



Seventh Grade Math Lesson Materials

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G7 Unit 5:

Expressions, Equations, and Inequalities

G7 U5 Lesson 1

Find unknown values in relationships, and interpret them as proportional and not proportional.

G7 U5 Lesson 1 - Students will find unknown values in relationships, and interpret them as proportional or not proportional.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we get to start a brand new unit. This unit is all about expressions, equations, and inequalities. What do you already know about expressions, equations, or inequalities? **Possible Student Answers, Key Points:**

- I know equations have an equal sign. Like $2 + 2 = 4$ is an equation. Sometimes equations have unknowns we have to solve for.
- I know expressions are made up of terms separated by symbols like $+$, $-$, \cdot , or \div .
- I know inequalities use the less than or greater than symbol.

It sounds like you already know a lot that will help us as we work through the next several lessons. We'll get started today by finding unknown values in relationships and determining whether the relationships we're looking at are proportional or not proportional. Time to get started!

Let's Talk (Slide 3): Let's head to the circus! Consider a situation where one box of popcorn costs \$3. How could this group of 4 people figure out how much it would cost for each person to have their own box of popcorn? **Possible Student Answers, Key Points:**

- They could add $3 + 3 + 3 + 3$, since all four of them are buying popcorn.
- They could write an equation like $3 \cdot p = ?$ to find the total.
- They could make a table to show the cost of 1 person buying popcorn, then 2 people, then 3, then 4...

#	\$
1	3
2	6
3	9
4	12

There are many ways to consider solving this problem. Let's draw a table. *(draw a simple t-chart labeled with number of people and cost)* Let's find the cost if 1, 2, 3, or 4 people purchase a box of popcorn. *(write 1 through 4 in the first column)* How much would it cost 1 person to buy popcorn? **(\$3)** *(fill in value, and continue filling in values as the student shares)* What would be the cost if 2 people bought popcorn? **(\$6)** What about 3 people? **(\$9)** What about 4 people? **(\$12)** The total cost for 4 people to each buy a box of popcorn would be \$12. We can see this clearly in the table.

#	\$
1	3
2	6
3	9
4	12

This relationship is considered proportional. The number of people is proportional to the total cost. The cost per box of popcorn stays constant. It's \$3 for 1 person. *(write $3/1 = 3$ next to that row of the table)* It's \$6 for 2 people. *(write $6/2 = 3$ next to that row of the table)* It's \$9 for 3 people. *(continue writing constant of proportionality as you narrate)* It's \$12 for 4 people. No matter what, we end up with 3. This 3 is called our constant of proportionality. *(highlight each 3)* When we divide the amount of money by the number of people, we always get the same quotient, or the same cost per person. In each row, we could multiply the number of people times the constant of proportionality, 3, to find the total cost. This relationship is proportional.

Let's Think (Slide 4): Now, let's think about a different set of relationships. I'll read this problem once through, then I want you to summarize what it is about. *(read problem)* What do you know? What is unknown? **Possible Student Answers, Key Points:**

- We know there are 2 bus companies. They charge different rates to rent buses.
- We don't know how much it costs to rent 1, 2, 3, or 4 buses from each company. We don't know which company is the better deal.

BB

D	\$
1	700
2	1400
3	2100
4	2800

Let's draw a table to represent each bus company's cost. I'll start by making a table to represent Barry's Bus Company. (sketch and label a t-chart and fill in 1, 2, 3, and 4 days) How much would it cost to rent from Barry's Bus Company for 1 day? 2 days? 3 days? 4 days? Possible Student Answers, Key Points:

- Each day, this company charges \$700. One day would be \$700, two days would be \$1,400, 3 days would be \$2,100, and 4 days would be \$2,800.

(fill in the table as student shares)

AA

D	\$
1	700
2	1200
3	1700
4	2200

Let's draw a table to represent Aaron's Auto Company next. Aaron's Auto Company charges their customers in a different way. They charge a one-time flat fee of \$200. Then, they charge \$500 per day on top of the flat fee. If I think about 1 day, that means I'll have to pay \$200 plus \$500 for the day. (fill in 700 on the chart) For 2 days, I'd still pay the \$200 fee then I'd have to pay \$500 for the first day and another \$500 for the second day. That's \$1,200 total. (fill 1,200 in the chart) How much does Aaron's Auto Company charge for 3 days? 4 days? (fill in the table as the student shares) Possible Student Answers, Key Points:

- For three days, I'd have to pay \$200 plus \$1,500. The total would be \$1,700.
- For four days, I'd have to pay \$200 plus \$2,000. The total would be \$2,200.

Great! We've done everything this problem asked us to do, but let's think about these two companies a bit more. One of these companies shows a proportional relationship, meaning the cost per day stays constant no matter what. One of these companies shows a relationship that is not proportional. Let's think about the cost per day in each scenario, starting with Barry's Bus Company.

D	\$	
1	700	$\frac{700}{1} = 700$
2	1400	$\frac{1400}{2} = 700$
3	2100	$\frac{2100}{3} = 700$
4	2800	$\frac{2800}{4} = 700$

To find the cost per day, I can divide the cost by the number of days. (as you narrate, write each constant of proportionality as a fraction then write the value next to each row) For 1 day, Barry's Bus Company charges \$700. I can think of the cost per day as $700/1$, which I know equals 700. For 2 days, I can think of the cost per day as $1400/2$, which I know equals 700. Help me find the cost per day for Day 3 and Day 4. Possible Student Answers, Key Points:

- For Day 3, I can write $2100/3$. I know 2100 divided by 3 is 700.
- For Day 4, the cost per day is also 700. I know $2800/4 = 700$.

Since each row of the table represents the same cost per day, we can say that 700 is the constant of proportionality. This relationship is proportional. What about Aaron's Auto?

D	\$	
1	700	$\frac{700}{1} = 700$
2	1200	$\frac{1200}{2} = 600$
3	1700	$\frac{1700}{3} \approx 567$
4	2200	$\frac{2200}{4} = 550$

(follow a similar process, labeling the cost per day next to each row as you narrate) The cost per day in the first row can be represented by $700/1$. The cost per day is \$700. What about the next three days? You're welcome to use a calculator if it's helpful. Possible Student Answers, Key Points:

- For Day 2, I can think of 1200 divided by 2. That's 600.
- For Day 3, I can write $1700/3$. 1700 divided by 3 is about 567.
- For Day 4, the cost per day is 550 dollars. 2200 divided by 4 is 550.

Notice how the cost per day does not remain constant when we look at Aaron's Auto Company. This means the relationship is NOT proportional.

To determine whether a relationship is proportional, we can use division to see if the relationship between values stays constant for each pair of values. Let's keep going.

Let's Think (Slide 5): This question wants us to find the cost per item at a store. The first table represents belts, and the second table represents scarves. Based on the cost per item, we'll determine which relationship is proportional.

Number of scarves	Cost in dollars
2	\$12
6	\$30
11	\$50

$\frac{12}{2} = 6$
 $\frac{30}{6} = 5$
 $\frac{50}{11} \approx 4.5$

Let's start by looking at the scarves. Let's find the cost per scarf by using the table. In the first row, I see 2 scarves cost \$12 total. I can find the cost per scarf by dividing the cost by the number of scarves. I know $12/6$ is equal to 6. *(label next to the row in the table, and continue doing so for the other rows as you narrate)*

The second row shows that the store charges \$30 to buy 6 scarves. I can find the cost per scarf by dividing the cost by the number of scarves. I know $30/6 = 5$. Hm, it looks like maybe the cost per scarf is not constant across the table,

but let's keep going to be sure...

How could I find the cost per scarf looking at the third row? You're welcome to use a calculator if you find it helpful. **Possible Student Answers, Key Points:**

- I can find the cost per scarf by dividing the total cost by the number of scarves. I can divide 50 by 11. My calculator shows a long decimal number, but it's about 4.5 if I round to the nearest tenth.

I can see that the cost per scarf is not constant for each pair of values. That means, the relationship between cost and number of scarves is NOT proportional.

Number of belts	Cost in dollars
2	\$5
6	\$15
11	\$27.50

$\frac{5}{2} = 2.5$
 $\frac{15}{6} = 2.5$
 $\frac{27.50}{11} = 2.5$

Let's now look at the belts. To find the cost per belt in the first row of this table, I can divide \$5 by 2 belts. $5/2$ is equal to 2.5, so I know each belt costs \$2.50. *(write $5/2 = 2.5$ next to the first row, and continue labeling as you and the student discuss the other rows)*

How can I find the cost per belt in the second row and third row of this table? If you want, you may use a calculator. **Possible Student Answers, Key Points:**

- I can divide \$15 by 6. I get \$2.50 per belt.
- I can divide \$27.50 by 11. I get \$2.50 per belt again.

Is this relationship between cost and number of belts proportional? How do you know? **Possible Student Answers, Key Points:**

- The relationship is proportional. We found that 2.5 is the constant of proportionality. In every row, the cost per belt was \$2.50.

The scarves did not represent a proportional relationship because the cost per scarf depended on how many scarves you bought. The belts did represent a proportional relationship, because no matter how many belts you bought, the cost per belt remained constant. The cost per belt was always \$2.50.

Let's Try it (Slides 6 - 7): Now let's work through a few more problems together, then you'll get a chance to practice on your own. We'll look closely at each relationship to determine whether it is proportional or not proportional. We can use division to see if the constant of proportionality remains consistent across the entire relationship. Let's go for it!

WARM WELCOME



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Today we will find unknown values in relationships, and interpret them as proportional and not proportional.

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Let's Talk:



Popcorn at the circus costs \$3.



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Let's Think:

Barry's Bus Company charges \$700 each day to rent a bus. Aaron's Auto Company charges a flat fee of \$200 and \$500 per day to rent a bus.

Sketch tables to represent the cost of renting 1, 2, 3, and 4 buses from each company.

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Let's Think:

A clothing store sells belts and scarves. Use the tables to find the cost per item. Which relationship is proportional?

Number of scarves	Cost in dollars
2	\$12
6	\$30
11	\$50

Number of belts	Cost in dollars
2	\$5
6	\$15
11	\$27.50

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Let's Try It:

Let's explore finding unknown values in relationships, and interpreting them as proportional or not proportional together.

Name: _____ G7 US Lesson 1 - Let's Try It

A bike-share service charges \$4 per hour to rent a bike.

- The \$2 in this story represents:
 - The cost to rent a bike once
 - The cost to rent a bike each hour.
- How much would it cost to rent the bike for 2 hours?
- How much would it cost to rent the bike for 8 hours?
- How much would it cost to rent the bike for 24 hours?
- Divide each cost you found (Questions #2 - 4) by the number of hours to find the cost per hour.
- The cost per mile...
 - is the same, so this is a proportional relationship.
 - is the same, so this is not a proportional relationship.
 - is not the same, so this is a proportional relationship.
 - is not the same, so this is not a proportional relationship.

The cost per hour in this problem is the **constant of proportionality**.

- The constant of proportionality in this scenario is _____.

A carnival charges an entry fee of \$5, and \$1 for every game or ride.

- What does \$5 represent in this situation?
 - The cost for each game or ride.
 - The cost to enter the carnival.
- What does \$1 represent in this situation?
 - The cost for each game or ride.
 - The cost to enter the carnival.

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- Determine how much it would cost for one person to go to the carnival and visit 2 games or rides. 8 games or rides? 24 games or rides?
- Divide each cost by the number of rides or games to find the cost per ride or game.
- Is the relationship proportional? How do you know?

Look at each relationship below. Determine whether each is proportional or not proportional. Show or explain how you know.

number of bagels	cost	number of muffins	cost	number of donuts	cost
2	\$1.80	3	\$5	6	\$3.60
7	\$6.30	5	\$7	10	\$6.00
10	\$9	7	\$9	20	\$11.40

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On your Own:

Now it's time to find unknown values in relationships, and interpret them as proportional and not proportional on your own.

Name: _____ G7 US Lesson 1 - Independent Work

1. A board game requires each player to start with \$10. Fill in the table to find how many dollars will be needed for 2, 3, 6, and 8 players. Then, fill in the column labeled dollars per player per player

Number of players	Number of dollars	Dollars per player
2		
3		
6		
8		

Is the relationship proportional? How do you know?

2. An amusement park offers a season pass where guests pay a flat fee of \$20, and then they only pay \$5 per visit to the park.

Number of visits	Total cost	Cost per visit
2		
5		
7		
10		

Is the relationship proportional? How do you know?

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3. Patrick mows lawns over the summer. He charges his customers an \$80 fee for the summer, and \$22 per lawnmowing.

- How much does Patrick charge to mow 1 lawn?
- How much does Patrick charge to mow 2 lawns?
- How much does Patrick charge to mow 3 lawns?
- Patrick said this relationship is proportional, because the cost increases \$22 every time. Is he correct? How do you know?

4. Determine whether each table represents a relationship that could be proportional.

Number of hats	Cost in dollars
20	152
2	18
8	66

Number of hats	Cost in dollars
6	60
11	110
12	120

Number of hats	Cost in dollars
4	24
5	33
20	108

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Name: _____

A bike-share service charges \$4 per hour to rent a bike.

1. The \$4 in this story represents:
 - a. The cost to rent a bike once
 - b. The cost to rent a bike each hour.

2. How much would it cost to rent the bike for 2 hours?

3. How much would it cost to rent the bike for 8 hours?

4. How much would it cost to rent the bike for 24 hours?

5. Divide each cost you found (Questions #2 - 4) by the number of hours to find the cost per hour.

6. The cost per mile...
 - a. is the same, so this is a proportional relationship.
 - b. is the same, so this is not a proportional relationship.
 - c. is not the same, so this is a proportional relationship.
 - d. is not the same, so this is not a proportional relationship.

The cost per hour in this problem is the constant of proportionality.

7. The constant of proportionality in this scenario is _____.

A carnival charges an entry fee of \$5, and \$1 for every game or ride.

8. What does \$5 represent in this situation?
 - a. The cost for each game or ride.
 - b. The cost to enter the carnival.

9. What does \$1 represent in this situation?
 - a. The cost for each game or ride.
 - b. The cost to enter the carnival.

10. Determine how much it would cost for one person to go to the carnival and visit 2 games or rides. 8 games or rides? 24 games or rides?

11. Divide each cost by the number of rides or games to find the cost per ride or game.

12. Is the relationship proportional? How do you know?

Look at each relationship below. Determine whether each is proportional or not proportional. Show or explain how you know.

13.

number of bagels	cost
2	\$1.80
7	\$6.30
10	\$9

14.

number muffins	cost
3	\$5
5	\$7
7	\$9

15.

number of donuts	cost
6	\$3.60
10	\$6.00
20	\$11.40

1. A board game requires each player to start with \$10. Fill in the table to find how many dollars will be needed for 2, 3, 6, and 8 players. Then, fill in the column labeled dollars per player

Number of players	Number of dollars	Dollars per player
2		
3		
6		
8		

Is the relationship proportional? How do you know?

2. An amusement park offers a season pass where guests pay a flat fee of \$20, and then they only pay \$5 per visit to the park.

Number of visits	Total cost	Cost per visit
2		
5		
7		
10		

Is the relationship proportional? How do you know?

3. Patrick mows lawns over the summer. He charges his customers an \$80 fee for the summer, and \$22 per lawnmowing.

- a. How much does Patrick charge to mow 1 lawn?

- b. How much does Patrick charge to mow 2 lawns?

- c. How much does Patrick charge to mow 3 lawns?

- d. Patrick said this relationship is proportional, because the cost increases \$22 every time. Is he correct? How do you know?

4. Determine whether each table represents a relationship that could be proportional.

Number of hats	Cost in dollars
20	162
2	18
8	66

Number of hats	Cost in dollars
6	60
11	110
12	120

Number of hats	Cost in dollars
4	24
5	33
20	108

Name: KEY

A bike-share service charges \$4 per hour to rent a bike.

1. The \$4 in this story represents:

- a. The cost to rent a bike once
- b. The cost to rent a bike each hour.

2. How much would it cost to rent the bike for 2 hours?

$\$8$

3. How much would it cost to rent the bike for 8 hours?

$\$32$

4. How much would it cost to rent the bike for 24 hours?

$$24 \times 4 = \$96$$

5. Divide each cost you found (Questions #2 - 4) by the number of hours to find the cost per hour.

$$\begin{aligned} 8 \div 2 &= 4 \\ 32 \div 8 &= 4 \\ 96 \div 24 &= 4 \end{aligned}$$

6. The cost per mile...

- a. is the same, so this is a proportional relationship.
- b. is the same, so this is not a proportional relationship.
- c. is ~~not the same~~, so this is a proportional relationship.
- d. is ~~not the same~~, so this is not a proportional relationship.

hr	cost
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32

The cost per hour in this problem is the constant of proportionality.

7. The constant of proportionality in this scenario is 4.

A carnival charges an entry fee of \$5, and \$1 for every game or ride.

8. What does \$5 represent in this situation?

- a. The cost for each game or ride.
- b. The cost to enter the carnival.

9. What does \$1 represent in this situation?

- a. The cost for each game or ride.
- b. The cost to enter the carnival.

10. Determine how much it would cost for one person to go to the carnival and visit 2 games or rides. 8 games or rides? 24 games or rides?

$$5 + 2(1)$$

$$\text{\$7}$$

$$5 + 8(1)$$

$$\text{\$13}$$

$$5 + 24(1)$$

$$\text{\$29}$$

11. Divide each cost by the number of rides or games to find the cost per ride or game.

$$7 \div 2 = \frac{7}{2} = 3\frac{1}{2}$$

$$13 \div 8 = \frac{13}{8} = 1\frac{5}{8}$$

$$29 \div 24 = \frac{29}{24} = 1\frac{5}{24}$$

12. Is the relationship proportional? How do you know?

No, this is not proportional. The cost per ride or game varies.

Look at each relationship below. Determine whether each is proportional or not proportional. Show or explain how you know.

13. *proportional*

number of bagels	cost
2	\\$1.80
7	\\$6.30
10	\\$9

$$1.80 \div 2 = 0.90 \checkmark$$

$$6.30 \div 7 = 0.90 \checkmark$$

$$9 \div 10 = 0.90 \checkmark$$

14. *not proportional*

number muffins	cost
3	\\$5
5	\\$7
7	\\$9

$$5 \div 3 = \frac{5}{3} = 1\frac{2}{3}$$

$$7 \div 5 = \frac{7}{5} = 1\frac{2}{5}$$

$$9 \div 7 = \frac{9}{7} = 1\frac{2}{7}$$

15. *not proportional*

number of donuts	cost
6	\\$3.60
10	\\$6.00
20	\\$11.40

$$3.60 \div 6 = 0.60 \checkmark$$

$$6 \div 10 = 0.60 \checkmark$$

$$11.40 \div 20 = 0.57 \times$$

1. A board game requires each player to start with \$10. Fill in the table to find how many dollars will be needed for 2, 3, 6, and 8 players. Then, fill in the column labeled dollars per player

Number of players	Number of dollars	Dollars per player
2	\$20	\$10
3	\$30	\$10
6	\$60	\$10
8	\$80	\$10

Is the relationship proportional? How do you know?

Yes. The dollars per player is consistent for any number of players.

2. An amusement park offers a season pass where guests pay a flat fee of \$20, and then they only pay \$5 per visit to the park.

Number of visits	Total cost	Cost per visit
2	\$30	\$15
5	\$45	\$9
7	\$55	$\$7\frac{2}{7}$
10	\$70	\$7

$$20 + 10 = 30$$

$$20 + 25 = 45$$

$$20 + 35 = 55$$

$$20 + 50 = 70$$

Is the relationship proportional? How do you know?

No. The cost per visit varies depending on the number of visits.

3. Patrick mows lawns over the summer. He charges his customers an \$80 fee for the summer, and \$22 per lawnmowing.

a. How much does Patrick charge to mow 1 lawn?

$$80 + 22 = \text{\$}102$$

$$102 \div 1 = 102$$

b. How much does Patrick charge to mow 2 lawns?

$$80 + 22 + 22$$

$$80 + 44 = \text{\$}124$$

$$124 \div 2 = 62$$

c. How much does Patrick charge to mow 3 lawns?

$$80 + 22(3)$$

$$80 + 66 = \text{\$}146$$

$$146 \div 3 = 48 \frac{2}{3}$$

d. Patrick said this relationship is proportional, because the cost increases \$22 every time. Is he correct? How do you know?

He is incorrect. It is not proportional, because the cost per lawn varies depending on the number of lawns.

4. Determine whether each table represents a relationship that could be proportional.

NOT!

Number of hats	Cost in dollars
20	162
2	18
8	66

$$162 \div 20 = 8 \frac{1}{10}$$

$$18 \div 2 = 9$$

$$66 \div 8 = 8 \frac{1}{4}$$

PROPORTIONAL!

Number of hats	Cost in dollars
6	60
11	110
12	120

$$60 \div 6 = 10$$

$$110 \div 11 = 10$$

$$120 \div 12 = 10$$

NOT!

Number of hats	Cost in dollars
4	24
5	33
20	108

$$24 \div 4 = 6$$

$$33 \div 5 = 6 \frac{3}{5}$$

$$108 \div 20 = 5 \frac{2}{5}$$

G7 U5 Lesson 2

Interpret tape diagrams that represent word problems, and use them to find unknown values.

G7 U5 Lesson 2 - Students will interpret tape diagrams that represent word problems, and use them to find unknown values.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our previous lesson, we looked at relationships that were proportional and not proportional. We used information from a story or a table to find unknown values based on the relationships. Today's work will still have use considering how we can find unknown values, but our work will mostly focus on using tape diagrams to help us find unknown values. Tape diagrams, sometimes called bar models, are useful tools to help us visualize what is happening in math problems. We can use the visual of a tape diagram to help us consider how to find any potential unknowns. Let's get started.

Let's Talk (Slide 3): Here we see two different examples of tape diagrams. Take a moment to look at them, then tell me what you notice and wonder about each. **Possible Student Answers, Key Points:**

- I notice they both show a total of 9. I notice one is split into equal boxes and the other is split into different-sized boxes. I notice they're rectangular. I notice the unknown is labeled with x.
- I wonder what x equals in both tape diagrams. I wonder if x is the same in both. I wonder what equations or stories could match the equations.

Most our tape diagrams today will be related to a story or a context. Even though we don't know the story behind these two tape diagrams, we can still use the structure of the tape diagram to help us consider how to find each unknown value.

$$\begin{aligned}x + x + x &= 9 \\3 \cdot x &= 9 \\9 \div 3 &= x\end{aligned}$$

Let's look at the top tape diagram. There are many different ways I can look at this tape diagram to consider the value of the unknown. I see three boxes that each have the same value, x. I see the total of all three boxes is labeled as 9. If I wanted to think about this mathematically, I might think $x + x + x = 9$. I know if I add the three boxes together, that should give me the total. (*write equations as you narrate*) Or maybe my brain sees this as 3 groups of x equals 9, so I write $3 \cdot x = 9$. Or even still, I could look

at this tape diagram as a total of 9 split into 3 equal boxes, so I write the equation $9 \div 3 = x$. Regardless of how I think of the tape diagram, what is the value of x? ($x = 3$) The tape diagram helps me visualize the solution pathway or pathways that most make sense to me.

$$\begin{aligned}x + 3 &= 9 \\9 - 3 &= x\end{aligned}$$

What about the other tape diagram? This one doesn't have equal groups like the last one.

What different ways could somebody look at this one to determine what the value of x is? (*write equations as student shares, supporting as necessary*) **Possible Student Answers, Key Points:**

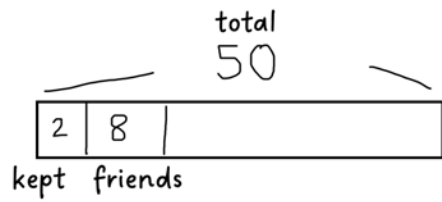
- I see two parts, x and 3, and they total up to 9. I can write $x + 3 = 9$ to represent the tape diagram.
- I can think of this as a total of 9. To find x, I could subtract out the part that is 3 to see what is left. So $9 - 3 = x$ could help me.

Great. Let's keep all this in mind as we look at some application problems where we can draw and then interpret tape diagrams to help us find unknowns.

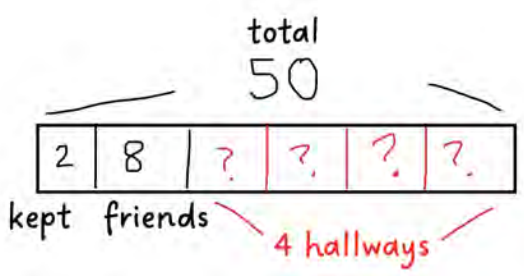
Let's Think (Slide 4): I'll read this problem once through while you read it to yourself. After, I want you to summarize what the story is about in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- Ron made 50 flyers for his campaign. He kept some, gave some to his friends, and then split the rest up evenly to put in hallways in his school.

- We know how much he made. We know how much he kept for himself and gave to his friends. We don't know the number that he put in each of the four hallways.



Before we try to solve, let's visualize this story using a tape diagram. (reread the first sentence) I know Ron made a total of 50 flyers. (draw as you narrate) I can draw a rectangle to represent the total, and I'll label the entire rectangle as 50. (reread the second sentence) If he kept 2 for himself, I'll partition a small rectangle to show that out of the 50 flyers, he kept these 2. I'll put a 2 in the box and label it, so I don't forget what that 2 means in the story. I'll do the same thing for the flyers he gives his friends. I'll partition, or cut, a slightly larger box to show that he gave a few more flyers to his friends. I'll put 8 in that box, and I'll label it as friends. (point to empty space remaining) This empty space in the rectangle represents the rest of the flyers.



I know Ron used the remaining flyers to hang equally along 4 hallways. How can I use this part of the tape diagram to show those 4 equal groups? (partition the remaining section into 4 equal parts as student shares) Possible Student Answers, Key Points:

- You can cut that part into fourths.
- You can partition the section into 4 equal-sized boxes to represent the amount in each hallway.

I'll label each box with a question mark since we don't know how many flyers he put in each hallway. I'll note that these four boxes represent the flyers that were put up in each of the four hallways. Now, we've represented the entire story. We can step back, look at the tape diagram, and start to think about how we can find those unknowns.

I see the total is 50, that's clear from the story and the model. I really only care about the flyers in the hallway, so I can remove the flyers that he kept and the flyers he gave to his friends. I could subtract $50 - 2$, and then subtract out the 8 he gave to his friends. Or, I also know that $2 + 8$ is an easy 10. Let's just subtract the 10 flyers we know about from our total. (write $50 - 10 = 40$) Once we remove those flyers, we can see that he has 40 flyers to hang up equal among the 4 hallways. What math can we do to figure out how many flyers went up in each hallway? Possible Student Answers, Key Points:

$$50 - 10 = 40$$

$$40 \div 4 = 10$$

flyers

- I know $4 \times ? = 40$, so he has to put 10 up in each hallway.
- I know 40 divided into 4 equal groups is 10.
- I know $10 + 10 + 10 + 10$ would be 40.

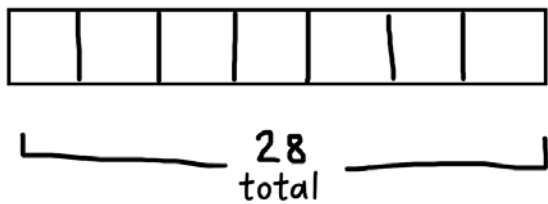
There are several ways to arrive at our answer. I'm going to think about 40 split into 4 equal groups. (write equation) I know 40 divided by 4 is equal to 10.

Ron hung 10 flyers up in each hallway. The tape diagram helped us visualize what was happening in the story so that we could think about the different parts in the story, the total, and what math we could do to eventually arrive at our unknown value. Let's try this thinking out with a different type of story.

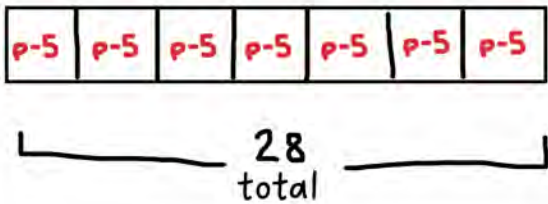
Let's Think (Slide 5): I'll read this problem once through while you read it to yourself. After, I want you to summarize what the story is about in your own words. What is known? What is unknown? Possible Student Answers, Key Points:

- Jai's unpacking plates after a move, and we're trying to figure out how many plates were in each box before she started unpacking.

- I know she has 7 boxes. I know she has removed 5 plates from each, and there are 28 left to unpack. We don't know how many were in each box to start.



Let's start sketching our tape diagram. I know Jai has 7 boxes. I'll begin by drawing 7 identical boxes. I find it easiest to draw a big rectangle and then split it into 7 equal groups. (*sketch tape diagram as you narrate*) The total number of plates left in those 7 boxes now is 28, so I'll use a bracket to show that.



Now, let's think about what we know about each box. Based on the story, I know each box had some plates in it, and then Jai unpacked or removed 5 from each. I can use the expression $p - 5$ to represent this. The variable p represents the number of plates to begin with, and I can subtract 5 from p to represent the plates she unpacked. (*label each box with $p - 5$*)

That's all the information we know. Let's step back and look at the tape diagram to think about an easy way to find the value of our unknown.

$$28 \div 7 = 4$$

$$p - 5 = 4$$

$$p = 9 \text{ plates in each box originally}$$

I *could* add up " $p - 5$ " seven times, because I know all 7 boxes have a total of 28. Picturing that as an equation feels like a bit much, though. What if we started by dividing the known total of 28 by the number of boxes, since we know each box is identical? What is 28 divided by 7 boxes? (4) (*write 28 divided by 7 = 4*) Nice! This means each box currently has 4 plates in it. Thinking about just one box, I know she started with some plates, p , she took out 5 plates, and now she has 4 plates in each box. I can write $p - 5 = 4$ to represent that. What math can

I consider to find p now? **Possible Student Answers, Key Points:**

- I know p is 9, because $9 - 5 = 4$.
- I can write a related addition equation to find the unknown. $4 + 5 = p$ helps me see that $p = 9$.

The value of the unknown is 9. Jai had 9 plates in each box before she began unpacking. The tape diagram helped us picture the story. It helped us see the total and each equal box so that we could strategically solve the problem one step at a time.

Let's Try it (Slides 6 - 7): Now it's our time to try a couple more examples together. Like we've been doing, we'll read each problem while carefully considering what we know and don't know. Based on what we know, we'll sketch and label a tape diagram to represent the story. The visual of the tape diagram will help us consider which operation or operations will best support us in planning our solution pathway. Let's dive in together, then you'll have some time to practice independently.

WARM WELCOME



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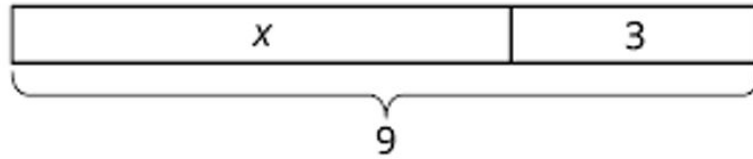
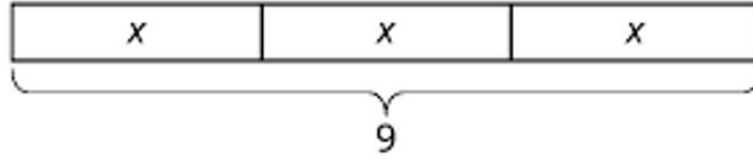
Today we will interpret tape diagrams that represent word problems, and use them to find unknown values.

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Let's Talk:

**What do you notice about the tape diagrams?
What do you wonder?**



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Let's Think:

Ron made 50 flyers for his campaign for student council president. He kept 2 for himself, gave 8 to friends, and divided the rest evenly along the 4 hallways of his school. How many flyers did he put in each hallway?

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Let's Think:

Jai packed p plates into each of 7 boxes when she moved. After unpacking 5 plates from each box, she has 28 plates left to unpack. How many plates did Jai originally pack in each box?

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Let's Try It:

Let's explore interpreting tape diagrams that represent word problems, and use them to find unknown values together.

Name: _____ G7 US Lesson 2 - Let's Try It

Marina's mom baked 72 cupcakes. She gave 30 cupcakes to Marina's teacher to give out to her class, and then divided the rest of them equally among 7 neighbors.

- What is happening with the cupcakes in this story?
 - They are baking cupcakes.
 - They are giving away cupcakes.
- How many total people did Marina's mom give cupcakes to? _____
- Did each person get the same amount of cards? Explain.

- Label the tape diagram with the values that are known from the story. Label the unknown(s) with c to represent cupcakes.
- What does c represent in this context?
 - The total number of cupcakes given away
 - The number of cupcakes the teacher got
 - The number of cupcakes each neighbor gets
- Use any strategy to find the value of c .

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Xavier is making 7 identical gift bags to give his friends after his birthday party. He puts 6 stickers into each gift bag. He also puts some chocolate bars in each bag. In all, he uses 56 items.

- We know...
- We don't know...
- Use the tape diagram below to label what is known. Use x to represent any unknowns.
- Use the tape diagram to find the number of chocolate bars in each bag.

Nina has 5 racks of shoes with p pairs of shoes on each rack. She wants to donate 1 pair of shoes from each rack. After donating, she has 10 pairs of shoes left.

- What is known? What is unknown?
- Draw a tape diagram to represent the story. Use p to represent the number of pairs of shoes Nina originally had on each rack.
- Use the tape diagram to find the value of p .

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On your Own:

Now it's time to interpret tape diagrams that represent word problems, and use them to find unknown values on your own.

Name: _____ G7 US Lesson 2 - Independent Work

1. Select ALL the stories that the tape diagram can represent.



- a. Henry's spends \$47 on 4 tickets to the school play and \$27 at the concession stand.
- b. Maria buys 4 bags of mints with 27 mints in each pack. She gives 47 mints to her brother.
- c. Lance has a collection of 47 rocks. He gives 27 rocks to his best friend and puts the other rocks equally into 4 boxes.
- d. Yusef made 47 friendship bracelets. He sells 27 of them, and he splits the remaining bracelets up between 4 friends.
- e. There are 47 dogs and 27 cats in the animal shelter. The cats are split equally into 5 rooms.

2. Ms. Catalano is grading 100 essays. So far, she has graded 75 essays. How many essays does she have left to grade?

- a. Draw a tape diagram to represent the story problem. Use x to represent the unknown.
- b. Use the tape diagram to determine how many essays Ms. Catalano has left to grade.

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3. Erin organizes her jewelry into boxes. She puts 3 bracelets and n necklaces into each box. After filling 6 boxes, she used a total of 42 pieces of jewelry.

- a. Draw a tape diagram to represent the story problem.
- b. Use the tape diagram to determine how many necklaces Erin puts in each box.

4. Mr. Guzman has 7 prize boxes for his students. Each box has p prizes. His students take 4 prizes from each box. Mr. Guzman had 21 prizes left in all. How many prizes were in each box to start with?

Draw a tape diagram as part of your solution pathway.

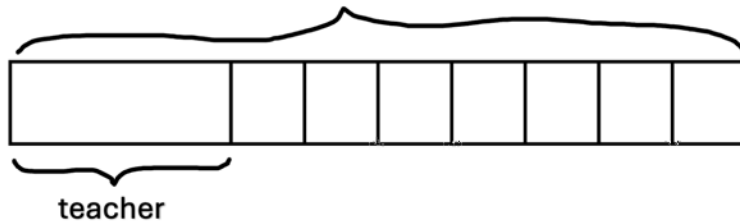
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Marina's mom baked 72 cupcakes. She gave 30 cupcakes to Marina's teacher to give out to her class, and then divided the rest of them equally among 7 neighbors.

1. What is happening with the cupcakes in this story?
 - a. They are baking cupcakes.
 - b. They are giving away cupcakes.
2. How many total people did Marina's mom give cupcakes to? _____
3. Did each person get the same amount of cards? Explain.

4. Label the tape diagram with the values that are known from the story. Label the unknown(s) with c to represent cupcakes.



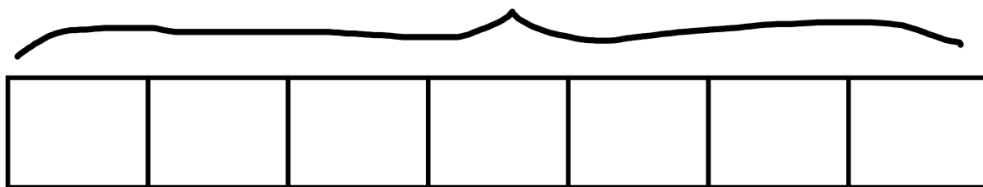
5. What does c represent in this context?
 - a. The total number of cupcakes given away
 - b. The number of cupcakes the teacher got
 - c. The number of cupcakes each neighbor gets
6. Use any strategy to find the value of c .

Xavier is making 7 identical gift bags to give his friends after his birthday party. He puts 6 stickers into each gift bag. He also puts some chocolate bars in each bag. In all, he uses 56 items.

7. We know...

8. We don't know...

9. Use the tape diagram below to label what is known. Use x to represent any unknowns.



10. Use the tape diagram to find the number of chocolate bars in each bag.

Nina has 5 racks of shoes with p pairs of shoes on each rack. She wants to donate 1 pair of shoes from each rack. After donating, she has 10 pairs of shoes left.

11. What is known? What is unknown?

12. Draw a tape diagram to represent the story. Use p to represent the number of pairs of shoes Nina originally had on each rack.

13. Use the tape diagram to find the value of p .

3. Erin organizes her jewelry into boxes. She puts 3 bracelets and n necklaces into each box. After filling 6 boxes, she used a total of 42 pieces of jewelry.

a. Draw a tape diagram to represent the story problem.

b. Use the tape diagram to determine how many necklaces Erin puts in each box.

4. Mr. Guzman has 7 prize boxes for his students. Each box has p prizes. His students take 4 prizes from each box. Mr. Guzman had 21 prizes left in all. How many prizes were in each box to start with?

Draw a tape diagram as part of your solution pathway.

Marina's mom baked 72 cupcakes. She gave 30 cupcakes to Marina's teacher to give out to her class, and then divided the rest of them equally among 7 neighbors.

1. What is happening with the cupcakes in this story?

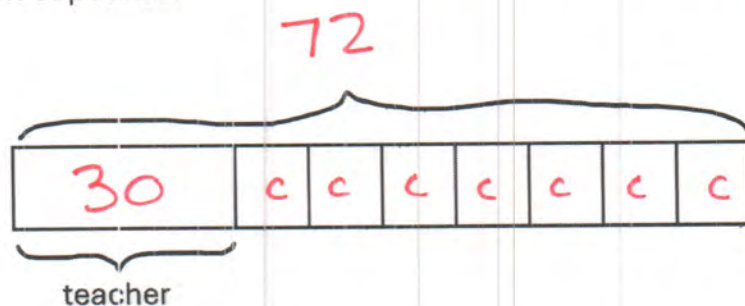
- a. They are baking cupcakes.
 b. They are giving away cupcakes.

2. How many total people did Marina's mom give cupcakes to? 8

3. Did each person get the same amount of cards? Explain.

The teacher gets 30 and the neighbors each get an unknown amount.

4. Label the tape diagram with the values that are known from the story. Label the unknown(s) with c to represent cupcakes.



5. What does c represent in this context?

- a. The total number of cupcakes given away
 b. The number of cupcakes the teacher got
 c. The number of cupcakes each neighbor gets

6. Use any strategy to find the value of c .

$$\begin{array}{r} 72 \\ -30 \\ \hline 42 \end{array}$$

$$42 \div 7 = 6$$

$c = 6$

Xavier is making 7 identical gift bags to give his friends after his birthday party. He puts 6 stickers into each gift bag. He also puts some chocolate bars in each bag. In all, he uses 56 items.

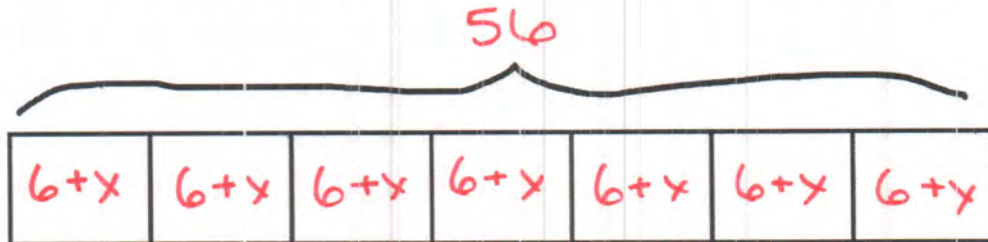
7. We know...

- there are 7 bags
- 6 stickers go in each
- there are 56 items in all

8. We don't know...

- how much chocolate goes in each

9. Use the tape diagram below to label what is known. Use x to represent any unknowns.



10. Use the tape diagram to find the number of chocolate bars in each bag.

$$56 \div 7 = 8$$

$$6 + x = 8$$

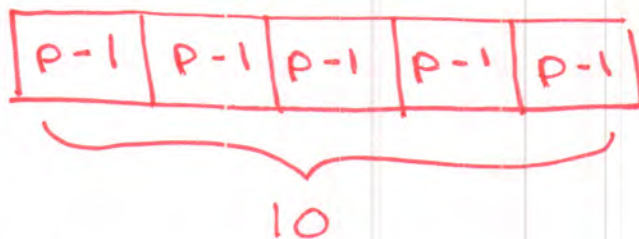
$$x = 2$$

Nina has 5 racks of shoes with p pairs of shoes on each rack. She wants to donate 1 pair of shoes from each rack. After donating, she has 10 pairs of shoes left.

11. What is known? What is unknown?

- 5 racks
- 10 left
- donates 1 per rack
- how many shoes started on each rack

12. Draw a tape diagram to represent the story. Use p to represent the number of pairs of shoes Nina originally had on each rack.



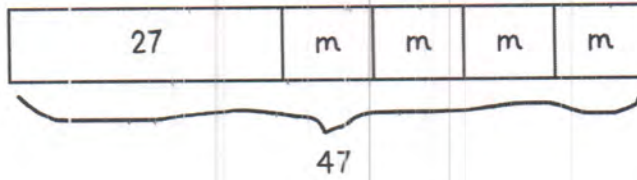
13. Use the tape diagram to find the value of p .

$$10 \div 5 = 2$$

$$p - 1 = 2$$

$$p = 3$$

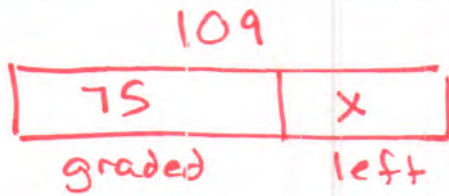
1. Select ALL the stories that the tape diagram can represent.



- a. Henry's spends \$47 on 4 tickets to the school play and \$27 at the concession stand.
- b. Maria buys 4 bags of mints with 27 mints in each pack. She gives 47 mints to her brother.
- c. Lance has a collection of 47 rocks. He gives 27 rocks to his best friend and puts the other rocks equally into 4 boxes.
- d. Yusef made 47 friendship bracelets. He sells 27 of them, and he splits the remaining bracelets up between 4 friends.
- e. There are 47 dogs and 27 cats in the animal shelter. The cats are split equally into 5 rooms.

2. Ms. Catalano is grading 109 essays. So far, she has graded 75 essays. How many essays does she have left to grade?

a. Draw a tape diagram to represent the story problem. Use x to represent the unknown.



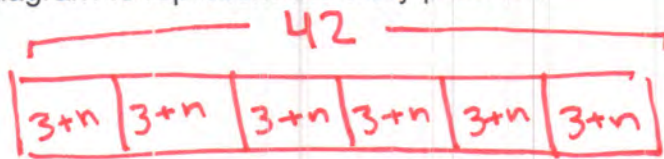
b. Use the tape diagram to determine how many essays Ms. Catalano has left to grade.

$$\begin{array}{r} 109 \\ - 75 \\ \hline 34 \end{array}$$

34 essays

3. Erin organizes her jewelry into boxes. She puts 3 bracelets and n necklaces into each box. After filling 6 boxes, she used a total of 42 pieces of jewelry.

a. Draw a tape diagram to represent the story problem.



b. Use the tape diagram to determine how many necklaces Erin puts in each box.

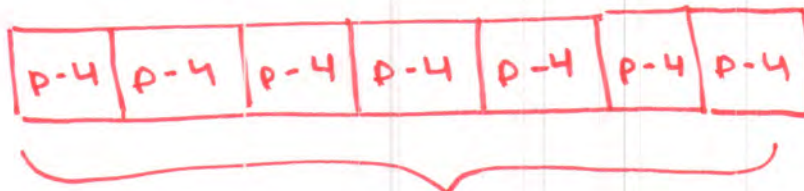
$$42 \div 6 = 7$$

$$3 + n = 7$$

$$n = 4$$

4. Mr. Guzman has 7 prize boxes for his students. Each box has p prizes. His students take 4 prizes from each box. Mr. Guzman had 21 prizes left in all. How many prizes were in each box to start with?

Draw a tape diagram as part of your solution pathway.



$$21 \div 7 = 3$$

$$p - 4 = 3$$

$$p = 7$$

G7 U5 Lesson 3

Write and match equations and tape diagrams that represent the same situation.

G7 U5 Lesson 3 - Students will write and match equations and tape diagrams that represent the same situation.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our previous lesson, we thought about how we can use tape diagrams to help us think about solutions to math problems. How did tape diagrams help us do that? **Possible Student Answers, Key Points:**

- Tape diagrams help us picture what is happening in a story. I can easily see parts and totals and consider how to find any unknowns.
- Based on how I draw my tape diagram, I can think of different ways to arrive at the same correct answer.

Today we're going to continue working with tape diagrams. Similar to how we saw multiple solution pathways given the same tape diagram in our previous lesson, today we'll see how we can write multiple equations to represent the same tape diagram. Let's get started.

Let's Talk (Slide 3): Look at the two tape diagrams shown here. What is the same about them? What is different? **Possible Student Answers, Key Points:**

- They're both the same length. They both have a 14, at least one 6, and some x's. They both show a rectangle split into smaller rectangles.
- The first one has three boxes inside, and the second only has two boxes. The second one has equal-sized boxes. The first one only has one term inside each box.

I notice both tape diagrams show a total of 14, but there is some variation in terms of how each box is composed. The first shows a 6 and x and an x. The second shows two equal groups of 6 plus x. Our work today will involve writing two or more equations to represent the same tape diagram. The first question we work on together is going to ask us to think about these two tape diagrams.

Let's Think (Slide 4): For this problem, we're tasked with writing 2 different, yet equivalent, expressions to represent each tape diagram.

$$6 + x + x = 14$$

$$6 + 2x = 14$$

Let's start by looking at the first tape diagram and thinking about how we can represent the visual with an equation. We already noted that this tape diagram has a total of 14 and that the boxes that compose the large rectangle show 6, x, and x. I know that if I combine those values, the total will equal 14. We can write that as an equation by simply writing $6 + x + x = 14$. (*write equation*) Another way I can think about the tape diagram is to think about the two x's as being two *groups of x*. In that

case, I might think 6 plus two equal groups of x is equal to 14. I can write that using multiplication. 6 plus 2x equals 14. (*write equation*) That's it! We just used two different equations to represent the same tape diagram.

Before we look at the next tape diagram, think back to our previous lesson. Can you use the tape diagram, or maybe one of the equations, to figure out what x must be equal to? **Possible Student Answers, Key Points:**

- I know the total is 14 and one part is 6, so I can take out 6 from 14 which leaves me with 8. Then I know $2x$ or $x + x$ has to equal 8, so x has to be 4.

$$14 - 6 = 8$$

$$2x = 8$$

$$x = 4$$

(*write equations as you narrate*) We can subtract the 6 part from the total of 14. 14 minus 6 equals 8. I know the 2 x-values now have to be equal to 8. $2x = 8$ or $x + x = 8$, so I know x has to be equal to 4.

Now let's do the same work with the other tape diagram. We already noted earlier that this tape diagram is a little different.

$$6 + x + 6 + x = 14$$

$$2(6 + x) = 14$$

I see two parts in this tape diagram: $6 + x$ and another $6 + x$. They're both the same parts. I know if I combine those two parts together, the total should be 14. One way to write that as an equation is to simply add everything inside the tape diagram. 6 plus x plus the other 6 plus x equals 14. *(write equations as you narrate)* Since both groups in this tape diagram are the same, I can also think of this as 2 groups of " $6 + x$ " have a total of 14. I can write that as 2

relationships we see in the tape diagram.

I wonder if you can use the tape diagram or equations to help you find x. What do you think? **Possible Student Answers, Key Points:**

- I see 14 split into two equal groups, so each group has to be equal to 7. I know because $7 + 7$ would equal 14. If each group is equal to 7, then I know x has to be 1. I know because $6 + 1 = 7$.

$$14 \div 2 = 7$$

$$6 + x = 7$$

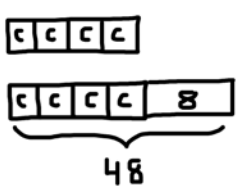
$$x = 1$$

We can easily see the equal groups in the tape diagram. If two equal groups have a total of 14, I know each group has a value of 7. There are a few ways to consider that, but I'll write 14 divided by 2, since my brain saw this as splitting 14 into 2 groups. *(write equations as you narrate)* If each group is worth 7, then I know $6 + x$ has to equal 7. From there, x must be 1. $6 + 1 = 7$.

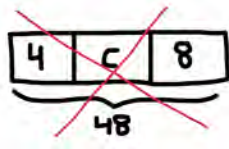
We just worked to write different, equivalent expressions to represent the first tape diagram. Then we did the same work with a different tape diagram.

Now let's see if we can create our own tape diagram based off of equations...

Let's Think (Slide 5): For this problem, we'll draw our own tape diagram to match the equation. Let's look at the first equation in red.



I can think of this equation as 4 groups of c plus 8 equals 48. I can start by drawing just that! I'll draw 4 groups of c by drawing four equal-sized rectangles. *(sketch as shown)* To that, I'll add 8 by drawing an adjacent rectangle and labeling that with the number 8. All four groups of c and the 8 should have a total of 48, so I'll use a bracket to show the total.



(sketch a tape diagram showing three adjacent rectangles labeled 4, c, and 8) Why would this tape diagram NOT represent the equation we were given? **Possible Student Answers, Key Points:**

- Our equation shows 4 groups of c. This equation only shows 1 group of c.
- This tape diagram shows $4 + c + 8 = 48$ instead of $4 \times c + 8 = 48$.

When we make our own tape diagrams, it's important to think about what the operations in our equation mean and how we can represent them accurately.

$$48 - 8 = 40$$

$$40 \div 4 = 10$$

$$c = 10$$

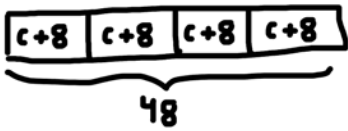
Before we look at our last equation, how could we use the correct tape diagram or the equation to find the solution? *(write equations as student shares, supporting as needed)*

Possible Student Answers, Key Points:

- I know the total is 48 and one part is 8. $48 - 8 = 40$, so I know all four equal groups of c must have a total of 40. $40 \div 4 = 10$ or $4 \times 10 = 40$ can help me see that $c = 10$.

Lastly, let's try sketching a tape diagram to represent the second equation in blue. What's the same and what's different about this equation? **Possible Student Answers, Key Points:**

- This equation also has a total of 48. The unknown is also labeled as c .
- This equation shows 4 groups of " $c + 8$ " instead of just 4 groups of c .



I can think of this equation as 4 groups of the value " $c + 8$ ". When I picture the tape diagram, I know that the 4 equal groups will have a total of 48. *(sketch a large rectangle, and partition it into 4 equal groups)* I'll use a bracket to show that the total is 48, and I can label $c + 8$ inside each of the 4 groups. *(label as narrated)* Here is our tape diagram that represents the equation.

$$48 \div 4 = 12$$

$$c + 8 = 12$$

$$c = 4$$

Could we use the tape diagram or equation to help us think about the solution? How so? *(write equations as student shares, supporting as needed)* **Possible Student Answers, Key Points:**

- I know the 48 is split into 4 groups, so each group has to equal 12. 48 divided by 4 equals 12. If each group equals 12, I can think of $c + 8 = 12$ to help me. I know c must be equal to 4.

Nicely done. Look at both tape diagrams we drew and their equations. What do you notice is different about the tape diagrams and why do you think that is? **Possible Student Answers, Key Points:**

- The first tape diagram has 4 c 's and only 1 8, because we were just adding 8 to the four groups of c .
- The second tape diagram has 4 c 's and 4 eights, because the equation shows 4 times the quantity of $c + 8$.

It's important to think carefully about what the operations in the equation mean, so that our tape diagram accurately reflects the problem.

Let's Try it (Slides 6 - 7): Now let's try out a few more similar problems. As we noticed today, whether we're writing an equation or a tape diagram, we want to carefully think about what operations mean. When we multiply, we're often thinking of it as equal groups. We can take equal groups of a single value or equal groups of an expression. Time to try out a few more before you get a chance to work on your own.

WARM WELCOME



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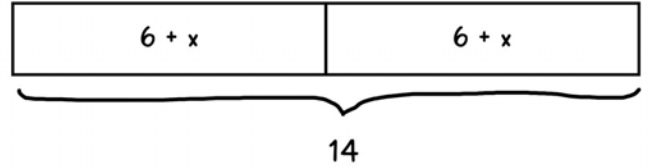
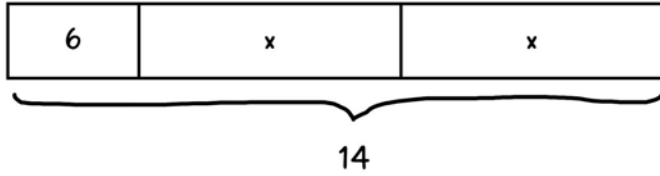
Today we will write and match equations and tape diagrams that represent the same situation.

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Let's Talk:

Look at the two tape diagrams.
What's the same? What is different?

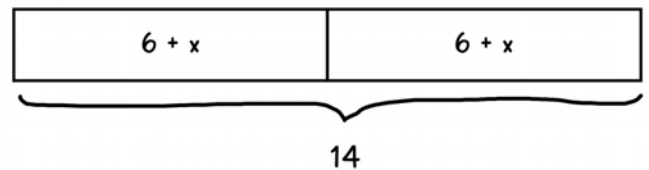
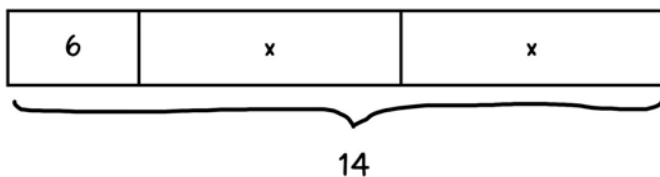


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Let's Think:

Write two different equations that could represent each tape diagram shown here.



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Let's Think:

Draw a tape diagram to match each equation.

$$4c + 8 = 48$$

$$4(c + 8) = 48$$

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Let's Try It:

Let's explore writing and matching equations and tape diagrams that represent the same situation together.

Name: _____ G7 U5 Lesson 3 - Let's Try It

Consider the tape diagram shown here.

- Write an equation to show that 28 is the sum of the parts in the tape diagram.
 $28 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$
- Rewrite the equation to show that 28 is equal to 10 plus 3 groups of n .
 $\underline{\quad} = \underline{\quad} + \underline{\quad}$
- Use either equation to find the value of n .

Consider the tape diagram shown here.

