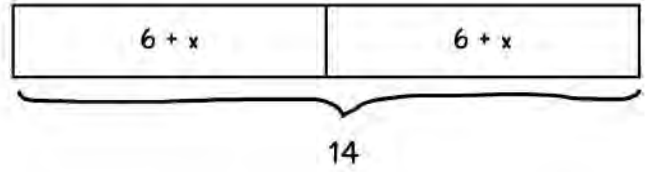
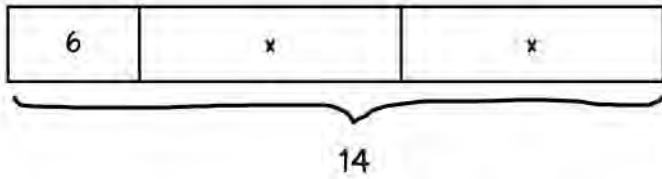
**Today we will write and match equations and tape diagrams that represent the same situation.**

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## Let's Talk:

Look at the two tape diagrams.  
What's the same? What is different?

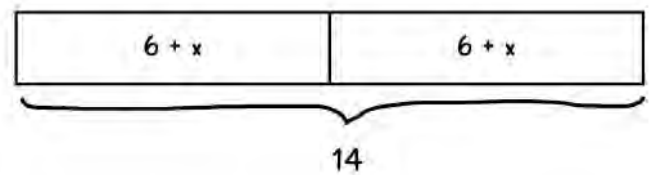
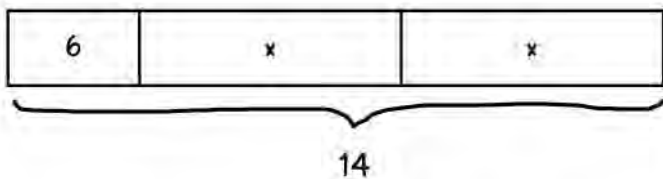


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## Let's Think:

Write two different equations that could represent each tape diagram shown here.



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# Let's Think:

Draw a tape diagram to match each equation.

$$4c + 8 = 48$$

$$4(c + 8) = 48$$

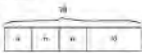
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# Let's Try It:

Let's explore writing and matching equations and tape diagrams that represent the same situation together.

Name: \_\_\_\_\_ G7 US Lesson 3 - Let's Try It

Consider the tape diagram shown here.

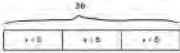


- Write an equation to show that 28 is the sum of the parts in the tape diagram.  

$$28 = \_ + \_ + \_ + \_$$
- Rewrite the equation to show that 28 is equal to 10 plus 3 groups of  $n$ .  

$$\_ = \_ + \_$$
- Use either equation to find the value of  $n$ .

Consider the tape diagram shown here.

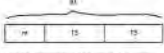


- Write an equation to show that 36 is the sum of the parts in the tape diagram.  

$$36 = \_ + \_ + \_$$
- Rewrite the equation to show that 36 is the sum of 3 groups of  $x + 5$ .
- Use either equation to find the value of  $x$ .

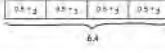
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Consider the tape diagram below.



- Write two different equations to represent the tape diagram.
- Use either equation you wrote to find the value of  $m$ .

Consider the tape diagram below.



- Write two different equations to represent the tape diagram.
- Use either equation you wrote to find the value of  $y$ .

Draw a tape diagram to match each equation. Then find the value of  $w$ .

- $11.76 = 4w + 16$
- $12.76 = 4(w + 4)$

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# On your Own:

Now it's time to write and match equations and tape diagrams that represent the same situation on your own.

Name: \_\_\_\_\_ 67 US Lesson 3 - Independent Work

1. Match the equations to the tape diagram that represents it. Then find the value of  $k$  for each equation.

$25 = 5k + 2$

$25 = 5k + 2$

2. Draw and label a tape diagram to represent each equation. Then find the value of  $v$  in each.

$64 = 4v + 8$        $64 = 4v + 8$

3. Write two different equations that could represent each tape diagram shown here. Find the value of the unknown.

$2 \cdot x$        $2 \cdot x$

24

$x$        $x$       2

24

4. Fran drew this tape diagram to represent  $9(\cdot + 8) = 62$ . Explain why this tape diagram does not represent her equation, and explain what she should do to correct her work.

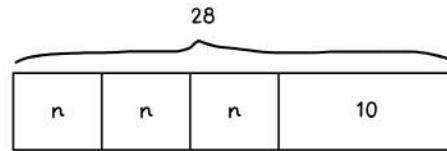
$\cdot$        $\cdot$        $\cdot$

62

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Consider the tape diagram shown here.



1. Write an equation to show that 28 is the sum of the parts in the tape diagram.

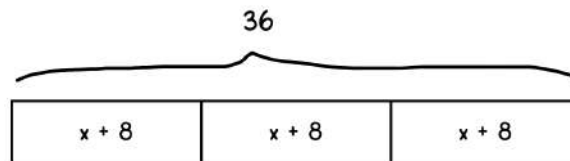
$$28 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

2. Rewrite the equation to show that 28 is equal to 10 plus 3 groups of  $n$ .

$$\underline{\quad} = \underline{\quad} + \underline{\quad}$$

3. Use either equation to find the value of  $n$ .

Consider the tape diagram shown here.



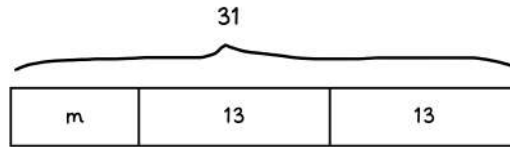
4. Write an equation to show that 36 is the sum of the parts in the tape diagram.

$$36 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

5. Rewrite the equation to show that 36 is the sum of 3 groups of  $x + 8$ .

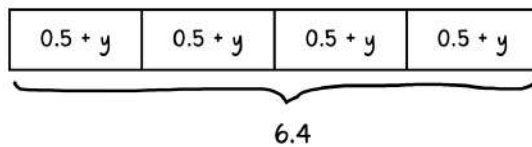
6. Use either equation to find the value of  $x$ .

Consider the tape diagram below.



- Write two different equations to represent the tape diagram.
- Use either equation you wrote to find the value of  $m$ .

Consider the tape diagram below.



- Write two different equations to represent the tape diagram.
- Use either equation you wrote to find the value of  $y$ .

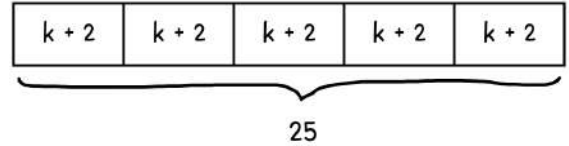
Draw a tape diagram to match each equation. Then find the value of  $w$ .

11.  $76 = 4w + 16$

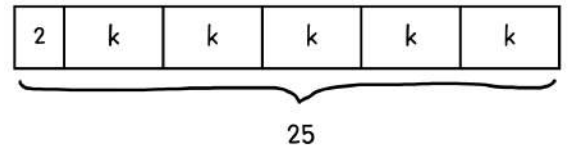
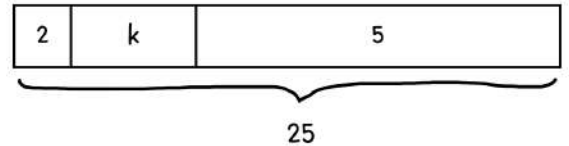
12.  $76 = 4(w + 4)$

1. Match the equations to the tape diagram that represents it. Then find the value of  $k$  for each equation.

$$25 = 5(k + 2)$$



$$25 = 5k + 2$$

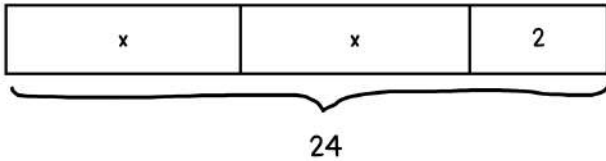
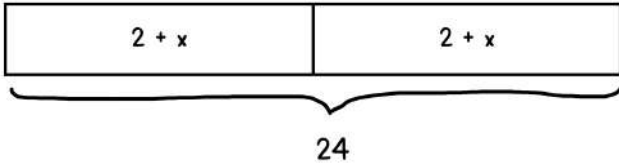


2. Draw and label a tape diagram to represent each equation. Then find the value of  $v$  in each.

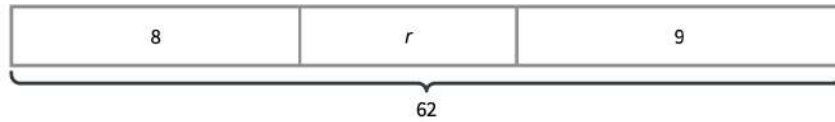
$$64 = 4v + 8$$

$$64 = 4(v + 8)$$

3. Write two different equations that could represent each tape diagram shown here. Find the value of the unknown.



4. Fran drew this tape diagram to represent  $9(r + 8) = 62$ . Explain why this tape diagram does not represent her equation, and explain what she should do to correct her work.



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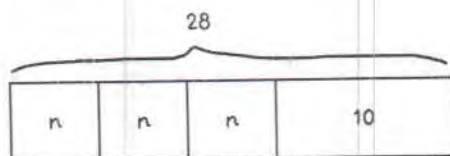
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Consider the tape diagram shown here.



1. Write an equation to show that 28 is the sum of the parts in the tape diagram.

$$28 = \underline{n} + \underline{n} + \underline{n} + \underline{10}$$

2. Rewrite the equation to show that 28 is equal to 10 plus 3 groups of  $n$ .

$$\underline{28} = \underline{3n} + \underline{10}$$

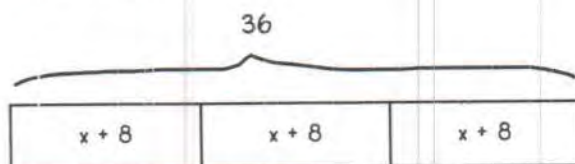
3. Use either equation to find the value of  $n$ .

$$\begin{array}{r} 28 - 10 \\ 18 \end{array}$$

$$\begin{array}{r} 18 \div 3 \\ 6 \end{array}$$

$$\boxed{n = 6}$$

Consider the tape diagram shown here.



4. Write an equation to show that 36 is the sum of the parts in the tape diagram.

$$36 = \underline{x} + \underline{8} + \underline{x} + \underline{8} + \underline{x} + \underline{8}$$

5. Rewrite the equation to show that 36 is the sum of 3 groups of  $x + 8$ .

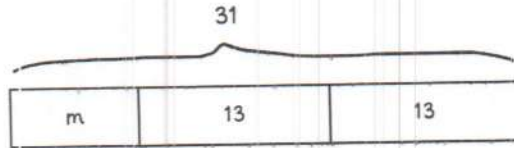
$$36 = 3(x + 8)$$

6. Use either equation to find the value of  $x$ .

$$\begin{array}{r} 36 \div 3 \\ 12 \end{array}$$

$$\begin{array}{r} x + 8 = 12 \\ \boxed{x = 4} \end{array}$$

Consider the tape diagram below.



7. Write two different equations to represent the tape diagram.

$$31 = m + 13 + 13$$

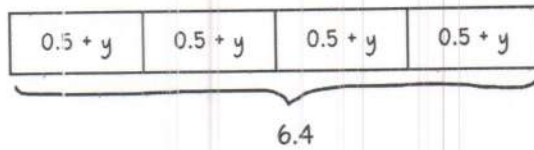
$$31 = m + 26$$

8. Use either equation you wrote to find the value of m.

$$\begin{array}{r} 31 \\ - 26 \\ \hline 5 \end{array}$$

$$m = 5$$

Consider the tape diagram below.



9. Write two different equations to represent the tape diagram.

$$6.4 = 4(0.5 + y)$$

$$6.4 = 0.5 + 0.5 + 0.5 + 0.5 + y + y + y + y$$

10. Use either equation you wrote to find the value of y.

$$6.4 \div 4 = 1.6$$

$$0.5 + y = 1.6$$

~~$$y = 1.1$$~~

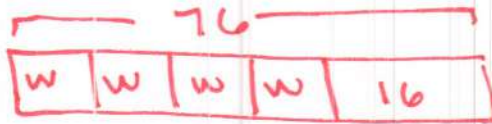
$$y = 1.1$$

Draw a tape diagram to match each equation. Then find the value of w.

11.  $76 = 4w + 16$

$$76 - 16 = 60$$

$$60 \div 4 = 15$$



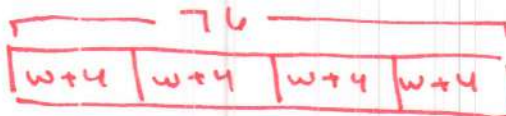
$$w = 15$$

12.  $76 = 4(w + 4)$

$$76 \div 4 = 19$$

$$w + 4 = 19$$

$$w = 15$$



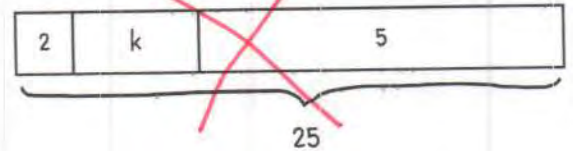
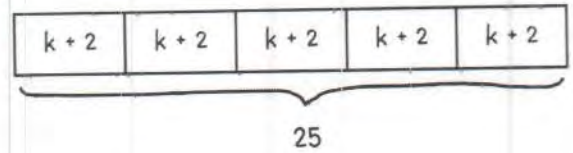
1. Match the equations to the tape diagram that represents it. Then find the value of  $k$  for each equation.

$$25 = 5(k + 2)$$

$$25 \div 5 = 5$$

$$k + 2 = 5$$

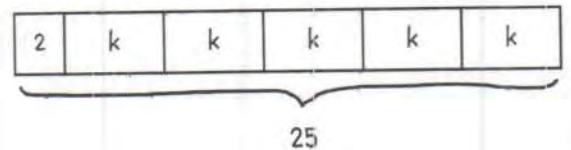
$$k = 3$$



$$25 = 5k + 2$$

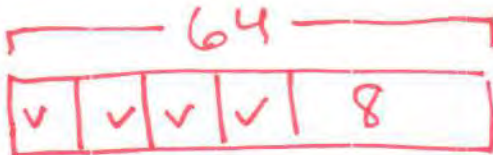
$$25 - 2 = 23$$

$$23 \div 5 = 4 \frac{3}{5} = k$$



2. Draw and label a tape diagram to represent each equation. Then find the value of  $v$  in each.

$$64 = 4v + 8$$

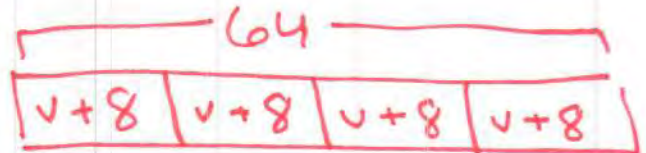


$$64 - 8 = 56$$

$$56 \div 4 = v$$

$$14 = v$$

$$64 = 4(v + 8)$$



$$64 \div 4 = 16$$

$$v + 8 = 16$$

$$v = 8$$



3. Write two different equations that could represent each tape diagram shown here. Find the value of the unknown.



$$2+x+2+x=24$$

$$2(2+x)=24$$

$$24 \div 2 = 12$$

$$2+x=12$$

$$x=10$$



$$x+x+2=24$$

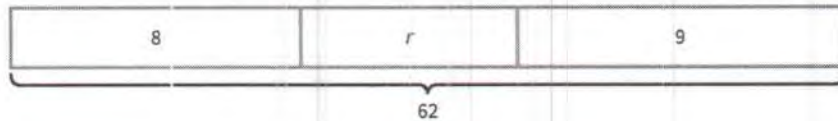
$$2x+2=24$$

$$24-2=22$$

$$22 \div 2 = 11$$

$$x=11$$

4. Fran drew this tape diagram to represent  $9(r+8) = 62$ . Explain why this tape diagram does not represent her equation, and explain what she should do to correct her work.



This tape diagram does not show 9 groups of " $r+8$ ". It shows one group of 8, one group of  $r$ , and one group of 9. A better equation would be  $8+r+9=62$  or  $r+17=62$ .

## **G7 U5 Lesson 4**

Coordinate tape diagrams, equations of the form  $px + q = r$ , and verbal descriptions of the situations, and reason about and interpret a solution.

**G7 U5 Lesson 4 - Students will coordinate tape diagrams, equations of the form  $px + q = r$ , and verbal descriptions of the situations, and reason about and interpret a solution.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous few lessons, we've explored how tape diagrams can help us represent and solve story problems or equations. We're going to continue that work today. As we work, since we often rely on the visual tape diagram to help us make sense of what is going on, it's important that our tape diagrams be neat and easy to interpret. What can we do today as we work to keep our tape diagrams clear and helpful? **Possible Student Answers, Key Points:**

- We can use word labels so we know what each part of the tape diagram means in a given context.
- We can use number labels so we can think about the values in the problem.
- We can try to keep our lines straight and any equal groups even so that the tape diagram is easy to read.

Those are great ideas. No tape diagram is perfect, but we still want to try to keep them as neat and accurate as possible so they can be helpful for interpreting any given scenario. Our goal today will be to think about how we can make sense of a particular type of story problem by using equations and tape diagrams. We'll want to make sure we can explain clearly how the tape diagram or equation accurately represents the given scenario.

Let's begin by being creative and thinking of our own story to match a tape diagram.

**Let's Talk (Slide 3):** Take a look at this tape diagram. Before we think about a story, pause to consider what you notice about this tape diagram. **Possible Student Answers, Key Points:**

- The total is 20. I see that is labeled with a bracket.
- I see three groups of  $m$  and one 2.

Can you think of a story that has those components? The story should have a total of 20, and the total needs to be composed of 3 groups of something and 2 more. **Possible Student Answers, Key Points:**

- There are 20 people on a playground. There are 2 teachers and 3 equal groups of students. How many students are in each group?
- I have 3 equal bags of M&Ms and 2 loose M&Ms. In all, I have 20 M&Ms. How many M&Ms are in each bag?

Any story that involve 3 groups of something and 2 more of that thing with a total of 20 can work to match this problem.

*(share one or two more examples like the ones below)*

- I have 3 bags of books and I'm holding 2 more books. How many books are in each bag if I have 20 books total?
- John spends \$20 at the store. He buys a pack of gum for \$2 and 3 bags of chips. How much does each bag of chips cost?

**Let's Think (Slide 4):** Let's use our thinking to answer this problem. It wants us to write an equation to represent the tape diagram, and then they want us to find the solution.

We just came up with several ideas for the tape diagram. Let's think about this one for the purposes of this problem: John spends \$20 at the store. He buys a pack of gum for \$2 and 3 bags of chips. How much does each bag of chips cost?

(change the context for this problem to match a student's context they came up with, or you can use the provided context)

$$3m + 2 = 20$$

How can we use the tape diagram and/or the context of the story to help us write a corresponding equation? I know we have a total of \$20, and that the bags of chips and the pack of gum compose that total. (write equation as you narrate) I can write "3m" to represent the cost of the bags of chips, and I can write + 2 to show the additional cost of the one pack of gum. This all should equal 20. How does the equation we just wrote match the tape diagram? Possible Student Answers, Key Points:

- The tape diagram shows 3 groups of m, which is the same as the 3m in our equation. The tape diagram also shows a value of 2 attached to the three m's, which matches the "+2" in our equation. And both the equation and the tape diagram show a total of 20.

$$3m + 2 = 20$$

$$20 - 2 = 18$$

$$3m = 18$$

$$m = 6$$

Excellent. We agree that both the tape diagram and the equation represent the story. Now let's solve the equation by thinking about the tape diagram. (write equations as you narrate) Let's start by subtracting the cost of the pack of gum from the \$20 total. What is  $20 - 2$ ? (18) That means the remaining 3 bags of chips should total \$18. If  $3m = 18$ , what does m have to equal? ( $m = 6$ ) Our solution is 6. Great job.

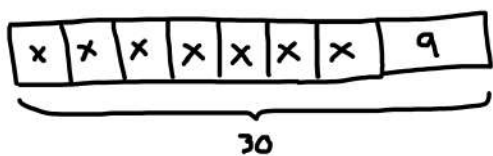
Before we close out on this problem, what does the 6 mean in the context of the story we're talking about? Possible Student Answers, Key Points:

- The solution of 6 in our story means that each bag of chips that John bought cost \$6.

**Let's Think (Slide 5):** For this problem, we have two word problems. For each word problem we will do three things. First, we'll draw a tape diagram to represent what is happening. Second, we'll write an equation to represent what is happening. Third, we'll solve to determine the unknown by using our tape diagram and equation.

Let's begin by looking at the first problem in red. I'll read it aloud, while you follow along. What is the problem about? What is known? What is unknown? Possible Student Answers, Key Points:

- We don't know how many crayons are in a pack.
- We know that he has 7 packs of crayons and 9 more crayons. We also know that he has 30 crayons in all.



We can represent this problem by drawing a tape diagram that shows 7 boxes of crayons and 9 more individual crayons. (sketch as you narrate) I can draw 7 equal-sized rectangles to represent the boxes of crayons. I'll put an x in each one, since the number of crayons in each pack is unknown. Since I know there are 9 additional crayons, I'll tack on another rectangle that shows 9

crayons. I can use a bracket to quickly show that the total number of crayons is 30. Thinking about the parts of the story I know and whether the parts are equal groups of one-off quantities, helps me make an accurate model of the problem.

$$7x + 9 = 30$$

Now all we need to think about is our equation. I think I have an idea of an equation we can write, but I'll want your help making sure each part of my equation makes sense.

(write  $7x + 9 = 30$ ) I've written  $7x + 9 = 30$ . How does each part of this equation connect back to the situation we represented with our tape diagram? Possible Student Answers, Key Points:

- The  $7x$  represents the 7 boxes of crayons. We don't know the number of crayons in each box, so that's represented by the x.
- The  $+ 9$  represents the 9 extra crayons. And the 30 is the total number of crayons.



$$30 - 9 = 21$$

$$7x = 21$$

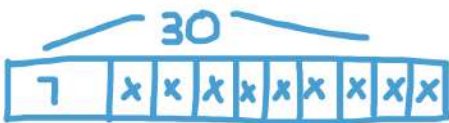
$$x = 3$$

Great thinking. Time to solve. I can see from the tape diagram, that it could be an efficient first step to remove the 9 crayons from my total of 30. What is 30 minus 9? (21) (write  $30 - 9 = 21$ ) So the 7 boxes of crayons have a total of 21 crayons inside them. I can show that as  $7x = 21$ . If 7 times  $x$  equals 21, then I know  $x$  has to equal 3 (write equation and solution) What does a solution of 3 mean in this context? Possible Student Answers, Key Points:

- A solution showing  $x = 3$  means that each box of crayons had 3 crayons in it.

Now let's take a second to do similar work with the second, blue word problem. Read the problem to yourself, and then describe what is known and unknown when you're ready. Possible Student Answers, Key Points:

- Farrah has yarn. She cuts a piece off, and then she cuts the remaining yarn into 9 equal-sized pieces.
- We don't know how long the 9 equal-sized pieces are. We can represent that with  $x$ .



Let's begin by drawing a tape diagram. What's the total length of yarn that Farrah used? (30 inches) (sketch tape diagram as you narrate) I'll draw a rectangle and use a bracket to show that the total is 30. How can we model what happens next with the yarn using our tape diagram? Possible Student Answers, Key Points:

- She cuts off 7 inches, so we can partition to make a box that represents that length.
- She cuts the rest into 9 equal pieces, so we can take the leftover section and divide it into 9 small rectangles of equal size. Those represent the nine pieces.

Excellent thinking. I can see the total length is 30. I see one piece she cut off is 7 inches, and then I see the 9 pieces of equal length that she cut with the remaining yarn. Now I need to think what this can look like as an equation. I know the parts that make up the yarn are the 7-inch piece and the other 9 pieces of unknown length. I can show this with an equation by showing 7 plus  $9x$  equals 30. (write equation) The 7 is the length she cut initially. The  $9x$  represents the 9 pieces, with  $x$  representing their unknown length. And we know the total length of yarn to begin with is 30.

$$7 + 9x = 30$$

$$30 - 7 = 23$$

$$9x = 23$$

$$x = \frac{23}{9} \text{ OR } 2\frac{5}{9}$$

Based on our tape diagram and the equation we just wrote, how could we find the value of  $x$ ? (write equations as student shares, supporting as needed) Possible Student Answers, Key Points:

- We can subtract the 7-inch piece from the original 30 inches.  $30 - 7 = 23$ .
- Then she cut the 23-inch piece into 9 equal pieces. We can write a related division fact or divide 23 by 9 to find that value.

Great. There were 23 inches that was cut into 9 equal pieces. 23 divided by 9 isn't a friendly number, so we can think of it as  $\frac{23}{9}$  or  $2\frac{5}{9}$  inches. Our solution means each of the 9 pieces of yarn measured  $2\frac{5}{9}$  inches.

**Let's Try it (Slides 6 - 7):** Now let's try some similar problems on our own. It can be helpful, when given a story problem, to show what is known in a tape diagram before attempting any computation. Anything that is unknown, we can represent with a variable. Once we have our tape diagram, we can write an equation that shows an equal groups and any additional values and set those equal to the total in the scenario. The tape diagram and equation help us think about a solution pathway that makes sense. I know you're going to do great. Let's begin.

# WARM WELCOME



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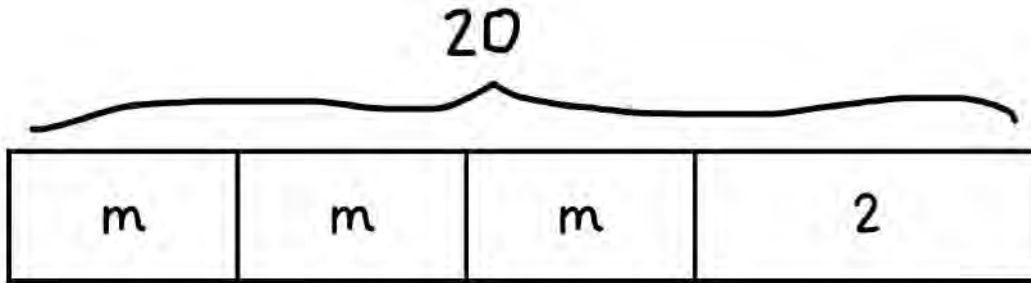
**Today we will coordinate tape diagrams, equations of the form  $px + q = r$ , and verbal descriptions of the situations, an reason about and interpret a solution.**

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Let's Talk:

Think of a story that could match the tape diagram below...

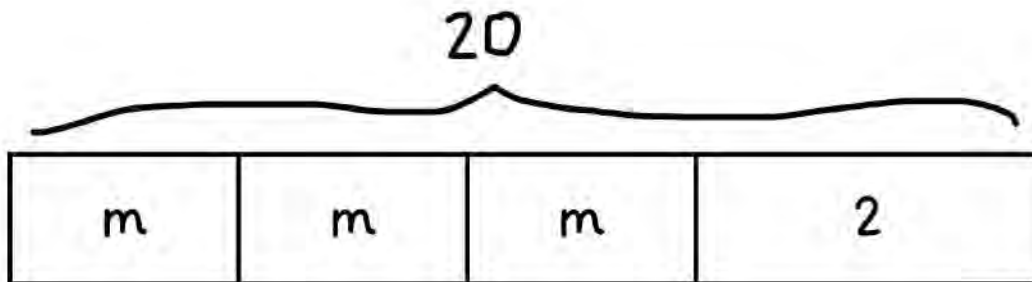


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Let's Think:

Write an equation to represent the tape diagram. Then find the solution to the equation.



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# Let's Think:

Draw a tape diagram to represent each situation. For each situation write and solve an equation.

**Darryl has 7 packs of crayons. Each pack has  $x$  crayons in it. His teacher gives him 9 more crayons, and now Darryl has 30 crayons.**

**Farrah has 30 inches of yarn. She cuts off 7 inches, and then cuts the remaining yarn into 9 equal length piece of  $x$  inches each.**

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# Let's Try It:

Let's explore coordinating tape diagrams, equations in the form  $px + q = r$ , and verbal descriptions of the situations together.

Name: \_\_\_\_\_ G7 US Lesson 4 - Let's Try It

Gary has 5 bunches of grapes. Each bunch has  $g$  grapes in it. His friend gives him 7 more grapes, and now Gary has 52 grapes in all.

- What is known in this story?
- What is unknown in this story?
- Circle the tape diagram that best represents this story.
 

$5g + 7 = 52$

$5g + 7 = 52$

$5g + 7 = 52$
- Use the tape diagram to complete the equation.
 

$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- Solve the equation for  $g$ .
- What does your solution mean in the context of the story?

Julianne is crafting with ribbon. She has 27 feet of ribbon. She cuts off 6 feet of ribbon, and then cuts the rest of the ribbon into 2 equal pieces of  $r$  feet.

- What is known in this story? What is unknown?

- Label the tape diagram below to represent the knowns and unknowns in the story.
- Use the tape diagram to write an equation that represents the story.
- Solve for  $r$ .
- What does your solution mean in the context of the story?

Match each situation to the tape diagram that best represents it. Then write a corresponding equation.

- Leo spends 3 hours at the gym. He spends 2 hours doing yoga, and then jogs 4 miles at a constant rate.
- Andy has 4 gallons of punch. She pours equal amounts for 3 of her friends, and has 2 gallons left over.
- Tessa spends \$4 on snacks. She bought 2 candy bars, and spent \$3 on popcorn.

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# On your Own:

Now it's time to coordinate tape diagrams, equations of the form  $px + q = r$ , and verbal descriptions of the situations on your own.

Name: \_\_\_\_\_ 67 US Lesson 4 - Independent Work

1. A bowling alley charges a family rate of \$10 for a round of bowling, plus a fee for each pair of bowling shoes rented. A family of 5 plays a round of bowling together, and each person rents a pair of bowling shoes. They pay a total of \$25.

a. Circle the tape diagram that best represents the situation.

b. What does each  $x$  represent in this problem?

c. Write and solve an equation to represent the problem.

2. Khyren buys 4 packs of tennis balls and 2 individual tennis balls. In all, he buys 34 tennis balls.

a. Label the tape diagram below to represent the situation.

b. Write and solve an equation to represent the problem.

3. A woodworker has a 28-foot plank of wood. He cuts off 4 feet of wood. Then, he cuts the remaining length into 8 equal lengths of  $w$  feet each. Draw a tape diagram and write an equation to represent this situation. Then find the value of  $w$ .

4. Match each story to the tape diagram that best represents it. Then find the value of  $x$  in two tape diagrams you select.

18

x

18

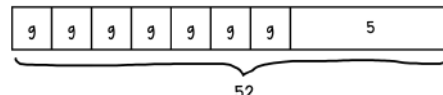
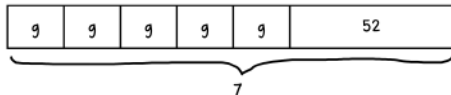
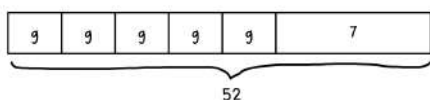
Jayden spends \$14 at a carnival. He bought 3 tickets to play games, and he paid \$5 to go on the ferris wheel.

Alicia has 14 cups of pancake batter. She pours equal amounts into 5 containers, and she has 3 cups leftover.

Name: \_\_\_\_\_

**Gary has 5 bunches of grapes. Each bunch has  $g$  grapes in it. His friend gives him 7 more grapes, and now Gary has 52 grapes in all.**

1. What is known in this story?
2. What is unknown in this story?
3. Circle the tape diagram that best represents this story.



4. Use the tape diagram to complete the equation.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

5. Solve the equation for  $g$ .

6. What does your solution mean in the context of the story?

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**Julianne is crafting with ribbon. She has 27 feet of ribbon. She cuts off 6 feet of ribbon, and then cuts the rest of the ribbon into 2 equal pieces of  $r$  feet.**

7. What is known in this story? What is unknown?



8. Label the tape diagram below to represent the knowns and unknowns in the story.



9. Use the tape diagram to write an equation that represents the story.

10. Solve for  $r$ .

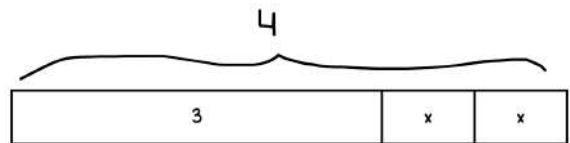
11. What does your solution mean in the context of the story?

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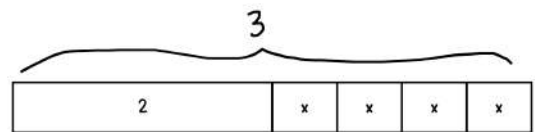
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**Match each situation to the tape diagram that best represents it. Then write a corresponding equation.**

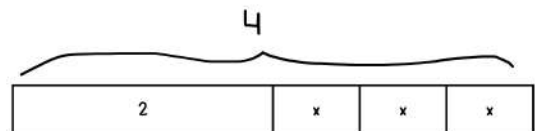
12. Leo spends 3 hours at the gym. He spends 2 hours doing yoga, and then jogs 4 miles at a constant rate.



13. Andy has 4 gallons of punch. She pours equal amounts for 3 of her friends, and has 2 gallons left over.

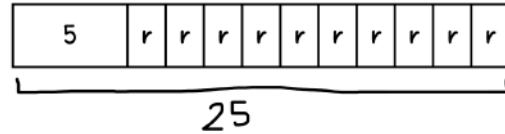
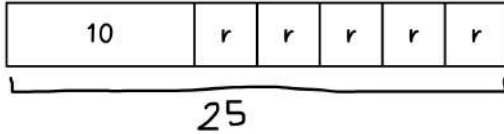


14. Tessa spends \$4 on snacks. She bought 2 candy bars, and spent \$3 on popcorn.



1. A bowling alley charges a family rate of \$10 for a round of bowling, plus a fee for each pair of bowling shoes rented. A family of 5 plays a round of bowling together, and each person rents a pair of bowling shoes. They pay a total of \$25.

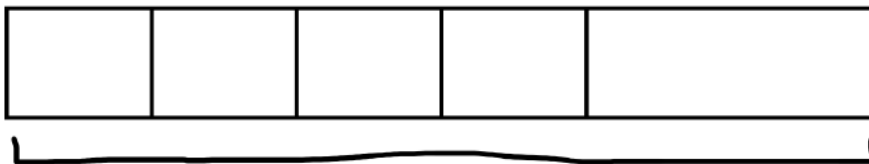
- a. Circle the tape diagram that best represents this situation.



- b. What does each  $r$  represent in this problem?
- c. Write and solve an equation to represent the problem.

2. Khyren buys 4 packs of tennis balls and 2 individual tennis balls. In all, he buys 34 tennis balls.

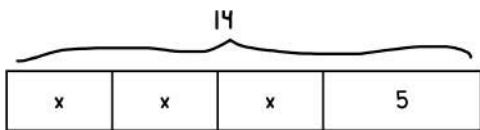
- a. Label the tape diagram below to represent the situation.



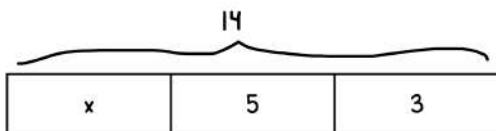
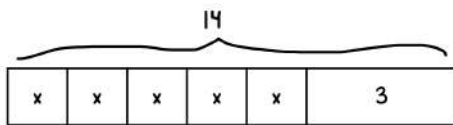
- b. Write and solve an equation to represent the problem.

3. A woodworker has a 28-foot plank of wood. He cuts off 4 feet of wood. Then, he cuts the remaining length into 8 equal lengths of  $w$  feet each. Draw a tape diagram and write an equation to represent this situation. Then find the value of  $w$ .

4. Match each story to the tape diagram that best represents it. Then find the value of  $x$  in two tape diagrams you select..



Jayden spends \$14 at a carnival. He bought 3 tickets to play games, and he paid \$5 to go on the ferris wheel.



Alicia has 14 cups of pancake batter. She pours equal amounts into 5 containers, and she has 3 cups leftover.

Name: KEY

Gary has 5 bunches of grapes. Each bunch has  $g$  grapes in it. His friend gives him 7 more grapes, and now Gary has 52 grapes in all.

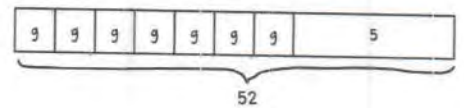
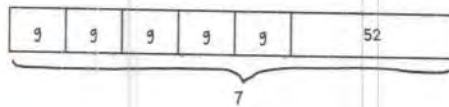
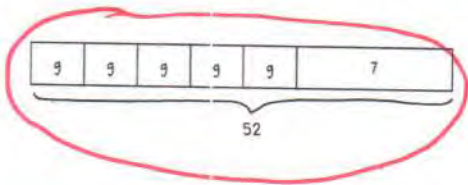
1. What is known in this story?

- 5 bunches
- He gets 7 more
- total of 52

2. What is unknown in this story?

- # of grapes in each bunch

3. Circle the tape diagram that best represents this story.



4. Use the tape diagram to complete the equation.

$$5g + 7 = 52$$

5. Solve the equation for  $g$ .

$$52 - 7 = 45 \quad 45 \div 5 = g$$
$$9 = g$$

6. What does your solution mean in the context of the story?

Each bunch of grapes has 9  
grapes.

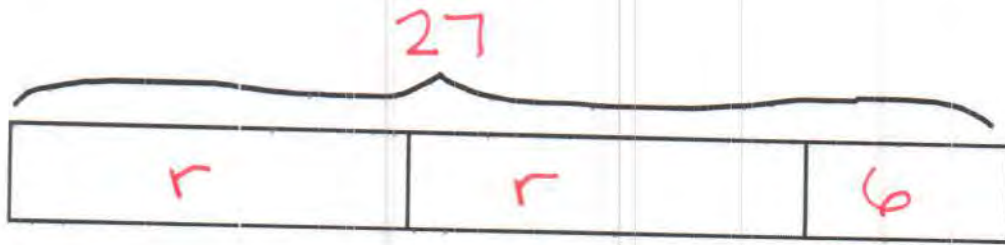
Julianne is crafting with ribbon. She has 27 feet of ribbon. She cuts off 6 feet of ribbon, and then cuts the rest of the ribbon into 2 equal pieces of  $r$  feet.

7. What is known in this story? What is unknown?

- 27 total feet
- cuts off 6 feet
- splits the rest 2 ways

- how long each of the 2 pieces are

8. Label the tape diagram below to represent the knowns and unknowns in the story.



9. Use the tape diagram to write an equation that represents the story.

$$2r + 6 = 27$$

10. Solve for  $r$ .

$$27 - 6 = 21$$

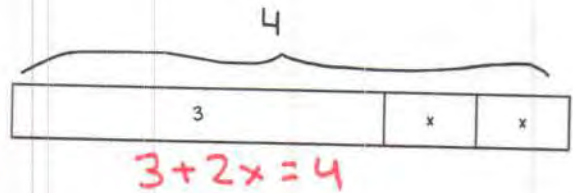
$$21 \div 2 = 10\frac{1}{2} \quad r = 10\frac{1}{2}$$

11. What does your solution mean in the context of the story?

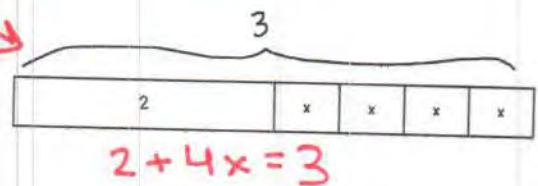
Each of the two pieces were  
 $10\frac{1}{2}$  feet long.

Match each situation to the tape diagram that best represents it. Then write a corresponding equation.

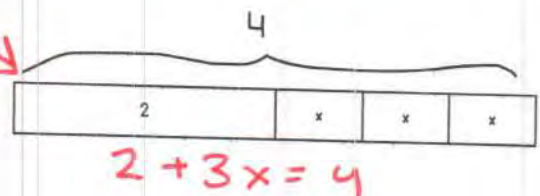
12. Leo spends 3 hours at the gym. He spends 2 hours doing yoga, and then jogs 4 miles at a constant rate.



13. Andy has 4 gallons of punch. She pours equal amounts for 3 of her friends, and has 2 gallons left over.



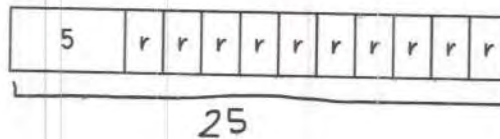
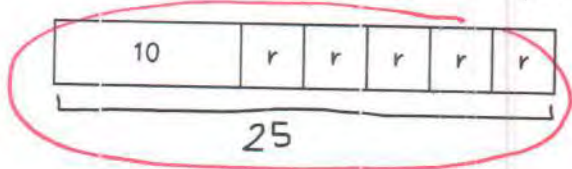
14. Tessa spends \$4 on snacks. She bought 2 candy bars, and spent \$3 on popcorn.





1. A bowling alley charges a family rate of \$10 for a round of bowling, plus a fee for each pair of bowling shoes rented. A family of 5 plays a round of bowling together, and each person rents a pair of bowling shoes. They pay a total of \$25.

a. Circle the tape diagram that best represents this situation.



b. What does each  $r$  represent in this problem?

the cost to rent bowling shoes

c. Write and solve an equation to represent the problem.

$$5r + 10 = 25$$

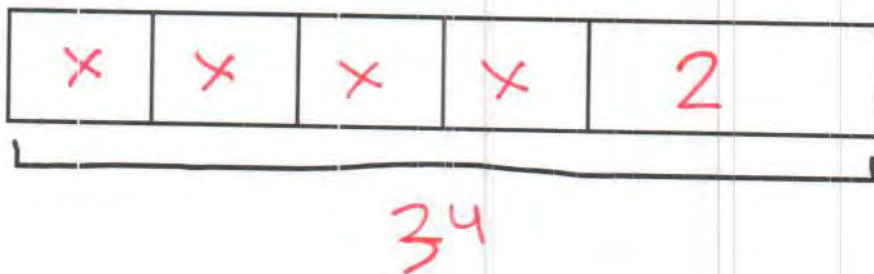
$$5r = 15$$

$$25 - 10 = 15$$

$$r = 3$$

2. Khyren buys 4 packs of tennis balls and 2 individual tennis balls. In all, he buys 34 tennis balls.

a. Label the tape diagram below to represent the situation.



b. Write and solve an equation to represent the problem.

$$4x + 2 = 34$$

~~$$4x + 2 = 34$$~~

$$34 - 2 = 32$$

$$4x = 32$$

$$32 \div 4 = x$$

$$8 = x$$

3. A woodworker has a 28-foot plank of wood. He cuts off 4 feet of wood. Then, he cuts the remaining length into 8 equal lengths of  $w$  feet each. Draw a tape diagram and write an equation to represent this situation. Then find the value of  $w$ .



$$4 + 8w = 28$$

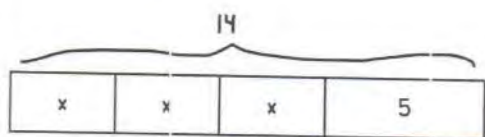
$$28 - 4 = 24$$

$$8w = 24$$

$$24 \div 8 = w$$

$$3 = w$$

4. Match each story to the tape diagram that best represents it. Then find the value of  $x$  in two tape diagrams you select..

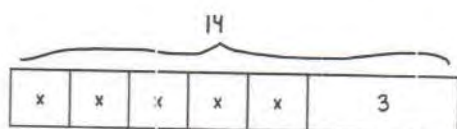


Jayden spends \$14 at a carnival. He bought 3 tickets to play games, and he paid \$5 to go on the ferris wheel.

$$3x + 5 = 14$$

$$3x = 9$$

$$x = 3$$



Alicia has 14 cups of pancake batter. She pours equal amounts into 5 containers, and she has 3 cups leftover.

$$5x + 3 = 14$$

$$5x = 11$$

$$x = 2\frac{1}{5}$$



## **G7 U5 Lesson 5**

Coordinate tape diagrams, equations of the form  $p(x + q) = r$ , and verbal descriptions of the situations, and reason about and interpret a solution.

**G7 U5 Lesson 5 - Students will coordinate tape diagrams, equations of the form  $p(x + q) = r$ , and verbal descriptions of the situations, and reason about and interpret a solution.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we worked hard to sketch tape diagrams and write equations to represent situations. We worked with different contexts, but you might have noticed that all of our problems had a similar structure. Today, we're actually going to do very similar work, but the structure of our problems will be slightly different. As we work today, I challenge you to consider what's the same about our work and what's different compared to the problem types we saw in our last lesson. Let's get going!

**Let's Talk (Slide 3):** I'm going to read this word problem. While I read, I want you to simply think about how you could summarize the story. What do we know? What don't we know? (*read problem*) **Possible Student Answers, Key Points:**

- Iris has some milk that she's pouring evenly into 5 bowls. She pours some into each bowl, then pours a little more into each bowl.
- We know the total amount of milk is 45 ounces. We know she has 5 bowls of milk. And we know that after she's poured some into the bowls, she pours 3 more ounces into each.
- We don't know how much she poured in each bowl to begin with.

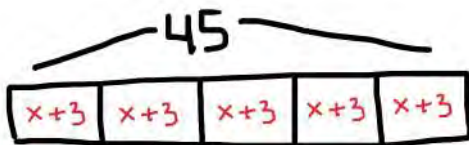
This problem feels a little different than the problems we looked at in our last lesson. In our last lesson, we saw problems where we had some equal groups and then another extra value of some sort. For instance, we saw a problem about 7 identical boxes of crayons plus 9 extra crayons. Or, we saw a problem about cutting of 7 inches of yarn and then splitting the rest into 9 equal pieces. Each problem involved a set of equal groups and another value in one way or another.

I notice that this problem feels a little different. Here we have some equal groups, the bowls of milk, but the extra value is also a part of those equal groups. Hm, I wonder how we can think about this...

Let's see if we can use some of the same thinking and modeling to help us tackle this different type of problem.

**Let's Think (Slide 4):** Our first problem deals directly with the story we just wondered about. Let's draw a tape diagram, write an equation, and then solve the equation related to this context.

Let's see if we can draw a tape diagram based on what we know. We know the total amount of milk is 45 ounces, so I'll start drawing a large rectangle and labeling it 45. (*sketch tape diagram as you narrate*) What can I do with this tape diagram to show that she poured the milk into 5 separate bowls? (**Partition the big rectangle into 5 equal sections.**) I'll draw four evenly spaced lines to cut the whole rectangle into 5 boxes. Each small box represents a bowl, so now I need to think about what I know about each bowl. I *don't* know how much she poured in the bowls to start, but I do know that she topped each bowl off with 3 more ounces. I can think of that as  $x + 3$  in each



bowl. I'll write  $x + 3$  in each rectangle. Take a look at the tape diagram. Does it accurately represent the story? How do you know? **Possible Student Answers, Key Points:**

- Yes. I see that 45 represents the total amount. I see the 5 boxes representing the bowls.
- Inside each bowl I see an unknown amount plus 3. This represents the original amount she poured in each bowl plus the 3 more ounces she poured in after.

$$5(x+3) = 45$$

Great! Let's use the story and our tape diagram to write an equation now. When I look at this tape diagram, I can see it as 5 groups of  $x + 3$  that have a total of 45. I know I can use multiplication to represent equal groups, so the equation I'll write is  $5(x + 3) = 45$ . (*write equation*) Does this equation accurately represent what is happening in the word problem? How do you know? [Possible Student Answers](#),

Key Points:

- Yes, it does! The  $5(x + 3)$  represents the 5 equal bowls. Inside each bowl there is an unknown amount plus 3 more ounces that were poured in later. The entire value of all 5 bowls is equal to 45 ounces.

$$45 \div 5 = 9$$

$$x + 3 = 9$$

$$x = 6$$

We have a tape diagram. We have an equation. Now we solve! Looking at the tape diagram, I know I can divide 45 by 5 to figure out the amount of milk in each bowl. (*write equations as you discuss*) What is 45 divided by 5? (9) Each bowl had 9 ounces in it once Iris was done. So to find the original amount she poured in, I can set any of the bowls equal to 9. I'll write  $x + 3 = 9$ , since each bowl is represented by  $x + 3$ . What must  $x$  equal? You can mentally substitute in a value or rewrite as a related subtraction equation to help you. (6)

So,  $x$  is equal to 6. What does a solution of 6 mean in the context of this problem? [Possible Student](#)

[Answers](#), [Key Points](#):

- We didn't know how much Iris poured in each bowl to start. A solution of 6 means that Iris poured 6 ounces into each bowl before going back to pour in 3 more ounces into each bowl.

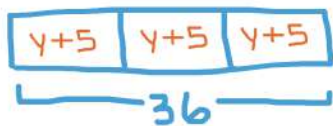
We just drew a tape diagram and then wrote and solved an equation to represent this real-world problem. Bravo! You're ready for another one.

**Let's Think (Slide 5):** Let's read this problem together. Before we solve it, we're going to assess another person's mistake. Doing this will help us avoid this mistake in the future. (*read problem*) What do we know in this story? What is unknown? [Possible Student Answers](#), [Key Points](#):

- We know there is a current total of 36 belts that are evenly arranged in 3 drawers. Wallace had some belts in each drawer already, and then put 5 more belts in each drawer.
- We don't know how many belts were in each drawer before Wallace added more in.

Picture what that might look like in your mind for a moment. Now look at Wallace's tape diagram. It's incorrect, and we'll help him fix it in a moment. Why does his tape diagram *not* match the story? [Possible Student Answers](#), [Key Points](#):

- He has a total of 36, but this story tells us they're split evenly into 3 drawers. Wallace's model is split into 4 boxes.
- Each drawer should have +5 in it, because he added 5 belts to each drawer. His tape diagram makes it look like he added 5 just one time.