- Write an equation to show that 36 is the sum of the parts in the tape diagram.
 $36 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$
- Rewrite the equation to show that 36 is the sum of 3 groups of $x + 8$.
- Use either equation to find the value of x .

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Consider the tape diagram below.

- Write two different equations to represent the tape diagram.
- Use either equation you wrote to find the value of m .

Consider the tape diagram below.

- Write two different equations to represent the tape diagram.
- Use either equation you wrote to find the value of y .

Draw a tape diagram to match each equation. Then find the value of w .

11. $76 = 4w + 16$

12. $76 = 4(w + 4)$

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On your Own:


Now it's time to write and match equations and tape diagrams that represent the same situation on your own.

Name: _____ G7 US Lesson 3 - Independent Work

1. Match the equations to the tape diagram that represents it. Then find the value of k for each equation.

$25 = 5(k + 2)$ 

$25 = 5k + 2$ 

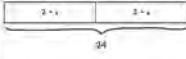


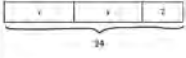
2. Draw and label a tape diagram to represent each equation. Then find the value of v in each.

$64 = 4v + 8$ $64 = 4(v + 8)$


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3. Write two different equations that could represent each tape diagram shown here. Find the value of the unknown.



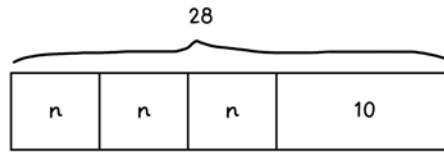


4. Fran drew this tape diagram to represent $9(r + 8) = 62$. Explain why this tape diagram does not represent her equation, and explain what she should do to correct her work.



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Consider the tape diagram shown here.



1. Write an equation to show that 28 is the sum of the parts in the tape diagram.

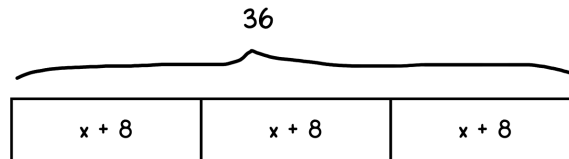
$$28 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

2. Rewrite the equation to show that 28 is equal to 10 plus 3 groups of n .

$$\underline{\quad} = \underline{\quad} + \underline{\quad}$$

3. Use either equation to find the value of n .

Consider the tape diagram shown here.



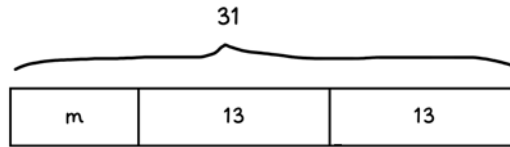
4. Write an equation to show that 36 is the sum of the parts in the tape diagram.

$$36 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}$$

5. Rewrite the equation to show that 36 is the sum of 3 groups of $x + 8$.

6. Use either equation to find the value of x .

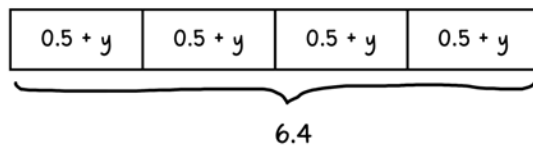
Consider the tape diagram below.



7. Write two different equations to represent the tape diagram.

8. Use either equation you wrote to find the value of m .

Consider the tape diagram below.



9. Write two different equations to represent the tape diagram.

10. Use either equation you wrote to find the value of y .

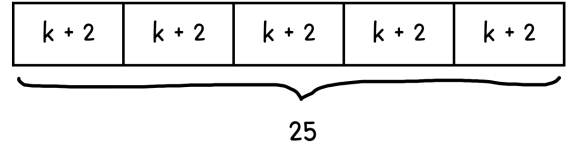
Draw a tape diagram to match each equation. Then find the value of w .

$$11.76 = 4w + 16$$

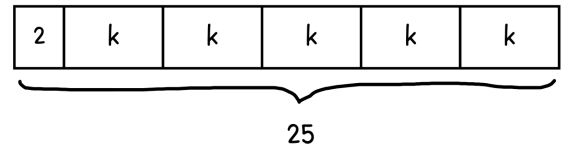
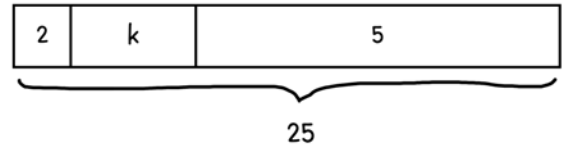
$$12.76 = 4(w + 4)$$

1. Match the equations to the tape diagram that represents it. Then find the value of k for each equation.

$$25 = 5(k + 2)$$



$$25 = 5k + 2$$

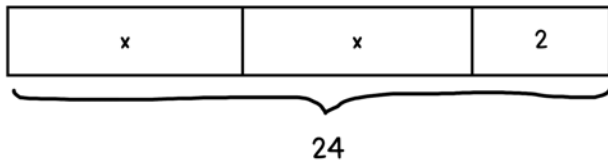
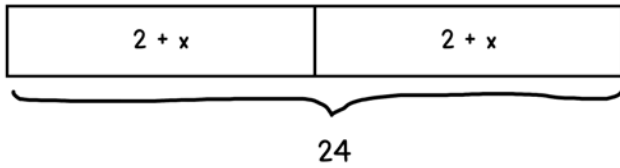


2. Draw and label a tape diagram to represent each equation. Then find the value of v in each.

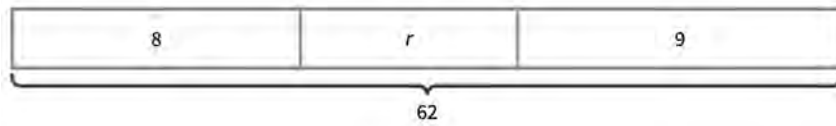
$$64 = 4v + 8$$

$$64 = 4(v + 8)$$

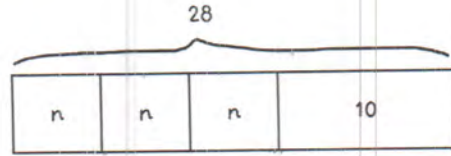
3. Write two different equations that could represent each tape diagram shown here. Find the value of the unknown.



4. Fran drew this tape diagram to represent $9(r + 8) = 62$. Explain why this tape diagram does not represent her equation, and explain what she should do to correct her work.



Consider the tape diagram shown here.



1. Write an equation to show that 28 is the sum of the parts in the tape diagram.

$$28 = n + n + n + 10$$

2. Rewrite the equation to show that 28 is equal to 10 plus 3 groups of n .

$$28 = 3n + 10$$

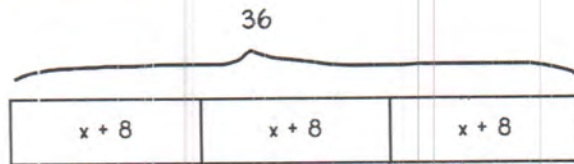
3. Use either equation to find the value of n .

$$\begin{array}{r} 28 - 10 \\ 18 \end{array}$$

$$\begin{array}{r} 18 \div 3 \\ 6 \end{array}$$

$$n = 6$$

Consider the tape diagram shown here.



4. Write an equation to show that 36 is the sum of the parts in the tape diagram.

$$36 = x + 8 + x + 8 + x + 8$$

5. Rewrite the equation to show that 36 is the sum of 3 groups of $x + 8$.

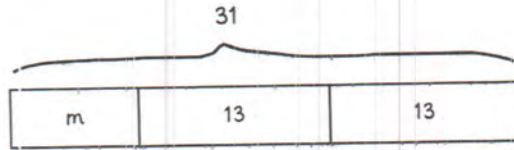
$$36 = 3(x + 8)$$

6. Use either equation to find the value of x .

$$\begin{array}{r} 36 \div 3 \\ 12 \end{array}$$

$$\begin{array}{r} x + 8 = 12 \\ x = 4 \end{array}$$

Consider the tape diagram below.



7. Write two different equations to represent the tape diagram.

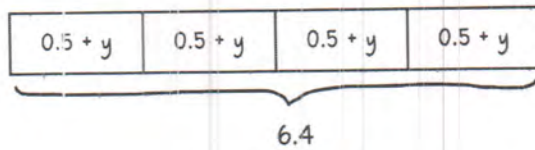
$$31 = m + 13 + 13 \qquad 31 = m + 26$$

8. Use either equation you wrote to find the value of m.

$$\begin{array}{r} 31 \\ - 26 \\ \hline 5 \end{array}$$

$$m = 5$$

Consider the tape diagram below.



9. Write two different equations to represent the tape diagram.

$$6.4 = 4(0.5 + y) \qquad 6.4 = 0.5 + 0.5 + 0.5 + 0.5 + y + y + y + y$$

10. Use either equation you wrote to find the value of y.

$$6.4 \div 4 = 1.6 \qquad 0.5 + y = 1.6$$

~~$$y = 1.1$$~~

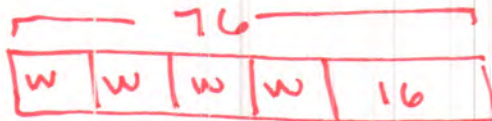
$$y = 1.1$$

Draw a tape diagram to match each equation. Then find the value of w.

$$11. 76 = 4w + 16$$

$$76 - 16 = 60$$

$$60 \div 4 = 15$$



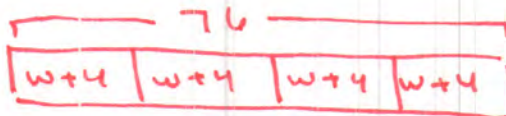
$$w = 15$$

$$12. 76 = 4(w + 4)$$

$$76 \div 4 = 19$$

$$w + 4 = 19$$

$$w = 15$$



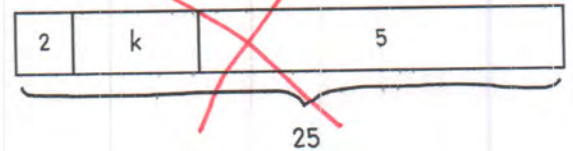
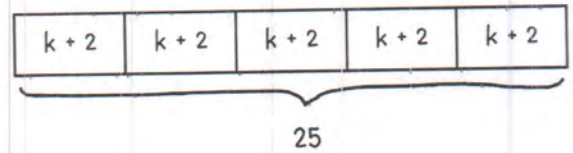
1. Match the equations to the tape diagram that represents it. Then find the value of k for each equation.

$$25 = 5(k + 2)$$

$$25 \div 5 = 5$$

$$k + 2 = 5$$

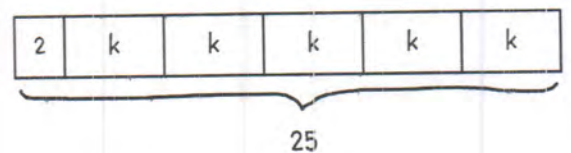
$$k = 3$$



$$25 = 5k + 2$$

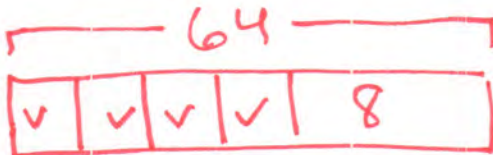
$$25 - 2 = 23$$

$$23 \div 5 = 4 \frac{3}{5} = k$$



2. Draw and label a tape diagram to represent each equation. Then find the value of v in each.

$$64 = 4v + 8$$

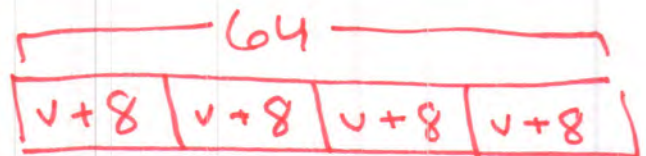


$$64 - 8 = 56$$

$$56 \div 4 = v$$

$$14 = v$$

$$64 = 4(v + 8)$$



$$64 \div 4 = 16$$

$$v + 8 = 16$$

$$v = 8$$

3. Write two different equations that could represent each tape diagram shown here. Find the value of the unknown.



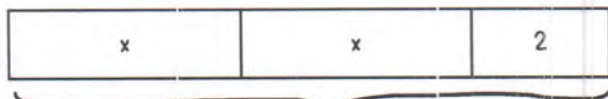
$$2 + x + 2 + x = 24$$

$$2(2 + x) = 24$$

$$24 \div 2 = 12$$

$$2 + x = 12$$

$$x = 10$$



$$x + x + 2 = 24$$

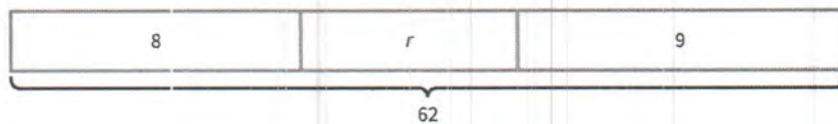
$$2x + 2 = 24$$

$$24 - 2 = 22$$

$$22 \div 2 = 11$$

$$x = 11$$

4. Fran drew this tape diagram to represent $9(r + 8) = 62$. Explain why this tape diagram does not represent her equation, and explain what she should do to correct her work.



This tape diagram does not show 9 groups of " $r + 8$ ". It shows one group of 8, one group of r , and one group of 9. A better equation would be $8 + r + 9 = 62$ or $r + 17 = 62$.

G7 U5 Lesson 4

Coordinate tape diagrams, equations of the form $px + q = r$, and verbal descriptions of the situations, and reason about and interpret a solution.

G7 U5 Lesson 4 - Students will coordinate tape diagrams, equations of the form $px + q = r$, and verbal descriptions of the situations, and reason about and interpret a solution.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our previous few lessons, we've explored how tape diagrams can help us represent and solve story problems or equations. We're going to continue that work today. As we work, since we often rely on the visual tape diagram to help us make sense of what is going on, it's important that our tape diagrams be neat and easy to interpret. What can we do today as we work to keep our tape diagrams clear and helpful? **Possible Student Answers, Key Points:**

- We can use word labels so we know what each part of the tape diagram means in a given context.
- We can use number labels so we can think about the values in the problem.
- We can try to keep our lines straight and any equal groups even so that the tape diagram is easy to read.

Those are great ideas. No tape diagram is perfect, but we still want to try to keep them as neat and accurate as possible so they can be helpful for interpreting any given scenario. Our goal today will be to think about how we can make sense of a particular type of story problem by using equations and tape diagrams. We'll want to make sure we can explain clearly how the tape diagram or equation accurately represents the given scenario.

Let's begin by being creative and thinking of our own story to match a tape diagram.

Let's Talk (Slide 3): Take a look at this tape diagram. Before we think about a story, pause to consider what you notice about this tape diagram. **Possible Student Answers, Key Points:**

- The total is 20. I see that is labeled with a bracket.
- I see three groups of m and one 2.

Can you think of a story that has those components? The story should have a total of 20, and the total needs to be composed of 3 groups of something and 2 more. **Possible Student Answers, Key Points:**

- There are 20 people on a playground. There are 2 teachers and 3 equal groups of students. How many students are in each group?
- I have 3 equal bags of M&Ms and 2 loose M&Ms. In all, I have 20 M&Ms. How many M&Ms are in each bag?

Any story that involve 3 groups of something and 2 more of that thing with a total of 20 can work to match this problem.

(share one or two more examples like the ones below)

- I have 3 bags of books and I'm holding 2 more books. How many books are in each bag if I have 20 books total?
- John spends \$20 at the store. He buys a pack of gum for \$2 and 3 bags of chips. How much does each bag of chips cost?

Let's Think (Slide 4): Let's use our thinking to answer this problem. It wants us to write an equation to represent the tape diagram, and then they want us to find the solution.

We just came up with several ideas for the tape diagram. Let's think about this one for the purposes of this problem: John spends \$20 at the store. He buys a pack of gum for \$2 and 3 bags of chips. How much does each bag of chips cost?

(change the context for this problem to match a student's context they came up with, or you can use the provided context)

$$3m + 2 = 20$$

How can we use the tape diagram and/or the context of the story to help us write a corresponding equation? I know we have a total of \$20, and that the bags of chips and the pack of gum compose that total. (write equation as you narrate) I can write "3m" to represent the cost of the bags of chips, and I can write + 2 to show the additional cost of the one pack of gum. This all should equal 20. How does the equation we just wrote match the tape diagram? Possible Student Answers, Key Points:

- The tape diagram shows 3 groups of m, which is the same as the 3m in our equation. The tape diagram also shows a value of 2 attached to the three m's, which matches the "+2" in our equation. And both the equation and the tape diagram show a total of 20.

$$3m + 2 = 20$$

$$20 - 2 = 18$$

$$3m = 18$$

$$m = 6$$

Excellent. We agree that both the tape diagram and the equation represent the story. Now let's solve the equation by thinking about the tape diagram. (write equations as you narrate) Let's start by subtracting the cost of the pack of gum from the \$20 total. What is $20 - 2$? (18) That means the remaining 3 bags of chips should total \$18. If $3m = 18$, what does m have to equal? ($m = 6$) Our solution is 6. Great job.

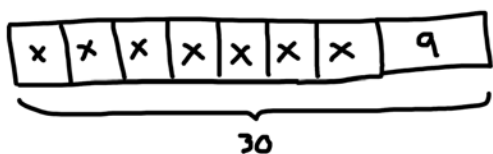
Before we close out on this problem, what does the 6 mean in the context of the story we're talking about? Possible Student Answers, Key Points:

- The solution of 6 in our story means that each bag of chips that John bought cost \$6.

Let's Think (Slide 5): For this problem, we have two word problems. For each word problem we will do three things. First, we'll draw a tape diagram to represent what is happening. Second, we'll write an equation to represent what is happening. Third, we'll solve to determine the unknown by using our tape diagram and equation.

Let's begin by looking at the first problem in red. I'll read it aloud, while you follow along. What is the problem about? What is known? What is unknown? Possible Student Answers, Key Points:

- We don't know how many crayons are in a pack.
- We know that he has 7 packs of crayons and 9 more crayons. We also know that he has 30 crayons in all.



We can represent this problem by drawing a tape diagram that shows 7 boxes of crayons and 9 more individual crayons. (sketch as you narrate) I can draw 7 equal-sized rectangles to represent the boxes of crayons. I'll put an x in each one, since the number of crayons in each pack is unknown. Since I know there are 9 additional crayons, I'll tack on another rectangle that shows 9

crayons. I can use a bracket to quickly show that the total number of crayons is 30. Thinking about the parts of the story I know and whether the parts are equal groups of one-off quantities, helps me make an accurate model of the problem.

$$7x + 9 = 30$$

Now all we need to think about is our equation. I think I have an idea of an equation we can write, but I'll want your help making sure each part of my equation makes sense.

(write $7x + 9 = 30$) I've written $7x + 9 = 30$. How does each part of this equation connect back to the situation we represented with our tape diagram? Possible Student Answers, Key Points:

- The $7x$ represents the 7 boxes of crayons. We don't know the number of crayons in each box, so that's represented by the x.
- The $+ 9$ represents the 9 extra crayons. And the 30 is the total number of crayons.

$$30 - 9 = 21$$

$$7x = 21$$

$$x = 3$$

Great thinking. Time to solve. I can see from the tape diagram, that it could be an efficient first step to remove the 9 crayons from my total of 30. What is 30 minus 9? (21) (write $30 - 9 = 21$) So the 7 boxes of crayons have a total of 21 crayons inside them. I can show that as $7x = 21$. If 7 times x equals 21, then I know x has to equal 3 (write equation and solution) What does a solution of 3 mean in this context? Possible Student Answers, Key Points:

- A solution showing $x = 3$ means that each box of crayons had 3 crayons in it.

Now let's take a second to do similar work with the second, blue word problem. Read the problem to yourself, and then describe what is known and unknown when you're ready. Possible Student Answers, Key Points:

- Farrah has yarn. She cuts a piece off, and then she cuts the remaining yarn into 9 equal-sized pieces.
- We don't know how long the 9 equal-sized pieces are. We can represent that with x .



Let's begin by drawing a tape diagram. What's the total length of yarn that Farrah used? (30 inches) (sketch tape diagram as you narrate) I'll draw a rectangle and use a bracket to show that the total is 30. How can we model what happens next with the yarn using our tape diagram? Possible Student Answers, Key Points:

- She cuts off 7 inches, so we can partition to make a box that represents that length.
- She cuts the rest into 9 equal pieces, so we can take the leftover section and divide it into 9 small rectangles of equal size. Those represent the nine pieces.

Excellent thinking. I can see the total length is 30. I see one piece she cut off is 7 inches, and then I see the 9 pieces of equal length that she cut with the remaining yarn. Now I need to think what this can look like as an equation. I know the parts that make up the yarn are the 7-inch piece and the other 9 pieces of unknown length. I can show this with an equation by showing 7 plus $9x$ equals 30. (write equation) The 7 is the length she cut initially. The $9x$ represents the 9 pieces, with x representing their unknown length. And we know the total length of yarn to begin with is 30.

$$7 + 9x = 30$$

$$30 - 7 = 23$$

$$9x = 23$$

$$x = \frac{23}{9} \text{ OR } 2\frac{5}{9}$$

Based on our tape diagram and the equation we just wrote, how could we find the value of x ? (write equations as student shares, supporting as needed) Possible Student Answers, Key Points:

- We can subtract the 7-inch piece from the original 30 inches. $30 - 7 = 23$.
- Then she cut the 23-inch piece into 9 equal pieces. We can write a related division fact or divide 23 by 9 to find that value.

Great. There were 23 inches that was cut into 9 equal pieces. 23 divided by 9 isn't a friendly number, so we can think of it as $\frac{23}{9}$ or $2\frac{5}{9}$ inches. Our solution means each of the 9 pieces of yarn measured $2\frac{5}{9}$ inches.

Let's Try it (Slides 6 - 7): Now let's try some similar problems on our own. It can be helpful, when given a story problem, to show what is known in a tape diagram before attempting any computation. Anything that is unknown, we can represent with a variable. Once we have our tape diagram, we can write an equation that shows an equal groups and any additional values and set those equal to the total in the scenario. The tape diagram and equation help us think about a solution pathway that makes sense. I know you're going to do great. Let's begin.

WARM WELCOME



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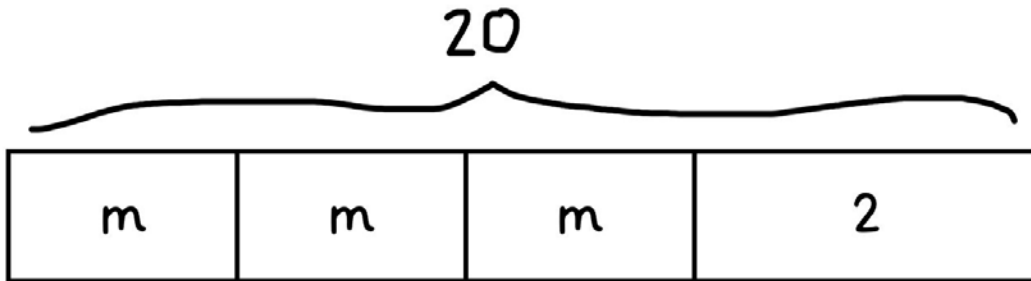
Today we will coordinate tape diagrams, equations of the form $px + q = r$, and verbal descriptions of the situations, an reason about and interpret a solution.

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Let's Talk:

Think of a story that could match the tape diagram below...

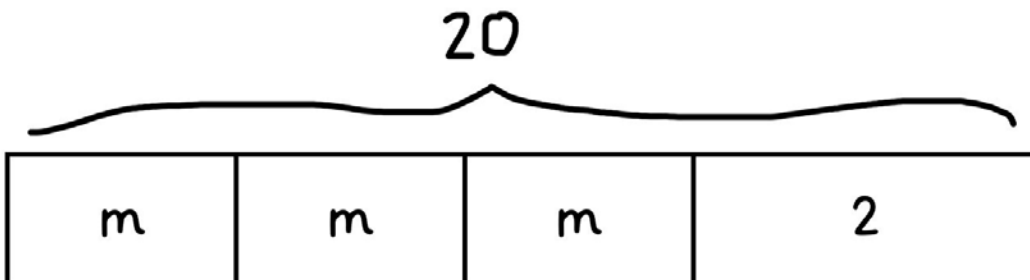


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Let's Think:

Write an equation to represent the tape diagram. Then find the solution to the equation.



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Let's Think:

Draw a tape diagram to represent each situation. For each situation write and solve an equation.

Darryl has 7 packs of crayons. Each pack has x crayons in it. His teacher gives him 9 more crayons, and now Darryl has 30 crayons.

Farrah has 30 inches of yarn. She cuts off 7 inches, and then cuts the remaining yarn into 9 equal length piece of x inches each.

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Let's Try It:

Let's explore coordinating tape diagrams, equations in the form $px + q = r$, and verbal descriptions of the situations together.

Name: _____ G7 US Lesson 4 - Let's Try It

Gary has 5 bunches of grapes. Each bunch has g grapes in it. His friend gives him 7 more grapes, and now Gary has 52 grapes in all.

- What is known in this story?
- What is unknown in this story?
- Circle the tape diagram that best represents this story.




- Use the tape diagram to complete the equation.


$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$
- Solve the equation for g .
- What does your solution mean in the context of the story?

Julianne is crafting with ribbon. She has 27 feet of ribbon. She cuts off 6 feet of ribbon, and then cuts the rest of the ribbon into 2 equal pieces of r feet.

- What is known in this story? What is unknown?

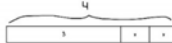
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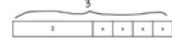
- Label the tape diagram below to represent the knowns and unknowns in the story.

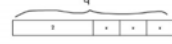

- Use the tape diagram to write an equation that represents the story.
- Solve for r .
- What does your solution mean in the context of the story?

Match each situation to the tape diagram that best represents it. Then write a corresponding equation.

- Leo spends 3 hours at the gym. He spends 2 hours doing yoga, and then jogs 4 miles at a constant rate.


- Andy has 4 gallons of punch. She pours equal amounts for 3 of her friends, and has 2 gallons left over.


- Tessa spends \$4 on snacks. She bought 2 candy bars, and spent \$3 on popcorn.



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On your Own:

Now it's time to coordinate tape diagrams, equations of the form $px + q = r$, and verbal descriptions of the situations on your own.

Name: _____ Q7 US Lesson 4 - Independent Work

1. A bowling alley charges a family rate of \$10 for a round of bowling, plus a fee for each pair of bowling shoes rented. A family of 5 plays a round of bowling together, and each person rents a pair of bowling shoes. They pay a total of \$25.

a. Circle the tape diagram that best represents this situation.

b. What does each r represent in this problem?

c. Write and solve an equation to represent the problem.

2. Khyren buys 4 packs of tennis balls and 2 individual tennis balls. In all, he buys 34 tennis balls.

a. Label the tape diagram below to represent the situation.

b. Write and solve an equation to represent the problem.

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3. A woodworker has a 28-foot plank of wood. He cuts off 4 feet of wood. Then, he cuts the remaining length into 8 equal lengths of w feet each. Draw a tape diagram and write an equation to represent this situation. Then find the value of w .

4. Match each story to the tape diagram that best represents it. Then find the value of x in two tape diagrams you select.

Jayden spends \$14 at a carnival. He bought 3 tickets to play games, and he paid \$5 to go on the ferris wheel.

Alicia has 14 cups of pancake batter. She pours equal amounts into 5 containers, and she has 3 cups leftover.

Alicia has 14 cups of pancake batter. She pours equal amounts into 5 containers, and she has 3 cups leftover.

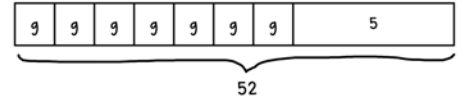
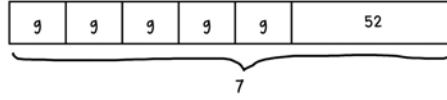
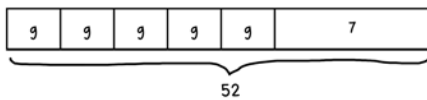
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Name: _____

Gary has 5 bunches of grapes. Each bunch has g grapes in it. His friend gives him 7 more grapes, and now Gary has 52 grapes in all.

1. What is known in this story?
2. What is unknown in this story?
3. Circle the tape diagram that best represents this story.



4. Use the tape diagram to complete the equation.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

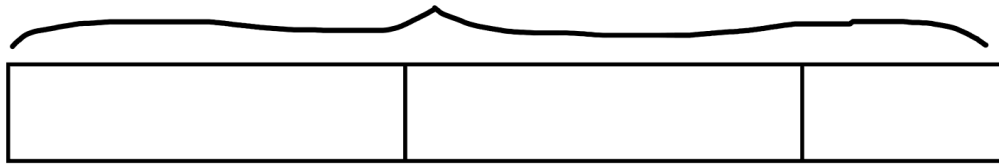
5. Solve the equation for g .

6. What does your solution mean in the context of the story?

Julianne is crafting with ribbon. She has 27 feet of ribbon. She cuts off 6 feet of ribbon, and then cuts the rest of the ribbon into 2 equal pieces of r feet.

7. What is known in this story? What is unknown?

8. Label the tape diagram below to represent the knowns and unknowns in the story.



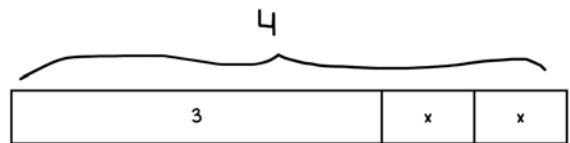
9. Use the tape diagram to write an equation that represents the story.

10. Solve for r .

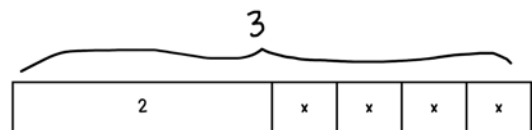
11. What does your solution mean in the context of the story?

Match each situation to the tape diagram that best represents it. Then write a corresponding equation.

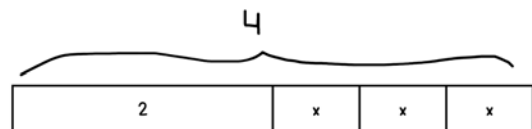
12. Leo spends 3 hours at the gym. He spends 2 hours doing yoga, and then jogs 4 miles at a constant rate.



13. Andy has 4 gallons of punch. She pours equal amounts for 3 of her friends, and has 2 gallons left over.

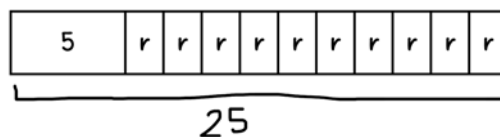
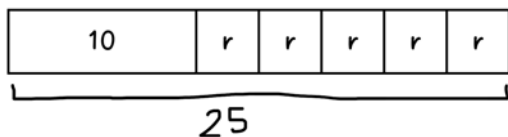


14. Tessa spends \$4 on snacks. She bought 2 candy bars, and spent \$3 on popcorn.



1. A bowling alley charges a family rate of \$10 for a round of bowling, plus a fee for each pair of bowling shoes rented. A family of 5 plays a round of bowling together, and each person rents a pair of bowling shoes. They pay a total of \$25.

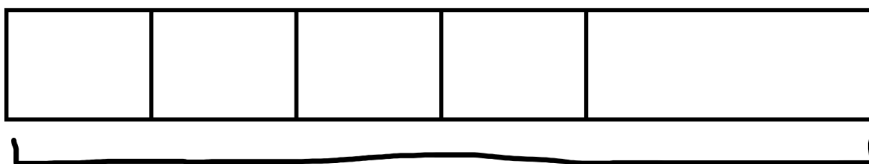
- a. Circle the tape diagram that best represents this situation.



- b. What does each r represent in this problem?
- c. Write and solve an equation to represent the problem.

2. Khyren buys 4 packs of tennis balls and 2 individual tennis balls. In all, he buys 34 tennis balls.

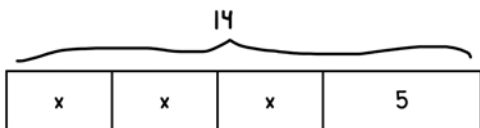
- a. Label the tape diagram below to represent the situation.



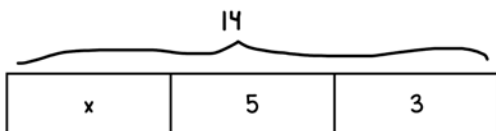
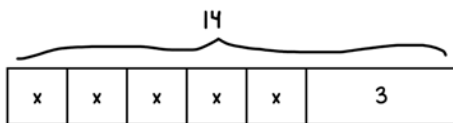
- b. Write and solve an equation to represent the problem.

3. A woodworker has a 28-foot plank of wood. He cuts off 4 feet of wood. Then, he cuts the remaining length into 8 equal lengths of w feet each. Draw a tape diagram and write an equation to represent this situation. Then find the value of w .

4. Match each story to the tape diagram that best represents it. Then find the value of x in two tape diagrams you select..



Jayden spends \$14 at a carnival. He bought 3 tickets to play games, and he paid \$5 to go on the ferris wheel.



Alicia has 14 cups of pancake batter. She pours equal amounts into 5 containers, and she has 3 cups leftover.

Name: KEY

Gary has 5 bunches of grapes. Each bunch has g grapes in it. His friend gives him 7 more grapes, and now Gary has 52 grapes in all.

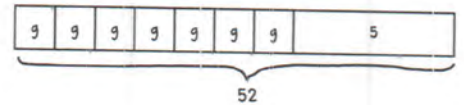
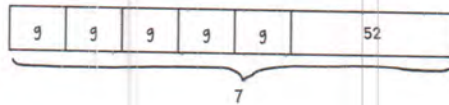
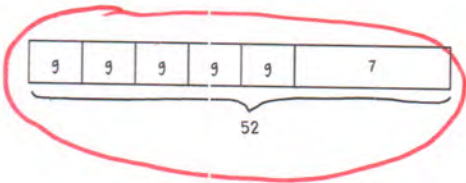
1. What is known in this story?

- 5 bunches
- He gets 7 more
- total of 52

2. What is unknown in this story?

- # of grapes in each bunch

3. Circle the tape diagram that best represents this story.



4. Use the tape diagram to complete the equation.

$$\underline{5g} + \underline{7} = \underline{52}$$

5. Solve the equation for g .

$$52 - 7 = 45 \quad 45 \div 5 = g$$
$$9 = g$$

6. What does your solution mean in the context of the story?

Each bunch of grapes has 9
grapes.

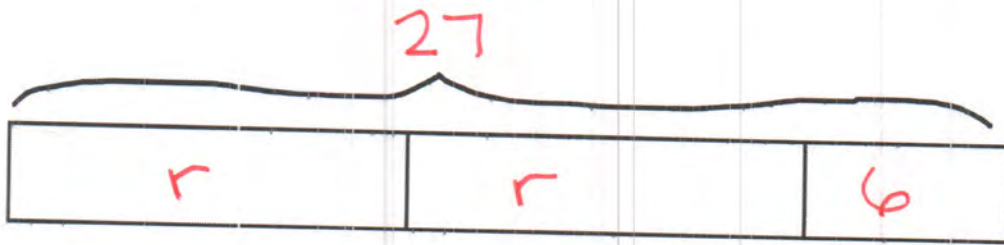
Julianne is crafting with ribbon. She has 27 feet of ribbon. She cuts off 6 feet of ribbon, and then cuts the rest of the ribbon into 2 equal pieces of r feet.

7. What is known in this story? What is unknown?

- 27 total feet
- cuts off 6 feet
- splits the rest 2 ways

- how long each of the 2 pieces are

8. Label the tape diagram below to represent the knowns and unknowns in the story.



9. Use the tape diagram to write an equation that represents the story.

$$2r + 6 = 27$$

10. Solve for r .

$$27 - 6 = 21$$

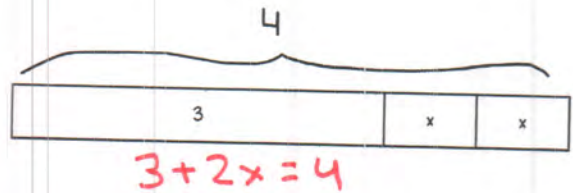
$$21 \div 2 = 10\frac{1}{2} \quad r = 10\frac{1}{2}$$

11. What does your solution mean in the context of the story?

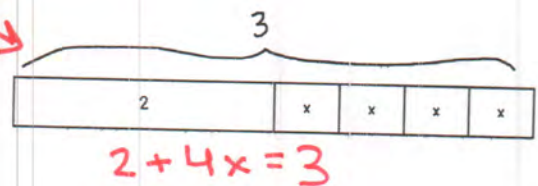
Each of the two pieces were
 $10\frac{1}{2}$ feet long.

Match each situation to the tape diagram that best represents it. Then write a corresponding equation.

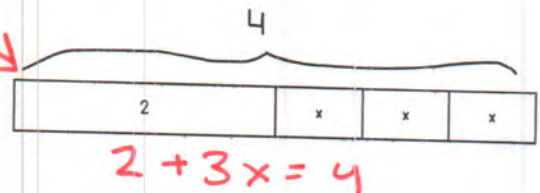
12. Leo spends 3 hours at the gym. He spends 2 hours doing yoga, and then jogs 4 miles at a constant rate.



13. Andy has 4 gallons of punch. She pours equal amounts for 3 of her friends, and has 2 gallons left over.

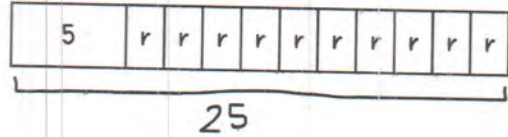
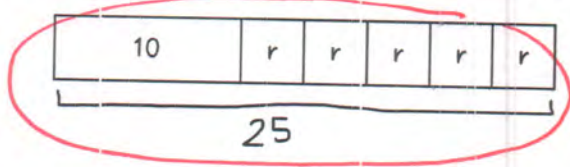


14. Tessa spends \$4 on snacks. She bought 2 candy bars, and spent \$3 on popcorn.



1. A bowling alley charges a family rate of \$10 for a round of bowling, plus a fee for each pair of bowling shoes rented. A family of 5 plays a round of bowling together, and each person rents a pair of bowling shoes. They pay a total of \$25.

a. Circle the tape diagram that best represents this situation.



b. What does each r represent in this problem?

the cost to rent bowling shoes

c. Write and solve an equation to represent the problem.

$$5r + 10 = 25$$

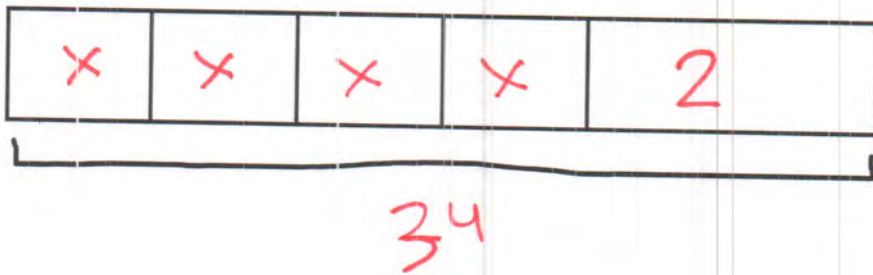
$$5r = 15$$

$$25 - 10 = 15$$

$$r = 3$$

2. Khyren buys 4 packs of tennis balls and 2 individual tennis balls. In all, he buys 34 tennis balls.

a. Label the tape diagram below to represent the situation.



b. Write and solve an equation to represent the problem.

$$4x + 2 = 34$$

~~$$4x + 2 = 34$$~~

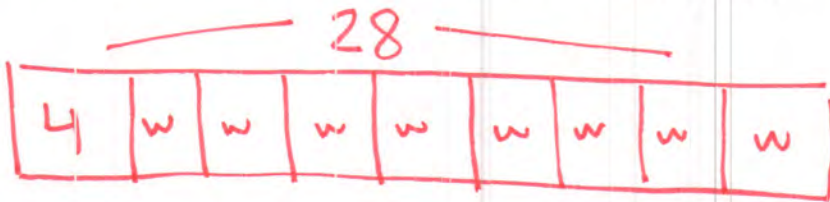
$$34 - 2 = 32$$

$$4x = 32$$

$$32 \div 4 = x$$

$$8 = x$$

3. A woodworker has a 28-foot plank of wood. He cuts off 4 feet of wood. Then, he cuts the remaining length into 8 equal lengths of w feet each. Draw a tape diagram and write an equation to represent this situation. Then find the value of w .



$$4 + 8w = 28$$

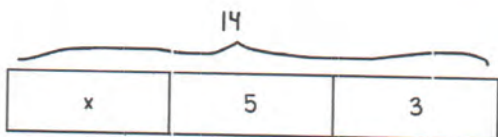
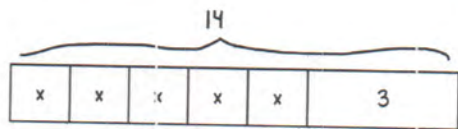
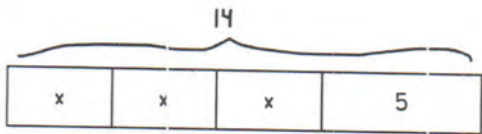
$$28 - 4 = 24$$

$$8w = 24$$

$$24 \div 8 = w$$

$$3 = w$$

4. Match each story to the tape diagram that best represents it. Then find the value of x in two tape diagrams you select..



Jayden spends \$14 at a carnival. He bought 3 tickets to play games, and he paid \$5 to go on the ferris wheel.

$$3x + 5 = 14$$

$$3x = 9$$

$$x = 3$$

Alicia has 14 cups of pancake batter. She pours equal amounts into 5 containers, and she has 3 cups leftover.

$$5x + 3 = 14$$

$$5x = 11$$

$$x = 2\frac{1}{5}$$

G7 U5 Lesson 5

Coordinate tape diagrams, equations of the form $p(x + q) = r$, and verbal descriptions of the situations, and reason about and interpret a solution.

G7 U5 Lesson 5 - Students will coordinate tape diagrams, equations of the form $p(x + q) = r$, and verbal descriptions of the situations, and reason about and interpret a solution.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our previous lesson, we worked hard to sketch tape diagrams and write equations to represent situations. We worked with different contexts, but you might have noticed that all of our problems had a similar structure. Today, we're actually going to do very similar work, but the structure of our problems will be slightly different. As we work today, I challenge you to consider what's the same about our work and what's different compared to the problem types we saw in our last lesson. Let's get going!

Let's Talk (Slide 3): I'm going to read this word problem. While I read, I want you to simply think about how you could summarize the story. What do we know? What don't we know? (*read problem*) **Possible Student Answers, Key Points:**

- Iris has some milk that she's pouring evenly into 5 bowls. She pours some into each bowl, then pours a little more into each bowl.
- We know the total amount of milk is 45 ounces. We know she has 5 bowls of milk. And we know that after she's poured some into the bowls, she pours 3 more ounces into each.
- We don't know how much she poured in each bowl to begin with.

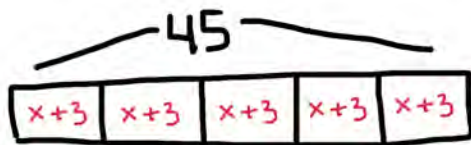
This problem feels a little different than the problems we looked at in our last lesson. In our last lesson, we saw problems where we had some equal groups and then another extra value of some sort. For instance, we saw a problem about 7 identical boxes of crayons plus 9 extra crayons. Or, we saw a problem about cutting of 7 inches of yarn and then splitting the rest into 9 equal pieces. Each problem involved a set of equal groups and another value in one way or another.

I notice that this problem feels a little different. Here we have some equal groups, the bowls of milk, but the extra value is also a part of those equal groups. Hm, I wonder how we can think about this...

Let's see if we can use some of the same thinking and modeling to help us tackle this different type of problem.

Let's Think (Slide 4): Our first problem deals directly with the story we just wondered about. Let's draw a tape diagram, write an equation, and then solve the equation related to this context.

Let's see if we can draw a tape diagram based on what we know. We know the total amount of milk is 45 ounces, so I'll start drawing a large rectangle and labeling it 45. (*sketch tape diagram as you narrate*) What can I do with this tape diagram to show that she poured the milk into 5 separate bowls? (**Partition the big rectangle into 5 equal sections.**) I'll draw four evenly spaced lines to cut the whole rectangle into 5 boxes. Each small box represents a bowl, so now I need to think about what I know about each bowl. I *don't* know how much she poured in the bowls to start, but I do know that she topped each bowl off with 3 more ounces. I can think of that as $x + 3$ in each



bowl. I'll write $x + 3$ in each rectangle. Take a look at the tape diagram. Does it accurately represent the story? How do you know? **Possible Student Answers, Key Points:**

- Yes. I see that 45 represents the total amount. I see the 5 boxes representing the bowls.
- Inside each bowl I see an unknown amount plus 3. This represents the original amount she poured in each bowl plus the 3 more ounces she poured in after.

$$5(x+3) = 45$$

Great! Let's use the story and our tape diagram to write an equation now. When I look at this tape diagram, I can see it as 5 groups of $x + 3$ that have a total of 45. I know I can use multiplication to represent equal groups, so the equation I'll write is $5(x + 3) = 45$. (*write equation*) Does this equation accurately represent what is happening in the word problem? How do you know? [Possible Student Answers](#),

Key Points:

- Yes, it does! The $5(x + 3)$ represents the 5 equal bowls. Inside each bowl there is an unknown amount plus 3 more ounces that were poured in later. The entire value of all 5 bowls is equal to 45 ounces.

$$45 \div 5 = 9$$

$$x + 3 = 9$$

$$x = 6$$

We have a tape diagram. We have an equation. Now we solve! Looking at the tape diagram, I know I can divide 45 by 5 to figure out the amount of milk in each bowl. (*write equations as you discuss*) What is 45 divided by 5? (9) Each bowl had 9 ounces in it once Iris was done. So to find the original amount she poured in, I can set any of the bowls equal to 9. I'll write $x + 3 = 9$, since each bowl is represented by $x + 3$. What must x equal? You can mentally substitute in a value or rewrite as a related subtraction equation to help you. (6)

So, x is equal to 6. What does a solution of 6 mean in the context of this problem? [Possible Student](#)

Answers, Key Points:

- We didn't know how much Iris poured in each bowl to start. A solution of 6 means that Iris poured 6 ounces into each bowl before going back to pour in 3 more ounces into each bowl.

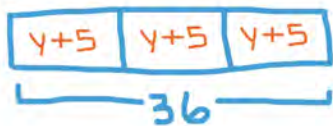
We just drew a tape diagram and then wrote and solved an equation to represent this real-world problem. Bravo! You're ready for another one.

Let's Think (Slide 5): Let's read this problem together. Before we solve it, we're going to assess another person's mistake. Doing this will help us avoid this mistake in the future. (*read problem*) What do we know in this story? What is unknown? [Possible Student Answers, Key Points:](#)

- We know there is a current total of 36 belts that are evenly arranged in 3 drawers. Wallace had some belts in each drawer already, and then put 5 more belts in each drawer.
- We don't know how many belts were in each drawer before Wallace added more in.

Picture what that might look like in your mind for a moment. Now look at Wallace's tape diagram. It's incorrect, and we'll help him fix it in a moment. Why does his tape diagram *not* match the story? [Possible Student Answers, Key Points:](#)

- He has a total of 36, but this story tells us they're split evenly into 3 drawers. Wallace's model is split into 4 boxes.
- Each drawer should have +5 in it, because he added 5 belts to each drawer. His tape diagram makes it look like he added 5 just one time.



It looks like Wallace maybe tried to represent the 3 drawers with each y , but he only added the 5 one time at the end. The problem clearly names that Wallace adds 5 belts to *each drawer*. Let's fix it. I'll still draw a rectangle with the total labeled as 36. (*sketch tape diagram as you narrate*) I'll cut the large rectangle into 3 sections to represent the drawers. We know there was an unknown amount in the drawers before Wallace adds 5 each. I'll write $y + 5$ in each

drawer to represent that. Our tape diagram is a more accurate representation of the details in the story than what Wallace tried. We've corrected his mistake, but let's now help him solve too!

$$3(y+5) = 36$$

If we think of this as 3 equal groups of $y + 5$, what equation can we write to represent the problem? (*write as student shares*) [Possible Student Answers, Key Points:](#)

- I can show the 3 drawers as $3(y + 5)$. All three drawers have a total of 36, so I can set $3(y + 5)$ equal to 36.

$$36 \div 3 = 12$$
$$y + 5 = 12$$
$$y = 7$$

Our equation shows that 3 equal drawers of “ $y + 5$ ” belts total up to 36. It’s time to solve. (*write equations as you discuss*) I can start by taking the total, 36, and dividing it by 3. This will help us think about just 1 drawer. What is 36 divided by 3? (12) Each drawer has 12 belts in it. So if 1 drawer is represented by $y + 5$, then I can think about $y + 5 = 12$ to solve for y . What is the value of y ? How do you know? Possible Student Answers, Key Points:

- I rewrote the equation as $12 - 5 = y$. I know $y = 7$.
- The solution is 7, because I know $7 + 5$ is equal to 12.

This means Wallace had 7 belts in each drawer before he put 5 more belts in each. Nice job fixing Wallace’s original error and helping him determine the value of the unknown using a tape diagram and an equation.

Let’s Try it (Slides 6 - 7): Now let’s try out a few more problems. Each of our problems today will involve equal groups, which we know we can represent in a tape diagram. We also know we can represent equal groups in an equation with the help of multiplication and parentheses. As we draw tape diagrams and write equations, I encourage you to constantly think back to the context of the story to make sure every aspect of your tape diagram and every part of your equation accurately tie back to what is happening in the given scenario. Time to get to work!

WARM WELCOME



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Today we will coordinate tape diagrams, equations of the form $p(x + q) = r$, and verbal descriptions of the situations, an reason about and interpret a solution.

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Let's Talk:

Iris has 45 ounces of milk. She pours the same amount of milk into 5 bowls, then adds 3 more ounces to each bowl.

What is known?
What is unknown?



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Let's Think:

Iris has 45 ounces of milk. She pours the same amount of milk into 5 bowls, then adds 3 more ounces to each bowl.

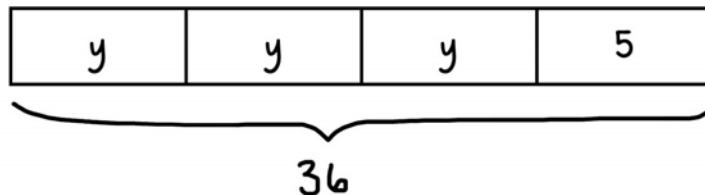
- Draw a tape diagram to represent the situation.
- Write an equation to represent the situation.
- Solve the equation.

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Let's Think:

Each of 3 drawers contains y belts. Wallace adds 5 more belts to each drawer. There are now 36 belts in all. He drew the diagram below.

Why does Wallace's tape diagram not represent the situation? Correct his thinking.



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Let's Try It:

Let's explore coordinating tape diagrams, equations, and verbal descriptions of the situations together.

Name: _____ G7 US Lesson 5 - Let's Try It

Five lifeguards each have a first aid kit that contains x bandages. Their manager gives each lifeguard 3 more bandages to put in their first aid kits. Altogether, the lifeguards have 45 bandages.

- What is known in this story?
- What is unknown in this story?
- Circle the tape diagram that best represents this situation.

45

5

45
- Use the tape diagram you selected to write an equation that represents this problem.
 $(\quad) = \quad$
- Solve the equation to find the unknown.
- What does your solution mean in this problem's context?

Caroline and her 3 sisters went to a concert. Tickets to the concert cost \$11 each, and each person also bought a poster for p dollars. In all, the 4 girls spent \$66.

- What is known in this story? What is unknown?

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- Use the information from Caroline's situation to label the tape diagram below.
- Write an equation to represent the scenario.
- Solve the equation for p .
- What does p mean in the context of this problem?

Read both situations below. Draw a tape diagram to represent each. Then, write and solve an equation that represents the situation.

12. A book club charges \$5 to attend plus the cost of the book. Yesterday, 6 people attended the book club. They spent a total of \$96.

13. A tour group spent \$44 total to go to a museum. They bought 5 tickets, and each person also spent \$2 to buy a map of the museum.

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On your Own:

Now it's time to coordinate tape diagrams, equations, and verbal descriptions of the situations on your own.

Name: _____ G7 US Lesson 5 - Independent Work

1. At the farmer's market, Aida fills 5 jars with x pickles. After the jars settle, she is able to add 3 more pickles to each jar. Aida ends up with 50 pickles.

a. Circle the tape diagram that best represents this situation.

b. What does each x represent in this problem?

c. Write and solve an equation to represent the problem.

2. Isaac takes his 6 dogs to the dog groomer. He pays c each, for the dogs to get their toenails trimmed. He also spends \$10 each to have his dogs shampooed. He spends \$210 in all.

a. Label the tape diagram below to represent the situation.

b. Write and solve an equation to represent the problem.

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3. There are four 7th grade homerooms at Mathy Middle School. Each homeroom had n students, and then 2 new students were added to each homeroom. In all, there are 92 7th graders at Mathy Middle School. Draw a tape diagram to represent the situation. Then write and solve an equation to show how many students were originally in each homeroom.

4. An ice skating rink charges x dollars for admission and \$4 for skate rentals. Six friends each by admission and a skate rental, and they spend a total of \$45.

Savannah drew this tape diagram to represent the situation.

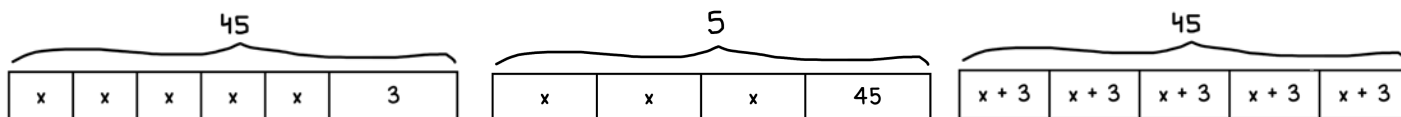
Explain why Savannah's tape diagram is incorrect. Show or explain how she could correct her tape diagram, then determine how much the ice skating rink charges for admission.

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Five lifeguards each have a first aid kit that contains x bandages. Their manager gives each lifeguard 3 more bandages to put in their first aid kits. Altogether, the lifeguards have 45 bandages.

1. What is known in this story?
2. What is unknown in this story?
3. Circle the tape diagram that best represents this situation.



4. Use the tape diagram you selected to write an equation that represents this problem.

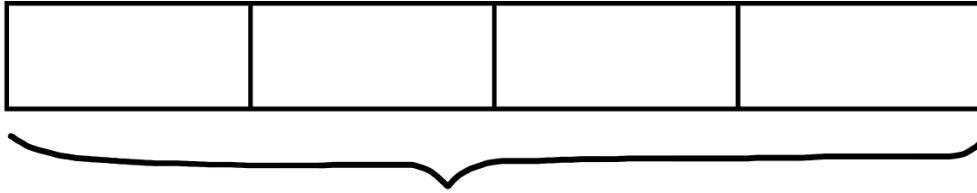
$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}}) = \underline{\hspace{2cm}}$$

5. Solve the equation to find the unknown.
6. What does your solution mean in this problem's context?

Caroline and her 3 sisters went to a concert. Tickets to the concert cost \$11 each, and each person also bought a poster for p dollars. In all, the 4 girls spent \$58.