It looks like Wallace maybe tried to represent the 3 drawers with each  $y$ , but he only added the 5 one time at the end. The problem clearly names that Wallace adds 5 belts to *each drawer*. Let's fix it. I'll still draw a rectangle with the total labeled as 36. (*sketch tape diagram as you narrate*) I'll cut the large rectangle into 3 sections to represent the drawers. We know there was an unknown amount in the drawers before Wallace adds 5 each. I'll write  $y + 5$  in each

drawer to represent that. Our tape diagram is a more accurate representation of the details in the story than what Wallace tried. We've corrected his mistake, but let's now help him solve too!

$$3(y+5) = 36$$

If we think of this as 3 equal groups of  $y + 5$ , what equation can we write to represent the problem? (*write as student shares*) [Possible Student Answers](#), [Key Points](#):

- I can show the 3 drawers as  $3(y + 5)$ . All three drawers have a total of 36, so I can set  $3(y + 5)$  equal to 36.

$$36 \div 3 = 12$$
$$y + 5 = 12$$
$$y = 7$$

Our equation shows that 3 equal drawers of “ $y + 5$ ” belts total up to 36. It’s time to solve. (*write equations as you discuss*) I can start by taking the total, 36, and dividing it by 3. This will help us think about just 1 drawer. What is 36 divided by 3? (12) Each drawer has 12 belts in it. So if 1 drawer is represented by  $y + 5$ , then I can think about  $y + 5 = 12$  to solve for  $y$ . What is the value of  $y$ ? How do you know? Possible Student Answers, Key Points:

- I rewrote the equation as  $12 - 5 = y$ . I know  $y = 7$ .
- The solution is 7, because I know  $7 + 5$  is equal to 12.

This means Wallace had 7 belts in each drawer before he put 5 more belts in each. Nice job fixing Wallace’s original error and helping him determine the value of the unknown using a tape diagram and an equation.

**Let’s Try it (Slides 6 - 7):** Now let’s try out a few more problems. Each of our problems today will involve equal groups, which we know we can represent in a tape diagram. We also know we can represent equal groups in an equation with the help of multiplication and parentheses. As we draw tape diagrams and write equations, I encourage you to constantly think back to the context of the story to make sure every aspect of your tape diagram and every part of your equation accurately tie back to what is happening in the given scenario. Time to get to work!

# WARM WELCOME



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**Today we will coordinate tape diagrams, equations of the form  $p(x + q) = r$ , and verbal descriptions of the situations, an reason about and interpret a solution.**

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## Let's Talk:

Iris has 45 ounces of milk. She pours the same amount of milk into 5 bowls, then adds 3 more ounces to each bowl.

What is known?  
What is unknown?



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## Let's Think:

Iris has 45 ounces of milk. She pours the same amount of milk into 5 bowls, then adds 3 more ounces to each bowl.

- Draw a tape diagram to represent the situation.
- Write an equation to represent the situation.
- Solve the equation.

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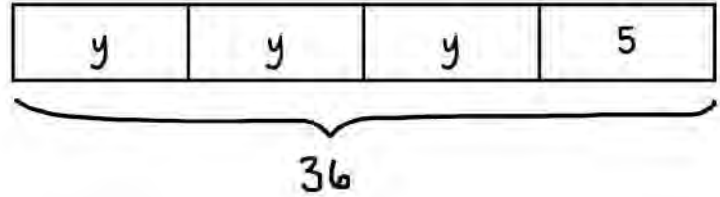




## Let's Think:

Each of 3 drawers contains  $y$  belts. Wallace adds 5 more belts to each drawer. There are now 36 belts in all. He drew the diagram below.

Why does Wallace's tape diagram not represent the situation?  
Correct his thinking.



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## Let's Try It:

Let's explore coordinating tape diagrams, equations, and verbal descriptions of the situations together.

Name: \_\_\_\_\_ G7 US Lesson 5 - Let's Try It

Five lifeguards each have a first aid kit that contains  $x$  bandages. Their manager gives each lifeguard 3 more bandages to put in their first aid kits. Altogether, the lifeguards have 45 bandages.

- What is known in this story?
- What is unknown in this story?
- Circle the tape diagram that best represents this situation.
 

48

5

45
- Use the tape diagram you selected to write an equation that represents this problem.  
 $(\quad) (\quad) = \quad$
- Solve the equation to find the unknown.
- What does your solution mean in this problem's context?

Caroline and her 3 sisters went to a concert. Tickets to the concert cost \$11 each, and each person also bought a poster for  $p$  dollars. In all, the 4 girls spent \$68.

- What is known in this story? What is unknown?

- Use the information from Caroline's situation to label the tape diagram below.
- Write an equation to represent the scenario.
- Solve the equation for  $p$ .
- What does  $p$  mean in the context of this problem?

**Read both situations below. Draw a tape diagram to represent each. Then, write and solve an equation that represents the situation.**

12. A book club charges \$5 to attend plus the cost of the book. Yesterday 5 people attended the book club. They spent a total of \$35.

13. A tour group spent \$44 total to go to a museum. They bought 5 tickets, and each person also spent \$2 to buy a map of the museum.

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# On your Own:

Now it's time to coordinate tape diagrams, equations, and verbal descriptions of the situations on your own.

Name: \_\_\_\_\_ G7 US Lesson 5 - Independent Work

1. At the farmer's market, Aida fills 5 jars with  $x$  pickles. After the jars settle, she is able to add 3 more pickles to each jar. Aida ends up with 50 pickles.

a. Circle the tape diagram that best represents this situation.

30

50

b. What does each  $x$  represent in this problem?

c. Write and solve an equation to represent the problem.

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2. Isaac takes his 6 dogs to the dog groomer. He pays  $c$  each, for the dogs to get their toenails trimmed. He also spends \$10 each to have his dogs shampooed. He spends \$210 in all.

a. Label the tape diagram below to represent the situation.

b. Write and solve an equation to represent the problem.

3. There are four 7th grade homerooms at Mitty Middle School. Each homeroom had 8 students, and then 2 new students were added to each homeroom. In all, there are 52 7th graders at Mitty Middle School. Draw a tape diagram to represent the situation. Then write and solve an equation to show how many students were originally in each homeroom.

4. An ice skating rink charges  $x$  dollars for admission and \$4 for skate rentals. Six friends each buy admission and a skate rental, and they spend a total of \$45.

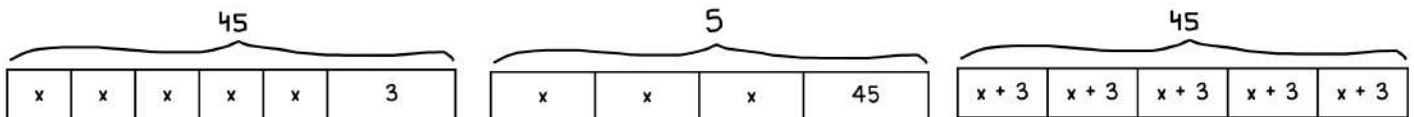
Savannah drew this tape diagram to represent the situation.

Explain why Savannah's tape diagram is incorrect. Show or explain how she could correct her tape diagram, then determine how much the ice skating rink charges for admission.

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**Five lifeguards each have a first aid kit that contains  $x$  bandages. Their manager gives each lifeguard 3 more bandages to put in their first aid kits. Altogether, the lifeguards have 45 bandages.**

1. What is known in this story?
2. What is unknown in this story?
3. Circle the tape diagram that best represents this situation.



4. Use the tape diagram you selected to write an equation that represents this problem.

$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$$

5. Solve the equation to find the unknown.
6. What does your solution mean in this problem's context?

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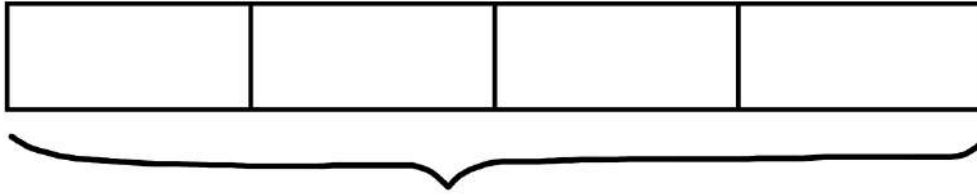


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**Caroline and her 3 sisters went to a concert. Tickets to the concert cost \$11 each, and each person also bought a poster for  $p$  dollars. In all, the 4 girls spent \$58.**

7. What is known in this story? What is unknown?

8. Use the information from Caroline's situation to label the tape diagram below.



9. Write an equation to represent the scenario.

10. Solve the equation for  $p$ .

11. What does  $p$  mean in the context of this problem?

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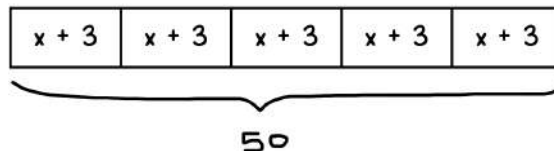
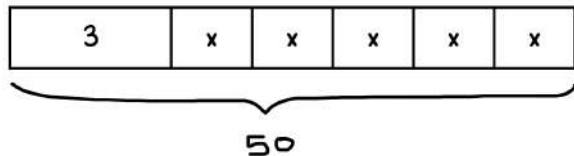
**Read both situations below. Draw a tape diagram to represent each. Then, write and solve an equation that represents the situation.**

12. A book club charges \$5 to attend plus the cost of the book. Yesterday, 6 people attended the book club. They spent a total of \$96.

13. A tour group spent \$44 total to go to a museum. They bought 5 tickets, and each person also spent \$2 to buy a map of the museum.

1. At the farmer's market, Aida fills 5 jars with  $x$  pickles. After the jars settle, she is able to add 3 more pickles to each jar. Aida ends up with 50 pickles.

a. Circle the tape diagram that best represents this situation.

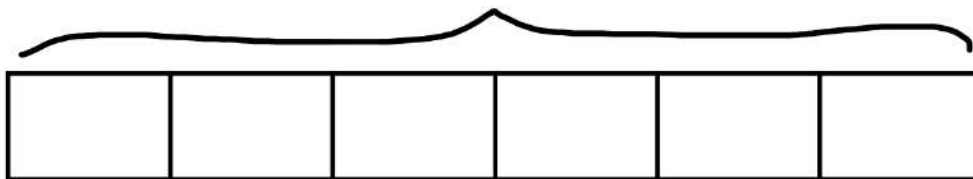


b. What does each  $x$  represent in this problem?

c. Write and solve an equation to represent the problem.

2. Isaac takes his 6 dogs to the dog groomer. He pays  $c$  each, for the dogs to get their toenails trimmed. He also spends \$10 each to have his dogs shampooed. He spends \$210 in all.

a. Label the tape diagram below to represent the situation.

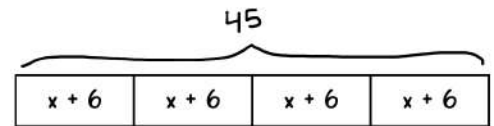


b. Write and solve an equation to represent the problem.

3. There are four 7th grade homerooms at Mathy Middle School. Each homeroom had  $n$  students, and then 2 new students were added to each homeroom. In all, there are 92 7th graders at Mathy Middle School. Draw a tape diagram to represent the situation. Then write and solve an equation to show how many students were originally in each homeroom.

4. An ice skating rink charges  $x$  dollars for admission and \$4 for skate rentals. Six friends each by admission and a skate rental, and they spend a total of \$45.

Savannah drew this tape diagram to represent the situation.



Explain why Savannah's tape diagram is incorrect. Show or explain how she could correct her tape diagram, then determine how much the ice skating rink charges for admission.

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Five lifeguards each have a first aid kit that contains  $x$  bandages. Their manager gives each lifeguard 3 more bandages to put in their first aid kits. Altogether, the lifeguards have 45 bandages.

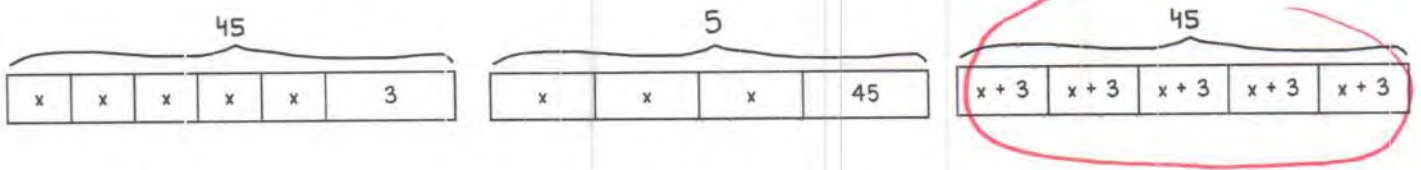
1. What is known in this story?

- 45 total bandages
- 5 lifeguards
- manager gives 3 more each

2. What is unknown in this story?

# of band-aids in each kit to start

3. Circle the tape diagram that best represents this situation.



4. Use the tape diagram you selected to write an equation that represents this problem.

$$5(x+3) = 45$$

5. Solve the equation to find the unknown.

$$45 \div 5 = 9 \quad x+3 = 9$$

$$x = 6$$

6. What does your solution mean in this problem's context?

There were 6 bandages in each kit to start.

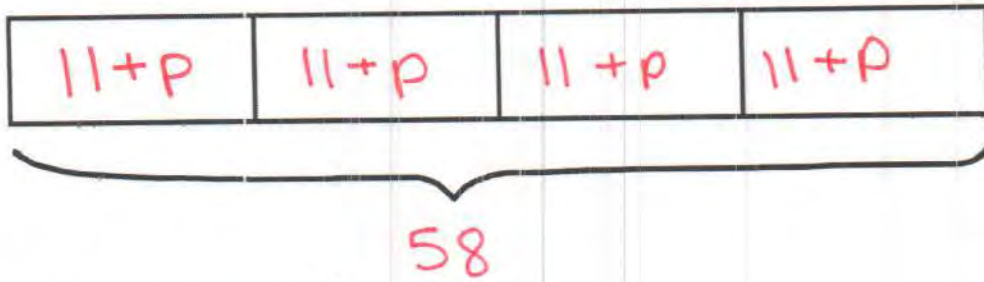
Caroline and her 3 sisters went to a concert. Tickets to the concert cost \$11 each, and each person also bought a poster for  $p$  dollars. In all, the 4 girls spent \$58.

7. What is known in this story? What is unknown?

- 4 people
- tickets cost \$11
- Spent \$58 in all

- cost for a poster

8. Use the information from Caroline's situation to label the tape diagram below.



9. Write an equation to represent the scenario.

$$4(11+p) = 58$$

10. Solve the equation for  $p$ .

$$58 \div 4 = 14\frac{1}{2}$$

$$11 + p = 14\frac{1}{2}$$

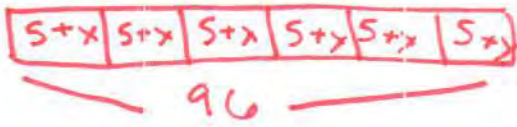
$$p = 3\frac{1}{2}$$

11. What does  $p$  mean in the context of this problem?

Each poster,  $p$ , costs \$3.50.

Read both situations below. Draw a tape diagram to represent each. Then, write and solve an equation that represents the situation.

12. A book club charges \$5 to attend plus the cost of the book. Yesterday, 6 people attended the book club. They spent a total of \$96.



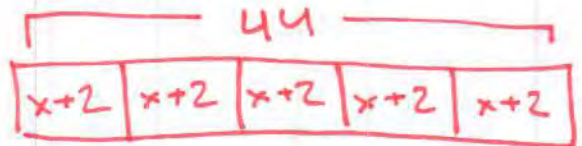
$$6(5+x) = 96$$

$$96 \div 6 = 16$$

$$5+x = 16$$

$$x = 11$$

13. A tour group spent \$44 total to go to a museum. They bought 5 tickets, and each person also spent \$2 to buy a map of the museum.



$$5(x+2) = 44$$

$$44 \div 5 = 8\frac{4}{5}$$

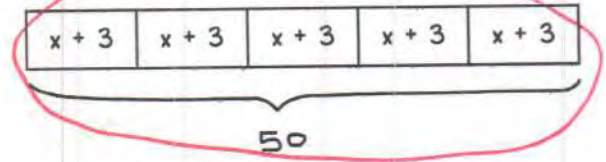
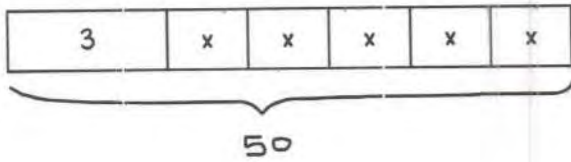
$$x+2 = 8\frac{4}{5}$$

$$x = 6\frac{4}{5} \text{ or } \$6.80$$



1. At the farmer's market, Aida fills 5 jars with  $x$  pickles. After the jars settle, she is able to add 3 more pickles to each jar. Aida ends up with 50 pickles.

- a. Circle the tape diagram that best represents this situation.



- b. What does each  $x$  represent in this problem?

the # of pickles in each jar to start

- c. Write and solve an equation to represent the problem.

$$5(x+3) = 50$$

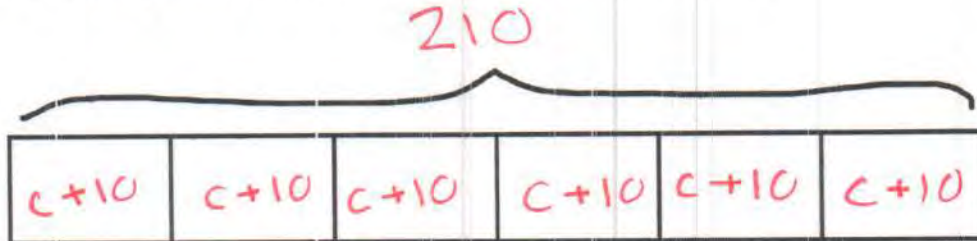
$$50 \div 5 = 10$$

$$x+3 = 10$$

$$x = 7$$

2. Isaac takes his 6 dogs to the dog groomer. He pays  $c$  each, for the dogs to get their toenails trimmed. He also spends \$10 each to have his dogs shampooed. He spends \$210 in all.

- a. Label the tape diagram below to represent the situation.



$$\begin{array}{r} 35 \\ 6 \overline{) 210} \\ \underline{-18} \phantom{0} \\ 30 \\ \underline{-30} \\ 0 \end{array}$$

- b. Write and solve an equation to represent the problem.

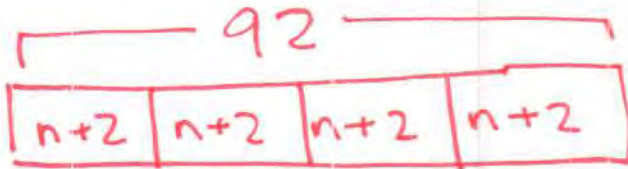
$$6(c+10) = 210$$

$$210 \div 6 = 35$$

$$c+10 = 35$$

$$c = 25$$

3. There are four 7th grade homerooms at Mathy Middle School. Each homeroom had  $n$  students, and then 2 new students were added to each homeroom. In all, there are 92 7th graders at Mathy Middle School. Draw a tape diagram to represent the situation. Then write and solve an equation to show how many students were originally in each homeroom.



$$4(n+2) = 92$$

$$92 \div 4 = 23$$

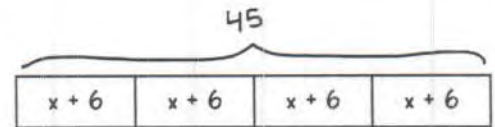
$$n+2 = 23$$

$$n = 21$$

$$\begin{array}{r} 23 \\ 4 \overline{)92} \\ \underline{-8} \phantom{0} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

4. An ice skating rink charges  $x$  dollars for admission and \$4 for skate rentals. Six friends each by admission and a skate rental, and they spend a total of \$45.

Savannah drew this tape diagram to represent the situation.



Explain why Savannah's tape diagram is incorrect. Show or explain how she could correct her tape diagram, then determine how much the ice skating rink charges for admission.

Savannah's story involves six groups of " $x+4$ ". Her model shows four groups of " $x+6$ " instead. She should redraw the model to show the correct groups.

$$6(x+4) = 45$$

$$45 \div 6 = 7\frac{1}{2}$$

$$x+4 = 7\frac{1}{2}$$

$$x = 3\frac{1}{2}$$

## **G7 U5 Lesson 6**

Write and categorize equations of the forms  $px + q = r$  and  $p(x + q) = r$  from situations and tape diagrams.



**G7 U5 Lesson 6 - Students will write and categorize equations of the forms  $px + q = r$  and  $p(x + q) = r$  from situations and tape diagrams.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We've spent the last two lessons together building tape diagrams and equations to represent different types of real-world problems. (*write  $px + q = r$  and  $p(x + q) = r$* ) Two lessons ago, all our stories were in the form  $px + q = r$ , meaning we had some equal groups (*point to  $px$* ) and an additional value that was not an equal group (*point to  $q$* ) that had a total of  $r$ . In the lesson after that, our problems had a different structure. In these problems, we had an unknown amount that was part of the equal groups. We'd have a number of equal groups (*point to  $p$* ), then within our equal groups we'd have a known and an unknown (*point to  $q$  and  $x$* ) that all had a total of  $r$ .

$$px + q = r$$
$$p(x + q) = r$$

It's not important that we memorize these equations or formulas, so don't worry about that. All we're going to do today is see a *mix* of stories and have to carefully craft a tape diagram and equation to match them. Since the problems won't all fall into the same structure, it will be extra important that we think about what is known and unknown carefully so that our representations are accurate and helpful.

**Let's Talk (Slide 3):** Look at the two equations shown here. What do you notice is the same? What is different? **Possible Student Answers, Key Points:**

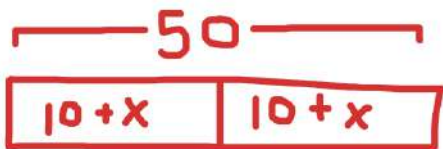
- They both have a 50, 20, 10 and  $x$ . They both have a + and an = sign. They both involve some equal groups.
- The first one shows 2 equal groups of " $x + 10$ ". The second one shows 2 equal groups of  $x$ , but then the 10 is not part of the equal groups.

These two equations are examples of the two problem types we've been exploring. Our work today will be to solve varied problem types using tape diagrams and equations to help us reason about real-world problems.

**Let's Think (Slide 4):** I'll read the problem once through while you follow along. After, summarize the story and tell me what information is known and what is unknown. **Possible Student Answers, Key Points:**

- This problem is about Minnie putting chocolate candy and fruit candy into 2 equal bags.
- We know there are 50 total pieces of candy that she's splitting evenly into two bags. 10 pieces in each bag are chocolate and the rest in each bag is fruit candy.
- We don't know how much fruit candy she put in each bag.

We've seen problems like this before. Let's tackle part A by sketching a tape diagram.



(*sketch as you narrate*) We know the total is 50, so I'll draw a long rectangle and label the total as 50. How can I show the amount in the two bags now that I've drawn the total? **Possible Student Answers, Key Points:**

- You can split the rectangle in half to show each bag.
- Each bag has 10 chocolate candies and an unknown number of fruit candies, so we can write  $10 + x$  in each bag.

Great. (*point to components as you name them*) I see the 50 total candies. I see the two bags represented by the smaller rectangles. I also see that each bag has 10 chocolate candies and " $x$ " fruit candies.

It's time to write an equation. I can see in the model, and I know from the story, that we have 2 groups of candy. I can think of those groups as each being worth " $10 + x$ ". Based on that, I can write the equation  $2(10 + x) = 50$ .

$$2(10+x) = 50$$

$$50 \div 2 = 25$$

$$10 + x = 25$$

$$x = 15$$

$+ x) = 50$ . (write equation) Two bags of candy that each have “10 + x” pieces of candy in them equal a total of 50 total pieces of candy. Writing the equation pretty easy when we’ve already drafted a tape diagram.

Part A and Part B are done, so let’s close this out with Part C. We’ll solve the equation. The tape diagram helps me see that I can divide 50 by 2 to figure out the number of candies in each bag. What is 50 divided by 2? (25) There are 25 pieces of candy per bag. I know 10 of those pieces are chocolate and x of those pieces are fruit candies. I’ll write  $10 + x = 25$  to represent that. How can I find the value of x? Possible Student Answers, Key Points:

- I just know 10 plus 15 makes 25, so there have to be 15 fruit candies.
- I can rewrite the equation as a related subtraction equation. I know  $25 - 10 = x$ , so x has to equal 15.

Our solution is 15. What does that mean when we connect it back to the context of the problem? Possible Student Answers, Key Points:

- We were trying to find how many fruit candies Minnie puts in each bag. If  $x = 15$ , that means there are 15 fruit candies in each bag.

Nice work. Let’s try another example.

**Let’s Think (Slide 5):** Again, I’ll read the problem once through while you follow along. After, summarize the story and tell me what information is known and what is unknown. Possible Student Answers, Key Points:

- This problem is also about 50 pieces of candy. This time, she’s putting some on display racks and then keeping some for herself.
- We know there is a total of 50. We know she’s splitting the same amount onto two displays. We know she’s keeping 10 of the 50 for herself.
- We don’t know how many she puts in each display.

I’ll start drawing this tape diagram similar to the previous problem, because they both involve totals of 50 pieces of candy. (sketch as you narrate) I’m tempted to split the total into two again, because I know she’s splitting some candy evenly onto 2 displays, but we realized she’s keeping some to herself in this problem. I’ll partition a rectangle at the end to show she’s keeping those 10 to herself. Now, I can split the remaining rectangle in half to show the two identical displays. Let’s put an x in each of those since we don’t know the value. Does my tape diagram show every part of the problem we’re trying to solve? (point to components that the student names)



Possible Student Answers, Key Points:

- Yes. I see the 50 total pieces of candy. I see the x in two boxes to represent the amount that she puts in each display. I also see the 10 pieces she keeps in the rectangle on the right side of the diagram.

For Part B, we’re tasked with writing an equation. What do you see in the tape diagram that can help us write the equation? Possible Student Answers, Key Points:

- I see the total is 50, so our equation should have “= 50” in it.
- I see two groups of x, which we can write as 2x. I also see 10, so we’ll need to add 10 to the 2x.

$$2x + 10 = 50$$

(write equation as you narrate) I’ll write  $2x + 10$  to represent the two groups of candy in the display plus the 10 pieces Minnie keeps to herself. I’ll set that equal to the total of 50. Nice work!



$$50 - 10 = 40$$

$$2x = 40$$

$$x = 20$$

Now we can use the tape diagram and the equation to solve for the unknown,  $x$ . (*write equations as you narrate*) Let's start by subtracting out the 10 pieces of candy that Minnie keeps. What is  $50 - 10$ ? (40) She has 40 pieces of candy that she wants to split evenly into two displays. I can think of that as 40 divided by 2 or as  $2x = 40$ . No matter how I think about it, I know she puts 20 candies onto each display.  $x$  is equal to 20.

We just did a lot of work! Even though the problem types were different, we were able to draw a tape diagram and equation to help us solve for unknowns. We'll now get some more practice.

**Let's Try it (Slides 6 - 7):** As we work through the next several examples, we'll want to be careful as we read each problem. It has been really helpful to read each problem, sometimes twice, and pause to think about the details. What do we know? What is unknown? This quick reflection made it much easier to draw accurate tape diagrams and write correct equations. It helps to make sense of a word problem before jumping into a solution strategy. Let's keep all this work in mind and try a few more together.

# WARM WELCOME



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**Today we will write and categorize equations of the forms  $px + q = r$  and  $p(x + q) = r$  from situations and tape diagrams.**

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## Let's Talk:

Look at the equations below. What is the same? What is different?

$$50 = 2(x + 10)$$

$$50 = 2x + 10$$

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## Let's Think:

**Minnie has 50 pieces of candy to put evenly into 2 bags. She puts 10 chocolate candies in each bag and  $x$  fruit candies in each bag.**

- Draw a tape diagram to represent the situation.
- Write an equation to represent the situation.
- Solve the equation.

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## Let's Think:

Minnie has 50 pieces of candy to put on display in her candy store. She puts  $x$  pieces of candy in each of 2 displays, then sets aside 10 pieces to take to her friend.

- Draw a tape diagram to represent the situation.
- Write an equation to represent the situation.
- Solve the equation.

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## Let's Try It:

Let's explore writing and categorizing equations of the form  $px + q = r$  and  $p(x + q) = r$  from situations and tape diagrams together.

Name: \_\_\_\_\_ (G7 US Lesson 6 - Let's Try It)

At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered the same number of chairs to each of four classrooms and had 12 chairs leftover.

- What is known in this situation? Unknown?
- Circle the tape diagram that best represents the situation.
 

$c \quad c \quad c \quad c \quad 12$   
84

$c + 12 \quad c + 12 \quad c + 12 \quad c + 12$   
84
- Use the tape diagram to write an equation using  $c$  to represent the unknown.
- Solve for  $c$ .
- What does your solution represent in this context?

At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered 12 chairs to each of four classrooms, and then he took the remaining chairs and divided them up equally between the classrooms.

- What is known in this situation? Unknown?
- Label the tape diagram to represent the situation.
 

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- Use the tape diagram to write and solve an equation. Use  $v$  to represent the unknown.
 

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- What does your solution represent in this context?

Think about the tape diagrams for each of the two situations you just considered.

- What is the same about the tape diagrams?
- What is different about the tape diagrams?

Match the story to the tape diagram that best represents it. Then find the value of the unknown in each story.

- Peter's soccer team has 60 ounces of Gatorade. He pours the Gatorade evenly into 5 bottles, and there is 4 ounces leftover. How much Gatorade did Peter pour in each bottle?
 

$n \quad n \quad n \quad n \quad n \quad 4$   
60

$n + 4 \quad n + 4 \quad n + 4 \quad n + 4 \quad n + 4$   
60
- Cher has 60 berries that she wants to share between 4 people. She starts by giving each person 6 berries, and then splits what is leftover evenly between each person. How many additional berries did each person receive?
 

$n + 6 \quad n + 6 \quad n + 6 \quad n + 6$   
60

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# On your Own:

Now it's time to write and categorize equations of the form  $px + q = r$  and  $p(x + q) = r$  from situations and tape diagrams on your own.

Name: \_\_\_\_\_ G7 US Lesson 6 | Independent Work

1. Dianne baked 72 muffins for her bakery. She put the same number of muffins into 4 display cases, and then boxed the 16 remaining muffins for a catering order.

Which tape diagram best represents the situation?

a.

b.

c.

Write and solve an equation to find the unknown.

2. Santiago was filling backpacks with pencils and notebooks to give to a local school. He had 95 total objects to fill 5 backpacks evenly. He put 11 pencils in each backpack and a notebook.

a. Draw a tape diagram to represent this situation.

b. Write and solve an equation to find the unknown quantity.

c. What does your solution mean in this context?

3. 74 students attended field day. Their teachers put them into 5 teams by putting  $x$  students on each team. Then, they made another team with 9 players on it. How many people did the teachers put on each team?

Use a tape diagram and an equation to represent this situation.

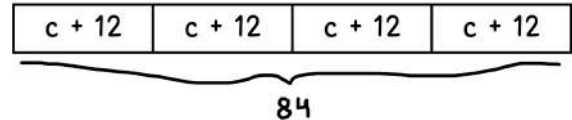
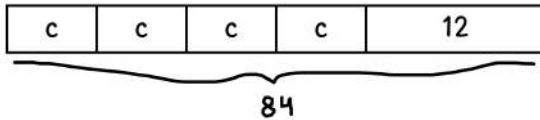
4. What is the same and different about the two sets of tape diagrams and equations shown here? Use the terms/phrases equal groups, total, and different-sized groups in your response.

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**At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered the same number of chairs to each of four classrooms and had 12 chairs leftover.**

1. What is known in this situation? Unknown?

2. Circle the tape diagram that best represents the situation.



3. Use the tape diagram to write an equation using  $c$  to represent the unknown.

4. Solve for  $c$ .

5. What does your solution represent in this context?

**At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered 12 chairs to each of four classrooms, and then he took the remaining chairs and divided them up equally between the classrooms.**

6. What is known in this situation? Unknown?

7. Label the tape diagram to represent the situation.





8. Use the tape diagram to write and solve an equation. Use  $c$  to represent the unknown.

9. What does your solution represent in this context?

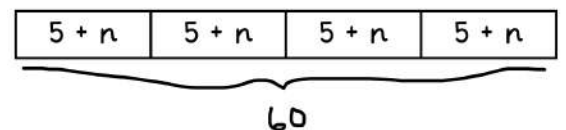
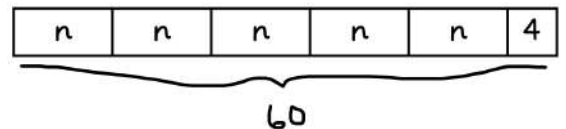
**Think about the tape diagrams for each of the two situations you just considered.**

10. What is the same about the tape diagrams?

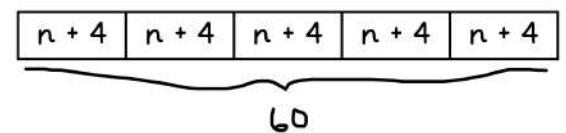
11. What is different about the tape diagrams?

**Match the story to the tape diagram that best represents it. Then find the value of the unknown in each story.**

12. Peter's soccer team has 60 ounces of Gatorade. He pours the Gatorade evenly into 5 bottles, and there is 4 ounces leftover. How much Gatorade did Peter pour in each bottle?

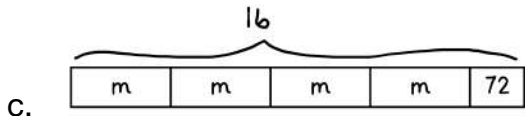
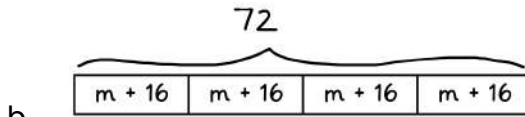
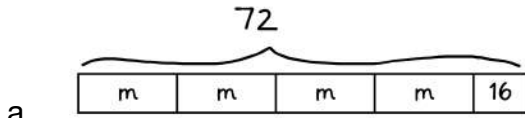


13. Cher has 60 berries that she wants to share between 4 people. She starts by giving each person 5 berries, and then splits what is leftover evenly between each person. How many additional berries did each person receive?



1. Dianne baked 72 muffins for her bakery. She put the same number of muffins into 4 display cases, and then boxed the 16 remaining muffins for a catering order.

Which tape diagram best represents the situation?



Write and solve an equation to find the unknown.

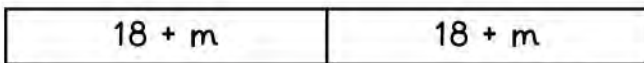
2. Santiago was filling backpacks with pencils and notebooks to give to a local school. He had 95 total objects to fill 5 backpacks evenly. He put 11 pencils in each backpack and  $n$  notebooks.

- a. Draw a tape diagram to represent this situation.
- b. Write and solve an equation to find the unknown quantity.
- c. What does your solution mean in this context?

3. 74 students attended field day. Their teachers put them into 5 teams by putting  $x$  students on each team. Then, they made another team with 9 players on it. How many people did the teachers put on each team?

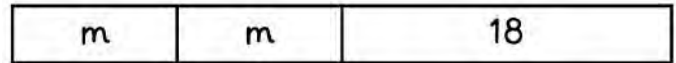
Use a tape diagram and an equation to represent this situation.

4. What is the same and different about the two sets of tape diagrams and equations shown here? Use the terms/phrases equal groups, total, and different-sized groups in your response.



$38$

$$2(18 + m) = 38$$



$38$

$$2m + 18 = 38$$

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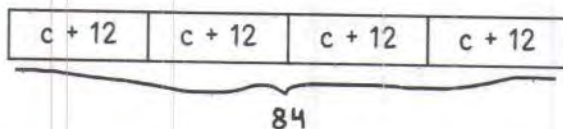
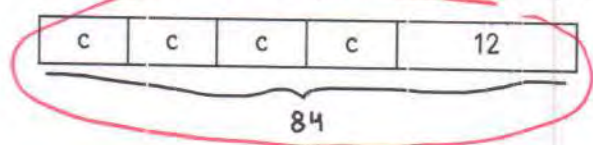
At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered the same number of chairs to each of four classrooms and had 12 chairs leftover.

1. What is known in this situation? Unknown?

- 84 total chairs
- 4 classrooms
- 12 left over

• # of chairs in each room

2. Circle the tape diagram that best represents the situation.



3. Use the tape diagram to write an equation using  $c$  to represent the unknown.

$$84 = 4c + 12$$

4. Solve for  $c$ .

$$84 - 12 = 72$$

$$72 \div 4 = c$$

$$18 = c$$

5. What does your solution represent in this context?

The custodian delivered 18 chairs to each room.

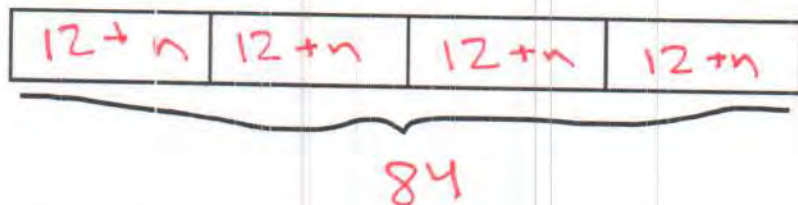
At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered 12 chairs to each of four classrooms, and then he took the remaining chairs and divided them up equally between the classrooms.

6. What is known in this situation? Unknown?

- 84 total chairs
- 12 chairs per room to start
- 4 classrooms

• # of chairs the custodian put in each room after the 12

7. Label the tape diagram to represent the situation.





8. Use the tape diagram to write and solve an equation. Use  $c$  to represent the unknown.

$$84 = 4(12 + c)$$

$$84 \div 4 = 21$$

$$12 + c = 21$$

$$c = 9$$

9. What does your solution represent in this context?

The custodian put 9 chairs in each room after the initial 12.

Think about the tape diagrams for each of the two situations you just considered.

10. What is the same about the tape diagrams?

- They had the same total.
- They each involved 4 classrooms.
- They each involved equal groups.

11. What is different about the tape diagrams?

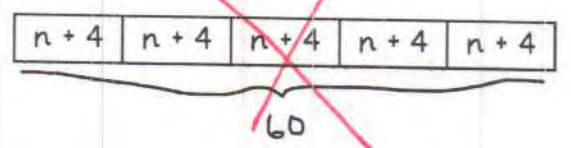
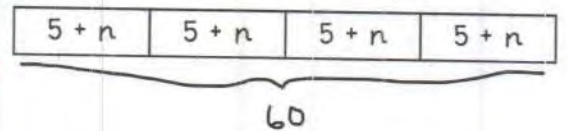
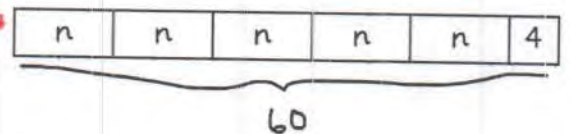
- The first situation had equal groups with an extra amount.
- The second situation involved only equal groups.

Match the story to the tape diagram that best represents it. Then find the value of the unknown in each story.

12. Peter's soccer team has 60 ounces of Gatorade. He pours the Gatorade evenly into 5 bottles, and there is 4 ounces leftover. How much Gatorade did Peter pour in each bottle?

$$60 - 4 = 56$$

$$56 \div 5 = 11 \frac{1}{5} \text{ oz}$$



13. Cher has 60 berries that she wants to share between 4 people. She starts by giving each person 5 berries, and then splits what is leftover evenly between each person. How many additional berries did each person receive?

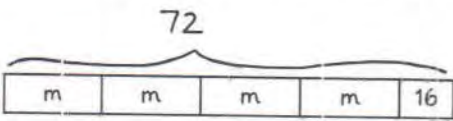
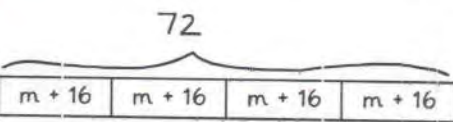
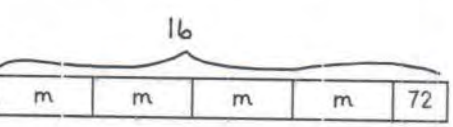
$$60 \div 4 = 15$$

$$5 + n = 15$$

$$n = 10$$

1. Dianne baked 72 muffins for her bakery. She put the same number of muffins into 4 display cases, and then boxed the 16 remaining muffins for a catering order.

Which tape diagram best represents the situation?

- a. 
- b. 
- c. 

Write and solve an equation to find the unknown.

$$72 = 4m + 16$$

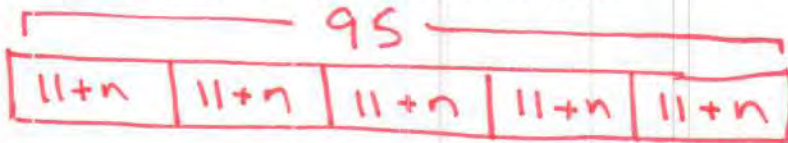
$$72 - 16 = 56$$

$$56 \div 4 = m$$

$$14 = m$$

2. Santiago was filling backpacks with pencils and notebooks to give to a local school. He had 95 total objects to fill 5 backpacks evenly. He put 11 pencils in each backpack and  $n$  notebooks.

- a. Draw a tape diagram to represent this situation.



- b. Write and solve an equation to find the unknown quantity.

$$95 = 5(11 + n)$$

$$95 \div 5 = 19$$

$$11 + n = 19$$

$$n = 8$$

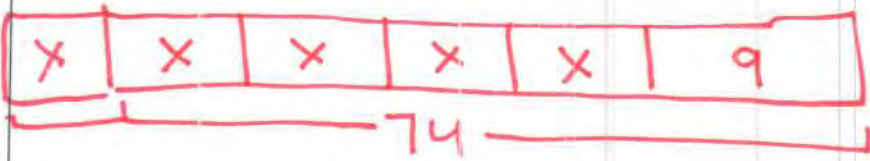
- c. What does your solution mean in this context?

Santiago puts 8 notebooks in each backpack.



3. 74 students attended field day. Their teachers put them into 5 teams by putting  $x$  students on each team. Then, they made another team with 9 players on it. How many people did the teachers put on each team?

Use a tape diagram and an equation to represent this situation.



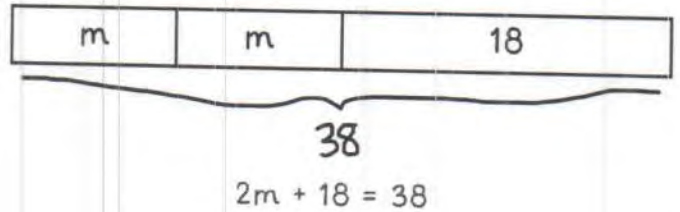
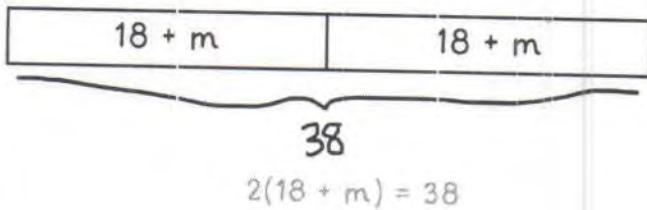
$$74 = 5x + 9$$

$$74 - 9 = 65$$

$$65 \div 5 = x$$

$$x = 13$$

4. What is the same and different about the two sets of tape diagrams and equations shown here? Use the terms/phrases equal groups, total, and different-sized groups in your response.



Both show totals of 38. The first shows 2 equal groups. The second has different-sized groups; there are 2 equal groups of  $m$  and 1 group of 18.

## **G7 U5 Lesson 7**

Use a balanced hanger diagram to reason about writing and solving equations in the form  $px + q = r$ .

**G7 U5 Lesson 7 - Students will use a balanced hanger diagram to reason about writing and solving equations in the form  $px + q = r$ .**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We've been putting a significant amount of time and effort into reasoning about real world problems and using what we know to draw tape diagrams and write equations that reflect the problems. Today, we're going to practice a cool way to think about solving equations that is very visual. We're going to work with hanger diagrams or hanger models. You might catch yourself feeling like today's problems feel more like mini-puzzles or games than like typical math problems.

**Let's Talk (Slide 3):** Check out this image. What do you notice? What do you wonder? [Possible Student Answers, Key Points:](#)

- I notice a clothes hanger. I notice there are cups tied with strings on both ends. I noticed it looks balanced, or the sides look even as if both sides weigh the same amount.
- I wonder what this has to do with math. I wonder what goes in the cups.

If we were to play around with this hanger in real life, what would happen if I put some marbles on the left side? (The hanger would droop to the left or go up on the right.) What if, instead, I put some marbles on the right side? (The hanger would droop to the right or go up on the left.) If I wanted to make any change to the hanger, what would I need to do to make sure the hanger stayed balanced? [Possible Student Answers, Key Points:](#)

- If you added something to one side, you'd need to add the same amount to the other side.
- If you took something out of one cup, you'd need to take the same amount out of the other cup.

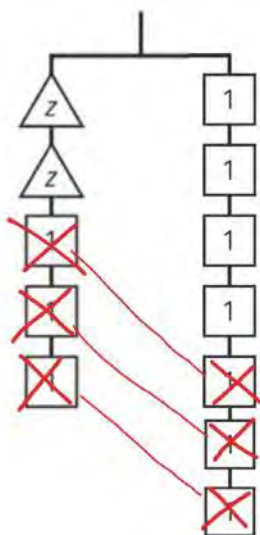
Great thinking. When trying to keep this hanger balanced, whatever I do to one side, I would have to do to the other.

Today, we'll see models that look similar to this hanger and work in a similar way. Let's look at one together.

**Let's Think (Slide 4):** This problem wants us to find the value of  $z$ . The model you see here is called a hanger diagram or a hanger model. Before we solve for  $z$ , what do you notice about the hanger diagram?

[Possible Student Answers, Key Points:](#)

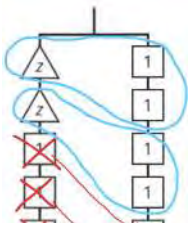
- It looks balanced, but it looks like the right side is longer than the left.
- It has some  $z$ 's and ones on the left. It only has a bunch of ones on the right.



Right now, the diagram has a lot on it. If we want to find just the value of the unknown,  $z$ , then it can be helpful to make our model a bit simpler. To do that, let's remove some quantities. Just like with the picture of the hanger, if we want to maintain balance, we have to do the same thing to both sides of the hanger. Since I see ones on both sides of the hanger, I can match up and remove some ones. (draw lines connecting three pairs of ones) There are enough ones that I can take three away from both sides. (cross out the matched ones) How do I know the hanger will remain balanced? [Possible Student Answers, Key Points:](#)

- You took the same thing off of both ends.
- If you take 3 from one side and 3 from the other, the hanger will stay balanced.

Now what is left on the hanger? (There are 2  $z$ 's on the left and 4 ones on the right.) Because the hanger is balanced, I know the value of these 2  $z$ 's on the left is equal to the value of the 4 ones on the right.



We can think of that on the model by matching each  $z$  to the same number of ones. I know I can match each  $z$  with two of the ones. (*circle each  $z$  with 2 ones as shown*) This makes it clear that each  $z$  is equal to 2.

That makes sense because if each  $z = 2$ , that means the left side of the hanger weighs 4 units, because  $2 + 2 = 4$ . That matches the weight of the right side of the hanger.

$$2z + 3 = 7$$

We can also think about the work we just did using an equation. If we think back to the original hanger diagram in this problem, I can use the equation  $2z + 3 = 7$  to represent it. (*write equation*) Why does  $2z + 3 = 7$  match the original hanger diagram?

**Possible Student Answers, Key Points:**

- The left side of the hanger shows a  $z$ , another  $z$ , and 3 ones. We can write that as  $2z + 3$ . The right side of the hanger shows 7 ones, which we can write as 7. Since the hanger is balanced,  $2z + 3 = 7$  makes sense.

$$2z = 4$$

$$z = 2$$

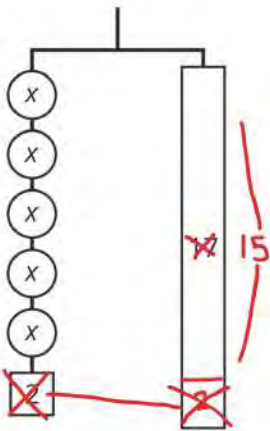
The first thing we did in the diagram was remove 3 from both sides. If we remove 3 from both sides of our equation, we're left with  $2z$  on the left and 4 on the right. (*write  $2z = 4$  underneath original equation*) Then, we divided each side into 2 groups when we matched the  $z$ 's with the ones. Dividing the left and right side of our equation by 2, leaves us with  $z = 2$ .

Both the hanger diagram and the equation help us see that the value of  $z$  is 2.

**Let's Think (Slide 5):** Let's look at one more. What do you notice is the same about this hanger model?

What do you notice is different? **Possible Student Answers, Key Points:**

- It's the same because it's balanced. It's the same because one side has variables and numbers and the other side has just numbers.
- It's different because the unknown here is  $x$ . It's also different because the numbers are bigger; it's not just a bunch of ones.



Even though this hanger model looks a little different, we can solve for the unknown using a similar approach.

Let's start by making the hanger model simpler. Let's try to get it so that we only have variables on the left. This is sometimes called isolating the variable. I can do that by taking 2 off of both sides of the hanger model. Since I can't match up ones, I'll just remove a chunk of 2 from both sides. I'll cross off the 2 on the left, and then I'll partition off 2 from the 17 on the right. What is  $17 - 2$ ? (15) Okay, let me show that. (*cross off the 2 on the left, and mark-up the 17 on the right as shown*)

The hanger is still balanced, because we removed 2 from both sides. What's left on the hanger for us to consider? (The left has five  $x$ 's. The right has 15.) If I know five  $x$ 's has to have the same value as 15, how can I determine the value of  $x$ ? **Possible**

**Student Answers, Key Points:**

- I know  $5 \times 3 = 15$ , so each  $x$  must be equal to 3.
- I can divide 15 into 5 groups, kind of like how we matched the ones to the  $z$ 's in the other problem. I know 15 divided into 5 equal groups is 3.

Each  $x$  must be equal to 3. Five groups of 3 is 15.

$$5x + 2 = 17$$

$$5x = 15$$

$$x = 3$$

Before we wrap up, let's think about how the work we did with the hanger model could look with an equation. What equation can I write to represent this hanger model? How do you know? *(write equation as student shares, supporting as needed)* Possible Student Answers, Key Points:

- We can write  $5x + 2 = 17$ .
- I see five x's and 2 on the left. I see 17 on the right.  $5x + 2$  must be equal to 17 for the hanger to be balanced.

*(write equations as you narrate)* The first thing we did in the model was remove 2 from both sides. If we think of doing that in the equation, we would be left with  $5x$  on the left and 15 on the right. We saw that in our hanger model too. Lastly, we thought of dividing the 15 into 5 equal groups to find the value of each of the x's. If we divide both sides of our equation by 5, we see that  $x = 3$ .

Nice work! Isolating a variable on the hanger model is a handy way to identify the value of the unknown. We can use equations to show similar thinking.

**Let's Try it (Slides 6 - 7):** It's time to try a few more examples collaboratively before we move into some time for independent work. As we look at each hanger model, it can be helpful to represent each side of the hanger as an expression in our equation. We ask ourselves what we can remove from both sides of the hanger that would keep it balanced. Once we've removed any known values, we can match up unknowns with anything leftover or reason about what value the unknown could be to make what's left on the hanger diagram balanced. We'll treat each of the problems like a quick balancing puzzle!

# WARM WELCOME



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**Today we will use a balanced hanger diagram to reason about writing and solving equations in the form  $px + q = r$ .**

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Let's Talk:

What do you notice?

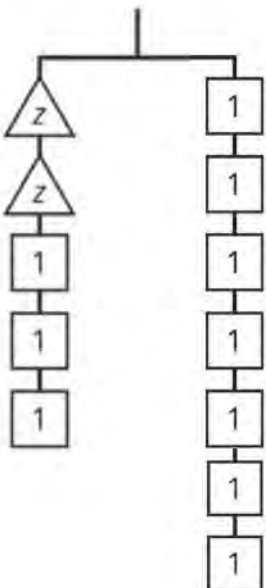
What do you wonder?



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Let's Think:

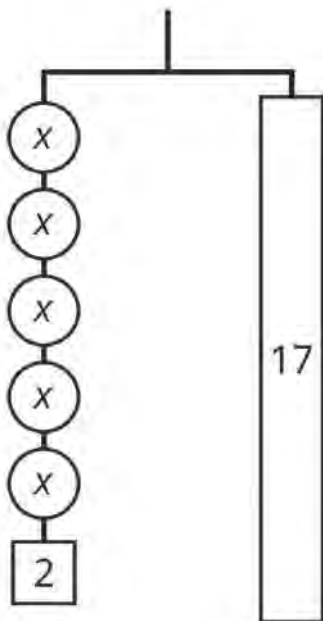
What is the value of z?



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# Let's Think:

What value of  $x$  keeps the hanger diagram balanced?



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# Let's Try It:

Let's explore using a balanced hanger diagram to reason about writing and solving equations in the form  $px + q = r$  together.

Name: \_\_\_\_\_ G7 US Lesson 7 - Let's Try It

**We can think of hanger diagrams like scales. Look at the scales below.**

- Which scale is balanced? How do you know?
- Which scale is not balanced? How do you know?

**The hanger diagram below is balanced.**

- Fill in the blanks to write an equation that represents the hanger diagram: \_\_\_\_\_ = \_\_\_\_\_
- What must be the value of  $x$  to keep the hanger in balance?

**Consider the balanced hanger diagram below.**

- Fill in the blanks to write an equation that represents the diagram: \_\_\_\_\_ = \_\_\_\_\_
- Remove 1 from both sides of the hanger diagram. Rewrite your equation: \_\_\_\_\_
- Group each  $w$  with the same number of ones. Rewrite your equation: \_\_\_\_\_
- What is the value of  $w$ ?

The hanger diagram below shows that the sum of  $z$ ,  $z$ , and 6 is equal to 14.

- Fill in the blanks to write an equation that represents the hanger diagram: \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_
- Remove 6 from both sides of the equation to maintain the hanger's balance. Rewrite the equation: \_\_\_\_\_
- What must be the value of  $z$  to keep the hanger in balance?

**Write an equation to represent each hanger diagram. Then find the value of each unknown. REMINDER: Remove the same amount from each side to keep the hanger balanced.**

12.

EQUATION: \_\_\_\_\_

$y =$  \_\_\_\_\_

13.

EQUATION: \_\_\_\_\_

$x =$  \_\_\_\_\_

14.

EQUATION: \_\_\_\_\_

$z =$  \_\_\_\_\_

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# On your Own:

Now it's time to use a balanced hanger diagram to reason about writing and solving equations in the form  $px + q = r$  on your own.

Name: \_\_\_\_\_ 57 U5 Lesson 7 - Independent Work

1. Write and solve an equation to find the value of the unknown in the hanger diagram?

2. The hanger diagram below is balanced. Each square has a value of 1, and each circle has a value of  $x$ . What is the value of  $x$ ?

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3. Write and solve an equation to represent each hanger diagram.

4. David started to solve for  $x$ . His first step is shown below. In your own words, explain what David has done. Then, describe what David can do next to find the value of  $x$ .

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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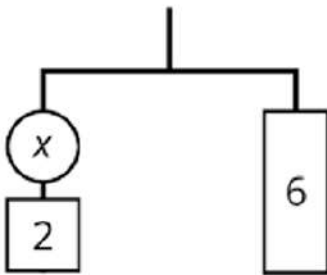
**We can think of hanger diagrams like scales. Look at the scales below.**

1. Which scale is balanced? How do you know?



2. Which scale is not balanced? How do you know?

**The hanger diagram below is balanced.**

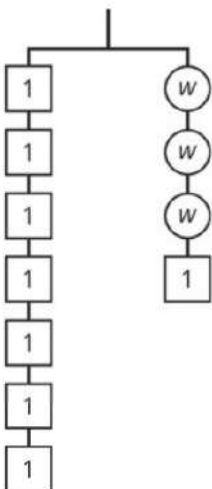


3. Fill in the blanks to write an equation that represents the hanger diagram.

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

4. What must be the value of  $x$  to keep the hanger in balance?

**Consider the balanced hanger diagram below.**



5. Fill in the blanks to write an equation that represents the diagram.

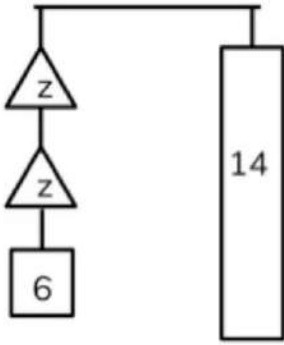
$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

6. Remove 1 from both sides of the hanger diagram. Rewrite your equation.

7. Group each  $w$  with the same number of ones. Rewrite your equation.

8. What is the value of  $w$ ?

The hanger diagram below shows that the sum of  $z$ ,  $z$ , and 6 is equal to 14.



9. Fill in the blanks to write an equation that represents the hanger diagram.

$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

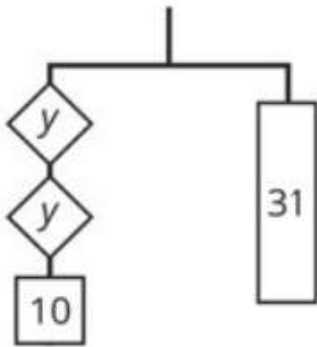
10. Remove 6 from both sides of the equation to maintain the hanger's balance. Rewrite the equation.

11. What must be the value of  $z$  to keep the hanger in balance?

**Write an equation to represent each hanger diagram. Then find the value of each unknown.**

REMINDER: Remove the same amount from each side to keep the hanger balanced.

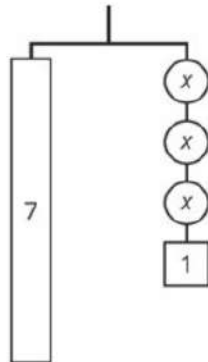
12.



EQUATION:

$$y = \underline{\quad}$$

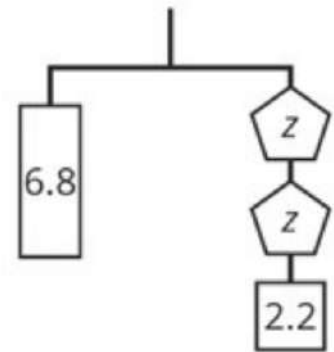
13.



EQUATION:

$$x = \underline{\quad}$$

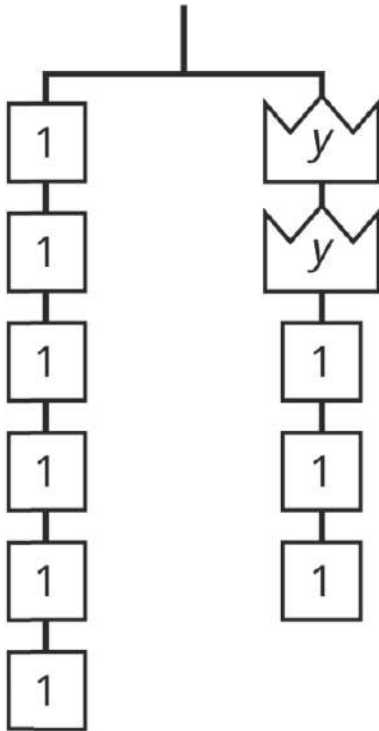
14.



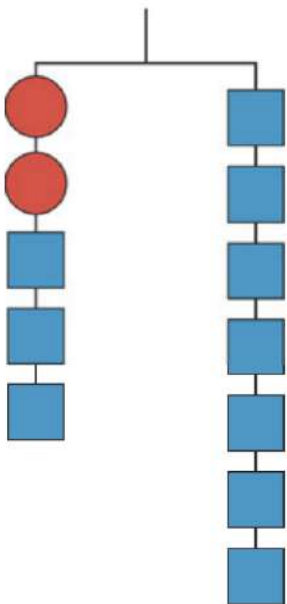
EQUATION:

$$z = \underline{\quad}$$

1. Write and solve an equation to find the value of the unknown in the hanger diagram.

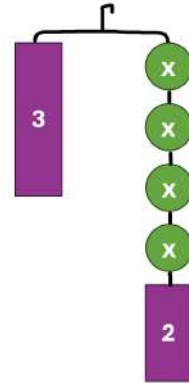
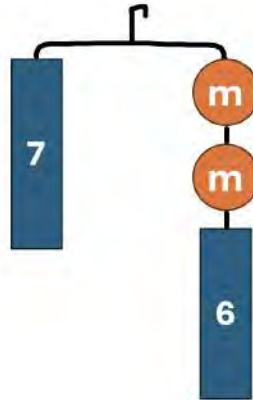
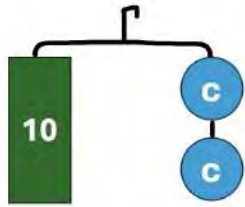


2. The hanger diagram below is balanced. Each square has a value of 1, and each circle has a value of  $x$ . What is the value of  $x$ ?

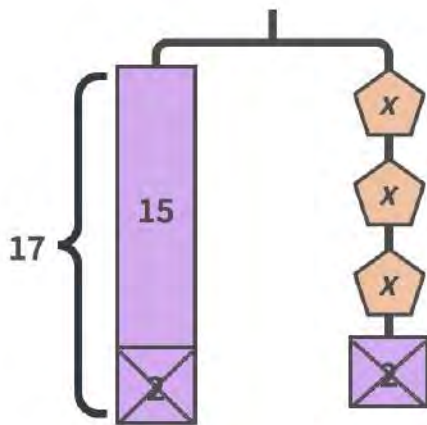




3. Write and solve an equation to represent each hanger diagram.



4. David started to solve for  $x$ . His first step is shown below. In your own words, explain what David has done. Then, describe what David can do next to find the value of  $x$ .




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We can think of hanger diagrams like scales. Look at the scales below.

1. Which scale is balanced? How do you know?

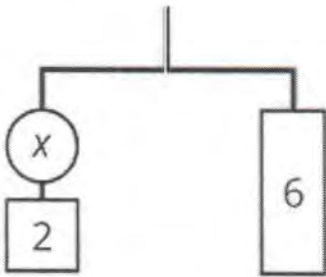
*A is balanced. The two sides are even.*



2. Which scale is not balanced? How do you know?

*Scale B is not balanced, because the grapes are heavier.*

The hanger diagram below is balanced.



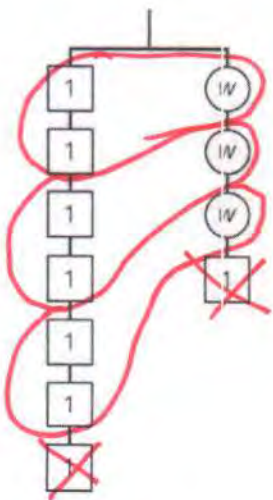
3. Fill in the blanks to write an equation that represents the hanger diagram.

$$\underline{x} + \underline{2} = \underline{6}$$

4. What must be the value of  $x$  to keep the hanger in balance?

$$\underline{x = 4}$$

Consider the balanced hanger diagram below.



5. Fill in the blanks to write an equation that represents the diagram.

$$\underline{7} = \underline{3w} + \underline{1}$$

6. Remove 1 from both sides of the hanger diagram. Rewrite your equation.

$$\underline{6} = \underline{3w}$$

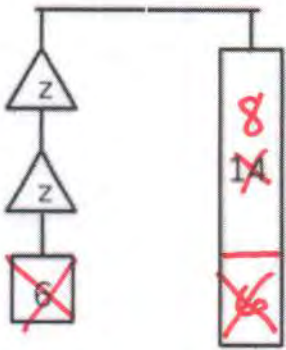
7. Group each  $w$  with the same number of ones. Rewrite your equation.

$$\underline{2} = \underline{w}$$

8. What is the value of  $w$ ?

$$\underline{2}$$

The hanger diagram below shows that the sum of  $z$ ,  $z$ , and  $6$  is equal to  $14$ .



9. Fill in the blanks to write an equation that represents the hanger diagram.

$$\underline{z} + \underline{z} + \underline{6} = \underline{14}$$

10. Remove  $6$  from both sides of the equation to maintain the hanger's balance. Rewrite the equation.

$$z + z = 8$$

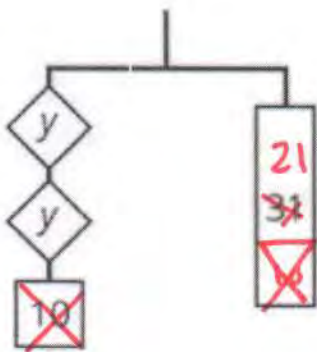
11. What must be the value of  $z$  to keep the hanger in balance?

$$z = 4$$

Write an equation to represent each hanger diagram. Then find the value of each unknown.

REMINDER: Remove the same amount from each side to keep the hanger balanced.

12.



EQUATION:

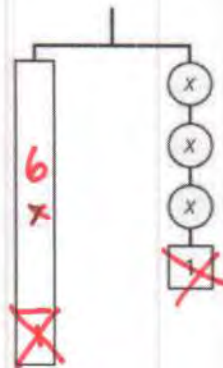
$$2y + 10 = 31$$

$$2y = 21$$

$$21 \div 2 = 10\frac{1}{2}$$

$$y = 10\frac{1}{2}$$

13.



EQUATION:

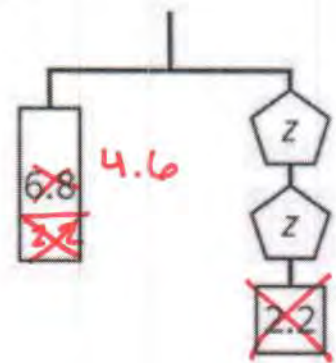
$$7 = 3x + 1$$

$$6 = 3x$$

$$2 = x$$

$$x = 2$$

14.



EQUATION:

$$6.8 = 2z + 2.2$$

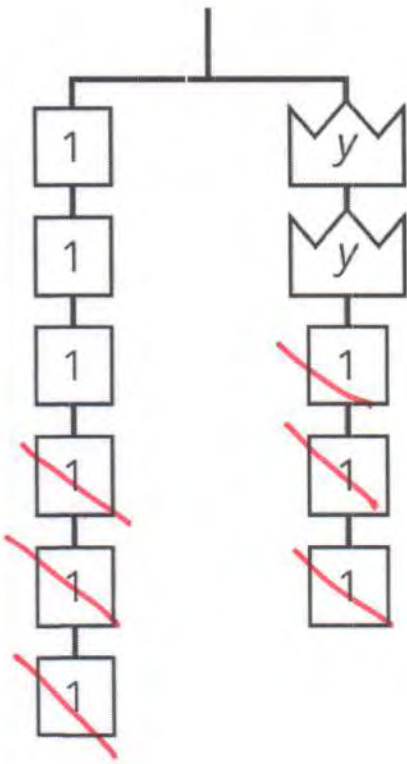
$$4.6 = 2z$$

$$4.6 \div 2 = 2.3$$

$$z = 2.3$$



1. Write and solve an equation to find the value of the unknown in the hanger diagram.



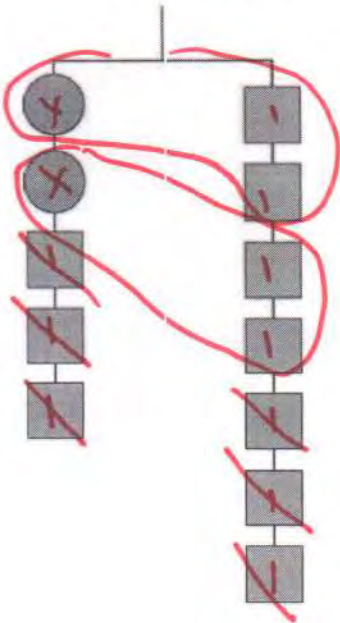
$$6 = 2y + 3$$

$$3 = 2y$$

$$3 \div 2 = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

$$y = 1\frac{1}{2}$$

2. The hanger diagram below is balanced. Each square has a value of 1, and each circle has a value of  $x$ . What is the value of  $x$ ?

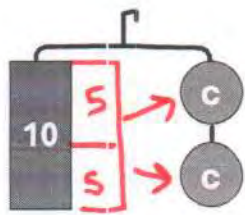


$$2x + 3 = 7$$

$$2x = 4$$

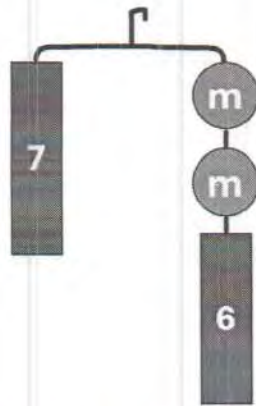
$$x = 2$$

3. Write and solve an equation to represent each hanger diagram.



$$10 = 2c$$

$$(5 = c)$$



$$7 = 2m + 6$$

$$1 = 2m$$

$$\left(\frac{1}{2} = m\right)$$

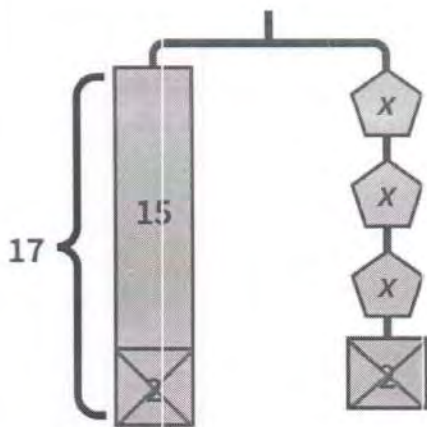


$$3 = 4x + 2$$

$$1 = 4x$$

$$\left(\frac{1}{4} = x\right)$$

4. David started to solve for  $x$ . His first step is shown below. In your own words, explain what David has done. Then, describe what David can do next to find the value of  $x$ .



David started by taking 2 off of each side of the balance.

He's left with  $15 = 3x$ ,

so he can divide 15 evenly between each  $x$ .  $15 \div 3$  means each  $x$  is worth 5.

## **G7 U5 Lesson 8**

Solve equations of the form  $px + q = r$  and  $p(x + q) = r$  that involve negative numbers.



**G7 U5 Lesson 8 - Students will solve equations of the form  $px + q = r$  and  $p(x + q) = r$  that involve negative numbers.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We have been working so hard to solve equations. Based on our past few lessons, what stands out to you when you think about solving equations?

Possible Student Answers, Key Points:

- We can use tape diagrams to help us write equations from word problems.
- Hanger diagrams are a visual way to think about equations. We have to do the same thing to both sides to keep the hanger or equation balanced.
- We can rewrite equations as related equations to help us solve them.

Do you think what we've learned so far also works if there are negative numbers in equations? Possible Student Answers, Key Points:

- I'm not sure. I've not seen a lot of negative numbers in equations.
- I don't think they'll work. It's hard to picture a tape diagram or hanger model with negative numbers.
- I think our same strategies will work, because negative numbers are still numbers.

Let's work together to find out if we can solve equations like we've been solving...but with negative numbers!

**Let's Talk (Slide 3):** What's the same and what's different about the two equations you see here? Possible Student Answers, Key Points:

- They both involve multiplication. They both have an  $x$  and an 8 in them. They're both short.
- They're different colors. The pink one has a  $-2$  rather than a  $+2$ . I think their answers are different.

$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

Let's think about the first equation in orange. One way to solve this would be to divide both sides of our equation by 2. (*write equation and divide both sides by 2*) I have to divide both sides by 2 because, like with the hanger diagrams, I have to keep the equation balanced. Dividing both sides by 2 isolates the variable, so that we can see  $x$  is equal to 4. (*write  $x = 4$* )

$$\frac{-2x}{-2} = \frac{8}{-2}$$
$$x = -4$$

We noticed that the second equation is similar, but the  $x$  has a coefficient of  $-2$ . If I want to isolate the variable, I couldn't do the exact same step we took in the last problem. If I divided both sides of this equation by just 2, we'd be left with  $-x = 4$ , which means the  $x$  would not be isolated. If I want to isolate the  $x$  in this equation, I'd have to divide both sides by *negative 2*. (*write equation and show dividing both sides by  $-2$* ) We end up with a solution of  $x = -4$ . I know 8 divided by  $-2$  is  $-4$ , because a positive number divided by a negative number results in a negative number.

I notice that how we tackled the equation with the negative number was not entirely different from how we solved the first equation with positive numbers. The only thing we had to do differently is pay attention to the sign of our numbers as we solved. The same steps we take to solve equations with positive numbers can help us solve equations with negative numbers.

**Let's Think (Slide 4):** Our first official problem today asks us to solve this equation. I notice there are some negative numbers involved, so we'll want to carefully keep track of signs as we take steps to solve.



I want to start by isolating the  $x$ . I can start by subtracting 7 from both sides of the equation. It is a little unusual to think about a hanger diagram with negative values, but if we think about the left and right side of our equation as items on a hanger diagram, I know removing 7 from both sides could help get the variable isolated. (*sketch a simple diagram as shown to clarify*)

$$\begin{array}{r} -2x + 7 = -5 \\ \underline{-7 \quad -7} \end{array}$$

$$\begin{array}{r} -2x \quad = -12 \\ \underline{-2 \quad -2} \end{array}$$

$$x = 6$$

Let's subtract 7 from both sides of the equation. (*write equation and subtract 7 from both sides*) What is -5 minus 7? (-12) When we subtract 7 from both sides, we're left with -2x on the left side of the equation and -12 on the right side of the equation. (*write -2x = -12*)

We're one step away from isolating the variable. If I want to get the x by itself, I need to divide both sides by -2. I'd do this because if I divide -2 by -2, I'll just be left with 1x on the left, which means we've isolated the variable. (*divide both sides of the equation by -2*)

I know if I divide the left side of the equation by -2, we're left with x. What do we get if we divide -12 by -2 on the right side of the equation? (6) I know x is

equal to positive 6.

$$-2(6) + 7 = -5$$

Let's make sure our answer is correct. I feel pretty good about it, but it never hurts to check. (*write equation with 6 substituted in place of x*) If I substitute 6 in for x, will the equation be true? How do you know? **Possible Student**

**Answers, Key Points:**

- I know -2 times 6 would be -12. And -12 plus 7 would be equal to -5
- Yes, the equation will be true. That means x = 6 is the correct solution.

The same thinking we do to solve equations with positive numbers can help us with negative numbers. We just have to keep a close eye on any signed numbers as we perform operations. Ignoring the signs on numbers could result in incorrect answers.

**Let's Think (Slide 5):** Let's solve one more problem together. We're going to solve this equation two ways so you can explore multiple ways to arrive at the correct solution. When we're done, I'll ask you to think about which method you prefer and why.

$$-2(4n - 1) = 10$$

$$-8n + 2 = 10$$

$$\begin{array}{r} -8n + 2 = 10 \\ \underline{-2 \quad -2} \\ -8n \quad = 8 \end{array}$$

$$n = -1$$

For our first attempt, we'll solve by distributing first. (*draw arrows to show distributing the -2 to the terms inside parentheses*) I'll multiply -2 times 4n and then -2 times -1. What is -2 times 4n? (-8n) What is -2 times -1? (+2) We can rewrite the equation as -8n + 2 = 10. (*rewrite*)

Now, let's work to isolate the variable, n. We'll start by subtracting 2 from both sides of the equation to keep the equation balanced. (*subtract 2 from both sides of the equation*) We're left with -8n on the left and 8 on the right. I'll rewrite our equation as -8n = 8. The variable is almost isolated. We just need to divide both sides by -8. (*divide both sides by -8*) We are left with n = -1.

In this method, we distributed the coefficient of -2 first, then solved the equation like other equations we've been working with.

$$\frac{-2(4n - 1)}{-2} = \frac{10}{-2}$$

Let's try another strategy to solve the same problem. This time, instead of distributing the -2, we'll divide both sides by -2. (*write equation and show division by -2 using horizontal fraction bars*) If I divide the left side by -2, I'll just be left with 4n - 1. What do we get on the right side if we divide 10 by -2? (-5) Dividing by -2 meant we didn't need to distribute the -2 like we did in our previous method. Let's keep going.

$$\begin{array}{r}
 4n - 1 = -5 \\
 \quad +1 \quad +1 \\
 \hline
 4n = -4 \\
 \frac{4n}{4} = \frac{-4}{4} \\
 n = -1
 \end{array}$$

(write  $4n - 1 = -5$  underneath) Our equation now reads  $4n - 1 = -5$ . What could I do next to both sides of the equation to isolate the variable? How do you know?

Possible Student Answers, Key Points:

- We can add 1 to both sides of the equation.
- Adding 1 to -1 on the left will mean we're just left with  $4n$ .

Let's add 1 to both sides of the equation. (show that step under the equation) Our equation is now  $4n = -4$ . We're so close! We'll just divide by 4 on both sides of the equation, and we're done. (divide both sides by 4) What is our solution? Possible

Student Answers, Key Points:

- Our solution is  $n = -1$ .
- I know 4 divided by 4 leaves us with  $n$  on the left. I know -4 divided by 4 leaves us with -1 on the right.

Instead of distributing first, we divided by the coefficient of -2 first in this method. No matter the method, we still ended up with  $n = -1$  as our solution.

We just solved this problem by distributing first *and* by dividing first. Both are valid methods. Which one did you prefer and why? Possible Student Answers, Key Points:

- I liked distributing first, because I think distributing is easy.
- I liked dividing first, because the numbers felt more manageable after that step compared to the previous strategy.

As you see problems like these, take a second to pause and consider which solution pathway will be most helpful for you. You might find that some problems lend themselves to one method more than the other.

**Let's Try it (Slides 6 - 7):** Now let's solve a few more equations with negative numbers together. We saw that solving equations with negative numbers isn't all that different from solving equations with positive numbers; you just have to be careful when operating with signed numbers. The wrong sign can mean the wrong answer! After we tackle these, you'll get a chance to try some independently.

# WARM WELCOME



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**Today we will solve equations of the form  $px + q = r$  and  $p(x + q) = r$  that involve negative numbers.**

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Let's Talk:

**What's the same? What's different?**

$$2x = 8$$

$$-2x = 8$$

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Let's Think:

**Solve the equation.**

$$-2x + 7 = -5$$

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## Let's Think:

Solve the equation both ways.

$$-2(4n - 1) = 10$$

**DISTRIBUTE -2 FIRST**

**DIVIDE BY -2 FIRST**

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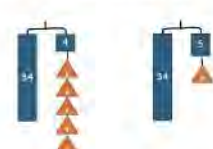
## Let's Try It:

Let's explore solving equations of the form  $px + q = r$  and  $p(x + q) = r$  that involve negative numbers together.

Name: \_\_\_\_\_ G7 US Lesson 8 - Let's Try It

**Consider the equation  $5x + 4 = 34$ .**

- Circle the hanger diagram that best matches the equation.



- Solve the equation. Begin by using the same operation on each side of the equation so that the  $5x$  remains.

- Describe the steps you took to get  $x$  on its own.

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We can solve equations with \_\_\_\_\_ similar to how we solve equations with positive numbers.

**Consider the equation  $-5x + 4 = 34$ .**

- What number can you subtract from both sides of the equation so that  $-5x$  remains on the left?
- Continue solving the equation. When you divide, make sure you divide both sides by  $-5$ .

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**Consider the equation  $3x + (-9) = 13$ .**

- What number can you subtract from both sides of the equation so that  $3x$  remains on the left?
- Continue solving the equation. Be careful with your signs!

**Consider the equation  $-2(y + 3) = -8$ .**

- Solve the equation by distributing the  $-2$  first.
- Solve the equation by dividing both sides by  $-2$ .

- What was the same about these two strategies? Different?

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# On your Own:

Now it's time to solve equations of the form  $px + q = r$  and  $p(x + q) = r$  that involve negative numbers on your own.

Name: \_\_\_\_\_ 07 US Lesson 6 - Independent Work

1. Solve each equation. Show your work.

$2a + 10 = 64$        $-7l = 10 = -38$

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2. Solve each equation. Show your work.

$3(-3c + 4) = 42$        $-7(v + 8) = -35$

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3. Mark attempted to solve the equation below. His work is shown. His teacher said he made a mistake. Identify Mark's mistake and show how to solve the equation correctly.

$$\begin{array}{r} -3n + 6 = -6 \\ -6 = -6 \\ \hline -3n = -12 \\ \hline n = 4 \end{array}$$


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4. Solve the equation below two ways.

$3(x - 4) = -12$

DISTRIBUTE 3 FIRST      DIVIDE BY 3 FIRST

Which strategy did you prefer in this example? Explain.

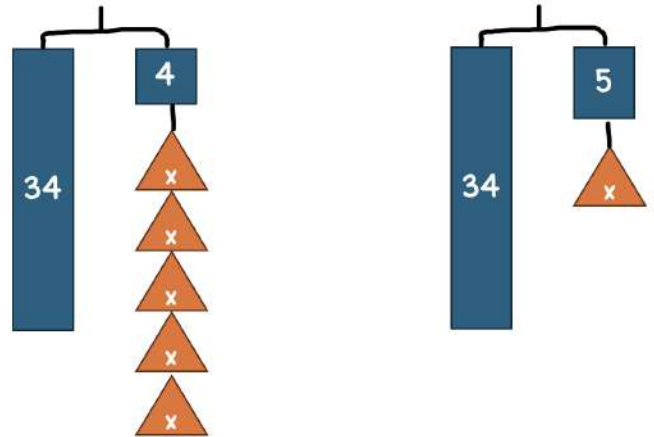
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Consider the equation  $5x + 4 = 34$ .

1. Circle the hanger diagram that best matches the equation.
2. Solve the equation. Begin by using the same operation on each side of the equation so that the  $5x$  remains.



3. Describe the steps you took to get  $x$  on its own.

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We can solve equations with \_\_\_\_\_ similar to how we solve equations with positive numbers.

Consider the equation  $-5x + 4 = 34$ .

4. What number can you subtract from both sides of the equation so that  $-5x$  remains on the left?
5. Continue solving the equation. When you divide, make sure you divide both sides by  $-5$ .

**Consider the equation  $3x + (-5) = 13$ .**

6. What number can you subtract from both sides of the equation so that  $3x$  remains on the left?
7. Continue solving the equation. Be careful with your signs!

**Consider the equation  $-2(y - 3) = -8$ .**

8. Solve the equation by distributing the  $-2$  first.
9. Solve the equation by dividing both sides by  $-2$ .

10. What was the same about these two strategies? Different?

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**1. Solve each equation. Show your work.**

$$2a + 10 = 64$$

$$-7h + 10 = -39$$

**2. Solve each equation. Show your work.**

$$3(-3c + 2) = 42$$

$$-7(-v + 3) = -35$$

3. Mark attempted to solve the equation below. His work is shown. His teacher said he made a mistake. Identify Mark's mistake and show how to solve the equation correctly.

$$\begin{array}{r}
 -3n + 6 \neq -6 \\
 \hline
 \begin{array}{r}
 -3n \quad \neq \quad 0 \\
 -3 \quad \quad | \quad -3 \\
 \hline
 n \quad \neq \quad 0
 \end{array}
 \end{array}$$

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4. Solve the equation below two ways.

$$3(x - 4) = -12$$

DISTRIBUTE 3 FIRST

DIVIDE BY 3 FIRST

Which strategy did you prefer in this example? Explain.

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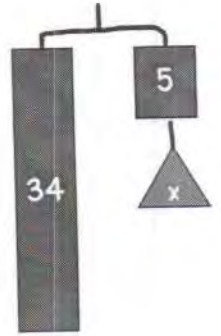
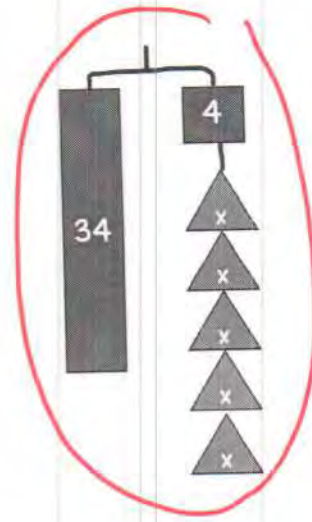
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Name: \_\_\_\_\_

**KEY**

Consider the equation  $5x + 4 = 34$ .

1. Circle the hanger diagram that best matches the equation.
2. Solve the equation. Begin by using the same operation on each side of the equation so that the  $5x$  remains.



$$\begin{array}{r}
 5x + 4 = 34 \\
 -4 \quad -4 \\
 \hline
 5x = 30 \\
 \frac{5x}{5} = \frac{30}{5} \\
 \boxed{x = 6}
 \end{array}$$

3. Describe the steps you took to get  $x$  on its own.

I subtracted 4 from both sides, then I divided both sides by 5.

We can solve equations with negative numbers similar to how we solve equations with positive numbers.

Consider the equation  $-5x + 4 = 34$ .

4. What number can you subtract from both sides of the equation so that  $-5x$  remains on the left?
5. Continue solving the equation. When you divide, make sure you divide both sides by  $-5$ .

$$\begin{array}{r}
 -5x + 4 = 34 \\
 -4 \quad -4 \\
 \hline
 -5x = 30 \\
 \frac{-5x}{-5} = \frac{30}{-5} \\
 \boxed{x = -6}
 \end{array}$$



Consider the equation  $3x + (-5) = 13$ .

6. What number can you subtract from both sides of the equation so that  $3x$  remains on the left?

$-5$

7. Continue solving the equation. Be careful with your signs!

$$\begin{array}{r} 3x + (-5) = 13 \\ \quad +5 \quad +5 \\ \hline 3x = 18 \end{array}$$

$$\begin{array}{r} 3x = 18 \\ \frac{3}{3} \quad \frac{3}{3} \\ \hline x = 6 \end{array}$$

Consider the equation  $-2(y - 3) = -8$ .

8. Solve the equation by distributing the  $-2$  first.

$$\begin{array}{r} -2(y-3) = -8 \\ -2y + 6 = -8 \\ \quad -6 \quad -6 \\ \hline -2y = -14 \\ \frac{-2}{-2} \quad \frac{-14}{-2} \\ \hline y = 7 \end{array}$$

9. Solve the equation by dividing both sides by  $-2$ .

$$\begin{array}{r} -2(y-3) = -8 \\ \frac{-2}{-2} \quad \frac{-8}{-2} \\ \hline y-3 = 4 \\ \quad +3 \quad +3 \\ \hline y = 7 \end{array}$$

10. What was the same about these two strategies? Different?

I got the same answer, and I isolated  
the variable in both. The steps to  
solve were different. With one, I distributed,  
and with the other I divided by the  
coefficient.

1. Solve each equation. Show your work.

$$\begin{array}{r}
 2a + 10 = 64 \\
 \underline{-10 \quad -10} \\
 2a = 54 \\
 \underline{\quad \quad 2} \\
 a = 27
 \end{array}$$

$$\begin{array}{r}
 -7h + 10 = -39 \\
 \underline{-10 \quad -10} \\
 -7h = -49 \\
 \underline{-7 \quad -7} \\
 h = 7
 \end{array}$$

2. Solve each equation. Show your work.

$$\begin{array}{r}
 \overbrace{3(-3c + 2)} = 42 \\
 -9c + 6 = 42 \\
 \underline{-6 \quad -6} \\
 -9c = 36 \\
 \underline{-9 \quad -9} \\
 c = -4
 \end{array}$$

$$\begin{array}{r}
 -7(-v + 3) = -35 \\
 \underline{-7 \quad -7} \\
 -v + 3 = 5 \\
 \underline{-3 \quad -3} \\
 -v = 2 \\
 \underline{-1 \quad -1} \\
 v = -2
 \end{array}$$

3. Mark attempted to solve the equation below. His work is shown. His teacher said he made a mistake. Identify Mark's mistake and show how to solve the equation correctly.

$$\begin{array}{r}
 -3n + 6 = -6 \\
 \underline{-6 \quad -6} \\
 -3n = 0 \\
 \underline{-3 \quad -3} \\
 n = 0
 \end{array}$$

-6 minus 6 is -12, not 0. If he did that step correct, he'd have  $-3n = -12$ . He could divide both sides by -3 to get a solution of  $n = 4$ .

4. Solve the equation below two ways.

$$3(x - 4) = -12$$

DISTRIBUTE 3 FIRST

$$\begin{array}{r}
 3x - 12 = -12 \\
 \underline{+12 \quad +12} \\
 3x = 0 \\
 \frac{3x}{3} = \frac{0}{3} \\
 x = 0
 \end{array}$$

DIVIDE BY 3 FIRST

$$\begin{array}{r}
 x - 4 = -4 \\
 \underline{+4 \quad +4} \\
 x = 0
 \end{array}$$

Which strategy did you prefer in this example? Explain. (answers may vary)

I liked dividing by 3 first, because it felt like fewer steps.

## **G7 U5 Lesson 9**

Decide between and use two common approaches for solving an equation of the form  $p(x + q) = r$  (expanding using the distributive property, or dividing each side by  $p$ )



**G7 U5 Lesson 9 - Students will decide between and use two common approaches for solving an equation of the form  $p(x + q) = r$  (expanding using the distributive property, or dividing each side by  $p$ ).**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today's work is going to feel like a choose-your-own-adventure story. We're going to look at equations and decide which strategy will be most efficient. Wisely picking our solution pathway can help us avoid overcomplicating a math problem. If we pause to think about the numbers we're going to work with, we can often find ways to make our lives a little easier when solving.

We did a little bit of this in our previous lesson, but today we're going to make choosing a smart solution pathway our primary focus. Let's start by looking at some student work samples.

**Let's Talk (Slide 3):** Here we see two student work samples. Both student work samples show a correct first step toward solving the same equation. What do you notice about their work? What do you wonder?

**Possible Student Answers, Key Points:**

- I notice they're in different colors. I notice they use a dotted line to separate each side of their equation. I notice the red work shows distributing first. I notice the blue work shows dividing by -3 first.
- I wonder what their next step will be. I wonder how they chose which solution pathway to use. I wonder what the correct answer is.

Looking at their work for this specific problem, would you personally prefer to use the first strategy where the student distributes the -3 or the second strategy where the student divides by -3? Why? **Possible Student Answers, Key Points:**

- For this problem I would choose to divide by -3, because the equation for the blue work looks so much easier to solve now. It's just  $n + 1 = -3$ . The red equation involves more steps to arrive at the solution.
- I think the distributive property seems easier, because I'm pretty good at distributing and the numbers don't seem too tricky.

Our equations today will share a similar structure to these. Each equation will have a coefficient outside of an expression in parentheses. It will be our job to consider which first step will make solving easier. Sometimes both strategies might be equally easy for you. Sometimes both strategies might present challenges. And we'll see several examples today where one strategy ends up being significantly easier than the other. Our job will be to pause, think ahead, and predict what we should do.

**Let's Think (Slide 4):** For many problems today, we'll get to choose our solution pathway. For this first one, we're going to solve it both ways and think about which way felt easier and why.

$$\begin{aligned} -11 &= 6\left(k + \frac{1}{6}\right) \\ -11 &= 6k + 1 \\ \frac{-11}{6} &= \frac{6k}{6} + \frac{1}{6} \\ \frac{-11}{6} - \frac{1}{6} &= k \\ \frac{-12}{6} &= k \\ -2 &= k \end{aligned}$$

Let's solve this equation first by using the distributive property. (*solve equation as shown as you narrate*) I'll begin by distributing the 6 to each term in the parentheses. 6 times  $k$  is  $6k$  and  $6 \times \frac{1}{6}$  is  $\frac{6}{6}$ . I'll just write that as 1 whole. Our equation now reads  $-11 = 6k + 1$ . I can subtract 1 from both sides to start isolating the variable. What is  $-11 - 1$ ?  $(-12)$  After subtracting 1 from both sides to keep the equation balanced, our equation reads  $-12 = 6k$ . I can divide by 6 on both sides to arrive at our solution. What is  $-12$  divided by 6?  $(-2)$  Our solution reads  $-2 = k$  or  $k = -2$ .

We had to be careful when operating with the signed values during this method, but otherwise, distributing felt fairly manageable with this equation.

$$\frac{-11}{6} = \frac{6(k + \frac{1}{6})}{6}$$

$$\frac{-11}{6} = k + \frac{1}{6}$$

$$\frac{-12}{6} = k$$

$$\boxed{-2 = k}$$

Let's solve the same equation, but instead of distributing, we'll divide both sides by 6 to start with.

(solve equation as shown as you narrate) Let's divide both sides by 6. -11 divided by 6 is a little unfriendly to think about. We can leave it as the fraction  $-\frac{11}{6}$ . Then the other side of our equation reads  $k + \frac{1}{6}$ . Our equation now reads  $-\frac{11}{6} = k + \frac{1}{6}$ . Let's isolate the variable. I can subtract  $\frac{1}{6}$  from both sides. What is  $-\frac{11}{6}$  minus  $\frac{1}{6}$ ? ( $-\frac{12}{6}$ ) Our equation now reads  $-\frac{12}{6} = k$ . I know  $-\frac{12}{6}$  is the same as -2 because -12 divided by 6 is -2. Our solution is  $-2 = k$  or  $k = -2$ .

We got the same answer using both strategies. They're both valid methods for solving. Based on the numbers in this equation and the steps we took to solve, which method would you have chosen to use to solve the equation if you only had to choose one method? Why? **Possible Student Answers, Key Points:**

- I would choose the first method. When we distributed, we avoided having to do any more work with fractions.
- I would choose the first method. The second method had us working with negative fractions and fractions greater than one. I know how to do the math, but it was definitely a little trickier in spots.

In this case, I think it was certainly a bit easier to distribute first. I'll warn you, that's not always the case. We'll see problems today where the other strategy of dividing by the coefficient make the work more manageable.

Nice work. Let's look at one more example.

**Let's Think (Slide 5):** For this problem, we get a choice. We don't have to do it both ways, so let's be strategic about choosing our method. Take a second to look at the numbers. Before we start solving, what stands out to you about the values in this equation? **Possible Student Answers, Key Points:**

- I notice some of the numbers are decimals, and one of the numbers is a whole number.
- I notice 4.21 is half of 8.42.

$$8.42 = 4.21(y - 7)$$

Let's consider using the distributive property. (draw arrows showing the 4.21 being distributed) Does this seem like an efficient first step? **Possible Student Answers, Key Points:**

- We can do it. 4.21 times y would just be 4.21y. I'm wondering if multiplying  $4.21 \times -7$  is the best idea though. That might take some time.

$$\frac{8.42}{4.21} = \frac{4.21(y - 7)}{4.21}$$

Hm, seems doable, but I wonder if the other strategy might be better. Let's think about dividing by 4.21 first before we decide. (rewrite equation and draw division bars to show dividing both sides by 4.21) Dividing with fractions can sometimes be painstaking, but I notice that 8.42 divided by 4.21 would just be 2 wholes, because  $4.21 + 4.21 = 8.42$ . Hmm...

$$\begin{array}{r} 2 = y - 7 \\ +7 \quad +7 \\ \hline \boxed{9 = y} \end{array}$$

Based on thinking about the first steps, it definitely seems like dividing by 4.21 might be the better way to go. After that step, our new equation would just be  $2 = y - 7$ . (write equation) How easy is that? We don't have to worry about decimals anymore. (solve equation) We can add 7 to both sides of the equation, and there we go!  $9 = y$  or  $y = 9$  is our solution.



Because we visualized where the numbers in this equation would go with each method, we were able to choose a solutions strategy that made solving the equation efficient.

What will you look for as you decide which method to choose when solving today's equations? [Possible Student Answers, Key Points:](#)

- I'll look for whichever strategy makes my numbers seem the smallest or easiest to work with.
- I'll look for whichever strategy can save me time by avoiding fractions or decimals.

**Let's Try it (Slides 6 - 7):** Practice makes perfect, so let's look at a few more. With each problem, we'll decide whether it makes more sense to distribute first or divide by the coefficient first. We know that either method will get us the correct solution, but sometimes one is more efficient than the other. When possible, it can be smart to avoid unfriendly numbers such as negative decimals or unwieldy fractions. We'll do a couple more together before you get the chance to try this on your own.

# WARM WELCOME



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**Today we will decide between and use two common approaches for solving an equation of the form  $p(x + q) = r$ .**

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# Let's Think:

Predict which strategy will work best. Then solve.

# $8.42 = 4.21(y - 7)$

**DISTRIBUTE FIRST**

**DIVIDE FIRST**

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# Let's Try It:

Let's explore deciding between and using two common approaches for solving an equation of the form  $p(x + q) = r$  together.

Name: \_\_\_\_\_ G7 US Lesson 9 - Let's Try It

Consider the equation  $4(y + 2) = 9.2$ .

1. Solve by dividing both sides by 4 first.      2. Solve by distributing the 4 first.

$\frac{4(y+2)}{4} = \frac{9.2}{4}$        $4(y+2) = 9.2$

3. Which strategy was easier for you with this equation? Why?

\_\_\_\_\_

\_\_\_\_\_

No matter which strategy you use, you get the same answer!

Consider the equation  $3.66 = 1.22(m - 5)$ .

4. Which first step do you predict will be easiest for this equation?

a. Distributing 1.22  
b. Dividing by 1.22

5. Solve by dividing first.      6. Solve by distributing first.

$3.66 = 1.22(m - 5)$        $3.66 = 1.22(m - 5)$

7. Which strategy was easier for you with this equation? Why?

\_\_\_\_\_

\_\_\_\_\_

Look at the numbers in each equation carefully. Then, decide whether you want to distribute or divide first. It's okay if you get part way through solving and change your strategy.

8. Solve.      9. Solve.

$4 = \frac{3}{2}(x + \frac{2}{3})$        $12(w - 13) = 36$

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## On your Own:

Now it's time to decide between and use two common approaches for solving an equation of the form  $p(x + q) = r$  on your own.

Name: \_\_\_\_\_ 07 US Lesson 6 - Independent Work

1. Solve the equation by dividing first. Then, solve the same equation by distributing first. Circle the strategy that you found easier in this problem.

$-15 = 3(a + 5)$        $-15 = 3(a + 5)$

2. Solve the equation by either dividing first or distributing first.

$-4(2.7 - 5) = 20$

Why did you choose your strategy?

\_\_\_\_\_

\_\_\_\_\_

3. Solve.	4. Solve.	5. Solve.
$13 = 2(c - 1.5)$	$-11(x - 5) = -44$	$8(x + 1) = 20$


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Consider the equation  $4(y + 2) = 9.2$ .

1. Solve by dividing both sides by 4 first.

$$\frac{4(y+2)}{4} = \frac{9.2}{4}$$

2. Solve by distributing the 4 first.


$$4(y + 2) = 9.2$$

3. Which strategy was easier for you with this equation? Why?

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**No matter which strategy you use, you get the same answer!**

Consider the equation  $3.66 = 1.22(m - 5)$ .

4. Which first step do you *predict* will be easiest for this equation?
- Distributing 1.22
  - Dividing by 1.22



5. Solve by dividing first.

$$3.66 = 1.22(m - 5)$$

6. Solve by distributing first.

$$3.66 = 1.22(m - 5)$$

7. Which strategy was easier for you with this equation? Why?

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**Look at the numbers in each equation carefully. Then, decide whether you want to distribute or divide first. It's okay if you get part way through solving and change your strategy.**

8. Solve.

$$4 = \frac{3}{2} \left( x + \frac{2}{3} \right)$$

9. Solve.

$$12(w - 13) = 36$$

1. Solve the equation by dividing first. Then, solve the same equation by distributing first. Circle the strategy that you found easier in this problem.

$$-15 = 3(a + \frac{1}{3})$$

$$-15 = 3(a + \frac{1}{3})$$

2. Solve the equation by either dividing first or distributing first.

$$4(2.7 + h) = 20$$

Why did you choose your strategy?

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**3. Solve.**

$$13 = 2(c + 1.5)$$

**4. Solve.**

$$-11(x - 5) = -44$$

**5. Solve.**

$$8(x + \frac{1}{2}) = 20$$

Consider the equation  $4(y + 2) = 9.2$ .