7. What is known in this story? What is unknown?

8. Use the information from Caroline's situation to label the tape diagram below.



9. Write an equation to represent the scenario.

10. Solve the equation for p .

11. What does p mean in the context of this problem?

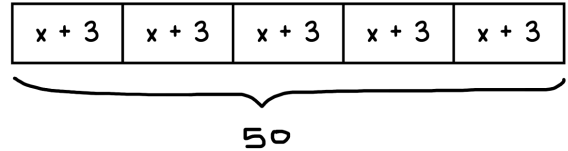
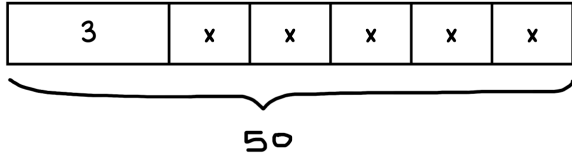
Read both situations below. Draw a tape diagram to represent each. Then, write and solve an equation that represents the situation.

12. A book club charges \$5 to attend plus the cost of the book. Yesterday, 6 people attended the book club. They spent a total of \$96.

13. A tour group spent \$44 total to go to a museum. They bought 5 tickets, and each person also spent \$2 to buy a map of the museum.

1. At the farmer’s market, Aida fills 5 jars with x pickles. After the jars settle, she is able to add 3 more pickles to each jar. Aida ends up with 50 pickles.

a. Circle the tape diagram that best represents this situation.

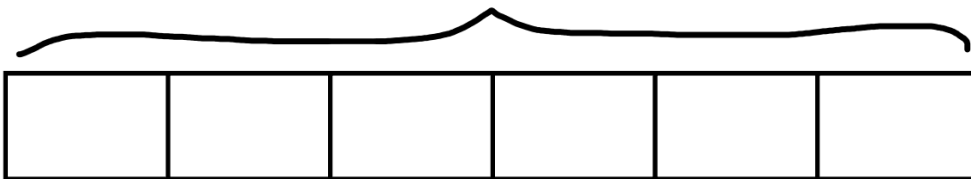


b. What does each x represent in this problem?

c. Write and solve an equation to represent the problem.

2. Isaac takes his 6 dogs to the dog groomer. He pays c each, for the dogs to get their toenails trimmed. He also spends \$10 each to have his dogs shampooed. He spends \$210 in all.

a. Label the tape diagram below to represent the situation.

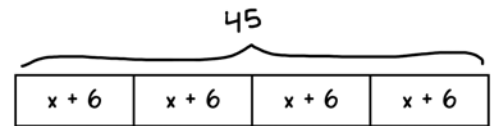


b. Write and solve an equation to represent the problem.

3. There are four 7th grade homerooms at Mathy Middle School. Each homeroom had n students, and then 2 new students were added to each homeroom. In all, there are 92 7th graders at Mathy Middle School. Draw a tape diagram to represent the situation. Then write and solve an equation to show how many students were originally in each homeroom.

4. An ice skating rink charges x dollars for admission and \$4 for skate rentals. Six friends each by admission and a skate rental, and they spend a total of \$45.

Savannah drew this tape diagram to represent the situation.



Explain why Savannah's tape diagram is incorrect. Show or explain how she could correct her tape diagram, then determine how much the ice skating rink charges for admission.

Five lifeguards each have a first aid kit that contains x bandages. Their manager gives each lifeguard 3 more bandages to put in their first aid kits. Altogether, the lifeguards have 45 bandages.

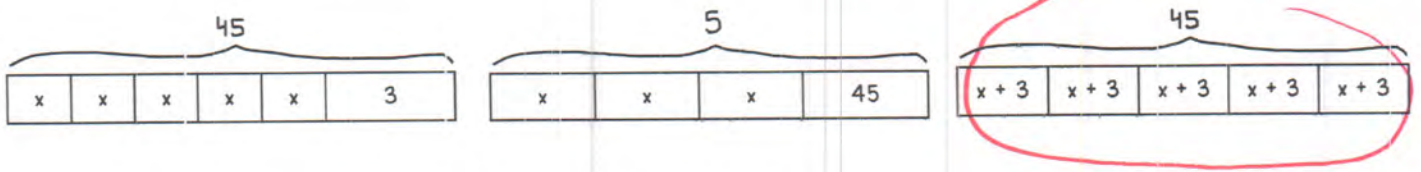
1. What is known in this story?

- 45 total bandages
- 5 lifeguards
- manager gives 3 more each

2. What is unknown in this story?

of band-aids in each kit to start

3. Circle the tape diagram that best represents this situation.



4. Use the tape diagram you selected to write an equation that represents this problem.

$$5(x + 3) = 45$$

5. Solve the equation to find the unknown.

$$45 \div 5 = 9 \quad x + 3 = 9$$

$$x = 6$$

6. What does your solution mean in this problem's context?

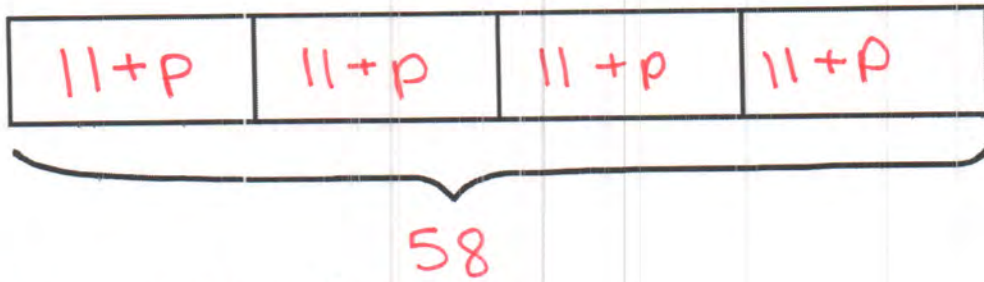
There were 6 bandages in each kit to start.

Caroline and her 3 sisters went to a concert. Tickets to the concert cost \$11 each, and each person also bought a poster for p dollars. In all, the 4 girls spent \$58.

7. What is known in this story? What is unknown?

- 4 people
- tickets cost \$11
- Spent \$58 in all
- cost for a poster

8. Use the information from Caroline's situation to label the tape diagram below.



9. Write an equation to represent the scenario.

$$4(11+p) = 58$$

10. Solve the equation for p .

$$58 \div 4 = 14\frac{1}{2}$$

$$11 + p = 14\frac{1}{2}$$

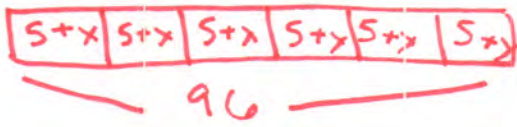
$$p = 3\frac{1}{2}$$

11. What does p mean in the context of this problem?

Each poster, p , costs \$3.50.

Read both situations below. Draw a tape diagram to represent each. Then, write and solve an equation that represents the situation.

12. A book club charges \$5 to attend plus the cost of the book. Yesterday, 6 people attended the book club. They spent a total of \$96.



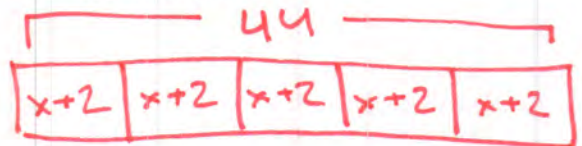
$$6(5+x) = 96$$

$$96 \div 6 = 16$$

$$5+x = 16$$

$$x = 11$$

13. A tour group spent \$44 total to go to a museum. They bought 5 tickets, and each person also spent \$2 to buy a map of the museum.



$$5(x+2) = 44$$

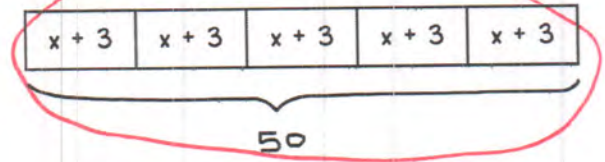
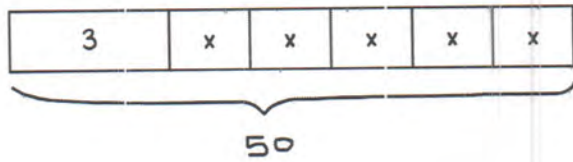
$$44 \div 5 = 8\frac{4}{5}$$

$$x+2 = 8\frac{4}{5}$$

$$x = 6\frac{4}{5} \text{ or } \$6.80$$

1. At the farmer's market, Aida fills 5 jars with x pickles. After the jars settle, she is able to add 3 more pickles to each jar. Aida ends up with 50 pickles.

- a. Circle the tape diagram that best represents this situation.



- b. What does each x represent in this problem?

the # of pickles in each jar to start

- c. Write and solve an equation to represent the problem.

$$5(x+3) = 50$$

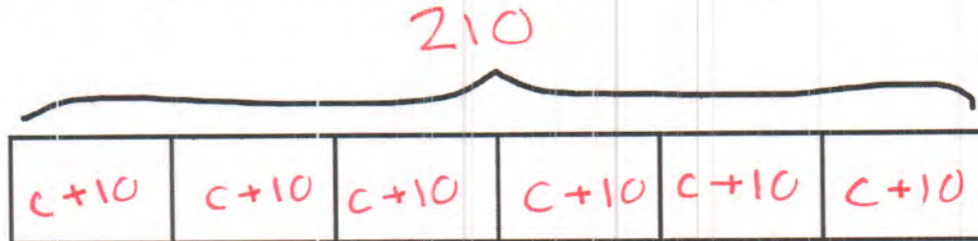
$$50 \div 5 = 10$$

$$x+3 = 10$$

$$x = 7$$

2. Isaac takes his 6 dogs to the dog groomer. He pays c each, for the dogs to get their toenails trimmed. He also spends \$10 each to have his dogs shampooed. He spends \$210 in all.

- a. Label the tape diagram below to represent the situation.



$$\begin{array}{r} 35 \\ 6 \overline{) 210} \\ \underline{-18} \\ 30 \\ \underline{-30} \\ 0 \end{array}$$

- b. Write and solve an equation to represent the problem.

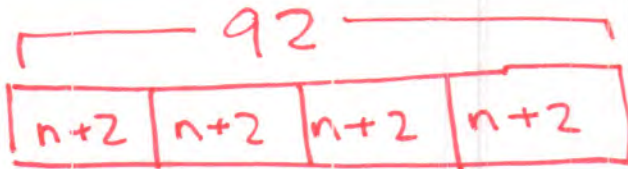
$$6(c+10) = 210$$

$$210 \div 6 = 35$$

$$c+10 = 35$$

$$c = 25$$

3. There are four 7th grade homerooms at Mathy Middle School. Each homeroom had n students, and then 2 new students were added to each homeroom. In all, there are 92 7th graders at Mathy Middle School. Draw a tape diagram to represent the situation. Then write and solve an equation to show how many students were originally in each homeroom.



$$\begin{array}{r} 23 \\ 4 \overline{)92} \\ \underline{-8} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

$$4(n+2) = 92$$

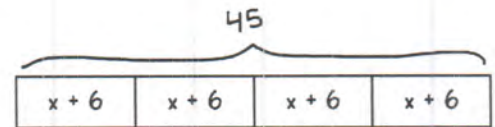
$$92 \div 4 = 23$$

$$n+2 = 23$$

$$n = 21$$

4. An ice skating rink charges x dollars for admission and \$4 for skate rentals. Six friends each buy admission and a skate rental, and they spend a total of \$45.

Savannah drew this tape diagram to represent the situation.



Explain why Savannah's tape diagram is incorrect. Show or explain how she could correct her tape diagram, then determine how much the ice skating rink charges for admission.

Savannah's story involves six groups of " $x+4$ ". Her model shows four groups of " $x+6$ " instead. She should redraw the model to show the correct groups.

$$6(x+4) = 45$$

$$45 \div 6 = 7\frac{1}{2}$$

$$x+4 = 7\frac{1}{2}$$

$$x = 3\frac{1}{2}$$

G7 U5 Lesson 6

Write and categorize equations of the forms $px + q = r$ and $p(x + q) = r$ from situations and tape diagrams.

G7 U5 Lesson 6 - Students will write and categorize equations of the forms $px + q = r$ and $p(x + q) = r$ from situations and tape diagrams.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We've spent the last two lessons together building tape diagrams and equations to represent different types of real-world problems. (*write $px + q = r$ and $p(x + q) = r$*) Two lessons ago, all our stories were in the form $px + q = r$, meaning we had some equal groups (*point to px*) and an additional value that was not an equal group (*point to q*) that had a total of r . In the lesson after that, our problems had a different structure. In these problems, we had an unknown amount that was part of the equal groups. We'd have a number of equal groups (*point to p*), then within our equal groups we'd have a known and an unknown (*point to q and x*) that all had a total of r .

$$px + q = r$$
$$p(x + q) = r$$

It's not important that we memorize these equations or formulas, so don't worry about that. All we're going to do today is see a *mix* of stories and have to carefully craft a tape diagram and equation to match them. Since the problems won't all fall into the same structure, it will be extra important that we think about what is known and unknown carefully so that our representations are accurate and helpful.

Let's Talk (Slide 3): Look at the two equations shown here. What do you notice is the same? What is different? **Possible Student Answers, Key Points:**

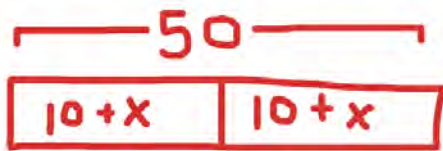
- They both have a 50, 20, 10 and x . They both have a + and an = sign. They both involve some equal groups.
- The first one shows 2 equal groups of " $x + 10$ ". The second one shows 2 equal groups of x , but then the 10 is not part of the equal groups.

These two equations are examples of the two problem types we've been exploring. Our work today will be to solve varied problem types using tape diagrams and equations to help us reason about real-world problems.

Let's Think (Slide 4): I'll read the problem once through while you follow along. After, summarize the story and tell me what information is known and what is unknown. **Possible Student Answers, Key Points:**

- This problem is about Minnie putting chocolate candy and fruit candy into 2 equal bags.
- We know there are 50 total pieces of candy that she's splitting evenly into two bags. 10 pieces in each bag are chocolate and the rest in each bag is fruit candy.
- We don't know how much fruit candy she put in each bag.

We've seen problems like this before. Let's tackle part A by sketching a tape diagram.



(*sketch as you narrate*) We know the total is 50, so I'll draw a long rectangle and label the total as 50. How can I show the amount in the two bags now that I've drawn the total? **Possible Student Answers, Key Points:**

- You can split the rectangle in half to show each bag.
- Each bag has 10 chocolate candies and an unknown number of fruit candies, so we can write $10 + x$ in each bag.

Great. (*point to components as you name them*) I see the 50 total candies. I see the two bags represented by the smaller rectangles. I also see that each bag has 10 chocolate candies and " x " fruit candies.

It's time to write an equation. I can see in the model, and I know from the story, that we have 2 groups of candy. I can think of those groups as each being worth " $10 + x$ ". Based on that, I can write the equation $2(10 + x) = 50$.

$$2(10+x) = 50$$

$$50 \div 2 = 25$$

$$10 + x = 25$$

$$x = 15$$

$+ x) = 50$. (write equation) Two bags of candy that each have “10 + x” pieces of candy in them equal a total of 50 total pieces of candy. Writing the equation pretty easy when we’ve already drafted a tape diagram.

Part A and Part B are done, so let’s close this out with Part C. We’ll solve the equation. The tape diagram helps me see that I can divide 50 by 2 to figure out the number of candies in each bag. What is 50 divided by 2? (25) There are 25 pieces of candy per bag. I know 10 of those pieces are chocolate and x of those pieces are fruit candies. I’ll write $10 + x = 25$ to represent that. How can I find the value of x? Possible Student Answers, Key Points:

- I just know 10 plus 15 makes 25, so there have to be 15 fruit candies.
- I can rewrite the equation as a related subtraction equation. I know $25 - 10 = x$, so x has to equal 15.

Our solution is 15. What does that mean when we connect it back to the context of the problem? Possible Student Answers, Key Points:

- We were trying to find how many fruit candies Minnie puts in each bag. If $x = 15$, that means there are 15 fruit candies in each bag.

Nice work. Let’s try another example.

Let’s Think (Slide 5): Again, I’ll read the problem once through while you follow along. After, summarize the story and tell me what information is known and what is unknown. Possible Student Answers, Key Points:

- This problem is also about 50 pieces of candy. This time, she’s putting some on display racks and then keeping some for herself.
- We know there is a total of 50. We know she’s splitting the same amount onto two displays. We know she’s keeping 10 of the 50 for herself.
- We don’t know how many she puts in each display.

I’ll start drawing this tape diagram similar to the previous problem, because they both involve totals of 50 pieces of candy. (sketch as you narrate) I’m tempted to split the total into two again, because I know she’s splitting some candy evenly onto 2 displays, but we realized she’s keeping some to herself in this problem. I’ll partition a rectangle at the end to show she’s keeping those 10 to herself. Now, I can split the remaining rectangle in half to show the two identical displays. Let’s put an x in each of those since we don’t know the value. Does my tape diagram show every part of the problem we’re trying to solve? (point to components that the student names)



Possible Student Answers, Key Points:

- Yes. I see the 50 total pieces of candy. I see the x in two boxes to represent the amount that she puts in each display. I also see the 10 pieces she keeps in the rectangle on the right side of the diagram.

For Part B, we’re tasked with writing an equation. What do you see in the tape diagram that can help us write the equation? Possible Student Answers, Key Points:

- I see the total is 50, so our equation should have “= 50” in it.
- I see two groups of x, which we can write as 2x. I also see 10, so we’ll need to add 10 to the 2x.

$$2x + 10 = 50$$

(write equation as you narrate) I’ll write $2x + 10$ to represent the two groups of candy in the display plus the 10 pieces Minnie keeps to herself. I’ll set that equal to the total of 50. Nice work!

$$50 - 10 = 40$$

$$2x = 40$$

$$x = 20$$

Now we can use the tape diagram and the equation to solve for the unknown, x . (*write equations as you narrate*) Let's start by subtracting out the 10 pieces of candy that Minnie keeps. What is $50 - 10$? (40) She has 40 pieces of candy that she wants to split evenly into two displays. I can think of that as 40 divided by 2 or as $2x = 40$. No matter how I think about it, I know she puts 20 candies onto each display. x is equal to 20.

We just did a lot of work! Even though the problem types were different, we were able to draw a tape diagram and equation to help us solve for unknowns. We'll now get some more practice.

Let's Try it (Slides 6 - 7): As we work through the next several examples, we'll want to be careful as we read each problem. It has been really helpful to read each problem, sometimes twice, and pause to think about the details. What do we know? What is unknown? This quick reflection made it much easier to draw accurate tape diagrams and write correct equations. It helps to make sense of a word problem before jumping into a solution strategy. Let's keep all this work in mind and try a few more together.

WARM WELCOME



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Today we will write and categorize equations of the forms $px + q = r$ and $p(x + q) = r$ from situations and tape diagrams.

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 **Let's Talk:**

Look at the equations below. What is the same? What is different?

$$50 = 2(x + 10)$$

$$50 = 2x + 10$$

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 **Let's Think:**

Minnie has 50 pieces of candy to put evenly into 2 bags. She puts 10 chocolate candies in each bag and x fruit candies in each bag.

- Draw a tape diagram to represent the situation.
- Write an equation to represent the situation.
- Solve the equation.

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Let's Think:

Minnie has 50 pieces of candy to put on display in her candy store. She puts x pieces of candy in each of 2 displays, then sets aside 10 pieces to take to her friend.

- Draw a tape diagram to represent the situation.
- Write an equation to represent the situation.
- Solve the equation.

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Let's Try It:

Let's explore writing and categorizing equations of the form $px + q = r$ and $p(x + q) = r$ from situations and tape diagrams together.

Name: _____ G7 US Lesson 6 - Let's Try It

At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered the same number of chairs to each of four classrooms and had 12 chairs leftover.

- What is known in this situation? Unknown?
- Circle the tape diagram that best represents the situation.
- Use the tape diagram to write an equation using c to represent the unknown.
- Solve for c .
- What does your solution represent in this context?

At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered 12 chairs to each of four classrooms, and then he took the remaining chairs and divided them up equally between the classrooms.

- What is known in this situation? Unknown?
- Label the tape diagram to represent the situation.

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- Use the tape diagram to write and solve an equation. Use c to represent the unknown.
- What does your solution represent in this context?

Think about the tape diagrams for each of the two situations you just considered.

- What is the same about the tape diagrams?
- What is different about the tape diagrams?

Match the story to the tape diagram that best represents it. Then find the value of the unknown in each story.

- Peter's soccer team has 60 ounces of Gatorade. He pours the Gatorade evenly into 5 bottles, and there is 4 ounces leftover. How much Gatorade did Peter pour in each bottle?
- Cher has 60 berries that she wants to share between 4 people. She starts by giving each person 5 berries, and then splits what is leftover evenly between each person. How many additional berries did each person receive?

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On your Own:

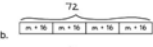
Now it's time to write and categorize equations of the form $px + q = r$ and $p(x + q) = r$ from situations and tape diagrams on your own.

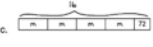
Name: _____ G7 US Lesson 6 - Independent Work

1. Dianne baked 72 muffins for her bakery. She put the same number of muffins into 4 display cases, and then boxed the 16 remaining muffins for a catering order.

Which tape diagram best represents the situation?

a. 

b. 

c. 

Write and solve an equation to find the unknown.

2. Santiago was filling backpacks with pencils and notebooks to give to a local school. He had 95 total objects to fill 5 backpacks evenly. He put 11 pencils in each backpack and n notebooks.

a. Draw a tape diagram to represent this situation.

b. Write and solve an equation to find the unknown quantity.

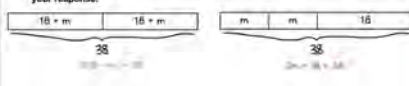
c. What does your solution mean in this context?

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3. 74 students attended field day. Their teachers put them into 5 teams by putting x students on each team. Then, they made another team with 9 players on it. How many people did the teachers put on each team?

Use a tape diagram and an equation to represent this situation.

4. What is the same and different about the two sets of tape diagrams and equations shown here? Use the terms/phrases equal groups, total, and different-sized groups in your response.



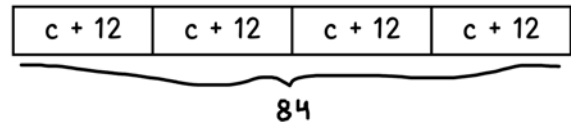
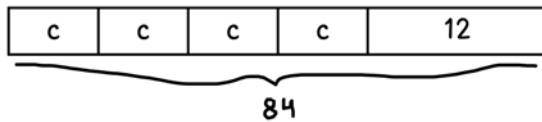
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At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered the same number of chairs to each of four classrooms and had 12 chairs leftover.

1. What is known in this situation? Unknown?

2. Circle the tape diagram that best represents the situation.



3. Use the tape diagram to write an equation using c to represent the unknown.

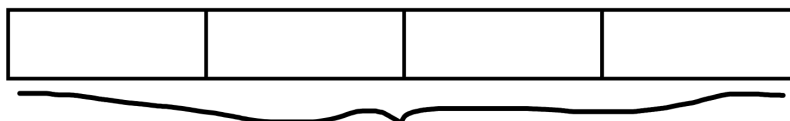
4. Solve for c .

5. What does your solution represent in this context?

At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered 12 chairs to each of four classrooms, and then he took the remaining chairs and divided them up equally between the classrooms.

6. What is known in this situation? Unknown?

7. Label the tape diagram to represent the situation.



8. Use the tape diagram to write and solve an equation. Use c to represent the unknown.

9. What does your solution represent in this context?

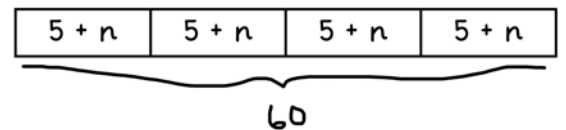
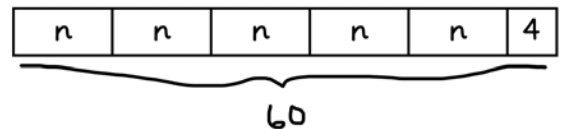
Think about the tape diagrams for each of the two situations you just considered.

10. What is the same about the tape diagrams?

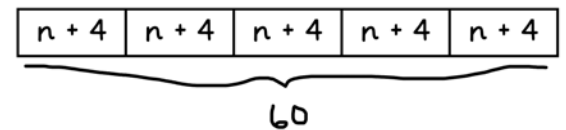
11. What is different about the tape diagrams?

Match the story to the tape diagram that best represents it. Then find the value of the unknown in each story.

12. Peter's soccer team has 60 ounces of Gatorade. He pours the Gatorade evenly into 5 bottles, and there is 4 ounces leftover. How much Gatorade did Peter pour in each bottle?

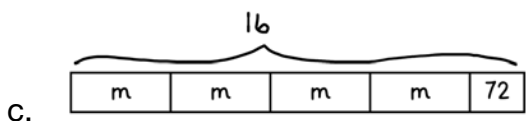
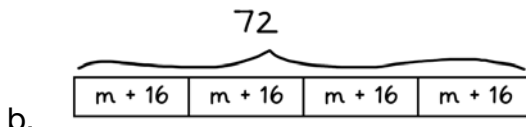
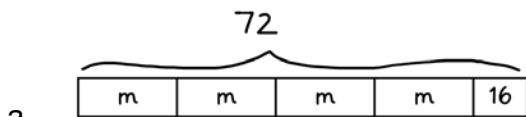


13. Cher has 60 berries that she wants to share between 4 people. She starts by giving each person 5 berries, and then splits what is leftover evenly between each person. How many additional berries did each person receive?



1. Dianne baked 72 muffins for her bakery. She put the same number of muffins into 4 display cases, and then boxed the 16 remaining muffins for a catering order.

Which tape diagram best represents the situation?



Write and solve an equation to find the unknown.

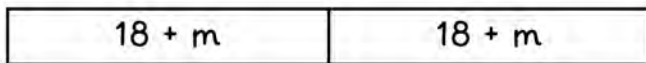
2. Santiago was filling backpacks with pencils and notebooks to give to a local school. He had 95 total objects to fill 5 backpacks evenly. He put 11 pencils in each backpack and n notebooks.

- a. Draw a tape diagram to represent this situation.
- b. Write and solve an equation to find the unknown quantity.
- c. What does your solution mean in this context?

3. 74 students attended field day. Their teachers put them into 5 teams by putting x students on each team. Then, they made another team with 9 players on it. How many people did the teachers put on each team?

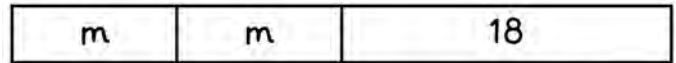
Use a tape diagram and an equation to represent this situation.

4. What is the same and different about the two sets of tape diagrams and equations shown here? Use the terms/phrases equal groups, total, and different-sized groups in your response.



38

$$2(18 + m) = 38$$



38

$$2m + 18 = 38$$

Name: KEY

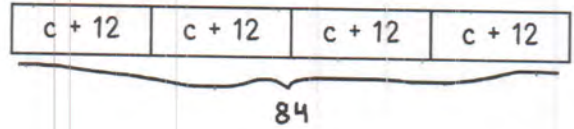
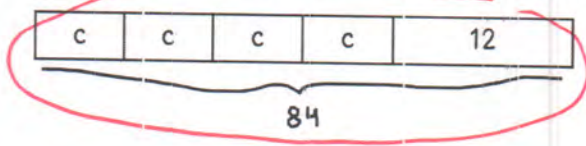
At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered the same number of chairs to each of four classrooms and had 12 chairs leftover.

1. What is known in this situation? Unknown?

- 84 total chairs
- 4 classrooms
- 12 left over

• # of chairs in each room

2. Circle the tape diagram that best represents the situation.



3. Use the tape diagram to write an equation using c to represent the unknown.

$$84 = 4c + 12$$

4. Solve for c .

$$84 - 12 = 72$$
$$72 \div 4 = c$$
$$18 = c$$

5. What does your solution represent in this context?

The custodian delivered 18 chairs to each room.

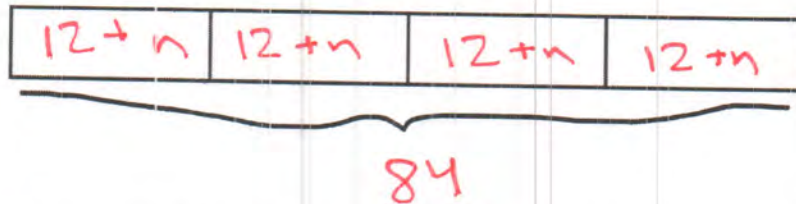
At the beginning of the school year, a custodian at a school had 84 student chairs in storage. He delivered 12 chairs to each of four classrooms, and then he took the remaining chairs and divided them up equally between the classrooms.

6. What is known in this situation? Unknown?

- 84 total chairs
- 12 chairs per room to start
- 4 classrooms

• # of chairs the custodian put in each room after the 12

7. Label the tape diagram to represent the situation.



8. Use the tape diagram to write and solve an equation. Use c to represent the unknown.

$$84 = 4(12 + c)$$

$$12 + c = 21$$

$$84 \div 4 = 21$$

$$c = 9$$

9. What does your solution represent in this context?

The custodian put 9 chairs in each room after the initial 12.

Think about the tape diagrams for each of the two situations you just considered.

10. What is the same about the tape diagrams?

- They had the same total.
- They each involved 4 classrooms.
- They each involved equal groups.

11. What is different about the tape diagrams?

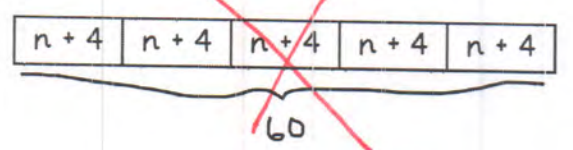
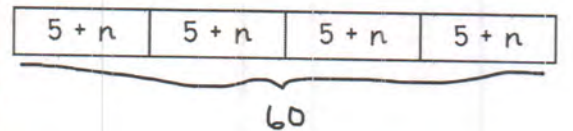
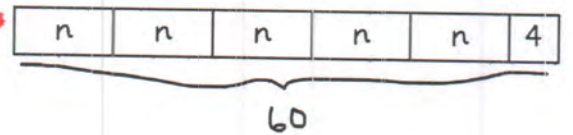
- The first situation had equal groups with an extra amount.
- The second situation involved only equal groups.

Match the story to the tape diagram that best represents it. Then find the value of the unknown in each story.

12. Peter's soccer team has 60 ounces of Gatorade. He pours the Gatorade evenly into 5 bottles, and there is 4 ounces leftover. How much Gatorade did Peter pour in each bottle?

$$60 - 4 = 56$$

$$56 \div 5 = 11 \frac{1}{5} \text{ oz}$$



13. Cher has 60 berries that she wants to share between 4 people. She starts by giving each person 5 berries, and then splits what is leftover evenly between each person. How many additional berries did each person receive?

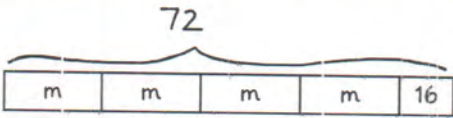
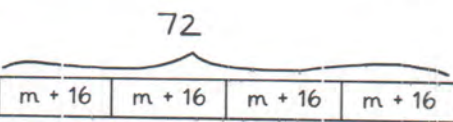
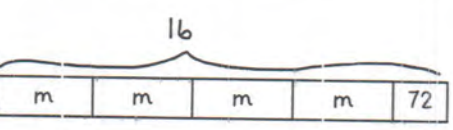
$$60 \div 4 = 15$$

$$5 + n = 15$$

$$n = 10$$

1. Dianne baked 72 muffins for her bakery. She put the same number of muffins into 4 display cases, and then boxed the 16 remaining muffins for a catering order.

Which tape diagram best represents the situation?

- a. 
- b. 
- c. 

Write and solve an equation to find the unknown.

$$72 = 4m + 16$$

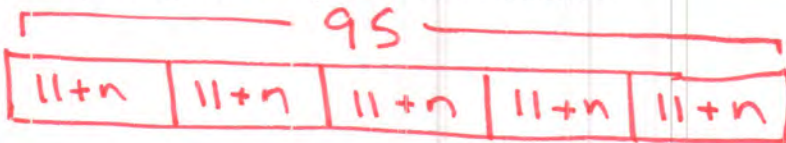
$$72 - 16 = 56$$

$$56 \div 4 = m$$

$$14 = m$$

2. Santiago was filling backpacks with pencils and notebooks to give to a local school. He had 95 total objects to fill 5 backpacks evenly. He put 11 pencils in each backpack and n notebooks.

a. Draw a tape diagram to represent this situation.



b. Write and solve an equation to find the unknown quantity.

$$95 = 5(11 + n)$$

$$95 \div 5 = 19$$

$$11 + n = 19$$

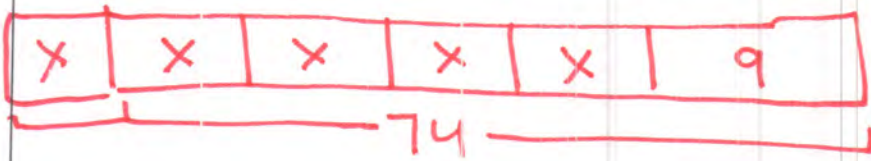
$$n = 8$$

c. What does your solution mean in this context?

Santiago puts 8 notebooks in each backpack.

3. 74 students attended field day. Their teachers put them into 5 teams by putting x students on each team. Then, they made another team with 9 players on it. How many people did the teachers put on each team?

Use a tape diagram and an equation to represent this situation.



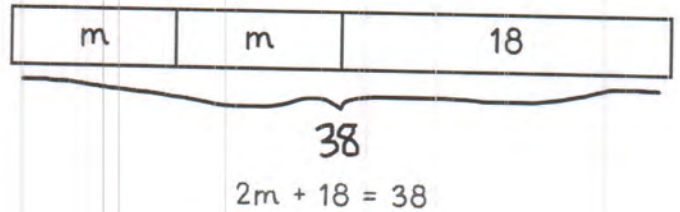
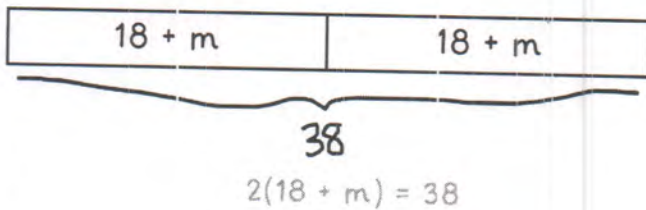
$$74 = 5x + 9$$

$$74 - 9 = 65$$

$$65 \div 5 = x$$

$$x = 13$$

4. What is the same and different about the two sets of tape diagrams and equations shown here? Use the terms/phrases equal groups, total, and different-sized groups in your response.



Both show totals of 38. The first shows 2 equal groups. The second has different-sized groups; there are 2 equal groups of m and 1 group of 18.

G7 U5 Lesson 7

Use a balanced hanger diagram to reason about writing and solving equations in the form $px + q = r$.

G7 U5 Lesson 7 - Students will use a balanced hanger diagram to reason about writing and solving equations in the form $px + q = r$.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We've been putting a significant amount of time and effort into reasoning about real world problems and using what we know to draw tape diagrams and write equations that reflect the problems. Today, we're going to practice a cool way to think about solving equations that is very visual. We're going to work with hanger diagrams or hanger models. You might catch yourself feeling like today's problems feel more like mini-puzzles or games than like typical math problems.

Let's Talk (Slide 3): Check out this image. What do you notice? What do you wonder? [Possible Student Answers, Key Points:](#)

- I notice a clothes hanger. I notice there are cups tied with strings on both ends. I noticed it looks balanced, or the sides look even as if both sides weigh the same amount.
- I wonder what this has to do with math. I wonder what goes in the cups.

If we were to play around with this hanger in real life, what would happen if I put some marbles on the left side? (The hanger would droop to the left or go up on the right.) What if, instead, I put some marbles on the right side? (The hanger would droop to the right or go up on the left.) If I wanted to make any change to the hanger, what would I need to do to make sure the hanger stayed balanced? [Possible Student Answers, Key Points:](#)

- If you added something to one side, you'd need to add the same amount to the other side.
- If you took something out of one cup, you'd need to take the same amount out of the other cup.

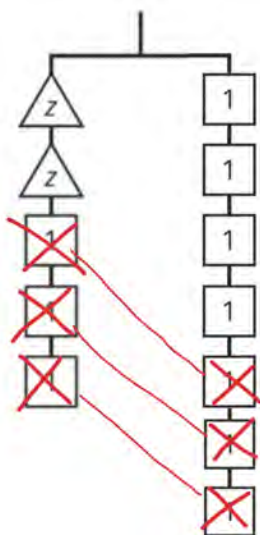
Great thinking. When trying to keep this hanger balanced, whatever I do to one side, I would have to do to the other.

Today, we'll see models that look similar to this hanger and work in a similar way. Let's look at one together.

Let's Think (Slide 4): This problem wants us to find the value of z . The model you see here is called a hanger diagram or a hanger model. Before we solve for z , what do you notice about the hanger diagram?

[Possible Student Answers, Key Points:](#)

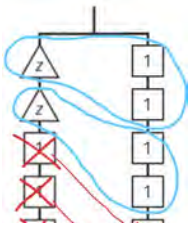
- It looks balanced, but it looks like the right side is longer than the left.
- It has some z 's and ones on the left. It only has a bunch of ones on the right.



Right now, the diagram has a lot on it. If we want to find just the value of the unknown, z , then it can be helpful to make our model a bit simpler. To do that, let's remove some quantities. Just like with the picture of the hanger, if we want to maintain balance, we have to do the same thing to both sides of the hanger. Since I see ones on both sides of the hanger, I can match up and remove some ones. (draw lines connecting three pairs of ones) There are enough ones that I can take three away from both sides. (cross out the matched ones) How do I know the hanger will remain balanced? [Possible Student Answers, Key Points:](#)

- You took the same thing off of both ends.
- If you take 3 from one side and 3 from the other, the hanger will stay balanced.

Now what is left on the hanger? (There are 2 z 's on the left and 4 ones on the right.) Because the hanger is balanced, I know the value of these 2 z 's on the left is equal to the value of the 4 ones on the right.



We can think of that on the model by matching each z to the same number of ones. I know I can match each z with two of the ones. (*circle each z with 2 ones as shown*) This makes it clear that each z is equal to 2.

That makes sense because if each $z = 2$, that means the left side of the hanger weighs 4 units, because $2 + 2 = 4$. That matches the weight of the right side of the hanger.

$$2z + 3 = 7$$

We can also think about the work we just did using an equation. If we think back to the original hanger diagram in this problem, I can use the equation $2z + 3 = 7$ to represent it. (*write equation*) Why does $2z + 3 = 7$ match the original hanger diagram?

Possible Student Answers, Key Points:

- The left side of the hanger shows a z , another z , and 3 ones. We can write that as $2z + 3$. The right side of the hanger shows 7 ones, which we can write as 7. Since the hanger is balanced, $2z + 3 = 7$ makes sense.

$$2z = 4$$

$$z = 2$$

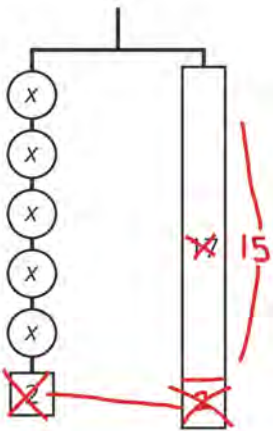
The first thing we did in the diagram was remove 3 from both sides. If we remove 3 from both sides of our equation, we're left with $2z$ on the left and 4 on the right. (*write $2z = 4$ underneath original equation*) Then, we divided each side into 2 groups when we matched the z 's with the ones. Dividing the left and right side of our equation by 2, leaves us with $z = 2$.

Both the hanger diagram and the equation help us see that the value of z is 2.

Let's Think (Slide 5): Let's look at one more. What do you notice is the same about this hanger model?

What do you notice is different? **Possible Student Answers, Key Points:**

- It's the same because it's balanced. It's the same because one side has variables and numbers and the other side has just numbers.
- It's different because the unknown here is x . It's also different because the numbers are bigger; it's not just a bunch of ones.



Even though this hanger model looks a little different, we can solve for the unknown using a similar approach.

Let's start by making the hanger model simpler. Let's try to get it so that we only have variables on the left. This is sometimes called isolating the variable. I can do that by taking 2 off of both sides of the hanger model. Since I can't match up ones, I'll just remove a chunk of 2 from both sides. I'll cross off the 2 on the left, and then I'll partition off 2 from the 17 on the right. What is $17 - 2$? (15) Okay, let me show that. (*cross off the 2 on the left, and mark-up the 17 on the right as shown*)

The hanger is still balanced, because we removed 2 from both sides. What's left on the hanger for us to consider? (The left has five x 's. The right has 15.) If I know five x 's has to have the same value as 15, how can I determine the value of x ? **Possible**

Student Answers, Key Points:

- I know $5 \times 3 = 15$, so each x must be equal to 3.
- I can divide 15 into 5 groups, kind of like how we matched the ones to the z 's in the other problem. I know 15 divided into 5 equal groups is 3.

Each x must be equal to 3. Five groups of 3 is 15.

$$5x + 2 = 17$$

$$5x = 15$$

$$x = 3$$

Before we wrap up, let's think about how the work we did with the hanger model could look with an equation. What equation can I write to represent this hanger model? How do you know? *(write equation as student shares, supporting as needed)* Possible Student Answers, Key Points:

- We can write $5x + 2 = 17$.
- I see five x's and 2 on the left. I see 17 on the right. $5x + 2$ must be equal to 17 for the hanger to be balanced.

(write equations as you narrate) The first thing we did in the model was remove 2 from both sides. If we think of doing that in the equation, we would be left with $5x$ on the left and 15 on the right. We saw that in our hanger model too. Lastly, we thought of dividing the 15 into 5 equal groups to find the value of each of the x's. If we divide both sides of our equation by 5, we see that $x = 3$.

Nice work! Isolating a variable on the hanger model is a handy way to identify the value of the unknown. We can use equations to show similar thinking.

Let's Try it (Slides 6 - 7): It's time to try a few more examples collaboratively before we move into some time for independent work. As we look at each hanger model, it can be helpful to represent each side of the hanger as an expression in our equation. We ask ourselves what we can remove from both sides of the hanger that would keep it balanced. Once we've removed any known values, we can match up unknowns with anything leftover or reason about what value the unknown could be to make what's left on the hanger diagram balanced. We'll treat each of the problems like a quick balancing puzzle!

WARM WELCOME



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Today we will use a balanced hanger diagram to reason about writing and solving equations in the form $px + q = r$.

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Let's Talk:

What do you notice?

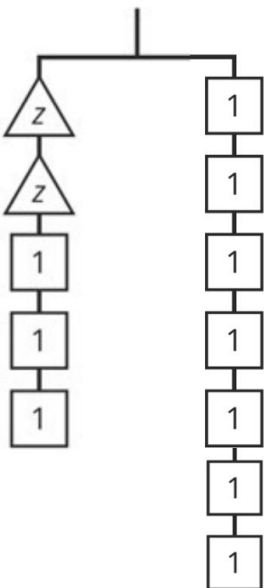
What do you wonder?



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Let's Think:

What is the value of z?

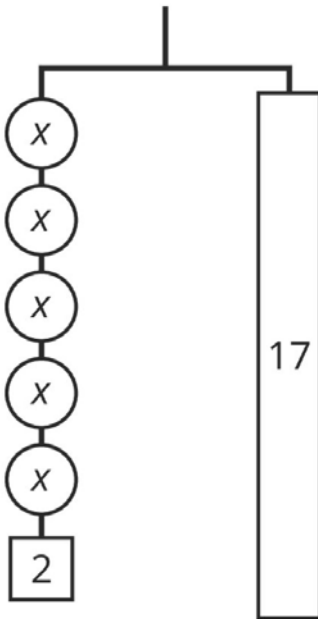


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Let's Think:

What value of x keeps the hanger diagram balanced?



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Let's Try It:

Let's explore using a balanced hanger diagram to reason about writing and solving equations in the form $px + q = r$ together.

Name: _____ 'G7 US Lesson 7 - Let's Try It'

We can think of hanger diagrams like scales. Look at the scales below.

- Which scale is balanced? How do you know?
- Which scale is not balanced? How do you know?

The hanger diagram below is balanced.

- Fill in the blanks to write an equation that represents the hanger diagram. _____ + _____ = _____
- What must be the value of x to keep the hanger in balance?

Consider the balanced hanger diagram below.

- Fill in the blanks to write an equation that represents the diagram. _____ = _____ + _____
- Remove 1 from both sides of the hanger diagram. Rewrite your equation.
- Group each w with the same number of ones. Rewrite your equation.
- What is the value of w ?

The hanger diagram below shows that the sum of z , z , and 6 is equal to 14.

- Fill in the blanks to write an equation that represents the hanger diagram. _____ + _____ + _____ = _____
- Remove 6 from both sides of the equation to maintain the hanger's balance. Rewrite the equation.
- What must be the value of z to keep the hanger in balance?

Write an equation to represent each hanger diagram. Then find the value of each unknown. REMINDER: Remove the same amount from each side to keep the hanger balanced.

- EQUATION: _____

$y =$ _____
- EQUATION: _____

$x =$ _____
- EQUATION: _____

$z =$ _____

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On your Own:

Now it's time to use a balanced hanger diagram to reason about writing and solving equations in the form $px + q = r$ on your own.

Name: _____ G7 US Lesson 7 - Independent Work

1. Write and solve an equation to find the value of the unknown in the hanger diagram?

2. The hanger diagram below is balanced. Each square has a value of 1, and each circle has a value of x . What is the value of x ?

3. Write and solve an equation to represent each hanger diagram.

4. David started to solve for x . His first step is shown below. In your own words, explain what David has done. Then, describe what David can do next to find the value of x .

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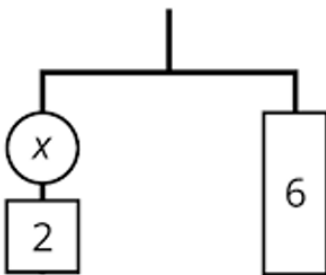
We can think of hanger diagrams like scales. Look at the scales below.

1. Which scale is balanced? How do you know?



2. Which scale is not balanced? How do you know?

The hanger diagram below is balanced.

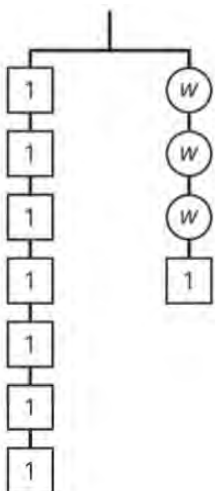


3. Fill in the blanks to write an equation that represents the hanger diagram.

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

4. What must be the value of x to keep the hanger in balance?

Consider the balanced hanger diagram below.



5. Fill in the blanks to write an equation that represents the diagram.

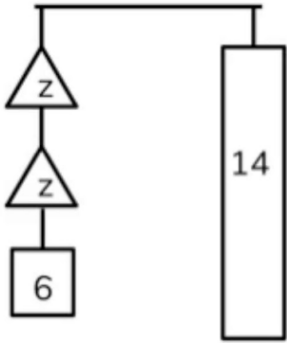
$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

6. Remove 1 from both sides of the hanger diagram. Rewrite your equation.

7. Group each w with the same number of ones. Rewrite your equation.

8. What is the value of w ?

The hanger diagram below shows that the sum of z , z , and 6 is equal to 14.



9. Fill in the blanks to write an equation that represents the hanger diagram.

$$\underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad}$$

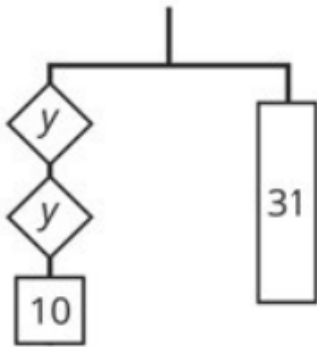
10. Remove 6 from both sides of the equation to maintain the hanger's balance. Rewrite the equation.

11. What must be the value of z to keep the hanger in balance?

Write an equation to represent each hanger diagram. Then find the value of each unknown.

REMINDER: Remove the same amount from each side to keep the hanger balanced.

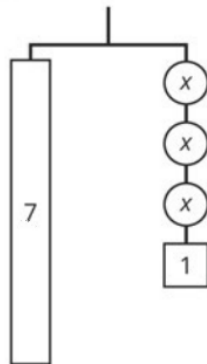
12.



EQUATION:

$$y = \underline{\quad}$$

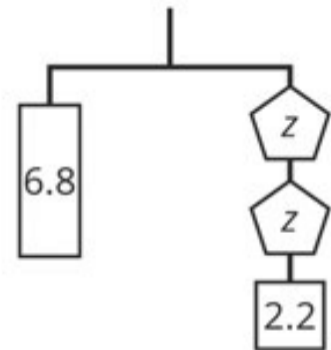
13.



EQUATION:

$$x = \underline{\quad}$$

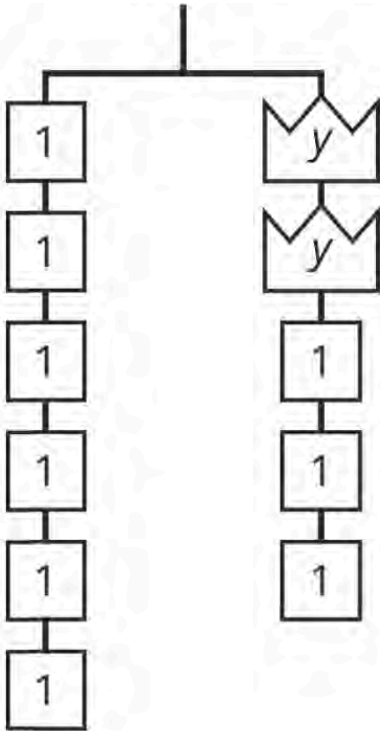
14.



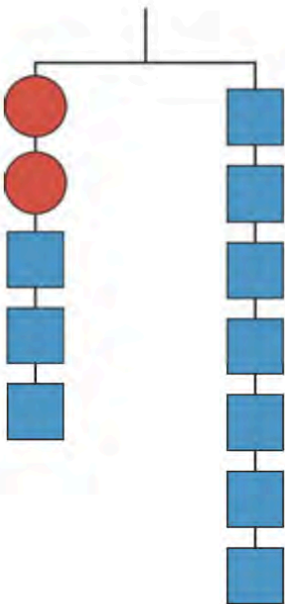
EQUATION:

$$z = \underline{\quad}$$

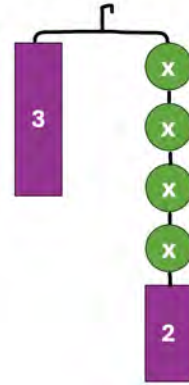
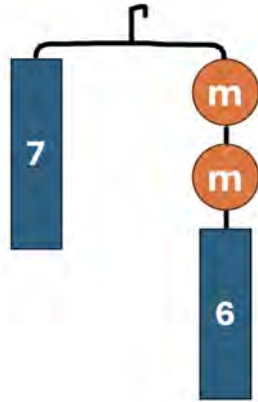
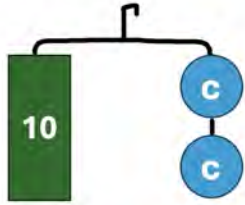
1. Write and solve an equation to find the value of the unknown in the hanger diagram.



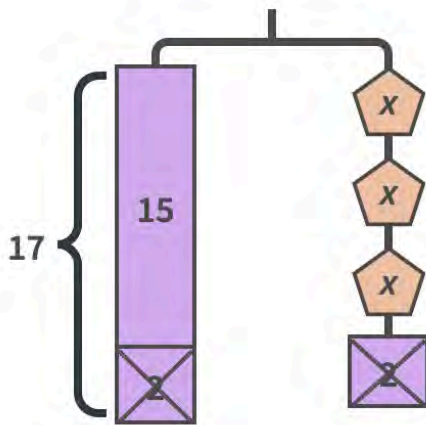
2. The hanger diagram below is balanced. Each square has a value of 1, and each circle has a value of x . What is the value of x ?



3. Write and solve an equation to represent each hanger diagram.



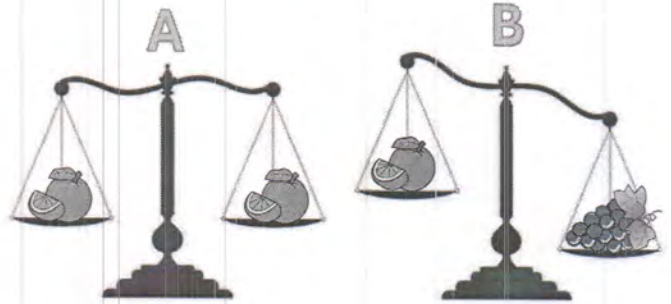
4. David started to solve for x . His first step is shown below. In your own words, explain what David has done. Then, describe what David can do next to find the value of x .



We can think of hanger diagrams like scales. Look at the scales below.

1. Which scale is balanced? How do you know?

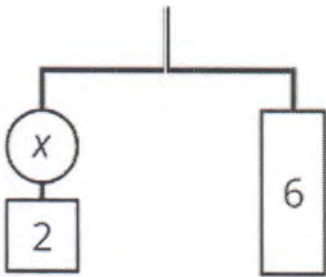
A is balanced. The two sides are even.



2. Which scale is not balanced? How do you know?

Scale B is not balanced, because the grapes are heavier.

The hanger diagram below is balanced.



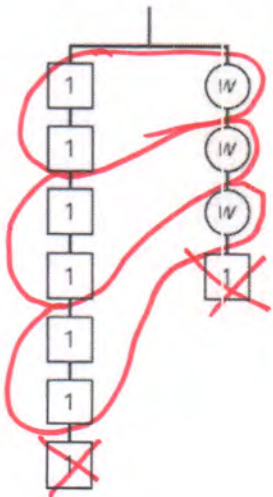
3. Fill in the blanks to write an equation that represents the hanger diagram.

$$x + 2 = 6$$

4. What must be the value of x to keep the hanger in balance?

$$x = 4$$

Consider the balanced hanger diagram below.



5. Fill in the blanks to write an equation that represents the diagram.

$$7 = 3w + 1$$

6. Remove 1 from both sides of the hanger diagram. Rewrite your equation.

$$6 = 3w$$

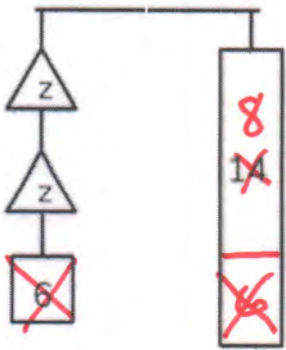
7. Group each w with the same number of ones. Rewrite your equation.

$$2 = w$$

8. What is the value of w ?

$$2$$

The hanger diagram below shows that the sum of z , z , and 6 is equal to 14.



9. Fill in the blanks to write an equation that represents the hanger diagram.

$$\underline{z} + \underline{z} + \underline{6} = \underline{14}$$

10. Remove 6 from both sides of the equation to maintain the hanger's balance. Rewrite the equation.

$$z + z = 8$$

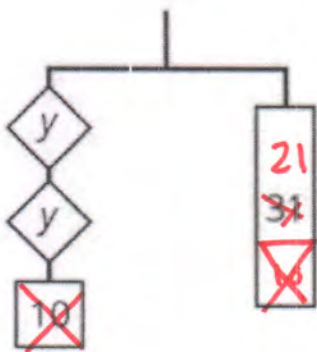
11. What must be the value of z to keep the hanger in balance?

$$z = 4$$

Write an equation to represent each hanger diagram. Then find the value of each unknown.