1. Solve by dividing both sides by 4 first.

$$\frac{4(y+2)}{4} = \frac{9.2}{4}$$

$$y+2 = 2.3$$

$$-2 \quad -2.0$$

$$\underline{y = 0.3}$$

$$\begin{array}{r} 2.3 \\ 4 \overline{)9.2} \\ \underline{8} \phantom{0} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

2. Solve by distributing the 4 first.

$$4(y+2) = 9.2$$

$$4y + 8 = 9.2$$

$$\underline{-8 \quad -8}$$

$$4y = 1.2$$

$$\underline{y = 0.3}$$

3. Which strategy was easier for you with this equation? Why?

I liked distributing in this case,  
because it made easier numbers to  
work with.

No matter which strategy you use, you get the same answer!

Consider the equation  $3.66 = 1.22(m - 5)$ .

4. Which first step do you *predict* will be easiest for this equation?

a. Distributing 1.22

b. Dividing by 1.22

5. Solve by dividing first.

$$\frac{3.66}{1.22} = \frac{1.22(m - 5)}{1.22}$$

$$\begin{array}{r} 3 = m - 5 \\ + 5 \quad \quad + 5 \\ \hline 8 = m \end{array}$$

6. Solve by distributing first.

$$3.66 = 1.22(m - 5)$$

$$\begin{array}{r} 3.66 = 1.22m - 6.10 \\ + 6.10 \quad \quad + 6.10 \\ \hline 9.76 = 1.22m \end{array}$$

$$\frac{9.76}{1.22} = \frac{1.22m}{1.22}$$

$$8 = m$$

$$\begin{array}{r} 1.22 \\ \times 5 \\ \hline 6.10 \end{array}$$

7. Which strategy was easier for you with this equation? Why?

I liked dividing first here. The numbers we worked with were easier.

Look at the numbers in each equation carefully. Then, decide whether you want to distribute or divide first. It's okay if you get part way through solving and change your strategy.

8. Solve.

$$\frac{4}{3/2} = \frac{3}{2} \left( x + \frac{2}{3} \right)$$

$$\begin{array}{r} \frac{8}{3} = x + \frac{2}{3} \\ - \frac{2}{3} \\ \hline \frac{6}{3} = x \rightarrow x = 2 \end{array}$$

9. Solve.

$$\frac{12(w - 13)}{12} = \frac{36}{12}$$

$$\begin{array}{r} w - 13 = 3 \\ + 13 \quad + 13 \\ \hline w = 16 \end{array}$$

1. Solve the equation by dividing first. Then, solve the same equation by distributing first. Circle the strategy that you found easier in this problem.

$$\begin{array}{r} -15 = 3(a + \frac{1}{3}) \\ \frac{-15}{3} = \frac{3(a + \frac{1}{3})}{3} \\ -5 = a + \frac{1}{3} \\ -\frac{1}{3} \quad -\frac{1}{3} \\ \hline -5\frac{1}{3} = a \end{array}$$

$$\begin{array}{r} -15 = 3(a + \frac{1}{3}) \\ -15 = 3a + 1 \\ -1 \quad -1 \\ \hline -16 = 3a \\ \frac{-16}{3} = \frac{3a}{3} \\ -5\frac{1}{3} = a \end{array}$$

2. Solve the equation by either dividing first or distributing first.

$$\begin{array}{r} 4(2.7 + h) = 20 \\ \frac{4(2.7 + h)}{4} = \frac{20}{4} \\ 2.7 + h = 5 \\ -2.7 \quad -2.7 \\ \hline h = 2.3 \end{array}$$

Why did you choose your strategy?

I knew I could quickly divide 20 by 4.



3. Solve.

$$13 = 2(c + 1.5)$$

$$\begin{array}{r} 13 = 2c + 3 \\ -3 \quad -3 \\ \hline \end{array}$$

$$\frac{10}{2} = \frac{2c}{2}$$

$$\boxed{5 = c}$$

4. Solve.

$$\frac{-11(x - 5)}{-11} = \frac{-44}{-11}$$

$$\begin{array}{r} x - 5 = 4 \\ +5 \quad +5 \\ \hline \end{array}$$

$$\boxed{x = 9}$$

5. Solve.

$$8(x + \frac{1}{2}) = 20$$

$$\begin{array}{r} 8x + 4 = 20 \\ -4 \quad -4 \\ \hline \end{array}$$

$$\frac{8x}{8} = \frac{16}{8}$$

$$\boxed{x = 2}$$

## **G7 U5 Lesson 10**

Use tape diagrams to translate verbal descriptions for situations into an equation of the form  $px + q = r$  or  $p(x + q) = r$ , and solve the resulting equation to determine an unknown quantity in the situation.

**G7 U5 Lesson 10 - Students will use tape diagrams to translate verbal descriptions for situations into an equation of the form  $px + q = r$  or  $p(x + q) = r$ , and solve the resulting equation to determine an unknown quantity in a situation.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We've been representing situations with models and equations and solving for unknowns for several lessons. We've seen a wide array of contexts and learned about different approaches to solving equations. Today we pull a lot of that thinking together. We'll take a situation from a story problem, create a tape diagram to help us reason about the relationships in the story, and then use that information to write and solve an equation. In a way, today might feel like a day to practice all the skills we've studied so far. around expressions and equations.

**Let's Talk (Slide 3):** Check out the tape diagrams here. Once you've analyzed them, share what you notice is the same and what you notice is different. **Possible Student Answers, Key Points:**

- They both show a rectangle cut into smaller rectangles. They both have a total of 39. They both include variables and the number 6. There are some equal groups inside each total.
- The variables are different. The second tape diagram has all equal groups. The first tape diagram only has one term in each small rectangle.

These tape diagrams represent different situations. We can see that because the relationship between the parts and the totals is different in each of them. As we're aware, we can write equations that correspond with each of these models. For example, I see the first tape diagram shows three groups of  $x$  plus a group of 6 have a total of 39. (*write equation*) I can write that as  $3x + 6 = y$ . What

$$39 = 3x + 6$$

$$39 = 3(x + 6)$$

do you see in the other tape diagram? What equation can match that? (*write equation as student shares*) **Possible Student Answers, Key Points:**

- I see 3 equal groups of  $y + 6$  equal a total of 39.
- I can write  $3(y + 6) = 39$ .

I wonder if we could think of our own story that could go with either of these. For the first tape diagram and equation, I know I need to think of a situation involving 3 groups of an unknown plus 6 more. Maybe my story could be something like: *I have 3 baskets of apples and 6 individual apples, and I have 39 apples in all. How many apples are in each basket?*

Can you think of a situation or a story problem to match the second tape diagram and equation? **Possible Student Answers, Key Points:**

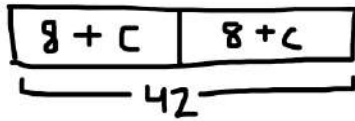
- I have 39 apples evenly distributed between 3 baskets. Each basket has some green apples and 6 red apples. How many green apples are in each basket?
- 39 people are attending a field trip on 3 buses. Each bus has 6 parents and some students. How many students are on each bus?

Today, we'll work to think about verbal descriptions, also known as word problems, and how tape diagrams and equations can help us solve for unknowns.

**Let's Think (Slide 4):** I'll read our first problem aloud as you follow along. When I'm done reading, summarize the story in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This problem is about the number of parents and children on a playground.
- We know the playground has 2 sections. We know there are 42 people on the playground. We know there are 8 adults in each section.
- We don't know how many children are in each section.

Let's draw a tape diagram and write an equation to help us solve and determine how many children are on each section of the playground.



I know there are 42 people on the playground, so I'll draw a tape diagram with a total of 42. *(draw and label tape diagram as you narrate)* I'll split the large rectangle in half to show the two sections of the playground. I know each section has 8 adults and an unknown amount of children. I'll use the expression  $8 + c$  to represent that inside each smaller rectangle.

Based on the verbal description and my tape diagram, what equation could we write to represent this problem? How do you know? **Possible Student Answers, Key Points:**

- In the tape diagram, I see two equal groups of  $8 + c$  have a total of 42.
- I can write  $2(8 + c) = 42$ .

$$42 = 2(8 + c)$$

$$42 = 16 + 2c$$

$$\begin{array}{r} -16 \quad -16 \\ \hline 26 = 2c \end{array}$$

$$\frac{26}{2} = \frac{2c}{2}$$

$$13 = c$$

*(write  $42 = 2(8 + c)$ )* We know we can solve this type of equation by distributing first or by dividing by 2 first. When I look at these numbers, I think either strategy can be efficient. Let's distribute the 2 first. *(draw arrows showing the distribution of the 2)* What is 2 times 8? (16) What is 2 times  $c$ ? ( $2c$ )

*(continue solving as you narrate)* We can rewrite the equation as  $42 = 16 + 2c$ . To isolate the variable, let's subtract 16 from both sides of the equation. 42 minus 16 leaves us with 26 on the left of our equation, and we have  $2c$  on the right of our equation. If I divide both sides by 2, we are left with our solution. I know  $13 = c$ . What does 13 mean in this context? **Possible Student Answers, Key Points:**

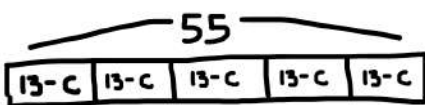
- We were trying to find how many children were in each section of the playground. There are 13 children in each section.

We just made a tape diagram then wrote and solved an equation to represent this verbal description. Nice work! To check my answer, I can pause to think if it seems reasonable in context. 13 kids in each section sounds reasonable based on the numbers in the story. It's not 1,000 kids. It's not -7 kids. It's not  $2\frac{1}{2}$  kids. It seems sensical. We could also substitute our value back into the original equation to check. *(substitute 13 in for  $c$  in the original equation)* I know  $8 + 13$  is 21, and  $2 \cdot 21$  would be 42. Our solution checks out. Nice work! Let's do one more.

$$42 = 2(8 + 13)$$

**Let's Think (Slide 5):** Like before, I'll read our first problem aloud as you follow along. When I'm done reading, summarize the story in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This problem is about a girl putting 55 cupcakes onto identical platters.
- We know there are 55 cupcakes. We know she has 5 platters. We know there were 13 cupcakes on each platter, but she took some off.
- We don't know how many cupcakes she took off each platter.



Let's model this problem using a tape diagram. How could you represent the total cupcakes and the amounts on each platter in a tape diagram? *(sketch as student shares, supporting as needed)* **Possible Student Answers, Key Points:**

- I'd draw a total of 55 along one large rectangle.
- I'd cut the rectangle into 5 equal pieces to show the platters.
- I'd write  $13 - c$  in each small section to show the 13 cupcakes with some removed.

This tape diagram makes sense. I see 55 total cupcakes split into 5 equal groups or platters. Each platter shows that it had 13 cupcakes, but some were taken off. Great work.

Now we can write an equation. (*write equation*) I can write  $55 = 5(13 - c)$  because it shows the total is 55 and that the total is equal to 5 groups of  $13 - c$ . It matches the tape diagram and the story. In our last example, we solved by distributing. In this example, I might choose to divide by 5 first, just because I noticed when I distribute I would have to multiply 5 by 13. I can do that, but dividing by 5 feels easier given that I know 55 divided by 5 with no problem. (*divide both sides by 5*) What's our new equation? ( $11 = 13 - c$ )

$$\frac{55}{5} = \frac{5(13 - c)}{5}$$

$$\frac{11}{-13} = \frac{13 - c}{-13}$$

$$\frac{-2}{-1} = \frac{-1c}{-1}$$

$$\boxed{2 = c}$$

(*rewrite equation*) We're left with  $11 = 13 - c$ . To balance the equation and isolate the variable, I'll remove 13 from both sides. (*subtract 13*) That leaves us with  $-2 = -c$ , or  $-2 = -1c$ . I'm not quite finished, because the variable is not totally by itself. Let's divide both sides by  $-1$ . What is  $-2$  divided by  $-1$ ? ( $+2$ )

Our solution is  $2 = c$  or  $c = 2$ . We did it!

How could I check to make sure our answer is correct? [Possible Student Answers, Key Points:](#)

- You can think if it feels reasonable in context. In this case, removing 2 cupcakes from each platter feels reasonable based on the story.
- I can substitute 2 in for the variable in the problem. I can think about  $55 = 5(13 - 2)$ .

We just used a tape diagram and an equation to help think about two different verbal descriptions and solve for an unknown. We're getting really good at this.

**Let's Try it (Slides 6 - 7):** Now let's practice! Just like we did in our last two examples, we'll pause to consider what is known and unknown in each verbal description. This will help us draw an accurate tape diagram, which we know makes it easy to write a corresponding equation. Once we're done solving any equation, we can always check our work to consider whether our answer is reasonable and/or by substituting our solution back in for the unknown in the original equation. Let's collaborate on a few more, and then you can practice some on your own.

# WARM WELCOME



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**Today we will use tape diagrams to represent verbal descriptions for situations with an equation of the form  $px + q = r$  or  $p(x + q) = r$ , and solve the resulting equation to determine an unknown quantity in the situation.**

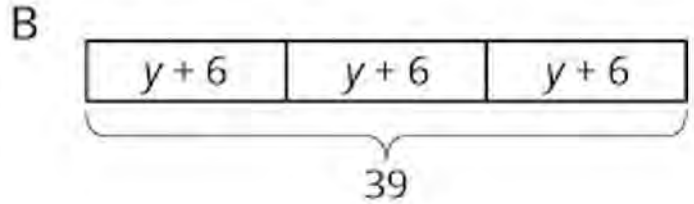
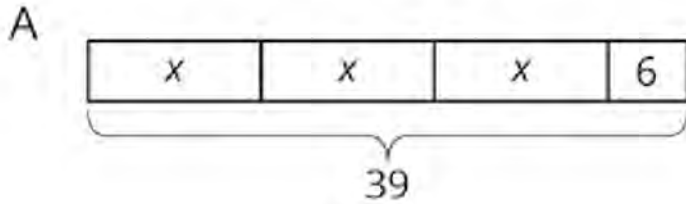
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## Let's Talk:

**What's the same? What's different?**



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## Let's Think:

**A playground has 2 sections. Each section has 8 adults, and an unknown number of children,  $c$ . If there are 42 total people at the playground, how many children are in each section?**

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## Let's Think:

Joy put 13 cupcakes on each of 5 platters. The platters looked too full, so she took the same number of cupcakes off each platter. After doing that, the platters had a total of 55 cupcakes. How many cupcakes did Joy take from each platter?

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## Let's Try It:

Let's explore using tape diagrams to represent verbal descriptions for situations with an equation of the form  $px + q = r$  or  $p(x + q) = r$  together.

Name: \_\_\_\_\_ G7.U5 Lesson 10 - Let's Try It

A P.E. teacher is filling bags to organize his equipment. He puts 7 soccer balls into each bag and  $f$  footballs into each bag. After filling 4 bags, he has organized a total of 60 items.

- What is the teacher doing?
  - Making bags that combine soccer balls and footballs.
  - Making separate bags of soccer balls and footballs.
- Did the teacher put the same number of footballs in each bag? \_\_\_\_\_
- How many items did the teacher organize after filling the 4 bags?
  - 60 items
  - An unknown number of items
- Circle the tape diagram that best matches the story.
 

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- Write an equation to represent the tape diagram.
- Solve the equation.

Dominique is at a party that has 5 plates of cookies. Each plate has the same amount on it. Dominique takes 1 cookie from each plate. There are now 35 total cookies on the plates. How many cookies were on each plate to start?

- What is known? What is unknown?

- Label the tape diagram below to represent the problem.
- Write an equation to represent the tape diagram. Solve the equation.
- What does your solution mean in the context of this problem?

Gerald buys 6 containers of laundry detergent. He has a coupon for \$0.50 off each container. If Gerald paid a total of \$21, what was the original cost of each container of laundry detergent?

- Draw a tape diagram to represent the problem. Use  $x$  to represent the original cost of each container.
- Write an equation. Solve.
- What does your solution mean in the context of this problem?

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# On your Own:

Now it's time to use tape diagrams to represent verbal descriptions for situations with an equation of the form  $px + q = r$  or  $p(x + q) = r$  on your own.

Name: \_\_\_\_\_ G7 US Lesson 10 – Independent Work

1. Kavan put the same number of flowers in 3 vases. He then adds 8 flowers to each vase. Kavan used a total of 67 flowers. How many flowers did Kavan put in each vase to begin with?

TAPE DIAGRAM \_\_\_\_\_ EQUATION \_\_\_\_\_

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2. A teacher has 66 students that she splits evenly onto 4 teams. On each team there are 5 girls and an unknown number of boys,  $x$ . How many boys are on each team?

TAPE DIAGRAM \_\_\_\_\_ EQUATION \_\_\_\_\_

3. Mr. Wallace has 3 identical boxes of chocolates. He removes 2 chocolates from each box, leaving a total of 66 chocolates remaining. How many chocolates were in each box to start?

a. Show or explain your work using a tape diagram and an equation.

\_\_\_\_\_

b. Explain what your answer means in the context of the problem.

\_\_\_\_\_

4. Write a story problem that could be represented using the tape diagram here. Then solve the problem.

$y + 3$	$y + 6$	$y + 6$
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36

\_\_\_\_\_

\_\_\_\_\_

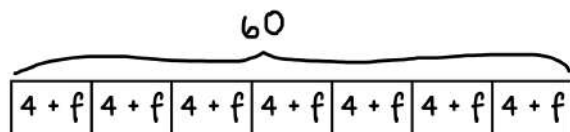
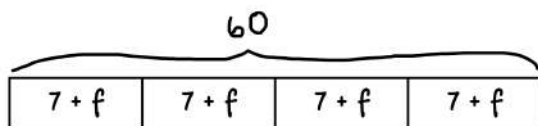
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**A P.E. teacher is filling bags to organize his equipment. He puts 7 soccer balls into each bag and  $f$  footballs into each bag. After filling 4 bags, he has organized a total of 60 items.**

1. What is the teacher doing?
  - a. Making bags that combine soccer balls and footballs.
  - b. Making separate bags of soccer balls and footballs.
2. Did the teacher put the same number of footballs in each bag? \_\_\_\_\_
3. How many items did the teacher organize after filling the 4 bags?
  - a. 60 items
  - b. An unknown number of items
4. Circle the tape diagram that best matches the story.



5. Write an equation to represent the tape diagram.
6. Solve the equation.

**Dominique is at a party that has 5 plates of cookies. Each plate has the same amount on it. Dominique takes 1 cookie from each plate. There are now 35 total cookies on the plates. How many cookies were on each plate to start?**

7. What is known? What is unknown?

8. Label the tape diagram below to represent the problem.



9. Write an equation to represent the tape diagram. Solve the equation.

10. What does your solution mean in the context of this problem?

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**Gerald buys 6 containers of laundry detergent. He has a coupon for \$0.50 off each container. If Gerald paid a total of \$21, what was the original cost of each container of laundry detergent?**

11. Draw a tape diagram to represent the problem. Use  $c$  to represent the original cost of each container.

12. Write an equation. Solve.

13. What does your solution mean in the context of this problem?

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- 1. Kevon put the same number of flowers in 3 vases. He then adds 8 flowers to each vase. Kevon used a total of 57 flowers. How many flowers did Kevon put in each vase to begin with?**

TAPE DIAGRAM

EQUATION

- 2. A teacher has 68 students that she splits evenly onto 4 teams. On each team there are 5 girls and an unknown number of boys,  $x$ . How many boys are on each team?**

TAPE DIAGRAM

EQUATION



3. Mr. Wallace has 3 identical boxes of chocolates. He removes 2 chocolates from each box, leaving a total of 66 chocolates remaining. How many chocolates were in each box to start?

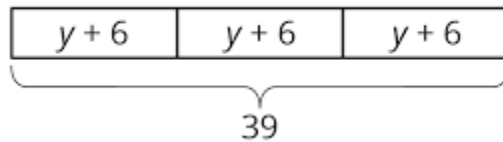
a. Show or explain your work using a tape diagram and an equation.

b. Explain what your answer means in the context of the problem.

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4. Write a story problem that could be represented using the tape diagram here. Then solve the problem.



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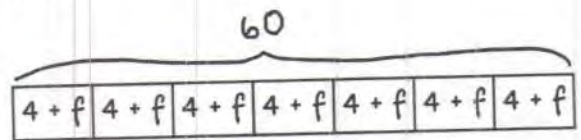
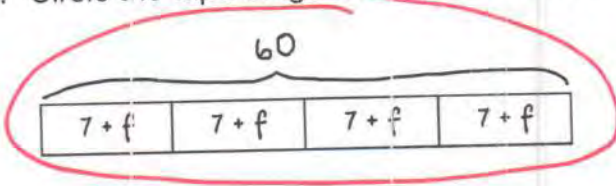
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A P.E. teacher is filling bags to organize his equipment. He puts 7 soccer balls into each bag and  $f$  footballs into each bag. After filling 4 bags, he has organized a total of 60 items.

- What is the teacher doing?
  - Making bags that combine soccer balls and footballs.
  - Making separate bags of soccer balls and footballs.
- Did the teacher put the same number of footballs in each bag? YES
- How many items did the teacher organize after filling the 4 bags?
  - 60 items
  - An unknown number of items
- Circle the tape diagram that best matches the story.



- Write an equation to represent the tape diagram.

$$60 = 4(7+f)$$

- Solve the equation.

$$\begin{aligned} \frac{60}{4} &= \frac{4(7+f)}{4} \\ 15 &= 7+f \\ -7 &-7 \\ \hline 8 &= f \end{aligned}$$

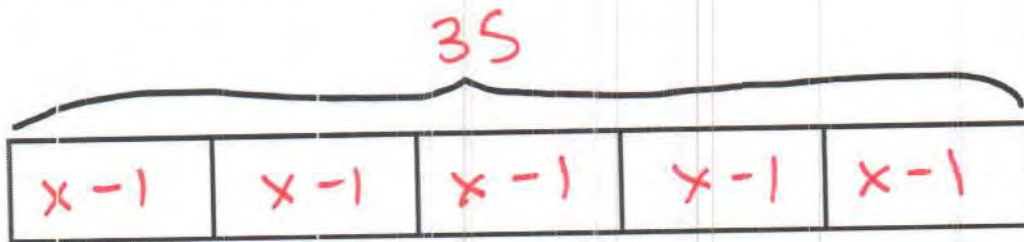
Dominique is at a party that has 5 plates of cookies. Each plate has the same amount on it. Dominique takes 1 cookie from each plate. There are now 35 total cookies on the plates. How many cookies were on each plate to start?

- What is known? What is unknown?

• 5 equal plates  
 • takes 1 from each  
 • 35 cookies left in all

How many cookies started out on each plate?

8. Label the tape diagram below to represent the problem.



9. Write an equation to represent the tape diagram. Solve the equation.

$$\frac{5(x-1)}{5} = \frac{35}{5}$$

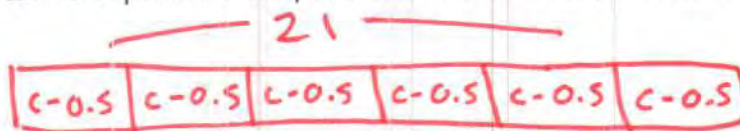
$$\begin{array}{r} x-1 = 7 \\ +1 \quad +1 \\ \hline x = 8 \end{array}$$

10. What does your solution mean in the context of this problem?

There were 8 cookies on each plate to start.

Gerald buys 6 containers of laundry detergent. He has a coupon for \$0.50 off each container. If Gerald paid a total of \$21, what was the original cost of each container of laundry detergent?

11. Draw a tape diagram to represent the problem. Use  $c$  to represent the original cost of each container.



12. Write an equation. Solve.

$$6(c-0.5) = 21$$

$$6c - 3 = 21$$

$$+3 \quad +3$$

$$\frac{6c}{6} = \frac{24}{6}$$

$$c = 4$$

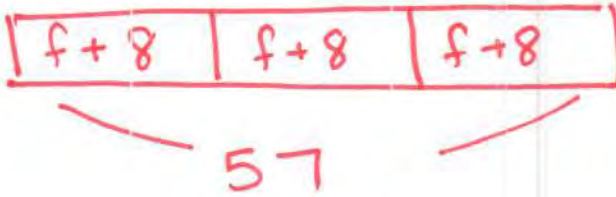
13. What does your solution mean in the context of this problem?

Each container costs \$4 without a coupon.



1. Kevon put the same number of flowers in 3 vases. He then adds 8 flowers to each vase. Kevon used a total of 57 flowers. How many flowers did Kevon put in each vase to begin with?

TAPE DIAGRAM



EQUATION

$$3(f+8) = 57$$

$$3f + 24 = 57$$

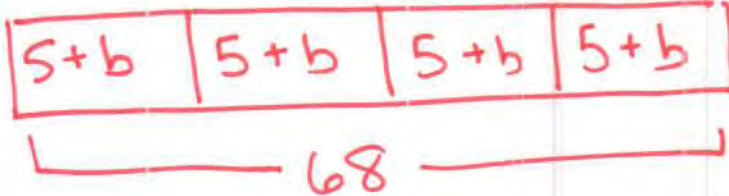
$$\begin{array}{r} -24 \quad -24 \\ \hline \end{array}$$

$$\frac{3f}{3} = \frac{33}{3}$$

$$f = 11$$

2. A teacher has 68 students that she splits evenly onto 4 teams. On each team there are 5 girls and an unknown number of boys, x. How many boys are on each team?

TAPE DIAGRAM



EQUATION

$$4(5+b) = 68$$

$$20 + 4b = 68$$

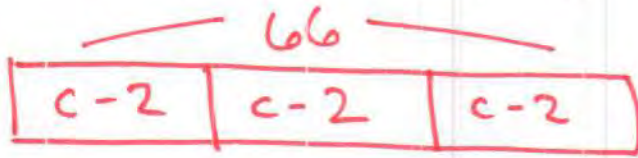
$$\begin{array}{r} -20 \quad -20 \\ \hline \end{array}$$

$$\frac{4b}{4} = \frac{48}{4}$$

$$b = 12$$

3. Mr. Wallace has 3 identical boxes of chocolates. He removes 2 chocolates from each box, leaving a total of 66 chocolates remaining. How many chocolates were in each box to start?

a. Show or explain your work using a tape diagram and an equation.

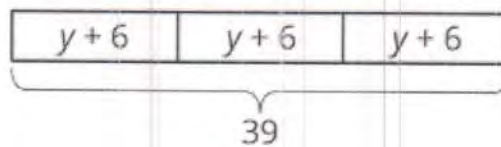


$$\begin{array}{r} 3(c-2) = 66 \\ \hline 3 \qquad 3 \\ c-2 = 22 \\ +2 \quad +2 \\ \hline c = 24 \end{array}$$

b. Explain what your answer means in the context of the problem.

Each box had 24 pieces to start with.

4. Write a story problem that could be represented using the tape diagram here. Then solve the problem.



$$\begin{array}{r} 3(y+6) = 39 \\ \hline 3 \qquad 3 \\ y+6 = 13 \\ -6 \quad -6 \\ \hline y = 7 \end{array}$$

There are 3 identical baskets of fruit. Each basket has  $y$  oranges and 6 pears. There are 39 total pieces of fruit. How many oranges are in each basket?

$$y = 7$$

# **G7 U5 Lesson 11**

Solve word problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.



**G7 U5 Lesson 11 - Students will solve word problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.**

*NOTE: Students are welcome to use calculators for some of the computation on this day.*

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We're reaching the end of the portion of this unit dedicated to equations. For our last lesson of this portion of the unit, we're going to look specifically at problems that deal with percent increase and percent decrease. We've worked with percent increase and decrease in a previous unit; today we're going to connect the percent thinking we've used before to the work we're currently considering around equations.

**Let's Talk (Slide 3):** Before we fully jump into the day's learning, let's consider a fairly common context involving percent change: grocery store prices. You've likely noticed that prices at the grocery store fluctuate. Sometimes they go up, sometimes they go down, sometimes things are on sale...they're constantly changing. Take a look at these two situations. What do you notice about them? What do you wonder? **Possible Student Answers, Key Points:**

- I notice the cost of bananas is going up and the cost of a loaf of bread is going down. I notice both problems have percents in them. I notice there is an equation under each one that uses a decimal.
- I wonder what the solutions are. I wonder why the equations use those decimals, since they're not the decimal equivalents of the percentages named in each problem.

$$\begin{array}{l} 100\% + 25\% = 125\% \\ \text{original cost} \quad \text{increase} \end{array}$$

We can use equations to find price mark-ups and mark-downs involving percents. If I know the cost of bananas is increasing 25%, that means I'll be paying the original cost of the bananas plus an extra 25% on top of that. I can think of that like  $100\% + 25\% = 125\%$ . The new cost is 125% of the original cost. (*write  $100\% + 25\% = 125\%$  as shown*) As an equation, I can write that as  $1.25(2) = c$ , because 125% in decimal form is 1.25. What is  $1.25 \cdot 2$ ? (**2.5**) Our solution is  $c = 2.5$ . That means the new cost of bananas is \$2.50.

Look at the other situation where the loaf of bread decreases in price by 25%. Why do you think the equation below can be used to represent a price *decreasing* 25%? **Possible Student Answers, Key Points:**

- If the price is decreasing 25%, that means it's going down 25% from the original 100%. 100% minus 25% is 75%. So, I'm paying 75% of the original cost.
- 75% written as a decimal is 0.75, so I can multiply  $2 \times 0.75$  to find 75% of the original cost.

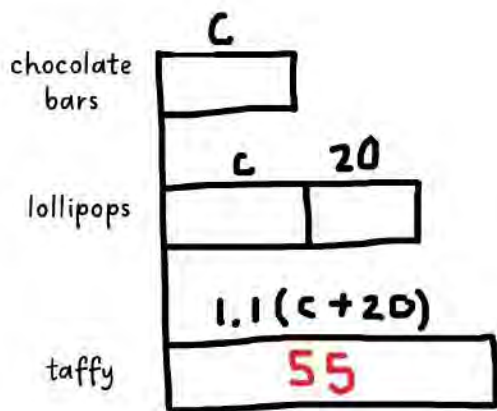
$$\begin{array}{l} 100\% - 25\% = 75\% \\ \text{original cost} \quad \text{decrease} \end{array}$$

Great thinking. If the price decreases by 25%, that means I was paying 100% originally but now I'm paying 25% less. (*write  $100\% - 25\% = 75\%$  as shown*) We can use  $0.75(2) = c$  to represent this as an equation, because 0.75 is the decimal equivalent of 75%. What is  $0.75 \cdot 2$ ? (**1.5**) If  $c = 1.5$ , that means the new cost of a loaf of bread is \$1.50.

Today, we'll use equations like these to consider problems involving percent increase and percent decrease.

**Let's Think (Slide 4):** I'll read the problem once through aloud as you follow along. When I'm done, summarize what the story is about. What is known and unknown? **Possible Student Answers, Key Points:**

- This problem is about a candy store that sells chocolate bars, lollipops, and taffy.
- We know they sell 55 pieces of taffy. We know they sell 10% more taffy than lollipops. We know they sell 20 more lollipops than chocolate bars.
- The unknown is the amount of chocolate bars sold,  $c$ .



We can use a tape diagram to help think about this story. I'll draw a vertical line and leave room for three rectangles to represent each of the types of candy, chocolate bars, lollipops, and taffy. *(sketch and tape diagram as you narrate)* We don't know the amount of chocolate bars sold, so I'll draw a rectangle labeled  $c$ .

I know they sell 20 more lollipops than chocolate bars. To show that, I'll draw another bar that is  $c$  long and attach another bar that represents the 20 more. I can think of the quantity of lollipops as being  $c + 20$ .

Lastly, I need to think about the taffy. I know they sell 10% more taffy than lollipops, so I'll draw the bar for taffy a little longer than the bar for lollipops. I also know the amount of taffy pieces sold

is 55, so I can label that inside the tape diagram. Let's think about this taffy quantity a little more. If I know the taffy is 10% more than the lollipop amount, how can I think of the 10% increase so that I can use it in the form of an equation? **Possible Student Answers, Key Points:**

- We can think of 100% of the lollipops plus 10% more.  $100\% + 10\% = 110\%$ .
- The amount of taffy is 110% the amount of lollipops. 110% as a decimal is 1.10 or 1.1.

The taffy is 110% of the lollipop amount. If the lollipop amount is  $c + 20$ , I can use the expression  $1.1(c + 20)$  to represent 110% of the lollipop amount. *(label the taffy rectangle as  $1.1(c + 20)$ )*

Now that we've thought through this multi-part tape diagram, we can work to solve. I know that  $1.1(c + 20)$  can be used to represent the amount of taffy. I also know that the exact amount of taffy is 55 pieces. I can set those equal to each other to set up an equation. *(write  $1.1(c + 20) = 55$ )* I know we can solve this equation type by distributing or by dividing by the coefficient. Let's divide both sides of the equation by 1.1, because I know dividing 55 by 11 tenths will be manageable. *(show division of 11 in equation)* What is 55 divided by 1.1? **(50)** *(rewrite equation)* The equation now reads  $c + 20 = 50$ . What can I do to isolate the variable? **(subtract 20 from both sides of the equation)** If I subtract 20 from both sides of the equation, I end up with a solution that reads  $c = 30$ . What does that mean in the context of this problem?

$$\begin{array}{r} 1.1(c + 20) = 55 \\ \hline 1.1 \quad 1.1 \\ c + 20 = 50 \\ \hline -20 \quad -20 \\ \hline c = 30 \end{array}$$

**Possible Student Answers, Key Points:**

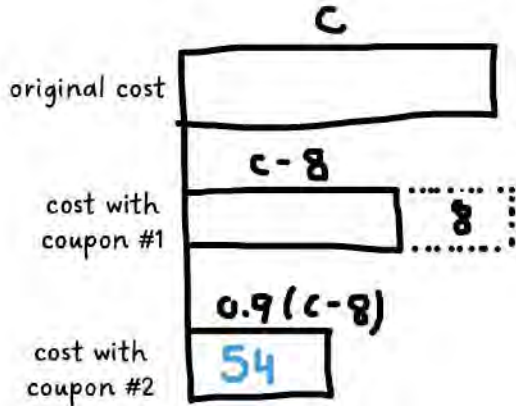
- We were trying to find the amount of chocolate bars the story sold. The solution  $c = 30$  means that the store sold 30 chocolate bars.

We carefully built out a multi-part tape diagram to visualize what was happening in the story. We wrote an equation, by thinking about the percent change as a decimal value, and then we solved to find our unknown. This question is a testament to all the work we've put in to understand solving equations and using percents.

**Let's Think (Slide 5):** Let's try one more just to make sure we're confident and ready to work more independently. I'll read the problem once through aloud as you follow along. When I'm done, summarize what the story is about. What is known and unknown? **Possible Student Answers, Key Points:**

- This problem is about Aaron and how he's trying to use two coupons to get a better deal on a pair of shoes.
- I know one coupon takes \$8 off and the other takes 10% off. I know the shoes end up costing \$54.
- I don't know the original cost of the shoes.

Let's start by drawing a tape diagram to visualize each part of this problem. I'll draw a vertical line with room to think about the original cost of the shoe, the cost after the first coupon, and the cost after the second coupon.



(sketch and label tape diagram as you narrate) If we're thinking about the cost of shoes *before* a sale, I know the cost will be more. I'll start by drawing a long rectangle labeled  $c$ . This represents the original cost of the shoes. The first coupon is for \$8 off of the price. I can show that by drawing a bar that is 8 dollars less than the original cost. I'll label this rectangle as  $c - 8$ , because it is the cost of the shoe minus 8 dollars.

The last coupon takes 10% off the discounted price. How can I think about a 10% decrease so that I can write an equation using a decimal? **Possible Student Answers, Key Points:**

- 10% off means 10% less than 100%.  $100\% - 10\% = 90\%$ . Aaron will pay 90% of the discounted price.
- I can write 90% as 0.90 or 0.9 in decimal form.

We can think of the final price of the shoes as being 90% of the discounted price. I'll use the expression  $0.9(c - 8)$  to represent the last rectangle on my tape diagram, because that's the same as saying 90% of  $c - 8$ . (label tape diagram) The problem also told me that the final cost was \$54, so I'll label that in the rectangle for the final cost of the shoes.

We now have all the information we need to solve. (write  $0.9(c - 8) = 54$ ) I know that 90% of the discounted price is equal to \$54. I can use the equation here to represent that.

$$\begin{array}{r} 0.9(c - 8) = 54 \\ \hline 0.9 \qquad 0.9 \\ c - 8 = 60 \\ \hline +8 \quad +8 \\ \hline c = 68 \end{array}$$

Like last time, I'm going to choose to divide by 0.9 instead of distribute. I chose that because I know I can divide 54 by 9 tenths without it getting too messy or complicated. (divide both sides by 0.9) We're left with  $c - 8 = 60$ . What would you do last to solve, and what would your solution mean in the context of the problem? **Possible Student Answers, Key Points:**

- I would add 8 to both sides of the equation to isolate the variable. My answer is  $c = 68$ .
- A solution showing  $c = 68$  means that the original cost of the shoes before the coupons was \$68.

We can use tape diagrams and equations to help us solve problems involving percent increase and percent decrease.

**Let's Try it (Slides 6 - 7):** Let's practice a few more together, and then you'll have time to work through some independently. For most problems, it will be useful to sketch a tape diagram first so we can visualize each part of the word problem. When we write equations, we'll want to think about how best to use decimals to represent the percent increase or decrease in the problem. Time to jump in!


# WARM WELCOME



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**Today we will solve word problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.**

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 **Let's Talk:**


**Bananas cost \$2. They increase in price by 25%.**

$$1.25(2) = ?$$

**A loaf of bread costs \$2. It decreases in price by 25%.**

$$0.75(2) = ?$$

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 **Let's Think:**

**A candy store sells  $c$  chocolate bars. The store sells 20 more lollipops than chocolate bars. The store sells 10% more pieces of taffy than lollipops. The store sells 55 pieces of taffy. How many chocolate bars does the candy store sell?**

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## Let's Think:

Aaron has two coupons for a pair of shoes he wants to buy that originally cost  $x$  dollars. He first applies a coupon that takes \$8 off the price of the shoes. Then, he applies a coupon that reduces that price by 10%. Aaron ends up paying \$54 for the shoes. How much did the shoes originally cost?

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## Let's Try It:

Let's explore solving word problems about percent increase or decrease together.

Name: \_\_\_\_\_ 07 US Lesson 11 : Let's Try It

Eve baked some brownies on Friday. On Saturday, she baked 10 more brownies than on Friday. On Sunday, she increased the number of brownies she baked by 20%, baking 48 brownies.

- What is Eve doing in this story?
- Do we know how many brownies she baked on Friday? \_\_\_\_\_
- The tape diagram below shows that Eve baked  $n$  brownies on Friday. Draw and label a tape diagram to represent the amount of brownies she baked on Saturday.

Friday	$n$
Saturday	
Sunday	

- Draw and label a tape diagram to show how on Sunday Eve baked 20% more than she did on Saturday.
- We know 20% more than the Saturday amount is equal to 48.
 
$$\begin{aligned} &20\% \text{ more than Saturday} = 48 \\ &20\% \text{ more than } \underline{\hspace{2cm}} = 48 \\ &\underline{\hspace{2cm}} ( \underline{\hspace{1cm}} + \underline{\hspace{1cm}} ) = 48 \end{aligned}$$
- Solve for  $n$  by either dividing first or distributing first.
- Eve baked \_\_\_\_\_ cupcakes on Friday.

There are  $x$  people in the theater club. There are 10 less people in the French club than in the theater club, and there are 50% more people in the robotics club than in the French club. There are 36 people in the robotics club.

- What is known in this situation? What is unknown?
- Draw a tape diagram to represent the people in the theater club. Then draw another tape diagram to represent the people in the French club. Then draw another tape diagram to represent the people in the robotics club.

theater	
French	
robotics	

- Write an equation to represent the number of students in the robotics club.
 
$$\begin{array}{c} \text{theater} \\ \text{French} \\ \text{robotics} \end{array} \quad \left( \begin{array}{c} \text{theater} \\ \text{French} \end{array} \right) \text{ is } \begin{array}{c} \text{robotics} \\ \text{French} \end{array}$$
- Solve for  $x$ .
- How many people are in the theater club?

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## On your Own:

Now it's time to solve word problems about percent increase or decrease on your own.

Name: \_\_\_\_\_ G7 L&S Lesson 11 – Independent Work

1. The number of students on-time to school increased 25% from last week to this week. The number of students on-time to school this week is 120. Draw a tape diagram and write an equation to help determine how many students were on-time to school last week.

2. A clothing store is having a sale where all jeans are discounted by 25%. Kieran has a coupon for \$10 off the regular price of jeans. The store first applies the coupon and then takes 25% off the reduced price. If Kieran ends up paying \$48 for the jeans, what was the original cost of the jeans?

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3. An animal shelter takes care of 12 more cats than bunnies. There are 10% fewer dogs than cats at the shelter. There are 81 dogs in the shelter. How many bunnies are in the shelter?

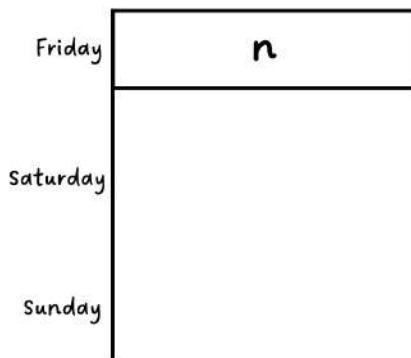
4. There are  $x$  students enrolled in Spanish class. There are 6 fewer students enrolled in French class than in Spanish class. There are 30% more students in Chinese class than in French class. There are 40 students enrolled in Chinese class. How many students are enrolled in Spanish?

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**Eve baked some brownies on Friday. On Saturday, she baked 10 more brownies than on Friday. On Sunday, she increased the number of brownies she baked by 20%, baking 48 brownies.**

1. What is Eve doing in this story?
2. Do we know how many brownies she baked on Friday? \_\_\_\_\_
3. The tape diagram below shows that Eve baked  $n$  brownies on Friday. Draw and label a tape diagram to represent the amount of brownies she baked on Saturday.



4. Draw and label a tape diagram to show how on Sunday Eve baked 20% more than she did on Saturday.
5. We know 20% more than the Saturday amount is equal to 48.

$$20\% \text{ more than Saturday} = 48$$

$$20\% \text{ more than } \underline{\hspace{2cm}} = 48$$

$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}}) = 48$$

6. Solve for  $n$  by either dividing first or distributing first.

7. Eve baked \_\_\_\_\_ cupcakes on Friday.

There are  $x$  people in the theater club. There are 10 less people in the French club than in the theater club, and there are 50% more people in the robotics club than in the French club. There are 36 people in the robotics club.

8. What is known in this situation? What is unknown?

9. Draw a tape diagram to represent the people in the theater club. Then draw another tape diagram to represent the people in the French club. Then draw another tape diagram to represent the people in the robotics club.



10. Write an equation to represent the number of students in the robotics club.

$$\text{50\% more} \left( \text{\# of people in French club} - \text{\# of people in robotics club} \right) = \text{\# of people in robotics club}$$

11. Solve for  $x$ .

12. How many people are in the theater club?

**1. The number of students on-time to school increased 25% from last week to this week. The number of students on-time to school this week is 120. Draw a tape diagram and write an equation to help determine how many students were on-time to school last week.**

**2. A clothing store is having a sale where all jeans are discounted by 25%. Kieran has a coupon for \$10 off the regular price of jeans. The store first applies the coupon and then takes 25% off the reduced price. If Kieran ends up paying \$48 for the jeans, what was the original cost of the jeans?**

**3. An animal shelter takes care of 12 more cats than bunnies. There are 10% fewer dogs than cats at the shelter. There are 81 dogs in the shelter. How many bunnies are in the shelter?**

**4. There are  $x$  students enrolled in Spanish class. There are 8 fewer students enrolled in French class than in Spanish class. There are 50% more students in Chinese class than in French class. There are 42 students enrolled in Chinese class. How many students are enrolled in Spanish?**

Eve baked some brownies on Friday. On Saturday, she baked 10 more brownies than on Friday. On Sunday, she increased the number of brownies she baked by 20%, baking 48 brownies.

1. What is Eve doing in this story?

She is baking brownies.

2. Do we know how many brownies she baked on Friday? NO

3. The tape diagram below shows that Eve baked  $n$  brownies on Friday. Draw and label a tape diagram to represent the amount of brownies she baked on Saturday.



4. Draw and label a tape diagram to show how on Sunday Eve baked 20% more than she did on Saturday.

5. We know 20% more than the Saturday amount is equal to 48.

$$20\% \text{ more than Saturday} = 48$$

$$20\% \text{ more than } \underline{n+10} = 48$$

$$\underline{1.2} (\underline{n} + \underline{10}) = 48$$

6. Solve for  $n$  by either dividing first or distributing first.

$$\frac{1.2(n+10)}{1.2} = \frac{48}{1.2}$$

$$n+10 = 40$$

$$n = 30$$

7. Eve baked (30) cupcakes on Friday.

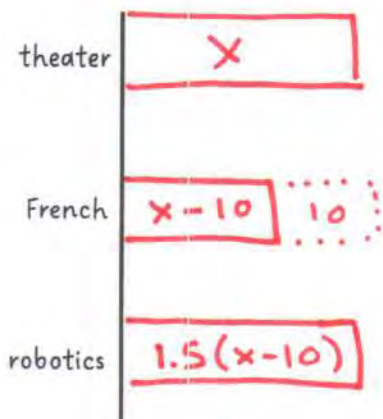


There are  $x$  people in the theater club. There are 10 less people in the French club than in the theater club, and there are 50% more people in the robotics club than in the French club. There are 36 people in the robotics club.

8. What is known in this situation? What is unknown?

- 10 less in French than theater  $\rightarrow$  # in theater club
- 50% more in robotics than French
- 36 people in robotics

9. Draw a tape diagram to represent the people in the theater club. Then draw another tape diagram to represent the people in the French club. Then draw another tape diagram to represent the people in the robotics club.



10. Write an equation to represent the number of students in the robotics club.

$$\underbrace{1.5}_{\text{50\% more}} \left( \underbrace{x - 10}_{\text{\# of people in French club}} \right) = \underbrace{36}_{\text{\# of people in robotics club}}$$

11. Solve for  $x$ .

$$\frac{1.5(x - 10)}{1.5} = \frac{36}{1.5}$$

$$x - 10 = 24$$

$$\begin{array}{r} x - 10 = 24 \\ +10 \quad +10 \\ \hline \end{array}$$

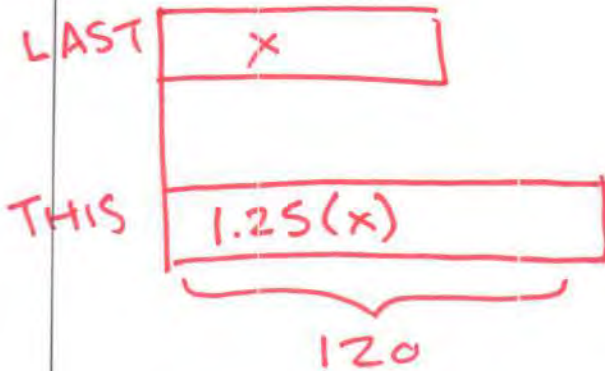
$$x = 34$$

$$\begin{array}{r} 24 \\ 15 \overline{) 360} \\ \underline{30} \phantom{0} \\ 60 \\ \underline{-60} \\ 0 \end{array}$$

12. How many people are in the theater club?

(34)

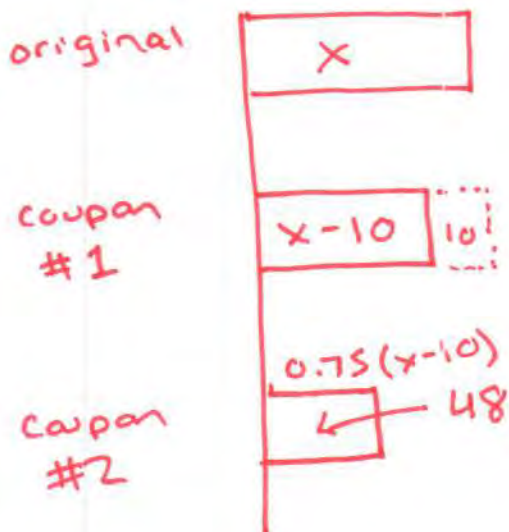
1. The number of students on-time to school increased 25% from last week to this week. The number of students on-time to school this week is 120. Draw a tape diagram and write an equation to help determine how many students were on-time to school last week.



$$\frac{1.25x}{1.25} = \frac{120}{1.25}$$

$$x = 96$$

2. A clothing store is having a sale where all jeans are discounted by 25%. Kieran has a coupon for \$10 off the regular price of jeans. The store first applies the coupon and then takes 25% off the reduced price. If Kieran ends up paying \$48 for the jeans, what was the original cost of the jeans?



$$\frac{0.75(x - 10)}{0.75} = \frac{48}{0.75}$$

$$x - 10 = 64$$

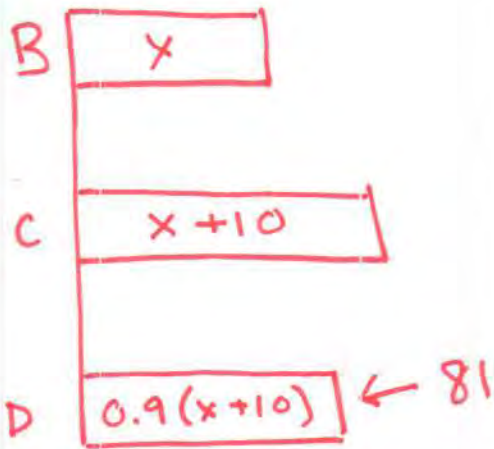
$$+10 \quad +10$$

$$x = 74$$

$$\text{\$74}$$



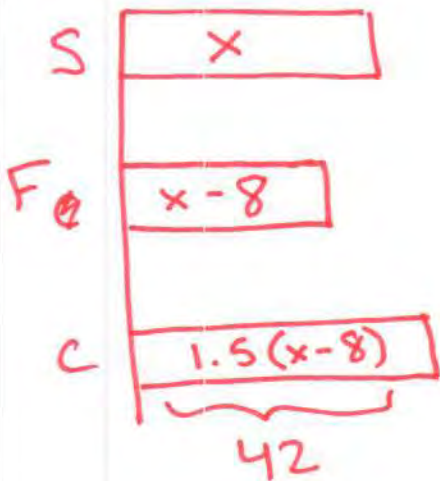
3. An animal shelter takes care of 12 more cats than bunnies. There are 10% fewer dogs than cats at the shelter. There are 81 dogs in the shelter. How many bunnies are in the shelter?



$$\frac{81}{0.9} = \frac{0.9(x+10)}{0.9}$$

$$\begin{array}{r} 90 = x + 10 \\ -10 \quad -10 \\ \hline 80 = x \end{array}$$

4. There are  $x$  students enrolled in Spanish class. There are 8 fewer students enrolled in French class than in Spanish class. There are 50% more students in Chinese class than in French class. There are 42 students enrolled in Chinese class. How many students are enrolled in Spanish?



$$1.5(x-8) = 42$$

$$\begin{array}{r} 1.5x - 12 = 42 \\ +12 \quad +12 \\ \hline \end{array}$$

$$\frac{1.5x}{1.5} = \frac{54}{1.5}$$

$$x = 36$$

## **G7 U5 Lesson 12**

Write inequality statements to represent inequality situations, and use substitution to determine whether a given value for a variable makes an inequality true.

**G7 U5 Lesson 12 - Students will write inequality statements to represent inequality situations, and use substitution to determine whether a given value for a variable makes an inequality true.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Since the beginning of this unit, we've been exploring various ways to think about and solve equations. Today, we get to use some of that thinking, but we're switching gears a bit. For the next several lessons, we'll be focusing on inequalities. An inequality is a math statement that, instead of using an equal sign, uses a less than symbol, a greater than symbol, a less than or equal to symbol, or a greater than or equal to symbol.

Let's start by thinking about what these symbols mean before we jump into today's problems.

**Let's Talk (Slide 3):** Here we have four different inequality statements. What do you notice about the four inequalities shown here? What do you wonder? **Possible Student Answers, Key Points:**

- I notice the red ones have  $x$  as the variable and the blue have  $y$  as the variable. I notice the red ones have the less than and greater than symbols. I notice the blue ones have the less than or equal to and the greater than or equal to symbols.
- I wonder which symbol is which, because they can be easy mix up. I wonder why we need a symbol that includes "equal to". I wonder if we can think about these the same as we think about equations.

The first red inequality reads as  $x$  is less than 5. That means,  $x$  can be any value less than 5. 4 could make the statement true. 0 could make the statement true. 2.99,  $\frac{1}{2}$ , -10 would all make the statement true. Do you think 5 would make the inequality true? **Possible Student Answers, Key Points:**

- No, 5 would not be a solution. 5 is not less than 5. It has the same value.

Look at the first blue inequality. It reads  $y$  is less than or equal to 10. Like the last example we saw, any value of  $y$  less than the number, in this case 10, would make the statement true. This symbol is special, because it also means that the solution could be equal to 10. So 10, and any number less than 10, are part of the solution.

What about the second red inequality? It reads  $x$  is greater than 5. What values of  $x$  would make the statement true? What values would not make the statement true? **Possible Student Answers, Key Points:**

- 6, 7, 8,  $9\frac{1}{2}$ , and 100 could all make the inequality true. As long as it's more than 5.
- 4, 3, 0.5, and -6 would all make the inequality false. It can't be a number less than 5.
- 5 would not be a solution, because 5 is not greater than 5.

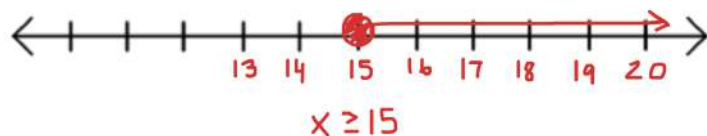
So then, let's consider the second blue inequality. It reads  $y$  is greater than or equal to 10. Would  $8\frac{1}{2}$  make the statement true? (No,  $8\frac{1}{2}$  is less than 10.) Would  $10\frac{1}{2}$  make the statement true? (Yes,  $10\frac{1}{2}$  is greater than 10.) Would 10 make the statement true? (Yes, because the inequality is true for any number greater than or equal to 10.)

It's important to pay attention to the symbols anytime we work with inequalities. If we see the  $\leq$  or the  $\geq$ , we know that the value the inequality is referring to can also be part of the solution.

Alright, I think it's time to look at some problems together.

**Let's Think (Slide 4):** This problem gives us two scenarios that can be represented with an inequality. They want us to write an inequality and model the solution on a number line. Let's start with Part A. What does it mean that Maryanne needs to get "at least" 15 math problems correct? (She needs 15 or more correct.) The phrase "at least" can be misleading, because it doesn't mean "less" like it sounds. It means the given number

or more. Let's think about that on a number line. (write 15 on the number line and label a few numbers above and below 15)



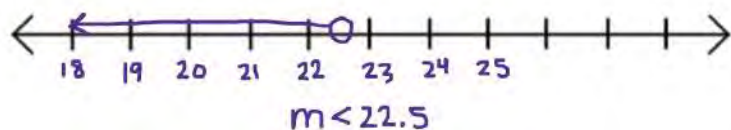
(circle, but don't shade a point on 15) I know Maryanne at least 15 questions correct, so is 15 part of the solution? (Yes.) She can get 15 or more. I'll shade in this point at 15 to note that 15 is included in our solution. (color in 15 point) So, I also know she could get 16 or 17 or 18 and still pass. She just can't get below 15. I'll model that

by drawing an arrow to the right to show that 15 and any number more than 15 could represent Maryanne's possible correct questions. (draw arrow pointing right)

I can represent this with an inequality by writing  $x \geq 15$ . If  $x$  is the number of questions Maryanne gets correct, I can think of the inequality as  $x$  is greater than or equal to 15.

Let's try the next one. What does it mean that Jacob has less than \$22.50 in his account? (He can have 22.49 or less.) He must have less than \$22.50. Notice in this case, \$22.50 would not make sense as a solution because \$22.50 is not less than \$22.50. Let's model this inequality.

(write 22 and 23 as tick marks and label a couple numbers above and below that) I'll mark \$22.50 between 22 and 23, since it's halfway between those values.



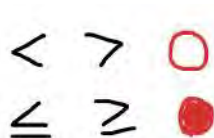
In this case, I'm not going to shade in the point at 22.5, because I don't want to include that in my answer. I'll leave that point open. I'll draw an arrow pointing left, since I know Jacob could have any value less than \$22.50. (draw arrow pointing left)

We can represent this as an inequality by writing  $m < 22.5$  or  $m < \$22.50$ . My inequality shows that Jacob's amount has to be less, but not equal to, \$22.50 to represent the scenario.

When modeling an inequality, it's important to think about whether values greater or less than the given number make the situation true. It's also important to consider whether or not the given number is part of the solution or not. How did we show on our model whether the given number was included in a situation?

Possible Student Answers, Key Points:

- We shaded the point in to include it.
- We left the point open when we did not want to include it.



(write < and > next to an open circle, and write ≤ and ≥ next to a closed circle) Any time we want to model a relationship involving values less than or greater than a given number, we use an open circle to show that the value is not part of the solution. If we do want to include the value, like when we use the ≤ or ≥ symbols, we color in the point to note that it is included in the solution.

**Let's Think (Slide 5):** For our last set of problems, we are given a table that shows an inequality. We are asked to test whether 25 or -25 make the statement true. To do this, we'll substitute values into the inequality and see if the math we do produces a true or false statement.

Let's start with  $100 \geq 4x$ . To test if 25 is a solution, let's substitute 25 in for  $x$ . (rewrite inequality with 25 in place of  $x$ )



$$100 \geq 4(25)$$

$$100 \geq 100 \quad \checkmark$$

I know  $4 \times 25$  is equal to 100. (*write  $100 \geq 100$* ) 100 is greater than or equal to 100, because it is equal to 100. (*put check next to inequality and write "true" in the table*) Since substituting 25 into the inequality resulted in a true statement, we know 25 is part of the solution set.

$$100 \geq 4(-25)$$

$$100 \geq -100 \quad \checkmark$$

What about -25? Substitute -25 in for  $x$  and see if -25 is part of the solution. Explain your work as you do it. (*write inequality and model work as shown, supporting as needed*) Possible Student Answers, Key Points:

- I can plug in -25 in place of  $x$ .
- I know  $4 \times -25$  is -100 because a positive times a negative results in a negative product.
- 100 is greater than or equal to -100, so -25 is a solution.

Since we ended up with an inequality that reads 100 is greater than or equal to -100, I know -25 is a solution to the inequality. (*write check next to the statement and write true in the table*) Both 25 and -25 made this inequality true.

$$25 - 30 \leq -10$$

$$-5 \leq -10 \quad \times$$

Let's do the same work with the other inequality,  $x$  minus 30 is less than or equal to -10. I'll substitute 25 in place of  $x$  and rewrite the inequality. (*rewrite inequality*) I know  $25 - 30$  is -5. (*rewrite inequality*) It now reads -5 is less than or equal to -10. That's false, -5 is greater than -10. That means 25 is not a solution to this inequality. (*write an X next to the statement and write false in the table*)

$$-25 - 30 \leq -10$$

$$-55 \leq -10 \quad \checkmark$$

You test out -25. Explain your thinking. (*write inequality and model work as shown, supporting as needed*) Possible Student Answers, Key Points:

- If substitute -25 in place of  $x$ . -25 minus 30 is -55.
- -55 is less than or equal to -10. That's true. -25 makes the inequality true.

Since -25 resulted in a true statement, -25 is a solution to the inequality. All we have to do to determine if a value makes an inequality true or false is substitute the value in place of the variable.

**Let's Try it (Slides 6 - 7):** Now it's time to practice together. As we consider inequalities, it's important that we think carefully about the situation or the symbol. We want to consider whether values that are greater than or less than the given number make a true inequality. It's also important that we think about whether the given number is included in the solution or excluded. It can be helpful to test values out by thinking about a number line and/or substituting values into a given inequality. I think you're ready to go!

# WARM WELCOME



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**Today we will write inequality statements to represent inequality situations, and use substitution to determine whether a given value for a variable makes an inequality true.**

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 **Let's Talk:**


$$x < 5$$

$$y \leq 10$$

$$x > 5$$

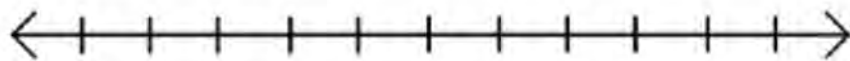
$$y \geq 10$$

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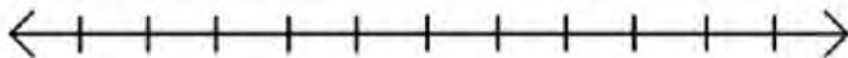
 **Let's Think:**

**Write an inequality to represent each situation.  
Model the inequality on a number line.**

- a. Maryanne needs to get at least 15 math problems correct on her exam to pass.



- b. Jacob has less than \$22.50 in his bank account.



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## Let's Think:

Decide whether each value makes the inequality true.

	$x = 25$	$x = -25$
$100 \geq 4x$		
$x - 30 \leq -10$		

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## Let's Try It:

Let's explore writing inequality statements to represent inequality situations and using substitution to determine whether a given value for a variable makes an inequality true together.

Name: \_\_\_\_\_ Q1 US Lesson 12 - Let's Try It

Match each phrase with the corresponding inequality symbol.

- Less than  $<$
- Greater than  $>$
- Less than or equal to  $\leq$
- Greater than or equal to  $\geq$

An office water cooler can hold up to 7 gallons of water.

5. Select all the amounts that the water cooler could hold.

- 1 1/2 gallons
- 5 gallons
- 7 gallons
- 7 3/4 gallons
- 10.5 gallons

6. What does it mean if the water cooler can hold "up to" 7 gallons?

7. Write an inequality to represent the situation. Use  $w$  to represent the gallons in the water cooler.

8. Use a number line to represent this relationship.

At an amusement park, you must be at least 48 inches tall to ride a roller coaster.

9. Which description best represents the heights a roller coaster rider must be?

- Any value less than 48
- Any value greater than 48
- Any value less than or equal to 48
- Any value greater than or equal to 48

10. Write an inequality to represent this situation. Let  $h$  be the height of a rider.

11. Sketch a number line to represent the relationship.

12. What would be different about this graph if the rule was riders had to be taller than 48 inches?

Consider the inequality  $25 \geq x$ . Substitute the given value to see if the inequality is true.

- Is the inequality true when  $x$  is 35?
- Is the inequality true when  $x$  is 12?
- Is the inequality true when  $x$  is 25?

Consider the inequality  $4y < 24$ . Substitute the given value to see if the inequality is true.

- Is the inequality true when  $y = 0$ ?
- Is the inequality true when  $y = 4$ ?
- Is the inequality true when  $y = -4$ ?

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# On your Own:

Now it's time to write inequality statements to represent inequality situations and use substitution to determine whether a given value for a variable makes an inequality true on your own.

Name: \_\_\_\_\_ 37 US Lesson 12 - Independent Work

1. Determine whether each inequality is true or false for the given values.

$a = -4$	$a = 4$	$a = 0$
$8 \leq 4 - a$		

$c = 15.99$	$c = 1$	$c = 16$
$c < 16$		

$m = 17$	$m = -9$	$m = 9$
$2m \geq 18$		

2. A coffee mug can hold up to 16 ounces of coffee.

Which inequality best represents the situation?

- $c > 16$
- $c < 16$
- $c \geq 16$
- $c \leq 16$

Graph the inequality on a number line.

Name a value that could represent the number of ounces in the mug: \_\_\_\_\_

Name a value that could NOT represent the number of ounces in the mug: \_\_\_\_\_

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3. Write an inequality to match each situation. Represent the inequality on a number line.

- Each classroom in a school can have up to 25 students.
- The temperature in the refrigerator must be less than 40 degrees.
- Customers must be at least 16 years old to have a gym membership.

4. Luke got a new electric scooter. The box said the scooter travels less than 13 miles per hour. Luke wrote the inequality below and modeled it on a number line. What mistake did Luke make? How should he correct his work?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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**Match each phrase with the corresponding inequality symbol.**

- |                             |        |
|-----------------------------|--------|
| 1. Less than                | $\leq$ |
| 2. Greater than             | $<$    |
| 3. Less than or equal to    | $\geq$ |
| 4. Greater than or equal to | $>$    |

**An office water cooler can hold up to 7 gallons of water.**

5. Select all the amounts that the water cooler could hold.
- a. 1  $\frac{1}{2}$  gallons
  - b. 5 gallons
  - c. 7 gallons
  - d. 7  $\frac{3}{4}$  gallons
  - e. 10.5 gallons

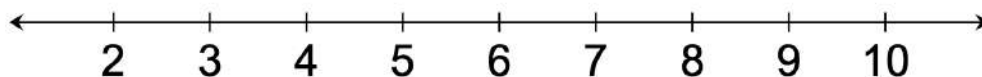
6. What does it mean if the water cooler can hold “up to” 7 gallons?

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7. Write an inequality to represent the situation. Use  $w$  to represent the gallons in the water cooler.

8. Use a number line to represent this relationship.



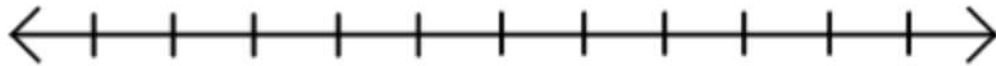


**At an amusement park, you must be at least 48 inches tall to ride a roller coaster.**

9. Which description best represents the heights a roller coaster rider must be?
- a. Any value less than 48
  - b. Any value greater than 48
  - c. Any value less than or equal to 48
  - d. Any value greater than or equal to 48

10. Write an inequality to represent this situation. Let  $h$  be the height of a rider.

11. Sketch a number line to represent the relationship.



12. What would be different about this graph if the rule was riders had to be taller than 48 inches?

---

---

**Consider the inequality  $25 \geq x$ . Substitute the given value to see if the inequality is true.**

13. Is the inequality true when  $x$  is 35?
14. Is the inequality true when  $x$  is 12?
15. Is the inequality true when  $x$  is 25?

**Consider the inequality  $4y < 24$ . Substitute the given value to see if the inequality is true.**

16. Is the inequality true when  $y = 0$ ?
17. Is the inequality true when  $y = 4$ ?
18. Is the inequality true when  $y = -4$ ?

1. Determine whether each inequality is true or false for the given values.

	$a = -4$	$a = 4$	$a = 0$
$8 \leq 4 - a$			

	$c = 15.99$	$c = 1$	$c = 16$
$c < 16$			

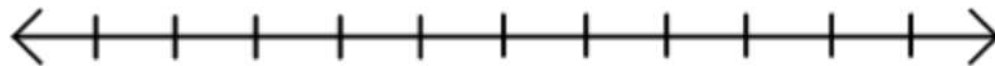
	$m = 17$	$m = -9$	$m = 9$
$2m \geq 18$			

2. A coffee mug can hold up to 16 ounces of coffee.

Which inequality best represents the situation?

- a.  $c > 16$
- b.  $c < 16$
- c.  $c \geq 16$
- d.  $c \leq 16$

Graph the inequality on a number line.



Name a value that could represent the number of ounces in the mug. \_\_\_\_\_

Name a value that could NOT represent the number of ounces in the mug. \_\_\_\_\_

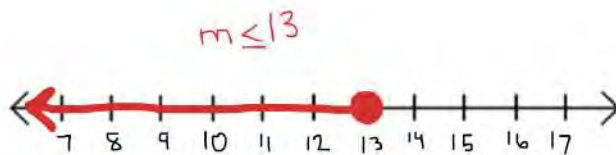
3. Write an inequality to match each situation. Represent the inequality on a number line.

a. Each classroom in a school can have up to 25 students.

b. The temperature in the refrigerator must be less than 40 degrees.

c. Customers must be at least 16 years old to have a gym membership.

4. Luke got a new electric scooter. The box said the scooter travels less than 13 miles per hour. Luke wrote the inequality below and modeled it on a number line. What mistake did Luke make? How should he correct his work?



Name: KEY

Match each phrase with the corresponding inequality symbol.

- |                             |              |                 |
|-----------------------------|--------------|-----------------|
| 1. Less than                | <del>→</del> | <del>≤</del>    |
| 2. Greater than             | <del>→</del> | <del>&lt;</del> |
| 3. Less than or equal to    | <del>→</del> | <del>≥</del>    |
| 4. Greater than or equal to | <del>→</del> | <del>&gt;</del> |

An office water cooler can hold up to 7 gallons of water.

5. Select all the amounts that the water cooler could hold.

- a. 1 ½ gallons
- b. 5 gallons
- c. 7 gallons
- d. 7 ¾ gallons
- e. 10.5 gallons

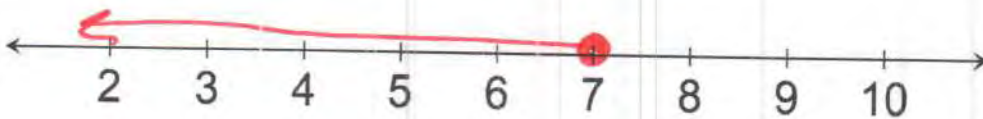
6. What does it mean if the water cooler can hold "up to" 7 gallons?

It means the most it can hold is  
7 gallons.

7. Write an inequality to represent the situation. Use  $w$  to represent the gallons in the water cooler.

$$w \leq 7$$

8. Use a number line to represent this relationship.



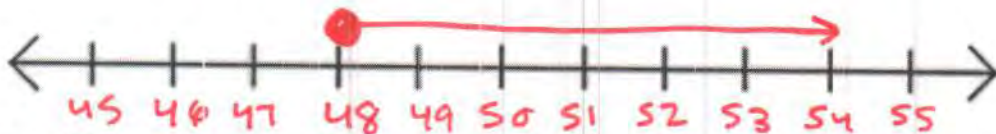
At an amusement park, you must be at least 48 inches tall to ride a roller coaster.

9. Which description best represents the heights a roller coaster rider must be?
- a. Any value less than 48
  - b. Any value greater than 48
  - c. Any value less than or equal to 48
  - d. Any value greater than or equal to 48

10. Write an inequality to represent this situation. Let  $h$  be the height of a rider.

$$h \geq 48$$

11. Sketch a number line to represent the relationship.



12. What would be different about this graph if the rule was riders had to be taller than 48 inches?

The number line would have an open circle instead of a closed one.

Consider the inequality  $25 \geq x$ . Substitute the given value to see if the inequality is true.

13. Is the inequality true when  $x$  is 35?  $25 \geq 35$   
NO
14. Is the inequality true when  $x$  is 12?  $25 \geq 12$   
YES
15. Is the inequality true when  $x$  is 25?  $25 \geq 25$   
YES

Consider the inequality  $4y < 24$ . Substitute the given value to see if the inequality is true.

16. Is the inequality true when  $y = 0$ ?  $4(0) < 24$   
 $0 < 24$   
YES
17. Is the inequality true when  $y = 4$ ?  $4(4) < 24$   
 $16 < 24$   
YES
18. Is the inequality true when  $y = -4$ ?  $4(-4) < 24$   
 $-16 < 24$   
YES



1. Determine whether each inequality is true or false for the given values.

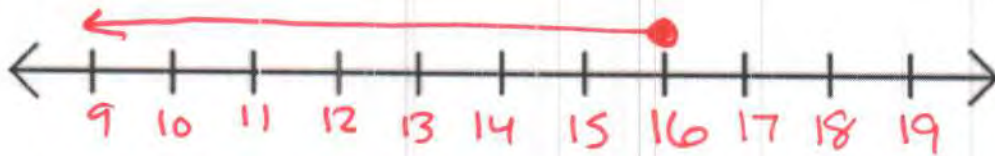
	$a = -4$	$a = 4$	$a = 0$
$8 \leq 4 - a$	TRUE $8 \leq 4 - (-4)$ $8 \leq 8$	FALSE $8 \leq 4 - 4$ $8 \leq 0$	FALSE $8 \leq 4 - 0$ $8 \leq 4$
	$c = 15.99$	$c = 1$	$c = 16$
$c < 16$	TRUE $15.99 < 16$	TRUE $1 < 16$	FALSE $16 < 16$
	$m = 17$	$m = -9$	$m = 9$
$2m \geq 18$	TRUE $34 \geq 18$	FALSE $-18 \geq 18$	TRUE $18 \geq 18$

2. A coffee mug can hold up to 16 ounces of coffee.

Which inequality best represents the situation?

- a.  $c > 16$
- b.  $c < 16$
- c.  $c \geq 16$
- d.  $c \leq 16$

Graph the inequality on a number line.



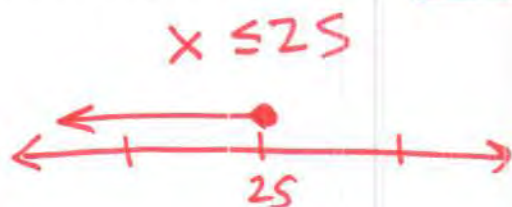
Name a value that could represent the number of ounces in the mug. (10)

Name a value that could NOT represent the number of ounces in the mug. (20)

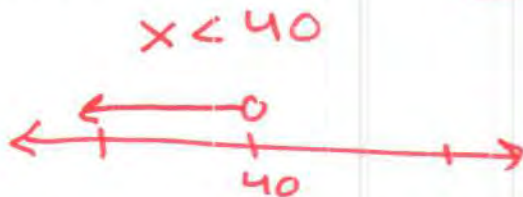


3. Write an inequality to match each situation. Represent the inequality on a number line.

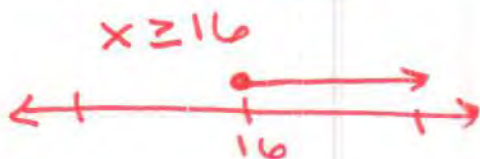
a. Each classroom in a school can have up to 25 students.



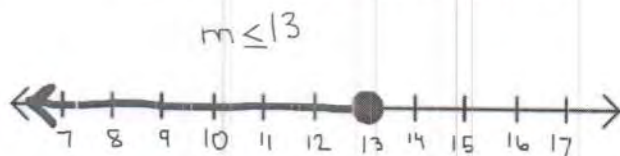
b. The temperature in the refrigerator must be less than 40 degrees.



c. Customers must be at least 16 years old to have a gym membership.



4. Luke got a new electric scooter. The box said the scooter travels less than 13 miles per hour. Luke wrote the inequality below and modeled it on a number line. What mistake did Luke make? How should he correct his work?



Luke should have used the  $<$  symbol and an open circle on the number line. The box said "less than 13 miles" which does not include 13. 13 is not less than 13.

## **G7 U5 Lesson 13**

Write inequalities that represent situations, and use substitution or reasoning about the context to find the solution.

**G7 U5 Lesson 13 - Students will write inequality statements to represent inequality situations, and use substitution or reasoning about the context to find the solution.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our last lesson, we started to do some work thinking about inequalities. We modeled simple situations on a number line and wrote inequalities to match the scenario. We also substituted values into an inequality to see if the value made a true statement. We'll see some similar work today as our goal is to write and solve inequalities that represent real-world situations. Once we solve, we'll make sure we understand what our solution means in the context of the given problem. Let's begin!

**Let's Talk (Slide 3):** Take a look at the two problems here. What is the same? What's different? **Possible Student Answers, Key Points:**

- The problems both have an  $x$ , a 5, and a 6 in them. The problems both involve addition.
- The problems are different colors. The first one has an equal sign, and the second one has a greater than sign. The second problem includes a number line. The first is an equation, and the second is an inequality.

The first problem is an equation, because it includes an equal sign. The value of  $x$  plus 5 is equal to 6. What is the value of  $x$ ? How do you know? **Possible Student Answers, Key Points:**

- The value of  $x$  is 1.
- I know the related equation  $6 - 5 = 1$ . I know that  $1 + 5$  is equal to 6.

The second problem is an inequality. Typically, there is only one solution to an equation. With an inequality, there are many values that could make the left side of the inequality greater than 6. Any number greater than 1 could work.  $2 + 5$  is greater than 6.  $3 + 5$  is greater than 6.  $4 + 5$  is greater than 6. Would 1 be a solution to this problem? **Possible Student Answers, Key Points:**

- No 1 is not a solution.
- $1 + 5$  is equal to 6.  $6 > 6$  is not a true statement.

As we work today, we'll see that we can solve inequalities similar to how we might solve an equation, but the way we reason about the solution is different in an inequality. The solution to an inequality can represent multiple, and sometimes unlimited solutions.

**Let's Think (Slide 4):** I'll read the problem aloud while you follow along. When I finish reading, I want you to summarize the gist of the story in your own words. What is known, and what is unknown? **Possible Student Answers, Key Points:**

- This problem is about Adam making gift bags for campers, and he's wondering how long it will take him to make all the bags.
- We know he's made 12 bags already. He can make 10 every hour, and he needs to make 152 in all.
- We don't know how long it will take him to finish the project.

$$\begin{array}{r} 12 + 10x = 152 \\ -12 \quad \quad -12 \\ \hline 10x = 140 \\ \frac{10x}{10} = \frac{140}{10} \\ x = 14 \end{array}$$

Let's not worry about an inequality for a moment. Let's just write an equation to represent this word problem, since that is what part A is asking for.

I can think of this as the 12 bags he completed plus 10 bags per hour,  $x$ . I know that must equal 152 so every camper gets a bag. (*write equation*) We can solve by subtracting 12 from both sides of the equation. (*subtract 12 from both sides and rewrite equation*) We're left with  $10x = 140$ . I know I can divide both sides by 10. Our equation's solution reads  $x = 14$ .

Now let's consider Part B. This part wants us to think about the same situation, but now Adam doesn't want exactly 152 bags. He wants to have some left over. What inequality symbol can we use to show that Adam wants bags left over? Explain how you know. **Possible Student Answers, Key Points:**

- We should use  $>$ , because he wants more than 152 bags.
- We can't use  $\geq$ , because if he made equal to 152 he would not have any leftover bags.

$$\begin{array}{r} 12 + 10x > 152 \\ -12 \quad -12 \\ \hline 10x > 140 \\ \frac{10x}{10} > \frac{140}{10} \\ x > 14 \end{array}$$

(write  $12 + 10x > 152$ ) If we rethink this scenario so that Adam can have leftovers, we can use  $12 + 10x > 152$ , so that we can find values that result in more than 152 bags.

I'll start by subtracting 12 from both sides of the inequality. (show underneath inequality) Our resulting inequality is  $10x > 140$ . From here, I can divide both sides by 10. (divide both sides by 10) The result reads  $x > 14$ , or  $x$  is greater than 14.

What does  $x > 14$  mean in the context of this particular problem? **Possible Student Answers, Key Points:**

- We were trying to find the number of hours it would take Adam to have leftover bags.
- The solution means that if Adam works for more than 14 hours, he'll end up with leftover bags.

If  $x$  is greater than 14, that means Adam will need to work for more than 14 hours in order to have leftover bags. Notice how solving an equation and solving an inequality felt fairly similar. We interpret our answers a bit different, but the process of solving is mostly the same. Let's see if that is true by looking at one more example.

**Let's Think (Slide 5):** Like last time, I'll read the problem aloud while you follow along. When I finish reading, I want you to summarize the gist of the story in your own words. What is known, and what is unknown?

**Possible Student Answers, Key Points:**

- This problem is about how many pairs of socks Lyric can buy if she has \$75 to buy a pair of shoes and some pairs of socks.
- We know her budget is \$75. We know the pair of shoes costs \$60 and she just buys the one pair of shoes. We know she buys 5 pairs of socks.
- We don't know how much the socks cost.

Now that we understand the scenario, we'll write an equation to represent it. I know Lyric has a budget of \$75, so her purchases will equal up to \$75. I'll write 60 to represent the shoes and  $5x$  to represent the fact that she buys 5 pairs of socks at an unknown price. (write  $60 + 5x = 75$ )

$$\begin{array}{r} 60 + 5x = 75 \\ -60 \quad -60 \\ \hline 5x = 15 \\ \frac{5x}{5} = \frac{15}{5} \\ x = 3 \end{array}$$

We've seen several equations that look similar in previous lessons. What steps would you take to solve? I'll follow along in writing as you share. **Possible Student Answers, Key Points:**

- We can subtract 60 from both sides. That leaves us with  $5x = 15$ .
- We can divide both sides by 5. That means our solution is  $x = 3$ .

If  $x = 3$ , that means each pair of socks cost Lyric \$3. Job well done!

For Part B, we'll think about the same scenario. This time, it wants us to use the same information, but we need to write an inequality. We can write an inequality, because this situation doesn't necessarily mean that Lyric needs to spend her entire budget. She could spend less than her budget. In some instances, it's a good thing to be under budget. Let's try it out.

What inequality symbol can we use if we think about the fact that she doesn't necessarily have to spend her entire budget? How do you know? [Possible Student Answers, Key Points:](#)

- We can use  $\leq$  because she can spend equal to her budget or less.
- I wouldn't want to use  $>$  or  $\geq$  because she probably can't go over budget.

$$\begin{array}{r} 60 + 5x \leq 75 \\ -60 \qquad -60 \\ \hline 5x \leq 15 \\ \frac{5x}{5} \leq \frac{15}{5} \\ x \leq 3 \end{array}$$

(write  $60 + 5x \leq 75$ ) This inequality shows that the shoes and the pairs of socks can end up costing less than or equal to \$75. I'll solve in the same way we solved the equation. (write equations as you narrate)

I'll subtract 60 from both sides of the inequality. That leaves us with a new inequality of  $5x \leq 15$ . From here, I know I can divide by 5 on both sides. The solution we're left with is  $x \leq 3$ .

What does a solution of  $x \leq 3$  mean in the context of this problem? If you're not sure, think about what was unknown to begin with. [Possible Student Answers, Key Points:](#)

- We did not know the cost of the pair of socks. This solution means the socks can cost less than or equal to \$3.

If the socks can cost less than or equal to \$3, what could be the cost of the socks besides \$3? (\$2.99, \$1, or any number less than 3) Could the cost of the socks be -\$1? (No, that doesn't make sense in real life.) -1 is technically less than 3, but some values within the solution set make more sense than others in context. We'll see more of that in future lessons.

Great work solving equation and inequalities that represent real-world scenarios. We were not only able to solve, but we were also able to talk about what our answers mean in context.

**Let's Try it (Slides 6 - 7):** Now let's do a few more together, before you get a chance to show what you know independently. As we saw, we can think of the steps of solving an inequality as similar to the steps of solving an equation. We'll work carefully to isolate the variable one step at a time. An important aspect of solving an inequality is that the solution usually includes many values, so we have to make sure we have a clear sense of what is happening in the story so we can accurately interpret the solution to an inequality. Let's give these next few problems a try. I know you're going to do great!

# WARM WELCOME



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**Today we will write inequalities that represent situations, and use substitution or reasoning about the context to find the solution.**

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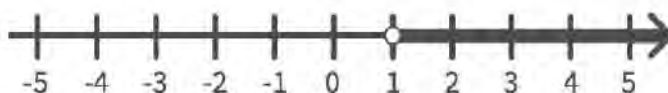


Let's Talk:

**What's the same? What's different?**

$$x + 5 = 6$$

$$x + 5 > 6$$



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Let's Think:

**Adam is making gift bags for new campers coming to summer camp. He has already made 12 bags, and he can make 10 bags every hour. If there are 152 campers, how many hours will it take Adam to make 152 bags?**

- Write and solve an equation to show how many hours it will take Adam to complete making exactly 152 bags.
- Write and solve an inequality to show how many hours it will take Adam to complete making 152 bags, if he wants to have some extras leftover.

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## Let's Think:

Lyric has a \$75 budget to buy shoes and socks. She needs 5 pairs of socks and 1 pair of shoes. The shoes she wants cost \$60. How much does Lyric have left to spend on each pair of socks?

- Write an equation to find exactly how much Lyric can spend on each pair of socks.
- Write an inequality to represent the situation, and name at least 2 other amounts Lyric could pay for each pair of socks.

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## Let's Try It:

Let's explore writing inequalities that represent situations and using substitution or reasoning about the context to find the solution together.

Name: \_\_\_\_\_ 07.15 Lesson 13 - Let's Try It

Alicia has \$20 in her bank account. She deposits \$9 every week. She wants to buy a new jacket that costs \$74.

- What expression can be used to represent the fact that Alicia earns 9 dollars every week,  $w$ ?
  - $9 = w$
  - $9w$
  - $9 - w$
  - $9 = w$
- Write an expression to show that Alicia starts with 20 dollars and deposits 9 dollars every week.
- Write an equation to show the exact amount of money Alicia will need to have in order to buy the jacket.
- Solve the equation to determine how many weeks it will take Alicia to earn exactly enough money to buy the jacket.
- Suppose Alicia wants to have extra money in her account after buying the jacket. Fill in the blank with an inequality symbol to represent this scenario.
 
$$20 + 9w \text{ \_\_\_\_\_\_ } 74$$
- Solve the inequality.
- What does the solution to the inequality mean in the context of this problem?

Omari is running laps for a charity race. Omari's uncle says he will donate \$50 just for running the race, and \$15 for every lap Omari runs.

- Write an equation to show how many laps,  $x$ , Omari needs to run in order to raise exactly \$170.
- Solve the equation. How many laps does Omari need to run to raise exactly \$170?
- Suppose Omari decides he wants to raise at least \$170. What does that mean?
- Write and solve an inequality to show how many laps Omari needs to run to raise at least \$170.

Consider the inequality  $4h + 21 \leq 41$ .

- Replace the inequality symbol with an equal sign and solve the equation.
- The value you found is called the **boundary point**. Is this boundary point a solution to this inequality?
- Choose a value less than your boundary point. Test to see if it is a solution to the inequality.
- Choose a value greater than the boundary point to see if it is a solution to the inequality.
- Write a solution statement for this inequality.

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# On your Own:

Now it's time to write inequalities that represent situations and use substitution or reasoning about the context to find the solution on your own.

Name: \_\_\_\_\_ 37. US Lesson 1.3 - Independent Work

1. Which values make the inequality true? Select all that apply.

$$8x + 20 < 98$$

a.  $x = -1$   
 b.  $x = 0$   
 c.  $x = 2$   
 d.  $x = 5$   
 e.  $x = 3.75$   
 f.  $x = 6$

If there were any values you did not select, explain why.

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2. Moriah has a balance of \$120 in her bank account. She wants to have at least \$300 in the account 5 weeks from now. She used the inequality shown here to find how much she needs to save each week to meet her goal.

$$5m + 120 \geq 300$$

a. What does  $5m$  represent?

b. Find at least three values for  $m$  that could work for Moriah.

c. Write an inequality to represent the answer to Moriah's question.

3. At a school carnival, students are allowed to be on the bumper cars for up to 12 minutes.

a. Write an inequality to represent this situation.

b. Name at least 2 values that make the inequality true, and name at least 2 values that make the inequality false.

c. Represent the inequality on a number line.

4. Zachary delivers pizzas for his family's restaurant. His dad gives him \$25 each day, plus \$3 for every pizza he delivers. Zachary hopes to make at least enough money today to buy a video game for \$56. Write and solve an inequality to represent this scenario. Use  $x$  to represent every pizza delivered.

If Zachary delivers 12 pizzas today, will he have enough to buy the video game?

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**Alicia has \$20 in her bank account. She deposits \$9 every week. She wants to buy a new jacket that costs \$74.**

1. What expression can be used to represent the fact that Alicia earns 9 dollars every week,  $w$ ?
  - a.  $9 + w$
  - b.  $9w$
  - c.  $9 - w$
  - d.  $9 \div w$
2. Write an expression to show that Alicia starts with 20 dollars and deposits 9 dollars every week.
3. Write an equation to show the exact amount of money Alicia will need to have in order to buy the jacket.
4. Solve the equation to determine how many weeks it will take Alicia to earn exactly enough money to buy the jacket.
5. Suppose Alicia wants to have extra money in her account after buying the jacket. Fill in the blank with an inequality symbol to represent this scenario.

$$20 + 9w \text{ \_\_\_\_ } 74$$

6. Solve the inequality.
7. What does the solution to the inequality mean in the context of this problem?

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**Omari is running laps for a charity race. Omari's uncle says he will donate \$50 just for running the race, and \$15 for every lap Omari runs.**

8. Write an equation to show how many laps,  $x$ , Omari needs to run in order to raise exactly \$170.

9. Solve the equation. How many laps does Omari need to run to raise exactly \$170?

10. Suppose Omari decides he wants to raise *at least* \$170. What does that mean?

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11. Write and solve an inequality to show how many laps Omari needs to run to raise at least \$170.

**Consider the inequality  $4h + 21 \leq 41$ .**

12. Replace the inequality symbol with an equal sign and solve the equation.

13. The value you found is called the boundary point. Is this boundary point a solution to this inequality?

14. Choose a value less than your boundary point. Test to see if it is a solution to the inequality.

15. Choose a value greater than the boundary point to see if it is a solution to the inequality.

16. Write a solution statement for this inequality.

**1. Which values make the inequality true? Select all that apply.**

$$6x + 20 < 38$$

- a.  $x = -1$
- b.  $x = 0$
- c.  $x = 2$
- d.  $x = 3$
- e.  $x = 3 \frac{1}{2}$
- f.  $x = 6$

If there were any values you did *not* select, explain why.

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**2. Moriah has a balance of \$120 in her bank account. She wants to have at least \$300 in the account 5 weeks from now. She used the inequality shown here to find how much she needs to save each week to meet her goal.**

$$5m + 120 \geq 300$$

- a. What does  $5m$  represent?
- b. Find at least three values for  $m$  that could work for Moriah.
- c. Write an inequality to represent the answer to Moriah's question.



**3. At a school carnival, students are allowed to be on the bumper cars for up to 12 minutes.**

a. Write an inequality to represent this situation.

b. Name at least 2 values that make the inequality true, and name at least 2 values that make the inequality false.

c. Represent the inequality on a number line.

**4. Zachary delivers pizzas for his family's restaurant. His dad gives him \$25 each day, plus \$3 for every pizza he delivers. Zachary hopes to make at least enough money today to buy a video game for \$58.** Write and solve an inequality to represent this scenario. Use  $x$  to represent every pizza delivered.

If Zachary delivers 12 pizzas today, will he have enough to buy the video game?

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Alicia has \$20 in her bank account. She deposits \$9 every week. She wants to buy a new jacket that costs \$74.

1. What expression can be used to represent the fact that Alicia earns 9 dollars every week,  $w$ ?

- a.  $9 + w$   
 b.  $9w$   
 c.  $9 - w$   
 d.  $9 \div w$

$9 \times$  # of weeks

2. Write an expression to show that Alicia starts with 20 dollars and deposits 9 dollars every week.

$$20 + 9x \text{ or } 20 + 9w$$

3. Write an equation to show the exact amount of money Alicia will need to have in order to buy the jacket.

$$20 + 9w = 74$$

4. Solve the equation to determine how many weeks it will take Alicia to earn exactly enough money to buy the jacket.

$$\begin{array}{r} 20 + 9w = 74 \\ -20 \quad -20 \\ \hline 9w = 54 \\ \frac{9}{9} \quad \frac{9}{9} \\ \hline w = 6 \end{array}$$

5. Suppose Alicia wants to have extra money in her account after buying the jacket. Fill in the blank with an inequality symbol to represent this scenario.

$$20 + 9w > 74$$

6. Solve the inequality.

$$\begin{array}{r} 20 + 9w > 74 \\ -20 \quad -20 \\ \hline 9w > 54 \\ \hline w > 6 \end{array}$$

7. What does the solution to the inequality mean in the context of this problem?

If she wants left over money, she'll need to save for more than 6 weeks.

Omari is running laps for a charity race. Omari's uncle says he will donate \$50 just for running the race, and \$15 for every lap Omari runs.

8. Write an equation to show how many laps,  $x$ , Omari needs to run in order to raise exactly \$170.

$$50 + 15x = 170$$

9. Solve the equation. How many laps does Omari need to run to raise exactly \$170?

$$\begin{array}{r} 50 + 15x = 170 \\ -50 \quad -50 \\ \hline 15x = 120 \\ \frac{15x}{15} = \frac{120}{15} \end{array} \quad (x = 8)$$

10. Suppose Omari decides he wants to raise at least \$170. What does that mean?

He wants to raise 170 or more.

11. Write and solve an inequality to show how many laps Omari needs to run to raise at least \$170.

$$\begin{array}{r} 50 + 15x \geq 170 \\ 15x \geq 120 \\ (x \geq 8) \end{array}$$

Consider the inequality  $4h + 21 \leq 41$ .

12. Replace the inequality symbol with an equal sign and solve the equation.

$$\begin{array}{r} 4h + 21 = 41 \\ -21 \quad -21 \\ \hline 4h = 20 \\ \frac{4h}{4} = \frac{20}{4} \\ (h = 5) \end{array}$$

13. The value you found is called the boundary point. Is this boundary point a solution to this inequality?

It is, because  $4(5) + 21 \leq 41$  is true.

14. Choose a value less than your boundary point. Test to see if it is a solution to the inequality.

$$\begin{array}{r} 4(0) + 21 \leq 41 \\ 21 \leq 41 \quad \checkmark \end{array}$$

15. Choose a value greater than the boundary point to see if it is a solution to the inequality.

$$\begin{array}{r} 4(10) + 21 \leq 41 \\ 40 + 21 \leq 41 \\ 61 \leq 41 \quad \times \end{array}$$

16. Write a solution statement for this inequality.  $(h \leq 5)$



1. Which values make the inequality true? Select all that apply.

$$6x + 20 < 38$$

- a.  $x = -1$   
 b.  $x = 0$   
 c.  $x = 2$   
 d.  $x = 3$   
 e.  $x = 3\frac{1}{2}$   
 f.  $x = 6$

a)  $-6 + 20 < 38$   
 $14 < 38$

c)  $12 + 20 < 38$   
 $32 < 38$

e)  $21 + 21 < 38$   
 $42 < 38$

b)  $0 + 20 < 38$   
 $20 < 38$

d)  $18 + 20 < 38$   
 $38 < 38$

f)  $36 + 20 < 38$   
 $56 < 38$

If there were any values you did *not* select, explain why.

When I substituted 3,  $3\frac{1}{2}$ , and 6 in for  $x$ , it did not end up making a true inequality statement.

2. Moriah has a balance of \$120 in her bank account. She wants to have at least \$300 in the account 5 weeks from now. She used the inequality shown here to find how much she needs to save each week to meet her goal.

$$5m + 120 \geq 300$$

- a. What does  $5m$  represent?

The amount earned in 5 weeks

- b. Find at least three values for  $m$  that could work for Moriah.

100, 200, 300

- c. Write an inequality to represent the answer to Moriah's question.

$$5m \geq 180$$

$$m \geq 36$$

3. At a school carnival, students are allowed to be on the bumper cars for up to 12 minutes.

a. Write an inequality to represent this situation.

$$x \leq 12$$

b. Name at least 2 values that make the inequality true, and name at least 2 values that make the inequality false.

$$\begin{array}{l} \text{TRUE} \\ 10 = x \\ 1 = x \end{array}$$

$$\begin{array}{l} \text{FALSE} \\ 13 = x \\ 100 = x \end{array}$$

c. Represent the inequality on a number line.



4. Zachary delivers pizzas for his family's restaurant. His dad gives him \$25 each day, plus \$3 for every pizza he delivers. Zachary hopes to make at least enough money today to buy a video game for \$58. Write and solve an inequality to represent this scenario. Use  $x$  to represent every pizza delivered.

$$\begin{array}{r} 25 + 3x \geq 58 \\ -25 \quad \quad -25 \\ \hline 3x \geq 33 \\ \textcircled{x \geq 11} \end{array}$$

If Zachary delivers 12 pizzas today, will he have enough to buy the video game?

Yes.  $25 + 3(12) = 25 + 36 = 61$ .

He will have \$61 which is more than enough.

# **G7 U5 Lesson 14**

Solve inequalities using the associated equation and testing values to determine the direction of the inequality in the solution.



**G7 U5 Lesson 14 - Students will solve inequalities using the associated equation and testing values to determine the direction of the inequality in the solution.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** This whole unit we've been working on solving equations and inequalities. Today, we'll keep our focus on solving inequalities, but we'll see how we can use associated equations to help us consider solutions to inequalities. Before we look at some problems, what do you already know is the same and different about equations compared to inequalities? **Possible Student Answers, Key Points:**

- Equations and inequalities are both types of math problems. Equations and inequalities can both involve unknowns. We can solve both.
- An equation uses an equal sign, while an inequality uses a symbol like  $<$ ,  $>$ ,  $\leq$ , or  $\geq$ . An equation typically has one solution while an inequality can have many solutions.

Let's use what we've learned about equations and inequalities to help us think through the next several problems.

**Let's Talk (Slide 3):** Here we see an inequality that reads  $x$  plus 1 is greater than 9. I know that means whatever number I substitute in for  $x$ , the total of  $x$  and 1 has to be more than 9. Let's test out a few values for practice. Consider each of the possible solutions, 0, 8, and 10. Use paper or mental math to determine whether each value is a solution to the inequality. Explain how you know. (write each inequality with the value substituted in for  $x$ ) **Possible Student Answers, Key Points:**

$$\begin{array}{l} 0 + 1 > 9 \quad \times \\ 8 + 1 > 9 \quad \times \\ 10 + 1 > 9 \quad \checkmark \end{array}$$

- 0 is not a solution.  $0 + 1 = 1$ , and 1 is not greater than 9.
- 8 is not a solution.  $8 + 1 = 9$ , and 9 is not greater than 9.
- 10 is not a solution.  $10 + 1 = 11$ , and 11 is greater than 9.

A number is a solution to an inequality if it makes the inequality a true statement when substituted in place of the unknown. We'll need this skill to help us with the problems we're going to look at today. Let's get going.

**Let's Think (Slide 4):** These two problems show inequalities involving addition and subtraction. We'll solve them using associated equations, and then use the number line to model the solutions. Let's start by looking at the blue inequality.

$$\begin{array}{r} 2 + x = -1 \\ -2 \quad -2 \\ \hline x = -3 \end{array}$$

The inequality reads 2 plus  $x$  is greater than 1. I'm going to rewrite the inequality as an equation, solve, and then we'll make sense of our solution using the inequality symbol. (write  $2 + x = -1$ ) This is the associated equation. Notice, I just switched the inequality symbol for an equal sign for the moment. How can I solve this equation? (subtract 2 from both sides of the equation) (show that, and write  $x = -3$ ) The solution to the associated equation is called our boundary point.

$$\begin{array}{l} 2 + (-4) > -1 \\ -2 > -1 \quad \times \end{array}$$

We'll test out a value less than our boundary point and a value greater than our boundary point to determine which inequality symbol makes most sense in our solution. I know  $-4$  is less than  $-3$ , so let's test out  $-4$  in our original inequality. (substitute  $-4$  in place of  $x$  and solve as you narrate) I know  $2 + (-4)$  equals  $-2$ .  $-2 > -1$  is a false statement, because  $-2$  is not greater than  $-1$ .