REMINDER: Remove the same amount from each side to keep the hanger balanced.

12.



EQUATION:

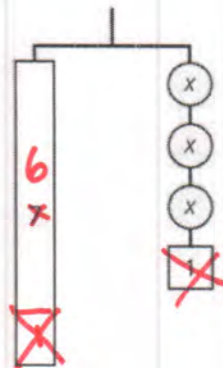
$$2y + 10 = 31$$

$$2y = 21$$

$$21 \div 2 = 10\frac{1}{2}$$

$$y = 10\frac{1}{2}$$

13.



EQUATION:

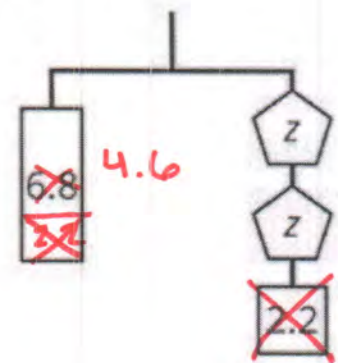
$$7 = 3x + 1$$

$$6 = 3x$$

$$2 = x$$

$$x = 2$$

14.



EQUATION:

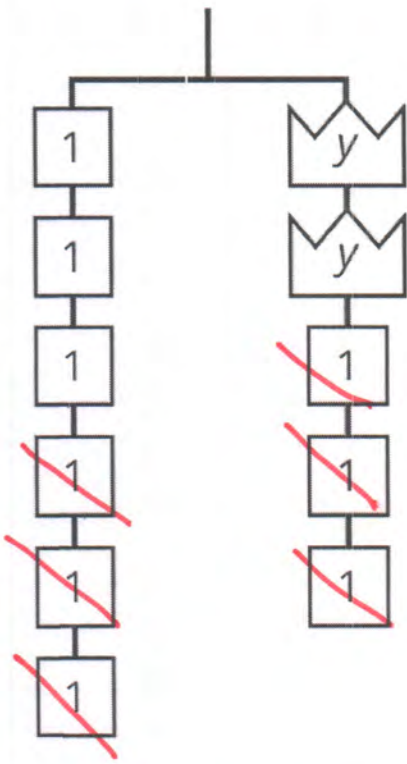
$$6.8 = 2z + 2.2$$

$$4.6 = 2z$$

$$4.6 \div 2 = 2.3$$

$$z = 2.3$$

1. Write and solve an equation to find the value of the unknown in the hanger diagram.



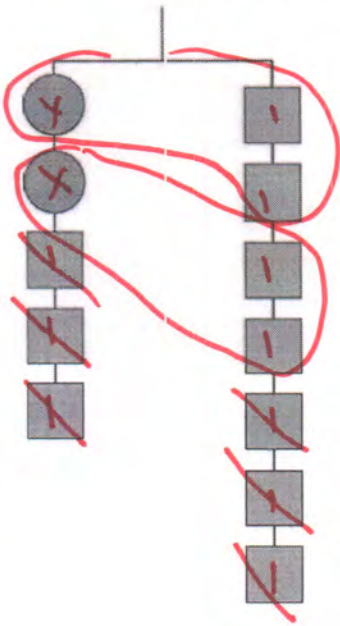
$$6 = 2y + 3$$

$$3 = 2y$$

$$3 \div 2 = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

$$y = 1\frac{1}{2}$$

2. The hanger diagram below is balanced. Each square has a value of 1, and each circle has a value of x . What is the value of x ?

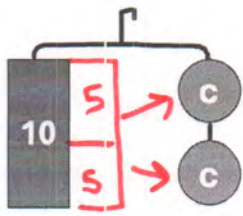


$$2x + 3 = 7$$

$$2x = 4$$

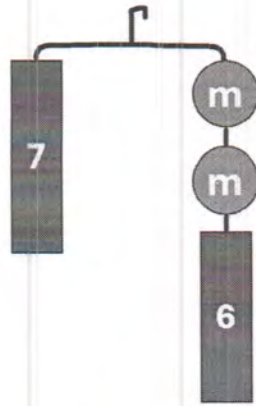
$$x = 2$$

3. Write and solve an equation to represent each hanger diagram.



$$10 = 2c$$

$$(5 = c)$$



$$7 = 2m + 6$$

$$1 = 2m$$

$$\left(\frac{1}{2} = m\right)$$

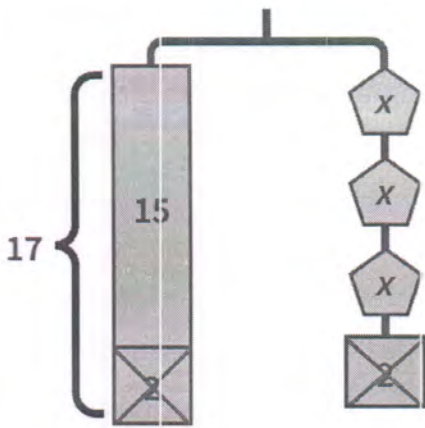


$$3 = 4x + 2$$

$$1 = 4x$$

$$\left(\frac{1}{4} = x\right)$$

4. David started to solve for x . His first step is shown below. In your own words, explain what David has done. Then, describe what David can do next to find the value of x .



David started by taking 2 off of each side of the balance.

He's left with $15 = 3x$,

so he can divide 15 evenly between each x . $15 \div 3$ means each x is worth 5.

G7 U5 Lesson 8

Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve negative numbers.

G7 U5 Lesson 8 - Students will solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve negative numbers.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We have been working so hard to solve equations. Based on our past few lessons, what stands out to you when you think about solving equations?

Possible Student Answers, Key Points:

- We can use tape diagrams to help us write equations from word problems.
- Hanger diagrams are a visual way to think about equations. We have to do the same thing to both sides to keep the hanger or equation balanced.
- We can rewrite equations as related equations to help us solve them.

Do you think what we've learned so far also works if there are negative numbers in equations? Possible Student Answers, Key Points:

- I'm not sure. I've not seen a lot of negative numbers in equations.
- I don't think they'll work. It's hard to picture a tape diagram or hanger model with negative numbers.
- I think our same strategies will work, because negative numbers are still numbers.

Let's work together to find out if we can solve equations like we've been solving...but with negative numbers!

Let's Talk (Slide 3): What's the same and what's different about the two equations you see here? Possible Student Answers, Key Points:

- They both involve multiplication. They both have an x and an 8 in them. They're both short.
- They're different colors. The pink one has a -2 rather than a $+2$. I think their answers are different.

$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

Let's think about the first equation in orange. One way to solve this would be to divide both sides of our equation by 2. (*write equation and divide both sides by 2*) I have to divide both sides by 2 because, like with the hanger diagrams, I have to keep the equation balanced. Dividing both sides by 2 isolates the variable, so that we can see x is equal to 4. (*write $x = 4$*)

$$\frac{-2x}{-2} = \frac{8}{-2}$$
$$x = -4$$

We noticed that the second equation is similar, but the x has a coefficient of -2 . If I want to isolate the variable, I couldn't do the exact same step we took in the last problem. If I divided both sides of this equation by just 2, we'd be left with $-x = 4$, which means the x would not be isolated. If I want to isolate the x in this equation, I'd have to divide both sides by *negative 2*. (*write equation and show dividing both sides by -2*) We end up with a solution of $x = -4$. I know 8 divided by -2 is -4 , because a positive number divided by a negative number results in a negative number.

I notice that how we tackled the equation with the negative number was not entirely different from how we solved the first equation with positive numbers. The only thing we had to do differently is pay attention to the sign of our numbers as we solved. The same steps we take to solve equations with positive numbers can help us solve equations with negative numbers.

Let's Think (Slide 4): Our first official problem today asks us to solve this equation. I notice there are some negative numbers involved, so we'll want to carefully keep track of signs as we take steps to solve.



I want to start by isolating the x . I can start by subtracting 7 from both sides of the equation. It is a little unusual to think about a hanger diagram with negative values, but if we think about the left and right side of our equation as items on a hanger diagram, I know removing 7 from both sides could help get the variable isolated. (*sketch a simple diagram as shown to clarify*)

$$\begin{array}{r} -2x + 7 = -5 \\ \underline{-7 \quad -7} \end{array}$$

$$\begin{array}{r} -2x = -12 \\ \underline{-2 \quad -2} \end{array}$$

$$x = 6$$

Let's subtract 7 from both sides of the equation. (*write equation and subtract 7 from both sides*) What is -5 minus 7? (-12) When we subtract 7 from both sides, we're left with -2x on the left side of the equation and -12 on the right side of the equation. (*write -2x = -12*)

We're one step away from isolating the variable. If I want to get the x by itself, I need to divide both sides by -2. I'd do this because if I divide -2 by -2, I'll just be left with 1x on the left, which means we've isolated the variable. (*divide both sides of the equation by -2*)

I know if I divide the left side of the equation by -2, we're left with x. What do we get if we divide -12 by -2 on the right side of the equation? (6) I know x is

equal to positive 6.

$$-2(6) + 7 = -5$$

Let's make sure our answer is correct. I feel pretty good about it, but it never hurts to check. (*write equation with 6 substituted in place of x*) If I substitute 6 in for x, will the equation be true? How do you know? **Possible Student**

Answers, Key Points:

- I know -2 times 6 would be -12. And -12 plus 7 would be equal to -5
- Yes, the equation will be true. That means x = 6 is the correct solution.

The same thinking we do to solve equations with positive numbers can help us with negative numbers. We just have to keep a close eye on any signed numbers as we perform operations. Ignoring the signs on numbers could result in incorrect answers.

Let's Think (Slide 5): Let's solve one more problem together. We're going to solve this equation two ways so you can explore multiple ways to arrive at the correct solution. When we're done, I'll ask you to think about which method you prefer and why.

$$-2(4n - 1) = 10$$

$$-8n + 2 = 10$$

$$\begin{array}{r} -8n + 2 = 10 \\ \underline{-2 \quad -2} \\ -8n = 8 \end{array}$$

$$n = -1$$

For our first attempt, we'll solve by distributing first. (*draw arrows to show distributing the -2 to the terms inside parentheses*) I'll multiply -2 times 4n and then -2 times -1. What is -2 times 4n? (-8n) What is -2 times -1? (+2) We can rewrite the equation as -8n + 2 = 10. (*rewrite*)

Now, let's work to isolate the variable, n. We'll start by subtracting 2 from both sides of the equation to keep the equation balanced. (*subtract 2 from both sides of the equation*) We're left with -8n on the left and 8 on the right. I'll rewrite our equation as -8n = 8. The variable is almost isolated. We just need to divide both sides by -8. (*divide both sides by -8*) We are left with n = -1.

In this method, we distributed the coefficient of -2 first, then solved the equation like other equations we've been working with.

$$\frac{-2(4n - 1)}{-2} = \frac{10}{-2}$$

Let's try another strategy to solve the same problem. This time, instead of distributing the -2, we'll divide both sides by -2. (*write equation and show division by -2 using horizontal fraction bars*) If I divide the left side by -2, I'll just be left with 4n - 1. What do we get on the right side if we divide 10 by -2? (-5) Dividing by -2 meant we didn't need to distribute the -2 like we did in our previous method. Let's keep going.

$$\begin{array}{r}
 4n - 1 = -5 \\
 \quad +1 \quad +1 \\
 \hline
 4n = -4 \\
 \frac{4n}{4} = \frac{-4}{4} \\
 n = -1
 \end{array}$$

(write $4n - 1 = -5$ underneath) Our equation now reads $4n - 1 = -5$. What could I do next to both sides of the equation to isolate the variable? How do you know?

Possible Student Answers, Key Points:

- We can add 1 to both sides of the equation.
- Adding 1 to -1 on the left will mean we're just left with $4n$.

Let's add 1 to both sides of the equation. (show that step under the equation) Our equation is now $4n = -4$. We're so close! We'll just divide by 4 on both sides of the equation, and we're done. (divide both sides by 4) What is our solution? Possible

Student Answers, Key Points:

- Our solution is $n = -1$.
- I know 4 divided by 4 leaves us with n on the left. I know -4 divided by 4 leaves us with -1 on the right.

Instead of distributing first, we divided by the coefficient of -2 first in this method. No matter the method, we still ended up with $n = -1$ as our solution.

We just solved this problem by distributing first *and* by dividing first. Both are valid methods. Which one did you prefer and why? Possible Student Answers, Key Points:

- I liked distributing first, because I think distributing is easy.
- I liked dividing first, because the numbers felt more manageable after that step compared to the previous strategy.

As you see problems like these, take a second to pause and consider which solution pathway will be most helpful for you. You might find that some problems lend themselves to one method more than the other.

Let's Try it (Slides 6 - 7): Now let's solve a few more equations with negative numbers together. We saw that solving equations with negative numbers isn't all that different from solving equations with positive numbers; you just have to be careful when operating with signed numbers. The wrong sign can mean the wrong answer! After we tackle these, you'll get a chance to try some independently.

WARM WELCOME



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Today we will solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve negative numbers.

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 Let's Talk:

What's the same? What's different?

$$2x = 8$$

$$-2x = 8$$

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 Let's Think:

Solve the equation.

$$-2x + 7 = -5$$

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Let's Think:

Solve the equation both ways.

$-2(4n - 1) = 10$

DISTRIBUTE -2 FIRST

DIVIDE BY -2 FIRST

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Let's Try It:

Let's explore solving equations of the form $px + q = r$ and $p(x + q) = r$ that involve negative numbers together.

Name: _____ G7 US Lesson 8 - Let's Try It

Consider the equation $5x + 4 = 34$.

- Circle the hanger diagram that best matches the equation.



- Solve the equation. Begin by using the same operation on each side of the equation so that the $5x$ remains.

- Describe the steps you took to get x on its own.

We can solve equations with _____ similar to how we solve equations with positive numbers.

Consider the equation $-5x + 4 = 34$.

- What number can you subtract from both sides of the equation so that $-5x$ remains on the left?
- Continue solving the equation. When you divide, make sure you divide both sides by -5 .

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Consider the equation $3x + (-9) = 13$.

- What number can you subtract from both sides of the equation so that $3x$ remains on the left?
- Continue solving the equation. Be careful with your signs!

Consider the equation $-2(y - 3) = -8$.

- Solve the equation by distributing the -2 first.
- Solve the equation by dividing both sides by -2 .

- What was the same about these two strategies? Different?

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On your Own:

Now it's time to solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve negative numbers on your own.

Name: _____ G7 US Lesson 8 - Independent Work

1. Solve each equation. Show your work.

$2a + 10 = 64$ $-7h + 10 = -39$

2. Solve each equation. Show your work.

$3(-3c + 2) = 42$ $-7(-v + 3) = -35$

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3. Mark attempted to solve the equation below. His work is shown. His teacher said he made a mistake. Identify Mark's mistake and show how to solve the equation correctly.

$$\begin{array}{r} -3n + 6\frac{1}{2} = -6 \\ -6 \quad | \quad -6 \\ \hline -3n \quad | \quad -12 \\ \div 3 \quad | \quad \div 3 \\ \hline n \quad | \quad -4 \end{array}$$

4. Solve the equation below two ways.

$3(x - 4) = -12$

DISTRIBUTE 3 FIRST DIVIDE BY 3 FIRST

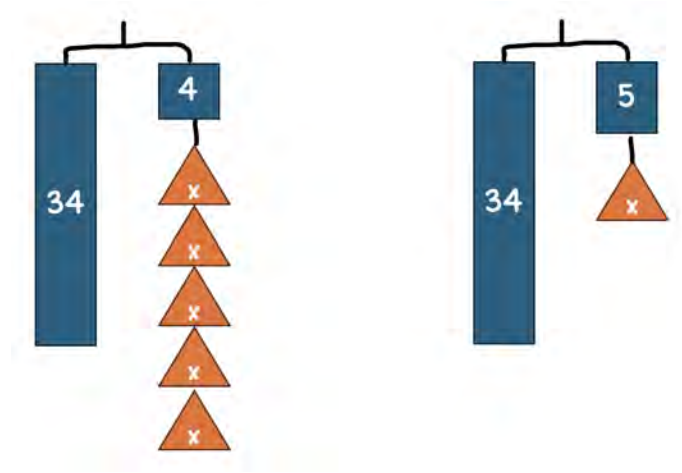
Which strategy did you prefer in this example? Explain.

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Consider the equation $5x + 4 = 34$.

1. Circle the hanger diagram that best matches the equation.
2. Solve the equation. Begin by using the same operation on each side of the equation so that the $5x$ remains.



3. Describe the steps you took to get x on its own.

We can solve equations with _____ similar to how we solve equations with positive numbers.

Consider the equation $-5x + 4 = 34$.

4. What number can you subtract from both sides of the equation so that $-5x$ remains on the left?
5. Continue solving the equation. When you divide, make sure you divide both sides by -5 .

Consider the equation $3x + (-5) = 13$.

6. What number can you subtract from both sides of the equation so that $3x$ remains on the left?
7. Continue solving the equation. Be careful with your signs!

Consider the equation $-2(y - 3) = -8$.

8. Solve the equation by distributing the -2 first.
9. Solve the equation by dividing both sides by -2 .

10. What was the same about these two strategies? Different?

Name: _____

1. Solve each equation. Show your work.

$$2a + 10 = 64$$

$$-7h + 10 = -39$$

2. Solve each equation. Show your work.

$$3(-3c + 2) = 42$$

$$-7(-v + 3) = -35$$

3. Mark attempted to solve the equation below. His work is shown. His teacher said he made a mistake. Identify Mark's mistake and show how to solve the equation correctly.

$$\begin{array}{r} -3n + 6 = -6 \\ \underline{-6 \quad -6} \\ -3n \quad = \quad 0 \\ \underline{-3 \quad \quad -3} \\ n \quad = \quad 0 \end{array}$$

4. Solve the equation below two ways.

$$3(x - 4) = -12$$

DISTRIBUTE 3 FIRST

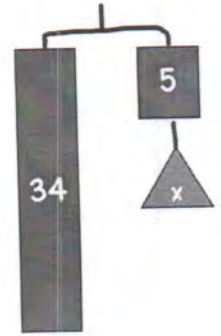
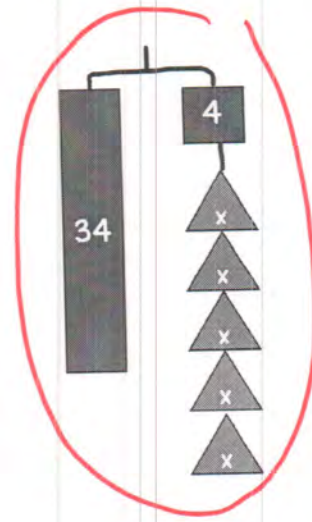
DIVIDE BY 3 FIRST

Which strategy did you prefer in this example? Explain.

Name: KEY

Consider the equation $5x + 4 = 34$.

1. Circle the hanger diagram that best matches the equation.
2. Solve the equation. Begin by using the same operation on each side of the equation so that the $5x$ remains.



$$\begin{array}{r} 5x + 4 = 34 \\ -4 \quad -4 \\ \hline 5x = 30 \\ \frac{5x}{5} = \frac{30}{5} \\ x = 6 \end{array}$$

3. Describe the steps you took to get x on its own.

I subtracted 4 from both sides, then I divided both sides by 5.

We can solve equations with negative numbers similar to how we solve equations with positive numbers.

Consider the equation $-5x + 4 = 34$.

4. What number can you subtract from both sides of the equation so that $-5x$ remains on the left?
(-4)
5. Continue solving the equation. When you divide, make sure you divide both sides by -5 .

$$\begin{array}{r} -5x + 4 = 34 \\ -4 \quad -4 \\ \hline -5x = 30 \\ \frac{-5x}{-5} = \frac{30}{-5} \\ x = -6 \end{array}$$

Consider the equation $3x + (-5) = 13$.

6. What number can you subtract from both sides of the equation so that $3x$ remains on the left?

-5

7. Continue solving the equation. Be careful with your signs!

$$\begin{array}{r} 3x + (-5) = 13 \\ +5 \quad +5 \\ \hline 3x = 18 \end{array}$$

$$\begin{array}{r} 3x = 18 \\ \frac{3}{3} \quad \frac{3}{3} \\ \hline x = 6 \end{array}$$

Consider the equation $-2(y - 3) = -8$.

8. Solve the equation by distributing the -2 first.

$$\begin{array}{r} -2(y-3) = -8 \\ -2y + 6 = -8 \\ -6 \quad -6 \\ \hline -2y = -14 \\ \frac{-2}{-2} \quad \frac{-14}{-2} \\ \hline y = 7 \end{array}$$

9. Solve the equation by dividing both sides by -2 .

$$\begin{array}{r} -2(y-3) = -8 \\ \frac{-2}{-2} \quad \frac{-8}{-2} \\ \hline y-3 = 4 \\ +3 \quad +3 \\ \hline y = 7 \end{array}$$

10. What was the same about these two strategies? Different?

I got the same answer, and I isolated
the variable in both. The steps to
solve were different. With one, I distributed,
and with the other I divided by the
coefficient.

1. Solve each equation. Show your work.

$$\begin{array}{r}
 2a + 10 = 64 \\
 \underline{-10 \quad -10} \\
 2a = 54 \\
 \underline{\quad \quad \quad 2} \\
 a = 27
 \end{array}$$

$$\begin{array}{r}
 -7h + 10 = -39 \\
 \underline{-10 \quad -10} \\
 -7h = -49 \\
 \underline{-7 \quad -7} \\
 h = 7
 \end{array}$$

2. Solve each equation. Show your work.

$$\begin{array}{r}
 \overbrace{3(-3c + 2)} = 42 \\
 -9c + 6 = 42 \\
 \underline{-6 \quad -6} \\
 -9c = 36 \\
 \underline{-9 \quad -9} \\
 c = -4
 \end{array}$$

$$\begin{array}{r}
 -7(-v + 3) = -35 \\
 \underline{-7 \quad -7} \\
 -v + 3 = 5 \\
 \underline{-3 \quad -3} \\
 -v = 2 \\
 \underline{-1 \quad -1} \\
 v = -2
 \end{array}$$

3. Mark attempted to solve the equation below. His work is shown. His teacher said he made a mistake. Identify Mark's mistake and show how to solve the equation correctly.

$$\begin{array}{r}
 -3n + 6 = -6 \\
 \underline{-6 \quad -6} \\
 -3n = 0 \\
 \underline{-3 \quad -3} \\
 n = 0
 \end{array}$$

-6 minus 6 is -12, not 0. If he did that step correct, he'd have $-3n = -12$. He could divide both sides by -3 to get a solution of $n = 4$.

4. Solve the equation below two ways.

$$3(x - 4) = -12$$

DISTRIBUTE 3 FIRST

$$\begin{array}{r}
 3x - 12 = -12 \\
 \underline{+12 \quad +12} \\
 3x = 0 \\
 \frac{3x}{3} = \frac{0}{3} \\
 x = 0
 \end{array}$$

DIVIDE BY 3 FIRST

$$\begin{array}{r}
 x - 4 = -4 \\
 \underline{+4 \quad +4} \\
 x = 0
 \end{array}$$

Which strategy did you prefer in this example? Explain. (answers may vary)

I liked dividing by 3 first, because it felt like fewer steps.

G7 U5 Lesson 9

Decide between and use two common approaches for solving an equation of the form $p(x + q) = r$ (expanding using the distributive property, or dividing each side by p)

G7 U5 Lesson 9 - Students will decide between and use two common approaches for solving an equation of the form $p(x + q) = r$ (expanding using the distributive property, or dividing each side by p).

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today's work is going to feel like a choose-your-own-adventure story. We're going to look at equations and decide which strategy will be most efficient. Wisely picking our solution pathway can help us avoid overcomplicating a math problem. If we pause to think about the numbers we're going to work with, we can often find ways to make our lives a little easier when solving.

We did a little bit of this in our previous lesson, but today we're going to make choosing a smart solution pathway our primary focus. Let's start by looking at some student work samples.

Let's Talk (Slide 3): Here we see two student work samples. Both student work samples show a correct first step toward solving the same equation. What do you notice about their work? What do you wonder?

Possible Student Answers, Key Points:

- I notice they're in different colors. I notice they use a dotted line to separate each side of their equation. I notice the red work shows distributing first. I notice the blue work shows dividing by -3 first.
- I wonder what their next step will be. I wonder how they chose which solution pathway to use. I wonder what the correct answer is.

Looking at their work for this specific problem, would you personally prefer to use the first strategy where the student distributes the -3 or the second strategy where the student divides by -3? Why? **Possible Student Answers, Key Points:**

- For this problem I would choose to divide by -3, because the equation for the blue work looks so much easier to solve now. It's just $n + 1 = -3$. The red equation involves more steps to arrive at the solution.
- I think the distributive property seems easier, because I'm pretty good at distributing and the numbers don't seem too tricky.

Our equations today will share a similar structure to these. Each equation will have a coefficient outside of an expression in parentheses. It will be our job to consider which first step will make solving easier. Sometimes both strategies might be equally easy for you. Sometimes both strategies might present challenges. And we'll see several examples today where one strategy ends up being significantly easier than the other. Our job will be to pause, think ahead, and predict what we should do.

Let's Think (Slide 4): For many problems today, we'll get to choose our solution pathway. For this first one, we're going to solve it both ways and think about which way felt easier and why.

$$\begin{aligned} -11 &= 6\left(k + \frac{1}{6}\right) \\ -11 &= 6k + 1 \\ \frac{-11}{-1} &= \frac{6k + 1}{-1} \\ \frac{-12}{6} &= \frac{6k}{6} \\ -2 &= k \end{aligned}$$

Let's solve this equation first by using the distributive property. (*solve equation as shown as you narrate*) I'll begin by distributing the 6 to each term in the parentheses. 6 times k is $6k$ and $6 \times \frac{1}{6}$ is $\frac{6}{6}$. I'll just write that as 1 whole. Our equation now reads $-11 = 6k + 1$. I can subtract 1 from both sides to start isolating the variable. What is $-11 - 1$? (-12) After subtracting 1 from both sides to keep the equation balanced, our equation reads $-12 = 6k$. I can divide by 6 on both sides to arrive at our solution. What is -12 divided by 6? (-2) Our solution reads $-2 = k$ or $k = -2$.

We had to be careful when operating with the signed values during this method, but otherwise, distributing felt fairly manageable with this equation.

$$\frac{-11}{6} = \frac{6(k + \frac{1}{6})}{6}$$

$$\frac{-11}{6} = k + \frac{1}{6}$$

$$\frac{-12}{6} = k$$

$$\boxed{-2 = k}$$

Let's solve the same equation, but instead of distributing, we'll divide both sides by 6 to start with.

(solve equation as shown as you narrate) Let's divide both sides by 6. -11 divided by 6 is a little unfriendly to think about. We can leave it as the fraction $-\frac{11}{6}$. Then the other side of our equation reads $k + \frac{1}{6}$. Our equation now reads $-\frac{11}{6} = k + \frac{1}{6}$. Let's isolate the variable. I can subtract $\frac{1}{6}$ from both sides. What is $-\frac{11}{6}$ minus $\frac{1}{6}$? ($-\frac{12}{6}$) Our equation now reads $-\frac{12}{6} = k$. I know $-\frac{12}{6}$ is the same as -2 because -12 divided by 6 is -2 . Our solution is $-2 = k$ or $k = -2$.

We got the same answer using both strategies. They're both valid methods for solving. Based on the numbers in this equation and the steps we took to solve, which method would you have chosen to use to solve the equation if you only had to choose one method? Why? **Possible Student Answers, Key Points:**

- I would choose the first method. When we distributed, we avoided having to do any more work with fractions.
- I would choose the first method. The second method had us working with negative fractions and fractions greater than one. I know how to do the math, but it was definitely a little trickier in spots.

In this case, I think it was certainly a bit easier to distribute first. I'll warn you, that's not always the case. We'll see problems today where the other strategy of dividing by the coefficient make the work more manageable.

Nice work. Let's look at one more example.

Let's Think (Slide 5): For this problem, we get a choice. We don't have to do it both ways, so let's be strategic about choosing our method. Take a second to look at the numbers. Before we start solving, what stands out to you about the values in this equation? **Possible Student Answers, Key Points:**

- I notice some of the numbers are decimals, and one of the numbers is a whole number.
- I notice 4.21 is half of 8.42.

$$8.42 = 4.21(y - 7)$$

Let's consider using the distributive property. (draw arrows showing the 4.21 being distributed) Does this seem like an efficient first step? **Possible Student Answers, Key Points:**

- We can do it. 4.21 times y would just be 4.21y. I'm wondering if multiplying 4.21×-7 is the best idea though. That might take some time.

$$\frac{8.42}{4.21} = \frac{4.21(y - 7)}{4.21}$$

Hm, seems doable, but I wonder if the other strategy might be better. Let's think about dividing by 4.21 first before we decide. (rewrite equation and draw division bars to show dividing both sides by 4.21) Dividing with fractions can sometimes be painstaking, but I notice that 8.42 divided by 4.21 would just be 2 wholes, because $4.21 + 4.21 = 8.42$. Hmm...

$$2 = y - 7$$

$$+7 \quad +7$$

$$\boxed{9 = y}$$

Based on thinking about the first steps, it definitely seems like dividing by 4.21 might be the better way to go. After that step, our new equation would just be $2 = y - 7$. (write equation) How easy is that? We don't have to worry about decimals anymore. (solve equation) We can add 7 to both sides of the equation, and there we go! $9 = y$ or $y = 9$ is our solution.

Because we visualized where the numbers in this equation would go with each method, we were able to choose a solutions strategy that made solving the equation efficient.

What will you look for as you decide which method to choose when solving today's equations? [Possible Student Answers, Key Points:](#)

- I'll look for whichever strategy makes my numbers seem the smallest or easiest to work with.
- I'll look for whichever strategy can save me time by avoiding fractions or decimals.

Let's Try it (Slides 6 - 7): Practice makes perfect, so let's look at a few more. With each problem, we'll decide whether it makes more sense to distribute first or divide by the coefficient first. We know that either method will get us the correct solution, but sometimes one is more efficient than the other. When possible, it can be smart to avoid unfriendly numbers such as negative decimals or unwieldy fractions. We'll do a couple more together before you get the chance to try this on your own.

WARM WELCOME



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Today we will decide between and use two common approaches for solving an equation of the form $p(x + q) = r$.

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 Let's Talk:

Two students started to solve the same problem. What do you notice and wonder about their work?

$$\begin{array}{r} \text{red arrows} \\ -3(n + 1) = 15 \\ \hline -3n - 3 \quad \neq 15 \end{array}$$

$$\begin{array}{r} -3(n + 1) \neq 15 \\ \hline -3 \quad \neq -3 \\ n + 1 \quad \neq -5 \end{array}$$

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 Let's Think:

Solve the equation two ways.

$$-11 = 6(k + \frac{1}{6})$$

$$-11 = 6(k + \frac{1}{6})$$

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Let's Think:

Predict which strategy will work best. Then solve.

$$8.42 = 4.21(y - 7)$$

DISTRIBUTE FIRST

DIVIDE FIRST

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Let's Try It:

Let's explore deciding between and using two common approaches for solving an equation of the form $p(x + q) = r$ together.

Name: _____ Q7 US Lesson 8 - Let's Try It!

Consider the equation $4(y + 2) = 9.2$.

1. Solve by dividing both sides by 4 first. 2. Solve by distributing the 4 first.

$\frac{4(y+2)}{4} = \frac{9.2}{4}$ $4(y+2) = 9.2$

3. Which strategy was easier for you with this equation? Why?

No matter which strategy you use, you get the same answer!

Consider the equation $3.66 = 1.22(m - 5)$.

4. Which first step do you predict will be easiest for this equation?

a. Distributing 1.24
b. Dividing by 1.24

5. Solve by dividing first. 6. Solve by distributing first.

$3.66 = 1.22(m - 5)$ $3.66 = 1.22(m - 5)$

7. Which strategy was easier for you with this equation? Why?

Look at the numbers in each equation carefully. Then, decide whether you want to distribute or divide first. It's okay if you get part way through solving and change your strategy.

8. Solve. 9. Solve.

$4 = \frac{3}{2}(x + \frac{2}{3})$ $12(w - 13) = 36$

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On your Own:

Now it's time to decide between and use two common approaches for solving an equation of the form $p(x + q) = r$ on your own.

Name: _____ G7 US Lesson 9 - Independent Work

1. Solve the equation by dividing first. Then, solve the same equation by distributing first. Circle the strategy that you found easier in this problem.

$-15 = 3(g + \frac{1}{2})$ $-15 = 3(g + \frac{1}{2})$

2. Solve the equation by either dividing first or distributing first.

$42.7 + h = 20$

Why did you choose your strategy?

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3. Solve. $13 = 2(c + 1.5)$	4. Solve. $-11(x - 5) = -44$	5. Solve. $8(x + \frac{1}{2}) = 20$
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Consider the equation $4(y + 2) = 9.2$.

1. Solve by dividing both sides by 4 first.

$$\frac{4(y+2)}{4} = \frac{9.2}{4}$$

2. Solve by distributing the 4 first.


$$4(y + 2) = 9.2$$

3. Which strategy was easier for you with this equation? Why?

No matter which strategy you use, you get the same answer!

Consider the equation $3.66 = 1.22(m - 5)$.

4. Which first step do you *predict* will be easiest for this equation?
- Distributing 1.22
 - Dividing by 1.22

5. Solve by dividing first.

$$3.66 = 1.22(m - 5)$$

6. Solve by distributing first.

$$3.66 = 1.22(m - 5)$$

7. Which strategy was easier for you with this equation? Why?

Look at the numbers in each equation carefully. Then, decide whether you want to distribute or divide first. It's okay if you get part way through solving and change your strategy.

8. Solve.

$$4 = \frac{3}{2} \left(x + \frac{2}{3} \right)$$

9. Solve.

$$12(w - 13) = 36$$

- 1. Solve the equation by dividing first. Then, solve the same equation by distributing first. Circle the strategy that you found easier in this problem.**

$$-15 = 3(a + \frac{1}{3})$$

$$-15 = 3(a + \frac{1}{3})$$

- 2. Solve the equation by either dividing first or distributing first.**

$$4(2.7 + h) = 20$$

Why did you choose your strategy?

3. Solve.

$$13 = 2(c + 1.5)$$

4. Solve.

$$-11(x - 5) = -44$$

5. Solve.

$$8(x + \frac{1}{2}) = 20$$

Name: _____

KEY

Consider the equation $4(y + 2) = 9.2$.

1. Solve by dividing both sides by 4 first.

$$\frac{4(y+2)}{4} = \frac{9.2}{4}$$

$$y+2 = 2.3$$
$$-2 \quad -2.0$$

$$y = 0.3$$

$$\begin{array}{r} 2.3 \\ 4 \overline{)9.2} \\ \underline{8} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

2. Solve by distributing the 4 first.

$$4(y + 2) = 9.2$$

$$4y + 8 = 9.2$$
$$ - 8 \quad - 8$$

$$4y = 1.2$$

$$y = 0.3$$

3. Which strategy was easier for you with this equation? Why?

I liked distributing in this case,
because it made easier numbers to
work with.

No matter which strategy you use, you get the same answer!

Consider the equation $3.66 = 1.22(m - 5)$.

4. Which first step do you *predict* will be easiest for this equation?

a. Distributing 1.22

b. Dividing by 1.22

5. Solve by dividing first.

$$\frac{3.66}{1.22} = \frac{1.22(m - 5)}{1.22}$$

$$\begin{array}{r} 3 = m - 5 \\ + 5 \quad + 5 \\ \hline 8 = m \end{array}$$

6. Solve by distributing first.

$$3.66 = 1.22(m - 5)$$

$$\begin{array}{r} 3.66 = 1.22m - 6.10 \\ + 6.10 \quad + 6.10 \\ \hline 9.76 = 1.22m \end{array}$$

$$\frac{9.76}{1.22} = \frac{1.22m}{1.22}$$

$$8 = m$$

$$\begin{array}{r} 1.22 \\ \times 5 \\ \hline 6.10 \end{array}$$

7. Which strategy was easier for you with this equation? Why?

I liked dividing first here. The numbers we worked with were easier.

Look at the numbers in each equation carefully. Then, decide whether you want to distribute or divide first. It's okay if you get part way through solving and change your strategy.

8. Solve.

$$\frac{4}{3/2} = \frac{3}{2} \left(x + \frac{2}{3} \right)$$

$$\begin{array}{r} \frac{8}{3} = x + \frac{2}{3} \\ - \frac{2}{3} \quad - \frac{2}{3} \\ \hline \frac{6}{3} = x \rightarrow x = 2 \end{array}$$

9. Solve.

$$\frac{12(w - 13)}{12} = \frac{36}{12}$$

$$\begin{array}{r} w - 13 = 3 \\ + 13 \quad + 13 \\ \hline \end{array}$$

$$w = 16$$

1. Solve the equation by dividing first. Then, solve the same equation by distributing first. Circle the strategy that you found easier in this problem.

$$\begin{array}{r} -15 = 3(a + \frac{1}{3}) \\ \frac{-15}{3} = \frac{3(a + \frac{1}{3})}{3} \\ -5 = a + \frac{1}{3} \\ -\frac{1}{3} \quad -\frac{1}{3} \\ \hline -5\frac{1}{3} = a \end{array}$$

$$\begin{array}{r} -15 = 3(a + \frac{1}{3}) \\ -15 = 3a + 1 \\ -1 \quad -1 \\ \hline -16 = 3a \\ \frac{-16}{3} = \frac{3a}{3} \\ -5\frac{1}{3} = a \end{array}$$

2. Solve the equation by either dividing first or distributing first.

$$\begin{array}{r} 4(2.7 + h) = 20 \\ \frac{4(2.7 + h)}{4} = \frac{20}{4} \\ 2.7 + h = 5 \\ -2.7 \quad -2.7 \\ \hline h = 2.3 \end{array}$$

Why did you choose your strategy?

I knew I could quickly divide 20
by 4.

3. Solve.

$$13 = 2(c + 1.5)$$

$$\begin{array}{r} 13 = 2c + 3 \\ -3 \quad -3 \\ \hline \end{array}$$

$$\frac{10}{2} = \frac{2c}{2}$$

$$\boxed{5 = c}$$

4. Solve.

$$\frac{-11(x - 5)}{-11} = \frac{-44}{-11}$$

$$\begin{array}{r} x - 5 = 4 \\ +5 \quad +5 \\ \hline \end{array}$$

$$\boxed{x = 9}$$

5. Solve.

$$8(x + \frac{1}{2}) = 20$$

$$\begin{array}{r} 8x + 4 = 20 \\ -4 \quad -4 \\ \hline \end{array}$$

$$\frac{8x}{8} = \frac{16}{8}$$

$$\boxed{x = 2}$$

G7 U5 Lesson 10

Use tape diagrams to translate verbal descriptions for situations into an equation of the form $px + q = r$ or $p(x + q) = r$, and solve the resulting equation to determine an unknown quantity in the situation.

G7 U5 Lesson 10 - Students will use tape diagrams to translate verbal descriptions for situations into an equation of the form $px + q = r$ or $p(x + q) = r$, and solve the resulting equation to determine an unknown quantity in a situation.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We've been representing situations with models and equations and solving for unknowns for several lessons. We've seen a wide array of contexts and learned about different approaches to solving equations. Today we pull a lot of that thinking together. We'll take a situation from a story problem, create a tape diagram to help us reason about the relationships in the story, and then use that information to write and solve an equation. In a way, today might feel like a day to practice all the skills we've studied so far. around expressions and equations.

Let's Talk (Slide 3): Check out the tape diagrams here. Once you've analyzed them, share what you notice is the same and what you notice is different. **Possible Student Answers, Key Points:**

- They both show a rectangle cut into smaller rectangles. They both have a total of 39. They both include variables and the number 6. There are some equal groups inside each total.
- The variables are different. The second tape diagram has all equal groups. The first tape diagram only has one term in each small rectangle.

These tape diagrams represent different situations. We can see that because the relationship between the parts and the totals is different in each of them. As we're aware, we can write equations that correspond with each of these models. For example, I see the first tape diagram shows three groups of x plus a group of 6 have a total of 39. (*write equation*) I can write that as $3x + 6 = y$. What

$$39 = 3x + 6$$

$$39 = 3(x + 6)$$

do you see in the other tape diagram? What equation can match that? (*write equation as student shares*) **Possible Student Answers, Key Points:**

- I see 3 equal groups of $y + 6$ equal a total of 39.
- I can write $3(y + 6) = 39$.

I wonder if we could think of our own story that could go with either of these. For the first tape diagram and equation, I know I need to think of a situation involving 3 groups of an unknown plus 6 more. Maybe my story could be something like: *I have 3 baskets of apples and 6 individual apples, and I have 39 apples in all. How many apples are in each basket?*

Can you think of a situation or a story problem to match the second tape diagram and equation? **Possible Student Answers, Key Points:**

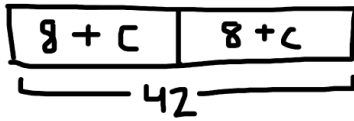
- I have 39 apples evenly distributed between 3 baskets. Each basket has some green apples and 6 red apples. How many green apples are in each basket?
- 39 people are attending a field trip on 3 buses. Each bus has 6 parents and some students. How many students are on each bus?

Today, we'll work to think about verbal descriptions, also known as word problems, and how tape diagrams and equations can help us solve for unknowns.

Let's Think (Slide 4): I'll read our first problem aloud as you follow along. When I'm done reading, summarize the story in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This problem is about the number of parents and children on a playground.
- We know the playground has 2 sections. We know there are 42 people on the playground. We know there are 8 adults in each section.
- We don't know how many children are in each section.

Let's draw a tape diagram and write an equation to help us solve and determine how many children are on each section of the playground.



I know there are 42 people on the playground, so I'll draw a tape diagram with a total of 42. *(draw and label tape diagram as you narrate)* I'll split the large rectangle in half to show the two sections of the playground. I know each section has 8 adults and an unknown amount of children. I'll use the expression $8 + c$ to represent that inside each smaller rectangle.

Based on the verbal description and my tape diagram, what equation could we write to represent this problem? How do you know? **Possible Student Answers, Key Points:**

- In the tape diagram, I see two equal groups of $8 + c$ have a total of 42.
- I can write $2(8 + c) = 42$.

$$42 = 2(8 + c)$$

$$42 = 16 + 2c$$

$$\begin{array}{r} -16 \quad -16 \\ \hline 26 = 2c \end{array}$$

$$\frac{26}{2} = \frac{2c}{2}$$

$$13 = c$$

(write $42 = 2(8 + c)$) We know we can solve this type of equation by distributing first or by dividing by 2 first. When I look at these numbers, I think either strategy can be efficient. Let's distribute the 2 first. *(draw arrows showing the distribution of the 2)* What is 2 times 8? (16) What is 2 times c ? ($2c$)

(continue solving as you narrate) We can rewrite the equation as $42 = 16 + 2c$. To isolate the variable, let's subtract 16 from both sides of the equation. 42 minus 16 leaves us with 26 on the left of our equation, and we have $2c$ on the right of our equation. If I divide both sides by 2, we are left with our solution. I know $13 = c$. What does 13 mean in this context? **Possible Student Answers, Key Points:**

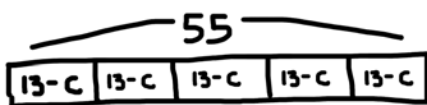
- We were trying to find how many children were in each section of the playground. There are 13 children in each section.

We just made a tape diagram then wrote and solved an equation to represent this verbal description. Nice work! To check my answer, I can pause to think if it seems reasonable in context. 13 kids in each section sounds reasonable based on the numbers in the story. It's not 1,000 kids. It's not -7 kids. It's not $2\frac{1}{2}$ kids. It seems sensical. We could also substitute our value back into the original equation to check. *(substitute 13 in for c in the original equation)* I know $8 + 13$ is 21, and $2 \cdot 21$ would be 42. Our solution checks out. Nice work! Let's do one more.

$$42 = 2(8 + 13)$$

Let's Think (Slide 5): Like before, I'll read our first problem aloud as you follow along. When I'm done reading, summarize the story in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This problem is about a girl putting 55 cupcakes onto identical platters.
- We know there are 55 cupcakes. We know she has 5 platters. We know there were 13 cupcakes on each platter, but she took some off.
- We don't know how many cupcakes she took off each platter.



Let's model this problem using a tape diagram. How could you represent the total cupcakes and the amounts on each platter in a tape diagram? *(sketch as student shares, supporting as needed)* **Possible Student Answers, Key Points:**

- I'd draw a total of 55 along one large rectangle.
- I'd cut the rectangle into 5 equal pieces to show the platters.
- I'd write $13 - c$ in each small section to show the 13 cupcakes with some removed.

This tape diagram makes sense. I see 55 total cupcakes split into 5 equal groups or platters. Each platter shows that it had 13 cupcakes, but some were taken off. Great work.

Now we can write an equation. (*write equation*) I can write $55 = 5(13 - c)$ because it shows the total is 55 and that the total is equal to 5 groups of $13 - c$. It matches the tape diagram and the story. In our last example, we solved by distributing. In this example, I might choose to divide by 5 first, just because I noticed when I distribute I would have to multiply 5 by 13. I can do that, but dividing by 5 feels easier given that I know 55 divided by 5 with no problem. (*divide both sides by 5*) What's our new equation? ($11 = 13 - c$)

$$\frac{55}{5} = \frac{5(13 - c)}{5}$$

$$\frac{11}{-13} = \frac{13 - c}{-13}$$

$$\frac{-2}{-1} = \frac{-1c}{-1}$$

$$\boxed{2 = c}$$

(*rewrite equation*) We're left with $11 = 13 - c$. To balance the equation and isolate the variable, I'll remove 13 from both sides. (*subtract 13*) That leaves us with $-2 = -c$, or $-2 = -1c$. I'm not quite finished, because the variable is not totally by itself. Let's divide both sides by -1 . What is -2 divided by -1 ? ($+2$)

Our solution is $2 = c$ or $c = 2$. We did it!

How could I check to make sure our answer is correct? [Possible Student Answers, Key Points:](#)

- You can think if it feels reasonable in context. In this case, removing 2 cupcakes from each platter feels reasonable based on the story.
- I can substitute 2 in for the variable in the problem. I can think about $55 = 5(13 - 2)$.

We just used a tape diagram and an equation to help think about two different verbal descriptions and solve for an unknown. We're getting really good at this.

Let's Try it (Slides 6 - 7): Now let's practice! Just like we did in our last two examples, we'll pause to consider what is known and unknown in each verbal description. This will help us draw an accurate tape diagram, which we know makes it easy to write a corresponding equation. Once we're done solving any equation, we can always check our work to consider whether our answer is reasonable and/or by substituting our solution back in for the unknown in the original equation. Let's collaborate on a few more, and then you can practice some on your own.

WARM WELCOME



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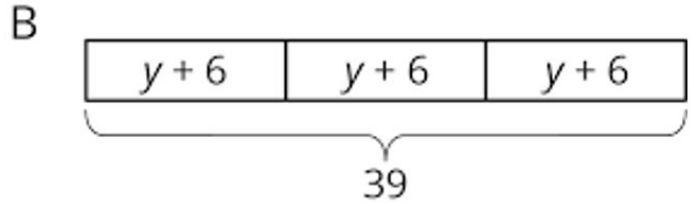
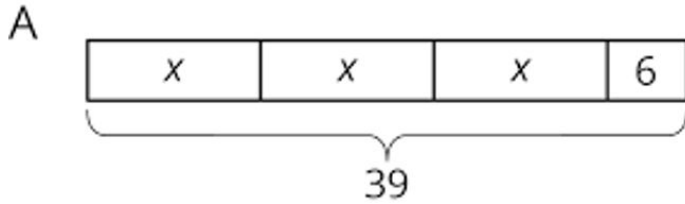
Today we will use tape diagrams to represent verbal descriptions for situations with an equation of the form $px + q = r$ or $p(x + q) = r$, and solve the resulting equation to determine an unknown quantity in the situation.

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Let's Talk:

What's the same? What's different?



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Let's Think:

A playground has 2 sections. Each section has 8 adults, and an unknown number of children, c . If there are 42 total people at the playground, how many children are in each section?

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Let's Think:

Joy put 13 cupcakes on each of 5 platters. The platters looked too full, so she took the same number of cupcakes off each platter. After doing that, the platters had a total of 55 cupcakes. How many cupcakes did Joy take from each platter?

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Let's Try It:

Let's explore using tape diagrams to represent verbal descriptions for situations with an equation of the form $px + q = r$ or $p(x + q) = r$ together.

Name: _____ G7 US Lesson 10 - Let's Try It

A P.E. teacher is filling bags to organize his equipment. He puts 7 soccer balls into each bag and f footballs into each bag. After filling 4 bags, he has organized a total of 60 items.

- What is the teacher doing?
 - Making bags that combine soccer balls and footballs.
 - Making separate bags of soccer balls and footballs.
- Did the teacher put the same number of footballs in each bag? _____
- How many items did the teacher organize after filling the 4 bags?
 - 60 items
 - An unknown number of items
- Circle the tape diagram that best matches the story.

$\cup \circ$

$\cup \circ$
- Write an equation to represent the tape diagram.
- Solve the equation.

Dominique is at a party that has 5 plates of cookies. Each plate has the same amount on it. Dominique takes 1 cookie from each plate. There are now 35 total cookies on the plates. How many cookies were on each plate to start?

- What is known? What is unknown?

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- Label the tape diagram below to represent the problem.
- Write an equation to represent the tape diagram. Solve the equation.
- What does your solution mean in the context of this problem?

Gerald buys 6 containers of laundry detergent. He has a coupon for \$0.50 off each container. If Gerald paid a total of \$21, what was the original cost of each container of laundry detergent?

- Draw a tape diagram to represent the problem. Use c to represent the original cost of each container.
- Write an equation. Solve.
- What does your solution mean in the context of this problem?

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On your Own:

Now it's time to use tape diagrams to represent verbal descriptions for situations with an equation of the form $px + q = r$ or $p(x + q) = r$ on your own.

Name: _____ G7 US Lesson 10 - Independent Work

1. Kevon put the same number of flowers in 3 vases. He then adds 8 flowers to each vase. Kevon used a total of 57 flowers. How many flowers did Kevon put in each vase to begin with?

TAPE DIAGRAM _____ EQUATION _____

2. A teacher has 68 students that she splits evenly onto 4 teams. On each team there are 5 girls and an unknown number of boys, x . How many boys are on each team?

TAPE DIAGRAM _____ EQUATION _____

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3. Mr. Wallace has 3 identical boxes of chocolates. He removes 2 chocolates from each box, leaving a total of 66 chocolates remaining. How many chocolates were in each box to start?

a. Show or explain your work using a tape diagram and an equation.

b. Explain what your answer means in the context of the problem.

4. Write a story problem that could be represented using the tape diagram here. Then solve the problem.

$y + 6$	$y + 6$	$y + 6$
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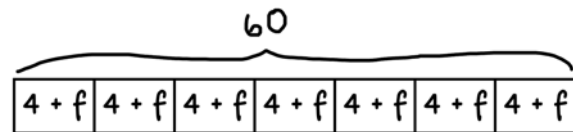
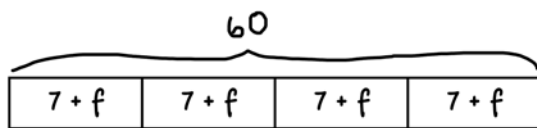
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A P.E. teacher is filling bags to organize his equipment. He puts 7 soccer balls into each bag and f footballs into each bag. After filling 4 bags, he has organized a total of 60 items.

1. What is the teacher doing?
 - a. Making bags that combine soccer balls and footballs.
 - b. Making separate bags of soccer balls and footballs.
2. Did the teacher put the same number of footballs in each bag? _____
3. How many items did the teacher organize after filling the 4 bags?
 - a. 60 items
 - b. An unknown number of items
4. Circle the tape diagram that best matches the story.



5. Write an equation to represent the tape diagram.
6. Solve the equation.

Dominique is at a party that has 5 plates of cookies. Each plate has the same amount on it. Dominique takes 1 cookie from each plate. There are now 35 total cookies on the plates. How many cookies were on each plate to start?

7. What is known? What is unknown?

8. Label the tape diagram below to represent the problem.



9. Write an equation to represent the tape diagram. Solve the equation.

10. What does your solution mean in the context of this problem?

Gerald buys 6 containers of laundry detergent. He has a coupon for \$0.50 off each container. If Gerald paid a total of \$21, what was the original cost of each container of laundry detergent?

11. Draw a tape diagram to represent the problem. Use c to represent the original cost of each container.

12. Write an equation. Solve.

13. What does your solution mean in the context of this problem?

- 1. Kevon put the same number of flowers in 3 vases. He then adds 8 flowers to each vase. Kevon used a total of 57 flowers. How many flowers did Kevon put in each vase to begin with?**

TAPE DIAGRAM

EQUATION

- 2. A teacher has 68 students that she splits evenly onto 4 teams. On each team there are 5 girls and an unknown number of boys, x . How many boys are on each team?**

TAPE DIAGRAM

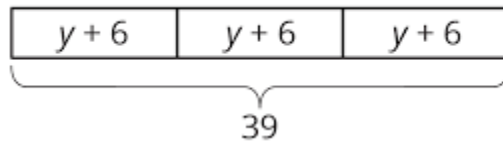
EQUATION

3. Mr. Wallace has 3 identical boxes of chocolates. He removes 2 chocolates from each box, leaving a total of 66 chocolates remaining. How many chocolates were in each box to start?

a. Show or explain your work using a tape diagram and an equation.

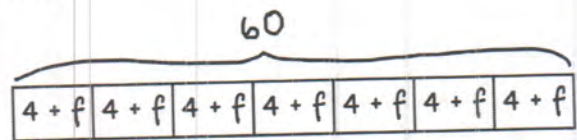
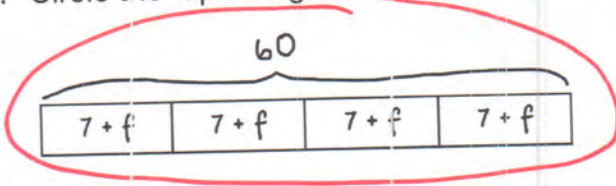
b. Explain what your answer means in the context of the problem.

4. Write a story problem that could be represented using the tape diagram here. Then solve the problem.



A P.E. teacher is filling bags to organize his equipment. He puts 7 soccer balls into each bag and f footballs into each bag. After filling 4 bags, he has organized a total of 60 items.

- What is the teacher doing?
 - Making bags that combine soccer balls and footballs.
 - Making separate bags of soccer balls and footballs.
- Did the teacher put the same number of footballs in each bag? YES
- How many items did the teacher organize after filling the 4 bags?
 - 60 items
 - An unknown number of items
- Circle the tape diagram that best matches the story.