Let's do the same thing, but test out a value greater than our boundary point. I know 0 is an easy number to work with that is greater than our boundary point of  $-3$ . Let's substitute 0 in for  $x$ , and see if it makes our

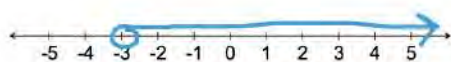
$$2 + (0) > -1$$

$$2 > -1 \checkmark$$

inequality true. (substitute 0 in for x and solve as you narrate) I know  $2 + 0$  equals 2.  $2 > -1$  is a true statement, because 2 is greater than 1.

$$x > -3$$

Since the value less than our boundary point made a false statement, and the value greater than our boundary point made a true statement, I can say that the value of x is any number *greater than* -3. (write  $x > -3$ )



(sketch on number line as you narrate) I can model this on a number line by circling -3 and drawing an arrow to the values greater than 3. I won't color in the point at -3, because I don't want to include that as a solution. -3 is not greater than -3

$$y - 3 = 0$$

$$\begin{array}{r} +3 \\ +3 \\ \hline y = 3 \end{array}$$

$$y \geq 3$$

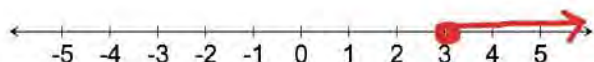
Let's look at the other inequality. This one reads y minus 3 is greater than or equal to 0. I'll start by writing the associated equation. (write  $y - 3 = 0$ ) How can I solve this equation? What is the solution? Possible Student Answers, Key Points:

- We can add 3 on both sides of the equal sign to keep it balanced.
- I know the solution is  $y = 3$ .

The solution to the equation is  $y = 3$ . That's our boundary point. To determine the inequality symbol we should use, we'll test out an easy value less than the boundary point and an easy value greater than the boundary point. I like to choose values that are easy for me to operate with. Let's choose 0 for a value less than our boundary point and 5 for the value greater than our boundary point. Does 0 or 5 make our inequality true? How do you know? Possible Student Answers, Key Points:

- If I substitute 0 in for y in the inequality, I end up with  $-3 \geq 0$ . That is false.
- If I substitute 5 in for y in the inequality, I end up with  $2 \geq 0$ . That is true.

Since the value greater than the boundary point made the inequality true, I can write the solution to the inequality as  $y \geq 3$ . (write  $y \geq 3$  underneath equation)



We can model this on a number line by marking a point at 3, and drawing an arrow to the values greater than 3. In this case, I'll color in the point at 3 because 3 is included in our solution  $3 \text{ is } \geq 3$  because 3 is equal to 3.

We just solved inequalities by using an associated equation to find the boundary point. Once we knew the boundary point, we tested values greater than and less than that point to determine the inequality symbol that can be used to represent the solution. We also graphed our solution set on a number line like we've seen previously. Let's try two more. We'll use the same thinking, but the inequalities look a bit different.

**Let's Think (Slide 5):** Let's solve two inequalities involving multiplication using the same approach.

$$5v = -10$$

$$\frac{5v}{5} = \frac{-10}{5}$$

$$v = -2$$

$$v < -2$$

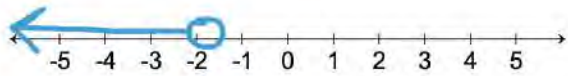
What associated equation can I write for the first inequality? What is the solution? (solve as shown while student shares, supporting as needed) Possible Student Answers, Key Points:

- We can write  $5v = -10$  instead of  $5v < -10$ .
- If I divide both sides by 5, I get  $v = -2$  as the solution. That's my boundary point.

If we solve the associated equation, we see the boundary point is -2. Let's choose a simple value less than -2 and a simple value greater than -2 to determine the symbol that completes the inequality correctly.

Let's use -10 and 10, since those are fairly easy numbers to think about. We'll substitute each value in for  $v$  to see if they make a true statement. Is -10 a solution to the inequality? Is 10 a solution to the inequality? How do you know? **Possible Student Answers, Key Points:**

- -10 is a solution, because  $5(-10)$  is -50. -50 is less than -10.
- 10 is not a solution, because  $5(10)$  is 50. 50 is not less than -10.



Since the value less than our boundary made the inequality true, we can say that our solution is  $v < -2$ . (*sketch on number line as you narrate*) I can model that on a number line by circling the point at -2 and drawing an arrow to the values less than -2. Why am I not going to color in the circle at the -2?

**Possible Student Answers, Key Points:**

- We don't color it in, because -2 is not part of the solution.  $-2 < -2$  is not a true statement.

$$\frac{-5v}{-5} = \frac{-10}{-5}$$

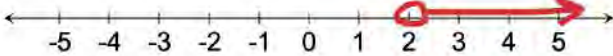
$$v = 2$$

$$v > 2$$

Let's think about our final inequality. The associated equation for  $-5v < -10$  is  $-5v = -10$ . (*write and solve*) I know  $v = 2$ , because -10 divided by -5 is positive 2. The boundary point in this problem is 2.

Let's use 0 as a value less than the boundary point and 4 as a value greater than the boundary point, since both of those values are pretty friendly to compute with. Which value makes a true statement? **Possible Student Answers, Key Points:**

- I know  $-5(0)$  is 0. The inequality  $0 < -10$  is not true.
- I know  $-5(4)$  is -20. The inequality  $-20 < -10$  is true. The value of 4 makes a true statement.



Since the value greater than our boundary point makes a true statement, the solution to the inequality can be written as  $v > 2$ . How can I model the solution set using a number line? (*sketch as student shares*) **Possible Student Answers, Key Points:**

- Mark a point at positive 2, but don't shade it in. 2 is not part of the solution, it's just the boundary.
- Draw an arrow to the right to represent the values greater than 2.

If you look back at the four inequalities we just solved, you'll notice that the inequality symbol used in the original problem sometimes matches the inequality used in the solution and sometimes does not. We always want to carefully consider values above and below the boundary point before placing an inequality symbol in the solution.

**Let's Try it (Slides 6 - 7):** Now let's try a few more on our own. We've seen that we can solve an associated equation to help us think about the solution to any inequality. Once we've found our boundary point, we can test values that are above or below the boundary point to determine which inequality symbol to use in our answer. As always, when dealing with inequalities, we want to think about whether the boundary point is or is not included in the solution set. A number line can help us represent all the values in our solution set. We'll try a few more together, and then you'll get a chance to show what you've learned independently.


# WARM WELCOME



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**Today we will solve inequalities using the associated equation and test values to determine the direction of the inequality in the solution.**

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 **Let's Talk:**


$$x + 1 > 9$$

**Is 0 a solution?**

**Is 8 a solution?**

**Is 10 a solution?**

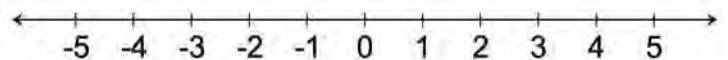
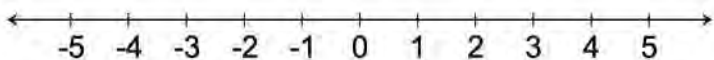
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 **Let's Think:**

**Solve each inequality. Use a number line to represent all possible solutions.**

$$2 + x > -1$$

$$y - 3 \geq 0$$



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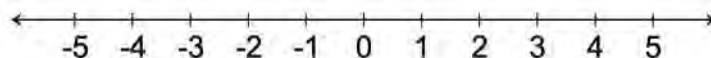
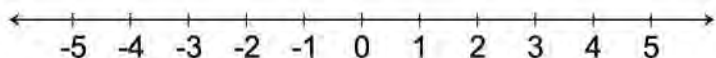


# Let's Think:

Solve each inequality. Use a number line to represent all possible solutions.

$$5v < -10$$

$$-5v < -10$$



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# Let's Try It:

Let's explore solving inequalities using the associated equation and testing values to determine the direction of the inequality in the solution together.

Name: \_\_\_\_\_ G7 US Lesson 14 - Let's Try It

Consider the inequality  $3a < 12$ .

- Which best describes the inequality?
  - 3 times  $a$  is greater than 8
  - 3 times  $a$  is less than 8
  - 3 times  $a$  is less than or equal to 8.
- Solve the associated equation,  $3a = 12$ . Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.

---

4. Complete the table to determine whether values less than the boundary (ex.  $a = 1$ ) or values greater than the boundary (ex.  $a = 5$ ) make the inequality true.

	$a = 1$	$a = 4$ (boundary)	$a = 5$
$3a < 12$			

- Write the inequality that represents every possible solution to  $3a < 12$ .
- Use the number line to represent all possible solutions to the inequality.

Consider the inequality  $-3a < 12$ .

- Solve the associated equation,  $-3a = 12$ . Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.
  - Yes
  - No

- Complete the table to determine whether values less than the boundary or values greater than the boundary make the inequality true.

	$a = -5$	$a = -4$ (boundary)	$a = 0$
$-3a < 12$			

- Write the inequality that represents every possible solution to  $-3a < 12$ .
- Sketch a number line to represent all possible solutions to the inequality.

Consider the inequality  $h - 5 \geq -8$ .

- Solve the associated equation. What is your boundary point? Is it a solution to the inequality?
- Test a value less than the boundary and greater than the value.
- Write the inequality that represents every possible solution to  $h - 5 \geq -8$ .
- Sketch a number line to represent all possible solutions to the inequality.

Consider the inequality  $-3x \leq 18$ .

- Write the inequality that represents the solution. Sketch a number line to represent all possible solutions.

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# On your Own:

Now it's time to solve inequalities using the associated equation and test values to determine the direction of the inequality in the solution on your own.

Name: \_\_\_\_\_ G7 US Lesson 14 / Independent Work

1.

a. Graph the solutions to  $9m \leq 54$  on the number line. Show all work.

b. Graph the solutions to  $-9m \leq 54$  on the number line. Show all work.

2. Find the solution to  $n + 8 > -8$ . Write the solution as an inequality.

Sketch a number line to represent all possible solutions to the inequality.

3. Jackie was trying to find the solution to  $-7k > -70$ . Her solution and number line are shown below.

Explain why Jackie's solution is unreasonable. Include the correct answer in your response.

4. Look at each inequality. Solve each. List at least 2 values that make each inequality true and at least 2 values that make each inequality false.

$y + 4 < 8$        $-2w \geq 20$

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**Consider the inequality  $3a < 12$ .**

- Which best describes the inequality?
  - 3 times  $a$  is greater than 8
  - 3 times  $a$  is less than 8
  - 3 times  $a$  is less than or equal to 8.
- Solve the associated equation,  $3a = 12$ . Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.

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- Complete the table to determine whether values less than the boundary (ex.  $a = 1$ ) or values greater than the boundary (ex.  $a = 5$ ) make the inequality true.

	$a = 1$	$a = 4$ (boundary)	$a = 5$
$3a < 12$			

- Write the inequality that represents every possible solution to  $3a < 12$ .
- Use the number line to represent all possible solutions to the inequality.

**Consider the inequality  $-3a < 12$ .**

- Solve the associated equation,  $-3a = 12$ . Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.
  - Yes
  - No

9. Complete the table to determine whether values less than the boundary or values greater than the boundary make the inequality true.

	$a = -5$	$a = -4$ (boundary)	$a = 0$
$-3a < 12$			

10. Write the inequality that represents every possible solution to  $-3a < 12$ .

11. Sketch a number line to represent all possible solutions to the inequality.

**Consider the inequality  $h - 5 \geq -9$ .**

12. Solve the associated equation. What is your boundary point? Is it a solution to the inequality?

13. Test a value less than the boundary and greater than the boundary.

14. Write the inequality that represents every possible solution to  $h - 5 \geq -9$ .

15. Sketch a number line to represent all possible solutions to the inequality.

**Consider the inequality  $-3x \leq 18$ .**

16. Write the inequality that represents the solution. Sketch a number line to represent all possible solutions.

1.

a. Graph the solutions to  $9m \leq 54$  on the number line. Show all work.



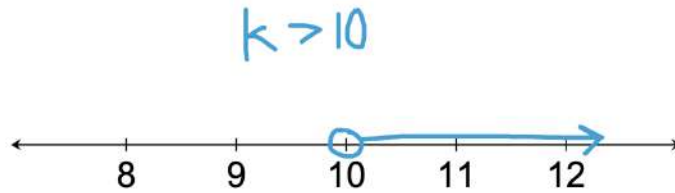
b. Graph the solutions to  $-9m \leq 54$  on the number line. Show all work.



2. Find the solution to  $n + 8 > -8$ . Write the solution as an inequality.

Sketch a number line to represent all possible solutions to the inequality.

3. Jackie was trying to find the solution to  $-7k > -70$ . Her solution and number line are shown below.



Explain why Jackie's solution is unreasonable. Include the correct answer in your response.

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4. **Look at each inequality. Solve each.** List at least 2 values that make each inequality true and at least 2 values that make each inequality false.

$$y + 4 < -6$$

$$-2w \geq 20$$

Consider the inequality  $3a < 12$ .

- Which best describes the inequality?
  - 3 times  $a$  is ~~greater~~ than 8
  - 3 times  $a$  is less than 8**
  - 3 times  $a$  is less than or ~~equal~~ to 8.
- Solve the associated equation,  $3a = 12$ . Your solution is the boundary point.
 
$$\frac{3a}{3} = \frac{12}{3}$$

$$a = 4$$
- Is your solution to the associated equation also a solution to the inequality? Explain.

**No.  $3(4) = 12$  and  $12 < 12$  is not a true statement.**

- Complete the table to determine whether values less than the boundary (ex.  $a = 1$ ) or values greater than the boundary (ex.  $a = 5$ ) make the inequality true.

$3(1) < 12$   
 $3 < 12$

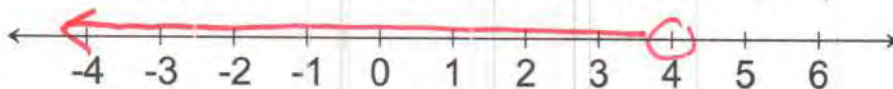
	$a = 1$	$a = 4$ (boundary)	$a = 5$
$3a < 12$	✓	X	X

$3(5) < 12$   
 $15 < 12$

- Write the inequality that represents every possible solution to  $3a < 12$ .

**$a < 4$**

- Use the number line to represent all possible solutions to the inequality.



Consider the inequality  $-3a < 12$ .

- Solve the associated equation,  $-3a = 12$ . Your solution is the boundary point.
 
$$\frac{-3a}{-3} = \frac{12}{-3}$$

$$a = -4$$
- Is your solution to the associated equation also a solution to the inequality? Explain.

- Yes
- No**

$-3(-4) < 12$   
 $12 < 12 \leftarrow \text{false}$



9. Complete the table to determine whether values less than the boundary or values greater than the boundary make the inequality true.

$$-3(-5) < 12$$

$$15 < 12$$

	$a = -5$	$a = -4$ (boundary)	$a = 0$
$-3a < 12$	X	X	✓

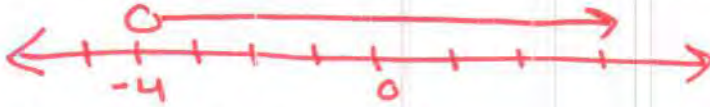
$$-3(0) < 12$$

$$0 < 12$$

10. Write the inequality that represents every possible solution to  $-3a < 12$ .

$$a > -4$$

11. Sketch a number line to represent all possible solutions to the inequality.



Consider the inequality  $h - 5 \geq -9$ .

12. Solve the associated equation. What is your boundary point? Is it a solution to the inequality?

$$h - 5 = -9$$

$$\begin{array}{r} +5 \quad +5 \\ \hline h = -4 \end{array}$$

$$h = -4$$

Yes, it is.

$$-4 - 5 \geq -9$$

$$-9 \geq -9$$

13. Test a value less than the boundary and greater than the value.

$$-5 - 5 \geq -9$$

$$-10 \geq -10 \quad \text{X}$$

$$0 - 5 \geq -9$$

$$-5 \geq -9 \quad \checkmark$$

14. Write the inequality that represents every possible solution to  $h - 5 \geq -9$ .

$$h \geq -4$$

15. Sketch a number line to represent all possible solutions to the inequality.



Consider the inequality  $-3x \leq 18$ .

16. Write the inequality that represents the solution. Sketch a number line to represent all possible solutions.

$$-3x \leq 18$$

$$\begin{array}{r} \div -3 \quad \div -3 \\ \hline x \geq -6 \end{array}$$

$$-3(-10) \leq 18$$

$$30 \leq 18 \quad \text{X}$$

$$-3(0) \leq 18$$

$$0 \leq 18 \quad \checkmark$$

$$x \geq -6$$



1.

a. Graph the solutions to  $9m \leq 54$  on the number line. Show all work.

$$\frac{9m}{9} = \frac{54}{9}$$

$$m = 6 \checkmark$$

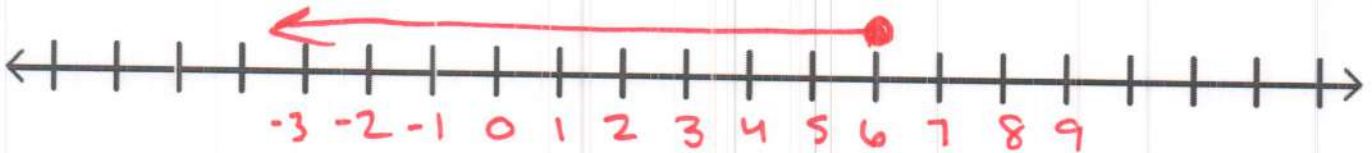
$$9(0) \leq 54 \checkmark$$

$$0 \leq 54$$

$$9(10) \leq 54 \text{ X}$$

$$90 \leq 54 \text{ X}$$

$$m \leq 6$$



b. Graph the solutions to  $-9m \leq 54$  on the number line. Show all work.

$$\frac{-9m}{-9} = \frac{54}{-9}$$

$$m = -6 \checkmark$$

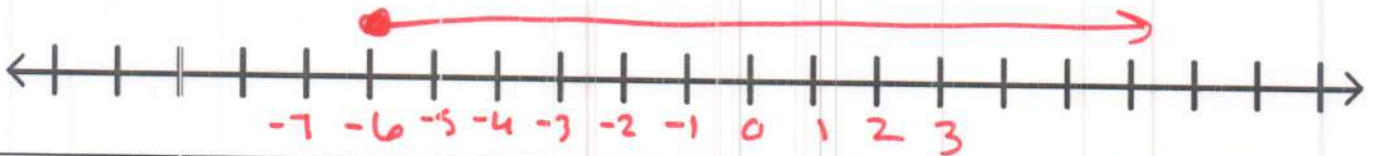
$$-9(-10) \leq 54$$

$$90 \leq 54 \text{ X}$$

$$-9(0) \leq 54$$

$$0 \leq 54 \checkmark$$

$$m \geq -6$$



2. Find the solution to  $n + 8 > -8$ . Write the solution as an inequality.

$$\frac{n+8}{-8} = \frac{-8}{-8}$$

$$n = -16 \text{ X}$$

$$-20 + 8 > -8$$

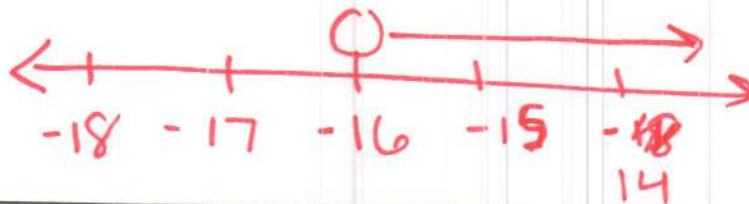
$$-12 > -8 \text{ X}$$

$$0 + 8 > -8$$

$$8 > -8 \checkmark$$

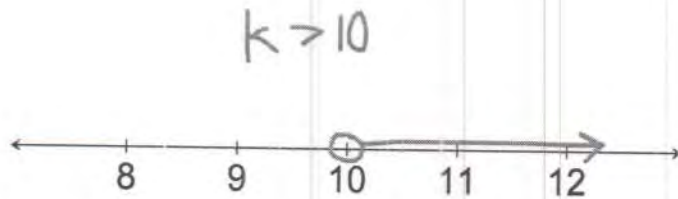
$$n > -16$$

Sketch a number line to represent all possible solutions to the inequality.





3. Jackie was trying to find the solution to  $-7k > -70$ . Her solution and number line are shown below.



Explain why Jackie's solution is unreasonable. Include the correct answer in your response.

If I substitute in 11 or 12, the **FALSE** inequality is not true.  $-7(11) > -70 \rightarrow -77 > -70$   
 The correct answer  $-7(12) > -70 \rightarrow -84 > -70$  is  $k < 10$ .

4. Look at each inequality. Solve each. List at least 2 values that make each inequality true and at least 2 values that make each inequality false.

$y + 4 < -6$	$-2w \geq 20$
$y < -10$	$w \leq -10$
<p><b>TRUE</b></p> <p><math>y = -100</math></p> <p><math>y = -11</math></p>	<p><math>w = -10</math></p> <p><math>w = -20</math></p>
<p><b>FALSE</b></p> <p><math>y = 10</math></p> <p><math>y = 0</math></p>	<p><math>w = 0</math></p> <p><math>w = 4</math></p>

# **G7 U5 Lesson 15**

Match an inequality to a situation it represents, explain what the parts of the inequality mean, solve it, and then interpret what the solution means in the situation.

**G7 U5 Lesson 15 - Students will match an inequality to a situation it represents, explain what the parts of the inequality mean, solve it, and then interpret what the solution means in the situation.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our previous lesson, we worked on solving inequalities by using the associated equation. For instance, if the inequality was  $x + 1 < 9$ , the associated equation would just be  $x + 1 = 9$ . (*write both inequality and equation, and solve as you narrate*) We could subtract 1 from both sides to see that  $x = 8$  is our boundary point. How did we use the boundary point to help us think about the solution to the inequality? **Possible Student Answers, Key Points:**

$$\begin{array}{r} x + 1 < 9 \\ x + 1 = 9 \\ \underline{-1 \quad -1} \\ x = 8 \end{array}$$

- The boundary point is the point we would start at on a number line model.
- We tested values above and below the boundary to know which inequality symbol best represents the solution.

We used the boundary point to test values greater than and less than the boundary point to determine the inequality symbol to use in the solution. If we mentally substitute 0 as a value less than the boundary point, I know  $0 + 1$  is less than 9. If I mentally substitute 10 as a value greater than the boundary point, I know  $10 + 1$  is *not less than* 9. That means the solution to the inequality would be  $x < 8$ , not  $x > 8$ .

Today, we'll use a lot of that same thinking, but we'll apply it to inequalities that represent real-world situations. As we work, we'll make sure to connect the math we're doing back to the context of the problem.

**Let's Talk (Slide 3):** Speaking of contexts, here is an example of a context we might see in inequality problems. Read it to yourself as I read it aloud. Then summarize what this scenario is about in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This situation is about the number of cakes a bakery sells on Saturday and Sunday.
- I know they sold 3 more cakes on Sunday than Saturday. I know they sold more than 12 cakes Sunday.
- I don't know how many cakes they sold on Saturday.

Thinking about the information in the story, take a second a look at the four inequalities we see here. Which one best matches the story? How do you know? If you're not 100% sure, are there any you can eliminate? **Possible Student Answers, Key Points:**

- I know the bakery sold greater than 12 cakes, so I know it won't be the blue or orange inequalities. Those symbols mean less than or less than and equal to.
- I think it's the green inequality,  $c + 3 > 12$ . The "c" represents the cakes on Saturday, the "+ 3" represents the 3 addition cakes made on Sunday, and the  $> 12$  shows that the value is more than 12.

The inequality  $c + 3 > 12$  best represents the information in this story. The value of  $c + 3$  must be greater than 12. The inequality  $c + 3 \geq 12$  doesn't make sense, since the story says they sold greater than, not equal to, 12 cakes on Sunday.

In today's problems, we'll read each problem carefully to write an inequality that matches the context. Then we'll use the associated equation to help us solve the inequality. Let's get going with our first of two problems.

**Let's Think (Slide 4):** Read this problem to yourself as I read it aloud. Then summarize what it's about in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This situation is about Kevin earning money from his daily wages and from commission from selling shoes.

- I know he earned more than \$250. I know he earned \$90 in wages. I know he sold 10 pairs of shoes and earns commission on each pair he sells.
- I don't know how many pairs of shoes he could have sold.

Based on the information in this story, we can write an inequality. I know he earns \$90. He also earns commission money from each pair of shoes he sells, and he sold 10 pairs. I can represent the amount he earns from commission with the expression  $10x$ . I know the total amount he makes is greater than \$250.

$$90 + 10x > 250$$

(write  $90 + 10x > 250$ ) This expression represents each part of the story. We see his wages plus the commission must total greater than 250 dollars.

$$\begin{array}{r} 90 + 10x = 250 \\ -90 \quad -90 \\ \hline \end{array}$$

Let's solve. What's the associated equation we can solve? ( $90 + 10x = 250$ )

$$\frac{10x}{10} = \frac{160}{10}$$

(write it and solve as you narrate) The associated equation is easy to write, because we can just swap the inequality symbol temporarily for an equal sign. I know in this equation I can subtract 90 from both sides. That leaves me with  $10x = 160$ . I can divide both sides by 10, and I see that  $x = 16$ . This value is our boundary point.

$$x = 16$$

Let's substitute a value less than 16 and a value more than 16 to determine which inequality symbol to use in our answer. I'll pick 0 and 20, since those are easy to work with. You can choose whichever numbers you like, as long as one is less than the boundary point and one is greater than the boundary point.

$$90 + 10(0) > 250$$

$$90 + 10(20) > 250 \checkmark$$

(rewrite the inequality substituting in 0 for  $x$ , and rewrite the inequality substituting 20 in for  $x$ ) Use mental math or scratch paper to determine which value makes the inequality true. Possible Student Answers, Key Points:

- 0 does not work, because  $10(0)$  is 0.  $90 + 0$  is not greater than 250.
- 20 does work, because  $10(20)$  is 200.  $90 + 200$  is greater than 250.

$$x > 16$$

Since the value greater than our boundary point made the inequality true, we can write the solution to this problem as  $x > 16$ . (write it) Think back to the word problem. What does a solution of  $x > 16$  mean in context? Possible Student Answers, Key Points:

- We didn't know how much Kevin could earn from each pair of shoes he sells. A solution of  $x > 16$  means that Kevin earns more than \$16 off of each pair of shoes sold.

Kevin earns greater than \$16 for each pair of shoes he sells. Based on the inequality, he could earn \$16.01, \$17, \$25, or any number greater than \$16. Nice work! Let's try another problem with a different context.

**Let's Think (Slide 5):** Read this second problem to yourself as I read it aloud. Then summarize what it's about in your own words. What is known? What is unknown? Possible Student Answers, Key Points:

- This situation is about how long it takes George to hike from one elevation to another elevation.
- I know his starting elevation is 21 feet. I know his elevation decreases 3 feet every minute. I know the trail ends at an elevation of -27 feet and that he hasn't quite made it there yet.
- I don't know how many minutes he hikes.

$$21 - 3m > -27$$

We can write an inequality to represent this situation. (write  $21 - 3m > -27$  as you narrate what each component means) He starts at +21 feet. He descends 3 feet per minute, which I can show as  $-3m$ . The problem says he hasn't reached the final elevation of -27 feet, so I'll write that he is *greater* than -27 feet since his elevation would be higher up.



$$\begin{array}{r}
 21 - 3m = -27 \\
 \underline{-21 \quad -21} \\
 -3m = -48 \\
 \underline{-3 \quad -3} \\
 m = 16
 \end{array}$$

Let's use an associated equation to help us solve the inequality. I can write  $21 - 3m = -27$  as the related equation. (*write and solve as you narrate*) I'll subtract 21 from both sides to isolate the variable. What is  $-27$  minus 21? ( $-48$ ) The updated equation now reads  $-3m = -48$ . What can I do next to isolate the variable? (*divide both sides of the equation by  $-3$* ) The solution to the associated equation is  $m = 16$ .

16 is the boundary point for this problem. Let's test any number less than 16 and any number greater than 16 to determine the solution to the inequality. Let's use 0 and 20 again, since those are easy numbers to work with that fit the criteria.

(*rewrite the inequality substituting in 0 for  $m$ , and rewrite the inequality substituting 20 in for  $m$* ) Use mental math or scratch paper to determine which value makes the inequality true.

$$\begin{array}{l}
 21 - 3(0) > -27 \quad \checkmark \\
 21 - 3(20) > -27
 \end{array}$$

Possible Student Answers, Key Points:

- 0 works, because  $3(0)$  is 0. 21 minus 0 is greater than  $-27$ .
- 20 does NOT work, because  $3(20)$  is 60. 21 minus 60 is  $-39$  which is not greater than  $-27$ .

$$m < 16$$

Since the value less than the boundary point makes the inequality true, I know the solution to the inequality is  $m < 16$ . (*write  $m < 16$* )

What does this solution mean in the context of the problem? What values would make the inequality true?

Possible Student Answers, Key Points:

- The inequality  $m < 16$  means that George hikes for less than 16 minutes.
- He could have hiked for 15 minutes, or 10 minutes, or 1 minute. Any value less than 16 makes sense for this problem.

Nice work!

**Let's Try it (Slides 6 - 7):** We just used what we know about a word problem to write an inequality that matches the story. We then used an associated equation to help us find the boundary point. Lastly, we used values less than and greater than the boundary point to test the inequality so we could pick the correct symbol for our final answer. Now let's do a few more together, before you have the opportunity to work independently.

# WARM WELCOME



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**Today we will match an inequality to a situation it represents, explain what the parts of the inequality mean, and then interpret what the solution means in the situation.**

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## Let's Talk:

A bakery sold some cakes on Saturday. They sold 3 more cakes on Sunday. They sold greater than 12 cakes on Sunday.

$$c + 3 > 12$$

$$c + 3 \geq 12$$

$$c + 3 < 12$$

$$c + 3 \leq 12$$

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## Let's Think:

Kevin earned more than \$250 yesterday working at a shoe store. He made \$90 in wages, plus  $p$  dollars in commission for each of the 10 pairs of shoes he helped sell. How much could Kevin have earned for each pair of shoes?

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## Let's Think:

George begins hiking on a trail at an elevation of 21 feet. He descends 3 feet per minute,  $m$ . He has not yet reached the trail's end which is at an elevation of -27 feet. How many minutes could George have been hiking so far?

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## Let's Try It:

Let's explore matching an inequality to a situation it represents, explaining what the parts of the inequality mean, and interpreting what the solution means together.

Name: \_\_\_\_\_ G7 US Lesson 15 - Let's Try It

A wedding reception can be arranged with one big table that has 15 seats surrounded by several smaller tables that each have 6 seats. If the room for reception can hold at most 81 people, how many smaller tables can be arranged in the room?

- What is known in this problem? What is unknown?
- Which represents a number of people that could attend a reception in this room?
  - a. 100
  - b. 85
  - c. 75
- Write an inequality to represent the situation.
- Solve the associated equation to find the boundary.
- Is your solution to the associated equation a solution to the inequality?
- Test one value below your boundary and one value above your boundary to determine which makes the inequality true. Then write the solution to the inequality.
- Describe what the solution means in context.

The temperature outside starts at 8 degrees Fahrenheit. It decreases 4 degrees every hour, but if it has not reached -32 degrees yet, how many hours could have passed?

- What is known in this problem? Unknown?
- Write an inequality to represent the number of hours that could have passed.
- Solve the associated equation.
- Test one value below your boundary and one value above your boundary to determine which makes the inequality true. Then write the solution to the inequality.
- Describe what the solution means in context.

Nakiyah uses a reloadable gift card to help her keep track of her coffee spending. There is \$75 dollars on the card currently, and she likes to keep at least \$20 on the card at all times. If each coffee costs \$3, how many coffees can Nakiyah buy and still have at least \$20 on the card?

- Write and solve an inequality to represent this situation.
- Describe what the solution means in context.

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## On your Own:

Now it's time to match an inequality to a situation it represents, explain what the parts of the inequality mean, and interpret what the solution means on your own.

Name: \_\_\_\_\_ (37.1.15) Lesson 15 - Independent Work

1. Haaver had \$50 in her savings account. She deposits \$12 every week into the account, and now she has more than 114 dollars in the account. How many weeks,  $w$ , could Haaver have been depositing money?

Could Haaver have been depositing money for 7 weeks? Explain.

2. A local post office can only ship packages that weigh no more than 45 pounds. Sasha is packing a box to ship a birthday gift and some books to her mother. The birthday gift weighs 8 pounds, and each book weighs 2 pounds. Write and solve an inequality to determine how many books Sasha can pack in the box.

What does your solution mean in the context of this problem?

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3. A vending machine contains 250 items. Every hour,  $h$ , 20 items are sold. How many hours could have passed if there are fewer than 80 items left in the machine? Write and solve an inequality to represent the situation.

4. Write a word problem that could be represented by the inequality below. Then solve and explain what your solution means in your chosen context.

$$8 + 1.5n > 50$$

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**A wedding reception can be arranged with one big table that has 15 seats surrounded by several smaller tables that each have 6 seats. If the room for reception can hold at most 81 people, how many smaller tables can be arranged in the room?**

1. What is known in this problem? What is unknown?
2. Which represents a number of people that could attend a reception in this room?
  - a. 100
  - b. 85
  - c. 75

3. Write an inequality to represent the situation.

$$\frac{\text{\# at large}}{\text{table}} + \frac{\text{\# at small}}{\text{tables}} \leq \frac{\text{maximum}}{\text{occupancy}} \frac{\text{of room}}{\text{of room}}$$

4. Solve the associated equation to find the boundary.

5. Is your solution to the associated equation a solution to this inequality?

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6. Test one value *below* your boundary and one value *above* your boundary to determine which makes the inequality true. Then write the solution to the inequality.

7. Describe what the solution means in context.

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**The temperature outside starts at 8 degrees Fahrenheit. It decreases 4 degrees every hour,  $h$ . If it has not reached  $-32$  degrees yet, how many hours could have passed?**

8. What is known in this problem? Unknown?
  
9. Write an inequality to represent the number of hours that could have passed.
  
10. Solve the associated equation.
  
11. Test one value *below* your boundary and one value *above* your boundary to determine which makes the inequality true. Then write the solution to the inequality.
  
12. Describe what the solution means in context.

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**Nakiyah uses a reloadable gift card to help her keep track of her coffee spending. There is \$75 dollars on the card currently, and she likes to keep at least \$20 on the card at all times. If each coffee costs \$3, how many coffees can Nakiyah buy and still have at least \$20 on the card?**

13. Write and solve an inequality to represent this situation.
  
  
  
  
  
  
  
  
  
  
14. Describe what the solution means in context.

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- 1. Heaven had \$30 in her savings account. She deposits \$12 every week into the account, and now she has more than 114 dollars in the account.** How many weeks,  $w$ , could Heaven have been depositing money?

Could Heaven have been depositing money for 7 weeks? Explain.

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- 2. A local post office can only ship packages that weigh no more than 45 pounds. Sasha is packing a box to ship a birthday gift and some books to her mother. The birthday gift weighs 8 pounds, and each book weighs 2 pounds.** Write and solve an inequality to determine how many books Sasha can pack in the box.

What does your solution mean in the context of this problem?

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**3. A vending machine contains 250 items. Every hour,  $h$ , 20 items are sold. How many hours could have passed if there are fewer than 90 items left in the machine? Write and solve an inequality to represent the situation.**

**4. Write a word problem that could be represented by the inequality below. Then solve and explain what your solution means in your chosen context.**

$$8 + 1.5n > 50$$

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A wedding reception can be arranged with one big table that has 15 seats surrounded by several smaller tables that each have 6 seats. If the room for reception can hold at most 81 people, how many smaller tables can be arranged in the room?

1. What is known in this problem? What is unknown?

- total most be 81 or less
- large table = 15 seats
- small tables = 6 seats

# of small tables that will fit

2. Which represents a number of people that could attend a reception in this room?

- a. 100
- b. 85
- c. 75

3. Write an inequality to represent the situation.

$$\frac{15}{\# \text{ at large table}} + \frac{6x}{\# \text{ at small tables}} \leq \frac{81}{\text{maximum occupancy of room}}$$

4. Solve the associated equation to find the boundary.

$$15 + 6x = 81$$

$$\underline{-15 \quad -15}$$

$$\frac{6x}{6} = \frac{66}{6}$$

$$x = 11$$

5. Is your solution to the associated equation a solution to this inequality?

Yes, the room could fit 11

small tables.

6. Test one value *below* your boundary and one value *above* your boundary to determine which makes the inequality true. Then write the solution to the inequality.

$$15 + 6(1) \leq 81$$

$$15 + 6(12) \leq 81$$

$$15 + 6 \leq 81$$

$$15 + 72 \leq 81$$

$$21 \leq 81 \checkmark$$

$$87 \leq 81 \times$$

$$x \leq 11$$

7. Describe what the solution means in context.

The room can handle 11 or fewer

small tables.

The temperature outside starts at 8 degrees Fahrenheit. It decreases 4 degrees every hour, h. If it has not reached -32 degrees yet, how many hours could have passed?

8. What is known in this problem? Unknown?

- starts at  $8^{\circ}$
  - decreases  $4^{\circ}$  each hour
  - not yet  $-32^{\circ}$
- # of hours passed

9. Write an inequality to represent the number of hours that could have passed.

$$8 - 4h > -32$$

10. Solve the associated equation.

$$\begin{array}{r} 8 - 4h = -32 \\ -8 \quad -8 \\ \hline -4h = -40 \end{array}$$

$$\begin{array}{r} -4h = -40 \\ \frac{-4h}{-4} = \frac{-40}{-4} \\ \hline h = 10 \end{array}$$

11. Test one value *below* your boundary and one value *above* your boundary to determine which makes the inequality true. Then write the solution to the inequality.

$$\begin{array}{l} 8 - 4(1) > -32 \\ 8 - 4 > -32 \\ 4 > -32 \checkmark \end{array}$$

$$\begin{array}{l} 8 - 4(11) > -32 \\ 8 - 44 > -32 \\ -36 > -32 \text{ X} \end{array}$$

$$h < 10$$

12. Describe what the solution means in context.

Less than 10 hours have passed if it's not  $-32^{\circ}$  yet.

Nakiyah uses a reloadable gift card to help her keep track of her coffee spending. There is \$75 dollars on the card currently, and she likes to keep at least \$20 on the card at all times. If each coffee costs \$3, how many coffees can Nakiyah buy and still have at least \$20 on the card?

13. Write and solve an inequality to represent this situation.

$$75 - 3x \geq 20$$

$$75 - 3(0) \geq 20$$

$$75 - 3(20) \geq 20$$

$$75 - 3x \geq 20$$

$$75 - 0 \geq 20 \checkmark$$

$$75 - 60 \geq 20$$

$$-3x = -55$$

$$x = 18\frac{1}{3}$$

$$x \leq 18\frac{1}{3}$$

$$15 \geq 20 \text{ X}$$

14. Describe what the solution means in context.

Nakiyah can buy less than or equal to  $18\frac{1}{3}$  cups to keep 20 or more dollars on the card.



1. Heaven had \$30 in her savings account. She deposits \$12 every week into the account, and now she has more than 114 dollars in the account. How many weeks,  $w$ , could Heaven have been depositing money?

$$30 + 12w > 114$$

$$30 + 12w = 114$$

$$12w = 84$$

$$w = 7 \text{ X}$$

$$30 + 12(0) > 114$$

$$30 + 0 > 114 \text{ X}$$

$$30 + 12(10) > 114$$

$$30 + 120 > 114$$

$$150 > 114 \checkmark$$

$$w > 7$$

Could Heaven have been depositing money for 7 weeks? Explain.

No  $7 > 7$  is not a true statement.

7 is not in the solution set.

2. A local post office can only ship packages that weigh no more than 45 pounds. Sasha is packing a box to ship a birthday gift and some books to her mother. The birthday gift weighs 8 pounds, and each book weighs 2 pounds. Write and solve an inequality to determine how many books Sasha can pack in the box.

$$8 + 2x \leq 45$$

$$8 + 2x = 45$$

$$2x = 37$$

$$x = 18\frac{1}{2} \checkmark$$

$$8 + 2(1) \leq 45$$

$$8 + 2 \leq 45 \checkmark$$

$$8 + 2(20) \leq 45$$

$$8 + 40 \leq 45 \text{ X}$$

$$x \leq 18\frac{1}{2}$$

What does your solution mean in the context of this problem?

Sasha can ship less than or equal to  $18\frac{1}{2}$

books, and meet the weight requirements.



3. A vending machine contains 250 items. Every hour,  $h$ , 20 items are sold. How many hours could have passed if there are fewer than 90 items left in the machine? Write and solve an inequality to represent the situation.

$$250 - 20h < 90$$

$$250 - 20h = 90$$

$$-20h = -160$$

$$h = 8 \quad \times$$

$$250 - 20(1) < 90$$

$$250 - 20 < 90 \quad \times$$

$$250 - 20(10) < 90$$

$$250 - 200 < 90 \quad \checkmark$$

$$h > 8$$

4. Write a word problem that could be represented by the inequality below. Then solve and explain what your solution means in your chosen context.

$$8 + 1.5n = 50$$

$$1.5n = 42$$

$$n = 28 \quad \times$$

$$8 + 1.5n > 50$$

$$8 + 1.5(10) > 50$$

$$8 + 15 > 50 \quad \times$$

$$8 + 1.5(100) > 50$$

$$8 + 150 > 50 \quad \checkmark$$

$$n > 28$$

I earned \$1.50 each hour I babysit my brother. If I already have \$8 saved, how many hours must I babysit to have more than 50 dollars?

# **G7 U5 Lesson 16**

Write and solve an inequality to solve real-world problems and critique the solution to an inequality.

## G7 U5 Lesson 16 - Students will write and solve inequality to solve real-world problems and critique the solution to an inequality.

*NOTE: It is permissible for students to use a calculator on this lesson to perform some computation.*

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today is going to feel like a continuation of the work we did in our previous lesson. If you remember, we took a word problem and wrote a corresponding inequality based on what was known and unknown. Once we had the inequality written, we solved an associated equation to find the boundary point. After testing out values less than and greater than the boundary point, we were able to write the solution with the correct inequality symbol.

The work we do today will be similar, but I'm going to challenge use to think extra carefully about the solution in context of the world around us. For instance, if I have a story about the number of dogs at a dog park, and my solution is  $d > 15$ ...would 16.5 make sense as a possible solution? Would 1,000,000 make sense as a possible solution? **Possible Student Answers, Key Points:**

- 16.5 wouldn't make sense, because you can't realistically have 16.5 dogs.
- 1,000,000 wouldn't make sense. It's greater than 15, so it is in the solution set for this inequality, but I highly doubt a dog park would have 1,000,000 dogs in it.

Today we'll work to reason carefully about the solutions to inequalities using what we know about the world around us.

**Let's Talk (Slide 3):** Before we look at today's problems let's briefly consider this story. How do you know that the inequality matches the story? **Possible Student Answers, Key Points:**

- The 3 represents the 3 kids on the playground to begin with. The  $k$  represents the unknown number of kids that joined them. The " $< 8$ " represents the fact that there are less than 8 kids on the playground now.

I can imagine solving this using an associated equation, and seeing that the boundary point is  $k = 5$ . I know the solution to the inequality would be  $k < 5$ . In math, we know  $k < 5$  means any value less than 5 makes the inequality true. Sometimes, because of the context, there are limits to what actually make sense in real life.

What numbers less than 5 could actually make sense in this case? What numbers less than 5 would NOT make sense in this case? **Possible Student Answers, Key Points:**

- 1, 2, 3, or 4 could make sense, because they could represent the number of kids that joined.
- 0 wouldn't make sense, because it says some kids joined.
- $4\frac{1}{2}$  or 1.5 wouldn't make sense because you can't have a fraction of a kid.
- -10 wouldn't make sense, because a negative number of kids doesn't exist.

As we work today, let's make sure we think about the meaning of our inequality in the real-world. We'll reason about which potential solutions make sense when tied to the context and which maybe don't make as much sense, even though they're mathematically correct.

**Let's Think (Slide 4):** Here is the first example we'll work through together. I'll read the problem out loud while you follow along. Then, as usual, I'll ask you to summarize the story in your own words. **Possible Student Answers, Key Points:**

- This problem is about Drake spending money on things for his dog.
- We know he buys dog food for \$15. We know he buys 4 dog toys. We know he spends less than \$36.
- We don't know how much each dog toy costs.

$$15 + 4d < 36$$

$$\begin{array}{r} 15 + 4d = 36 \\ -15 \quad -15 \\ \hline 4d = 21 \\ \frac{4d}{4} = \frac{21}{4} \\ d = 5.25 \end{array}$$

Let's start by writing an inequality that matches this information. (*write inequality as you name each component*) I know he spends \$15 on dog food. In addition, he buys 4 dog toys that cost an unknown amount. I can show that with the expression  $4d$ . I know the total of these items comes out to be less than 36, so I'll write  $< 36$ .

What is the associated equation for the inequality? ( $15 + 4d = 36$ ) Let's solve it to find the boundary point. (*write equation and solve as you narrate*) I'll subtract 15 from both sides of the equal sign. We end up with  $4d = 21$ . If I divide both sides by 4, I know  $d = 5.25$  or  $5 \frac{1}{4}$ . I'll leave it as 5.25 since this problem is about money.

The boundary point is 5.25. Let's test out an easy value less than 5.25 and greater than 5.25 by substituting each value into the original inequality we wrote. I'll use 0 and 6, since 0 is less than 5.25 and 6 is greater than 5.25. Like you've seen me do before, I like to pick numbers that I find are easy to calculate with.

$$15 + 4(0) < 36 \quad \checkmark$$

$$15 + 4(6) < 36$$

than 36.

(*write the inequality twice, substituting in the stated values*) Use scratch paper or mental math to determine which value makes the inequality true. How do you know? **Possible Student Answers, Key Points:**

- 0 makes the inequality true. 4 times 0 is 0, and  $15 + 0$  is less than 36.
- 6 does not make the inequality true. 4 times 6 is 24, and  $15 + 24$  is not less

Since the value less than the boundary point made the original inequality true, I know the solution is  $d < 5.25$ . (*write  $d < 5.25$* ) What does that mean in the context of the problem? **Possible Student Answers, Key Points:**

- We were trying to find the cost for each dog toy. The inequality means the cost of each dog toy must be less than \$5.25.

$$d < 5.25$$

The mathematical meaning of this answer means that any value less than 5.25 is a solution. Can you think of some values that are less than 5.25 that could make sense? Are there values less than 5.25 that do NOT make sense in real life? **Possible Student Answers, Key Points:**

- Any price lower than \$5.25 could make sense. Each toy could cost \$4.99. Each toy could cost \$1. There are a lot of possibilities.
- A price of \$0 might not make sense, because I'm not sure how the store would make any money. A negative number also wouldn't make sense, because the problem is asking about money values.

Great work! Sometimes how we think about an inequality answer mathematically is a little different than how we might apply the inequality in real life. Let's do another.

**Let's Think (Slide 5):** Here is our next problem. I'll read it out loud while you follow along. Then, as usual, I'll ask you to summarize the story in your own words. **Possible Student Answers, Key Points:**

- This problem is about the temperature dropping at a constant rate.
- We know the temperature starts at 105 degrees. We know the temperature drops 8 degrees every hour. We know the temperature should be above 80 degrees.
- We don't know how long it will take the temperature to drop.

$$105 - 8h > 80$$

Let's write an inequality to represent the information in the story. (*write inequality as you talk through each component*) The temperature starts at 105. It drops 8

degrees every hour, I can show that by subtracting 8h. The temperature ends up being above 80 degrees, so I'll write  $> 80$  to finish the inequality.

$$\begin{array}{r} 105 - 8h = 80 \\ -105 \quad -105 \\ \hline -8h = -25 \\ -8 \quad -8 \\ \hline h = 3\frac{1}{8} \end{array}$$

We can now solve this by thinking about an associated equation. The associated equation is  $105 - 8h = 80$ . Talk me through how you would solve this, and I'll write out what you say. (solve equation as student shares, supporting as needed) [Possible Student Answers, Key Points:](#)

- We can subtract 105 from both sides first. 80 minus 105 is -25. Rewrite the equation as  $-8h = -25$ .
- I can divide both sides by -8. -25 divided by -8 is 3.125 or  $3\frac{1}{8}$ .

The solution to the associated equation is 3.125 or  $3\frac{1}{8}$ . That's the boundary point. Before we write the answer to the inequality with a symbol, let's test a value less than and a value greater than the boundary point so we know which sign to use in the answer. Let's use 0 and 5, since those are easy to work with.

(write the inequality twice, substituting in the stated values) Use scratch paper or mental math to determine which value makes the inequality true. How do you know? [Possible Student Answers, Key Points:](#)

$$\begin{array}{l} 105 - 8(0) > 80 \checkmark \\ 105 - 8(5) > 80 \end{array}$$

- 0 makes the inequality true. 8 times 0 is 0, and 105 - 0 is greater than 80.
- 5 does not make the inequality true. 8 times 5 is 40, and 105 minus 40 is not greater than 80.

$$h < 3\frac{1}{8}$$

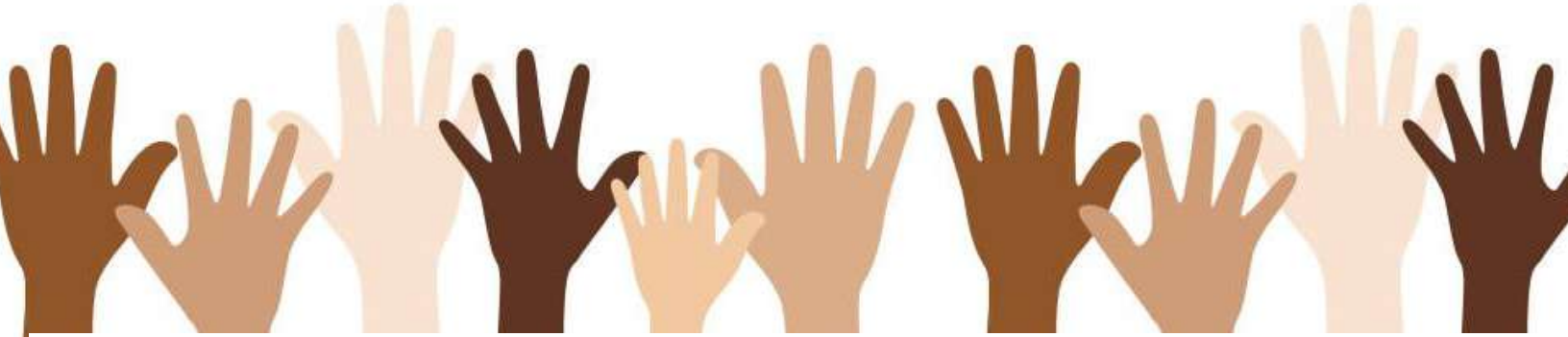
The solution to the inequality is  $h < 3\frac{1}{8}$  or  $h < 3.125$ . (write solution) We know this means that our solution can be any number less than  $3\frac{1}{8}$ . Thinking about the problem's context, can you think of any possible solutions that wouldn't make sense in reality? [Possible Student Answers, Key Points:](#)

- A negative number would be confusing to think about, because our unknown was the number of hours that have gone by. An answer like -10 is technically in a solution, but doesn't actually make much sense when I think about the story.

You did a fantastic job helping me write and solve the inequalities. In addition to the solving work, we also challenged ourselves to think critically about the meaning of possible solutions as they relate to the limitations of the world around us.

**Let's Try it (Slides 6 - 7):** Now let's try out a few more similar problems together. With each problem, it will be important to carefully think about what is known and unknown, so that we can write an accurate inequality. We'll solve the inequality by using an associated equation and thinking about the boundary point. In some cases, we'll also have to think about whether the mathematical solution has any limits based on the context of the story. Sometimes an answer is mathematically correct, but does not make sense in the real-world. When we're done with these next few examples, you'll get a chance to try some independently.

# WARM WELCOME



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**Today we will write and solve an inequality to solve real-world problems and critique the solution to an inequality.**

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**Let's Talk:**

**3 kids were on the playground. Some more kids showed up. Now, there are less than 8 kids on the playground.**

$$3 + k < 8$$

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**Let's Think:**

**Drake buys dog food for \$15 and 4 dog toys for  $d$  dollars each. He spent less than \$36. Write and solve an inequality to represent the cost of each dog toy.**

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## Let's Think:

The temperature in Phoenix, Arizona this afternoon was 105 degrees. The temperature decreased at a rate of 8 degrees per hour. How many hours could have passed if the temperature is above 80 degrees?

Does -2 make sense as a solution in this context? Does 0 make sense as a solution?

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## Let's Try It:

Let's explore writing and solving an inequality to solve real-world problems and critiquing the solution to an inequality together.

Name: \_\_\_\_\_ G7 US Lesson 16 - Let's Try It

A freight elevator can hold a maximum of 2,000 pounds. A delivery person weighs 160 pounds. Each package he loads onto the elevator weighs 23 pounds. How many packages can the delivery person load onto the elevator with him safely?

1. What is known in this problem?
2. What is unknown in this problem?
3. Complete the inequality to represent this problem:
 

$\frac{\text{weight of}}{\text{delivery}} \times$	$\frac{\text{weight of}}{\text{package}}$	$\leq$	$\frac{\text{maximum}}{\text{weight for}} \frac{\text{elevator}}$
---	---	--------	---
4. Solve the inequality.
5. Represent the solution on a number line:
6. What does your solution mean in the context of this problem?
7. Does 70  $\frac{1}{2}$  make sense as a possible solution to this problem? Explain.

Mr. Robinson has a budget of \$128 to spend on supplies for a class celebration. Every cupcake costs \$4 and he wants to have at least \$50 left over for decorations. How many cupcakes can Mr. Robinson order?

8. What is known in this problem? Unknown?
9. Write an inequality to represent the problem.
10. Solve the inequality.
11. Sketch a number line to represent the solution set.
12. Does -10 make sense as a solution in this problem? Explain.

Rachel has 66 inches of trim that she wants to use to form the border of a rectangular piece of artwork. She does not necessarily need to use all the trim. If she wants the artwork to be 7 inches tall, what widths can she choose for the piece?

13. Write and solve an inequality to represent the situation.
14. Does 0 make sense as a solution in this problem? Explain.

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## On your Own:

Now it's time to write and solve an inequality to solve real-world problems and critique the solution to an inequality on your own.

Name: \_\_\_\_\_ G7 U5 Lesson 16 - Independent Work

1. Solve each inequality. Sketch a number line to model each inequality's solution.

a.  $-8n > -64$

b.  $-6n < -60$

c.  $5n > -60$

2. Mrs. Dzija is making ice cubes. The water in the ice cube trays is originally 72 degrees Fahrenheit. In the freezer, the water cools at a rate of 7 degrees per hour. She takes the ice cube tray out when the temperature is below 30 degrees. How many hours could Mrs. Dzija have had the ice cube trays in the freezer? Write and solve an inequality.

Does 6 hours make sense as a solution to this problem? Explain.

\_\_\_\_\_

\_\_\_\_\_

3. A company charges \$50 to rent a motorcycle per day in addition to a one-time fee of \$25 for insurance. How many days can Vance rent a motorcycle if he wants to spend no more than \$385?

Does -1 make sense as a solution to this problem? Explain.

\_\_\_\_\_

\_\_\_\_\_

4. Beatriz wants to know how many episodes of her favorite TV show she can download onto her tablet. The tablet can store 64 gigabytes of data, but 20 GB are already being used by other applications. Each TV episode is 2 GB. Beatriz used the inequality  $28 + 2x \geq 64$  to determine how many episodes she can download.

a. Explain why her inequality will not work to solve the problem.

\_\_\_\_\_

\_\_\_\_\_

b. Correct her work.

\_\_\_\_\_

\_\_\_\_\_

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**A freight elevator can hold a maximum of 2,000 pounds. A delivery person weighs 160 pounds. Each package he loads onto the elevator weighs 23 pounds. How many packages can the delivery person load onto the elevator with him safely?**

1. What is known in this problem?

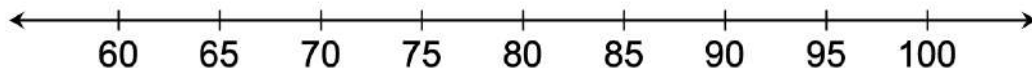
2. What is unknown in this problem?

3. Complete the inequality to represent this problem.

$$\frac{\text{weight of}}{\text{delivery}} \frac{\text{person}}{\text{}} + \frac{\text{weight of}}{\text{packages}} \frac{\text{}}{\text{}} \leq \frac{\text{maximum}}{\text{weight for}} \frac{\text{elevator}}{\text{}}$$

4. Solve the inequality.

5. Represent the solution on a number line.



6. What does your solution mean in the context of this problem?

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7. Does  $70 \frac{1}{2}$  make sense as a possible solution to this problem? Explain.

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**Mr. Robinson has a budget of \$128 to spend on supplies for a class celebration. Every cupcake costs \$4 and he wants to have at least \$50 left over for decorations. How many cupcakes can Mr. Robinson order?**

8. What is known in this problem? Unknown?

9. Write an inequality to represent this problem.

10. Solve the inequality.

11. Sketch a number line to represent the solution set.

12. Does -10 make sense as a solution in this problem? Explain.

---

---

**Rachel has 65 inches of trim that she wants to use to form the border of a rectangular piece of artwork. She does not necessarily need to use all the trim. If she wants the artwork to be 7 inches tall, what widths can she choose for the piece?**

13. Write and solve an inequality to represent the situation.

14. Does 0 make sense as a solution in this problem? Explain.

---

---

**1. Solve each inequality. Sketch a number line to model each inequality's solution.**

a.  $-6n \geq -60$

b.  $-6n < -60$

c.  $6n > -60$

**2. Mrs. Dzija is making ice cubes. The water in the ice cube trays is originally 72 degrees Fahrenheit. In the freezer, the water cools at a rate of 7 degrees per hour. She takes the ice cube tray out when the temperature is below 30 degrees. How many hours could Mrs. Dzija have had the ice cube trays in the freezer? Write and solve an inequality.**

Does 6 hours make sense as a solution to this problem? Explain.

---

---



**3. A company charges \$60 to rent a motorcycle per day in addition to a one-time fee of \$25 for insurance.** How many days can Vance rent a motorcycle if he wants to spend no more than \$385?

Does -1 make sense as a solution to this problem? Explain.

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---

**4. Beatriz wants to know how many episodes of her favorite TV show she can download onto her tablet. The tablet can store 64 gigabytes of data, but 29 GB are already being used by other applications. Each TV episode is 2 GB. Beatriz used the inequality  $29 + 2x \geq 64$  to determine how many episodes she can download.**

a. Explain why her inequality will not work to solve the problem.

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b. Correct her work.

A freight elevator can hold a maximum of 2,000 pounds. A delivery person weighs 160 pounds. Each package he loads onto the elevator weighs 23 pounds. How many packages can the delivery person load onto the elevator with him safely?

1. What is known in this problem?

- elevator holds  $\leq 2000$  lbs
- delivery person = 160 lbs
- boxes weigh 23 lbs

2. What is unknown in this problem?

How many boxes can safely fit on the elevator.

3. Complete the inequality to represent this problem.

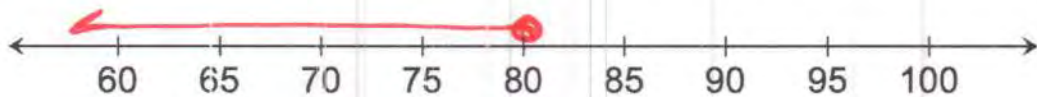
$$\frac{160}{\text{weight of delivery person}} + \frac{23x}{\text{weight of packages}} \leq \frac{2000}{\text{maximum weight for elevator}}$$

4. Solve the inequality.

$$\begin{array}{r} 160 + 23x \leq 2000 \\ -160 \quad -160 \\ \hline 23x \leq 1840 \\ \frac{23x}{23} \leq \frac{1840}{23} \end{array}$$

$$x \leq 80$$

5. Represent the solution on a number line.



6. What does your solution mean in the context of this problem?

The elevator can hold less than or equal to 80 boxes safely.

7. Does  $70\frac{1}{2}$  make sense as a possible solution to this problem? Explain.

It does mathematically, but not in real life.  $70\frac{1}{2}$  "packages" doesn't make much sense.

Mr. Robinson has a budget of \$128 to spend on supplies for a class celebration. Every cupcake costs \$4 and he wants to have at least \$50 left over for decorations. How many cupcakes can Mr. Robinson order?

8. What is known in this problem? Unknown?

- budget = \$128
  - cupcakes cost \$4
  - must have  $\geq 50$  dollars left
- # of cupcakes

9. Write an inequality to represent this problem.

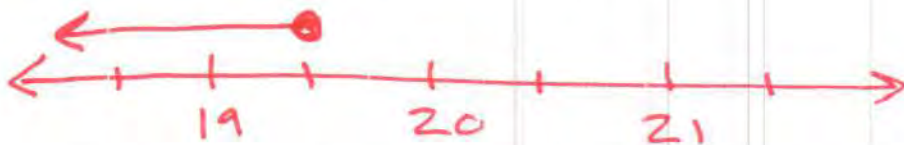
$$128 - 4x \geq 50$$

10. Solve the inequality.

$$\begin{array}{r} 128 - 4x \geq 50 \\ -128 \quad -128 \\ \hline -4x \geq -78 \end{array}$$

$$x \leq 19\frac{1}{2}$$

11. Sketch a number line to represent the solution set.



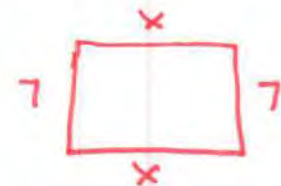
12. Does -10 make sense as a solution in this problem? Explain.

It does not, because -10 cupcakes is odd to consider.

Rachel has 65 inches of trim that she wants to use to form the border of a rectangular piece of artwork. She does not necessarily need to use all the trim. If she wants the artwork to be 7 inches tall, what widths can she choose for the piece?

13. Write and solve an inequality to represent the situation.

$$\begin{array}{r} 2x + 14 \leq 65 \\ 2x \leq 51 \\ x \leq 25\frac{1}{2} \end{array}$$



14. Does 0 make sense as a solution in this problem? Explain.

No, because the art can't have a width of 0.

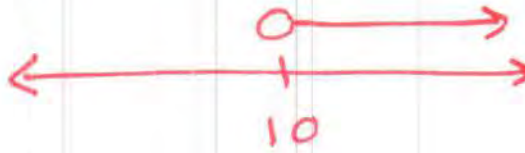


1. Solve each inequality. Sketch a number line to model each inequality's solution.

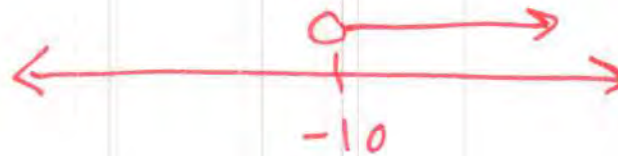
a.  $\frac{-6n}{-6} \geq \frac{-60}{-6}$   
 $n \leq 10$



b.  $\frac{-6n}{-6} < \frac{-60}{-6}$   
 $n > 10$



c.  $\frac{6n}{6} > \frac{-60}{6}$   
 $n > -10$



2. Mrs. Dzija is making ice cubes. The water in the ice cube trays is originally 72 degrees Fahrenheit. In the freezer, the water cools at a rate of 7 degrees per hour. She takes the ice cube tray out when the temperature is below 30 degrees. How many hours could Mrs. Dzija have had the ice cube trays in the freezer? Write and solve an inequality.

$$\begin{array}{r} 72 - 7h < 30 \\ -72 \quad -72 \\ \hline -7h < -42 \\ \frac{-7}{-7} \quad \frac{-7}{-7} \\ \hline n > 6 \end{array}$$

Does 6 hours make sense as a solution to this problem? Explain.

No, because 6 is not greater than 6.

3. A company charges \$60 to rent a motorcycle per day in addition to a one-time fee of \$25 for insurance. How many days can Vance rent a motorcycle if he wants to spend no more than \$385?

$$\begin{array}{r} 60d + 25 \leq 385 \\ -25 \quad -25 \\ \hline 60d \leq 360 \\ \underline{60} \quad \underline{60} \\ d \leq 6 \end{array}$$

Does -1 make sense as a solution to this problem? Explain.

No, because -1 can't represent a number of days.

4. Beatriz wants to know how many episodes of her favorite TV show she can download onto her tablet. The tablet can store 64 gigabytes of data, but 29 GB are already being used by other applications. Each TV episode is 2 GB. Beatriz used the inequality  $29 + 2x \geq 64$  to determine how many episodes she can download.

- a. Explain why her inequality will not work to solve the problem.

The tablet can hold  $\leq 64$  gigabytes, not  $\geq 64$  like she wrote.

- b. Correct her work.

$$\begin{array}{r} 29 + 2x \leq 64 \\ 2x \leq 35 \\ \underline{\quad} \quad \underline{\quad} \\ x \leq 17.5 \end{array}$$

# **G7 U5 Lesson 17**

Extend the distributive property to expressions with negative coefficients.



**G7 U5 Lesson 17 - Students will extend the distributive property to expressions with negative coefficients.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** For the past several lessons, we've been working to solve inequalities. Today will feel like we're switching gears a bit, as we start our last series of lessons in this unit. The work we explore today and in subsequent lessons will have us thinking about different ways to write equivalent expressions. Our focus for today will be on a property you've likely worked with since elementary school: The Distributive Property. Let's get started!

**Let's Talk (Slide 3):** For part of our work today, it will be helpful to use addition. This is because addition is commutative, meaning we can rearrange the order of terms without impacting the value of an expression. That's not true when we subtract, so it's often easier to rearrange expressions with addition.

As we've seen previously, we can rewrite subtraction expressions by using addition. Let's refresh on that idea, so we're ready to tackle today's problems.

$$\begin{array}{l} 8-7 \\ 8+(-7) \end{array} \quad \begin{array}{l} -2.4-3.5 \\ -2.4+(-3.5) \end{array}$$
$$\begin{array}{l} 1-6 \\ 1+(-6) \end{array} \quad \begin{array}{l} 1\frac{1}{2}-\frac{3}{4} \\ 1\frac{1}{2}+(-\frac{3}{4}) \end{array}$$

Here we see 4 subtraction expressions. How could I use the additive inverse to help me rewrite each as a subtraction expression? (*rewrite each as shown while the student shares out, supporting as needed*). Possible Student

Answers, Key Points:

- I can rewrite  $8-7$  as  $8+(-7)$ . Adding a negative is the same as subtracting.
- I can rewrite  $1-6$  as  $1+(-6)$ . I can rewrite  $-2.4-3.5$  as  $-2.4+(-3.5)$ . I can rewrite  $1\frac{1}{2}-\frac{3}{4}$  as  $1\frac{1}{2}+(-\frac{3}{4})$ .

Let's keep this handy skill in mind as we tackle our first problem...

**Let's Think (Slide 4):** This problem wants us to use the distributive property to write an equivalent expression. When we use the distributive property, we distribute, or multiply, the term outside parentheses by each term inside the parentheses.

$$\begin{array}{l} 5(-2y-4) \\ 5(-2y+(-4)) \end{array}$$
$$\begin{array}{|c|c|} \hline -10y & -20 \\ \hline \end{array}$$
$$-10y + -20$$
$$\textcircled{-10y - 20}$$

It's not always required, but let's start by rewriting the subtraction expression inside parentheses as addition. Then it will be easy to organize our work along a tape diagram. What is  $-2y-4$  as an addition expression? ( $-2y+(-4)$ ) (*rewrite expression*)

(*draw a rectangular area model partitioned into two smaller rectangles*) Let's label this area model using the expression. I'll place the coefficient of 5 on the left, and the two terms we're adding in parentheses on top of each rectangle. (*label as described*)

Now, we can multiply. What is 5 times  $-2y$ ? ( $-10y$ ) What is 5 times  $-4$ ? ( $-20$ )

We can write those inside the rectangles. I can combine those two products to write our equivalent expression. (*write  $-10y + -20$* ) I can also write this as  $-10y - 20$ .

We just successfully used the distributive property by organizing our work in an area model to rewrite the original expression as an equivalent expression. Great job!

**Let's Think (Slide 5):** Let's look at one more example. We're going to do pretty much the same process, but this problem looks different. What do you notice is the same and different about this problem compared to the previous example? **Possible Student Answers, Key Points:**

- It's similar because there is a number outside of the parentheses that we can distribute.
- It's different because I see the coefficient is a fraction. The coefficient is a negative number. There are three terms inside parentheses.

$$-\frac{1}{2}(10 - 6w - 4)$$

$$-\frac{1}{2}(10 + (-6w) + (-4))$$

Let's use what we know to tackle this one. I'll start by rewriting the expression inside parentheses as an addition expression. (*rewrite as shown*)  $10 - 6w$  is the same as 10 plus negative 6w. And instead of  $-4$ , I can write plus  $-4$ .

$-\frac{1}{2}$	$10$ $-5$	$-6w$ $+3w$	$-4$ $+2$
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$-5 + 3w + 2$   
 $3w + -5 + 2$   
 $3w - 3$

We need to distribute  $-\frac{1}{2}$  to each term inside parentheses. We can set our problem up with an area model to help keep track of each part. I'll draw a box and partition it into three parts so that each term has its own dedicated space. How can I set up the area model with the numbers from the problem? (*label area model as student shares*)

**Possible Student Answers, Key Points:**

- Put the coefficient of  $-\frac{1}{2}$  on the side of the area model.
- Put 10 on top of the first box,  $-6w$  on the second, and  $-4$  on the third.

Now, we multiply. (*fill each product in the corresponding rectangle*) I know  $-\frac{1}{2}$  times 10 is  $-10/2$  or just  $-5$ . What is  $-\frac{1}{2}$  times  $-6w$ ? ( $3w$ ) What is  $-\frac{1}{2}$  times  $-4$ ? ( $2$ ) We've multiplied each term by the negative coefficient, so now let's write our expression.

(*write expressions as you narrate*) The expression I can write is  $-5 + 3w + 2$ . I notice we can combine the number terms, so I'll rearrange the terms so the numbers are near each other. I can write that as  $3w + -5 + 2$ . From here, I can combine  $-5 + 2$  which is  $-3$ . My expression can be  $3w + -3$  or just  $3w - 3$ .

Even though this expression had a fractions, a negative coefficient, and more terms than the previously problem, the same thinking helped us arrive at an equivalent, simpler expression.

**Let's Try it (Slides 6 - 7):** We'll collaborate on a few more problems together before you get a chance to show what you know independently. When we rewrite expressions using the distributive property, it can help to rewrite the expression inside the parentheses as an addition problem to help us set up an area model. Then, we carefully multiply each term inside parentheses by the term on the outside. If we have terms that we can combine, which isn't always the case, we can rearrange the resulting expression to help us easily combine them. Let's use what we've practiced on a few more examples.


# WARM WELCOME



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**Today we will extend the distributive property to expressions with negative coefficients.**

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 **Let's Talk:**

$$8 - 7$$

$$1 - 6$$

$$-2.4 - 3.5$$

$$1 \frac{1}{2} - \frac{3}{4}$$

**Can we write these expressions using addition instead of subtraction?**

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 **Let's Think:**

**Use the distributive property to write an expression equivalent to  $5(-2y - 4)$ .**

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# Let's Think:

Use the distributive property to write an expression equivalent to  $\frac{1}{2}(10 - 6w - 4)$ .



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# Let's Try It:

Let's explore extending the distributive property to expressions with negative coefficients together.

Name: \_\_\_\_\_ G7 US Lesson 17 - Let's Try It

**Rewrite each subtraction expression as an addition expression.**

1.  $6 - 15$       2.  $8\frac{1}{4} - 2\frac{1}{4}$       3.  $-34.2 - 17.9$

**Consider the expression  $6\frac{1}{4} + 14 - 3\frac{1}{4}$ .**

4. Rewrite the expression so it uses only addition. Use the commutative property of addition to rearrange the expression so like units are next to each other.

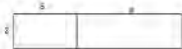
5. Find the value of the expression.

6. How did rewriting the expression help you arrive at the answer?

7. Use a similar strategy to find the value of the expression below:  
 $\frac{1}{4} + 1\frac{1}{2} - 5\frac{3}{4}$

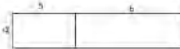
**Look at the dimensions labeled on the composed rectangle.**

8. Complete the expressions to represent the area of the large rectangle.



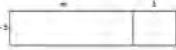
$2 \cdot ( \quad + \quad )$   
 $( \quad + \quad ) + ( \quad + \quad )$   
 $( \quad ) + ( \quad )$

9. Look at the new rectangle. Notice how the width is labeled as  $-2$  now. Use the distributive property, like in #11, to write and evaluate a similar expression.



**This area model has an unknown value,  $m$ .**

10. Complete the expressions to represent the area of the large rectangle.



$( \quad ) + ( \quad )$   
 $( \quad ) + ( \quad )$

**Liam wrote the expression  $4(-3y - 9 + 2)$  to represent an area model.**

11. Rewrite the expression using addition.

12. Draw an area model to represent the equation.

13. Use the distributive property to write a simplified, equivalent expression.

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# On your Own:

Now it's time to extend the distributive property to expressions with negative coefficients on your own.

Name: \_\_\_\_\_ 37 US Lesson 17 - Independence Work

1. Rewrite each expression using addition.

a.  $8 + 5$

b.  $-11.3 + 14.58$

c.  $3\frac{1}{4} + 4\frac{1}{4} + 9\frac{3}{4}$

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2. Sketch an area model to represent the expression  $4(6 + 4)$ . Then write a simplified, equivalent expression.


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3. Rewrite the expression as an addition expression. Then write a simplified, equivalent expression.

$7(-2y - 9)$

4. Use the distributive property to write a simplified, equivalent expression based on each area model.

$4$     $2x$     $5$        $2x$     $-6$     $-2y$

$-4$        $+$

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5. Chloé drew an area model and wrote an equivalent expression to represent  $-3(5x - 2)$ . Look at her work below.

$5x$     $2$

$-3$        $(-3 \cdot 5x) + (-3 \cdot 2)$

$= -15x + -6$

$= -15x - 6$

What mistake did Chloé make? Include the correct work in your response.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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**Rewrite each subtraction expression as an addition expression.**

1.  $6 - 15$

2.  $8 \frac{3}{4} - 2 \frac{1}{5}$

3.  $-34.2 - 17.9$

**Consider the expression  $6 \frac{1}{5} + \frac{1}{4} - 3 \frac{1}{5}$ .**

4. Rewrite the expression so it uses only addition. Use the commutative property of addition to rearrange the expression so like units are next to each other.

5. Find the value of the expression.

6. How did rewriting the expression help you arrive at the answer?

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7. Use a similar strategy to find the value of the expression below.

$$\frac{1}{3} + 1\frac{1}{2} - 5\frac{1}{3}$$

**Look at the dimensions labeled on the composed rectangle.**

8. Complete the expressions to represent the area of the large rectangle.



$$\underline{\quad} (\underline{\quad} + \underline{\quad})$$

$$(\underline{\quad} \cdot \underline{\quad}) + (\underline{\quad} \cdot \underline{\quad})$$

$$(\underline{\quad}) + (\underline{\quad})$$

\_\_\_\_\_

9. Look at the new rectangle. Notice how the width is labeled as -2 now. Use the distributive property, like in #11, to write and evaluate a similar expression.



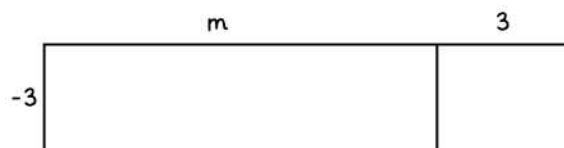
**This area model has an unknown value,  $m$ .**

10. Complete the expressions to represent the area of the large rectangle.

\_\_\_\_\_ ( \_\_\_\_\_ + \_\_\_\_\_ )

\_\_\_\_\_ + \_\_\_\_\_

\_\_\_\_\_



**Liam wrote the expression  $4(-3y - 9 + 2)$  to represent an area model.**

11. Rewrite the expression using addition.

12. Draw an area model to represent the equation.

13. Use the distributive property to write a simplified, equivalent expression.

**1. Rewrite each expression using addition.**

a.  $8 - 5$

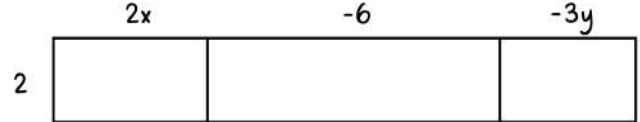
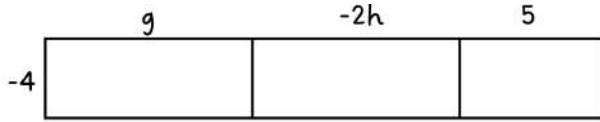
b.  $-11.3 - 14.58$

c.  $3 \frac{1}{2} + 4 \frac{1}{4} - 9 \frac{3}{4}$

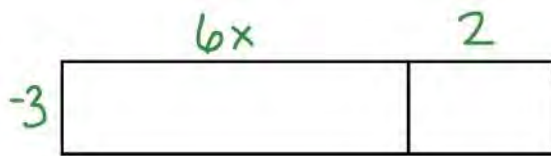
**2. Sketch an area model to represent the expression  $4(4n + 4)$ . Then write a simplified, equivalent expression.****3. Rewrite the expression as an addition expression. Then write a simplified, equivalent expression.**

$$7(-2y - 9)$$

4. Use the distributive property to write a simplified, equivalent expression based on each area model.



5. Chloe drew an area model and wrote an equivalent expression to represent  $-3(6x - 2)$ . Look at her work below.



$$\begin{aligned} &(-3 \cdot 6x) + (-3 \cdot 2) \\ &-18x + -6 \\ &-18x - 6 \end{aligned}$$

What mistake did Chloe make? Include the correct work in your response.

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Name: KEY

Rewrite each subtraction expression as an addition expression.

1.  $6 - 15$

$6 + (-15)$

2.  $8\frac{3}{4} - 2\frac{1}{3}$

$8\frac{3}{4} + (-2\frac{1}{3})$

3.  $-34.2 - 17.9$

$-34.2 + (-17.9)$

Consider the expression  $6\frac{1}{5} + \frac{1}{4} - 3\frac{1}{5}$ .

4. Rewrite the expression so it uses only addition. Use the commutative property of addition to rearrange the expression so like units are next to each other.

$6\frac{1}{5} + \frac{1}{4} + (-3\frac{1}{5})$

$6\frac{1}{5} + (-3\frac{1}{5}) + \frac{1}{4}$

5. Find the value of the expression.

$3 + \frac{1}{4} = 3\frac{1}{4}$

6. How did rewriting the expression help you arrive at the answer?

Rearranging made it easy to efficiently combine terms with like units.

7. Use a similar strategy to find the value of the expression below.

$\frac{1}{3} + 1\frac{1}{2} - 5\frac{1}{3}$

$\frac{1}{3} + 1\frac{1}{2} + (-5\frac{1}{3})$

$\frac{1}{3} + (-5\frac{1}{3}) + 1\frac{1}{2} \rightarrow -5 + 1\frac{1}{2} \rightarrow 3\frac{1}{2}$

Look at the dimensions labeled on the composed rectangle.

8. Complete the expressions to represent the area of the large rectangle.

2 (5 + 6)

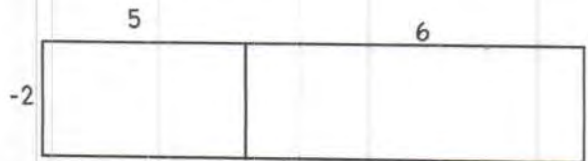
(2 · 5) + (2 · 6)

(10) + (12)

22



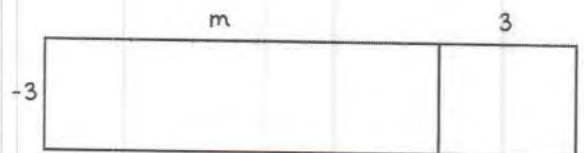
9. Look at the new rectangle. Notice how the width is labeled as -2 now. Use the distributive property, like in #11, to write and evaluate a similar expression.



$$\begin{aligned} &(-2 \cdot 5) + (-2 \cdot 6) \\ &-10 + (-12) \\ &\underline{-22} \end{aligned}$$

This area model has an unknown value,  $m$ .

10. Complete the expressions to represent the area of the large rectangle.



$$\begin{aligned} &\underline{-3} (\underline{m} + \underline{3}) \\ &\underline{-3m} + \underline{-9} \\ &\underline{-3m - 9} \end{aligned}$$

Liam wrote the expression  $4(-3y - 9 + 2)$  to represent an area model.

11. Rewrite the expression using addition.

$$4(-3y + -9 + 2)$$

12. Draw an area model to represent the equation.



13. Use the distributive property to write a simplified, equivalent expression.

$$\begin{aligned} &(4 \cdot -3y) + (4 \cdot -9) + (4 \cdot 2) \\ &-12y - 36 + 8 \\ &\underline{-12y - 28} \end{aligned}$$



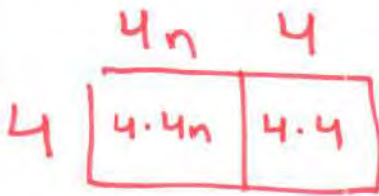
1. Rewrite each expression using addition.

a.  $8 - 5$       $8 + (-5)$

b.  $-11.3 - 14.58$       $-11.3 + (-14.58)$

c.  $3\frac{1}{2} + 4\frac{1}{4} - 9\frac{3}{4}$       $3\frac{1}{2} + 4\frac{1}{4} + (-9\frac{3}{4})$

2. Sketch an area model to represent the expression  $4(4n + 4)$ . Then write a simplified, equivalent expression.



$$16n + 16$$

3. Rewrite the expression as an addition expression. Then write a simplified, equivalent expression.

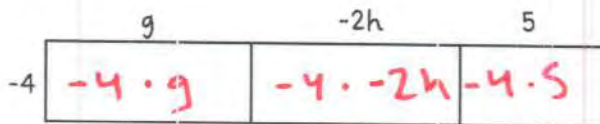
$$7(-2y - 9)$$

$$7(-2y + -9)$$

$$-14y + -63$$

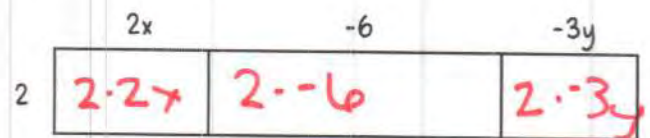
$$-14y - 63$$

4. Use the distributive property to write a simplified, equivalent expression based on each area model.



$$-4g + 8h + -20$$

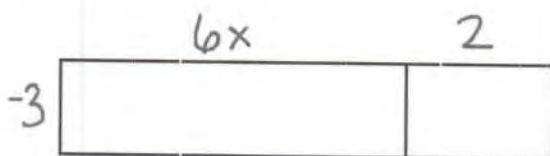
$$-4g + 8h - 20$$



$$4x + -12 + -6y$$

$$4x - 6y - 12$$

5. Chloe drew an area model and wrote an equivalent expression to represent  $-3(6x - 2)$ . Look at her work below.



$$(-3 \cdot 6x) + (-3 \cdot 2)$$

$$-18x + -6$$

$$-18x - 6$$

What mistake did Chloe make? Include the correct work in your response.

Her expression isn't  $-3(6x+2)$  like her area model shows. The 2 should be  $-2$ .

$$(-3 \cdot 6x) + (-3 \cdot -2)$$

$$-18x + +6$$

$$-18x + 6$$

# **G7 U5 Lesson 18**

Use the distributive property to find equivalent expressions by expanding or factoring.

**G7 U5 Lesson 18 - Students will use the distributive property to find equivalent expressions by expanding or factoring.**

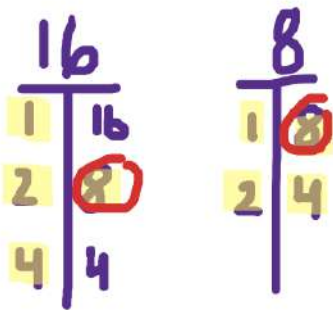
**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** In our last lesson together, we explored writing equivalent expressions using the distributive property. What are some things you know or remember about using the distributive property? **Possible Student Answers, Key Points:**

- When we use the distributive property we multiply the number outside the parentheses by each term inside the parentheses.
- We can use an area model to organize our work when using the distributive property.

We saw that we can use the distributive property even if we have a negative coefficient outside of the parentheses. We used area models to help organize our thinking. Today, we'll continue thinking about the distributive property, but almost in reverse. Today, we're going to divide out a factor to write an equivalent expression. Before we officially start, let's revisit what it means when we're asked to find factors.

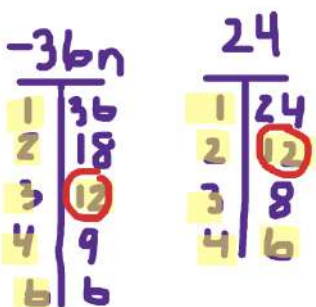
**Let's Talk (Slide 3):** Factors are numbers we can multiply together to get another number. This slide prompts us to find the greatest common factor of two terms.



To do that, let's start by finding all factor pairs for each number. *(sketch a t-chart to organize factors of 16 and another to organize factors for 8)* I'll systematically think through all factors pairs for 16. *(write them in the t-chart as you think aloud)* I know 1 times 16 is 16, so 1 and 16 are a factor pair. I know 2 times 8 is 16. I know 4 times 4 is 16. These are all the factor pairs for 16. Help me find all the factor pairs that can make 8. *(complete chart as student shares thinking)* **Possible Student Answers, Key Points:**

- 1 times 8 equals 8, so 1 and 8 are factors.
- 2 and 4 are factors, because  $2 \times 4 = 8$ .

We listed out all factor pairs for each term. Now we can find the factors they have in common. *(highlight all common factors)* I see both terms have factors of 1, 2, 4, and 8. These are all common factors. Since the prompt asks us to find the *greatest* common factor, or GCF, I just need to pick the common factor with the greatest value. The greatest common factor of 16 and 8 is 8. The GCF is 8.



Let's try one more. This one has a term that is negative and has a variable. For the purposes of today, we can focus just on factoring the number, since the other term doesn't have a variable or a negative in common. I'm going to systematically list all the factors of 36 in a t-chart. *(list as you narrate)* I know  $1 \times 36 = 36$ . I know  $2 \times 18 = 36$ . I know  $3 \times 12 = 36$ . I know  $4 \times 9 = 36$ . There isn't a 5 fact that makes 36, so I can skip to the next number. I know  $6 \times 6$  equals 36. 36 has a lot of factors, so it was helpful to think through them sequentially so I didn't miss any.

What are the factors of 24? *(list in t-chart as student shares)* **Possible Student Answers, Key Points:**

- The factor pairs for 24 are 1 and 24, 2 and 12, 3 and 8, and 4 and 6.

*(highlight all the common factors)* 36 and 24 have many common factors. Which would you say is the greatest common factor, or the GCF? (12) Great! Of all the factors 36 and 24 have in common, 12 is the greatest common factor. The GCF of  $-36n$  and 24 is 12.

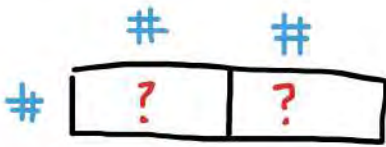


How would you describe to a friend how to find the greatest common factor of two terms? [Possible Student Answers, Key Points:](#)

- To find the greatest common factor, it can help to list out all the factor pairs for each term.
- Once you have all the factor pairs, identify the factors the terms have in common. The greatest common factor will be the common factor with the greatest value.

Now, let's use this skill to factor some expressions. In a way, it might feel like we're distributing in reverse.

**Let's Think (Slide 4):** This problem wants us to use the distributive property to write the expression in factored form. You'll notice, this looks different than other problems that have asked us to use the distributive property. Usually, we multiply a number outside parentheses by an expression inside parentheses. Here, we're actually going to use these terms to pull out a greatest common factor. An area model can help show you what I mean.



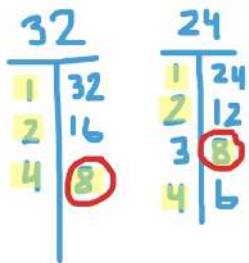
*(sketch and label area model as shown in first example)* When we've used an area model to distribute before, we've had a coefficient on the side of the area model and the terms of our expression on top of the area model. Then, we multiply each term by the coefficient to write the equivalent expression. We use multiplication to find what goes inside the area model.



*(sketch and label area model as shown in second example)* Today, we're being given the expression that typically goes inside the area model. We're going to find and factor out the GCF to rewrite the expression. It's like we know what goes inside our area model, and we'll work backwards to find out

the factors on the outside.

Let's start by factoring  $-32h$  and  $24$  to find the greatest common factor, or GCF. Like earlier, when factoring  $-32h$ , let's just focus on the number in the term. *(sketch t-charts for factors of 32 and 24)* What are the factor pairs for 32? Try to work in order so that you don't miss any. [Possible Student](#)



[Answers, Key Points:](#)

- I know  $1 \times 32$  is 32,  $2 \times 16$  is 32, and  $4 \times 8$  is 32. The factors are 1, 2, 4, 8, 16, and 32.

*(fill in the factors for 32)* We already found the factors for 24 in a previous example, so I'll just copy those into the chart. *(fill in factors for 24)* What is the greatest common factor of the two terms? (8)

*(write  $-32h$  and  $24$  in the inner boxes of an area model and 8 outside on the left of the area model)* Our original expression was  $-32h + 24$ , so I'll write that inside the area model. We found the GCF is 8, so I know I can factor 8 out of both terms. I'll write that on the side of the area model where the coefficient usually goes. All we have left to do is think about what other factors remain.



*(fill in missing factors as you narrate)* To find the first factor, I can think  $8 \times ?$  is equal to  $-32h$ . Or I could think  $-32h$  divided by 8 is equal to what? I know the missing factor is  $-4h$ . To find the second factor, I can think  $8 \times ? = 24$  or 24 divided by 8 equals what? The other missing factor is 3.

$$8(-4h + 3)$$

So, I can rewrite this expression as  $8(-4h + 3)$ . This is the factored form of the expression we started with. *(write expression)*

We just factored out the GCF of the two terms in the expression we were given to rewrite it using the distributive property. Let's try one more example using the same thinking.

**Let's Think (Slide 5):** This problem gives us the same directions with a different expression.

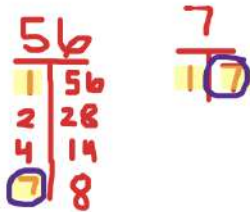
$$56x + -7$$

Let's start by rewriting this expression as an addition expression. What would that look like? **Possible Student Answers, Key Points:**

- I know subtracting 7 is the same as adding -7.
- We can rewrite the expression as  $56x + -7$ .



Since we're going to factor this expression, I'll write each term inside its own section of an area model. *(sketch a rectangular area model partitioned into two sections, and write one term in each)*



Now we'll figure out the greatest common factor for each term. Working systematically, what are the factors of 56 and 7? *(list each set of factors in a t-chart as student shares)* **Possible Student Answers, Key Points:**

- The factors of 56 are 1 and 56, 2 and 28, 4 and 14, and 7 and 8.
- The factors of 7 are just 1 and 7.

The common factors are 1 and 7, which means the greatest common factor for our two terms is 7.



$$7(8x - 1)$$

We can factor a 7 out of both terms, so I'll write 7 on the side of the area model. *(label 7 on the left side of the area model)* To find the first missing factor, I can think 7 times what equals  $56x$ , or I can think  $56x$  divided by 7 equals what? The missing factor is  $8x$ . *(label area model)* How could I find the other missing factor? **Possible Student Answers, Key Points:**

- I can think  $7 \times ? = -7$ . The missing factor is -1.
- I can think  $-7$  divided by  $7 = ?$ . The missing factor is -1.

We factored out the GCF of 7, and were left with  $8x$  and  $-1$  as the other factors. We can write the equivalent expression as  $7(8x + -1)$  or just  $7(8x - 1)$ .

When the terms in an expression have a common factor, we can write an equivalent fraction by factoring out the common factor using the distributive property. An area model can help organize our work and help us keep track of the factors.

**Let's Try it (Slides 6 - 7):** Now let's do a few more examples where we factor to write equivalent expressions using the distributive property. As we factor expressions, make sure to systematically work to find the greatest common factor of both terms. We saw today that an area model can be a helpful way to keep track of our thinking. After we do the next few problems together, you'll get a chance to show what you know on your own.




# WARM WELCOME



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**Today we will use the distributive property to find equivalent expressions by expanding or factoring.**

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 Let's Talk:

**16, 8**

**-36n, 24**

**What is the  
greatest common  
factor?**

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 Let's Think:

**Use the distributive property to write the expression  
in factored form.**

**-32h + 24**

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# Let's Think:

Use the distributive property to write the expression in factored form.

# $56x - 7$

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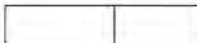


# Let's Try It:

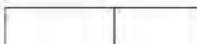
Let's explore using the distributive property to find equivalent expressions by expanding or factoring together.

Name: \_\_\_\_\_ G7 US Lesson 18 - Let's Try It

**Consider the expression  $3(c + 5)$ .**

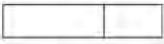
- Label the area model to represent the expression. 
- Use the distributive property to write an equivalent expression.
- What are the two factors in your rewritten expression?

**Consider the expression  $10n + 35$ .**

- Write the terms inside the area model. 
- What factor do  $10n$  and  $35$  have in common?
  - 2
  - 5
  - 10
- Write that common factor on the left side of the area model.
- 5 times what number has a product of  $10n$ ? Write that above the first rectangle.
- 5 times what number has the product of  $35$ ? Write that above the second rectangle.
- Fill in the blanks based off of your work to write an equivalent expression to  $10n + 35$ .  
 $\underline{\hspace{1cm}} (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$

**You just factored  $10n + 35$  to write an equivalent expression!**

**Consider the expression  $9x - 12$ .**

- Rewrite the expression as an addition expression.  
 $\underline{\hspace{1cm}} + \underline{\hspace{1cm}}$
- Fill in the diagram with the expression. 
- Find the greatest common factor of  $9x$  and  $-12$ . Label it on the side of the area model.
- Find the other factors based on what you know. Write them above each smaller rectangle in the area model.
- Write the factors in a new, equivalent expression.  
 $\underline{\hspace{1cm}} (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$
- Write this expression in another equivalent way using subtraction.  
 $\underline{\hspace{1cm}} (\underline{\hspace{1cm}} - \underline{\hspace{1cm}})$

**Consider the expression  $-24w + 6$ .**

- Sketch an area model and write the two terms of the expression inside each rectangle.
- Find the greatest common factor. Then find the other two factors. Label your area model.
- Write the equivalent expression to  $-24w + 6$  based on the factors you found.

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# On your Own:

Now it's time to use the distributive property to find equivalent expressions by expanding or factoring on your own.

Name: \_\_\_\_\_ G7 US Lesson 18 - Independent Work

1. Find the greatest common factor of each pair of terms.

a. 14, 12

b. 21, 29

c.  $-9y$ , 23

2. Rewrite each subtraction expression as an addition expression. Then use the distributive property to write an equivalent expression. Sketch an area model if that helps you arrive at your answer.

$-(4y - 2)$                        $-(9n + 17)$

3. For each area model, find the missing factors. Then, write an equivalent expression.

a.

$16x$	$6$
-------	-----

b.

$-4k$	$10$
-------	------

4. Jim was trying to use the distributive property to factor the expression  $49x - 28$ . He started making the area model below, but realized he made a mistake.

$49x$	$-28$
-------	-------

What mistake did Jim make? Correct this mistake, and then use the distributive property to factor the expression.

\_\_\_\_\_

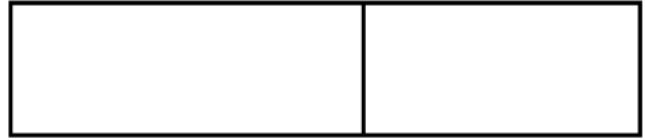
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**Consider the expression  $3(c + 5)$ .**

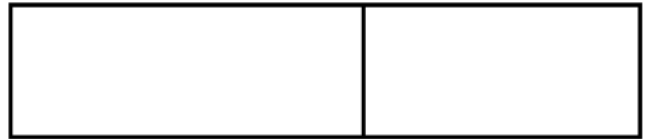
1. Label the area model to represent the expression.
2. Use the distributive property to write an equivalent expression.



3. What are the two terms in your rewritten expression?

**Consider the expression  $10n + 35$ .**

4. Write the terms inside the area model.
5. What factor do  $10n$  and  $35$  have in common?
  - a. 2
  - b. 5
  - c. 10



6. Write that common factor on the left side of the area model.
7. 5 times what number has a product of  $10n$ ? Write that above the first rectangle.
8. 5 times what number has the product of  $35$ ? Write that above the second rectangle.
9. Fill in the blanks based off of your work to write an equivalent expression to  $10n + 35$ .  
\_\_\_\_\_ ( \_\_\_\_\_ + \_\_\_\_\_ )

**You just factored  $10n + 35$  to write an equivalent expression!**

**Consider the expression  $9x - 12$ .**

10. Rewrite the expression as an addition expression.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

11. Fill in the diagram with the expression.



12. Find the greatest common factor of  $9x$  and  $-12$ .  
Label it on the side of the area model.

13. Find the other factors based on what you know. Write them above each smaller rectangle in the area model.

14. Write the factors in a new, equivalent expression.

$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}})$$

15. Write this expression in another equivalent way using subtraction.

$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}} - \underline{\hspace{2cm}})$$

**Consider the expression  $-24w + 6$ .**

16. Sketch an area model and write the two terms of this expression inside each rectangle.

17. Find the greatest common factor. Then find the other two factors. Label your area model.

18. Write the equivalent expression to  $-24w + 6$  based on the factors you found.



**1. Find the greatest common factor of each pair of terms.**

a. 14, 12

b. 21, 28

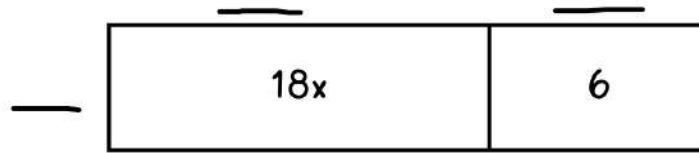
c.  $-6y$ , 33**2. Rewrite each subtraction expression as an addition expression. Then use the distributive property to write an equivalent expression. Sketch an area model if that helps you arrive at your answer.**

$$4(-3y - 2)$$

$$-5(m - 17)$$

3. For each area model, find the missing factors. Then, write an equivalent expression.

a.



b.



4. Jim was trying to use the distributive property to factor the expression  $49x - 28$ . He started making the area model below, but realized he made a mistake.



What mistake did Jim make? Correct the mistake, and then use the distributive property to factor the expression.

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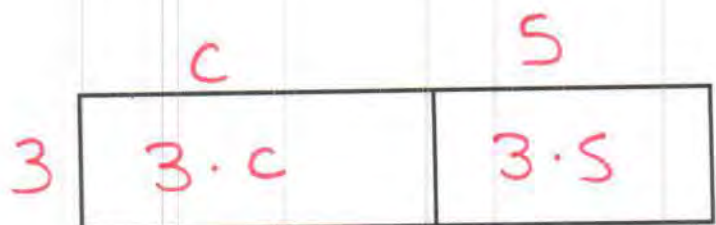
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Name: KEY

Consider the expression  $3(c + 5)$ .

1. Label the area model to represent the expression.



2. Use the distributive property to write an equivalent expression.

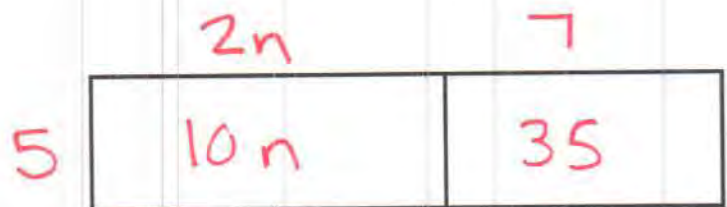
$$3c + 15$$

3. What are the two <sup>terms</sup> factors in your rewritten expression?

$3c$  and  $15$

Consider the expression  $10n + 35$ .