5. Write an equation to represent the tape diagram.

$$60 = 4(7 + f)$$

6. Solve the equation.

$$\frac{60}{4} = \frac{4(7+f)}{4}$$

$$\frac{15}{1} = \frac{7+f}{1}$$

$$8 = f$$

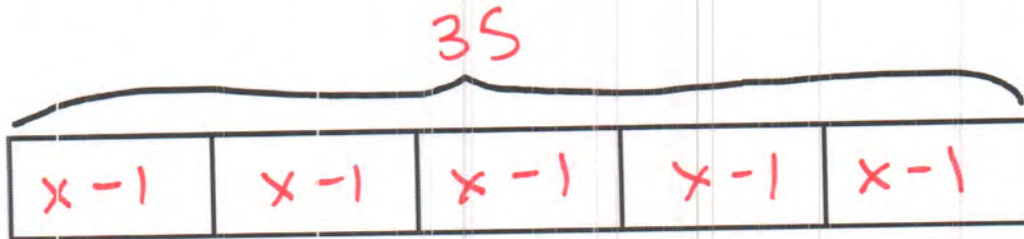
Dominique is at a party that has 5 plates of cookies. Each plate has the same amount on it. Dominique takes 1 cookie from each plate. There are now 35 total cookies on the plates. How many cookies were on each plate to start?

7. What is known? What is unknown?

- 5 equal plates
- takes 1 from each
- 35 cookies left in all

How many cookies started out on each plate?

8. Label the tape diagram below to represent the problem.



9. Write an equation to represent the tape diagram. Solve the equation.

$$\frac{5(x-1)}{5} = \frac{35}{5}$$

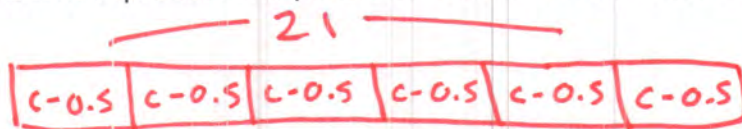
$$\begin{array}{r} x-1 = 7 \\ +1 \quad +1 \\ \hline x = 8 \end{array}$$

10. What does your solution mean in the context of this problem?

There were 8 cookies on each plate to start.

Gerald buys 6 containers of laundry detergent. He has a coupon for \$0.50 off each container. If Gerald paid a total of \$21, what was the original cost of each container of laundry detergent?

11. Draw a tape diagram to represent the problem. Use c to represent the original cost of each container.



12. Write an equation. Solve.

$$6(c-0.5) = 21$$

$$\begin{array}{r} 6c - 3 = 21 \\ +3 \quad +3 \\ \hline 6c = 24 \end{array}$$

$$\frac{6c}{6} = \frac{24}{6}$$

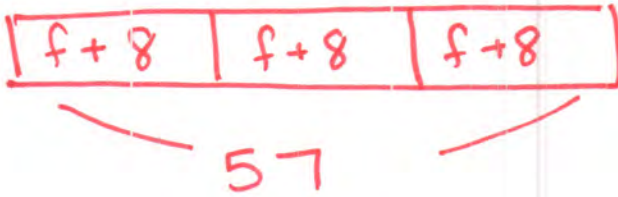
$$c = 4$$

13. What does your solution mean in the context of this problem?

Each container costs \$4 without a coupon.

1. Kevon put the same number of flowers in 3 vases. He then adds 8 flowers to each vase. Kevon used a total of 57 flowers. How many flowers did Kevon put in each vase to begin with?

TAPE DIAGRAM



EQUATION

$$3(f+8) = 57$$

$$3f + 24 = 57$$

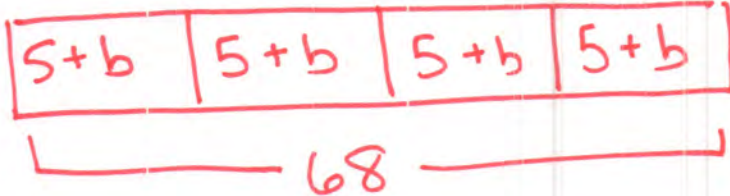
$$\begin{array}{r} -24 \quad -24 \\ \hline \end{array}$$

$$\frac{3f}{3} = \frac{33}{3}$$

$$f = 11$$

2. A teacher has 68 students that she splits evenly onto 4 teams. On each team there are 5 girls and an unknown number of boys, x. How many boys are on each team?

TAPE DIAGRAM



EQUATION

$$4(5+b) = 68$$

$$20 + 4b = 68$$

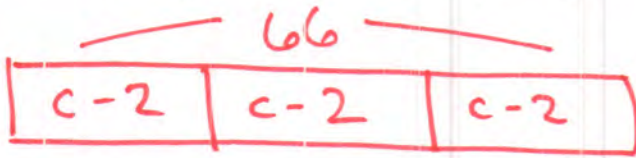
$$\begin{array}{r} -20 \quad -20 \\ \hline \end{array}$$

$$\frac{4b}{4} = \frac{48}{4}$$

$$b = 12$$

3. Mr. Wallace has 3 identical boxes of chocolates. He removes 2 chocolates from each box, leaving a total of 66 chocolates remaining. How many chocolates were in each box to start?

a. Show or explain your work using a tape diagram and an equation.

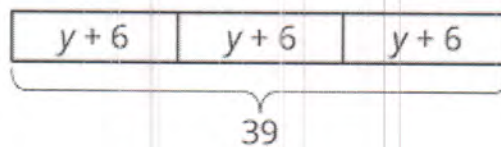


$$\begin{array}{r} 3(c-2) = 66 \\ \hline 3 \qquad 3 \\ c-2 = 22 \\ +2 \quad +2 \\ \hline c = 24 \end{array}$$

b. Explain what your answer means in the context of the problem.

Each box had 24 pieces to start with.

4. Write a story problem that could be represented using the tape diagram here. Then solve the problem.



$$\begin{array}{r} 3(y+6) = 39 \\ \hline 3 \qquad 3 \\ y+6 = 13 \\ -6 \quad -6 \\ \hline y = 7 \end{array}$$

There are 3 identical baskets of fruit. Each basket has y oranges and 6 pears. There are 39 total pieces of fruit. How many oranges are in each basket?

$$y = 7$$

G7 U5 Lesson 11

Solve word problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.

G7 U5 Lesson 11 - Students will solve word problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.

NOTE: Students are welcome to use calculators for some of the computation on this day.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We're reaching the end of the portion of this unit dedicated to equations. For our last lesson of this portion of the unit, we're going to look specifically at problems that deal with percent increase and percent decrease. We've worked with percent increase and decrease in a previous unit; today we're going to connect the percent thinking we've used before to the work we're currently considering around equations.

Let's Talk (Slide 3): Before we fully jump into the day's learning, let's consider a fairly common context involving percent change: grocery store prices. You've likely noticed that prices at the grocery store fluctuate. Sometimes they go up, sometimes they go down, sometimes things are on sale...they're constantly changing. Take a look at these two situations. What do you notice about them? What do you wonder? **Possible Student Answers, Key Points:**

- I notice the cost of bananas is going up and the cost of a loaf of bread is going down. I notice both problems have percents in them. I notice there is an equation under each one that uses a decimal.
- I wonder what the solutions are. I wonder why the equations use those decimals, since they're not the decimal equivalents of the percentages named in each problem.

$$100\% + 25\% = 125\%$$

original cost increase

We can use equations to find price mark-ups and mark-downs involving percents. If I know the cost of bananas is increasing 25%, that means I'll be paying the original cost of the bananas plus an extra 25% on top of that. I can think of that like $100\% + 25\% = 125\%$. The new cost is 125% of the original cost. (write $100\% + 25\% = 125\%$ as shown) As an equation, I can write that as $1.25(2) = c$, because 125% in decimal form is 1.25. What is $1.25 \cdot 2$? (2.5) Our solution is $c = 2.5$. That means the new cost of bananas is \$2.50.

Look at the other situation where the loaf of bread decreases in price by 25%. Why do you think the equation below can be used to represent a price *decreasing* 25%? **Possible Student Answers, Key Points:**

- If the price is decreasing 25%, that means it's going down 25% from the original 100%. 100% minus 25% is 75%. So, I'm paying 75% of the original cost.
- 75% written as a decimal is 0.75, so I can multiply 2×0.75 to find 75% of the original cost.

$$100\% - 25\% = 75\%$$

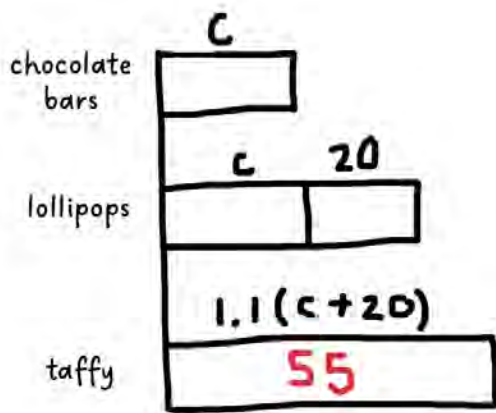
original cost decrease

Great thinking. If the price decreases by 25%, that means I was paying 100% originally but now I'm paying 25% less. (write $100\% - 25\% = 75\%$ as shown) We can use $0.75(2) = c$ to represent this as an equation, because 0.75 is the decimal equivalent of 75%. What is $0.75 \cdot 2$? (1.5) If $c = 1.5$, that means the new cost of a loaf of bread is \$1.50.

Today, we'll use equations like these to consider problems involving percent increase and percent decrease.

Let's Think (Slide 4): I'll read the problem once through aloud as you follow along. When I'm done, summarize what the story is about. What is known and unknown? **Possible Student Answers, Key Points:**

- This problem is about a candy store that sells chocolate bars, lollipops, and taffy.
- We know they sell 55 pieces of taffy. We know they sell 10% more taffy than lollipops. We know they sell 20 more lollipops than chocolate bars.
- The unknown is the amount of chocolate bars sold, c .



We can use a tape diagram to help think about this story. I'll draw a vertical line and leave room for three rectangles to represent each of the types of candy, chocolate bars, lollipops, and taffy. *(sketch and tape diagram as you narrate)* We don't know the amount of chocolate bars sold, so I'll draw a rectangle labeled c .

I know they sell 20 more lollipops than chocolate bars. To show that, I'll draw another bar that is c long and attach another bar that represents the 20 more. I can think of the quantity of lollipops as being $c + 20$.

Lastly, I need to think about the taffy. I know they sell 10% more taffy than lollipops, so I'll draw the bar for taffy a little longer than the bar for lollipops. I also know the amount of taffy pieces sold is 55, so I can label that inside the tape diagram. Let's think about this taffy quantity a little more. If I know the taffy is 10% more than the lollipop amount, how can I think of the 10% increase so that I can use it in the form of an equation? **Possible Student Answers, Key Points:**

- We can think of 100% of the lollipops plus 10% more. $100\% + 10\% = 110\%$.
- The amount of taffy is 110% the amount of lollipops. 110% as a decimal is 1.10 or 1.1.

The taffy is 110% of the lollipop amount. If the lollipop amount is $c + 20$, I can use the expression $1.1(c + 20)$ to represent 110% of the lollipop amount. *(label the taffy rectangle as $1.1(c + 20)$)*

Now that we've thought through this multi-part tape diagram, we can work to solve. I know that $1.1(c + 20)$ can be used to represent the amount of taffy. I also know that the exact amount of taffy is 55 pieces. I can set those equal to each other to set up an equation. *(write $1.1(c + 20) = 55$)* I know we can solve this equation type by distributing or by dividing by the coefficient. Let's divide both sides of the equation by 1.1, because I know dividing 55 by 11 tenths will be manageable. *(show division of 11 in equation)* What is 55 divided by 1.1? **(50)** *(rewrite equation)* The equation now reads $c + 20 = 50$. What can I do to isolate the variable? **(subtract 20 from both sides of the equation)** If I subtract 20 from both sides of the equation, I end up with a solution that reads $c = 30$. What does that mean in the context of this problem?

$$\begin{array}{r} 1.1(c + 20) = 55 \\ \hline 1.1 \quad 1.1 \\ c + 20 = 50 \\ \hline -20 \quad -20 \\ \hline c = 30 \end{array}$$

Possible Student Answers, Key Points:

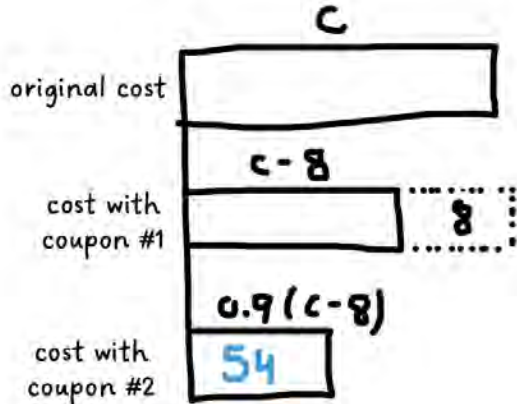
- We were trying to find the amount of chocolate bars the story sold. The solution $c = 30$ means that the store sold 30 chocolate bars.

We carefully built out a multi-part tape diagram to visualize what was happening in the story. We wrote an equation, by thinking about the percent change as a decimal value, and then we solved to find our unknown. This question is a testament to all the work we've put in to understand solving equations and using percents.

Let's Think (Slide 5): Let's try one more just to make sure we're confident and ready to work more independently. I'll read the problem once through aloud as you follow along. When I'm done, summarize what the story is about. What is known and unknown? **Possible Student Answers, Key Points:**

- This problem is about Aaron and how he's trying to use two coupons to get a better deal on a pair of shoes.
- I know one coupon takes \$8 off and the other takes 10% off. I know the shoes end up costing \$54.
- I don't know the original cost of the shoes.

Let's start by drawing a tape diagram to visualize each part of this problem. I'll draw a vertical line with room to think about the original cost of the shoe, the cost after the first coupon, and the cost after the second coupon.



(sketch and label tape diagram as you narrate) If we're thinking about the cost of shoes *before* a sale, I know the cost will be more. I'll start by drawing a long rectangle labeled c . This represents the original cost of the shoes. The first coupon is for \$8 off of the price. I can show that by drawing a bar that is 8 dollars less than the original cost. I'll label this rectangle as $c - 8$, because it is the cost of the shoe minus 8 dollars.

The last coupon takes 10% off the discounted price. How can I think about a 10% decrease so that I can write an equation using a decimal? **Possible Student Answers, Key Points:**

- 10% off means 10% less than 100%. $100\% - 10\% = 90\%$. Aaron will pay 90% of the discounted price.
- I can write 90% as 0.90 or 0.9 in decimal form.

We can think of the final price of the shoes as being 90% of the discounted price. I'll use the expression $0.9(c - 8)$ to represent the last rectangle on my tape diagram, because that's the same as saying 90% of $c - 8$. (label tape diagram) The problem also told me that the final cost was \$54, so I'll label that in the rectangle for the final cost of the shoes.

We now have all the information we need to solve. (write $0.9(c - 8) = 54$) I know that 90% of the discounted price is equal to \$54. I can use the equation here to represent that.

$$\begin{array}{r} 0.9(c - 8) = 54 \\ \hline 0.9 \quad 0.9 \\ c - 8 = 60 \\ \hline +8 \quad +8 \\ \hline c = 68 \end{array}$$

Like last time, I'm going to choose to divide by 0.9 instead of distribute. I chose that because I know I can divide 54 by 9 tenths without it getting too messy or complicated. (divide both sides by 0.9) We're left with $c - 8 = 60$. What would you do last to solve, and what would your solution mean in the context of the problem? **Possible Student Answers, Key Points:**

- I would add 8 to both sides of the equation to isolate the variable. My answer is $c = 68$.
- A solution showing $c = 68$ means that the original cost of the shoes before the coupons was \$68.

We can use tape diagrams and equations to help us solve problems involving percent increase and percent decrease.

Let's Try it (Slides 6 - 7): Let's practice a few more together, and then you'll have time to work through some independently. For most problems, it will be useful to sketch a tape diagram first so we can visualize each part of the word problem. When we write equations, we'll want to think about how best to use decimals to represent the percent increase or decrease in the problem. Time to jump in!

WARM WELCOME



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Today we will solve word problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.

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 **Let's Talk:**


Bananas cost \$2. They increase in price by 25%.

$$1.25(2) = ?$$

A loaf of bread costs \$2. It decreases in price by 25%.

$$0.75(2) = ?$$

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 **Let's Think:**

A candy store sells c chocolate bars. The store sells 20 more lollipops than chocolate bars. The store sells 10% more pieces of taffy than lollipops. The store sells 55 pieces of taffy. How many chocolate bars does the candy store sell?

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Let's Think:

Aaron has two coupons for a pair of shoes he wants to buy that originally cost x dollars. He first applies a coupon that takes \$8 off the price of the shoes. Then, he applies a coupon that reduces that price by 10%. Aaron ends up paying \$54 for the shoes. How much did the shoes originally cost?

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Let's Try It:

Let's explore solving word problems about percent increase or decrease together.

Name: _____ G7 US Lesson 11 - Let's Try It

Eve baked some brownies on Friday. On Saturday, she baked 10 more brownies than on Friday. On Sunday, she increased the number of brownies she baked by 20%, baking 48 brownies.

- What is Eve doing in this story?
- Do we know how many brownies she baked on Friday? _____
- The tape diagram below shows that Eve baked n brownies on Friday. Draw and label a tape diagram to represent the amount of brownies she baked on Saturday.

Friday	n
Saturday	
Sunday	

- Draw and label a tape diagram to show how on Sunday Eve baked 20% more than she did on Saturday.
- We know 20% more than the Saturday amount is equal to 48.

$$20\% \text{ more than Saturday} = 48$$

$$20\% \text{ more than } \underline{\hspace{2cm}} = 48$$

$$\underline{\hspace{2cm}} (\underline{\hspace{1cm}} + \underline{\hspace{1cm}}) = 48$$
- Solve for n by either dividing first or distributing first.
- Eve baked _____ cupcakes on Friday.

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There are x people in the theater club. There are 10 less people in the French club than in the theater club, and there are 50% more people in the robotics club than in the French club. There are 36 people in the robotics club.

- What is known in this situation? What is unknown?
- Draw a tape diagram to represent the people in the theater club. Then draw another tape diagram to represent the people in the French club. Then draw another tape diagram to represent the people in the robotics club.

theater	
French	
robotics	

- Write an equation to represent the number of students in the robotics club.

$$\text{50\% more } (\underline{\hspace{2cm}} + \underline{\hspace{1cm}}) = \underline{\hspace{2cm}}$$

of people in French club # of people in robotics club
- Solve for x .
- How many people are in the theater club?

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On your Own:

Now it's time to solve word problems about percent increase or decrease on your own.

Name: _____ G7 US Lesson 11 - Independent Work

1. The number of students on-time to school increased 25% from last week to this week. The number of students on-time to school this week is 120. Draw a tape diagram and write an equation to help determine how many students were on-time to school last week.

2. A clothing store is having a sale where all jeans are discounted by 25%. Kieran has a coupon for \$10 off the regular price of jeans. The store first applies the coupon and then takes 25% off the reduced price. If Kieran ends up paying \$48 for the jeans, what was the original cost of the jeans?

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3. An animal shelter takes care of 12 more cats than bunnies. There are 10% fewer dogs than cats at the shelter. There are 81 dogs in the shelter. How many bunnies are in the shelter?

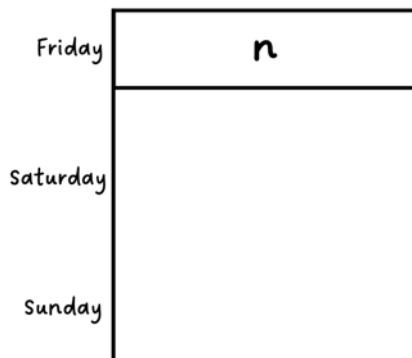
4. There are x students enrolled in Spanish class. There are 8 fewer students enrolled in French class than in Spanish class. There are 50% more students in Chinese class than in French class. There are 40 students enrolled in Chinese class. How many students are enrolled in Spanish?

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Eve baked some brownies on Friday. On Saturday, she baked 10 more brownies than on Friday. On Sunday, she increased the number of brownies she baked by 20%, baking 48 brownies.

1. What is Eve doing in this story?
2. Do we know how many brownies she baked on Friday? _____
3. The tape diagram below shows that Eve baked n brownies on Friday. Draw and label a tape diagram to represent the amount of brownies she baked on Saturday.



4. Draw and label a tape diagram to show how on Sunday Eve baked 20% more than she did on Saturday.
5. We know 20% more than the Saturday amount is equal to 48.

$$20\% \text{ more than Saturday} = 48$$

$$20\% \text{ more than } \underline{\hspace{2cm}} = 48$$

$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}}) = 48$$

6. Solve for n by either dividing first or distributing first.

7. Eve baked _____ cupcakes on Friday.

There are x people in the theater club. There are 10 less people in the French club than in the theater club, and there are 50% more people in the robotics club than in the French club. There are 36 people in the robotics club.

8. What is known in this situation? What is unknown?

9. Draw a tape diagram to represent the people in the theater club. Then draw another tape diagram to represent the people in the French club. Then draw another tape diagram to represent the people in the robotics club.



10. Write an equation to represent the number of students in the robotics club.

$$\text{50\% more} \left(\text{\# of people in French club} - \text{\# of people in French club} \right) = \text{\# of people in robotics club}$$

11. Solve for x .

12. How many people are in the theater club?

1. The number of students on-time to school increased 25% from last week to this week. The number of students on-time to school this week is 120. Draw a tape diagram and write an equation to help determine how many students were on-time to school last week.

2. A clothing store is having a sale where all jeans are discounted by 25%. Kieran has a coupon for \$10 off the regular price of jeans. The store first applies the coupon and then takes 25% off the reduced price. If Kieran ends up paying \$48 for the jeans, what was the original cost of the jeans?

3. An animal shelter takes care of 12 more cats than bunnies. There are 10% fewer dogs than cats at the shelter. There are 81 dogs in the shelter. How many bunnies are in the shelter?

4. There are x students enrolled in Spanish class. There are 8 fewer students enrolled in French class than in Spanish class. There are 50% more students in Chinese class than in French class. There are 42 students enrolled in Chinese class. How many students are enrolled in Spanish?

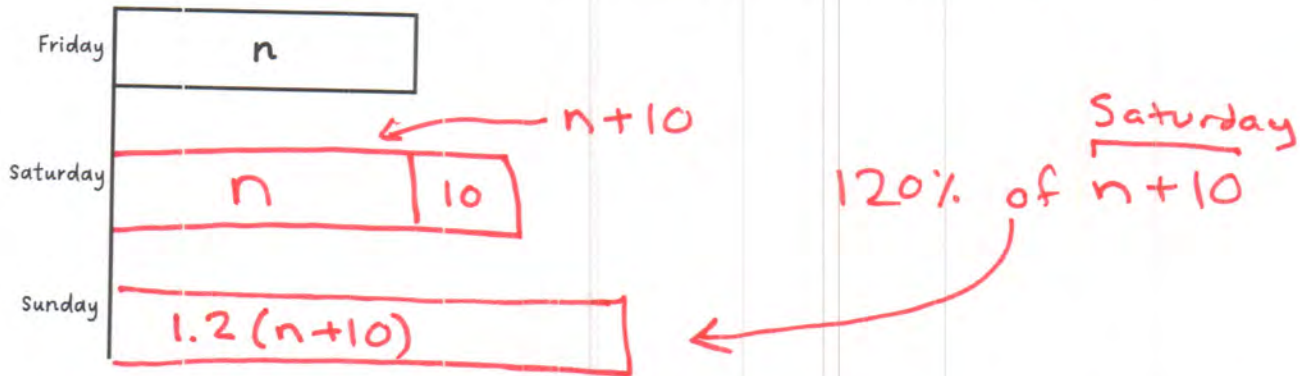
Eve baked some brownies on Friday. On Saturday, she baked 10 more brownies than on Friday. On Sunday, she increased the number of brownies she baked by 20%, baking 48 brownies.

1. What is Eve doing in this story?

She is baking brownies.

2. Do we know how many brownies she baked on Friday? NO

3. The tape diagram below shows that Eve baked n brownies on Friday. Draw and label a tape diagram to represent the amount of brownies she baked on Saturday.



4. Draw and label a tape diagram to show how on Sunday Eve baked 20% more than she did on Saturday.

5. We know 20% more than the Saturday amount is equal to 48.

$$20\% \text{ more than Saturday} = 48$$

$$20\% \text{ more than } \underline{n+10} = 48$$

$$\underline{1.2} (\underline{n} + \underline{10}) = 48$$

6. Solve for n by either dividing first or distributing first.

$$\frac{1.2(n+10)}{1.2} = \frac{48}{1.2}$$

$$n+10 = 40$$

$$n = 30$$

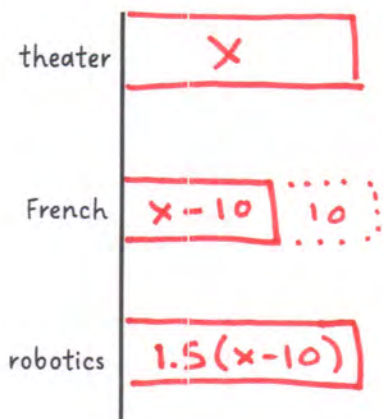
7. Eve baked (30) cupcakes on Friday.

There are x people in the theater club. There are 10 less people in the French club than in the theater club, and there are 50% more people in the robotics club than in the French club. There are 36 people in the robotics club.

8. What is known in this situation? What is unknown?

- 10 less in French than theater \rightarrow # in theater club
- 50% more in robotics than French
- 36 people in robotics

9. Draw a tape diagram to represent the people in the theater club. Then draw another tape diagram to represent the people in the French club. Then draw another tape diagram to represent the people in the robotics club.



10. Write an equation to represent the number of students in the robotics club.

$$\underbrace{1.5}_{\text{50\% more}} \left(\underbrace{x - 10}_{\text{\# of people in French club}} \right) = \underbrace{36}_{\text{\# of people in robotics club}}$$

11. Solve for x .

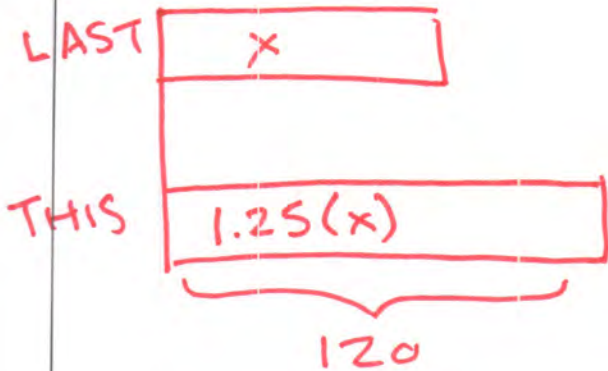
$$\begin{aligned} \frac{1.5(x - 10)}{1.5} &= \frac{36}{1.5} \\ x - 10 &= 24 \\ \underline{+10 \quad +10} & \\ x &= 34 \end{aligned}$$

$$\begin{array}{r} 24 \\ 15 \overline{) 360} \\ \underline{30} \\ 60 \\ \underline{-60} \\ 0 \end{array}$$

12. How many people are in the theater club?

34

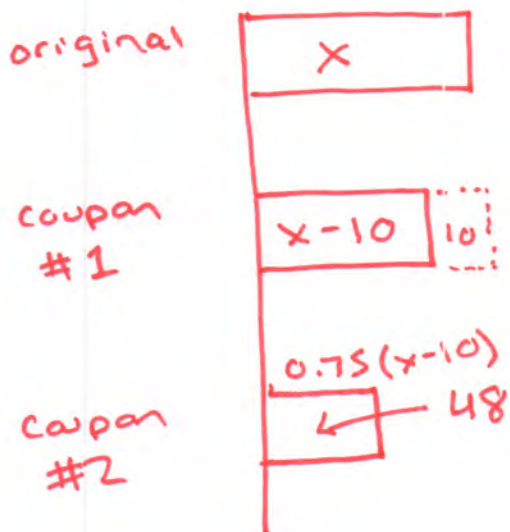
1. The number of students on-time to school increased 25% from last week to this week. The number of students on-time to school this week is 120. Draw a tape diagram and write an equation to help determine how many students were on-time to school last week.



$$\frac{1.25x = 120}{1.25 \quad 1.25}$$

$$x = 96$$

2. A clothing store is having a sale where all jeans are discounted by 25%. Kieran has a coupon for \$10 off the regular price of jeans. The store first applies the coupon and then takes 25% off the reduced price. If Kieran ends up paying \$48 for the jeans, what was the original cost of the jeans?



$$\frac{0.75(x - 10) = 48}{0.75 \quad 0.75}$$

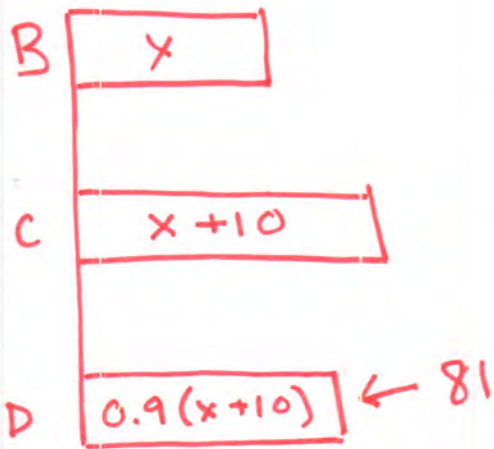
$$x - 10 = 64$$

$$+10 \quad +10$$

$$x = 74$$

$$\text{\$74}$$

3. An animal shelter takes care of 12 more cats than bunnies. There are 10% fewer dogs than cats at the shelter. There are 81 dogs in the shelter. How many bunnies are in the shelter?

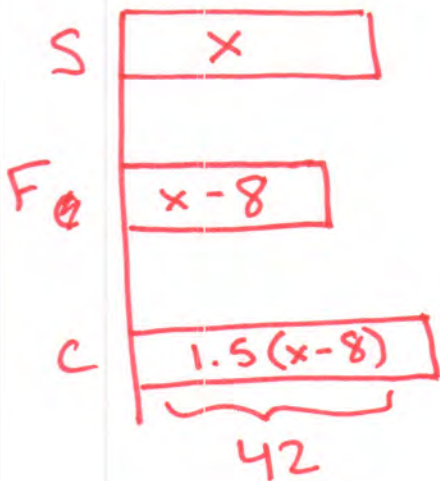


$$\frac{81}{0.9} = \frac{0.9(x+10)}{0.9}$$

$$\frac{90}{-10} = \frac{x+10}{-10}$$

$$\boxed{80 = x}$$

4. There are x students enrolled in Spanish class. There are 8 fewer students enrolled in French class than in Spanish class. There are 50% more students in Chinese class than in French class. There are 42 students enrolled in Chinese class. How many students are enrolled in Spanish?



$$1.5(x-8) = 42$$

$$\frac{1.5x - 12}{+12} = \frac{42}{+12}$$

$$\frac{1.5x}{1.5} = \frac{54}{1.5}$$

$$\boxed{x = 36}$$

G7 U5 Lesson 12

Write inequality statements to represent inequality situations, and use substitution to determine whether a given value for a variable makes an inequality true.

G7 U5 Lesson 12 - Students will write inequality statements to represent inequality situations, and use substitution to determine whether a given value for a variable makes an inequality true.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Since the beginning of this unit, we've been exploring various ways to think about and solve equations. Today, we get to use some of that thinking, but we're switching gears a bit. For the next several lessons, we'll be focusing on inequalities. An inequality is a math statement that, instead of using an equal sign, uses a less than symbol, a greater than symbol, a less than or equal to symbol, or a greater than or equal to symbol.

Let's start by thinking about what these symbols mean before we jump into today's problems.

Let's Talk (Slide 3): Here we have four different inequality statements. What do you notice about the four inequalities shown here? What do you wonder? **Possible Student Answers, Key Points:**

- I notice the red ones have x as the variable and the blue have y as the variable. I notice the red ones have the less than and greater than symbols. I notice the blue ones have the less than or equal to and the greater than or equal to symbols.
- I wonder which symbol is which, because they can be easy mix up. I wonder why we need a symbol that includes "equal to". I wonder if we can think about these the same as we think about equations.

The first red inequality reads as x is less than 5. That means, x can be any value less than 5. 4 could make the statement true. 0 could make the statement true. 2.99, $\frac{1}{2}$, -10 would all make the statement true. Do you think 5 would make the inequality true? **Possible Student Answers, Key Points:**

- No, 5 would not be a solution. 5 is not less than 5. It has the same value.

Look at the first blue inequality. It reads y is less than or equal to 10. Like the last example we saw, any value of y less than the number, in this case 10, would make the statement true. This symbol is special, because it also means that the solution could be equal to 10. So 10, and any number less than 10, are part of the solution.

What about the second red inequality? It reads x is greater than 5. What values of x would make the statement true? What values would not make the statement true? **Possible Student Answers, Key Points:**

- 6, 7, 8, $9\frac{1}{2}$, and 100 could all make the inequality true. As long as it's more than 5.
- 4, 3, 0.5, and -6 would all make the inequality false. It can't be a number less than 5.
- 5 would not be a solution, because 5 is not greater than 5.

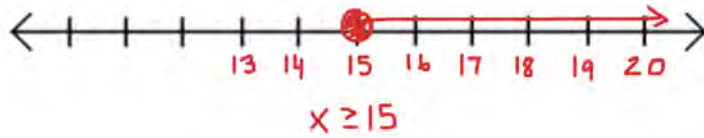
So then, let's consider the second blue inequality. It reads y is greater than or equal to 10. Would $8\frac{1}{2}$ make the statement true? (No, $8\frac{1}{2}$ is less than 10.) Would $10\frac{1}{2}$ make the statement true? (Yes, $10\frac{1}{2}$ is greater than 10.) Would 10 make the statement true? (Yes, because the inequality is true for any number greater than or equal to 10.)

It's important to pay attention to the symbols anytime we work with inequalities. If we see the \leq or the \geq , we know that the value the inequality is referring to can also be part of the solution.

Alright, I think it's time to look at some problems together.

Let's Think (Slide 4): This problem gives us two scenarios that can be represented with an inequality. They want us to write an inequality and model the solution on a number line. Let's start with Part A. What does it mean that Maryanne needs to get "at least" 15 math problems correct? (She needs 15 or more correct.) The phrase "at least" can be misleading, because it doesn't mean "less" like it sounds. It means the given number

or more. Let's think about that on a number line. (write 15 on the number line and label a few numbers above and below 15)



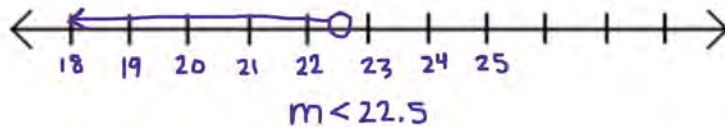
(circle, but don't shade a point on 15) I know Maryanne at least 15 questions correct, so is 15 part of the solution? (Yes.) She can get 15 or more. I'll shade in this point at 15 to note that 15 is included in our solution. (color in 15 point) So, I also know she could get 16 or 17 or 18 and still pass. She just can't get below 15. I'll model that

by drawing an arrow to the right to show that 15 and any number more than 15 could represent Maryanne's possible correct questions. (draw arrow pointing right)

I can represent this with an inequality by writing $x \geq 15$. If x is the number of questions Maryanne gets correct, I can think of the inequality as x is greater than or equal to 15.

Let's try the next one. What does it mean that Jacob has less than \$22.50 in his account? (He can have 22.49 or less.) He must have less than \$22.50. Notice in this case, \$22.50 would not make sense as a solution because \$22.50 is not less than \$22.50. Let's model this inequality.

(write 22 and 23 as tick marks and label a couple numbers above and below that) I'll mark \$22.50 between 22 and 23, since it's halfway between those values.



In this case, I'm not going to shade in the point at 22.5, because I don't want to include that in my answer. I'll leave that point open. I'll draw an arrow pointing left, since I know Jacob could have any value less than \$22.50. (draw arrow pointing left)

We can represent this as an inequality by writing $m < 22.5$ or $m < \$22.50$. My inequality shows that Jacob's amount has to be less, but not equal to, \$22.50 to represent the scenario.

When modeling an inequality, it's important to think about whether values greater or less than the given number make the situation true. It's also important to consider whether or not the given number is part of the solution or not. How did we show on our model whether the given number was included in a situation?

Possible Student Answers, Key Points:

- We shaded the point in to include it.
- We left the point open when we did not want to include it.



(write $<$ and $>$ next to an open circle, and write \leq and \geq next to a closed circle) Any time we want to model a relationship involving values less than or greater than a given number, we use an open circle to show that the value is not part of the solution. If we do want to include the value, like when we use the \leq or \geq symbols, we color in the point to note that it is included in the solution.

Let's Think (Slide 5): For our last set of problems, we are given a table that shows an inequality. We are asked to test whether 25 or -25 make the statement true. To do this, we'll substitute values into the inequality and see if the math we do produces a true or false statement.

Let's start with $100 \geq 4x$. To test if 25 is a solution, let's substitute 25 in for x . (rewrite inequality with 25 in place of x)

$$100 \geq 4(25)$$

$$100 \geq 100 \quad \checkmark$$

I know 4×25 is equal to 100. (*write $100 \geq 100$*) 100 is greater than or equal to 100, because it is equal to 100. (*put check next to inequality and write "true" in the table*) Since substituting 25 into the inequality resulted in a true statement, we know 25 is part of the solution set.

$$100 \geq 4(-25)$$

$$100 \geq -100 \quad \checkmark$$

What about -25? Substitute -25 in for x and see if -25 is part of the solution. Explain your work as you do it. (*write inequality and model work as shown, supporting as needed*) Possible Student Answers, Key Points:

- I can plug in -25 in place of x .
- I know 4×-25 is -100 because a positive times a negative results in a negative product.
- 100 is greater than or equal to -100, so -25 is a solution.

Since we ended up with an inequality that reads 100 is greater than or equal to -100, I know -25 is a solution to the inequality. (*write check next to the statement and write true in the table*) Both 25 and -25 made this inequality true.

$$25 - 30 \leq -10$$

$$-5 \leq -10 \quad \times$$

Let's do the same work with the other inequality, x minus 30 is less than or equal to -10. I'll substitute 25 in place of x and rewrite the inequality. (*rewrite inequality*) I know $25 - 30$ is -5. (*rewrite inequality*) It now reads -5 is less than or equal to -10. That's false, -5 is greater than -10. That means 25 is not a solution to this inequality. (*write an X next to the statement and write false in the table*)

$$-25 - 30 \leq -10$$

$$-55 \leq -10 \quad \checkmark$$

You test out -25. Explain your thinking. (*write inequality and model work as shown, supporting as needed*) Possible Student Answers, Key Points:

- If substitute -25 in place of x . -25 minus 30 is -55.
- -55 is less than or equal to -10. That's true. -25 makes the inequality true.

Since -25 resulted in a true statement, -25 is a solution to the inequality. All we have to do to determine if a value makes an inequality true or false is substitute the value in place of the variable.

Let's Try it (Slides 6 - 7): Now it's time to practice together. As we consider inequalities, it's important that we think carefully about the situation or the symbol. We want to consider whether values that are greater than or less than the given number make a true inequality. It's also important that we think about whether the given number is included in the solution or excluded. It can be helpful to test values out by thinking about a number line and/or substituting values into a given inequality. I think you're ready to go!

WARM WELCOME



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Today we will write inequality statements to represent inequality situations, and use substitution to determine whether a given value for a variable makes an inequality true.

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Let's Talk:

$$x < 5$$

$$y \leq 10$$

$$x > 5$$

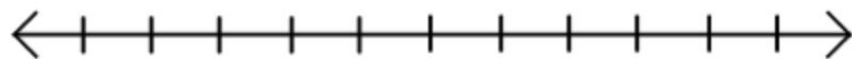
$$y \geq 10$$

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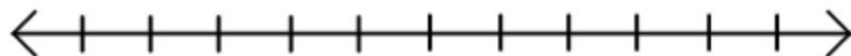
Let's Think:

**Write an inequality to represent each situation.
Model the inequality on a number line.**

- a. Maryanne needs to get at least 15 math problems correct on her exam to pass.



- b. Jacob has less than \$22.50 in his bank account.



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Let's Think:

Decide whether each value makes the inequality true.

	$x = 25$	$x = -25$
$100 \geq 4x$		
$x - 30 \leq -10$		

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Let's Try It:

Let's explore writing inequality statements to represent inequality situations and using substitution to determine whether a given value for a variable makes an inequality true together.

Name: _____ G7 US Lesson 12 - Let's Try It

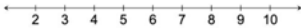
Match each phrase with the corresponding inequality symbol.

- Less than \leq
- Greater than $<$
- Less than or equal to \geq
- Greater than or equal to $>$

An office water cooler can hold up to 7 gallons of water.

- Select all the amounts that the water cooler could hold.
 - 1 1/2 gallons
 - 5 gallons
 - 7 gallons
 - 7 1/4 gallons
 - 10.5 gallons
- What does it mean if the water cooler can hold "up to" 7 gallons?


- Write an inequality to represent the situation. Use w to represent the gallons in the water cooler.

- Use a number line to represent this relationship.
 

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At an amusement park, you must be at least 48 inches tall to ride a roller coaster.

- Which description best represents the heights a roller coaster rider must be?
 - Any value less than 48
 - Any value greater than 48
 - Any value less than or equal to 48
 - Any value greater than or equal to 48
- Write an inequality to represent this situation. Let h be the height of a rider.

- Sketch a number line to represent the relationship.
 
- What would be different about this graph if the rule was riders had to be taller than 48 inches?

Consider the inequality $25 \geq x$. Substitute the given value to see if the inequality is true.

- Is the inequality true when x is 35?
- Is the inequality true when x is 12?
- Is the inequality true when x is 25?

Consider the inequality $4y < 24$. Substitute the given value to see if the inequality is true.

- Is the inequality true when $y = 0$?
- Is the inequality true when $y = 4$?
- Is the inequality true when $y = -4$?

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Match each phrase with the corresponding inequality symbol.

- | | |
|-----------------------------|--------|
| 1. Less than | \leq |
| 2. Greater than | $<$ |
| 3. Less than or equal to | \geq |
| 4. Greater than or equal to | $>$ |

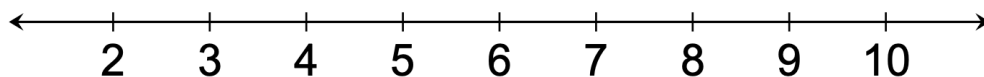
An office water cooler can hold up to 7 gallons of water.

5. Select all the amounts that the water cooler could hold.
- a. 1 $\frac{1}{2}$ gallons
 - b. 5 gallons
 - c. 7 gallons
 - d. 7 $\frac{3}{4}$ gallons
 - e. 10.5 gallons

6. What does it mean if the water cooler can hold “up to” 7 gallons?

7. Write an inequality to represent the situation. Use w to represent the gallons in the water cooler.

8. Use a number line to represent this relationship.

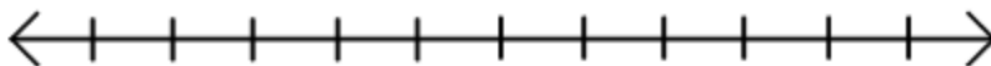


At an amusement park, you must be at least 48 inches tall to ride a roller coaster.

9. Which description best represents the heights a roller coaster rider must be?
- a. Any value less than 48
 - b. Any value greater than 48
 - c. Any value less than or equal to 48
 - d. Any value greater than or equal to 48

10. Write an inequality to represent this situation. Let h be the height of a rider.

11. Sketch a number line to represent the relationship.



12. What would be different about this graph if the rule was riders had to be taller than 48 inches?

Consider the inequality $25 \geq x$. Substitute the given value to see if the inequality is true.

13. Is the inequality true when x is 35?
14. Is the inequality true when x is 12?
15. Is the inequality true when x is 25?

Consider the inequality $4y < 24$. Substitute the given value to see if the inequality is true.

16. Is the inequality true when $y = 0$?
17. Is the inequality true when $y = 4$?
18. Is the inequality true when $y = -4$?

1. Determine whether each inequality is true or false for the given values.

	$a = -4$	$a = 4$	$a = 0$
$8 \leq 4 - a$			

	$c = 15.99$	$c = 1$	$c = 16$
$c < 16$			

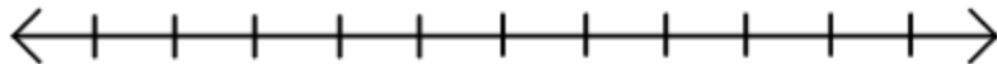
	$m = 17$	$m = -9$	$m = 9$
$2m \geq 18$			

2. A coffee mug can hold up to 16 ounces of coffee.

Which inequality best represents the situation?

- a. $c > 16$
- b. $c < 16$
- c. $c \geq 16$
- d. $c \leq 16$

Graph the inequality on a number line.



Name a value that could represent the number of ounces in the mug. _____

Name a value that could NOT represent the number of ounces in the mug. _____

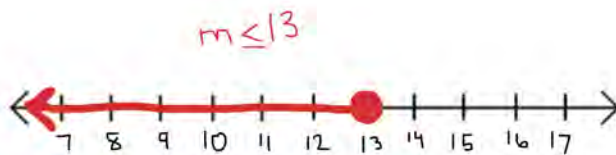
3. Write an inequality to match each situation. Represent the inequality on a number line.

a. Each classroom in a school can have up to 25 students.

b. The temperature in the refrigerator must be less than 40 degrees.

c. Customers must be at least 16 years old to have a gym membership.

4. Luke got a new electric scooter. The box said the scooter travels less than 13 miles per hour. Luke wrote the inequality below and modeled it on a number line. What mistake did Luke make? How should he correct his work?



Name: KEY

Match each phrase with the corresponding inequality symbol.

- | | | |
|-----------------------------|--------------|-----------------|
| 1. Less than | → | ≤ |
| 2. Greater than | → | < |
| 3. Less than or equal to | → | ≥ |
| 4. Greater than or equal to | → | > |

An office water cooler can hold up to 7 gallons of water.

5. Select all the amounts that the water cooler could hold.

- a. 1 ½ gallons
- b. 5 gallons
- c. 7 gallons
- d. 7 ¾ gallons
- e. 10.5 gallons

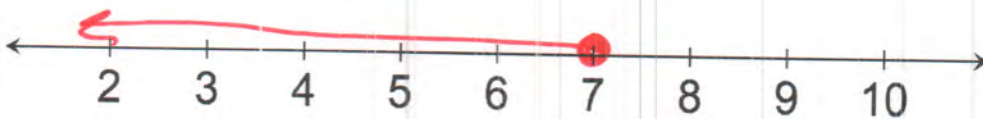
6. What does it mean if the water cooler can hold "up to" 7 gallons?

It means the most it can hold is
7 gallons.

7. Write an inequality to represent the situation. Use w to represent the gallons in the water cooler.

$$w \leq 7$$

8. Use a number line to represent this relationship.



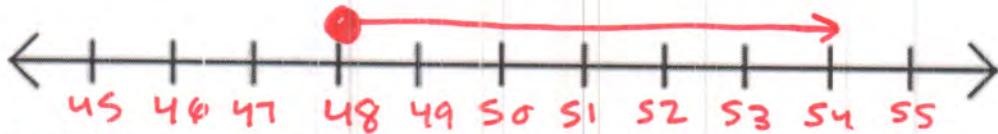
At an amusement park, you must be at least 48 inches tall to ride a roller coaster.

9. Which description best represents the heights a roller coaster rider must be?
- a. Any value less than 48
 - b. Any value greater than 48
 - c. Any value less than or equal to 48
 - d. Any value greater than or equal to 48

10. Write an inequality to represent this situation. Let h be the height of a rider.

$$h \geq 48$$

11. Sketch a number line to represent the relationship.



12. What would be different about this graph if the rule was riders had to be taller than 48 inches?

The number line would have an open circle instead of a closed one.

Consider the inequality $25 \geq x$. Substitute the given value to see if the inequality is true.

13. Is the inequality true when x is 35? $25 \geq 35$
NO

14. Is the inequality true when x is 12? $25 \geq 12$
YES

15. Is the inequality true when x is 25? $25 \geq 25$
YES

Consider the inequality $4y < 24$. Substitute the given value to see if the inequality is true.

16. Is the inequality true when $y = 0$? $4(0) < 24$
 $0 < 24$
YES

17. Is the inequality true when $y = 4$? $4(4) < 24$
 $16 < 24$
YES

18. Is the inequality true when $y = -4$? $4(-4) < 24$
 $-16 < 24$
YES

1. Determine whether each inequality is true or false for the given values.

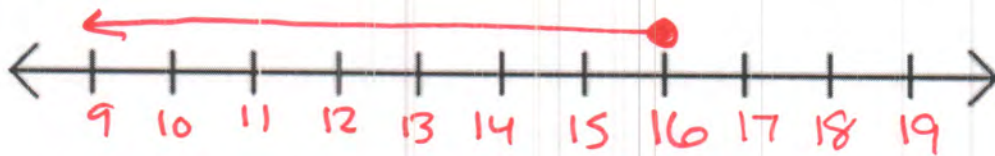
	$a = -4$	$a = 4$	$a = 0$
$8 \leq 4 - a$	TRUE $8 \leq 4 - (-4)$ $8 \leq 8$	FALSE $8 \leq 4 - 4$ $8 \leq 0$	FALSE $8 \leq 4 - 0$ $8 \leq 4$
	$c = 15.99$	$c = 1$	$c = 16$
$c < 16$	TRUE $15.99 < 16$	TRUE $1 < 16$	FALSE $16 < 16$
	$m = 17$	$m = -9$	$m = 9$
$2m \geq 18$	TRUE $34 \geq 18$	FALSE $-18 \geq 18$	TRUE $18 \geq 18$

2. A coffee mug can hold up to 16 ounces of coffee.

Which inequality best represents the situation?

- a. $c > 16$
- b. $c < 16$
- c. $c \geq 16$
- d. $c \leq 16$**

Graph the inequality on a number line.

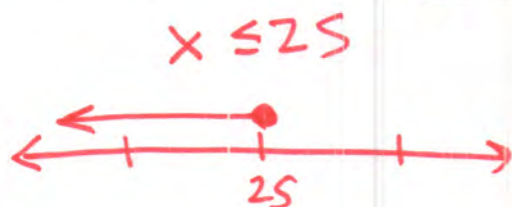


Name a value that could represent the number of ounces in the mug. (10)

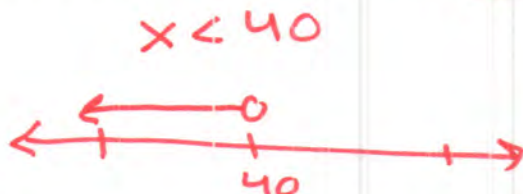
Name a value that could NOT represent the number of ounces in the mug. (20)

3. Write an inequality to match each situation. Represent the inequality on a number line.

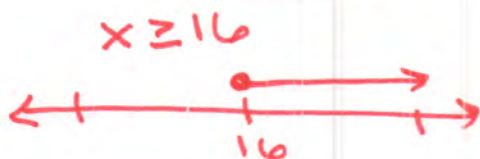
a. Each classroom in a school can have up to 25 students.



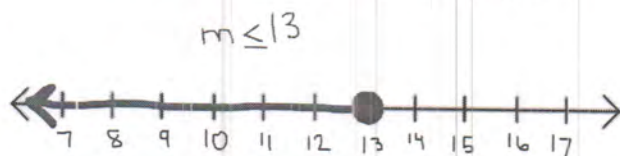
b. The temperature in the refrigerator must be less than 40 degrees.



c. Customers must be at least 16 years old to have a gym membership.



4. Luke got a new electric scooter. The box said the scooter travels less than 13 miles per hour. Luke wrote the inequality below and modeled it on a number line. What mistake did Luke make? How should he correct his work?



Luke should have used the $<$ symbol and an open circle on the number line. The box said "less than 13 miles" which does not include 13. 13 is not less than 13.

G7 U5 Lesson 13

Write inequalities that represent situations, and use substitution or reasoning about the context to find the solution.

G7 U5 Lesson 13 - Students will write inequality statements to represent inequality situations, and use substitution or reasoning about the context to find the solution.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our last lesson, we started to do some work thinking about inequalities. We modeled simple situations on a number line and wrote inequalities to match the scenario. We also substituted values into an inequality to see if the value made a true statement. We'll see some similar work today as our goal is to write and solve inequalities that represent real-world situations. Once we solve, we'll make sure we understand what our solution means in the context of the given problem. Let's begin!

Let's Talk (Slide 3): Take a look at the two problems here. What is the same? What's different? **Possible Student Answers, Key Points:**

- The problems both have an x , a 5, and a 6 in them. The problems both involve addition.
- The problems are different colors. The first one has an equal sign, and the second one has a greater than sign. The second problem includes a number line. The first is an equation, and the second is an inequality.

The first problem is an equation, because it includes an equal sign. The value of x plus 5 is equal to 6. What is the value of x ? How do you know? **Possible Student Answers, Key Points:**

- The value of x is 1.
- I know the related equation $6 - 5 = 1$. I know that $1 + 5$ is equal to 6.

The second problem is an inequality. Typically, there is only one solution to an equation. With an inequality, there are many values that could make the left side of the inequality greater than 6. Any number greater than 1 could work. $2 + 5$ is greater than 6. $3 + 5$ is greater than 6. $4 + 5$ is greater than 6. Would 1 be a solution to this problem? **Possible Student Answers, Key Points:**

- No 1 is not a solution.
- $1 + 5$ is equal to 6. $6 > 6$ is not a true statement.

As we work today, we'll see that we can solve inequalities similar to how we might solve an equation, but the way we reason about the solution is different in an inequality. The solution to an inequality can represent multiple, and sometimes unlimited solutions.

Let's Think (Slide 4): I'll read the problem aloud while you follow along. When I finish reading, I want you to summarize the gist of the story in your own words. What is known, and what is unknown? **Possible Student Answers, Key Points:**

- This problem is about Adam making gift bags for campers, and he's wondering how long it will take him to make all the bags.
- We know he's made 12 bags already. He can make 10 every hour, and he needs to make 152 in all.
- We don't know how long it will take him to finish the project.

$$\begin{array}{r} 12 + 10x = 152 \\ -12 \quad \quad -12 \\ \hline 10x = 140 \\ \frac{10x}{10} = \frac{140}{10} \\ x = 14 \end{array}$$

Let's not worry about an inequality for a moment. Let's just write an equation to represent this word problem, since that is what part A is asking for.

I can think of this as the 12 bags he completed plus 10 bags per hour, x . I know that must equal 152 so every camper gets a bag. (*write equation*) We can solve by subtracting 12 from both sides of the equation. (*subtract 12 from both sides and rewrite equation*) We're left with $10x = 140$. I know I can divide both sides by 10. Our equation's solution reads $x = 14$.

Now let's consider Part B. This part wants us to think about the same situation, but now Adam doesn't want exactly 152 bags. He wants to have some left over. What inequality symbol can we use to show that Adam wants bags left over? Explain how you know. **Possible Student Answers, Key Points:**

- We should use $>$, because he wants more than 152 bags.
- We can't use \geq , because if he made equal to 152 he would not have any leftover bags.

$$\begin{array}{r} 12 + 10x > 152 \\ -12 \quad -12 \\ \hline 10x > 140 \\ \frac{10x}{10} > \frac{140}{10} \\ x > 14 \end{array}$$

(write $12 + 10x > 152$) If we rethink this scenario so that Adam can have leftovers, we can use $12 + 10x > 152$, so that we can find values that result in more than 152 bags.

I'll start by subtracting 12 from both sides of the inequality. (show underneath inequality) Our resulting inequality is $10x > 140$. From here, I can divide both sides by 10. (divide both sides by 10) The result reads $x > 14$, or x is greater than 14.

What does $x > 14$ mean in the context of this particular problem? **Possible Student Answers, Key Points:**

- We were trying to find the number of hours it would take Adam to have leftover bags.
- The solution means that if Adam works for more than 14 hours, he'll end up with leftover bags.

If x is greater than 14, that means Adam will need to work for more than 14 hours in order to have leftover bags. Notice how solving an equation and solving an inequality felt fairly similar. We interpret our answers a bit different, but the process of solving is mostly the same. Let's see if that is true by looking at one more example.

Let's Think (Slide 5): Like last time, I'll read the problem aloud while you follow along. When I finish reading, I want you to summarize the gist of the story in your own words. What is known, and what is unknown?

Possible Student Answers, Key Points:

- This problem is about how many pairs of socks Lyric can buy if she has \$75 to buy a pair of shoes and some pairs of socks.
- We know her budget is \$75. We know the pair of shoes costs \$60 and she just buys the one pair of shoes. We know she buys 5 pairs of socks.
- We don't know how much the socks cost.

Now that we understand the scenario, we'll write an equation to represent it. I know Lyric has a budget of \$75, so her purchases will equal up to \$75. I'll write 60 to represent the shoes and $5x$ to represent the fact that she buys 5 pairs of socks at an unknown price. (write $60 + 5x = 75$)

$$\begin{array}{r} 60 + 5x = 75 \\ -60 \quad -60 \\ \hline 5x = 15 \\ \frac{5x}{5} = \frac{15}{5} \\ x = 3 \end{array}$$

We've seen several equations that look similar in previous lessons. What steps would you take to solve? I'll follow along in writing as you share. **Possible Student Answers, Key Points:**

- We can subtract 60 from both sides. That leaves us with $5x = 15$.
- We can divide both sides by 5. That means our solution is $x = 3$.

If $x = 3$, that means each pair of socks cost Lyric \$3. Job well done!

For Part B, we'll think about the same scenario. This time, it wants us to use the same information, but we need to write an inequality. We can write an inequality, because this situation doesn't necessarily mean that Lyric needs to spend her entire budget. She could spend less than her budget. In some instances, it's a good thing to be under budget. Let's try it out.

What inequality symbol can we use if we think about the fact that she doesn't necessarily have to spend her entire budget? How do you know? [Possible Student Answers, Key Points:](#)

- We can use \leq because she can spend equal to her budget or less.
- I wouldn't want to use $>$ or \geq because she probably can't go over budget.

$$\begin{array}{r} 60 + 5x \leq 75 \\ -60 \qquad -60 \\ \hline 5x \leq 15 \\ \frac{5x}{5} \leq \frac{15}{5} \\ x \leq 3 \end{array}$$

(write $60 + 5x \leq 75$) This inequality shows that the shoes and the pairs of socks can end up costing less than or equal to \$75. I'll solve in the same way we solved the equation. (write equations as you narrate)

I'll subtract 60 from both sides of the inequality. That leaves us with a new inequality of $5x \leq 15$. From here, I know I can divide by 5 on both sides. The solution we're left with is $x \leq 3$.

What does a solution of $x \leq 3$ mean in the context of this problem? If you're not sure, think about what was unknown to begin with. [Possible Student Answers, Key Points:](#)

- We did not know the cost of the pair of socks. This solution means the socks can cost less than or equal to \$3.

If the socks can cost less than or equal to \$3, what could be the cost of the socks besides \$3? (\$2.99, \$1, or any number less than 3) Could the cost of the socks be -\$1? (No, that doesn't make sense in real life.) -1 is technically less than 3, but some values within the solution set make more sense than others in context. We'll see more of that in future lessons.

Great work solving equation and inequalities that represent real-world scenarios. We were not only able to solve, but we were also able to talk about what our answers mean in context.

Let's Try it (Slides 6 - 7): Now let's do a few more together, before you get a chance to show what you know independently. As we saw, we can think of the steps of solving an inequality as similar to the steps of solving an equation. We'll work carefully to isolate the variable one step at a time. An important aspect of solving an inequality is that the solution usually includes many values, so we have to make sure we have a clear sense of what is happening in the story so we can accurately interpret the solution to an inequality. Let's give these next few problems a try. I know you're going to do great!

WARM WELCOME



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Today we will write inequalities that represent situations, and use substitution or reasoning about the context to find the solution.

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 Let's Talk:


What's the same? What's different?

$$x + 5 = 6$$

$$x + 5 > 6$$



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 Let's Think:

Adam is making gift bags for new campers coming to summer camp. He has already made 12 bags, and he can make 10 bags every hour. If there are 152 campers, how many hours will it take Adam to make 152 bags?

- Write and solve an equation to show how many hours it will take Adam to complete making exactly 152 bags.
- Write and solve an inequality to show how many hours it will take Adam to complete making 152 bags, if he wants to have some extras leftover.

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Let's Think:

Lyric has a \$75 budget to buy shoes and socks. She needs 5 pairs of socks and 1 pair of shoes. The shoes she wants cost \$60. How much does Lyric have left to spend on each pair of socks?

- Write an equation to find exactly how much Lyric can spend on each pair of socks.
- Write an inequality to represent the situation, and name at least 2 other amounts Lyric could pay for each pair of socks.

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Let's Try It:

Let's explore writing inequalities that represent situations and using substitution or reasoning about the context to find the solution together.

Name: _____ G7 US Lesson 13 - Let's Try It

Alicia has \$20 in her bank account. She deposits \$9 every week. She wants to buy a new jacket that costs \$74.

- What expression can be used to represent the fact that Alicia earns 9 dollars every week, w ?
 - $9 = w$
 - $9w$
 - $9 - w$
 - $9 = w$
- Write an expression to show that Alicia starts with 20 dollars and deposits 9 dollars every week.
- Write an equation to show the exact amount of money Alicia will need to have in order to buy the jacket.
- Solve the equation to determine how many weeks it will take Alicia to earn exactly enough money to buy the jacket.
- Suppose Alicia wants to have extra money in her account after buying the jacket. Fill in the blank with an inequality symbol to represent this scenario:

$$20 + 9w \text{ _____ } 74$$
- Solve the inequality.
- What does the solution to the inequality mean in the context of this problem?

Omari is running laps for a charity race. Omari's uncle says he will donate \$50 just for running the race, and \$15 for every lap Omari runs.

- Write an equation to show how many laps, x , Omari needs to run in order to raise exactly \$170.
- Solve the equation. How many laps does Omari need to run to raise exactly \$170?
- Suppose Omari decides he wants to raise at least \$170. What does that mean?
- Write and solve an inequality to show how many laps Omari needs to run to raise at least \$170.

Consider the inequality $4h + 21 \leq 41$.

- Replace the inequality symbol with an equal sign and solve the equation.
- The value you found is called the **boundary point**. Is this boundary point a solution to this inequality?
- Choose a value less than your boundary point. Test to see if it is a solution to the inequality.
- Choose a value greater than the boundary point to see if it is a solution to the inequality.
- Write a solution statement for this inequality.

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On your Own:

Now it's time to write inequalities that represent situations and use substitution or reasoning about the context to find the solution on your own.

Name: _____ G7 US Lesson 13 – Independent Work

1. Which values make the inequality true? Select all that apply.

$$6x + 20 < 38$$

a. $x = -1$
 b. $x = 0$
 c. $x = 2$
 d. $x = 3$
 e. $x = 3\frac{1}{2}$
 f. $x = 6$

If there were any values you did not select, explain why.

2. Moriah has a balance of \$120 in her bank account. She wants to have at least \$300 in the account 5 weeks from now. She used the inequality shown here to find how much she needs to save each week to meet her goal.

$$5m + 120 \geq 300$$

a. What does $5m$ represent?

b. Find at least three values for m that could work for Moriah.

c. Write an inequality to represent the answer to Moriah's question.

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3. At a school carnival, students are allowed to be on the bumper cars for up to 12 minutes.

a. Write an inequality to represent this situation.

b. Name at least 2 values that make the inequality true, and name at least 2 values that make the inequality false.

c. Represent the inequality on a number line.

4. Zachary delivers pizzas for his family's restaurant. His dad gives him \$25 each day, plus \$3 for every pizza he delivers. Zachary hopes to make at least enough money today to buy a video game for \$56. Write and solve an inequality to represent this scenario. Use x to represent every pizza delivered.

If Zachary delivers 12 pizzas today, will he have enough to buy the video game?

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Alicia has \$20 in her bank account. She deposits \$9 every week. She wants to buy a new jacket that costs \$74.

1. What expression can be used to represent the fact that Alicia earns 9 dollars every week, w ?
 - a. $9 + w$
 - b. $9w$
 - c. $9 - w$
 - d. $9 \div w$
2. Write an expression to show that Alicia starts with 20 dollars and deposits 9 dollars every week.
3. Write an equation to show the exact amount of money Alicia will need to have in order to buy the jacket.
4. Solve the equation to determine how many weeks it will take Alicia to earn exactly enough money to buy the jacket.
5. Suppose Alicia wants to have extra money in her account after buying the jacket. Fill in the blank with an inequality symbol to represent this scenario.

$$20 + 9w \text{ ____ } 74$$

6. Solve the inequality.
7. What does the solution to the inequality mean in the context of this problem?

Omari is running laps for a charity race. Omari's uncle says he will donate \$50 just for running the race, and \$15 for every lap Omari runs.

8. Write an equation to show how many laps, x , Omari needs to run in order to raise exactly \$170.

9. Solve the equation. How many laps does Omari need to run to raise exactly \$170?

10. Suppose Omari decides he wants to raise *at least* \$170. What does that mean?

11. Write and solve an inequality to show how many laps Omari needs to run to raise at least \$170.

Consider the inequality $4h + 21 \leq 41$.

12. Replace the inequality symbol with an equal sign and solve the equation.

13. The value you found is called the boundary point. Is this boundary point a solution to this inequality?

14. Choose a value less than your boundary point. Test to see if it is a solution to the inequality.

15. Choose a value greater than the boundary point to see if it is a solution to the inequality.

16. Write a solution statement for this inequality.

1. Which values make the inequality true? Select all that apply.

$$6x + 20 < 38$$

- a. $x = -1$
- b. $x = 0$
- c. $x = 2$
- d. $x = 3$
- e. $x = 3 \frac{1}{2}$
- f. $x = 6$

If there were any values you did *not* select, explain why.

2. Moriah has a balance of \$120 in her bank account. She wants to have at least \$300 in the account 5 weeks from now. She used the inequality shown here to find how much she needs to save each week to meet her goal.

$$5m + 120 \geq 300$$

- a. What does $5m$ represent?
- b. Find at least three values for m that could work for Moriah.
- c. Write an inequality to represent the answer to Moriah's question.

3. At a school carnival, students are allowed to be on the bumper cars for up to 12 minutes.

a. Write an inequality to represent this situation.

b. Name at least 2 values that make the inequality true, and name at least 2 values that make the inequality false.

c. Represent the inequality on a number line.

4. Zachary delivers pizzas for his family's restaurant. His dad gives him \$25 each day, plus \$3 for every pizza he delivers. Zachary hopes to make at least enough money today to buy a video game for \$58. Write and solve an inequality to represent this scenario. Use x to represent every pizza delivered.

If Zachary delivers 12 pizzas today, will he have enough to buy the video game?

Alicia has \$20 in her bank account. She deposits \$9 every week. She wants to buy a new jacket that costs \$74.

1. What expression can be used to represent the fact that Alicia earns 9 dollars every week, w ?

- a. $9 + w$
 b. $9w$
 c. $9 - w$
 d. $9 \div w$

$9 \times$ # of weeks

2. Write an expression to show that Alicia starts with 20 dollars and deposits 9 dollars every week.

$$20 + 9x \text{ or } 20 + 9w$$

3. Write an equation to show the exact amount of money Alicia will need to have in order to buy the jacket.

$$20 + 9w = 74$$

4. Solve the equation to determine how many weeks it will take Alicia to earn exactly enough money to buy the jacket.

$$\begin{array}{r} 20 + 9w = 74 \\ -20 \quad -20 \\ \hline 9w = 54 \\ \frac{9}{9} \quad \frac{54}{9} \\ \hline w = 6 \end{array}$$

5. Suppose Alicia wants to have extra money in her account after buying the jacket. Fill in the blank with an inequality symbol to represent this scenario.

$$20 + 9w \overset{>}{\neq} 74$$

6. Solve the inequality.

$$\begin{array}{r} 20 + 9w \overset{>}{\neq} 74 \\ -20 \quad -20 \\ \hline 9w \overset{>}{\neq} 54 \\ \hline w \overset{>}{\neq} 6 \end{array}$$

7. What does the solution to the inequality mean in the context of this problem?

If she wants left over money, she'll need to save for more than 6 weeks.

Omari is running laps for a charity race. Omari's uncle says he will donate \$50 just for running the race, and \$15 for every lap Omari runs.

8. Write an equation to show how many laps, x , Omari needs to run in order to raise exactly \$170.

$$50 + 15x = 170$$

9. Solve the equation. How many laps does Omari need to run to raise exactly \$170?

$$\begin{array}{r} 50 + 15x = 170 \\ -50 \quad -50 \\ \hline 15x = 120 \\ \frac{15x}{15} = \frac{120}{15} \end{array} \quad (x = 8)$$

10. Suppose Omari decides he wants to raise *at least* \$170. What does that mean?

He wants to raise 170 or more.

11. Write and solve an inequality to show how many laps Omari needs to run to raise at least \$170.

$$\begin{array}{r} 50 + 15x \geq 170 \\ 15x \geq 120 \\ (x \geq 8) \end{array}$$

Consider the inequality $4h + 21 \leq 41$.

12. Replace the inequality symbol with an equal sign and solve the equation.

$$\begin{array}{r} 4h + 21 = 41 \\ -21 \quad -21 \\ \hline 4h = 20 \\ \frac{4h}{4} = \frac{20}{4} \\ (h = 5) \end{array}$$

13. The value you found is called the boundary point. Is this boundary point a solution to this inequality?

It is, because $4(5) + 21 \leq 41$ is true.

14. Choose a value less than your boundary point. Test to see if it is a solution to the inequality.

$$\begin{array}{r} 4(0) + 21 \leq 41 \\ 21 \leq 41 \quad \checkmark \end{array}$$

15. Choose a value greater than the boundary point to see if it is a solution to the inequality.

$$\begin{array}{r} 4(10) + 21 \leq 41 \\ 40 + 21 \leq 41 \\ 61 \leq 41 \quad \times \end{array}$$

16. Write a solution statement for this inequality. $(h \leq 5)$

1. Which values make the inequality true? Select all that apply.

$$6x + 20 < 38$$

a. $x = -1$

b. $x = 0$

c. $x = 2$

d. $x = 3$

e. $x = 3\frac{1}{2}$

f. $x = 6$

Ⓐ $-6 + 20 < 38$
 $14 < 38$

Ⓒ $12 + 20 < 38$
 $32 < 38$

Ⓔ $21 + 21 < 38$
 $42 < 38$

Ⓑ $0 + 20 < 38$
 $20 < 38$

Ⓓ $18 + 20 < 38$
 $38 < 38$

Ⓕ $36 + 20 < 38$
 $56 < 38$

If there were any values you did *not* select, explain why.

When I substituted 3, $3\frac{1}{2}$, and 6 in for x , it did not end up making a true inequality statement.

2. Moriah has a balance of \$120 in her bank account. She wants to have at least \$300 in the account 5 weeks from now. She used the inequality shown here to find how much she needs to save each week to meet her goal.

$$5m + 120 \geq 300$$

- a. What does $5m$ represent?

The amount earned in 5 weeks

- b. Find at least three values for m that could work for Moriah.

100, 200, 300

- c. Write an inequality to represent the answer to Moriah's question.

$$5m \geq 180$$

$$m \geq 36$$

3. At a school carnival, students are allowed to be on the bumper cars for up to 12 minutes.

a. Write an inequality to represent this situation.

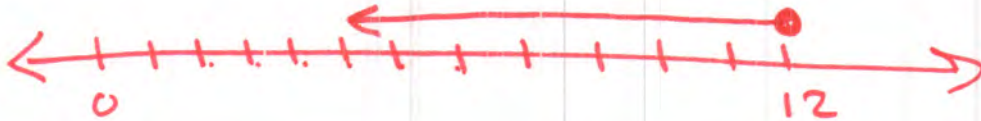
$$x \leq 12$$

b. Name at least 2 values that make the inequality true, and name at least 2 values that make the inequality false.

$$\begin{array}{l} \text{TRUE} \\ 10 = x \\ 1 = x \end{array}$$

$$\begin{array}{l} \text{FALSE} \\ 13 = x \\ 100 = x \end{array}$$

c. Represent the inequality on a number line.



4. Zachary delivers pizzas for his family's restaurant. His dad gives him \$25 each day, plus \$3 for every pizza he delivers. Zachary hopes to make at least enough money today to buy a video game for \$58. Write and solve an inequality to represent this scenario. Use x to represent every pizza delivered.

$$\begin{array}{r} 25 + 3x \geq 58 \\ -25 \quad \quad -25 \\ \hline 3x \geq 33 \\ \textcircled{x \geq 11} \end{array}$$

If Zachary delivers 12 pizzas today, will he have enough to buy the video game?

Yes. $25 + 3(12) = 25 + 36 = 61.$

He will have \$61 which is more than enough.

G7 U5 Lesson 14

Solve inequalities using the associated equation and testing values to determine the direction of the inequality in the solution.

G7 U5 Lesson 14 - Students will solve inequalities using the associated equation and testing values to determine the direction of the inequality in the solution.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): This whole unit we've been working on solving equations and inequalities. Today, we'll keep our focus on solving inequalities, but we'll see how we can use associated equations to help us consider solutions to inequalities. Before we look at some problems, what do you already know is the same and different about equations compared to inequalities? **Possible Student Answers, Key Points:**

- Equations and inequalities are both types of math problems. Equations and inequalities can both involve unknowns. We can solve both.
- An equation uses an equal sign, while an inequality uses a symbol like $<$, $>$, \leq , or \geq . An equation typically has one solution while an inequality can have many solutions.

Let's use what we've learned about equations and inequalities to help us think through the next several problems.

Let's Talk (Slide 3): Here we see an inequality that reads x plus 1 is greater than 9. I know that means whatever number I substitute in for x , the total of x and 1 has to be more than 9. Let's test out a few values for practice. Consider each of the possible solutions, 0, 8, and 10. Use paper or mental math to determine whether each value is a solution to the inequality. Explain how you know. *(write each inequality with the value substituted in for x)* **Possible Student Answers, Key Points:**

$$\begin{array}{l} 0 + 1 > 9 \quad \times \\ 8 + 1 > 9 \quad \times \\ 10 + 1 > 9 \quad \checkmark \end{array}$$

- 0 is not a solution. $0 + 1 = 1$, and 1 is not greater than 9.
- 8 is not a solution. $8 + 1 = 9$, and 9 is not greater than 9.
- 10 is not a solution. $10 + 1 = 11$, and 11 is greater than 9.

A number is a solution to an inequality if it makes the inequality a true statement when substituted in place of the unknown. We'll need this skill to help us with the problems we're going to look at today. Let's get going.

Let's Think (Slide 4): These two problems show inequalities involving addition and subtraction. We'll solve them using associated equations, and then use the number line to model the solutions. Let's start by looking at the blue inequality.

$$\begin{array}{r} 2 + x = -1 \\ -2 \quad -2 \\ \hline x = -3 \end{array}$$

The inequality reads 2 plus x is greater than 1. I'm going to rewrite the inequality as an equation, solve, and then we'll make sense of our solution using the inequality symbol. *(write $2 + x = -1$)* This is the associated equation. Notice, I just switched the inequality symbol for an equal sign for the moment. How can I solve this equation? *(subtract 2 from both sides of the equation)* *(show that, and write $x = -3$)* The solution to the associated equation is called our boundary point.

$$\begin{array}{l} 2 + (-4) > -1 \\ -2 > -1 \quad \times \end{array}$$

We'll test out a value less than our boundary point and a value greater than our boundary point to determine which inequality symbol makes most sense in our solution. I know -4 is less than -3 , so let's test out -4 in our original inequality. *(substitute -4 in place of x and solve as you narrate)* I know $2 + (-4)$ equals -2 . $-2 > -1$ is a false statement, because -2 is not greater than -1 .

Let's do the same thing, but test out a value greater than our boundary point. I know 0 is an easy number to work with that is greater than our boundary point of -3 . Let's substitute 0 in for x , and see if it makes our

$$2 + (0) > -1$$

$$2 > -1 \checkmark$$

inequality true. (substitute 0 in for x and solve as you narrate) I know $2 + 0$ equals 2. $2 > -1$ is a true statement, because 2 is greater than 1.

$$x > -3$$

Since the value less than our boundary point made a false statement, and the value greater than our boundary point made a true statement, I can say that the value of x is any number *greater than* -3. (write $x > -3$)



(sketch on number line as you narrate) I can model this on a number line by circling -3 and drawing an arrow to the values greater than 3. I won't color in the point at -3, because I don't want to include that as a solution. -3 is not greater than -3

$$y - 3 = 0$$

$$\begin{array}{r} +3 \\ +3 \\ \hline y = 3 \end{array}$$

$$y \geq 3$$

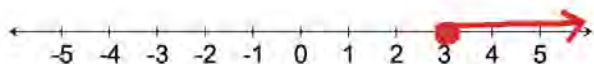
Let's look at the other inequality. This one reads y minus 3 is greater than or equal to 0. I'll start by writing the associated equation. (write $y - 3 = 0$) How can I solve this equation? What is the solution? Possible Student Answers, Key Points:

- We can add 3 on both sides of the equal sign to keep it balanced.
- I know the solution is $y = 3$.

The solution to the equation is $y = 3$. That's our boundary point. To determine the inequality symbol we should use, we'll test out an easy value less than the boundary point and an easy value greater than the boundary point. I like to choose values that are easy for me to operate with. Let's choose 0 for a value less than our boundary point and 5 for the value greater than our boundary point. Does 0 or 5 make our inequality true? How do you know? Possible Student Answers, Key Points:

- If I substitute 0 in for y in the inequality, I end up with $-3 \geq 0$. That is false.
- If I substitute 5 in for y in the inequality, I end up with $2 \geq 0$. That is true.

Since the value greater than the boundary point made the inequality true, I can write the solution to the inequality as $y \geq 3$. (write $y \geq 3$ underneath equation)



We can model this on a number line by marking a point at 3, and drawing an arrow to the values greater than 3. In this case, I'll color in the point at 3 because 3 is included in our solution $3 \text{ is } \geq 3$ because 3 is equal to 3.

We just solved inequalities by using an associated equation to find the boundary point. Once we knew the boundary point, we tested values greater than and less than that point to determine the inequality symbol that can be used to represent the solution. We also graphed our solution set on a number line like we've seen previously. Let's try two more. We'll use the same thinking, but the inequalities look a bit different.

Let's Think (Slide 5): Let's solve two inequalities involving multiplication using the same approach.

$$5v = -10$$

$$v = -2$$

$$v < -2$$

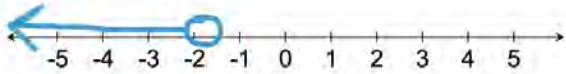
What associated equation can I write for the first inequality? What is the solution? (solve as shown while student shares, supporting as needed) Possible Student Answers, Key Points:

- We can write $5v = -10$ instead of $5v < -10$.
- If I divide both sides by 5, I get $v = -2$ as the solution. That's my boundary point.

If we solve the associated equation, we see the boundary point is -2. Let's choose a simple value less than -2 and a simple value greater than -2 to determine the symbol that completes the inequality correctly.

Let's use -10 and 10, since those are fairly easy numbers to think about. We'll substitute each value in for v to see if they make a true statement. Is -10 a solution to the inequality? Is 10 a solution to the inequality? How do you know? **Possible Student Answers, Key Points:**

- -10 is a solution, because $5(-10)$ is -50. -50 is less than -10.
- 10 is not a solution, because $5(10)$ is 50. 50 is not less than -10.



Since the value less than our boundary made the inequality true, we can say that our solution is $v < -2$. (*sketch on number line as you narrate*) I can model that on a number line by circling the point at -2 and drawing an arrow to the values less than -2. Why am I not going to color in the circle at the -2?

Possible Student Answers, Key Points:

- We don't color it in, because -2 is not part of the solution. $-2 < -2$ is not a true statement.

$$\frac{-5v}{-5} = \frac{-10}{-5}$$

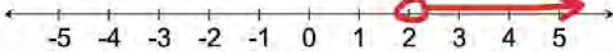
$$v = 2$$

$$v > 2$$

Let's think about our final inequality. The associated equation for $-5v < -10$ is $-5v = -10$. (*write and solve*) I know $v = 2$, because -10 divided by -5 is positive 2. The boundary point in this problem is 2.

Let's use 0 as a value less than the boundary point and 4 as a value greater than the boundary point, since both of those values are pretty friendly to compute with. Which value makes a true statement? **Possible Student Answers, Key Points:**

- I know $-5(0)$ is 0. The inequality $0 < -10$ is not true.
- I know $-5(4)$ is -20. The inequality $-20 < -10$ is true. The value of 4 makes a true statement.



Since the value greater than our boundary point makes a true statement, the solution to the inequality can be written as $v > 2$. How can I model the solution set using a number line? (*sketch as student shares*) **Possible Student Answers, Key Points:**

- Mark a point at positive 2, but don't shade it in. 2 is not part of the solution, it's just the boundary.
- Draw an arrow to the right to represent the values greater than 2.

If you look back at the four inequalities we just solved, you'll notice that the inequality symbol used in the original problem sometimes matches the inequality used in the solution and sometimes does not. We always want to carefully consider values above and below the boundary point before placing an inequality symbol in the solution.

Let's Try it (Slides 6 - 7): Now let's try a few more on our own. We've seen that we can solve an associated equation to help us think about the solution to any inequality. Once we've found our boundary point, we can test values that are above or below the boundary point to determine which inequality symbol to use in our answer. As always, when dealing with inequalities, we want to think about whether the boundary point is or is not included in the solution set. A number line can help us represent all the values in our solution set. We'll try a few more together, and then you'll get a chance to show what you've learned independently.

WARM WELCOME



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Today we will solve inequalities using the associated equation and test values to determine the direction of the inequality in the solution.

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 **Let's Talk:**


$$x + 1 > 9$$

Is 0 a solution?

Is 8 a solution?

Is 10 a solution?

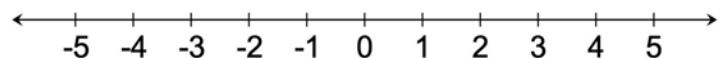
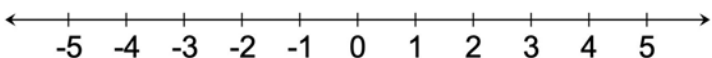
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 **Let's Think:**

Solve each inequality. Use a number line to represent all possible solutions.

$$2 + x > -1$$

$$y - 3 \geq 0$$



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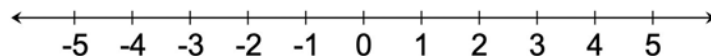
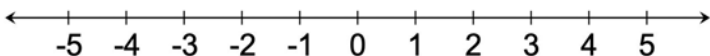


Let's Think:

Solve each inequality. Use a number line to represent all possible solutions.

$$5v < -10$$

$$-5v < -10$$



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Let's Try It:

Let's explore solving inequalities using the associated equation and testing values to determine the direction of the inequality in the solution together.

Name: _____ G7 US Lesson 14 - Let's Try It

Consider the inequality $3a < 12$.

- Which best describes the inequality?
 - 3 times a is greater than 8
 - 3 times a is less than 8
 - 3 times a is less than or equal to 8.
- Solve the associated equation, $3a = 12$. Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.

- Complete the table to determine whether values less than the boundary (ex. $a = 1$) or values greater than the boundary (ex. $a = 5$) make the inequality true.

	$a = 1$	$a = 4$ (boundary)	$a = 5$
$-3a + 12$			
- Write the inequality that represents every possible solution to $3a < 12$.
- Use the number line to represent all possible solutions to the inequality.

Consider the inequality $-3a < 12$.

- Solve the associated equation, $-3a = 12$. Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.
 - Yes
 - No

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- Complete the table to determine whether values less than the boundary or values greater than the boundary make the inequality true.

	$a = -5$	$a = -4$ (boundary)	$a = 0$
$-3a + 12$			
- Write the inequality that represents every possible solution to $-3a < 12$.
- Sketch a number line to represent all possible solutions to the inequality.

Consider the inequality $h - 5 \geq -9$.

- Solve the associated equation. What is your boundary point? Is it a solution to the inequality?
- Test a value less than the boundary and greater than the value.
- Write the inequality that represents every possible solution to $h - 5 \geq -9$.
- Sketch a number line to represent all possible solutions to the inequality.

Consider the inequality $-3x \leq 18$.

- Write the inequality that represents the solution. Sketch a number line to represent all possible solutions.

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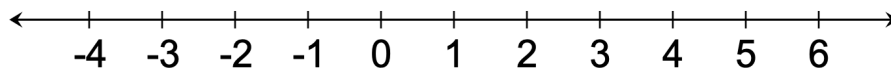
Consider the inequality $3a < 12$.

- Which best describes the inequality?
 - 3 times a is greater than 8
 - 3 times a is less than 8
 - 3 times a is less than or equal to 8.
- Solve the associated equation, $3a = 12$. Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.

- Complete the table to determine whether values less than the boundary (ex. $a = 1$) or values greater than the boundary (ex. $a = 5$) make the inequality true.

	$a = 1$	$a = 4$ (boundary)	$a = 5$
$3a < 12$			

- Write the inequality that represents every possible solution to $3a < 12$.
- Use the number line to represent all possible solutions to the inequality.

**Consider the inequality $-3a < 12$.**

- Solve the associated equation, $-3a = 12$. Your solution is the boundary point.
- Is your solution to the associated equation also a solution to the inequality? Explain.
 - Yes
 - No

9. Complete the table to determine whether values less than the boundary or values greater than the boundary make the inequality true.

	$a = -5$	$a = -4$ (boundary)	$a = 0$
$-3a < 12$			

10. Write the inequality that represents every possible solution to $-3a < 12$.

11. Sketch a number line to represent all possible solutions to the inequality.

Consider the inequality $h - 5 \geq -9$.

12. Solve the associated equation. What is your boundary point? Is it a solution to the inequality?

13. Test a value less than the boundary and greater than the boundary.

14. Write the inequality that represents every possible solution to $h - 5 \geq -9$.

15. Sketch a number line to represent all possible solutions to the inequality.

Consider the inequality $-3x \leq 18$.

16. Write the inequality that represents the solution. Sketch a number line to represent all possible solutions.

1.

a. Graph the solutions to $9m \leq 54$ on the number line. Show all work.



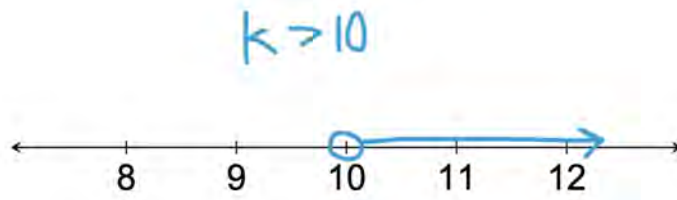
b. Graph the solutions to $-9m \leq 54$ on the number line. Show all work.



2. Find the solution to $n + 8 > -8$. Write the solution as an inequality.

Sketch a number line to represent all possible solutions to the inequality.

3. Jackie was trying to find the solution to $-7k > -70$. Her solution and number line are shown below.



Explain why Jackie's solution is unreasonable. Include the correct answer in your response.

4. **Look at each inequality. Solve each.** List at least 2 values that make each inequality true and at least 2 values that make each inequality false.

$$y + 4 < -6$$

$$-2w \geq 20$$

Consider the inequality $3a < 12$.

- Which best describes the inequality?
 - 3 times a is ~~greater~~ than 8
 - 3 times a is less than 8**
 - 3 times a is less than or equal to 8.
- Solve the associated equation, $3a = 12$. Your solution is the boundary point.

$$\frac{3a}{3} = \frac{12}{3}$$

$$a = 4$$
- Is your solution to the associated equation also a solution to the inequality? Explain.

No. $3(4) = 12$ and $12 < 12$ is not a true statement.

- Complete the table to determine whether values less than the boundary (ex. $a = 1$) or values greater than the boundary (ex. $a = 5$) make the inequality true.

$3(1) < 12$
 $3 < 12$

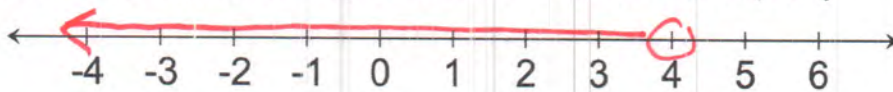
	$a = 1$	$a = 4$ (boundary)	$a = 5$
$3a < 12$	✓	X	X

$3(5) < 12$
 $15 < 12$

- Write the inequality that represents every possible solution to $3a < 12$.

$a < 4$

- Use the number line to represent all possible solutions to the inequality.



Consider the inequality $-3a < 12$.

- Solve the associated equation, $-3a = 12$. Your solution is the boundary point.

$$\frac{-3a}{-3} = \frac{12}{-3}$$

$$a = -4$$
- Is your solution to the associated equation also a solution to the inequality? Explain.

- Yes
- No**

$-3(-4) < 12$
 $12 < 12 \leftarrow \text{false}$

9. Complete the table to determine whether values less than the boundary or values greater than the boundary make the inequality true.

$$\begin{aligned} -3(-5) &< 12 \\ 15 &< 12 \end{aligned}$$

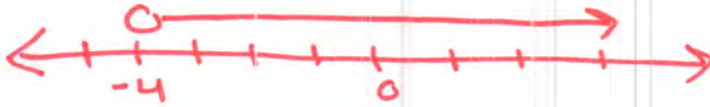
	$a = -5$	$a = -4$ (boundary)	$a = 0$
$-3a < 12$	X	X	✓

$$\begin{aligned} -3(0) &< 12 \\ 0 &< 12 \end{aligned}$$

10. Write the inequality that represents every possible solution to $-3a < 12$.

$$a > -4$$

11. Sketch a number line to represent all possible solutions to the inequality.



Consider the inequality $h - 5 \geq -9$.

12. Solve the associated equation. What is your boundary point? Is it a solution to the inequality?

$$\begin{array}{r} h - 5 = -9 \\ +5 \quad +5 \\ \hline h = -4 \end{array}$$

$$h = -4$$

Yes, it is.

$$\begin{aligned} -4 - 5 &\geq -9 \\ -9 &\geq -9 \end{aligned}$$

13. Test a value less than the boundary and greater than the value.

$$\begin{aligned} -5 - 5 &\geq -9 \\ -10 &\geq -10 \quad \text{X} \end{aligned}$$

$$\begin{aligned} 0 - 5 &\geq -9 \\ -5 &\geq -9 \quad \checkmark \end{aligned}$$

14. Write the inequality that represents every possible solution to $h - 5 \geq -9$.

$$h \geq -4$$

15. Sketch a number line to represent all possible solutions to the inequality.



Consider the inequality $-3x \leq 18$.

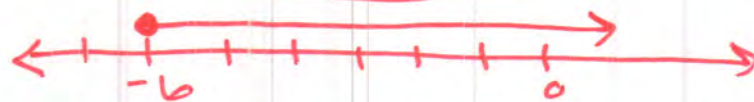
16. Write the inequality that represents the solution. Sketch a number line to represent all possible solutions.

$$\begin{array}{r} -3x \leq 18 \\ \frac{-3x}{-3} \leq \frac{18}{-3} \\ x \geq -6 \end{array}$$

$$\begin{aligned} -3(-10) &\leq 18 \\ 30 &\leq 18 \quad \text{X} \end{aligned}$$

$$\begin{aligned} -3(0) &\leq 18 \\ 0 &\leq 18 \quad \checkmark \end{aligned}$$

$$x \geq -6$$



1.

a. Graph the solutions to $9m \leq 54$ on the number line. Show all work.

$$\frac{9m}{9} = \frac{54}{9}$$

$$m = 6 \checkmark$$

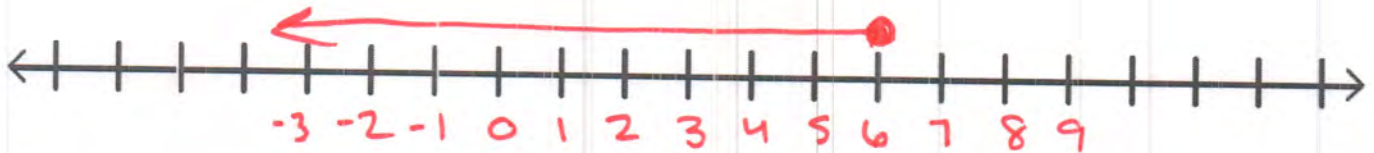
$$9(0) \leq 54 \checkmark$$

$$0 \leq 54$$

$$9(10) \leq 54 \text{ X}$$

$$90 \leq 54$$

$$m \leq 6$$



b. Graph the solutions to $-9m \leq 54$ on the number line. Show all work.

$$\frac{-9m}{-9} = \frac{54}{-9}$$

$$m = -6 \checkmark$$

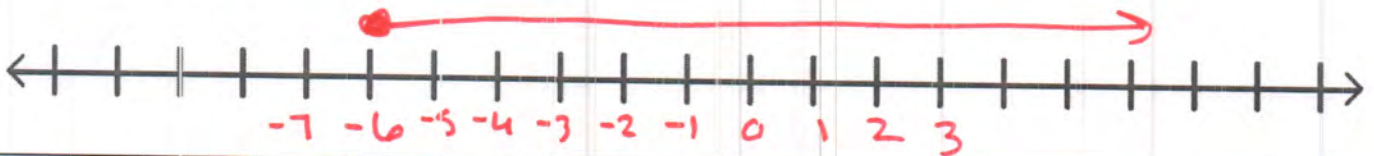
$$-9(-10) \leq 54$$

$$90 \leq 54 \text{ X}$$

$$-9(0) \leq 54$$

$$0 \leq 54 \checkmark$$

$$m \geq -6$$



2. Find the solution to $n + 8 > -8$. Write the solution as an inequality.

$$\begin{array}{r} n + 8 = 8 \\ -8 \quad -8 \\ \hline n = -16 \text{ X} \end{array}$$

$$-20 + 8 > -8$$

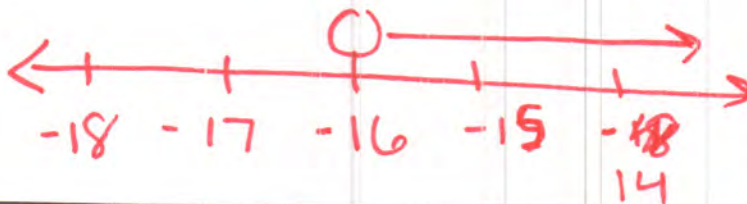
$$-12 > -8 \text{ X}$$

$$0 + 8 > -8$$

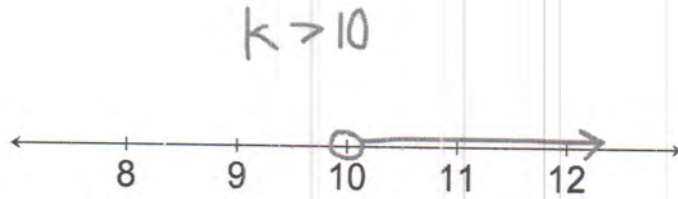
$$8 > -8 \checkmark$$

$$n > -16$$

Sketch a number line to represent all possible solutions to the inequality.



3. Jackie was trying to find the solution to $-7k > -70$. Her solution and number line are shown below.



Explain why Jackie's solution is unreasonable. Include the correct answer in your response.

If I substitute in 11 or 12, the **FALSE** inequality is not true. $-7(11) > -70 \rightarrow -77 > -70$
 The correct answer $-7(12) > -70 \rightarrow -84 > -70$ is $k < 10$.

4. Look at each inequality. Solve each. List at least 2 values that make each inequality true and at least 2 values that make each inequality false.

$$y + 4 < -6$$

$$y < -10$$

TRUE

$$y = -100$$

$$y = -11$$

FALSE

$$y = 10$$

$$y = 0$$

$$-2w \geq 20$$

$$w \leq -10$$

$$w = -10$$

$$w = -20$$

$$w = 0$$

$$w = 4$$

G7 U5 Lesson 15

Match an inequality to a situation it represents, explain what the parts of the inequality mean, solve it, and then interpret what the solution means in the situation.

G7 U5 Lesson 15 - Students will match an inequality to a situation it represents, explain what the parts of the inequality mean, solve it, and then interpret what the solution means in the situation.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our previous lesson, we worked on solving inequalities by using the associated equation. For instance, if the inequality was $x + 1 < 9$, the associated equation would just be $x + 1 = 9$. (*write both inequality and equation, and solve as you narrate*) We could subtract 1 from both sides to see that $x = 8$ is our boundary point. How did we use the boundary point to help us think about the solution to the inequality? **Possible Student Answers, Key Points:**

$$\begin{array}{r} x + 1 < 9 \\ x + 1 = 9 \\ \underline{-1 \quad -1} \\ x = 8 \end{array}$$

- The boundary point is the point we would start at on a number line model.
- We tested values above and below the boundary to know which inequality symbol best represents the solution.

We used the boundary point to test values greater than and less than the boundary point to determine the inequality symbol to use in the solution. If we mentally substitute 0 as a value less than the boundary point, I know $0 + 1$ is less than 9. If I mentally substitute 10 as a value greater than the boundary point, I know $10 + 1$ is *not less than* 9. That means the solution to the inequality would be $x < 8$, not $x > 8$.

Today, we'll use a lot of that same thinking, but we'll apply it to inequalities that represent real-world situations. As we work, we'll make sure to connect the math we're doing back to the context of the problem.

Let's Talk (Slide 3): Speaking of contexts, here is an example of a context we might see in inequality problems. Read it to yourself as I read it aloud. Then summarize what this scenario is about in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This situation is about the number of cakes a bakery sells on Saturday and Sunday.
- I know they sold 3 more cakes on Sunday than Saturday. I know they sold more than 12 cakes Sunday.
- I don't know how many cakes they sold on Saturday.

Thinking about the information in the story, take a second a look at the four inequalities we see here. Which one best matches the story? How do you know? If you're not 100% sure, are there any you can eliminate? **Possible Student Answers, Key Points:**

- I know the bakery sold greater than 12 cakes, so I know it won't be the blue or orange inequalities. Those symbols mean less than or less than and equal to.
- I think it's the green inequality, $c + 3 > 12$. The "c" represents the cakes on Saturday, the "+ 3" represents the 3 addition cakes made on Sunday, and the > 12 shows that the value is more than 12.

The inequality $c + 3 > 12$ best represents the information in this story. The value of $c + 3$ must be greater than 12. The inequality $c + 3 \geq 12$ doesn't make sense, since the story says they sold greater than, not equal to, 12 cakes on Sunday.

In today's problems, we'll read each problem carefully to write an inequality that matches the context. Then we'll use the associated equation to help us solve the inequality. Let's get going with our first of two problems.

Let's Think (Slide 4): Read this problem to yourself as I read it aloud. Then summarize what it's about in your own words. What is known? What is unknown? **Possible Student Answers, Key Points:**

- This situation is about Kevin earning money from his daily wages and from commission from selling shoes.

- I know he earned more than \$250. I know he earned \$90 in wages. I know he sold 10 pairs of shoes and earns commission on each pair he sells.
- I don't know how many pairs of shoes he could have sold.

Based on the information in this story, we can write an inequality. I know he earns \$90. He also earns commission money from each pair of shoes he sells, and he sold 10 pairs. I can represent the amount he earns from commission with the expression $10x$. I know the total amount he makes is greater than \$250.

$$90 + 10x > 250$$

(write $90 + 10x > 250$) This expression represents each part of the story. We see his wages plus the commission must total greater than 250 dollars.

$$\begin{array}{r} 90 + 10x = 250 \\ -90 \quad -90 \\ \hline \end{array}$$

Let's solve. What's the associated equation we can solve? ($90 + 10x = 250$)

$$\frac{10x}{10} = \frac{160}{10}$$

(write it and solve as you narrate) The associated equation is easy to write, because we can just swap the inequality symbol temporarily for an equal sign. I know in this equation I can subtract 90 from both sides. That leaves me with $10x = 160$. I can divide both sides by 10, and I see that $x = 16$. This value is our boundary point.

$$x = 16$$

Let's substitute a value less than 16 and a value more than 16 to determine which inequality symbol to use in our answer. I'll pick 0 and 20, since those are easy to work with. You can choose whichever numbers you like, as long as one is less than the boundary point and one is greater than the boundary point.

$$90 + 10(0) > 250$$

$$90 + 10(20) > 250 \checkmark$$

(rewrite the inequality substituting in 0 for x , and rewrite the inequality substituting 20 in for x) Use mental math or scratch paper to determine which value makes the inequality true. Possible Student Answers, Key Points:

- 0 does not work, because $10(0)$ is 0. $90 + 0$ is not greater than 250.
- 20 does work, because $10(20)$ is 200. $90 + 200$ is greater than 250.

$$x > 16$$

Since the value greater than our boundary point made the inequality true, we can write the solution to this problem as $x > 16$. (write it) Think back to the word problem. What does a solution of $x > 16$ mean in context? Possible Student Answers, Key Points:

- We didn't know how much Kevin could earn from each pair of shoes he sells. A solution of $x > 16$ means that Kevin earns more than \$16 off of each pair of shoes sold.

Kevin earns greater than \$16 for each pair of shoes he sells. Based on the inequality, he could earn \$16.01, \$17, \$25, or any number greater than \$16. Nice work! Let's try another problem with a different context.

Let's Think (Slide 5): Read this second problem to yourself as I read it aloud. Then summarize what it's about in your own words. What is known? What is unknown? Possible Student Answers, Key Points:

- This situation is about how long it takes George to hike from one elevation to another elevation.
- I know his starting elevation is 21 feet. I know his elevation decreases 3 feet every minute. I know the trail ends at an elevation of -27 feet and that he hasn't quite made it there yet.
- I don't know how many minutes he hikes.

$$21 - 3m > -27$$

We can write an inequality to represent this situation. (write $21 - 3m > -27$ as you narrate what each component means) He starts at +21 feet. He descends 3 feet per minute, which I can show as $-3m$. The problem says he hasn't reached the final elevation of -27 feet, so I'll write that he is *greater* than -27 feet since his elevation would be higher up.

$$\begin{array}{r}
 21 - 3m = -27 \\
 \underline{-21 \quad -21} \\
 -3m = -48 \\
 \underline{-3 \quad -3} \\
 m = 16
 \end{array}$$

Let's use an associated equation to help us solve the inequality. I can write $21 - 3m = -27$ as the related equation. (*write and solve as you narrate*) I'll subtract 21 from both sides to isolate the variable. What is -27 minus 21? (-48) The updated equation now reads $-3m = -48$. What can I do next to isolate the variable? (*divide both sides of the equation by -3*) The solution to the associated equation is $m = 16$.

16 is the boundary point for this problem. Let's test any number less than 16 and any number greater than 16 to determine the solution to the inequality. Let's use 0 and 20 again, since those are easy numbers to work with that fit the criteria.

(*rewrite the inequality substituting in 0 for m , and rewrite the inequality substituting 20 in for m*) Use mental math or scratch paper to determine which value makes the inequality true.

$$\begin{array}{l}
 21 - 3(0) > -27 \quad \checkmark \\
 21 - 3(20) > -27
 \end{array}$$

Possible Student Answers, Key Points:

- 0 works, because $3(0)$ is 0. 21 minus 0 is greater than -27 .
- 20 does NOT work, because $3(20)$ is 60. 21 minus 60 is -39 which is not greater than -27 .

$$m < 16$$

Since the value less than the boundary point makes the inequality true, I know the solution to the inequality is $m < 16$. (*write $m < 16$*)

What does this solution mean in the context of the problem? What values would make the inequality true?

Possible Student Answers, Key Points:

- The inequality $m < 16$ means that George hikes for less than 16 minutes.
- He could have hiked for 15 minutes, or 10 minutes, or 1 minute. Any value less than 16 makes sense for this problem.

Nice work!

Let's Try it (Slides 6 - 7): We just used what we know about a word problem to write an inequality that matches the story. We then used an associated equation to help us find the boundary point. Lastly, we used values less than and greater than the boundary point to test the inequality so we could pick the correct symbol for our final answer. Now let's do a few more together, before you have the opportunity to work independently.

WARM WELCOME



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Today we will match an inequality to a situation it represents, explain what the parts of the inequality mean, and then interpret what the solution means in the situation.

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Let's Talk:

A bakery sold some cakes on Saturday. They sold 3 more cakes on Sunday. They sold greater than 12 cakes on Sunday.

$$c + 3 > 12$$

$$c + 3 \geq 12$$

$$c + 3 < 12$$

$$c + 3 \leq 12$$

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Let's Think:

Kevin earned more than \$250 yesterday working at a shoe store. He made \$90 in wages, plus p dollars in commission for each of the 10 pairs of shoes he helped sell. How much could Kevin have earned for each pair of shoes?

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Let's Think:

George begins hiking on a trail at an elevation of 21 feet. He descends 3 feet per minute, m . He has not yet reached the trail's end which is at an elevation of -27 feet. How many minutes could George have been hiking so far?

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Let's Try It:

Let's explore matching an inequality to a situation it represents, explaining what the parts of the inequality mean, and interpreting what the solution means together.

Name: _____ G7 US Lesson 15 - Let's Try It

A wedding reception can be arranged with one big table that has 15 seats surrounded by several smaller tables that each have 6 seats. If the room for reception can hold at most 81 people, how many smaller tables can be arranged in the room?

- What is known in this problem? What is unknown?
- Which represents a number of people that could attend a reception in this room?
 - 100
 - 85
 - 75
- Write an inequality to represent the situation.

$\frac{15}{\# \text{ of large table}}$	+	$\frac{6}{\# \text{ of small tables}}$	=	$\frac{81}{\text{maximum occupancy of room}}$
--	---	--	---	---
- Solve the associated equation to find the boundary.
- Is your solution to the associated equation a solution to this inequality?

- Test one value below your boundary and one value above your boundary to determine which makes the inequality true. Then write the solution to the inequality.
- Describe what the solution means in context.

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The temperature outside starts at 8 degrees Fahrenheit. It decreases 4 degrees every hour, h . If it has not reached -32 degrees yet, how many hours could have passed?

- What is known in this problem? Unknown?
- Write an inequality to represent the number of hours that could have passed.
- Solve the associated equation.
- Test one value below your boundary and one value above your boundary to determine which makes the inequality true. Then write the solution to the inequality.
- Describe what the solution means in context.

Nakiyah uses a reloadable gift card to help her keep track of her coffee spending. There is \$75 dollars on the card currently, and she likes to keep at least \$20 on the card at all times. If each coffee costs \$3, how many coffees can Nakiyah buy and still have at least \$20 on the card?

- Write and solve an inequality to represent this situation.
- Describe what the solution means in context.

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A wedding reception can be arranged with one big table that has 15 seats surrounded by several smaller tables that each have 6 seats. If the room for reception can hold at most 81 people, how many smaller tables can be arranged in the room?

1. What is known in this problem? What is unknown?
2. Which represents a number of people that could attend a reception in this room?
 - a. 100
 - b. 85
 - c. 75

3. Write an inequality to represent the situation.

$$\frac{\text{\# at large}}{\text{table}} + \frac{\text{\# at small}}{\text{tables}} \leq \frac{\text{maximum}}{\text{occupancy}} \frac{\text{of room}}{\text{of room}}$$

4. Solve the associated equation to find the boundary.

5. Is your solution to the associated equation a solution to this inequality?

6. Test one value *below* your boundary and one value *above* your boundary to determine which makes the inequality true. Then write the solution to the inequality.

7. Describe what the solution means in context.

The temperature outside starts at 8 degrees Fahrenheit. It decreases 4 degrees every hour, h . If it has not reached -32 degrees yet, how many hours could have passed?

8. What is known in this problem? Unknown?

9. Write an inequality to represent the number of hours that could have passed.

10. Solve the associated equation.

11. Test one value *below* your boundary and one value *above* your boundary to determine which makes the inequality true. Then write the solution to the inequality.

12. Describe what the solution means in context.

Nakiyah uses a reloadable gift card to help her keep track of her coffee spending. There is \$75 dollars on the card currently, and she likes to keep at least \$20 on the card at all times. If each coffee costs \$3, how many coffees can Nakiyah buy and still have at least \$20 on the card?

13. Write and solve an inequality to represent this situation.

14. Describe what the solution means in context.

- 1. Heaven had \$30 in her savings account. She deposits \$12 every week into the account, and now she has more than 114 dollars in the account.** How many weeks, w , could Heaven have been depositing money?

Could Heaven have been depositing money for 7 weeks? Explain.

- 2. A local post office can only ship packages that weigh no more than 45 pounds. Sasha is packing a box to ship a birthday gift and some books to her mother. The birthday gift weighs 8 pounds, and each book weighs 2 pounds.** Write and solve an inequality to determine how many books Sasha can pack in the box.

What does your solution mean in the context of this problem?

3. A vending machine contains 250 items. Every hour, h , 20 items are sold. How many hours could have passed if there are fewer than 90 items left in the machine? Write and solve an inequality to represent the situation.

4. Write a word problem that could be represented by the inequality below. Then solve and explain what your solution means in your chosen context.

$$8 + 1.5n > 50$$

A wedding reception can be arranged with one big table that has 15 seats surrounded by several smaller tables that each have 6 seats. If the room for reception can hold at most 81 people, how many smaller tables can be arranged in the room?

1. What is known in this problem? What is unknown?

- total most be 81 or less
- large table = 15 seats
- small tables = 6 seats

of small tables that will fit

2. Which represents a number of people that could attend a reception in this room?

- a. 100
- b. 85
- c. 75

3. Write an inequality to represent the situation.

$$\frac{15}{\# \text{ at large table}} + \frac{6x}{\# \text{ at small tables}} \leq \frac{81}{\text{maximum occupancy of room}}$$

4. Solve the associated equation to find the boundary.

$$15 + 6x = 81$$

$$\underline{-15 \quad -15}$$

$$\frac{6x}{6} = \frac{66}{6}$$

$$x = 11$$

5. Is your solution to the associated equation a solution to this inequality?

Yes, the room could fit 11

small tables.

6. Test one value *below* your boundary and one value *above* your boundary to determine which makes the inequality true. Then write the solution to the inequality.

$$15 + 6(1) \leq 81$$

$$15 + 6(12) \leq 81$$

$$15 + 6 \leq 81$$

$$15 + 72 \leq 81$$

$$21 \leq 81 \checkmark$$

$$87 \leq 81 \times$$

$$x \leq 11$$

7. Describe what the solution means in context.

The room can handle 11 or fewer

small tables.

The temperature outside starts at 8 degrees Fahrenheit. It decreases 4 degrees every hour, h. If it has not reached -32 degrees yet, how many hours could have passed?