4. Write the terms inside the area model.



5. What factor do  $10n$  and  $35$  have in common?

- a. 2
- b. 5**
- c. 10

6. Write that common factor on the left side of the area model. ✓

7. 5 times what number has a product of  $10n$ ? Write that above the first rectangle.

$$5 \times ? = 10n$$

8. 5 times what number has the product of  $35$ ? Write that above the second rectangle.

$$5 \times ? = 35$$

9. Fill in the blanks based off of your work to write an equivalent expression to  $10n + 35$ .

$$\underline{5} (\underline{2n} + \underline{7})$$

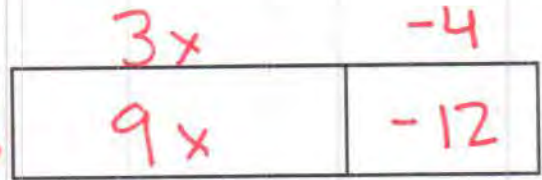
**You just factored  $10n + 35$  to write an equivalent expression!**

Consider the expression  $9x - 12$ .

10. Rewrite the expression as an addition expression.

$$\underline{9x} + \underline{-12}$$

11. Fill in the diagram with the expression.



12. Find the greatest common factor of  $9x$  and  $-12$ .

Label it on the side of the area model.

13. Find the other factors based on what you know. Write them above each smaller rectangle in the area model.

14. Write the factors in a new, equivalent expression.

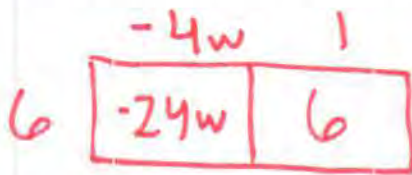
$$\underline{3} (\underline{3x} + \underline{-4})$$

15. Write this expression in another equivalent way using subtraction.

$$\underline{3} (\underline{3x} - \underline{4})$$

Consider the expression  $-24w + 6$ .

16. Sketch an area model and write the two terms of this expression inside each rectangle.



17. Find the greatest common factor. Then find the other two factors. Label your area model.

18. Write the equivalent expression to  $-24w + 6$  based on the factors you found.

$$6(-4w + 1)$$

1. Find the greatest common factor of each pair of terms.

a. 14, 12

$$\begin{array}{r} 14 \\ 1 \overline{) 14} \\ \underline{14} \\ 0 \end{array}$$

$$\begin{array}{r} 12 \\ 1 \overline{) 12} \\ \underline{12} \\ 0 \end{array}$$

b. 21, 28

$$\begin{array}{r} 21 \\ 1 \overline{) 21} \\ \underline{21} \\ 0 \end{array}$$

$$\begin{array}{r} 28 \\ 1 \overline{) 28} \\ \underline{28} \\ 0 \end{array}$$

c.  $-6y$ , 33

$$\begin{array}{r} 6 \\ 1 \overline{) 6} \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{r} 33 \\ 1 \overline{) 33} \\ \underline{33} \\ 0 \end{array}$$

2. Rewrite each subtraction expression as an addition expression. Then use the distributive property to write an equivalent expression. Sketch an area model if that helps you arrive at your answer.

$$4(-3y - 2)$$

$$4(-3y + -2)$$

$$4 \begin{array}{|c|c|} \hline -3y & -2 \\ \hline -12y & -8 \\ \hline \end{array}$$

$$-12y + -8$$

$$(-12y - 8)$$

$$-5(m - 17)$$

$$-5(m + -17)$$

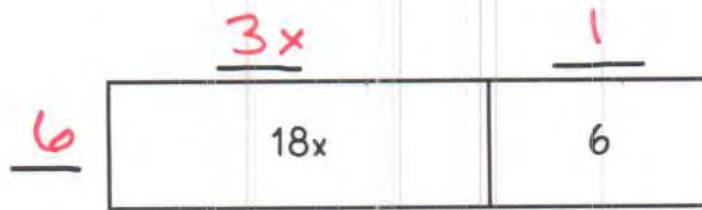
$$-5 \begin{array}{|c|c|} \hline m & -17 \\ \hline -5m & +85 \\ \hline \end{array}$$

$$(-5m + 85)$$



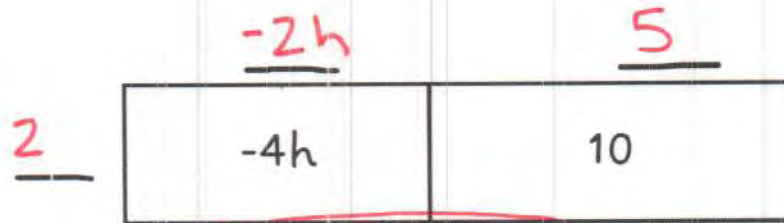
3. For each area model, find the missing factors. Then, write an equivalent expression.

a.



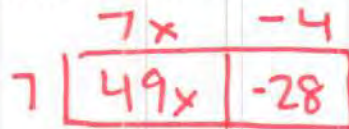
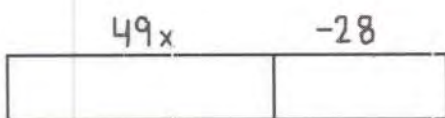
$$6(3x+1)$$

b.



$$2(-2h+5)$$

4. Jim was trying to use the distributive property to factor the expression  $49x - 28$ . He started making the area model below, but realized he made a mistake.



What mistake did Jim make? Correct the mistake, and then use the distributive property to factor the expression.

He wrote the terms in the wrong place.

They go inside the area model when you're

trying to factor. The correct

answer is  $7(7x-4)$ .



## **G7 U5 Lesson 19**

Given an expression, write an equivalent expression with fewer terms using properties of operations, and explain why the expressions are equivalent.

**G7 U5 Lesson 19 - Students will, given an expression, write an equivalent expression with fewer terms using properties of operations, and explain why the expressions are equivalent.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** We've been thinking about and writing equivalent expressions for the past couple lessons. We know that expressions are equivalent when they always represent the same value. For example, I know  $1 + x$  is equivalent to  $x + 1$ . They look a little different, but they'll always represent the same value. Or, when we use the distributive property, we know our expressions end up looking different, but they represent the same value.

Today, we'll continue working to write equivalent expressions. Most of our problems today will involve combining like terms to write expressions with as few terms as possible. Let me show you what I mean...

**Let's Talk (Slide 3):** What do you notice about the expressions shown here? What do you wonder?.

Possible Student Answers, Key Points:

- I notice they all have a, b, and c in them. I notice the first one doesn't show any numbers. I notice one is longer than the other. I notice they represent the same thing.
- I wonder if they are equivalent. I wonder why one only has variables and one has variables and numbers. I wonder what a, b, and c can equal.

These two expressions don't look identical, but they represent equivalent expressions. The first is expanded to show every a, b, and c. The second combines like terms and uses multiplication to show how many groups of a, b, and c there are. *(highlight each variable in a different color)* I see 3 groups of a in the first expression, which is equivalent to  $3a$  in the second expression. I see 4 groups of b in the first expression, which is equivalent to  $4b$

$$a + a + a + b + b + b + b + c$$

in the second expression. I see 1 group of c in both expressions. Both expressions are equivalent, and both are valid ways to write the expression. Why might you want to use one expression over the other? Possible Student Answers, Key Points:

- I might use the first expression if I wanted to clearly see each term.
- I might use the second expression to save time. The numbers also help avoid having to count up the number of each variable.

Today we'll work to combine like terms to write equivalent expressions with as few terms as possible.

**Let's Think (Slide 4):** This prompt wants us to name whether these expressions are equivalent or not. At first glance, they look quite different. Let's see if we can rewrite the longer expressions with fewer terms to determine whether the expressions are equivalent.

$$q + 2r + 3q + 3r$$

$$1q + 3q + 2r + 3r$$

$$4q + 5r$$

*(highlight like terms in similar colors)* I know each term with a "q" represents groups of q. I know each term with an "r" represents groups of r. I can combine the groups of q together, and I can combine the groups of r together. I'll rewrite the expression so that these like terms are adjacent. *(write  $1q + 3q + 2r + 3r$  in similar colors to how they were highlighted)* Now, I'll combine like terms. What is q, or  $1q$ , plus  $3q$ ?  $(4q)$  What is  $2r$  plus  $3r$ ?  $(5r)$  I can rewrite the expression as  $4q + 5r$ . *(write  $4q + 5r$ )*

By rearranging the expression and combining like terms, we were able to write an expression with fewer terms.

Based on our work here, are the two expressions equivalent? How do you know? [Possible Student Answers, Key Points:](#)

- The two expressions are equivalent, because after we combined like terms, we were left with identical expression.
- They are equivalent. Each expression shows 4 groups of  $q$  and 5 groups of  $r$ .

**Let's Think (Slide 5):** Our second problem wants us to write an equivalent expression using as few terms as possible.

$$6g + h + 8 - 2h + 2g - 1$$
$$6g + 2g + h + -2h + 8 + -1$$
$$8g \quad -1h \quad +7$$
$$8g - h + 7$$

I'll start by highlighting like terms that I can combine. I see terms that represent groups of  $g$ , terms that represent groups of  $h$ , and I see some numbers without variables that I can combine. *(highlight each set of like terms using a different color)*

I notice that this expression involves subtraction. To help rearrange terms, let's rewrite the expression using addition in place of subtraction. *(using color-coding, write  $6g + h + 8 + -2h + 2g + -1$ )* Now we can rearrange the terms so like terms are adjacent. *(write  $6g + 2g + h + -2h + 8 + -1$ )*

Now the terms I can combine are side-by-side. How can I combine the terms? [Possible Student Answers, Key Points:](#)

- I know  $6g + 2g$  is  $8g$ .
- I know  $h + -2h$  is  $-h$  or  $-1h$ .
- I know  $8 + -1$  is  $7$ .

*(write  $8g - h + 7$ )* I can write  $8g + -1h + 7$  as  $8g - h + 7$ . I know we've written an equivalent expression with as few terms as possible, because there is nothing left to combine. I can't combine groups of  $g$  with groups of  $h$ , for examples, because they're different units. We took our long expression that we started with that had six different terms, and we rearranged it so we could combine like terms. Our equivalent expressions has just three terms.

**Let's Try it (Slides 6 - 7):** We'll work through a few more examples together, before you get a chance for some independent time. It will be helpful to rearrange expressions to group like terms, and we can color-code the terms in our expressions if we want. Writing any subtraction as addition can make it easier for us to rearrange terms. Let's keep these pointers in mind as we continue working together.

# WARM WELCOME



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**Today we will, given an expression, write an equivalent expression with fewer terms using properties of operations and explain why the expressions are equivalent.**

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**Let's Talk:**

**What do you notice?  
What do you wonder?**

$$a + a + a + b + b + b + b + c$$

$$3a + 4b + c$$

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**Let's Think:**

**Are the expressions equivalent?**

$$4q + 5r$$

$$q + 2r + 3q + 3r$$

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## Let's Think:

Rewrite the expression using as few terms as possible.

$$6g + h + 8 - 2h + 2g - 1$$

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## Let's Try It:

Let's explore writing an equivalent expression with fewer terms using properties of operations together.

Name: \_\_\_\_\_ G7 US Lesson 10 - Let's Try It

Consider the two expressions below. Let's determine if they are equivalent.

$$6x + 3y \qquad 4x + 2y + 2x + y$$

- Evaluate  $6x + 3y$  when  $x = 2$  and  $y = 3$ .
- Evaluate  $4x + 2y + 2x + y$  when  $x = 2$  and  $y = 3$ .
- The two expressions have...
  - the same value.
  - different values.
- Evaluate both expressions using new values for  $x$  and  $y$ . Choose your own values this time.
- The two expressions seem equivalent based on the values we've substituted. To be sure they're equivalent, let's think about the expression  $4x + 2y + 2x + y$ . Rewrite the expression by using repeated addition in place of multiplication.
- Put parentheses around like variables in the expression. Then rewrite the expression combining all the like variables.
- Consider the work you just did, and look back at the original expressions. Are they equivalent? How do you know?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

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Consider the expression.

$$7g + h + 2h - 2g$$

- Expand the expression using repeated addition.
- Rewrite the expression to group all the like variables together.
- Rewrite the expression using the fewest possible terms.

Consider the expression.

$$8n + 6m - 5n + 2m$$

- Rewrite the expression replacing subtraction with adding the opposite.
- Rewrite the addends so the like variables are grouped together.
- Write an equivalent expression with only two terms.

Consider each expression. Rewrite each with the fewest possible terms.

- $3a - 4b + 7a + 2b$
- $-5 + x - 2y + 5x + 8 + 4y$

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Consider the two expressions below. Let's determine if they are equivalent.

$$6x + 3y$$

$$4x + 2y + 2x + y$$

1. Evaluate  $6x + 3y$  when  $x = 2$  and  $y = 3$ .
2. Evaluate  $4x + 2y + 2x + y$  when  $x = 2$  and  $y = 3$ .
3. The two expressions have...
  - a. the same value.
  - b. different values.
4. Evaluate both expressions using new values for  $x$  and  $y$ . Choose your own values this time.
5. The two expressions seem equivalent based on the values we've substituted. To be sure they're equivalent, let's think about the expression  $4x + 2y + 2x + y$ . Rewrite the expression by using repeated addition in place of multiplication.
6. Put parentheses around like variables in the expression. Then rewrite the expression combining all the like variables.
7. Consider the work you just did, and look back at the original expressions. Are they equivalent? How do you know?

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**Consider the expression.**

$$7g + h + 2h - 2g$$

8. Expand the expression using repeated addition.
9. Rewrite the expression to group all the like variables together.
10. Rewrite the expression using the fewest possible terms.

**Consider the expression.**

$$6n + 6m - 5n + 2m$$

11. Rewrite the expression replacing subtraction with adding the opposite.
12. Rewrite the addends so the like variables are grouped together.
13. Write an equivalent expression with only two terms.

**Consider each expression. Rewrite each with the fewest possible terms.**

14.  $3a - 4b + 7a + 2b$

15.  $-5 + x - 2y + 5x + 8 + 4y$

**1. Are the expressions below equivalent?**

$$8a + 3c - 9c - 5a$$

$$3a - 6c$$

How do you know?

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**2. Which expressions are equivalent to the one below? Select all that apply.**

$$4x + 2y + 3x + y$$

- A.  $7x + 3y$
- B.  $7x + 2y$
- C.  $x + x + x + x + x + x + x + y + y + y$
- D.  $x + y + 9$
- E.  $x + y + 10$
- F.  $9xy$
- G.  $10xy$

**3. Consider the expression below.**

$$2g - 1h + 4g + 4h$$

- a. Expand the expression to show repeated addition in place of multiplication. Rearrange terms so that all like variables are grouped.
- b. Write the expression using the fewest terms possible.

**4. Write each expression using as fewest terms as possible.**

$$4n + m - n + 5m$$

$$6x + 3 + 8y - 2x + 7y - 9$$

Consider the two expressions below. Let's determine if they are equivalent.

$$6x + 3y$$

$$4x + 2y + 2x + y$$

1. Evaluate  $6x + 3y$  when  $x = 2$  and  $y = 3$ .

$$6(2) + 3(3) \\ 12 + 9 = 21$$

2. Evaluate  $4x + 2y + 2x + y$  when  $x = 2$  and  $y = 3$ .

$$4(2) + 2(3) + 2(2) + 3 \\ 8 + 6 + 4 + 3 = 21$$

3. The two expressions have...

- a. the same value.  
b. different values.

4. Evaluate both expressions using new values for  $x$  and  $y$ . Choose your own values this time.

$$\begin{array}{l} x=0 \quad y=1 \\ 6(0) + 3(1) \\ 0 + 3 \\ 3 \end{array} \qquad \begin{array}{l} 4(0) + 2(1) + 2(0) + 1 \\ 0 + 2 + 0 + 1 \\ 3 \end{array}$$

5. The two expressions seem equivalent based on the values we've substituted. To be sure they're equivalent, let's think about the expression  $4x + 2y + 2x + y$ . Rewrite the expression by using repeated addition in place of multiplication.

$$(x+x+x+x) + (y+y) + (x+x) + (y)$$

6. Put parentheses around like variables in the expression. Then rewrite the expression combining all the like variables.

$$6x + 3y$$

7. Consider the work you just did, and look back at the original expressions. Are they equivalent? How do you know?

Yes! They both represent 6 groups of  $x$  and 3 groups of  $y$ .



Consider the expression.

$$7g + h + 2h - 2g$$

8. Expand the expression using repeated addition.

$$g + g + g + g + g + g + g + h + h + h + (-g) + (-g)$$

9. Rewrite the expression to group all the like variables together.

$$g + g + g + g + g + g + g + (-g) + (-g) + h + h + h$$

10. Rewrite the expression using the fewest possible terms.

$$5g + 3h$$

Consider the expression.

$$6n + 6m - 5n + 2m$$

11. Rewrite the expression replacing subtraction with adding the opposite.

$$6n + 6m + (-5n) + 2m$$

12. Rewrite the addends so the like variables are grouped together.

$$6n + (-5n) + 6m + 2m$$

13. Write an equivalent expression with only two terms.

$$n + 8m$$

Consider each expression. Rewrite each with the fewest possible terms.

14.  $3a - 4b + 7a + 2b$

$$3a + 7a - 4b + 2b$$
$$(10a - 2b)$$

15.  $-5 + x - 2y + 5x + 8 + 4y$

$$-5 + 8 + x + 5x - 2y + 4y$$
$$(3 + 6x + 2y)$$

1. Are the expressions below equivalent?

$$8a + 3c - 9c - 5a$$

$$3a - 6c$$

$$\underline{8a} + \underline{3c} + \underline{-9c} + \underline{-5a}$$

$$3a + -6c$$

$$3a - 6c$$

How do you know?

Yes! Both expressions show 3 groups of "a" and -6 groups of "c".

2. Which expressions are equivalent to the one below? Select all that apply.

$$\underline{4x} + \underline{2y} + \underline{3x} + \underline{y}$$

A.  $7x + 3y$

B.  $7x + 2y$

C.  $x + x + x + x + x + x + x + y + y + y$

D.  $x + y + 9$

E.  $x + y + 10$

F.  $9xy$

G.  $10xy$

$$7x + 3y$$

3. Consider the expression below.

$$2g - 1h + 4g + 4h$$

- a. Expand the expression to show repeated addition in place of multiplication. Rearrange terms so that all like variables are grouped.

$$\underbrace{g+g+g+g+g+g} + \underbrace{h+h+h+h} + \underbrace{-h}$$

- b. Write the expression using the fewest terms possible.

$$(6g + 3h)$$

4. Write each expression using as fewest terms as possible.

$$4n + m - n + 5m$$

$$4n - n + m + 5m$$

$$(3n + 6m)$$

$$6x + 3 + 8y - 2x + 7y - 9$$

$$6x - 2x + 8y + 7y + 3 - 9$$

$$(4x + 15y - 6)$$

## **G7 U5 Lesson 20**

Write expressions with fewer terms that are equivalent to a given expression that includes negative coefficients and parentheses.

**G7 U5 Lesson 20 - Students will write expressions with fewer terms that are equivalent to a given expression that included negative coefficients and parentheses.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** Today we're going to combine some of the thinking from the past few lessons. We've worked to write equivalent expressions using the distributive property. We've worked to write equivalent expressions by combining like terms. Today, we'll see expressions where we can do *both* steps within the same expression. Let's start with a warm-up dealing with the distributive property.

**Let's Talk (Slide 3):** Take a look at these two expressions. Are they equivalent? How do you know?

Possible Student Answers, Key Points:

- Yes, they're equivalent. I can factor a 4 from the first expression and rewrite it as  $4(a + 2b)$
- Yes, they're equivalent. I can distribute the 4 in the second expression and rewrite it as  $4a + 8b$ .

The expressions shown here are equivalent. We can factor a 4 out of the first expression to give us the second expression, or we can distribute the coefficient of 4 in the second expression to give us the first expression.

Let's distribute the 4 in the second expression, just to make sure. Walk me through how we can distribute the coefficient of 4. (*show arrows to distribute the 4 and rewrite the expression as the student shares*) Possible Student Answers, Key Points:

- Multiply 4 times a. That equals 4a. Multiply 4 times 2b. That equals 8b.
- When you distribute 4 to each term, you end up with  $4a + 8b$ .

Today, we'll use the distributive property, along with other mathematical properties, to help us write equivalent expressions.

**Let's Think (Slide 4):** For this problem, we'll use the distributive property to write an equivalent expression.

(*draw arrows from -9 to 2c and 5*) Let's start by multiplying each term inside parentheses by the coefficient of -9. I'll use an area model to keep our work organized. (*sketch partitioned rectangle for the area model, and label the sides with -9, 2c, and 5 as shown*)

What is -9 times 2c? ( $-18c$ ) What is -9 times 5? ( $-45$ ) We can rewrite this expression as  $-18c + -45$ . (*write expression*) How can we write this expression as a subtraction expression? ( $-18c - 45$ ) We can write it as  $-18c - 45$  since adding negative 45 is the same as subtracting 45.

Are we finished? Can we combine  $-18c$  minus 45? Possible Student Answers, Key Points:

- We cannot combine  $-18c$  and  $-45$ , because they are not like terms.
- We are finished, because I can't combine a term with a c on it with a term without a c.

We just used the distributive property to rewrite  $-9(2c + 5)$  as the equivalent expression  $-18c - 45$ . The area model helped us keep our work organized, and we paid close attention to the signs of our numbers as we rewrote the expression. Let's try one more that's a little similar and a little different.



**Let's Think (Slide 5):** What do you notice is the same or different about this problem compared to the one we just completed? **Possible Student Answers, Key Points:**

- It has numbers and variables. We can use the distributive property to help us rewrite the expression. The directions are the same.
- It has more terms. It only has subtraction. The variable is different.

$$7 - 2(4 - x)$$
$$7 - 2(4 + -x)$$
$$7 - 8 + 2x$$

$$\begin{array}{c} -1 + 2x \\ \text{or} \\ 2x - 1 \end{array}$$

Let's rewrite this expression to use as few terms as possible. I might be tempted to start by subtracting  $7 - 2$ , but the order of operations tells me I should distribute the  $-2$  first. Before I do that, let's rewrite the expression using addition instead of subtraction. Instead of  $7 - 2$ , I can write  $7 + -2$ . Instead of  $4 - x$ , I can write  $4 + -x$ . (*rewrite expression*)

Now we'll distribute  $-2$ . What is  $-2$  times  $4$ ? ( $-8$ ) What is  $-2$  times  $-x$ ? ( $2x$ ) (*rewrite expression as  $7 + -8 + 2x$* ) The expression we just wrote is equivalent to the original expression. I notice the directions said to write the expressions using as few terms as possible. What terms in this expression can I combine? How do you know? **Possible Student Answers, Key Points:**

- You can combine  $7$  and  $-8$ , because they're just numbers.
- You can't combine the  $2x$  with any other term, because no other term represents a group of  $x$ .

I know  $7$  plus  $-8$  is  $-1$ . When I combine those terms, we end up with the expression  $-1 + 2x$ . We could also write this as  $2x + -1$  or  $2x - 1$ .

We just used the distributive property to rewrite this expression. Once we distributed, we noticed we could combine some terms to make a simpler expression. We can combine like terms and use the distributive property to write equivalent expressions.

**Let's Try it (Slides 6 - 7):** Now we'll try a few more problems together. We'll write equivalent expressions using the distributive property. In some cases, we'll combine like terms to write an expression with as few terms as possible. We noticed that it can be helpful to rewrite subtraction as addition, so it's easier to manipulate the terms in an expression. Not only can this help when using the distributive property, but it can also make rearranging terms using the commutative property a bit easier. Once we're done with the next few examples, you'll get a chance to try some out on your own.



# WARM WELCOME



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**Today we will write expressions with fewer terms that are equivalent to a given expression that include negative coefficients and parentheses.**

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**Let's Talk:**

**Are the expressions equivalent?  
How do you know?**

$$4a + 8b$$

$$4(a + 2b)$$

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**Let's Think:**

**Use the distributive property to write an equivalent expression using as few terms as possible.**

$$-9(2c + 5)$$

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# Let's Think:

Use the distributive property to write an equivalent expression using as few terms as possible.

$$7 - 2(4 - x)$$

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# Let's Try It:

Let's explore writing equivalent expressions with fewer terms that are equivalent to a given expression that include negative coefficients and parentheses together.

Name: \_\_\_\_\_ G7 US Lesson 20 Let's Try It

**Consider the expression  $-5(3 + 2x)$ .**

1. Fill in the blanks on the area model to represent the expression.

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2. Use the distributive property to write an equivalent expression.

$( \quad \cdot \quad ) + ( \quad \cdot \quad )$

$\quad + \quad$

$\quad - \quad$

**Consider the expression  $-5(3 - 2x)$ . Note that this expression has subtraction inside the parentheses.**

3. Rewrite the expression using addition.

$-5( \quad + \quad )$

4. Fill in the blanks on the area model to represent the expression.

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5. Use the distributive property to write an equivalent expression.

$( \quad \cdot \quad ) + ( \quad \cdot \quad )$

$\quad + \quad$

$\quad - \quad$

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**Consider the expression  $8 - 4(2 - m)$ .**

6. Which step should you do first to begin writing equivalent expressions? Remember to consider the order of operations.

- Subtract  $4 \cdot 2$
- Multiply to distribute the  $-4$

7. Rewrite the expression to show subtraction as adding the opposite.

$\quad + \quad ( \quad - \quad )$

8. Use the distributive property to write an equivalent expression.

$\quad + ( \quad \cdot \quad ) + ( \quad \cdot \quad )$

$\quad + \quad + \quad$

$\quad - \quad$

**Consider the expression  $8n - 2(2n - 7)$ .**

9. Rewrite the expression using addition.

10. Use the distributive property to write an equivalent expression with the fewest possible terms.

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## On your Own:

Now it's time to write expressions with fewer terms that are equivalent to a given expression that include negative coefficients and parentheses on your own.

Name: \_\_\_\_\_ 07 US1 Lesson 20 – Independent Work

1. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$-6(8 + 3n)$

2. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$-7(1 - 7k)$

3. Marcel rewrote the expression  $5 - 4(6z + 2)$  as  $-1(6z + 2)$ . Explain Marcel's mistake, and show how he could write an equivalent expression using as few terms as possible.

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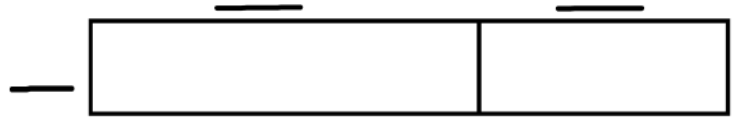
4. Which expressions are equivalent to  $11 - 5(4v - 2)$ ? Select all that apply.

- a.  $6(4v - 2)$
- b.  $11 - 20v + 10$
- c.  $5 + 2v$
- d.  $32v$
- e.  $23(4 - 2)$
- f.  $21 - 20v$
- g.  $-20v + 1$

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Consider the expression  $-5(3 + 2x)$ .

1. Fill in the blanks on the area model to represent the expression.



2. Use the distributive property to write an equivalent expression.

$$(\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

$$\text{_____} + \text{_____}$$

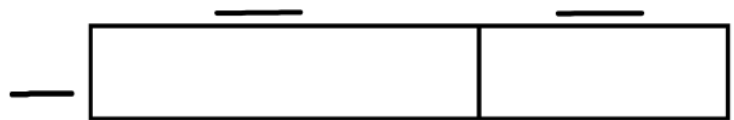
$$\text{_____} - \text{_____}$$

Consider the expression  $-5(3 - 2x)$ . Note that this expression has subtraction inside the parentheses.

3. Rewrite the expression using addition.

$$-5(\text{_____} + \text{_____})$$

4. Fill in the blanks on the area model to represent the expression.



5. Use the distributive property to write an equivalent expression.

$$(\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

$$\text{_____} + \text{_____}$$

Consider the expression  $8 - 4(2 - m)$ .

6. Which step should you do first to begin writing equivalent expressions? Remember to consider the order of operations.
- Subtract 8 - 4
  - Multiply to distribute the -4
7. Rewrite the expression to show subtraction as adding the opposite.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}})$$

8. Use the distributive property to write an equivalent expression.

$$\underline{\hspace{2cm}} + (\underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}}) + (\underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}})$$

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

**Consider the expression  $8n - 2(2n - 7)$ .**

9. Rewrite the expression using addition.
10. Use the distributive property to write an equivalent expression with the fewest possible terms.



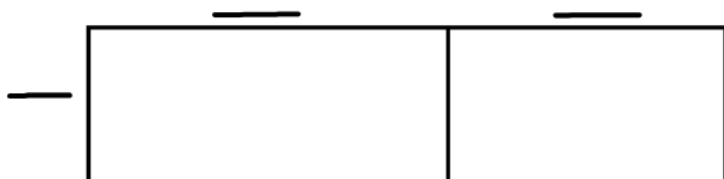
1. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-6(8 + 3m)$$



2. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-7(1 - 7k)$$



**3. Marcel rewrote the expression  $5 - 4(6z + 2)$  as  $-1(6z + 2)$ . Explain Marcel's mistake, and show how he could write an equivalent expression using as few terms as possible.**

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**4. Which expressions are equivalent to  $11 - 5(4v - 2)$ ? Select all that apply.**

- a.  $6(4v - 2)$
- b.  $11 - 20v + 10$
- c.  $6 \cdot 2v$
- d.  $22v$
- e.  $-20v + 21$
- f.  $21 - 20v$
- g.  $-20v + 1$

Consider the expression  $-5(3 + 2x)$ .

1. Fill in the blanks on the area model to represent the expression.



2. Use the distributive property to write an equivalent expression.

$$\begin{aligned} & (\underline{-5} \cdot \underline{3}) + (\underline{-5} \cdot \underline{2x}) \\ & \underline{-15} + \underline{-10x} \\ & \underline{-15} - \underline{10x} \end{aligned}$$

Consider the expression  $-5(3 - 2x)$ . Note that this expression has subtraction inside the parentheses.

3. Rewrite the expression using addition.

$$-5(\underline{3} + \underline{-2x})$$

4. Fill in the blanks on the area model to represent the expression.



5. Use the distributive property to write an equivalent expression.

$$\begin{aligned} & (\underline{-5} \cdot \underline{3}) + (\underline{-5} \cdot \underline{-2x}) \\ & \underline{-15} + \underline{10x} \end{aligned}$$

Consider the expression  $8 - 4(2 - m)$ .

6. Which step should you do first to begin writing equivalent expressions? Remember to consider the order of operations.

a. Subtract  $8 - 4$

b. Multiply to distribute the  $-4$

7. Rewrite the expression to show subtraction as adding the opposite.

$$\underline{8} + \underline{4} (\underline{2} + \underline{-m})$$

8. Use the distributive property to write an equivalent expression.

$$\underline{8} + (\underline{4} \cdot \underline{2}) + (\underline{4} \cdot \underline{-m})$$

$$\underline{8} + \underline{8} + \underline{-4m}$$

$$\underline{16} + \underline{-4m}$$

Consider the expression  $8n - 2(2n - 7)$ .

9. Rewrite the expression using addition.

$$8n - 2(2n + -7)$$

10. Use the distributive property to write an equivalent expression with the fewest possible terms.

$$8n - 4n + 14$$

$$\begin{array}{c} \vee \\ \boxed{4n + 14} \end{array}$$

1. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-6(8 + 3m)$$

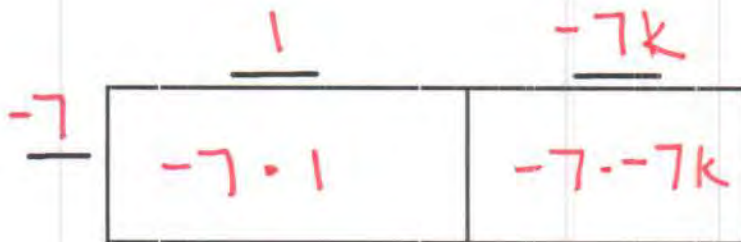


$$-48 + -18m$$

$$(-48 - 18m)$$

2. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-7(1 - 7k)$$



$$(-7 + 49k)$$



3. Marcel rewrote the expression  $5 - 4(6z + 2)$  as  $-1(6z + 2)$ . Explain Marcel's mistake, and show how he could write an equivalent expression using as few terms as possible.

$$5 + \overbrace{-4(6z + 2)}$$

$$5 + -24z + -8$$

$$5 - 24z - 8$$

$$-3 - 24z$$

Marcel did not follow the order of operations. He should distribute before subtracting. The correct expression is  $-3 - 24z$ .

4. Which expressions are equivalent to  $11 - 5(4v - 2)$ ? Select all that apply.

~~a.  $6(4v - 2)$~~

b.  $11 - 20v + 10$

~~c.  $6 + 2v$~~

~~d.  $22v$~~

e.  $-20v + 21$

f.  $21 - 20v$

~~g.  $-20v + 1$~~

$$11 - 20v + 10$$

$$21 - 20v$$



# **G7 U5 Lesson 21**

Write equivalent expressions with fewer terms by combining like terms.

**G7 U5 Lesson 21 - Students will write equivalent expressions with fewer terms by combining like terms.**

**Warm Welcome (Slide 1):** Tutor choice

**Frame the Learning/Connect to Prior Learning (Slide 2):** This is our last lesson of this unit. We've spent a lot of time learning about equations and inequalities. More recently, we've worked through some lessons where we've used properties to write equivalent expressions. What do you remember or what stands out to you as important when thinking about equivalent expressions? **Possible Student Answers, Key Points:**

- Equivalent expressions might look different, but they have the same value.
- We can use the distributive property to rewrite expressions. This can involve multiplying a coefficient by terms inside parentheses, or it can involve factoring out a common factor.
- We can combine like terms to write an equivalent expression with fewer terms.

Great thinking! We've combined like terms, we've factored, and we've multiplied using the distributive property. Today, we'll pull all of this thinking together to rewrite expressions in equivalent forms. Let's get going.

**Let's Talk (Slide 3):** Take a look at the expression here. Which student's work shows an equivalent expression? How do you know? **Possible Student Answers, Key Points:**

- I think Bryson's work is equivalent.
- If you distribute -2 to 3x, you end up with -6x. -2 times -7y is +14y.

$$\begin{array}{l} -2(3x + -7y) \\ -6x + 14y \end{array}$$

Bryson's work correctly shows the distributive property. To help us think about this carefully, we can rewrite  $3x - 7y$  using addition. (*write  $-2(3x + -7y)$* ) Adding  $-7y$  is the same as subtracting  $7y$ . (*draw arrows to represent distributing the  $-2$  to each term*) I know  $-2$  times  $3x$  is  $-6x$ . I know  $-2$  times  $-7y$  is  $14y$ . We can rewrite the expression as  $-6x + 14y$ .

Bryson's work is correct. Amy's thinking is close, but she might have benefitted from paying closer attention to the values of her signed numbers.

Errors with signs can be common if we rush, so we'll want to be extra cautious as we work through today's examples. Let's get started.

**Let's Think (Slide 4):** Here we have two problems that are slightly different. For both, we're asked to rewrite the expressions without parentheses. Let's start with the first one.

$$\begin{array}{l} y + 1(3x + 2y) \\ y + 3x + 2y \\ y + 2y + 3x \\ \textcircled{3y + 3x} \end{array}$$

I notice there are parentheses, but there does not appear to be a coefficient directly outside of the parentheses. We can then assume that this problem is referring to 1 group of  $3x + 2y$ . I'll rewrite the expression placing a 1 as the coefficient to the expression in parentheses. (*rewrite expression as shown*) How can we think about distributing the 1? (*rewrite expression student shares*)

**Possible Student Answers, Key Points:**

- 1 times  $3x$  is  $3x$ . 1 times  $2y$  is  $2y$ .
- We're left with  $y + 3x + 2y$ .

After distributing, we have  $y + 2y + 3x$ . I can combine each of the  $y$  terms, and write their total as  $3y$ . The equivalent expression is  $3y + 3x$ . We distributed and combined like terms to arrive at our answer.

I also see in this example, we could factor out a 3 from both terms. The directions don't ask us to do that, but we'll see problems later today that ask us to think about factoring an expression after we take other steps to rewrite it.

Now look at the second expression. What's the same? What's different? [Possible Student Answers, Key Points:](#)

- It uses the same variables. It has parentheses. It starts with y.
- It's has a minus sign after the instead of a plus sign. Everything else is the same.

Since, like our last example, it appears as though there is not a coefficient directly next to the parentheses, we can assume this expression only has one group of  $3x + 2y$ . I can think of this as  $y - 1(3x + 2y)$ , but it's probably easier to consider the subtraction as addition.

$$\begin{array}{l}
 y + -1(3x + 2y) \\
 y + -3x + -2y \\
 y + -2y + -3x \\
 \textcircled{-y - 3x}
 \end{array}$$

(write  $y + -1(3x + 2y)$ , and draw arrows to show the distributive property) We can use the expression and distribute the -1 to each term in parentheses. What is -1 times  $3x$ ?  $(-3x)$  What is -1 times  $2y$ ?  $(-2y)$  Great, let's rewrite the entire expression as  $y + -3x + -2y$ . I'll rearrange the y-terms so that they're adjacent to each other. (rewrite so y terms are adjacent)

From here, we can combine like terms. I know y plus  $-2y$  is  $-1y$  or  $-y$ . There is no other x-term, so the equivalent expression is  $-y + -3x$ , or  $-y - 3x$ .

We distributed the -1 to each term in parentheses and then combined like terms to write a simpler, equivalent expression.

In both of these examples, there was not a clearly labeled coefficient outside of the parentheses. Mathematicians don't always put a 1 in front of a group if they don't have to, since a single group of something doesn't require a coefficient. If it's helpful, rewrite similar expressions with a coefficient of 1 to make thinking about the distributive property easier.

**Let's Think (Slide 5):** We have one more problem to tackle. This expression looks long, since it has four terms in it and two sets of parentheses. That's not a problem! We'll use the same thinking to help us write an equivalent expression.

$$\begin{array}{l}
 (2m + 1n) + -1(4n + 12m) \\
 2m + 1n + -4n + -12m \\
 2m + -12m + 1n + -4n \\
 \textcircled{-10m - 3n}
 \end{array}$$

Let's start by rewriting the subtraction of these two parts of the expression as addition. Instead of subtracting one group of  $4n + 12m$ , I'll add -1 group of  $4n + 12m$ . (write  $+ -1$  between sets of parentheses instead of the minus symbol) Now we can distribute the -1 to each term in the second set of parentheses. What is -1 times  $4n$ ?  $(-4n)$  What is -1 times  $12m$ ?  $(-12m)$

We can write the expression as  $2m + 1n + -4n + -12m$ . How could you rearrange and combine like terms to write an equivalent expression with as few terms as possible? (rewrite expression as student shares, supporting as needed) [Possible Student Answers, Key Points:](#)

- I can start by writing the m-terms next to each other and the n-terms next to each other.
- I know  $2m$  plus  $-12m$  is  $-10m$ . I know  $1n$  plus  $-4n$  is  $-3n$ .
- The expression can be written as  $-10m + -3n$  or  $10m - 3n$ .

The equivalent expression is  $-10m - 3n$ . I couldn't combine any more terms, since they don't share a variable. I also couldn't factor anything meaningful out of  $-10m - 3n$ , because 10 and 3 only share a factor of 1.

Nice work! We just used everything we've been learning the past several lessons to help us rewrite expressions as equivalent expressions using as few terms as possible.

**Let's Try it (Slides 6 - 7):** It's time to collaborate on a few problems together, then you can have a chance to show everything you've learned independently. We'll look at each expression we're given to see if we can distribute, combine like terms, or possibly both. We should also keep an eye out for a few other unique situations. For example, if there does not appear to be a coefficient in front of parentheses, we can assume there is a 1. Also, we can keep an eye out for opportunities to factor expressions. Let's dive in!

# WARM WELCOME



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**Today we will write equivalent expressions with fewer terms by combining like terms.**

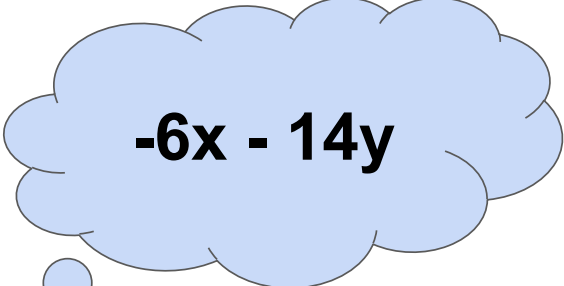
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Let's Talk:

Who is correct?

$$-2(3x - 7y)$$


$$-6x - 14y$$

AMY


$$-6x + 14y$$

BRYSON

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Let's Think:

Write an equivalent expression without parentheses.

$$y + (3x + 2y)$$

$$y - (3x + 2y)$$

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## Let's Think:

Write an equivalent expression with as few terms as possible.

$$(2m + n) - (4n + 12m)$$

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## Let's Try It:

Let's explore writing equivalent expressions with fewer terms by combining like terms together.

Name: \_\_\_\_\_ G7 US Lesson 21 - Let's Try It

**Consider the expression  $6b + (4a + 5b)$ .**

- Rewrite the expression with a 1 in front of the parentheses to think of the expression as  $6b$  plus 1 group of  $4a + 5b$ .
- Distribute the 1 to each term within the parentheses.  

$$6b + (\underline{\quad} \cdot \underline{\quad}) + (\underline{\quad} \cdot \underline{\quad})$$
- Rewrite the expression using as few terms as possible.  

$$6b + \underline{\quad} + \underline{\quad}$$

**Consider the expression  $6b - (4a + 5b)$ .**

- Rewrite the expression to show that we can think of this as  $6b$  plus 1 negative group of  $4a + 5b$ .
- Distribute the -1 to each term within the parentheses.  

$$6b + (\underline{\quad} \cdot \underline{\quad}) + (\underline{\quad} \cdot \underline{\quad})$$
- Rewrite the expression using as few terms as possible.  

$$6b + \underline{\quad} + \underline{\quad}$$

**Consider the expression  $(5n + 3m) + (2n - 8m)$ .**

- Use the distributive property to write an equivalent expression without parentheses.
- Rearrange the expression to group like terms.
- Write the expression using as few terms as possible.

**Consider the expression  $(3g + 5h) - (4h + 7g)$ .**

- Use the distributive property to write an equivalent expression without parentheses.
- Rearrange the expression to group like terms.
- Write the expression using as few terms as possible.
- Now let's factor the expression. What is the greatest common factor of each term?
- Rewrite the expression by factoring out the greatest common factor.

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# On your Own:

Now it's time to write equivalent expressions with fewer terms by combining like terms on your own.

Name: \_\_\_\_\_ G7/G8 Lesson 21 - Independent Work

1. Use the distributive property to rewrite each expression as an equivalent expression without parentheses. Use as few terms as possible.

$8x + (12y - 6x)$        $4g - (g + 2f)$

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2. Which expressions are equivalent to the one below? Select all that apply.

$(10a + 5b) - (2a + 8b)$

a.  $13a + 5b - 2a - 8b$   
b.  $8a - 4b$   
c.  $-4b + 8a$   
d.  $10a - 2a + 5b - 8b$   
e.  $-4b + 8a$   
f.  $10a + 5b - (2a + 8b)$

3. Which expression is equivalent to the one below? Show your work.

$(10x + 6y) - (4x - 6y)$

a.  $6x - 12y$   
b.  $6x - 2y$   
c.  $6x + 12y$   
d.  $12x - 2y$

Rewrite the expression you selected by factoring out the greatest common factor.

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4. Fiona says that  $(10w + 6v) - (7w + 9v)$  is equivalent to  $3(w - v)$ . Is she correct? Show or explain how you know.

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**Consider the expression  $6b + (4a + 5b)$ .**

1. Rewrite the expression with a 1 in front of the parentheses to think of the expression as  $6b$  plus 1 group of  $4a + 5b$ .
2. Distribute the 1 to each term within the parentheses.

$$6b + (\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

3. Rewrite the expression using as few terms as possible.

$$6b + \text{_____} + \text{_____}$$

$$\text{_____} + \text{_____}$$

**Consider the expression  $6b - (4a + 5b)$ .**

4. Rewrite the expression to show that we can think of this as  $6b$  plus 1 *negative* group of  $4a + 5b$ .
5. Distribute the  $-1$  to each term within the parentheses.

$$6b + (\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

6. Rewrite the expression using as few terms as possible.

$$6b + \text{_____} + \text{_____}$$

$$\text{_____} + \text{_____}$$

$$\text{_____} - \text{_____}$$

**Consider the expression  $(5n + 3m) + (2n - 6m)$ .**

7. Use the distributive property to write an equivalent expression without parentheses.
8. Rearrange the expression to group like terms.
9. Write the expression using as few terms as possible.

**Consider the expression  $(3g + 8h) - (4h + 7g)$ .**

10. Use the distributive property to write an equivalent expression without parentheses.
11. Rearrange the expression to group like terms.
12. Write the expression using as few terms as possible.
13. Now let's factor the expression. What is the greatest common factor of each term?
14. Rewrite the expression by factoring out the greatest common factor.

- 1. Use the distributive property to rewrite each expression as an equivalent expression without parentheses. Use as few terms as possible.**

$$9x + (12y - 4x)$$

$$4g - (g + 20)$$

- 2. Which expressions are equivalent to the one below? Select all that apply.**

$$(10a + 5b) - (2a + 9b)$$

- a.  $10a + 5b - 2a - 9b$
- b.  $8a - 4b$
- c.  $-4b + 8a$
- d.  $10a - 2a + 5b - 9b$
- e.  $4(2a - b)$
- f.  $10a + 5b - 1(2a + 9b)$

3. Which expression is equivalent to the one below? Show your work.

$$(10x + 6y) - (4x - 6y)$$

- a.  $6x - 12y$
- b.  $6x - 2y$
- c.  $6x + 12y$
- d.  $12x - 2y$

Rewrite the expression you selected by factoring out the greatest common factor.

4. Fiona says that  $(10w + 6v) - (7w + 9v)$  is equivalent to  $3(w - v)$ . Is she correct? Show or explain how you know.

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Name: KEY

Consider the expression  $6b + (4a + 5b)$ .

1. Rewrite the expression with a 1 in front of the parentheses to think of the expression as 6b plus 1 group of 4a + 5b.

$$6b + 1(4a + 5b)$$

2. Distribute the 1 to each term within the parentheses.

$$6b + (\underline{1} \cdot \underline{4a}) + (\underline{1} \cdot \underline{5b})$$

3. Rewrite the expression using as few terms as possible.

$$\begin{array}{r} 6b + \underline{4a} + \underline{5b} \\ \underline{11b} + \underline{4a} \end{array}$$

Consider the expression  $6b - (4a + 5b)$ .

4. Rewrite the expression to show that we can think of this as 6b plus 1 *negative* group of 4a + 5b.

$$6b - 1(4a + 5b)$$

5. Distribute the -1 to each term within the parentheses.

$$6b + (\underline{-1} \cdot \underline{4a}) + (\underline{-1} \cdot \underline{5b})$$

6. Rewrite the expression using as few terms as possible.

$$\begin{array}{r} 6b + \underline{-4a} + \underline{-5b} \\ \underline{b} + \underline{-4a} \\ \underline{b} - \underline{4a} \end{array}$$

Consider the expression  $(5n + 3m) + (2n - 6m)$ .

7. Use the distributive property to write an equivalent expression without parentheses.

$$5n + 3m + 2n - 6m$$

8. Rearrange the expression to group like terms.

$$5n + 2n + 3m - 6m$$

9. Write the expression using as few terms as possible.

$$(7n - 3m)$$

Consider the expression  $(3g + 8h) - (4h + 7g)$ .

10. Use the distributive property to write an equivalent expression without parentheses.

$$3g + 8h - 4h - 7g$$

11. Rearrange the expression to group like terms.

$$3g - 7g + 8h - 4h$$

12. Write the expression using as few terms as possible.

$$-4g + 4h$$

13. Now let's factor the expression. What is the greatest common factor of each term?

$$4$$

14. Rewrite the expression by factoring out the greatest common factor.

$$4(-g + h)$$

1. Use the distributive property to rewrite each expression as an equivalent expression without parentheses. Use as few terms as possible.

$$9x + (12y - 4x)$$

$$\begin{aligned} &9x + 12y - 4x \\ &9x - 4x + 12y \\ &\underline{5x + 12y} \end{aligned}$$

$$4g - (g + 20)$$

$$\begin{aligned} &4g - g - 20 \\ &\underline{3g - 20} \end{aligned}$$

2. Which expressions are equivalent to the one below? Select all that apply.

$$(10a + 5b) - (2a + 9b)$$

- a.  $10a + 5b - 2a - 9b$   
 b.  $8a - 4b$   
 c.  $-4b + 8a$   
 d.  $10a - 2a + 5b - 9b$   
 e.  $4(2a - b)$   
 f.  $10a + 5b - 1(2a + 9b)$

$$\begin{aligned} &10a + 5b - 2a - 9b \\ &10a - 2a + 5b - 9b \\ &8a - 4b \\ &4(2a - b) \end{aligned}$$



3. Which expression is equivalent to the one below? Show your work.

$$(10x + 6y) - (4x - 6y)$$

- a.  $6x - 12y$
- b.  $6x - 2y$
- c.  $6x + 12y$
- d.  $12x - 2y$

$$\begin{aligned}10x + 6y - 4x + 6y \\10x - 4x + 6y + 6y \\6x + 12y\end{aligned}$$

Rewrite the expression you selected by factoring out the greatest common factor.

$$\text{GCF: } 6$$

$$6(x + 2y)$$

4. Fiona says that  $(10w + 6v) - (7w + 9v)$  is equivalent to  $3(w - v)$ . Is she correct? Show or explain how you know.

$$\begin{aligned}10w + 6v - 7w - 9v \\10w - 7w + 6v - 9v \\3w - 3v \\3(w - v)\end{aligned}$$

I agree. I rewrote the expression with as few terms as possible, and I got  $3w - 3v$ . Then, you can factor out a 3 from both terms to get  $3(w - v)$ .