8. What is known in this problem? Unknown?

- starts at 8°
 - decreases 4° each hour
 - not yet -32°
- # of hours passed

9. Write an inequality to represent the number of hours that could have passed.

$$8 - 4h > -32$$

10. Solve the associated equation.

$$\begin{array}{r} 8 - 4h = -32 \\ -8 \quad -8 \\ \hline -4h = -40 \end{array}$$

$$\begin{array}{r} -4h = -40 \\ \frac{-4h}{-4} = \frac{-40}{-4} \\ \hline h = 10 \end{array}$$

11. Test one value below your boundary and one value above your boundary to determine which makes the inequality true. Then write the solution to the inequality.

$$\begin{array}{l} 8 - 4(1) > -32 \\ 8 - 4 > -32 \\ 4 > -32 \checkmark \end{array}$$

$$\begin{array}{l} 8 - 4(11) > -32 \\ 8 - 44 > -32 \\ -36 > -32 \text{ X} \end{array}$$

$$h < 10$$

12. Describe what the solution means in context.

Less than 10 hours have passed if it's not -32° yet.

Nakiyah uses a reloadable gift card to help her keep track of her coffee spending. There is \$75 dollars on the card currently, and she likes to keep at least \$20 on the card at all times. If each coffee costs \$3, how many coffees can Nakiyah buy and still have at least \$20 on the card?

13. Write and solve an inequality to represent this situation.

$$75 - 3x \geq 20$$

$$75 - 3(0) \geq 20$$

$$75 - 3(20) \geq 20$$

$$75 - 3x \geq 20$$

$$75 - 0 \geq 20 \checkmark$$

$$75 - 60 \geq 20$$

$$-3x = -55$$

$$x = 18\frac{1}{3}$$

$$x \leq 18\frac{1}{3}$$

$$15 \geq 20 \text{ X}$$

14. Describe what the solution means in context.

Nakiyah can buy less than or equal to $18\frac{1}{3}$ cups to keep 20 or more dollars on the card.

1. Heaven had \$30 in her savings account. She deposits \$12 every week into the account, and now she has more than 114 dollars in the account. How many weeks, w , could Heaven have been depositing money?

$$30 + 12w > 114$$

$$30 + 12w = 114$$

$$12w = 84$$

$$w = 7 \text{ X}$$

$$30 + 12(0) > 114$$

$$30 + 0 > 114 \text{ X}$$

$$30 + 12(10) > 114$$

$$30 + 120 > 114$$

$$150 > 114 \checkmark$$

$$w > 7$$

Could Heaven have been depositing money for 7 weeks? Explain.

No $7 > 7$ is not a true statement.

7 is not in the solution set.

2. A local post office can only ship packages that weigh no more than 45 pounds. Sasha is packing a box to ship a birthday gift and some books to her mother. The birthday gift weighs 8 pounds, and each book weighs 2 pounds. Write and solve an inequality to determine how many books Sasha can pack in the box.

$$8 + 2x \leq 45$$

$$8 + 2x = 45$$

$$2x = 37$$

$$x = 18\frac{1}{2} \checkmark$$

$$8 + 2(1) \leq 45$$

$$8 + 2 \leq 45 \checkmark$$

$$8 + 2(20) \leq 45$$

$$8 + 40 \leq 45 \text{ X}$$

$$x \leq 18\frac{1}{2}$$

What does your solution mean in the context of this problem?

Sasha can ship less than or equal to $18\frac{1}{2}$

books, and meet the weight requirements.

3. A vending machine contains 250 items. Every hour, h , 20 items are sold. How many hours could have passed if there are fewer than 90 items left in the machine? Write and solve an inequality to represent the situation.

$$250 - 20h < 90$$

$$250 - 20h = 90$$

$$-20h = -160$$

$$h = 8 \quad \times$$

$$250 - 20(1) < 90$$

$$250 - 20 < 90 \quad \times$$

$$250 - 20(10) < 90$$

$$250 - 200 < 90 \quad \checkmark$$

$$h > 8$$

4. Write a word problem that could be represented by the inequality below. Then solve and explain what your solution means in your chosen context.

$$8 + 1.5n = 50$$

$$1.5n = 42$$

$$n = 28 \quad \times$$

$$8 + 1.5n > 50$$

$$8 + 1.5(10) > 50$$

$$8 + 15 > 50 \quad \times$$

$$8 + 1.5(100) > 50$$

$$8 + 150 > 50 \quad \checkmark$$

$$n > 28$$

I earned \$1.50 each hour I babysit my brother. If I already have \$8 saved, how many hours must I babysit to have more than 50 dollars?

G7 U5 Lesson 16

Write and solve an inequality to solve real-world problems and critique the solution to an inequality.

G7 U5 Lesson 16 - Students will write and solve inequality to solve real-world problems and critique the solution to an inequality.

NOTE: It is permissible for students to use a calculator on this lesson to perform some computation.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today is going to feel like a continuation of the work we did in our previous lesson. If you remember, we took a word problem and wrote a corresponding inequality based on what was known and unknown. Once we had the inequality written, we solved an associated equation to find the boundary point. After testing out values less than and greater than the boundary point, we were able to write the solution with the correct inequality symbol.

The work we do today will be similar, but I'm going to challenge use to think extra carefully about the solution in context of the world around us. For instance, if I have a story about the number of dogs at a dog park, and my solution is $d > 15$...would 16.5 make sense as a possible solution? Would 1,000,000 make sense as a possible solution? **Possible Student Answers, Key Points:**

- 16.5 wouldn't make sense, because you can't realistically have 16.5 dogs.
- 1,000,000 wouldn't make sense. It's greater than 15, so it is in the solution set for this inequality, but I highly doubt a dog park would have 1,000,000 dogs in it.

Today we'll work to reason carefully about the solutions to inequalities using what we know about the world around us.

Let's Talk (Slide 3): Before we look at today's problems let's briefly consider this story. How do you know that the inequality matches the story? **Possible Student Answers, Key Points:**

- The 3 represents the 3 kids on the playground to begin with. The k represents the unknown number of kids that joined them. The " < 8 " represents the fact that there are less than 8 kids on the playground now.

I can imagine solving this using an associated equation, and seeing that the boundary point is $k = 5$. I know the solution to the inequality would be $k < 5$. In math, we know $k < 5$ means any value less than 5 makes the inequality true. Sometimes, because of the context, there are limits to what actually make sense in real life.

What numbers less than 5 could actually make sense in this case? What numbers less than 5 would NOT make sense in this case? **Possible Student Answers, Key Points:**

- 1, 2, 3, or 4 could make sense, because they could represent the number of kids that joined.
- 0 wouldn't make sense, because it says some kids joined.
- $4\frac{1}{2}$ or 1.5 wouldn't make sense because you can't have a fraction of a kid.
- -10 wouldn't make sense, because a negative number of kids doesn't exist.

As we work today, let's make sure we think about the meaning of our inequality in the real-world. We'll reason about which potential solutions make sense when tied to the context and which maybe don't make as much sense, even though they're mathematically correct.

Let's Think (Slide 4): Here is the first example we'll work through together. I'll read the problem out loud while you follow along. Then, as usual, I'll ask you to summarize the story in your own words. **Possible Student Answers, Key Points:**

- This problem is about Drake spending money on things for his dog.
- We know he buys dog food for \$15. We know he buys 4 dog toys. We know he spends less than \$36.
- We don't know how much each dog toy costs.

$$15 + 4d < 36$$

$$\begin{array}{r} 15 + 4d = 36 \\ -15 \quad -15 \\ \hline 4d = 21 \\ \frac{4d}{4} = \frac{21}{4} \\ d = 5.25 \end{array}$$

Let's start by writing an inequality that matches this information. (*write inequality as you name each component*) I know he spends \$15 on dog food. In addition, he buys 4 dog toys that cost an unknown amount. I can show that with the expression $4d$. I know the total of these items comes out to be less than 36, so I'll write < 36 .

What is the associated equation for the inequality? ($15 + 4d = 36$) Let's solve it to find the boundary point. (*write equation and solve as you narrate*) I'll subtract 15 from both sides of the equal sign. We end up with $4d = 21$. If I divide both sides by 4, I know $d = 5.25$ or $5 \frac{1}{4}$. I'll leave it as 5.25 since this problem is about money.

The boundary point is 5.25. Let's test out an easy value less than 5.25 and greater than 5.25 by substituting each value into the original inequality we wrote. I'll use 0 and 6, since 0 is less than 5.25 and 6 is greater than 5.25. Like you've seen me do before, I like to pick numbers that I find are easy to calculate with.

$$15 + 4(0) < 36 \quad \checkmark$$

$$15 + 4(6) < 36$$

than 36.

(*write the inequality twice, substituting in the stated values*) Use scratch paper or mental math to determine which value makes the inequality true. How do you know? **Possible Student Answers, Key Points:**

- 0 makes the inequality true. 4 times 0 is 0, and $15 + 0$ is less than 36.
- 6 does not make the inequality true. 4 times 6 is 24, and $15 + 24$ is not less

Since the value less than the boundary point made the original inequality true, I know the solution is $d < 5.25$. (*write $d < 5.25$*) What does that mean in the context of the problem? **Possible Student Answers, Key Points:**

- We were trying to find the cost for each dog toy. The inequality means the cost of each dog toy must be less than \$5.25.

$$d < 5.25$$

The mathematical meaning of this answer means that any value less than 5.25 is a solution. Can you think of some values that are less than 5.25 that could make sense? Are there values less than 5.25 that do NOT make sense in real life? **Possible Student Answers, Key Points:**

- Any price lower than \$5.25 could make sense. Each toy could cost \$4.99. Each toy could cost \$1. There are a lot of possibilities.
- A price of \$0 might not make sense, because I'm not sure how the store would make any money. A negative number also wouldn't make sense, because the problem is asking about money values.

Great work! Sometimes how we think about an inequality answer mathematically is a little different than how we might apply the inequality in real life. Let's do another.

Let's Think (Slide 5): Here is our next problem. I'll read it out loud while you follow along. Then, as usual, I'll ask you to summarize the story in your own words. **Possible Student Answers, Key Points:**

- This problem is about the temperature dropping at a constant rate.
- We know the temperature starts at 105 degrees. We know the temperature drops 8 degrees every hour. We know the temperature should be above 80 degrees.
- We don't know how long it will take the temperature to drop.

$$105 - 8h > 80$$

Let's write an inequality to represent the information in the story. (*write inequality as you talk through each component*) The temperature starts at 105. It drops 8

degrees every hour, I can show that by subtracting 8h. The temperature ends up being above 80 degrees, so I'll write > 80 to finish the inequality.

$$\begin{array}{r} 105 - 8h = 80 \\ -105 \quad -105 \\ \hline -8h = -25 \\ -8 \quad -8 \\ \hline h = 3\frac{1}{8} \end{array}$$

We can now solve this by thinking about an associated equation. The associated equation is $105 - 8h = 80$. Talk me through how you would solve this, and I'll write out what you say. (solve equation as student shares, supporting as needed) [Possible Student Answers, Key Points:](#)

- We can subtract 105 from both sides first. 80 minus 105 is -25. Rewrite the equation as $-8h = -25$.
- I can divide both sides by -8. -25 divided by -8 is 3.125 or $3\frac{1}{8}$.

The solution to the associated equation is 3.125 or $3\frac{1}{8}$. That's the boundary point. Before we write the answer to the inequality with a symbol, let's test a value less than and a value greater than the boundary point so we know which sign to use in the answer. Let's use 0 and 5, since those are easy to work with.

(write the inequality twice, substituting in the stated values) Use scratch paper or mental math to determine which value makes the inequality true. How do you know? [Possible Student Answers, Key Points:](#)

$$\begin{array}{l} 105 - 8(0) > 80 \checkmark \\ 105 - 8(5) > 80 \end{array}$$

- 0 makes the inequality true. 8 times 0 is 0, and 105 - 0 is greater than 80.
- 5 does not make the inequality true. 8 times 5 is 40, and 105 minus 40 is not greater than 80.

$$h < 3\frac{1}{8}$$

The solution to the inequality is $h < 3\frac{1}{8}$ or $h < 3.125$. (write solution) We know this means that our solution can be any number less than $3\frac{1}{8}$. Thinking about the problem's context, can you think of any possible solutions that wouldn't make sense in reality? [Possible Student Answers, Key Points:](#)

- A negative number would be confusing to think about, because our unknown was the number of hours that have gone by. An answer like -10 is technically in a solution, but doesn't actually make much sense when I think about the story.

You did a fantastic job helping me write and solve the inequalities. In addition to the solving work, we also challenged ourselves to think critically about the meaning of possible solutions as they relate to the limitations of the world around us.

Let's Try it (Slides 6 - 7): Now let's try out a few more similar problems together. With each problem, it will be important to carefully think about what is known and unknown, so that we can write an accurate inequality. We'll solve the inequality by using an associated equation and thinking about the boundary point. In some cases, we'll also have to think about whether the mathematical solution has any limits based on the context of the story. Sometimes an answer is mathematically correct, but does not make sense in the real-world. When we're done with these next few examples, you'll get a chance to try some independently.

WARM WELCOME



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Today we will write and solve an inequality to solve real-world problems and critique the solution to an inequality.

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Let's Talk:

3 kids were on the playground. Some more kids showed up. Now, there are less than 8 kids on the playground.

$$3 + k < 8$$

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Let's Think:

Drake buys dog food for \$15 and 4 dog toys for d dollars each. He spent less than \$36. Write and solve an inequality to represent the cost of each dog toy.

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Let's Think:

The temperature in Phoenix, Arizona this afternoon was 105 degrees. The temperature decreased at a rate of 8 degrees per hour. How many hours could have passed if the temperature is above 80 degrees?

Does -2 make sense as a solution in this context? Does 0 make sense as a solution?

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Let's Try It:

Let's explore writing and solving an inequality to solve real-world problems and critiquing the solution to an inequality together.

Name: _____ G7 US Lesson 16 - Let's Try It

A freight elevator can hold a maximum of 2,000 pounds. A delivery person weighs 160 pounds. Each package he loads onto the elevator weighs 23 pounds. How many packages can the delivery person load onto the elevator with him safely?

- What is known in this problem?
- What is unknown in this problem?
- Complete the inequality to represent this problem. $\frac{\text{weight of delivery person}}{\text{weight of package}} + \frac{\text{weight of packages}}{\text{weight of package}} \leq \frac{\text{maximum weight for elevator}}{\text{weight of package}}$
- Solve the inequality.
- Represent the solution on a number line.
- What does your solution mean in the context of this problem?

- Does 70 $\frac{1}{2}$ make sense as a possible solution to this problem? Explain.

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Mr. Robinson has a budget of \$128 to spend on supplies for a class celebration. Every cupcake costs \$4 and he wants to have at least \$50 left over for decorations. How many cupcakes can Mr. Robinson order?

- What is known in this problem? Unknown?
- Write an inequality to represent this problem.
- Solve the inequality.
- Sketch a number line to represent the solution set.
- Does -10 make sense as a solution in this problem? Explain.

Rachel has 65 inches of trim that she wants to use to form the border of a rectangular piece of artwork. She does not necessarily need to use all the trim. If she wants the artwork to be 7 inches tall, what widths can she choose for the piece?

- Write and solve an inequality to represent the situation.
- Does 0 make sense as a solution in this problem? Explain.

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On your Own:

Now it's time to write and solve an inequality to solve real-world problems and critique the solution to an inequality on your own.

Name: _____ G7 US Lesson 16 - Independent Work

1. Solve each inequality. Sketch a number line to model each inequality's solution.

a. $-6n \geq -60$

b. $-6n < -60$

c. $6n > -60$

2. Mrs. Dzija is making ice cubes. The water in the ice cube trays is originally 72 degrees Fahrenheit. In the freezer, the water cools at a rate of 7 degrees per hour. She takes the ice cube tray out when the temperature is below 30 degrees. How many hours could Mrs. Dzija have had the ice cube trays in the freezer? Write and solve an inequality.

Does 6 hours make sense as a solution to this problem? Explain.

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3. A company charges \$60 to rent a motorcycle per day in addition to a one-time fee of \$25 for insurance. How many days can Vance rent a motorcycle if he wants to spend no more than \$385?

Does -1 make sense as a solution to this problem? Explain.

4. Beatriz wants to know how many episodes of her favorite TV show she can download onto her tablet. The tablet can store 64 gigabytes of data, but 29 GB are already being used by other applications. Each TV episode is 2 GB. Beatriz used the inequality $29 + 2x \geq 64$ to determine how many episodes she can download.

a. Explain why her inequality will not work to solve the problem.

b. Correct her work.

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A freight elevator can hold a maximum of 2,000 pounds. A delivery person weighs 160 pounds. Each package he loads onto the elevator weighs 23 pounds. How many packages can the delivery person load onto the elevator with him safely?

1. What is known in this problem?

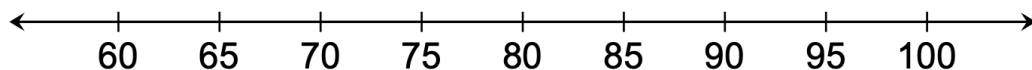
2. What is unknown in this problem?

3. Complete the inequality to represent this problem.

$$\frac{\text{weight of}}{\text{delivery}} \frac{\text{person}}{\text{}} + \frac{\text{weight of}}{\text{packages}} \frac{\text{}}{\text{}} \leq \frac{\text{maximum}}{\text{weight for}} \frac{\text{elevator}}{\text{}}$$

4. Solve the inequality.

5. Represent the solution on a number line.



6. What does your solution mean in the context of this problem?

7. Does $70 \frac{1}{2}$ make sense as a possible solution to this problem? Explain.

Mr. Robinson has a budget of \$128 to spend on supplies for a class celebration. Every cupcake costs \$4 and he wants to have at least \$50 left over for decorations. How many cupcakes can Mr. Robinson order?

8. What is known in this problem? Unknown?

9. Write an inequality to represent this problem.

10. Solve the inequality.

11. Sketch a number line to represent the solution set.

12. Does -10 make sense as a solution in this problem? Explain.

Rachel has 65 inches of trim that she wants to use to form the border of a rectangular piece of artwork. She does not necessarily need to use all the trim. If she wants the artwork to be 7 inches tall, what widths can she choose for the piece?

13. Write and solve an inequality to represent the situation.

14. Does 0 make sense as a solution in this problem? Explain.

1. Solve each inequality. Sketch a number line to model each inequality's solution.

a. $-6n \geq -60$

b. $-6n < -60$

c. $6n > -60$

2. Mrs. Dzija is making ice cubes. The water in the ice cube trays is originally 72 degrees Fahrenheit. In the freezer, the water cools at a rate of 7 degrees per hour. She takes the ice cube tray out when the temperature is below 30 degrees. How many hours could Mrs. Dzija have had the ice cube trays in the freezer? Write and solve an inequality.

Does 6 hours make sense as a solution to this problem? Explain.

3. A company charges \$60 to rent a motorcycle per day in addition to a one-time fee of \$25 for insurance. How many days can Vance rent a motorcycle if he wants to spend no more than \$385?

Does -1 make sense as a solution to this problem? Explain.

4. Beatriz wants to know how many episodes of her favorite TV show she can download onto her tablet. The tablet can store 64 gigabytes of data, but 29 GB are already being used by other applications. Each TV episode is 2 GB. Beatriz used the inequality $29 + 2x \geq 64$ to determine how many episodes she can download.

a. Explain why her inequality will not work to solve the problem.

b. Correct her work.

A freight elevator can hold a maximum of 2,000 pounds. A delivery person weighs 160 pounds. Each package he loads onto the elevator weighs 23 pounds. How many packages can the delivery person load onto the elevator with him safely?

1. What is known in this problem?

- elevator holds ≤ 2000 lbs
- delivery person = 160 lbs
- boxes weigh 23 lbs

2. What is unknown in this problem?

How many boxes can safely fit on the elevator.

3. Complete the inequality to represent this problem.

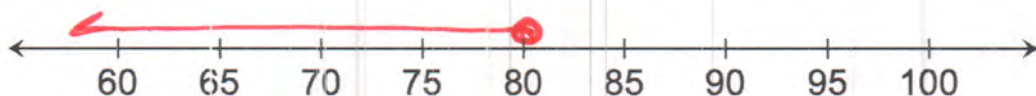
$$\frac{160}{\text{weight of delivery person}} + \frac{23x}{\text{weight of packages}} \leq \frac{2000}{\text{maximum weight for elevator}}$$

4. Solve the inequality.

$$\begin{array}{r} 160 + 23x \leq 2000 \\ -160 \quad -160 \\ \hline 23x \leq 1840 \\ \frac{23x}{23} \leq \frac{1840}{23} \end{array}$$

$$x \leq 80$$

5. Represent the solution on a number line.



6. What does your solution mean in the context of this problem?

The elevator can hold less than or equal to 80 boxes safely.

7. Does $70\frac{1}{2}$ make sense as a possible solution to this problem? Explain.

It does mathematically, but not in real life. $70\frac{1}{2}$ "packages" doesn't make much sense.

Mr. Robinson has a budget of \$128 to spend on supplies for a class celebration. Every cupcake costs \$4 and he wants to have at least \$50 left over for decorations. How many cupcakes can Mr. Robinson order?

8. What is known in this problem? Unknown?

- budget = \$128
 - cupcakes cost \$4
 - must have ≥ 50 dollars left
- # of cupcakes

9. Write an inequality to represent this problem.

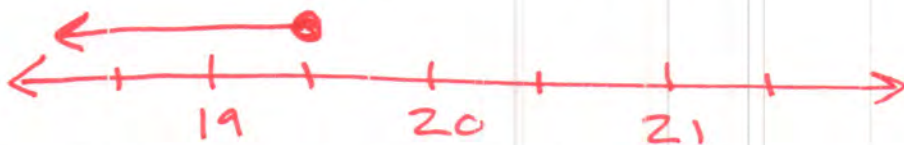
$$128 - 4x \geq 50$$

10. Solve the inequality.

$$\begin{array}{r} 128 - 4x \geq 50 \\ -128 \quad -128 \\ \hline -4x \geq -78 \end{array}$$

$$x \leq 19\frac{1}{2}$$

11. Sketch a number line to represent the solution set.



12. Does -10 make sense as a solution in this problem? Explain.

It does not, because -10 cupcakes is odd to consider.

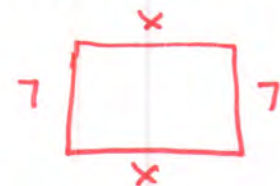
Rachel has 65 inches of trim that she wants to use to form the border of a rectangular piece of artwork. She does not necessarily need to use all the trim. If she wants the artwork to be 7 inches tall, what widths can she choose for the piece?

13. Write and solve an inequality to represent the situation.

$$2x + 14 \leq 65$$

$$2x \leq 51$$

$$x \leq 25\frac{1}{2}$$

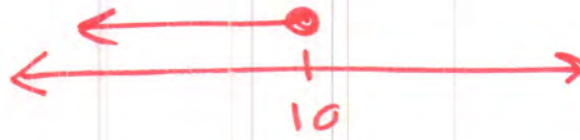


14. Does 0 make sense as a solution in this problem? Explain.

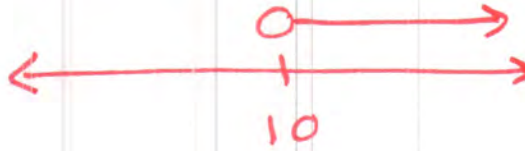
No, because the art can't have a width of 0.

1. Solve each inequality. Sketch a number line to model each inequality's solution.

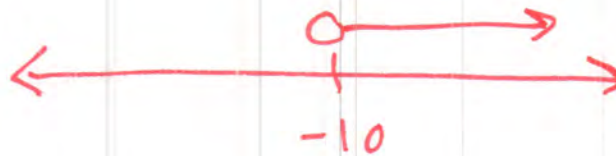
a. $\frac{-6n}{-6} \geq \frac{-60}{-6}$
 $n \leq 10$



b. $\frac{-6n}{-6} < \frac{-60}{-6}$
 $n > 10$



c. $\frac{6n}{6} > \frac{-60}{6}$
 $n > -10$



2. Mrs. Dzija is making ice cubes. The water in the ice cube trays is originally 72 degrees Fahrenheit. In the freezer, the water cools at a rate of 7 degrees per hour. She takes the ice cube tray out when the temperature is below 30 degrees. How many hours could Mrs. Dzija have had the ice cube trays in the freezer? Write and solve an inequality.

$$\begin{array}{r} 72 - 7h < 30 \\ -72 \quad -72 \\ \hline -7h < -42 \\ \frac{-7h}{-7} < \frac{-42}{-7} \\ \hline n > 6 \end{array}$$

Does 6 hours make sense as a solution to this problem? Explain.

No, because 6 is not greater than 6.

3. A company charges \$60 to rent a motorcycle per day in addition to a one-time fee of \$25 for insurance. How many days can Vance rent a motorcycle if he wants to spend no more than \$385?

$$\begin{array}{r} 60d + 25 \leq 385 \\ -25 \quad -25 \\ \hline 60d \leq 360 \\ \underline{60} \quad \underline{60} \\ d \leq 6 \end{array}$$

Does -1 make sense as a solution to this problem? Explain.

No, because -1 can't represent a number of days.

4. Beatriz wants to know how many episodes of her favorite TV show she can download onto her tablet. The tablet can store 64 gigabytes of data, but 29 GB are already being used by other applications. Each TV episode is 2 GB. Beatriz used the inequality $29 + 2x \geq 64$ to determine how many episodes she can download.

- a. Explain why her inequality will not work to solve the problem.

The tablet can hold ≤ 64 gigabytes, not ≥ 64 like she wrote.

- b. Correct her work.

$$\begin{array}{r} 29 + 2x \leq 64 \\ 2x \leq 35 \\ \underline{\quad} \quad \underline{\quad} \\ x \leq 17.5 \end{array}$$

G7 U5 Lesson 17

Extend the distributive property to expressions with negative coefficients.

G7 U5 Lesson 17 - Students will extend the distributive property to expressions with negative coefficients.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): For the past several lessons, we've been working to solve inequalities. Today will feel like we're switching gears a bit, as we start our last series of lessons in this unit. The work we explore today and in subsequent lessons will have us thinking about different ways to write equivalent expressions. Our focus for today will be on a property you've likely worked with since elementary school: The Distributive Property. Let's get started!

Let's Talk (Slide 3): For part of our work today, it will be helpful to use addition. This is because addition is commutative, meaning we can rearrange the order of terms without impacting the value of an expression. That's not true when we subtract, so it's often easier to rearrange expressions with addition.

As we've seen previously, we can rewrite subtraction expressions by using addition. Let's refresh on that idea, so we're ready to tackle today's problems.

$$\begin{array}{l} 8 - 7 \\ 8 + (-7) \end{array} \quad \begin{array}{l} -2.4 - 3.5 \\ -2.4 + (-3.5) \end{array}$$
$$\begin{array}{l} 1 - 6 \\ 1 + (-6) \end{array} \quad \begin{array}{l} 1\frac{1}{2} - \frac{3}{4} \\ 1\frac{1}{2} + (-\frac{3}{4}) \end{array}$$

Here we see 4 subtraction expressions. How could I use the additive inverse to help me rewrite each as a subtraction expression? (*rewrite each as shown while the student shares out, supporting as needed*). Possible Student

Answers, Key Points:

- I can rewrite $8 - 7$ as $8 + (-7)$. Adding a negative is the same as subtracting.
- I can rewrite $1 - 6$ as $1 + (-6)$. I can rewrite $-2.4 - 3.5$ as $-2.4 + (-3.5)$. I can rewrite $1\frac{1}{2} - \frac{3}{4}$ as $1\frac{1}{2} + (-\frac{3}{4})$.

Let's keep this handy skill in mind as we tackle our first problem...

Let's Think (Slide 4): This problem wants us to use the distributive property to write an equivalent expression. When we use the distributive property, we distribute, or multiply, the term outside parentheses by each term inside the parentheses.

$$\begin{array}{l} 5(-2y - 4) \\ 5(-2y + (-4)) \end{array}$$
$$\begin{array}{l} -10y + -20 \\ -10y - 20 \end{array}$$

It's not always required, but let's start by rewriting the subtraction expression inside parentheses as addition. Then it will be easy to organize our work along a tape diagram. What is $-2y - 4$ as an addition expression? ($-2y + (-4)$) (*rewrite expression*)

(*draw a rectangular area model partitioned into two smaller rectangles*) Let's label this area model using the expression. I'll place the coefficient of 5 on the left, and the two terms we're adding in parentheses on top of each rectangle. (*label as described*)

Now, we can multiply. What is 5 times $-2y$? ($-10y$) What is 5 times -4 ? (-20)

We can write those inside the rectangles. I can combine those two products to write our equivalent expression. (*write $-10y + -20$*) I can also write this as $-10y - 20$.

We just successfully used the distributive property by organizing our work in an area model to rewrite the original expression as an equivalent expression. Great job!

Let's Think (Slide 5): Let's look at one more example. We're going to do pretty much the same process, but this problem looks different. What do you notice is the same and different about this problem compared to the previous example? **Possible Student Answers, Key Points:**

- It's similar because there is a number outside of the parentheses that we can distribute.
- It's different because I see the coefficient is a fraction. The coefficient is a negative number. There are three terms inside parentheses.

$$-\frac{1}{2}(10 - 6w - 4)$$

$$-\frac{1}{2}(10 + (-6w) + (-4))$$

Let's use what we know to tackle this one. I'll start by rewriting the expression inside parentheses as an addition expression. (*rewrite as shown*) $10 - 6w$ is the same as 10 plus negative 6w. And instead of -4 , I can write plus -4 .

$-\frac{1}{2}$	10	$-6w$	-4
	-5	$+3w$	$+2$

$-5 + 3w + 2$
 $3w + -5 + 2$
 $3w - 3$

We need to distribute $-\frac{1}{2}$ to each term inside parentheses. We can set our problem up with an area model to help keep track of each part. I'll draw a box and partition it into three parts so that each term has its own dedicated space. How can I set up the area model with the numbers from the problem? (*label area model as student shares*)

Possible Student Answers, Key Points:

- Put the coefficient of $-\frac{1}{2}$ on the side of the area model.
- Put 10 on top of the first box, $-6w$ on the second, and -4 on the third.

Now, we multiply. (*fill each product in the corresponding rectangle*) I know $-\frac{1}{2}$ times 10 is $-10/2$ or just -5 . What is $-\frac{1}{2}$ times $-6w$? ($3w$) What is $-\frac{1}{2}$ times -4 ? (2) We've multiplied each term by the negative coefficient, so now let's write our expression.

(*write expressions as you narrate*) The expression I can write is $-5 + 3w + 2$. I notice we can combine the number terms, so I'll rearrange the terms so the numbers are near each other. I can write that as $3w + -5 + 2$. From here, I can combine $-5 + 2$ which is -3 . My expression can be $3w + -3$ or just $3w - 3$.

Even though this expression had a fractions, a negative coefficient, and more terms than the previously problem, the same thinking helped us arrive at an equivalent, simpler expression.

Let's Try it (Slides 6 - 7): We'll collaborate on a few more problems together before you get a chance to show what you know independently. When we rewrite expressions using the distributive property, it can help to rewrite the expression inside the parentheses as an addition problem to help us set up an area model. Then, we carefully multiply each term inside parentheses by the term on the outside. If we have terms that we can combine, which isn't always the case, we can rearrange the resulting expression to help us easily combine them. Let's use what we've practiced on a few more examples.

WARM WELCOME



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Today we will extend the distributive property to expressions with negative coefficients.

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 Let's Talk:

$$8 - 7$$


$$1 - 6$$

$$-2.4 - 3.5$$

$$1 \frac{1}{2} - \frac{3}{4}$$

Can we write these expressions using addition instead of subtraction?

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 Let's Think:

Use the distributive property to write an expression equivalent to $5(-2y - 4)$.

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Let's Think:

Use the distributive property to write an expression equivalent to $\frac{1}{2}(10 - 6w - 4)$.



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Let's Try It:

Let's explore extending the distributive property to expressions with negative coefficients together.

Name: _____ G7 US Lesson 17 - Let's Try It

Rewrite each subtraction expression as an addition expression.

1. $6 - 15$ 2. $84 - 2 \frac{1}{2}$ 3. $-34.2 - 17.9$

Consider the expression $6 \frac{1}{2} + \frac{1}{2} - 3 \frac{1}{2}$.

4. Rewrite the expression so it uses only addition. Use the commutative property of addition to rearrange the expression so like units are next to each other.

5. Find the value of this expression.

6. How did rewriting the expression help you arrive at the answer?

7. Use a similar strategy to find the value of the expression below.

$\frac{7}{8} + 1\frac{1}{8} - 5 \frac{7}{8}$

Look at the dimensions labeled on the composed rectangle.

8. Complete the expressions to represent the area of the large rectangle.

(_____ + _____)

(_____ - _____) + (_____ + _____)

(_____) + (_____)

9. Look at the new rectangle. Notice how the width is labeled as -2 now. Use the distributive property, like in #11, to write and evaluate a similar expression.

This area model has an unknown value, m.

10. Complete the expressions to represent the area of the large rectangle.

(_____ + _____)

(_____ + _____)

Liam wrote the expression $4(-3y - 9 + 2)$ to represent an area model.

11. Rewrite the expression using addition.

12. Draw an area model to represent the equation.

13. Use the distributive property to write a simplified, equivalent expression.

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On your Own:

Now it's time to extend the distributive property to expressions with negative coefficients on your own.

Name: _____ G7 US Lesson 17 - Independent Work

1. Rewrite each expression using addition.

a. $8 - 5$

b. $-11.3 - 14.58$

c. $3\frac{1}{2} + 4\frac{1}{4} - 9\frac{3}{8}$

2. Sketch an area model to represent the expression $4(6 + 4)$. Then write a simplified, equivalent expression.

3. Rewrite the expression as an addition expression. Then write a simplified, equivalent expression.

$7(-2y - 9)$

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4. Use the distributive property to write a simplified, equivalent expression based on each area model.

-4

--	--	--

 $-8x$ 5

--	--	--

 $2x$ -6 $-5y$

5. Chloe drew an area model and wrote an equivalent expression to represent $-3(6x - 2)$. Look at her work below.

-3

--	--

 $6x$ -2 $(-3 \cdot 6x) + (-3 \cdot -2)$
 $= -18x + 6$
 $= -18x - 6$

What mistake did Chloe make? Include the correct work in your response.

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Rewrite each subtraction expression as an addition expression.

1. $6 - 15$

2. $8 \frac{3}{4} - 2 \frac{1}{5}$

3. $-34.2 - 17.9$

Consider the expression $6 \frac{1}{5} + \frac{1}{4} - 3 \frac{1}{5}$.

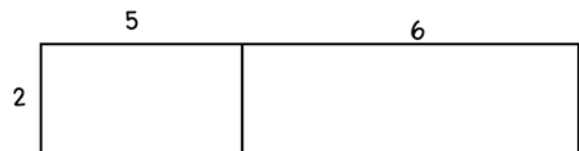
- Rewrite the expression so it uses only addition. Use the commutative property of addition to rearrange the expression so like units are next to each other.
- Find the value of the expression.
- How did rewriting the expression help you arrive at the answer?

- Use a similar strategy to find the value of the expression below.

$$\frac{1}{3} + 1\frac{1}{2} - 5\frac{1}{3}$$

Look at the dimensions labeled on the composed rectangle.

- Complete the expressions to represent the area of the large rectangle.

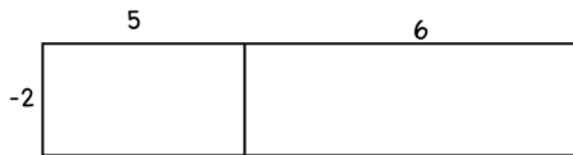


$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}})$$

$$(\underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}}) + (\underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}})$$

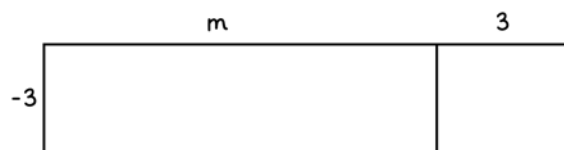
$$(\underline{\hspace{2cm}}) + (\underline{\hspace{2cm}})$$

9. Look at the new rectangle. Notice how the width is labeled as -2 now. Use the distributive property, like in #11, to write and evaluate a similar expression.



This area model has an unknown value, m .

10. Complete the expressions to represent the area of the large rectangle.



_____ (_____ + _____)

_____ + _____

Liam wrote the expression $4(-3y - 9 + 2)$ to represent an area model.

11. Rewrite the expression using addition.

12. Draw an area model to represent the equation.

13. Use the distributive property to write a simplified, equivalent expression.

1. Rewrite each expression using addition.

a. $8 - 5$

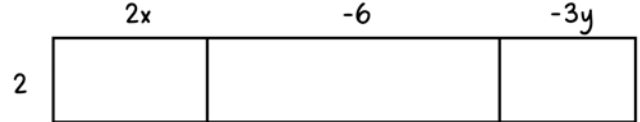
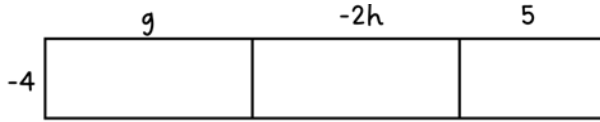
b. $-11.3 - 14.58$

c. $3 \frac{1}{2} + 4 \frac{1}{4} - 9 \frac{3}{4}$

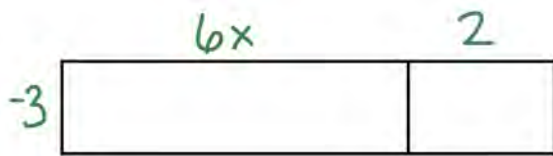
2. Sketch an area model to represent the expression $4(4n + 4)$. Then write a simplified, equivalent expression.**3. Rewrite the expression as an addition expression. Then write a simplified, equivalent expression.**

$$7(-2y - 9)$$

4. Use the distributive property to write a simplified, equivalent expression based on each area model.



5. Chloe drew an area model and wrote an equivalent expression to represent $-3(6x - 2)$. Look at her work below.



$$\begin{aligned}
 &(-3 \cdot 6x) + (-3 \cdot 2) \\
 &-18x + -6 \\
 &-18x - 6
 \end{aligned}$$

What mistake did Chloe make? Include the correct work in your response.

Name: KEY

Rewrite each subtraction expression as an addition expression.

1. $6 - 15$

$6 + (-15)$

2. $8\frac{3}{4} - 2\frac{1}{3}$

$8\frac{3}{4} + (-2\frac{1}{3})$

3. $-34.2 - 17.9$

$-34.2 + (-17.9)$

Consider the expression $6\frac{1}{5} + \frac{1}{4} - 3\frac{1}{5}$.

4. Rewrite the expression so it uses only addition. Use the commutative property of addition to rearrange the expression so like units are next to each other.

$6\frac{1}{5} + \frac{1}{4} + (-3\frac{1}{5})$

$6\frac{1}{5} + (-3\frac{1}{5}) + \frac{1}{4}$

5. Find the value of the expression.

$3 + \frac{1}{4} = 3\frac{1}{4}$

6. How did rewriting the expression help you arrive at the answer?

Rearranging made it easy to efficiently combine terms with like units.

7. Use a similar strategy to find the value of the expression below.

$\frac{1}{3} + 1\frac{1}{2} - 5\frac{1}{3}$

$\frac{1}{3} + 1\frac{1}{2} + (-5\frac{1}{3})$

$\frac{1}{3} + (-5\frac{1}{3}) + 1\frac{1}{2} \rightarrow -5 + 1\frac{1}{2} \rightarrow 3\frac{1}{2}$

Look at the dimensions labeled on the composed rectangle.

8. Complete the expressions to represent the area of the large rectangle.



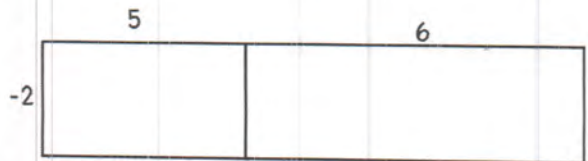
$2 \cdot (5 + 6)$

$(2 \cdot 5) + (2 \cdot 6)$

$(10) + (12)$

22

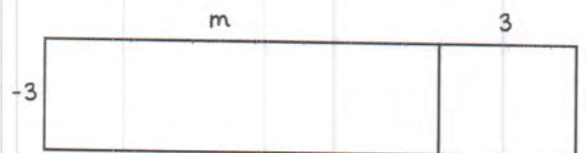
9. Look at the new rectangle. Notice how the width is labeled as -2 now. Use the distributive property, like in #11, to write and evaluate a similar expression.



$$\begin{aligned} & (-2 \cdot 5) + (-2 \cdot 6) \\ & -10 + (-12) \\ & \underline{-22} \end{aligned}$$

This area model has an unknown value, m .

10. Complete the expressions to represent the area of the large rectangle.



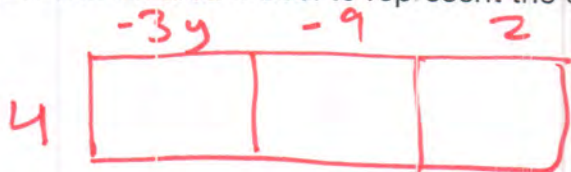
$$\begin{aligned} & \underline{-3} (\underline{m} + \underline{3}) \\ & \underline{-3m} + \underline{-9} \\ & \underline{-3m - 9} \end{aligned}$$

Liam wrote the expression $4(-3y - 9 + 2)$ to represent an area model.

11. Rewrite the expression using addition.

$$4(-3y + -9 + 2)$$

12. Draw an area model to represent the equation.



13. Use the distributive property to write a simplified, equivalent expression.

$$\begin{aligned} & (4 \cdot -3y) + (4 \cdot -9) + (4 \cdot 2) \\ & -12y - 36 + 8 \\ & \underline{-12y - 28} \end{aligned}$$

1. Rewrite each expression using addition.

a. $8 - 5$

$$8 + (-5)$$

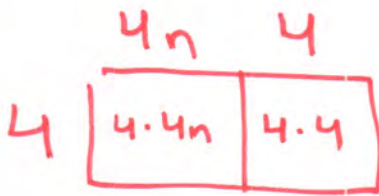
b. $-11.3 - 14.58$

$$-11.3 + (-14.58)$$

c. $3\frac{1}{2} + 4\frac{1}{4} - 9\frac{3}{4}$

$$3\frac{1}{2} + 4\frac{1}{4} + (-9\frac{3}{4})$$

2. Sketch an area model to represent the expression $4(4n + 4)$. Then write a simplified, equivalent expression.



$$16n + 16$$

3. Rewrite the expression as an addition expression. Then write a simplified, equivalent expression.

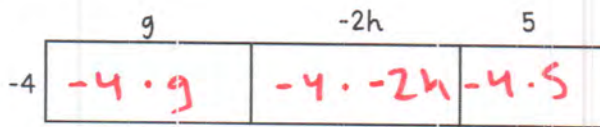
$$7(-2y - 9)$$

$$7(-2y + -9)$$

$$-14y + -63$$

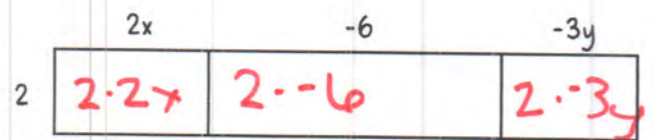
$$-14y - 63$$

4. Use the distributive property to write a simplified, equivalent expression based on each area model.



$$-4g + 8h + -20$$

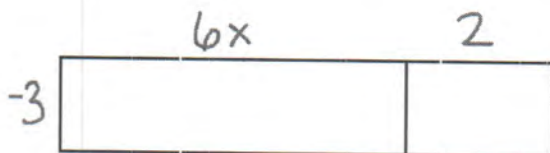
$$\boxed{-4g + 8h - 20}$$



$$4x + -12 + -6y$$

$$\boxed{4x - 6y - 12}$$

5. Chloe drew an area model and wrote an equivalent expression to represent $-3(6x - 2)$. Look at her work below.



$$(-3 \cdot 6x) + (-3 \cdot 2)$$

$$-18x + -6$$

$$-18x - 6$$

What mistake did Chloe make? Include the correct work in your response.

Her expression isn't $-3(6x+2)$ like her area model shows. The 2 should be -2 .

$$(-3 \cdot 6x) + (-3 \cdot -2)$$

$$-18x + +6$$

$$\boxed{-18x + 6}$$

G7 U5 Lesson 18

Use the distributive property to find equivalent expressions by expanding or factoring.

G7 U5 Lesson 18 - Students will use the distributive property to find equivalent expressions by expanding or factoring.

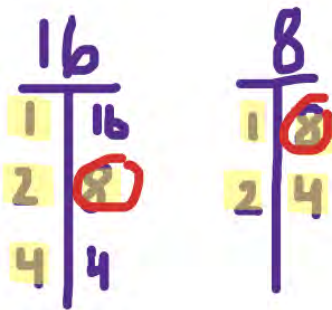
Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): In our last lesson together, we explored writing equivalent expressions using the distributive property. What are some things you know or remember about using the distributive property? **Possible Student Answers, Key Points:**

- When we use the distributive property we multiply the number outside the parentheses by each term inside the parentheses.
- We can use an area model to organize our work when using the distributive property.

We saw that we can use the distributive property even if we have a negative coefficient outside of the parentheses. We used area models to help organize our thinking. Today, we'll continue thinking about the distributive property, but almost in reverse. Today, we're going to divide out a factor to write an equivalent expression. Before we officially start, let's revisit what it means when we're asked to find factors.

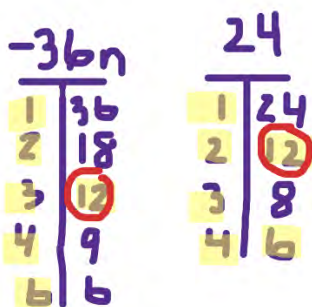
Let's Talk (Slide 3): Factors are numbers we can multiply together to get another number. This slide prompts us to find the greatest common factor of two terms.



To do that, let's start by finding all factor pairs for each number. (*sketch a t-chart to organize factors of 16 and another to organize factors for 8*) I'll systematically think through all factors pairs for 16. (*write them in the t-chart as you think aloud*) I know 1 times 16 is 16, so 1 and 16 are a factor pair. I know 2 times 8 is 16. I know 4 times 4 is 16. These are all the factor pairs for 16. Help me find all the factor pairs that can make 8. (*complete chart as student shares thinking*) **Possible Student Answers, Key Points:**

- 1 times 8 equals 8, so 1 and 8 are factors.
- 2 and 4 are factors, because $2 \times 4 = 8$.

We listed out all factor pairs for each term. Now we can find the factors they have in common. (*highlight all common factors*) I see both terms have factors of 1, 2, 4, and 8. These are all common factors. Since the prompt asks us to find the *greatest* common factor, or GCF, I just need to pick the common factor with the greatest value. The greatest common factor of 16 and 8 is 8. The GCF is 8.



Let's try one more. This one has a term that is negative and has a variable. For the purposes of today, we can focus just on factoring the number, since the other term doesn't have a variable or a negative in common. I'm going to systematically list all the factors of 36 in a t-chart. (*list as you narrate*) I know $1 \times 36 = 36$. I know $2 \times 18 = 36$. I know $3 \times 12 = 36$. I know $4 \times 9 = 36$. There isn't a 5 fact that makes 36, so I can skip to the next number. I know 6×6 equals 36. 36 has a lot of factors, so it was helpful to think through them sequentially so I didn't miss any.

What are the factors of 24? (*list in t-chart as student shares*) **Possible Student Answers, Key Points:**

- The factor pairs for 24 are 1 and 24, 2 and 12, 3 and 8, and 4 and 6.

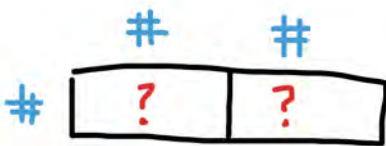
(*highlight all the common factors*) 36 and 24 have many common factors. Which would you say is the greatest common factor, or the GCF? (12) Great! Of all the factors 36 and 24 have in common, 12 is the greatest common factor. The GCF of $-36n$ and 24 is 12.

How would you describe to a friend how to find the greatest common factor of two terms? [Possible Student Answers, Key Points:](#)

- To find the greatest common factor, it can help to list out all the factor pairs for each term.
- Once you have all the factor pairs, identify the factors the terms have in common. The greatest common factor will be the common factor with the greatest value.

Now, let's use this skill to factor some expressions. In a way, it might feel like we're distributing in reverse.

Let's Think (Slide 4): This problem wants us to use the distributive property to write the expression in factored form. You'll notice, this looks different than other problems that have asked us to use the distributive property. Usually, we multiply a number outside parentheses by an expression inside parentheses. Here, we're actually going to use these terms to pull out a greatest common factor. An area model can help show you what I mean.



(sketch and label area model as shown in first example) When we've used an area model to distribute before, we've had a coefficient on the side of the area model and the terms of our expression on top of the area model. Then, we multiply each term by the coefficient to write the equivalent expression. We use multiplication to find what goes inside the area model.



(sketch and label area model as shown in second example) Today, we're being given the expression that typically goes inside the area model. We're going to find and factor out the GCF to rewrite the expression. It's like we know what goes inside our area model, and we'll work backwards to find out

the factors on the outside.

Let's start by factoring $-32h$ and 24 to find the greatest common factor, or GCF. Like earlier, when factoring $-32h$, let's just focus on the number in the term. *(sketch t-charts for factors of 32 and 24)* What are the factor pairs for 32? Try to work in order so that you don't miss any. [Possible Student](#)

32	
1	32
2	16
4	8

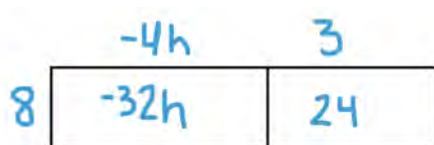
24	
1	24
2	12
3	8
4	6

[Answers, Key Points:](#)

- I know 1×32 is 32, 2×16 is 32, and 4×8 is 32. The factors are 1, 2, 4, 8, 16, and 32.

(fill in the factors for 32) We already found the factors for 24 in a previous example, so I'll just copy those into the chart. *(fill in factors for 24)* What is the greatest common factor of the two terms? (8)

(write $-32h$ and 24 in the inner boxes of an area model and 8 outside on the left of the area model) Our original expression was $-32h + 24$, so I'll write that inside the area model. We found the GCF is 8, so I know I can factor 8 out of both terms. I'll write that on the side of the area model where the coefficient usually goes. All we have left to do is think about what other factors remain.



(fill in missing factors as you narrate) To find the first factor, I can think $8 \times ?$ is equal to $-32h$. Or I could think $-32h$ divided by 8 is equal to what? I know the missing factor is $-4h$. To find the second factor, I can think $8 \times ? = 24$ or 24 divided by 8 equals what? The other missing factor is 3.

$$8(-4h + 3)$$

So, I can rewrite this expression as $8(-4h + 3)$. This is the factored form of the expression we started with. *(write expression)*

We just factored out the GCF of the two terms in the expression we were given to rewrite it using the distributive property. Let's try one more example using the same thinking.

Let's Think (Slide 5): This problem gives us the same directions with a different expression.

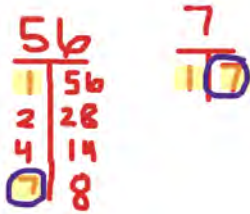
$$56x + -7$$

Let's start by rewriting this expression as an addition expression. What would that look like? **Possible Student Answers, Key Points:**

- I know subtracting 7 is the same as adding -7.
- We can rewrite the expression as $56x + -7$.



Since we're going to factor this expression, I'll write each term inside its own section of an area model. *(sketch a rectangular area model partitioned into two sections, and write one term in each)*



Now we'll figure out the greatest common factor for each term. Working systematically, what are the factors of 56 and 7? *(list each set of factors in a t-chart as student shares)* **Possible Student Answers, Key Points:**

- The factors of 56 are 1 and 56, 2 and 28, 4 and 14, and 7 and 8.
- The factors of 7 are just 1 and 7.

The common factors are 1 and 7, which means the greatest common factor for our two terms is 7.



$$7(8x - 1)$$

We can factor a 7 out of both terms, so I'll write 7 on the side of the area model. *(label 7 on the left side of the area model)* To find the first missing factor, I can think 7 times what equals $56x$, or I can think $56x$ divided by 7 equals what? The missing factor is $8x$. *(label area model)* How could I find the other missing factor? **Possible Student Answers, Key Points:**

- I can think $7 \times ? = -7$. The missing factor is -1.
- I can think -7 divided by $7 = ?$. The missing factor is -1.

We factored out the GCF of 7, and were left with $8x$ and -1 as the other factors. We can write the equivalent expression as $7(8x + -1)$ or just $7(8x - 1)$.

When the terms in an expression have a common factor, we can write an equivalent fraction by factoring out the common factor using the distributive property. An area model can help organize our work and help us keep track of the factors.

Let's Try it (Slides 6 - 7): Now let's do a few more examples where we factor to write equivalent expressions using the distributive property. As we factor expressions, make sure to systematically work to find the greatest common factor of both terms. We saw today that an area model can be a helpful way to keep track of our thinking. After we do the next few problems together, you'll get a chance to show what you know on your own.

WARM WELCOME



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Today we will use the distributive property to find equivalent expressions by expanding or factoring.

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 Let's Talk:

16, 8

-36n, 24

**What is the
greatest common
factor?**

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 Let's Think:

**Use the distributive property to write the expression
in factored form.**

-32h + 24

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Let's Think:

Use the distributive property to write the expression in factored form.

$56x - 7$

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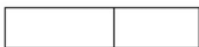


Let's Try It:


Let's explore using the distributive property to find equivalent expressions by expanding or factoring together.

Name: _____ G7 US Lesson 18 - Let's Try It

Consider the expression $3(c + 5)$.

- Label the area model to represent the expression. 
- Use the distributive property to write an equivalent expression.
- What are the two factors in your rewritten expression?


Consider the expression $10n + 35$.

- Write the terms inside the area model. 
- What factor do $10n$ and 35 have in common?
 - 2
 - 5
 - 10
- Write that common factor on the left side of the area model.
- 5 times what number has a product of $10n$? Write that above the first rectangle.
- 5 times what number has the product of 35 ? Write that above the second rectangle.
- Fill in the blanks based off of your work to write an equivalent expression to $10n + 35$.
 ____ (____ + ____)

You just factored $10n + 35$ to write an equivalent expression!

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Consider the expression $9x - 12$.

- Rewrite the expression as an addition expression.
 ____ + ____
- Fill in the diagram with the expression. 
- Find the greatest common factor of $9x$ and -12 . Label it on the side of the area model.
- Find the other factors based on what you know. Write them above each smaller rectangle in the area model.
- Write the factors in a new, equivalent expression.
 ____ (____ + ____)
- Write this expression in another equivalent way using subtraction.
 ____ (____ - ____)

Consider the expression $-24w + 6$.

- Sketch an area model and write the two terms of this expression inside each rectangle.
- Find the greatest common factor. Then find the other two factors. Label your area model.
- Write the equivalent expression to $-24w + 6$ based on the factors you found.

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On your Own:

Now it's time to use the distributive property to find equivalent expressions by expanding or factoring on your own.

Name: _____ G7 US Lesson 18 - Independent Work

1. Find the greatest common factor of each pair of terms.

a. 14, 12

b. 21, 28

c. $-9y$, 33

2. Rewrite each subtraction expression as an addition expression. Then use the distributive property to write an equivalent expression. Sketch an area model if that helps you arrive at your answer.

$4(-3y - 2)$ $-5(m - 17)$

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3. For each area model, find the missing factors. Then, write an equivalent expression.

a.

$15x$	6
-------	-----

b.

$-4h$	10
-------	------

4. Jim was trying to use the distributive property to factor the expression $48z - 28$. He started making the area model below, but realized he made a mistake.

$12z$	-7
-------	------

What mistake did Jim make? Correct the mistake, and then use the distributive property to factor the expression.

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Consider the expression $3(c + 5)$.

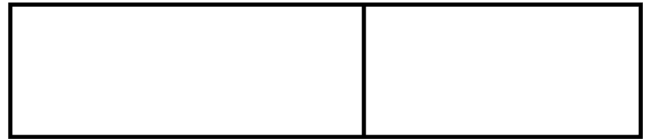
1. Label the area model to represent the expression.
2. Use the distributive property to write an equivalent expression.



3. What are the two terms in your rewritten expression?

Consider the expression $10n + 35$.

4. Write the terms inside the area model.
5. What factor do $10n$ and 35 have in common?
 - a. 2
 - b. 5
 - c. 10



6. Write that common factor on the left side of the area model.
7. 5 times what number has a product of $10n$? Write that above the first rectangle.
8. 5 times what number has the product of 35 ? Write that above the second rectangle.
9. Fill in the blanks based off of your work to write an equivalent expression to $10n + 35$.
_____ (_____ + _____)

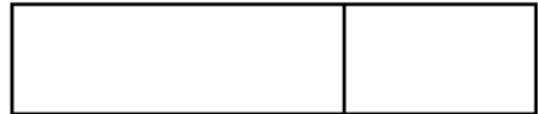
You just factored $10n + 35$ to write an equivalent expression!

Consider the expression $9x - 12$.

10. Rewrite the expression as an addition expression.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

11. Fill in the diagram with the expression.



12. Find the greatest common factor of $9x$ and -12 .
Label it on the side of the area model.

13. Find the other factors based on what you know. Write them above each smaller rectangle in the area model.

14. Write the factors in a new, equivalent expression.

$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}})$$

15. Write this expression in another equivalent way using subtraction.

$$\underline{\hspace{2cm}} (\underline{\hspace{2cm}} - \underline{\hspace{2cm}})$$

Consider the expression $-24w + 6$.

16. Sketch an area model and write the two terms of this expression inside each rectangle.

17. Find the greatest common factor. Then find the other two factors. Label your area model.

18. Write the equivalent expression to $-24w + 6$ based on the factors you found.

1. Find the greatest common factor of each pair of terms.

a. 14, 12

b. 21, 28

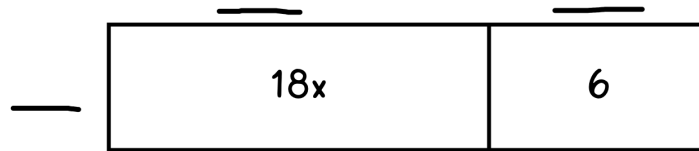
c. $-6y$, 33**2. Rewrite each subtraction expression as an addition expression. Then use the distributive property to write an equivalent expression. Sketch an area model if that helps you arrive at your answer.**

$$4(-3y - 2)$$

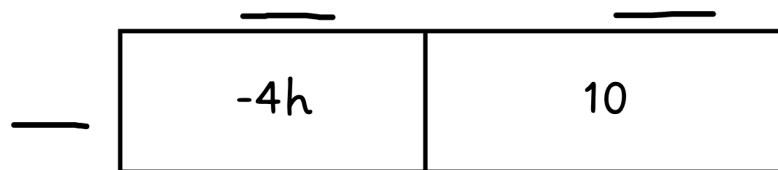
$$-5(m - 17)$$

3. For each area model, find the missing factors. Then, write an equivalent expression.

a.



b.



4. Jim was trying to use the distributive property to factor the expression $49x - 28$. He started making the area model below, but realized he made a mistake.



What mistake did Jim make? Correct the mistake, and then use the distributive property to factor the expression.

Name: KEY

Consider the expression $3(c + 5)$.

1. Label the area model to represent the expression.



2. Use the distributive property to write an equivalent expression.

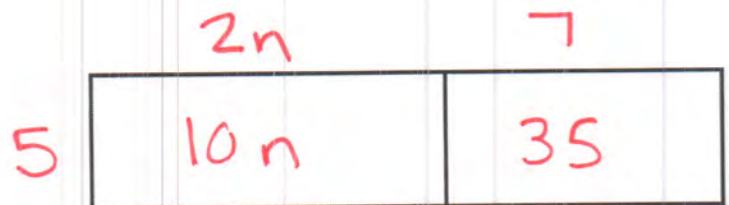
$$3c + 15$$

3. What are the two ^{terms} factors in your rewritten expression?

$3c$ and 15

Consider the expression $10n + 35$.

4. Write the terms inside the area model.



5. What factor do $10n$ and 35 have in common?

- a. 2
- b. 5**
- c. 10

6. Write that common factor on the left side of the area model. ✓

7. 5 times what number has a product of $10n$? Write that above the first rectangle.

$$5 \times ? = 10n$$

8. 5 times what number has the product of 35 ? Write that above the second rectangle.

$$5 \times ? = 35$$

9. Fill in the blanks based off of your work to write an equivalent expression to $10n + 35$.

$$\underline{5} (\underline{2n} + \underline{7})$$

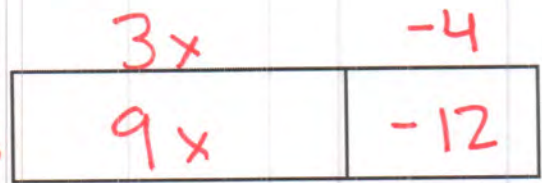
You just factored $10n + 35$ to write an equivalent expression!

Consider the expression $9x - 12$.

10. Rewrite the expression as an addition expression.

$$\underline{9x} + \underline{-12}$$

11. Fill in the diagram with the expression.



12. Find the greatest common factor of $9x$ and -12 .

Label it on the side of the area model.

13. Find the other factors based on what you know. Write them above each smaller rectangle in the area model.

14. Write the factors in a new, equivalent expression.

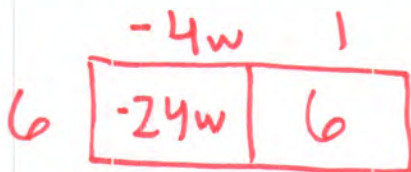
$$\underline{3} (\underline{3x} + \underline{-4})$$

15. Write this expression in another equivalent way using subtraction.

$$\underline{3} (\underline{3x} - \underline{4})$$

Consider the expression $-24w + 6$.

16. Sketch an area model and write the two terms of this expression inside each rectangle.



17. Find the greatest common factor. Then find the other two factors. Label your area model.

18. Write the equivalent expression to $-24w + 6$ based on the factors you found.

$$6(-4w + 1)$$

1. Find the greatest common factor of each pair of terms.

a. 14, 12

$$\begin{array}{r} 14 \\ 1 \overline{) 14} \\ \underline{14} \\ 0 \end{array}$$

$$\begin{array}{r} 12 \\ 1 \overline{) 12} \\ \underline{12} \\ 0 \end{array}$$

b. 21, 28

$$\begin{array}{r} 21 \\ 1 \overline{) 21} \\ \underline{21} \\ 0 \end{array}$$

$$\begin{array}{r} 28 \\ 1 \overline{) 28} \\ \underline{28} \\ 0 \end{array}$$

c. -6y, 33

$$\begin{array}{r} 6 \\ 1 \overline{) 6} \\ \underline{6} \\ 0 \end{array}$$

$$\begin{array}{r} 33 \\ 1 \overline{) 33} \\ \underline{33} \\ 0 \end{array}$$

2. Rewrite each subtraction expression as an addition expression. Then use the distributive property to write an equivalent expression. Sketch an area model if that helps you arrive at your answer.

$$4(-3y - 2)$$

$$4(-3y + -2)$$

$$4 \begin{array}{|c|c|} \hline -3y & -2 \\ \hline \end{array}$$

$$-12y + -8$$

$$\boxed{-12y - 8}$$

$$-5(m - 17)$$

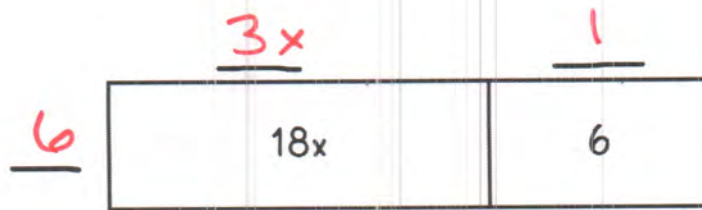
$$-5(m + -17)$$

$$-5 \begin{array}{|c|c|} \hline m & -17 \\ \hline \end{array}$$

$$\boxed{-5m + 85}$$

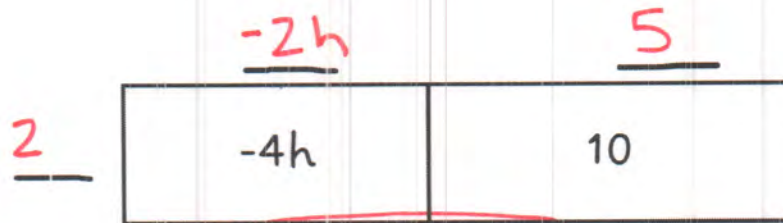
3. For each area model, find the missing factors. Then, write an equivalent expression.

a.



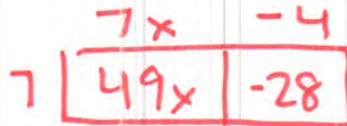
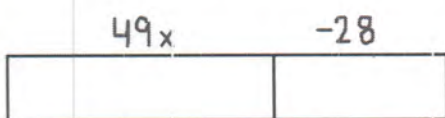
$$6(3x+1)$$

b.



$$2(-2h+5)$$

4. Jim was trying to use the distributive property to factor the expression $49x - 28$. He started making the area model below, but realized he made a mistake.



What mistake did Jim make? Correct the mistake, and then use the distributive property to factor the expression.

He wrote the terms in the wrong place.

They go inside the area model when you're

trying to factor. The correct

answer is $7(7x-4)$.

G7 U5 Lesson 19

Given an expression, write an equivalent expression with fewer terms using properties of operations, and explain why the expressions are equivalent.

G7 U5 Lesson 19 - Students will, given an expression, write an equivalent expression with fewer terms using properties of operations, and explain why the expressions are equivalent.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): We've been thinking about and writing equivalent expressions for the past couple lessons. We know that expressions are equivalent when they always represent the same value. For example, I know $1 + x$ is equivalent to $x + 1$. They look a little different, but they'll always represent the same value. Or, when we use the distributive property, we know our expressions end up looking different, but they represent the same value.

Today, we'll continue working to write equivalent expressions. Most of our problems today will involve combining like terms to write expressions with as few terms as possible. Let me show you what I mean...

Let's Talk (Slide 3): What do you notice about the expressions shown here? What do you wonder?.

Possible Student Answers, Key Points:

- I notice they all have a, b, and c in them. I notice the first one doesn't show any numbers. I notice one is longer than the other. I notice they represent the same thing.
- I wonder if they are equivalent. I wonder why one only has variables and one has variables and numbers. I wonder what a, b, and c can equal.

These two expressions don't look identical, but they represent equivalent expressions. The first is expanded to show every a, b, and c. The second combines like terms and uses multiplication to show how many groups of a, b, and c there are. *(highlight each variable in a different color)* I see 3 groups of a in the first expression, which is equivalent to $3a$ in the second expression. I see 4 groups of b in the first expression, which is equivalent to $4b$

$$a + a + a + b + b + b + b + c$$

in the second expression. I see 1 group of c in both expressions. Both expressions are equivalent, and both are valid ways to write the expression. Why might you want to use one expression over the other? Possible Student Answers, Key Points:

- I might use the first expression if I wanted to clearly see each term.
- I might use the second expression to save time. The numbers also help avoid having to count up the number of each variable.

Today we'll work to combine like terms to write equivalent expressions with as few terms as possible.

Let's Think (Slide 4): This prompt wants us to name whether these expressions are equivalent or not. At first glance, they look quite different. Let's see if we can rewrite the longer expressions with fewer terms to determine whether the expressions are equivalent.

$$q + 2r + 3q + 3r$$

$$1q + 3q + 2r + 3r$$

$$4q + 5r$$

(highlight like terms in similar colors) I know each term with a "q" represents groups of q. I know each term with an "r" represents groups of r. I can combine the groups of q together, and I can combine the groups of r together. I'll rewrite the expression so that these like terms are adjacent. *(write $1q + 3q + 2r + 3r$ in similar colors to how they were highlighted)* Now, I'll combine like terms. What is q, or $1q$, plus $3q$? $(4q)$ What is $2r$ plus $3r$? $(5r)$ I can rewrite the expression as $4q + 5r$. *(write $4q + 5r$)*

By rearranging the expression and combining like terms, we were able to write an expression with fewer terms.

Based on our work here, are the two expressions equivalent? How do you know? [Possible Student Answers, Key Points:](#)

- The two expressions are equivalent, because after we combined like terms, we were left with identical expression.
- They are equivalent. Each expression shows 4 groups of q and 5 groups of r .

Let's Think (Slide 5): Our second problem wants us to write an equivalent expression using as few terms as possible.

$$6g + h + 8 - 2h + 2g - 1$$

$$6g + 2g + h + -2h + 8 + -1$$

$$8g \quad -1h \quad +7$$

$$8g - h + 7$$

I'll start by highlighting like terms that I can combine. I see terms that represent groups of g , terms that represent groups of h , and I see some numbers without variables that I can combine. *(highlight each set of like terms using a different color)*

I notice that this expression involves subtraction. To help rearrange terms, let's rewrite the expression using addition in place of subtraction. *(using color-coding, write $6g + h + 8 + -2h + 2g + -1$)* Now we can rearrange the terms so like terms are adjacent. *(write $6g + 2g + h + -2h + 8 + -1$)*

Now the terms I can combine are side-by-side. How can I combine the terms? [Possible Student Answers, Key Points:](#)

- I know $6g + 2g$ is $8g$.
- I know $h + -2h$ is $-h$ or $-1h$.
- I know $8 + -1$ is 7 .

(write $8g - h + 7$) I can write $8g + -1h + 7$ as $8g - h + 7$. I know we've written an equivalent expression with as few terms as possible, because there is nothing left to combine. I can't combine groups of g with groups of h , for examples, because they're different units. We took our long expression that we started with that had six different terms, and we rearranged it so we could combine like terms. Our equivalent expressions has just three terms.

Let's Try it (Slides 6 - 7): We'll work through a few more examples together, before you get a chance for some independent time. It will be helpful to rearrange expressions to group like terms, and we can color-code the terms in our expressions if we want. Writing any subtraction as addition can make it easier for us to rearrange terms. Let's keep these pointers in mind as we continue working together.

WARM WELCOME



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Today we will, given an expression, write an equivalent expression with fewer terms using properties of operations and explain why the expressions are equivalent.

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Let's Talk:

**What do you notice?
What do you wonder?**

$$a + a + a + b + b + b + b + c$$

$$3a + 4b + c$$

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Let's Think:

Are the expressions equivalent?

$$4q + 5r$$

$$q + 2r + 3q + 3r$$

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Let's Think:

Rewrite the expression using as few terms as possible.

$$6g + h + 8 - 2h + 2g - 1$$

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Let's Try It:

Let's explore writing an equivalent expression with fewer terms using properties of operations together.

Name: _____ G7 US Lesson 19 - Let's Try It

Consider the two expressions below. Let's determine if they are equivalent.
 $6x + 3y$ $4x + 2y + 2x + y$

- Evaluate $6x + 3y$ when $x = 2$ and $y = 3$.
- Evaluate $4x + 2y + 2x + y$ when $x = 2$ and $y = 3$.
- The two expressions have...
 - the same value.
 - different values.
- Evaluate both expressions using new values for x and y . Choose your own values this time.
- The two expressions seem equivalent based on the values we've substituted. To be sure they're equivalent, let's think about the expression $4x + 2y + 2x + y$. Rewrite the expression by using repeated addition in place of multiplication.
- Put parentheses around like variables in the expression. Then rewrite the expression combining all the like variables.
- Consider the work you just did, and look back at the original expressions. Are they equivalent? How do you know?

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Consider the expression.

$$7g + h + 2h - 2g$$

- Expand the expression using repeated addition.
- Rewrite the expression to group all the like variables together.
- Rewrite the expression using the fewest possible terms.

Consider the expression.

$$6n + 6m - 5n + 2m$$

- Rewrite the expression replacing subtraction with adding the opposite.
- Rewrite the addends so the like variables are grouped together.
- Write an equivalent expression with only two terms.

Consider each expression. Rewrite each with the fewest possible terms.

14. $3a - 4b + 7a + 2b$

15. $-5 + x - 2y + 5x + 8 + 4y$

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On your Own:

Now it's time to write an equivalent expression with fewer terms using properties of operations on your own.

Name: _____ G7 U5 Lesson 19 - Independent Work

1. Are the expressions below equivalent?

$$8a + 3c - 9c - 5a \qquad 3a - 6c$$

How do you know?

2. Which expressions are equivalent to the one below? Select all that apply.

$$4x + 2y + 3x + y$$

A. $7x + 3y$
B. $7x + 2y$
C. $x + x + x + x + x + x + x + y + y$
D. $x + y + 9$
E. $x + y + 10$
F. $9xy$
G. $10xy$

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3. Consider the expression below.

$$2g - 1h + 4g + 4h$$

a. Expand the expression to show repeated addition in place of multiplication. Rearrange terms so that all like variables are grouped.

b. Write the expression using the fewest terms possible.

4. Write each expression using as fewest terms as possible.

$$4n + m - n + 5m \qquad 6x + 3 + 8y - 2x + 7y - 9$$

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Consider the two expressions below. Let's determine if they are equivalent.

$$6x + 3y$$

$$4x + 2y + 2x + y$$

1. Evaluate $6x + 3y$ when $x = 2$ and $y = 3$.
2. Evaluate $4x + 2y + 2x + y$ when $x = 2$ and $y = 3$.
3. The two expressions have...
 - a. the same value.
 - b. different values.
4. Evaluate both expressions using new values for x and y . Choose your own values this time.
5. The two expressions seem equivalent based on the values we've substituted. To be sure they're equivalent, let's think about the expression $4x + 2y + 2x + y$. Rewrite the expression by using repeated addition in place of multiplication.
6. Put parentheses around like variables in the expression. Then rewrite the expression combining all the like variables.
7. Consider the work you just did, and look back at the original expressions. Are they equivalent? How do you know?

Consider the expression.

$$7g + h + 2h - 2g$$

8. Expand the expression using repeated addition.
9. Rewrite the expression to group all the like variables together.
10. Rewrite the expression using the fewest possible terms.

Consider the expression.

$$6n + 6m - 5n + 2m$$

11. Rewrite the expression replacing subtraction with adding the opposite.
12. Rewrite the addends so the like variables are grouped together.
13. Write an equivalent expression with only two terms.

Consider each expression. Rewrite each with the fewest possible terms.

14. $3a - 4b + 7a + 2b$

15. $-5 + x - 2y + 5x + 8 + 4y$

1. Are the expressions below equivalent?

$$8a + 3c - 9c - 5a$$

$$3a - 6c$$

How do you know?

2. Which expressions are equivalent to the one below? Select all that apply.

$$4x + 2y + 3x + y$$

- A. $7x + 3y$
- B. $7x + 2y$
- C. $x + x + x + x + x + x + x + x + y + y + y$
- D. $x + y + 9$
- E. $x + y + 10$
- F. $9xy$
- G. $10xy$

3. Consider the expression below.

$$2g - 1h + 4g + 4h$$

- a. Expand the expression to show repeated addition in place of multiplication. Rearrange terms so that all like variables are grouped.
- b. Write the expression using the fewest terms possible.

4. Write each expression using as fewest terms as possible.

$$4n + m - n + 5m$$

$$6x + 3 + 8y - 2x + 7y - 9$$

Consider the two expressions below. Let's determine if they are equivalent.

$$6x + 3y$$

$$4x + 2y + 2x + y$$

1. Evaluate $6x + 3y$ when $x = 2$ and $y = 3$.

$$6(2) + 3(3)$$

$$12 + 9 = 21$$

2. Evaluate $4x + 2y + 2x + y$ when $x = 2$ and $y = 3$.

$$4(2) + 2(3) + 2(2) + 3$$

$$8 + 6 + 4 + 3 = 21$$

3. The two expressions have...

a. the same value.

b. different values.

4. Evaluate both expressions using new values for x and y . Choose your own values this time.

$$x = 0 \quad y = 1$$

$$6(0) + 3(1)$$

$$0 + 3$$

$$3$$

$$4(0) + 2(1) + 2(0) + 1$$

$$0 + 2 + 0 + 1$$

$$3$$

5. The two expressions seem equivalent based on the values we've substituted. To be sure they're equivalent, let's think about the expression $4x + 2y + 2x + y$. Rewrite the expression by using repeated addition in place of multiplication.

$$(x + x + x + x) + (y + y) + (x + x) + (y)$$

6. Put parentheses around like variables in the expression. Then rewrite the expression combining all the like variables.

$$6x + 3y$$

7. Consider the work you just did, and look back at the original expressions. Are they equivalent? How do you know?

Yes! They both represent 6 groups of x and 3 groups of y .

Consider the expression.

$$7g + h + 2h - 2g$$

8. Expand the expression using repeated addition.

$$g + g + g + g + g + g + g + h + h + h + (-g) + (-g)$$

9. Rewrite the expression to group all the like variables together.

$$g + g + g + g + g + g + g + (-g) + (-g) + h + h + h$$

10. Rewrite the expression using the fewest possible terms.

$$5g + 3h$$

Consider the expression.

$$6n + 6m - 5n + 2m$$

11. Rewrite the expression replacing subtraction with adding the opposite.

$$6n + 6m + (-5n) + 2m$$

12. Rewrite the addends so the like variables are grouped together.

$$6n + (-5n) + 6m + 2m$$

13. Write an equivalent expression with only two terms.

$$n + 8m$$

Consider each expression. Rewrite each with the fewest possible terms.

14. $3a - 4b + 7a + 2b$

$$3a + 7a - 4b + 2b$$
$$(10a - 2b)$$

15. $-5 + x - 2y + 5x + 8 + 4y$

$$-5 + 8 + x + 5x - 2y + 4y$$
$$(3 + 6x + 2y)$$

1. Are the expressions below equivalent?

$$8a + 3c - 9c - 5a$$

$$3a - 6c$$

$$\underline{8a} + \underline{3c} + \underline{-9c} + \underline{-5a}$$

$$3a + -6c$$

$$3a - 6c$$

How do you know?

Yes! Both expressions show 3 groups of "a" and -6 groups of "c".

2. Which expressions are equivalent to the one below? Select all that apply.

$$\underline{4x} + \underline{2y} + \underline{3x} + \underline{y}$$

A. $7x + 3y$

B. $7x + 2y$

C. $x + x + x + x + x + x + x + y + y + y$

D. $x + y + 9$

E. $x + y + 10$

F. $9xy$

G. $10xy$

$$7x + 3y$$

3. Consider the expression below.

$$2g - 1h + 4g + 4h$$

- a. Expand the expression to show repeated addition in place of multiplication. Rearrange terms so that all like variables are grouped.

$$\underbrace{g+g+g+g+g+g} + \underbrace{h+h+h+h} + \underbrace{-h}$$

- b. Write the expression using the fewest terms possible.

$$(6g + 3h)$$

4. Write each expression using as fewest terms as possible.

$$4n + m - n + 5m$$

$$4n - n + m + 5m$$

$$(3n + 6m)$$

$$6x + 3 + 8y - 2x + 7y - 9$$

$$6x - 2x + 8y + 7y + 3 - 9$$

$$(4x + 15y - 6)$$

G7 U5 Lesson 20

Write expressions with fewer terms that are equivalent to a given expression that includes negative coefficients and parentheses.

G7 U5 Lesson 20 - Students will write expressions with fewer terms that are equivalent to a given expression that included negative coefficients and parentheses.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): Today we're going to combine some of the thinking from the past few lessons. We've worked to write equivalent expressions using the distributive property. We've worked to write equivalent expressions by combining like terms. Today, we'll see expressions where we can do *both* steps within the same expression. Let's start with a warm-up dealing with the distributive property.

Let's Talk (Slide 3): Take a look at these two expressions. Are they equivalent? How do you know?

Possible Student Answers, Key Points:

- Yes, they're equivalent. I can factor a 4 from the first expression and rewrite it as $4(a + 2b)$
- Yes, they're equivalent. I can distribute the 4 in the second expression and rewrite it as $4a + 8b$.

The expressions shown here are equivalent. We can factor a 4 out of the first expression to give us the second expression, or we can distribute the coefficient of 4 in the second expression to give us the first expression.

$$4(a + 2b)$$
$$4a + 8b \checkmark$$

Let's distribute the 4 in the second expression, just to make sure. Walk me through how we can distribute the coefficient of 4. (*show arrows to distribute the 4 and rewrite the expression as the student shares*) Possible Student Answers, Key Points:

- Multiply 4 times a. That equals 4a. Multiply 4 times 2b. That equals 8b.
- When you distribute 4 to each term, you end up with $4a + 8b$.

Today, we'll use the distributive property, along with other mathematical properties, to help us write equivalent expressions.

Let's Think (Slide 4): For this problem, we'll use the distributive property to write an equivalent expression.

$$-9(2c + 5)$$

	2c	5
-9	-18c	-45

$$-18c + -45$$
$$\textcircled{-18c - 45}$$

(*draw arrows from -9 to 2c and 5*) Let's start by multiplying each term inside parentheses by the coefficient of -9. I'll use an area model to keep our work organized. (*sketch partitioned rectangle for the area model, and label the sides with -9, 2c, and 5 as shown*)

What is -9 times 2c? ($-18c$) What is -9 times 5? (-45) We can rewrite this expression as $-18c + -45$. (*write expression*) How can we write this expression as a subtraction expression? ($-18c - 45$) We can write it as $-18c - 45$ since adding negative 45 is the same as subtracting 45.

Are we finished? Can we combine $-18c$ minus 45? Possible Student Answers, Key Points:

- We cannot combine $-18c$ and -45 , because they are not like terms.
- We are finished, because I can't combine a term with a c on it with a term without a c.

We just used the distributive property to rewrite $-9(2c + 5)$ as the equivalent expression $-18c - 45$. The area model helped us keep our work organized, and we paid close attention to the signs of our numbers as we rewrote the expression. Let's try one more that's a little similar and a little different.

Let's Think (Slide 5): What do you notice is the same or different about this problem compared to the one we just completed? **Possible Student Answers, Key Points:**

- It has numbers and variables. We can use the distributive property to help us rewrite the expression. The directions are the same.
- It has more terms. It only has subtraction. The variable is different.

$$7 + 2(4 - x)$$
$$7 + 2(4 + -x)$$
$$7 + -8 + 2x$$

$$\begin{array}{c} -1 + 2x \\ \text{or} \\ 2x - 1 \end{array}$$

Let's rewrite this expression to use as few terms as possible. I might be tempted to start by subtracting $7 - 2$, but the order of operations tells me I should distribute the -2 first. Before I do that, let's rewrite the expression using addition instead of subtraction. Instead of $7 - 2$, I can write $7 + -2$. Instead of $4 - x$, I can write $4 + -x$. (*rewrite expression*)

Now we'll distribute -2 . What is -2 times 4 ? (-8) What is -2 times $-x$? ($2x$) (*rewrite expression as $7 + -8 + 2x$*) The expression we just wrote is equivalent to the original expression. I notice the directions said to write the expressions using as few terms as possible. What terms in this expression can I combine? How do you know? **Possible Student Answers, Key Points:**

- You can combine 7 and -8 , because they're just numbers.
- You can't combine the $2x$ with any other term, because no other term represents a group of x .

I know 7 plus -8 is -1 . When I combine those terms, we end up with the expression $-1 + 2x$. We could also write this as $2x + -1$ or $2x - 1$.

We just used the distributive property to rewrite this expression. Once we distributed, we noticed we could combine some terms to make a simpler expression. We can combine like terms and use the distributive property to write equivalent expressions.

Let's Try it (Slides 6 - 7): Now we'll try a few more problems together. We'll write equivalent expressions using the distributive property. In some cases, we'll combine like terms to write an expression with as few terms as possible. We noticed that it can be helpful to rewrite subtraction as addition, so it's easier to manipulate the terms in an expression. Not only can this help when using the distributive property, but it can also make rearranging terms using the commutative property a bit easier. Once we're done with the next few examples, you'll get a chance to try some out on your own.

WARM WELCOME



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Today we will write expressions with fewer terms that are equivalent to a given expression that include negative coefficients and parentheses.

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Let's Talk:

**Are the expressions equivalent?
How do you know?**

$$4a + 8b$$

$$4(a + 2b)$$

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Let's Think:

Use the distributive property to write an equivalent expression using as few terms as possible.

$$-9(2c + 5)$$

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Let's Think:

Use the distributive property to write an equivalent expression using as few terms as possible.

$$7 - 2(4 - x)$$

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Let's Try It:

Let's explore writing equivalent expressions with fewer terms that are equivalent to a given expression that include negative coefficients and parentheses together.

Name: _____ G7 US Lesson 20 - Let's Try It

Consider the expression $-5(3 + 2x)$.

- Fill in the blanks on the area model to represent the expression. 
- Use the distributive property to write an equivalent expression.


$$(\text{---} \cdot \text{---}) + (\text{---} \cdot \text{---})$$

$$\text{---} + \text{---}$$

$$\text{---} - \text{---}$$

Consider the expression $-5(3 - 2x)$. Note that this expression has subtraction inside the parentheses.

- Rewrite the expression using addition.

$$-5(\text{---} + \text{---})$$
- Fill in the blanks on the area model to represent the expression. 
- Use the distributive property to write an equivalent expression.

$$(\text{---} \cdot \text{---}) + (\text{---} \cdot \text{---})$$

$$\text{---} + \text{---}$$

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Consider the expression $8 - 4(2 - m)$.

- Which step should you do first to begin writing equivalent expressions? Remember to consider the order of operations.
 - Subtract $8 - 4$
 - Multiply to distribute the -4
- Rewrite the expression to show subtraction as adding the opposite.

$$\text{---} + \text{---}(\text{---} + \text{---})$$
- Use the distributive property to write an equivalent expression.

$$\text{---} + (\text{---} \cdot \text{---}) + (\text{---} \cdot \text{---})$$

$$\text{---} + \text{---} + \text{---}$$

$$\text{---} + \text{---}$$

Consider the expression $8n - 2(2n - 7)$.

- Rewrite the expression using addition.
- Use the distributive property to write an equivalent expression with the fewest possible terms.

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On your Own:

Now it's time to write expressions with fewer terms that are equivalent to a given expression that include negative coefficients and parentheses on your own.

Name: _____ G7 US Lesson 20 - Independent Work

1. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$-6(8 + 3m)$

2. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$-7(1 - 7k)$

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3. Marcel rewrote the expression $5 - 4(8z + 2)$ as $-1(8z + 2)$. Explain Marcel's mistake, and show how he could write an equivalent expression using as few terms as possible.

4. Which expressions are equivalent to $11 - 5(4v - 2)$? Select all that apply.

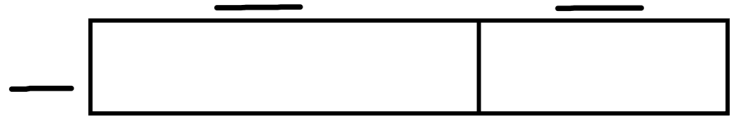
- a. $6(4v - 2)$
- b. $11 - 20v + 10$
- c. $6 + 2v$
- d. $22v$
- e. $-20v + 21$
- f. $21 - 20v$
- g. $-20v + 1$

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Consider the expression $-5(3 + 2x)$.

1. Fill in the blanks on the area model to represent the expression.



2. Use the distributive property to write an equivalent expression.

$$(\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

$$\text{_____} + \text{_____}$$

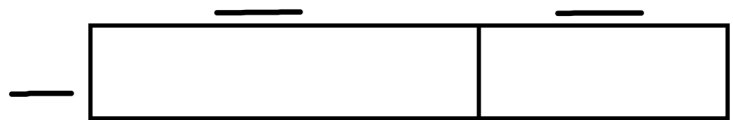
$$\text{_____} - \text{_____}$$

Consider the expression $-5(3 - 2x)$. Note that this expression has subtraction inside the parentheses.

3. Rewrite the expression using addition.

$$-5(\text{_____} + \text{_____})$$

4. Fill in the blanks on the area model to represent the expression.



5. Use the distributive property to write an equivalent expression.

$$(\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

$$\text{_____} + \text{_____}$$

Consider the expression $8 - 4(2 - m)$.

6. Which step should you do first to begin writing equivalent expressions? Remember to consider the order of operations.
- Subtract 8 - 4
 - Multiply to distribute the -4
7. Rewrite the expression to show subtraction as adding the opposite.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} (\underline{\hspace{2cm}} + \underline{\hspace{2cm}})$$

8. Use the distributive property to write an equivalent expression.

$$\underline{\hspace{2cm}} + (\underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}}) + (\underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}})$$

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

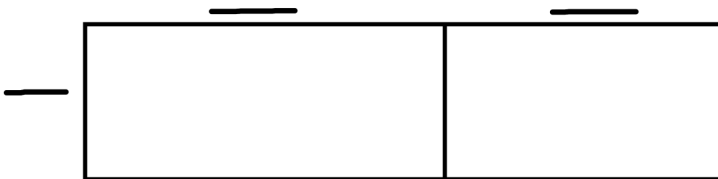
$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}}$$

Consider the expression $8n - 2(2n - 7)$.

9. Rewrite the expression using addition.
10. Use the distributive property to write an equivalent expression with the fewest possible terms.

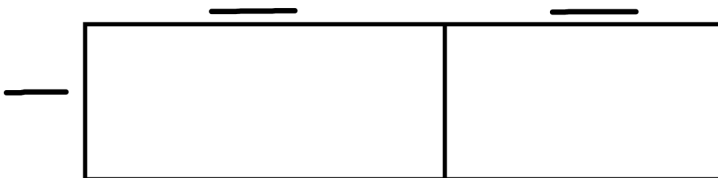
1. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-6(8 + 3m)$$



2. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-7(1 - 7k)$$



3. Marcel rewrote the expression $5 - 4(6z + 2)$ as $-1(6z + 2)$. Explain Marcel's mistake, and show how he could write an equivalent expression using as few terms as possible.

4. Which expressions are equivalent to $11 - 5(4v - 2)$? Select all that apply.

- a. $6(4v - 2)$
- b. $11 - 20v + 10$
- c. $6 \cdot 2v$
- d. $22v$
- e. $-20v + 21$
- f. $21 - 20v$
- g. $-20v + 1$

Name: KEY

Consider the expression $-5(3 + 2x)$.

1. Fill in the blanks on the area model to represent the expression.



2. Use the distributive property to write an equivalent expression.

$$\begin{aligned} & (\underline{-5} \cdot \underline{3}) + (\underline{-5} \cdot \underline{2x}) \\ & \underline{-15} + \underline{-10x} \\ & \underline{-15} - \underline{10x} \end{aligned}$$

Consider the expression $-5(3 - 2x)$. Note that this expression has subtraction inside the parentheses.

3. Rewrite the expression using addition.

$$-5(\underline{3} + \underline{-2x})$$

4. Fill in the blanks on the area model to represent the expression.



5. Use the distributive property to write an equivalent expression.

$$\begin{aligned} & (\underline{-5} \cdot \underline{3}) + (\underline{-5} \cdot \underline{-2x}) \\ & \underline{-15} + \underline{10x} \end{aligned}$$

Consider the expression $8 - 4(2 - m)$.

6. Which step should you do first to begin writing equivalent expressions? Remember to consider the order of operations.

a. Subtract $8 - 4$

b. Multiply to distribute the -4

7. Rewrite the expression to show subtraction as adding the opposite.

$$\underline{8} + \underline{4} (\underline{2} + \underline{-m})$$

8. Use the distributive property to write an equivalent expression.

$$\underline{8} + (\underline{4} \cdot \underline{2}) + (\underline{4} \cdot \underline{-m})$$

$$\underline{8} + \underline{8} + \underline{-4m}$$

$$\underline{16} + \underline{-4m}$$

Consider the expression $8n - 2(2n - 7)$.

9. Rewrite the expression using addition.

$$8n - 2(2n + -7)$$

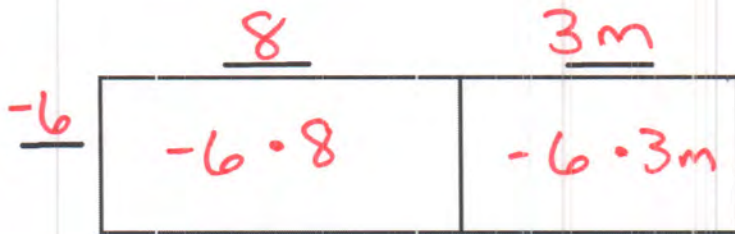
10. Use the distributive property to write an equivalent expression with the fewest possible terms.

$$8n - 4n + 14$$

$$\begin{array}{c} \vee \\ \boxed{4n + 14} \end{array}$$

1. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-6(8 + 3m)$$

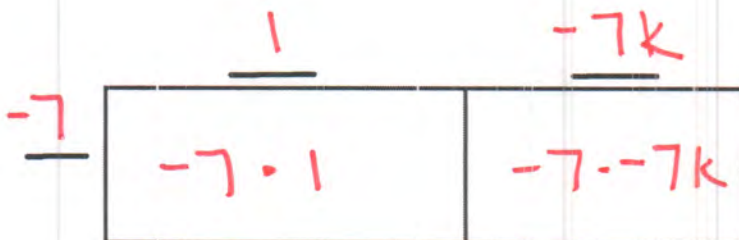


$$-48 + -18m$$

$$(-48 - 18m)$$

2. Use the distributive property to write an equivalent expression. Use the area model to help show your thinking.

$$-7(1 - 7k)$$



$$(-7 + 49k)$$

3. Marcel rewrote the expression $5 - 4(6z + 2)$ as $-1(6z + 2)$. Explain Marcel's mistake, and show how he could write an equivalent expression using as few terms as possible.

$$5 + \overbrace{-4(6z + 2)}$$

$$5 + -24z + -8$$

$$5 - 24z - 8$$

$$-3 - 24z$$

Marcel did not follow the order of operations. He should distribute before subtracting. The correct expression is $-3 - 24z$.

4. Which expressions are equivalent to $11 - 5(4v - 2)$? Select all that apply.

~~a. $6(4v - 2)$~~

b. $11 - 20v + 10$

~~c. $6 + 2v$~~

~~d. $22v$~~

e. $-20v + 21$

f. $21 - 20v$

~~g. $-20v + 1$~~

$$11 - 20v + 10$$

$$21 - 20v$$

G7 U5 Lesson 21

Write equivalent expressions with fewer terms by combining like terms.

G7 U5 Lesson 21 - Students will write equivalent expressions with fewer terms by combining like terms.

Warm Welcome (Slide 1): Tutor choice

Frame the Learning/Connect to Prior Learning (Slide 2): This is our last lesson of this unit. We've spent a lot of time learning about equations and inequalities. More recently, we've worked through some lessons where we've used properties to write equivalent expressions. What do you remember or what stands out to you as important when thinking about equivalent expressions? **Possible Student Answers, Key Points:**

- Equivalent expressions might look different, but they have the same value.
- We can use the distributive property to rewrite expressions. This can involve multiplying a coefficient by terms inside parentheses, or it can involve factoring out a common factor.
- We can combine like terms to write an equivalent expression with fewer terms.

Great thinking! We've combined like terms, we've factored, and we've multiplied using the distributive property. Today, we'll pull all of this thinking together to rewrite expressions in equivalent forms. Let's get going.

Let's Talk (Slide 3): Take a look at the expression here. Which student's work shows an equivalent expression? How do you know? **Possible Student Answers, Key Points:**

- I think Bryson's work is equivalent.
- If you distribute -2 to 3x, you end up with -6x. -2 times -7y is +14y.

$$\begin{array}{l} -2(3x + -7y) \\ -6x + 14y \end{array}$$

Bryson's work correctly shows the distributive property. To help us think about this carefully, we can rewrite $3x - 7y$ using addition. (*write $-2(3x + -7y)$)* Adding $-7y$ is the same as subtracting $7y$. (*draw arrows to represent distributing the -2 to each term*) I know -2 times $3x$ is $-6x$. I know -2 times $-7y$ is $14y$. We can rewrite the expression as $-6x + 14y$.

Bryson's work is correct. Amy's thinking is close, but she might have benefitted from paying closer attention to the values of her signed numbers.

Errors with signs can be common if we rush, so we'll want to be extra cautious as we work through today's examples. Let's get started.

Let's Think (Slide 4): Here we have two problems that are slightly different. For both, we're asked to rewrite the expressions without parentheses. Let's start with the first one.

$$\begin{array}{l} y + 1(3x + 2y) \\ y + 3x + 2y \\ y + 2y + 3x \\ \boxed{3y + 3x} \end{array}$$

I notice there are parentheses, but there does not appear to be a coefficient directly outside of the parentheses. We can then assume that this problem is referring to 1 group of $3x + 2y$. I'll rewrite the expression placing a 1 as the coefficient to the expression in parentheses. (*rewrite expression as shown*) How can we think about distributing the 1? (*rewrite expression student shares*)

Possible Student Answers, Key Points:

- 1 times $3x$ is $3x$. 1 times $2y$ is $2y$.
- We're left with $y + 3x + 2y$.

After distributing, we have $y + 2y + 3x$. I can combine each of the y terms, and write their total as $3y$. The equivalent expression is $3y + 3x$. We distributed and combined like terms to arrive at our answer.

I also see in this example, we could factor out a 3 from both terms. The directions don't ask us to do that, but we'll see problems later today that ask us to think about factoring an expression after we take other steps to rewrite it.

Now look at the second expression. What's the same? What's different? [Possible Student Answers, Key Points:](#)

- It uses the same variables. It has parentheses. It starts with y.
- It's has a minus sign after the instead of a plus sign. Everything else is the same.

Since, like our last example, it appears as though there is not a coefficient directly next to the parentheses, we can assume this expression only has one group of $3x + 2y$. I can think of this as $y - 1(3x + 2y)$, but it's probably easier to consider the subtraction as addition.

$$\begin{array}{l}
 y + -1(3x + 2y) \\
 y + -3x + -2y \\
 y + -2y + -3x \\
 \textcircled{-y - 3x}
 \end{array}$$

(write $y + -1(3x + 2y)$, and draw arrows to show the distributive property) We can use the expression and distribute the -1 to each term in parentheses. What is -1 times $3x$? $(-3x)$ What is -1 times $2y$? $(-2y)$ Great, let's rewrite the entire expression as $y + -3x + -2y$. I'll rearrange the y -terms so that they're adjacent to each other. (rewrite so y terms are adjacent)

From here, we can combine like terms. I know y plus $-2y$ is $-1y$ or $-y$. There is no other x -term, so the equivalent expression is $-y + -3x$, or $-y - 3x$.

We distributed the -1 to each term in parentheses and then combined like terms to write a simpler, equivalent expression.

In both of these examples, there was not a clearly labeled coefficient outside of the parentheses. Mathematicians don't always put a 1 in front of a group if they don't have to, since a single group of something doesn't require a coefficient. If it's helpful, rewrite similar expressions with a coefficient of 1 to make thinking about the distributive property easier.

Let's Think (Slide 5): We have one more problem to tackle. This expression looks long, since it has four terms in it and two sets of parentheses. That's not a problem! We'll use the same thinking to help us write an equivalent expression.

$$\begin{array}{l}
 (2m + 1n) + -1(4n + 12m) \\
 2m + 1n + -4n + -12m \\
 2m + -12m + 1n + -4n \\
 \textcircled{-10m - 3n}
 \end{array}$$

Let's start by rewriting the subtraction of these two parts of the expression as addition. Instead of subtracting one group of $4n + 12m$, I'll add -1 group of $4n + 12m$. (write $+ -1$ between sets of parentheses instead of the minus symbol) Now we can distribute the -1 to each term in the second set of parentheses. What is -1 times $4n$? $(-4n)$ What is -1 times $12m$? $(-12m)$

We can write the expression as $2m + 1n + -4n + -12m$. How could you rearrange and combine like terms to write an equivalent expression with as few terms as possible? (rewrite expression as student shares, supporting as needed) [Possible Student Answers, Key Points:](#)

- I can start by writing the m -terms next to each other and the n -terms next to each other.
- I know $2m$ plus $-12m$ is $-10m$. I know $1n$ plus $-4n$ is $-3n$.
- The expression can be written as $-10m + -3n$ or $10m - 3n$.

The equivalent expression is $-10m - 3n$. I couldn't combine any more terms, since they don't share a variable. I also couldn't factor anything meaningful out of $-10m - 3n$, because 10 and 3 only share a factor of 1.

Nice work! We just used everything we've been learning the past several lessons to help us rewrite expressions as equivalent expressions using as few terms as possible.

Let's Try it (Slides 6 - 7): It's time to collaborate on a few problems together, then you can have a chance to show everything you've learned independently. We'll look at each expression we're given to see if we can distribute, combine like terms, or possibly both. We should also keep an eye out for a few other unique situations. For example, if there does not appear to be a coefficient in front of parentheses, we can assume there is a 1. Also, we can keep an eye out for opportunities to factor expressions. Let's dive in!

WARM WELCOME



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Today we will write equivalent expressions with fewer terms by combining like terms.

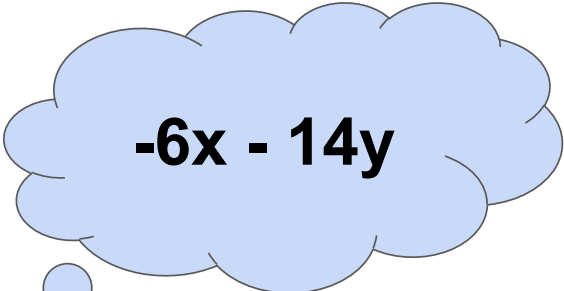
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Let's Talk:

Who is correct?

$$-2(3x - 7y)$$


$$-6x - 14y$$

AMY


$$-6x + 14y$$

BRYSON

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Let's Think:

Write an equivalent expression without parentheses.

$$y + (3x + 2y)$$

$$y - (3x + 2y)$$

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Let's Think:

Write an equivalent expression with as few terms as possible.

$$(2m + n) - (4n + 12m)$$

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Let's Try It:

Let's explore writing equivalent expressions with fewer terms by combining like terms together.

Name: _____ G7 US Lesson 21 - Let's Try It

Consider the expression $6b + (4a + 5b)$.

- Rewrite the expression with a 1 in front of the parentheses to think of the expression as 6b plus 1 group of $4a + 5b$.
- Distribute the 1 to each term within the parentheses.
 $6b + (\underline{\quad} \cdot \underline{\quad}) + (\underline{\quad} \cdot \underline{\quad})$
- Rewrite the expression using as few terms as possible.
 $6b + \underline{\quad} + \underline{\quad}$
 $\underline{\quad} + \underline{\quad}$

Consider the expression $6b - (4a + 5b)$.

- Rewrite the expression to show that we can think of this as 6b plus 1 negative group of $4a + 5b$.
- Distribute the -1 to each term within the parentheses.
 $6b + (\underline{\quad} \cdot \underline{\quad}) + (\underline{\quad} \cdot \underline{\quad})$
- Rewrite the expression using as few terms as possible.
 $6b + \underline{\quad} + \underline{\quad}$
 $\underline{\quad} + \underline{\quad}$
 $\underline{\quad} - \underline{\quad}$

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Consider the expression $(5n + 3m) + (2n - 6m)$.

- Use the distributive property to write an equivalent expression without parentheses.
- Rearrange the expression to group like terms.
- Write the expression using as few terms as possible.

Consider the expression $(3g + 8h) - (4h + 7g)$.

- Use the distributive property to write an equivalent expression without parentheses.
- Rearrange the expression to group like terms.
- Write the expression using as few terms as possible.
- Now let's factor the expression. What is the greatest common factor of each term?
- Rewrite the expression by factoring out the greatest common factor.

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On your Own:

Now it's time to write equivalent expressions with fewer terms by combining like terms on your own.

Name: _____ G7 US Lesson 21 - Independent Work

1. Use the distributive property to rewrite each expression as an equivalent expression without parentheses. Use as few terms as possible.

$9x + (12y - 4x)$ $4g - (g + 20)$

2. Which expressions are equivalent to the one below? Select all that apply.

$(10a + 5b) - (2a + 9b)$

a. $10a + 5b - 2a - 9b$
b. $8a - 4b$
c. $-4b + 8a$
d. $10a - 2a + 5b - 9b$
e. $4(2a - b)$
f. $10a + 5b - 1(2a + 9b)$

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3. Which expression is equivalent to the one below? Show your work.

$(10x + 6y) - (4x - 6y)$

a. $6x - 12y$
b. $6x - 2y$
c. $6x + 12y$
d. $12x - 2y$

Rewrite the expression you selected by factoring out the greatest common factor.

4. Fiona says that $(10w + 6v) - (7w + 9v)$ is equivalent to $3(w - v)$. Is she correct? Show or explain how you know.

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Consider the expression $6b + (4a + 5b)$.

1. Rewrite the expression with a 1 in front of the parentheses to think of the expression as $6b$ plus 1 group of $4a + 5b$.
2. Distribute the 1 to each term within the parentheses.

$$6b + (\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

3. Rewrite the expression using as few terms as possible.

$$6b + \text{_____} + \text{_____}$$

$$\text{_____} + \text{_____}$$

Consider the expression $6b - (4a + 5b)$.

4. Rewrite the expression to show that we can think of this as $6b$ plus 1 *negative* group of $4a + 5b$.
5. Distribute the -1 to each term within the parentheses.

$$6b + (\text{_____} \cdot \text{_____}) + (\text{_____} \cdot \text{_____})$$

6. Rewrite the expression using as few terms as possible.

$$6b + \text{_____} + \text{_____}$$

$$\text{_____} + \text{_____}$$

$$\text{_____} - \text{_____}$$

Consider the expression $(5n + 3m) + (2n - 6m)$.

7. Use the distributive property to write an equivalent expression without parentheses.
8. Rearrange the expression to group like terms.
9. Write the expression using as few terms as possible.

Consider the expression $(3g + 8h) - (4h + 7g)$.

10. Use the distributive property to write an equivalent expression without parentheses.
11. Rearrange the expression to group like terms.
12. Write the expression using as few terms as possible.
13. Now let's factor the expression. What is the greatest common factor of each term?
14. Rewrite the expression by factoring out the greatest common factor.

- 1. Use the distributive property to rewrite each expression as an equivalent expression without parentheses. Use as few terms as possible.**

$$9x + (12y - 4x)$$

$$4g - (g + 20)$$

- 2. Which expressions are equivalent to the one below? Select all that apply.**

$$(10a + 5b) - (2a + 9b)$$

- a. $10a + 5b - 2a - 9b$
- b. $8a - 4b$
- c. $-4b + 8a$
- d. $10a - 2a + 5b - 9b$
- e. $4(2a - b)$
- f. $10a + 5b - 1(2a + 9b)$

3. Which expression is equivalent to the one below? Show your work.

$$(10x + 6y) - (4x - 6y)$$

- a. $6x - 12y$
- b. $6x - 2y$
- c. $6x + 12y$
- d. $12x - 2y$

Rewrite the expression you selected by factoring out the greatest common factor.

4. Fiona says that $(10w + 6v) - (7w + 9v)$ is equivalent to $3(w - v)$. Is she correct? Show or explain how you know.

Name: KEY

Consider the expression $6b + (4a + 5b)$.

1. Rewrite the expression with a 1 in front of the parentheses to think of the expression as 6b plus 1 group of 4a + 5b.

$$6b + 1(4a + 5b)$$

2. Distribute the 1 to each term within the parentheses.

$$6b + (\underline{1} \cdot \underline{4a}) + (\underline{1} \cdot \underline{5b})$$

3. Rewrite the expression using as few terms as possible.

$$\begin{array}{r} 6b + \underline{4a} + \underline{5b} \\ \underline{11b} + \underline{4a} \end{array}$$

Consider the expression $6b - (4a + 5b)$.

4. Rewrite the expression to show that we can think of this as 6b plus 1 *negative* group of 4a + 5b.

$$6b - 1(4a + 5b)$$

5. Distribute the -1 to each term within the parentheses.

$$6b + (\underline{-1} \cdot \underline{4a}) + (\underline{-1} \cdot \underline{5b})$$

6. Rewrite the expression using as few terms as possible.

$$\begin{array}{r} 6b + \underline{-4a} + \underline{-5b} \\ \underline{b} + \underline{-4a} \\ \underline{b} - \underline{4a} \end{array}$$

Consider the expression $(5n + 3m) + (2n - 6m)$.

7. Use the distributive property to write an equivalent expression without parentheses.

$$5n + 3m + 2n - 6m$$

8. Rearrange the expression to group like terms.

$$5n + 2n + 3m - 6m$$

9. Write the expression using as few terms as possible.

$$(7n - 3m)$$

Consider the expression $(3g + 8h) - (4h + 7g)$.

10. Use the distributive property to write an equivalent expression without parentheses.

$$3g + 8h - 4h - 7g$$

11. Rearrange the expression to group like terms.

$$3g - 7g + 8h - 4h$$

12. Write the expression using as few terms as possible.

$$-4g + 4h$$

13. Now let's factor the expression. What is the greatest common factor of each term?

$$4$$

14. Rewrite the expression by factoring out the greatest common factor.

$$4(-g + h)$$

1. Use the distributive property to rewrite each expression as an equivalent expression without parentheses. Use as few terms as possible.

$$9x + (12y - 4x)$$

$$9x + 12y - 4x$$

$$9x - 4x + 12y$$

$$(5x + 12y)$$

$$4g - (g + 20)$$

$$4g - g - 20$$

$$(3g - 20)$$

2. Which expressions are equivalent to the one below? Select all that apply.

$$(10a + 5b) - (2a + 9b)$$

- a. $10a + 5b - 2a - 9b$
 b. $8a - 4b$
 c. $-4b + 8a$
 d. $10a - 2a + 5b - 9b$
 e. $4(2a - b)$
 f. $10a + 5b - 1(2a + 9b)$

$$10a + 5b - 2a - 9b$$

$$10a - 2a + 5b - 9b$$

$$8a - 4b$$

$$4(2a - b)$$

3. Which expression is equivalent to the one below? Show your work.

$$(10x + 6y) - (4x - 6y)$$

- a. $6x - 12y$
- b. $6x - 2y$
- c. $6x + 12y$
- d. $12x - 2y$

$$\begin{aligned} 10x + 6y - 4x + 6y \\ 10x - 4x + 6y + 6y \\ 6x + 12y \end{aligned}$$

Rewrite the expression you selected by factoring out the greatest common factor.

$$\text{GCF: } 6$$

$$6(x + 2y)$$

4. Fiona says that $(10w + 6v) - (7w + 9v)$ is equivalent to $3(w - v)$. Is she correct? Show or explain how you know.

$$\begin{aligned} 10w + 6v - 7w - 9v \\ 10w - 7w + 6v - 9v \\ 3w - 3v \\ 3(w - v) \end{aligned}$$

I agree. I rewrote the expression with as few terms as possible, and I got $3w - 3v$. Then, you can factor out a 3 from both terms to get $3(w - v)$.